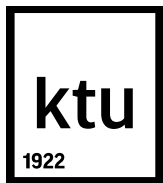


# P170M109 Computational Intelligence and Decision Making

Unsupervised Learning



# Machine Learning Approaches

- **Supervised learning** – learning from input-output pairs to map from input to output
- **Unsupervised learning** - learning patterns without explicit feedback
- **Reinforcement learning** – learning from a series of reinforcements (rewards or punishments)
- **Semi-supervised learning** – learning from few labeled examples and a large collection of unlabeled examples
- ...

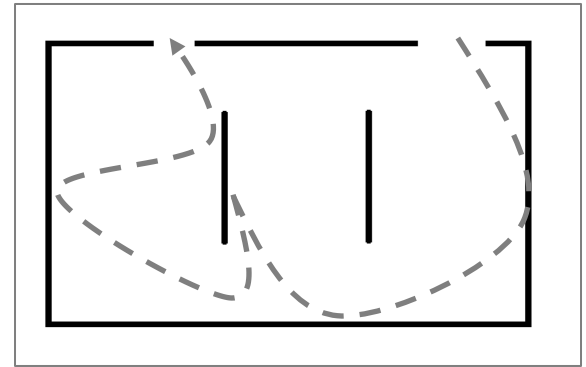
# Machine Learning Approaches

	Supervised Learning	Unsupervised Learning	Reinforcement Learning
<b>Dataset</b>	fixed; correct (expected answer is given)	fixed; no label or class information given; no explicit feedback	incremental; feedback (reward) about the value of the action in environment
<b>Objective</b>	predict answers for new examples	extracting information	find a suite of actions to maximize its accumulated reward
<b>Applications</b>	regression (prediction); classification; translation	feature extraction; data compression; series modeling; clustering; anomaly detection	game playing; robot navigation

# Advantages of Unsupervised Learning

- **No labels needed** – annotating (manual labeling) large datasets is expensive;
- **No prior knowledge is needed** about how many or what classes the data should be divided into.
- **Enables to gain insight into the structure of data** (correlations and relationships of attributes) before designing a model in supervised learning.

# Example



## **Customer segmentation.**

**Given:** data on the buying patterns of all customers of a grocery store.

**Objective:** suggest special offers or plan store layout.

Unsupervised learning is applied to identify buying patterns.

For example, those who:

- buy coloring pencils, also buy children's books
- buy frozen food, also buy snacks

# Clustering

- Partitioning the entire dataset based on some similarity criterion (distance)
- Finding unknown groups of elements so that data within a group (or else, cluster) are similar to each other and dissimilar to data from other clusters

## Applied in:

- Detecting groups of data across multiple attributes
- Data reduction tasks (data compression, noise smoothing, outlier detection, dataset partition).

# Classes of Clustering Methods

- **Hierarchical methods**
  - Agglomerative clustering (*start with a singleton clusters and merge with respect to the similarity criterion*)
  - Divisive clustering (*start with all elements in one cluster and divide in several partitions in each step*)
- **K-means methods**
- **Graph theory methods**

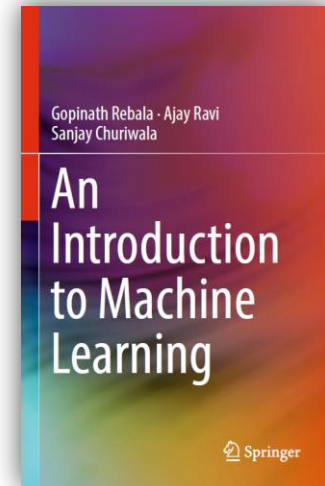
# Literature

**Rebala, Gopinath, Ajay Ravi, and Sanjay Churiwala. *An Introduction to Machine Learning*. Springer, 2019.**

...

*Chapter 6, Clustering*

...



<https://link.springer.com/book/10.1007%2F978-3-030-15729-6>

Use KTU VPN or perform search through <https://vb.ktu.edu> (uses SSO login and proxy to access full text document)



# K-Means Clustering

## Given:

$S$  – set of elements

$K$  – Number of clusters

## Method:

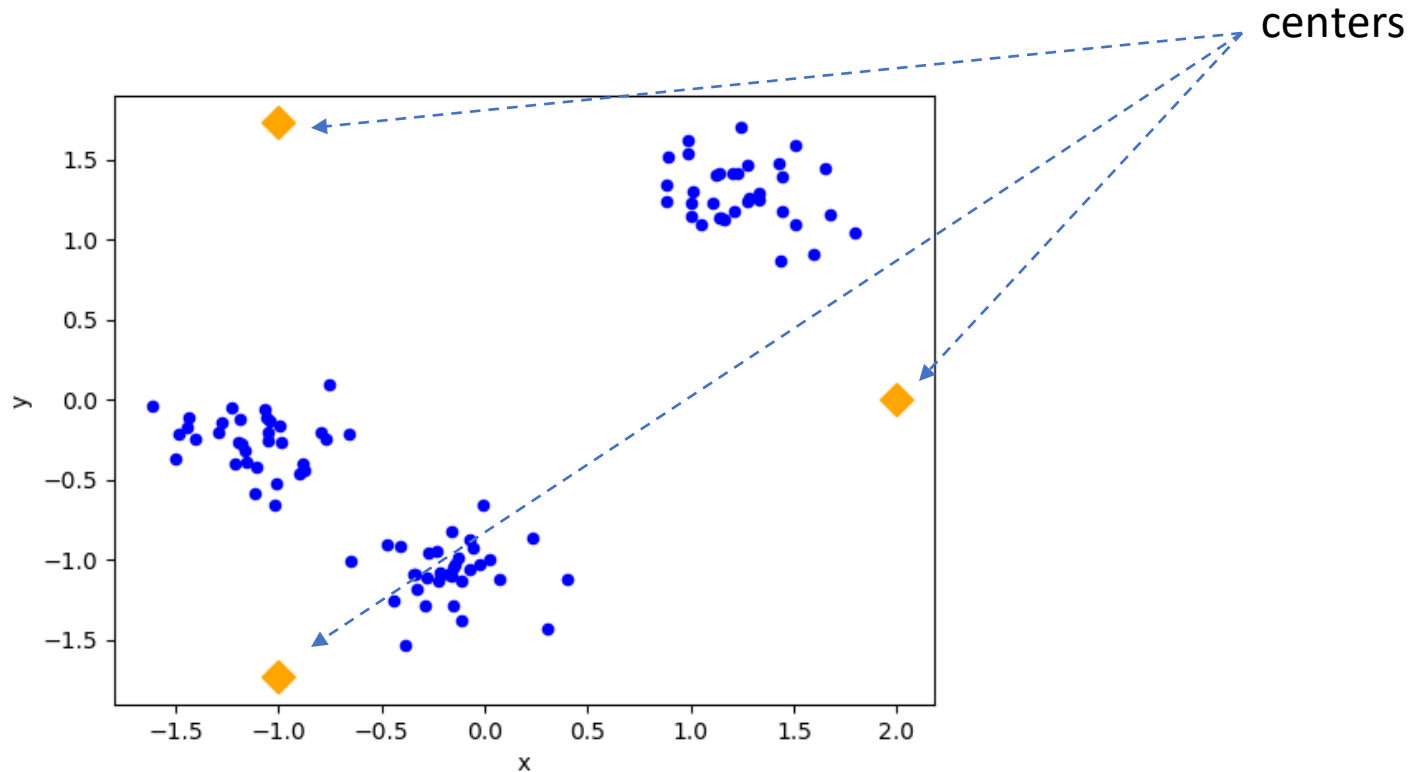
**Step 1:** Select  $K$  initial centers of clusters  $C_1, C_2, \dots, C_K$ .

**Step 2:** Assign each element  $X \in S$  to the cluster  $C_i$  ( $1 \leq i \leq K$ ) with the closest center.

**Step 3:** Recalculate the centroids in each cluster  $C_j$  ( $1 \leq j \leq K$ ) in which element was added or removed.

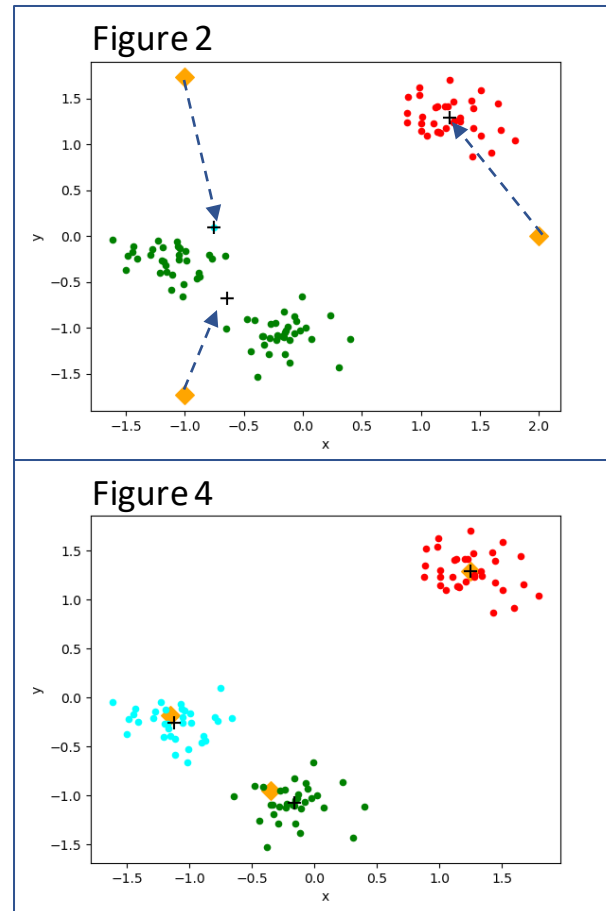
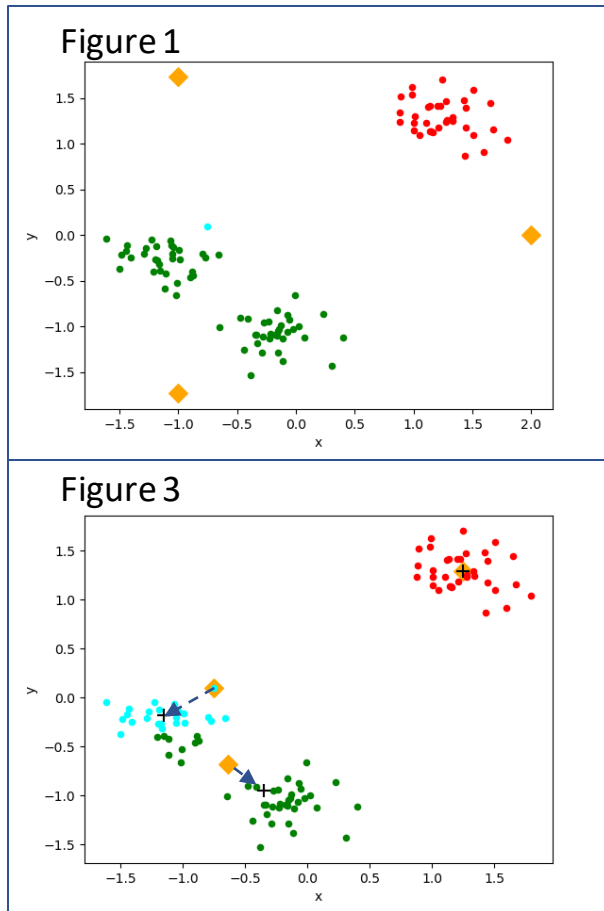
**Step 4:** Repeat the steps 2 and 3 until the algorithm converges.

# K-Means Clustering



Number of clusters = 3

# K-Means Clustering



◆ current centers  
+ updated centers

# K-Means Clustering

## How to...

- evaluate similarity (distance, cost function)?
- choose number of clusters?
- initialize centroids?

# K-Means Clustering. How to evaluate similarity?

Euclidean distance:

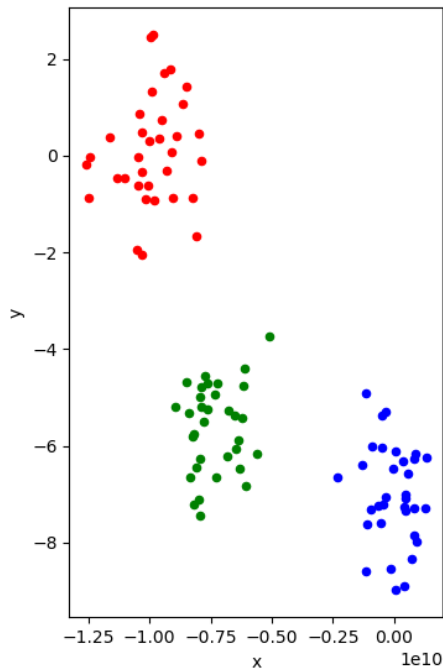
$$d_j = \sqrt{\sum_{i=1} (X_i - C_i^j)^2}$$

$d_j$  - distance of element X to the center  $C^j$

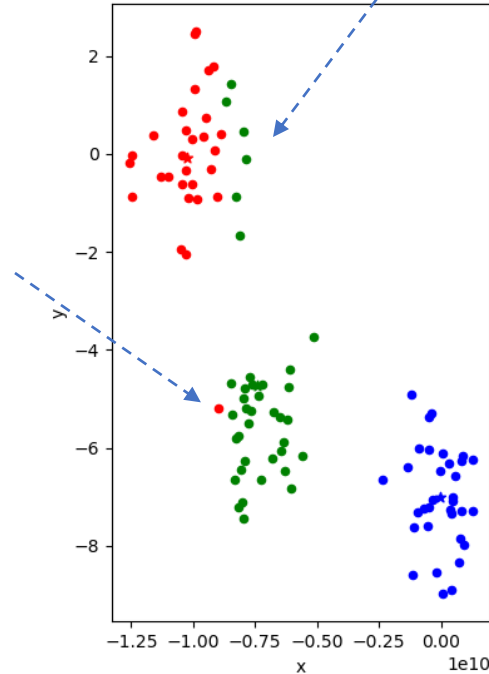
# K-Means Clustering.

Standardization (or normalization) is important!

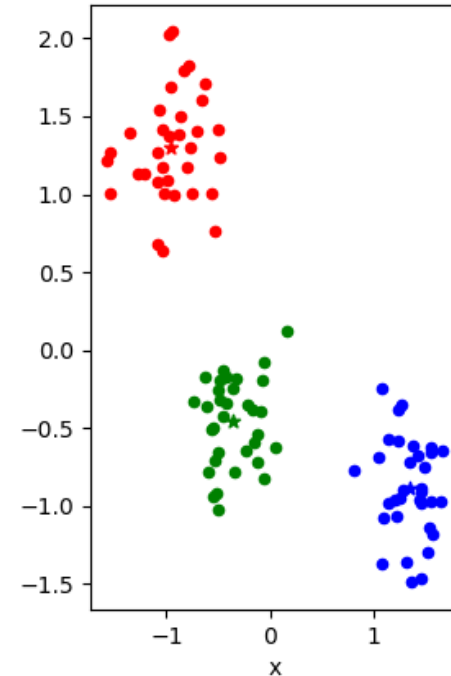
Simulated clusters



K-means result



K-means result for  
standardized dataset



# **K-Means Clustering.** How to choose number of clusters?

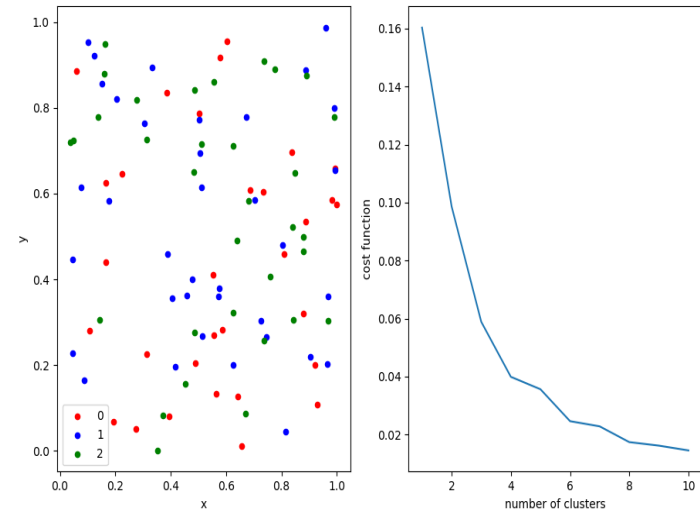
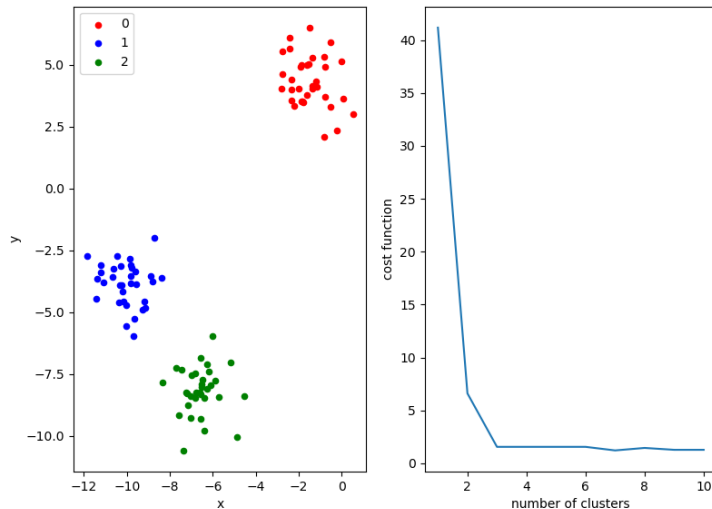
- Elbow method
- Silhouette method
- Gap statistic method
- ...

# K-Means Clustering. How to choose number of clusters?

Elbow method:

- plot cost function vs number of clusters;
- look for a change of slope from steep to shallow.

Inexact!



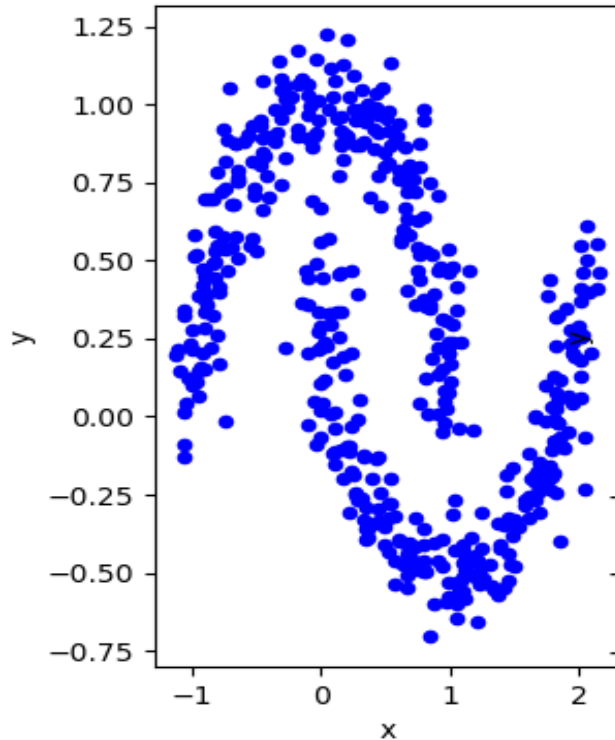
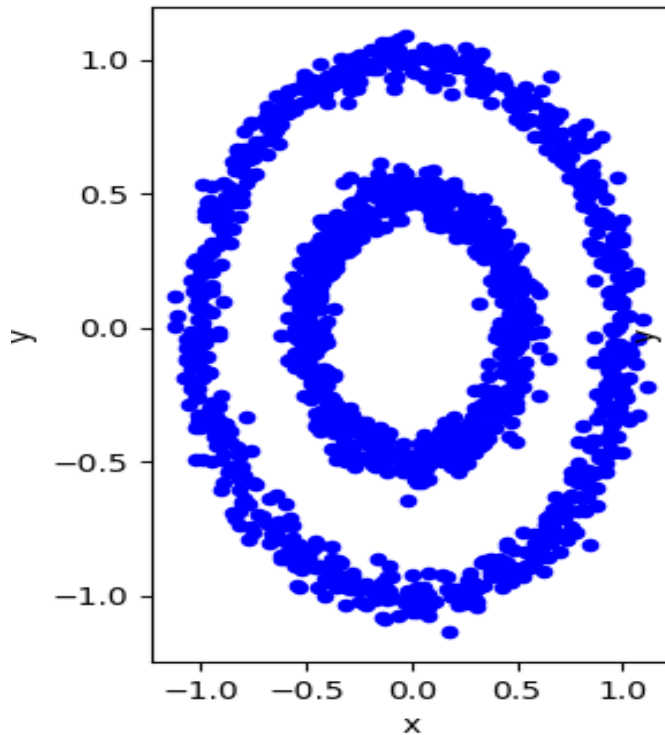


# K-Means Clustering. How to initialize centroids?

## Suggested solutions:

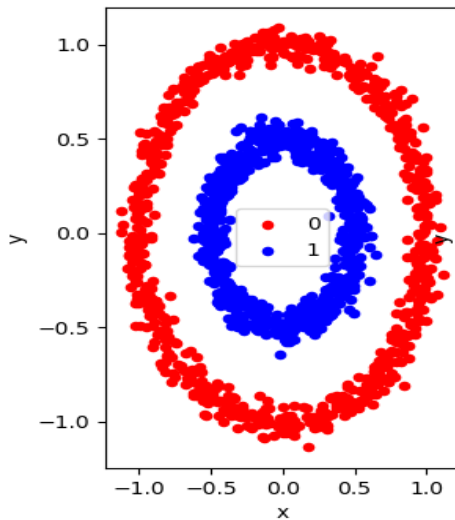
- Randomly partition data into  $K$  non-empty clusters and calculate centroids.
- Randomly select  $K$  elements from the dataset to represent clusters.
- Select  $K$  elements from the dataset to represent clusters. The elements should be as far as possible from each other – difficult to implement in high dimensions.

# K-Means Clustering. Is it suitable for all datasets?

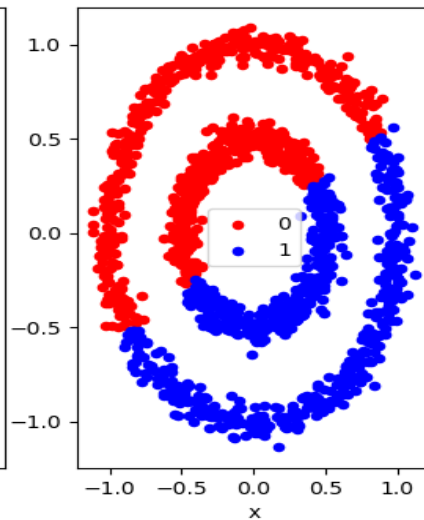


# K-Means Clustering. Is it suitable for all datasets?

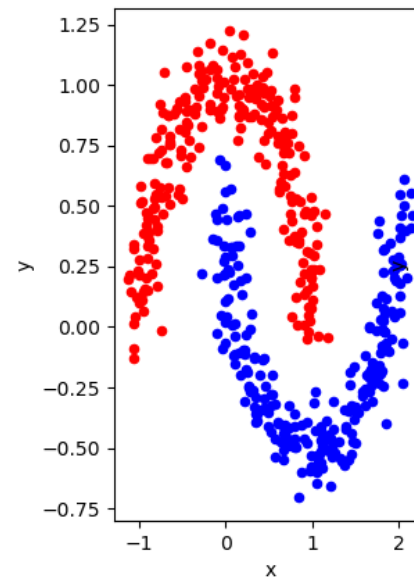
Simulated clusters



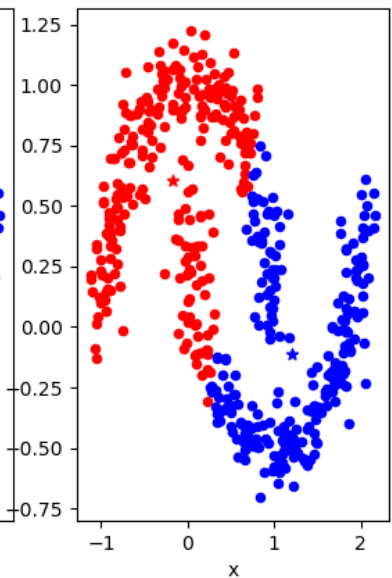
K-means result



Simulated clusters



K-means result



# K-Means Clustering

## Strengths:

- simple implementation.

## Weaknesses:

- applicable only to data objects in a continuous space.
- the number of clusters  $K$  must be specified in advance.
- not suitable to discover clusters with non-convex shapes as it can only find hyper-spherical clusters.
- sensitive to outliers.

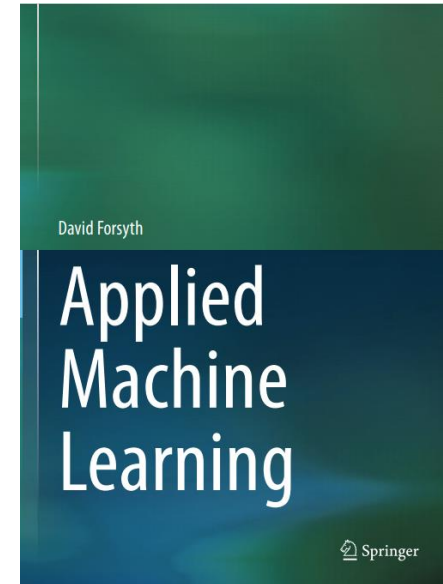
# Literature

David Forsyth. *Applied Machine Learning*. Springer, 2019.

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8.2.2 Clustering -> Soft Assignment

...



<https://www.springer.com/gp/book/9783030181130>

# Soft Assignment (K-Means Clustering)

Let's say, You are clustering images based on it's extracted numerical features, and the results are:



What difficulties may occur during data analysis?

# Soft Assignment (K-Means Clustering)

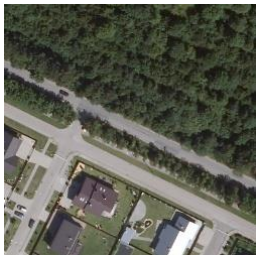


Using standard k-means clustering each point (feature vector) must belong to exactly one cluster!

# Soft Assignment (K-Means Clustering)

Let's assign point  $\mathbf{x}_i = \{x_1, \dots, x_n\}$  to cluster centers with weights  $\omega_{i,j}$ . Here

- $\mathbf{x}_i$  – point represented features vector, where  $i \in [1, N]$ , here  $N$  – number of points
- $\omega_{i,j}$  - connects point  $i$  with cluster center  $j$ .
- $j \in [1, c]$ , where  $c$  is total number of clusters
- all weights are non-negative:  $\omega_{i,j} \geq 0$
- each point should carry a total weight of 1. It means:  $\sum_j \omega_{i,j} = 1$



i.e. we have image represented by it's features as  $\mathbf{x}_i$  vector. Here one part of image is urban area, another part is forest. One of the scenarios after clustering that image would have similar weights in two clusters, it means:

$\omega_i = \{0, \dots, \omega_k, \dots, \omega_l, \dots 0\}$  – here  $k, l$  is the clusters represented forest and urban area (in this case  $\omega_k \approx \omega_l \approx 0.5$ ).



# Soft Assignment (K-Means Clustering)

Objective to minimize:  $\Phi(\omega, \mathbf{c}) = \sum_{i,j} \omega_{i,j} \left[ (\mathbf{x}_i - \mathbf{c}_j)^T (\mathbf{x}_i - \mathbf{c}_j) \right]$

---

Relation between  $\omega, \mathbf{c}$ :

$d_{i,j} = \|(\mathbf{x}_i - \mathbf{c}_j)\|$  - distance between point and cluster center

$\sigma$  – scaling parameter ( $\sigma > 0$ )

$$s_{i,j} = e^{-\frac{d_{i,j}^2}{2\sigma^2}}$$



**affinity** between the point  $i$  and the center  $j$ . It is large, if they are close in  $\sigma$  units, and small if they are far apart

# Soft Assignment (K-Means Clustering)

Now to obtain weights of a particular point, we ensure that sum of cluster weights is equal to 1

$$\omega_{i,j} = \frac{S_{i,j}}{\sum_{l=1}^k S_{i,l}}$$

---

Finally, re-estimated cluster centers could be as:

$$\frac{\sum_i \omega_{i,j} \mathbf{x}_i}{\sum_i \omega_{i,j}}$$

---

*Basically, k-means algorithm is a special case of soft assignment, when  $\sigma \rightarrow 0$*

# Soft Assignment (K-Means Clustering)

## Algorithm

Choose  $k$  initial points  $\mathbf{c}_j$  and assign it as initial cluster centers and:

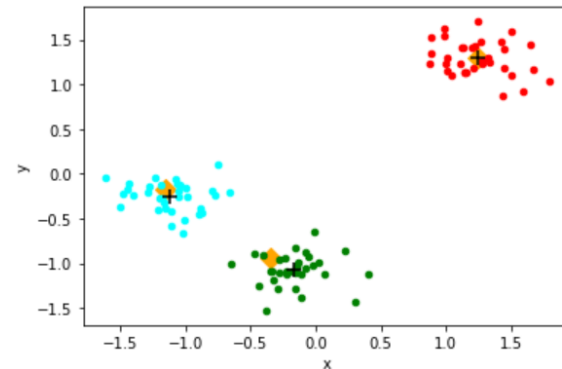
- Repeat until converge
1. For each pair of data point and cluster compute affinity  $S_{i,j} = e^{-\frac{d_{i,j}^2}{2\sigma^2}}$
  2. Compute soft-weight connections  $\omega_{i,j} = \frac{S_{i,j}}{\sum_{l=1}^k S_{i,l}}$
  3. Get new center for each cluster  $\mathbf{c}_i = \frac{\sum_i \omega_{i,j} \mathbf{x}_i}{\sum_i \omega_{i,j}}$

# Code example

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k-means clustering example

*kMeansClusteringExample.ipynb*

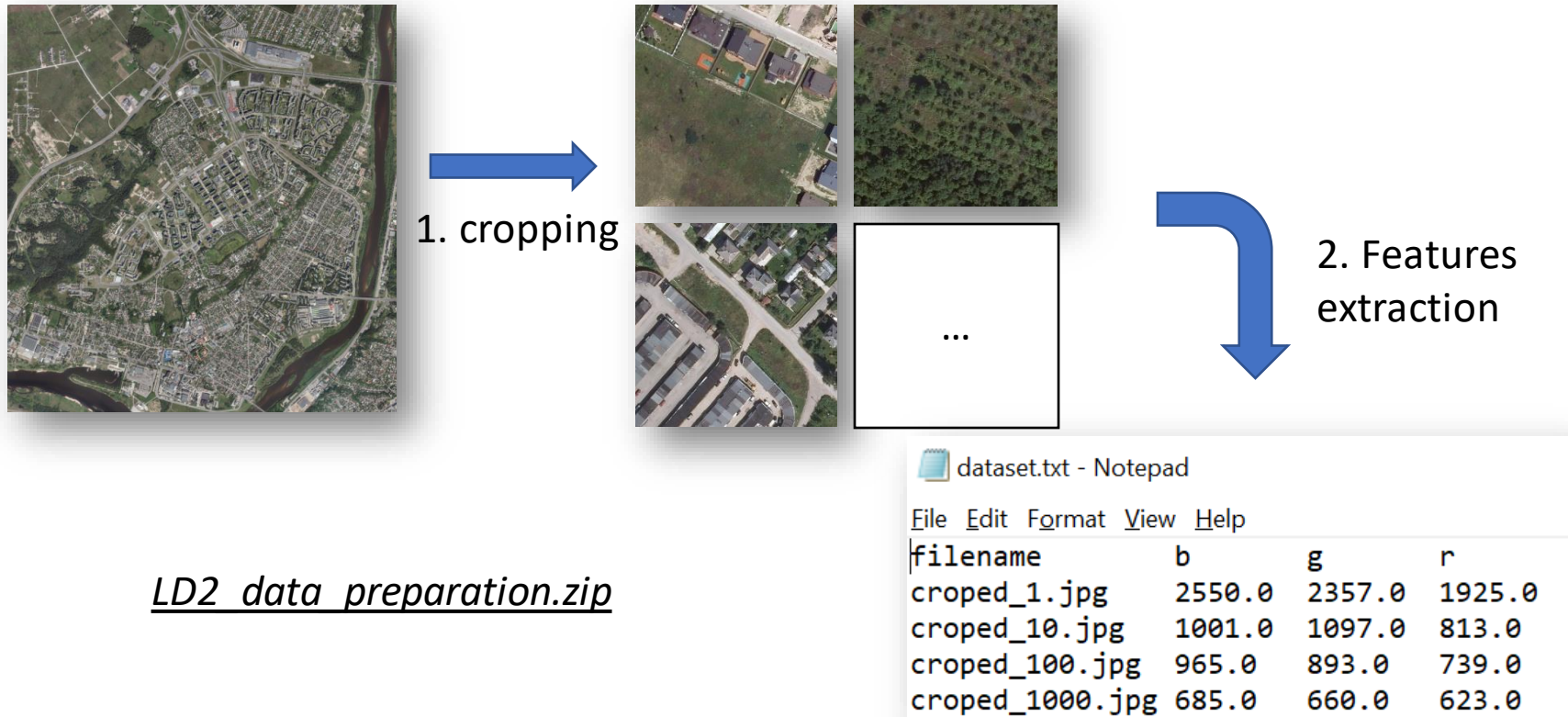


TODO:

1. use different initial data distributions
2. try random centroid initialization
3. try clustering with real dataset
4. implement selection of number of clusters

# Code example

Dataset preparation example for LD2 (image cropping and feature extraction)



# Questions?

