UNIVERSITY OF PADUA
DEGREE COURSE IN
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DEPARTMENT OF INFORMATION ENGINEERING
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1^{st} Homework

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PROBLEM 1

Given a single realization of 800 i.i.d. samples of the r.p $\{x(k)\}$ defined by:

$$x(k) = e^{j(2\pi f_1 k + \varphi_1)} + 0.8 e^{j(2\pi f_2 k + \varphi_2)} + w(k)$$

where $w(k) \sim \mathcal{CN}(0, \sigma_w^2)$, $f_1 = 0.17$, $f_2 = 0.78$, $\sigma_w^2 = 2.0$ $\varphi_{1,2} \sim \mathcal{U}(0, 2\pi)$ are statistically independent, we want to estimate the $\mathcal{PSD}$ of $\{x(k)\}$ using the following models:

BLACKMAN AND TUKEY CORRELOGRAM

$$\mathcal{P}(F) = T_c \sum_{n=-L}^{L} w(n) \hat{r}_x(n) e^{-j2\pi f n T_c} \tag{1}$$

where $\hat{r}_x(n)$ is the autocorrelation of x taken from -L to L and $w(n)$ is a Hamming window of length 2L+1. The autocorrelation function is evaluated using the *unbiased estimate* give by:

$$\hat{r}_x(n) = \frac{1}{K-n} \sum_{k=n}^{K-1} x(k) x^*(k-n) \qquad n = 0, 1, ..., K-1$$

where K is the number of samples. In ordert to reduce the variance of the estimate, we chose $L = \lfloor \frac{K}{5} \rfloor$. The hamming window is defined by:

$$w(k) = \begin{cases} 0.54 + 0.46 \cos\left(2\pi \frac{k - \frac{D-1}{2}}{D-1}\right) & k = 0, 1, ..., D-1 \\ 0 & elsewhere \end{cases}$$

PERIODOGRAM

$$\mathcal{P}_{PER}(f) = \frac{1}{KT_c} |\tilde{\mathcal{X}}(f)| \tag{2}$$

where $\tilde{\mathcal{X}}(f) = T_c \mathcal{X}(f)$, $\mathcal{X}(f)$ is the Fourier Transform of $\{x(k)\}$, $k = 0, 1, ..., K-1$.

WELCH PERIODOGRAM

$$\mathcal{P}_{WE}(F) = \frac{1}{N_s} \sum_{x=0}^{N_s-1} \mathcal{P}_{PER}^{(s)}(f)$$

where $\mathcal{P}_{PER}^{(s)}(F)$ is the periodogram estimate given by Eq.2. The model extracs different subsequences of consecutive D samples which eventually overlap. If $x^{(s)}$ is the s-th subsequence, characterized by S samples in common with the previous subsequence $x^{(s-1)}$ and with the following $x^{(s+1)}$, the total number of subsequences is $N_s = \lfloor \frac{K-D}{D-S} - 1 \rfloor$. In our analysis, we chose S=?? and D=??.