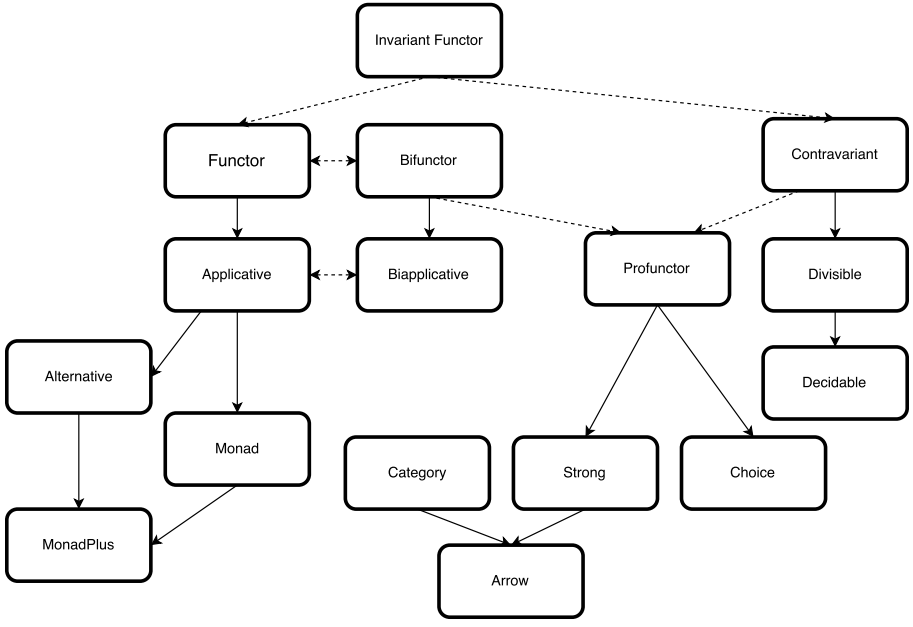
Contravariant: The Other Side of the Coin

George Wilson

Data61/CSIRO

george.wilson@data61.csiro.au

22nd May 2018


Invariant Functor
Functor
Bifunctor
Contravariant
Applicative
Biapplicative
Profunctor
Divisible
Alternative
Monad
Decidable
Category
Strong
Choice
MonadPlus
Arrow

Contravariant

```haskell
newtype Predicate a =
  Predicate { runPredicate :: a -> Bool}
```

```haskell
newtype Predicate a =
  Predicate { runPredicate :: a -> Bool}


evenP :: Predicate Int
evenP = Predicate (\i -> i `mod` 2 == 0)
```

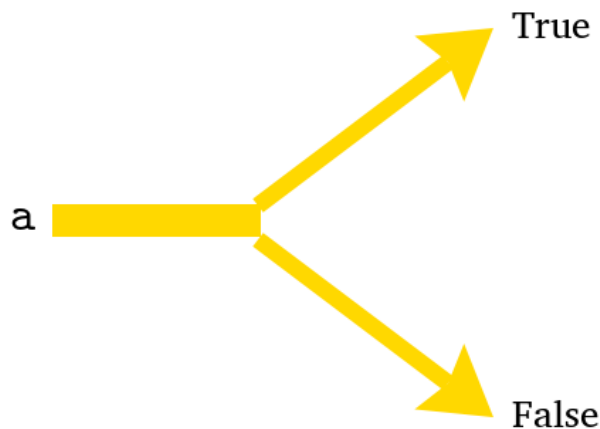
```haskell
newtype Predicate a =
  Predicate { runPredicate :: a -> Bool}


evenP :: Predicate Int
evenP = Predicate (\i -> i `mod` 2 == 0)


x :: Bool
x = runPredicate evenP 7
```

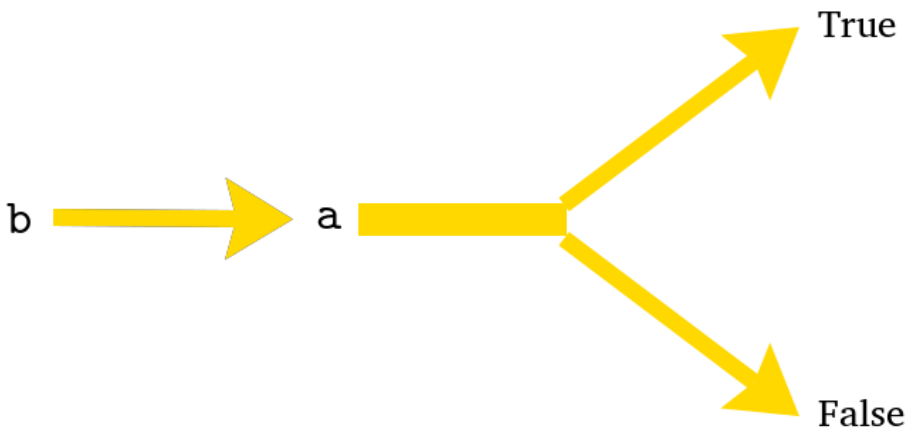
```haskell
gt5 :: Predicate Int
gt5 = Predicate (\i ->
                   i > 5)
```

```haskell
gt5 :: Predicate Int
gt5    = Predicate (\i ->
                      i > 5)

lenGT5 :: Predicate String
lenGT5 = Predicate (\str ->
                      let i = length str
                      in  i > 5)
```

```haskell
gt5 :: Predicate Int
gt5    = Predicate (\i ->
                      i > 5)

lenGT5 :: Predicate String
lenGT5 = Predicate (\str ->
                      let i = length str
                      in  i > 5)

mapP :: (b -> a) -> Predicate a -> Predicate b
mapP ba (Predicate abool) =
  Predicate (\b ->
               let a = ba b
               in  abool a)
```

```haskell
gt5 :: Predicate Int
gt5    = Predicate (\i ->
                      i > 5)


lenGT5' :: Predicate String
lenGT5' = mapP length gt5


mapP :: (b -> a) -> Predicate a -> Predicate b
mapP ba (Predicate abool) =
  Predicate (\b ->
               let a = ba b
               in  abool a)
```

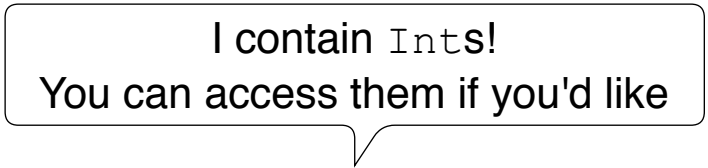

True
a
False


b
a
True
False

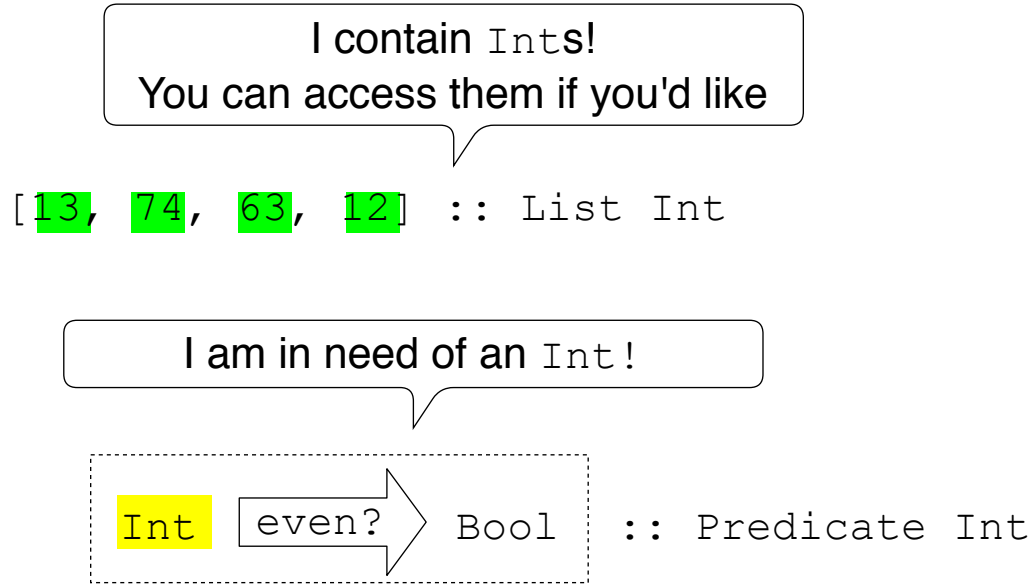
Is `Predicate` a `Functor`?

Is `Predicate` a `Functor`?

```
mapP :: (b -> a) -> Predicate a -> Predicate b
fmap :: (a -> b) -> Predicate a -> Predicate b
```




```
[13, 74, 63, 12] :: List Int
```


I contain Ints!
You can access them if you'd like
[13, 74, 63, 12] :: List Int
I am in need of an Int!
Int
even?
Bool
:: Predicate Int

```haskell
class Contravariant f where
  contramap :: (b -> a) -> f a -> f b
```

```haskell
class Contravariant f where
  contramap :: (b -> a) -> f a -> f b
```

Laws:

```haskell
contramap id = id
contramap f . contramap g = contramap (g . f)
```

```haskell
class Contravariant f where
  contramap :: (b -> a) -> f a -> f b


instance Contravariant Predicate where
  contramap :: (b -> a) -> Predicate a -> Predicate b
  contramap ba (Predicate abool) = Predicate (abool . ba)
```

We need more power!

```
class Functor f => Applicative  f where
  (<*>)  :: f (a -> b)    -> f a -> f b
  pure   :: a -> f a
```

```haskell
class Functor f => ApplicativeL f where
  liftA2 :: ((a, b) -> c) -> f a -> f b -> f c
  pure   :: a -> f a
```

```haskell
class Functor f => ApplicativeL f where
  liftA2 :: ((a, b) -> c) -> f a -> f b -> f c
  pure   :: a -> f a

(<*>) :: ApplicativeL f => f (a -> b) -> f a -> f b
(<*>) fab fa = liftA2 (\(ab,a) -> ab a) fab fa
```

```haskell
class Contravariant f => Divisible f where
  divide  :: (c -> (a, b)) -> f a -> f b -> f c
  conquer :: f a
```

```haskell
class Contravariant f => Divisible f where
  divide  :: (c -> (a, b)) -> f a -> f b -> f c
  conquer :: f a
```

Laws:

```haskell
divide f m conquer = contramap (fst . f) m

divide f conquer m = contramap (snd . f) m

divide f (divide g m n) o = divide f' m (divide id n o)
  where
    f' a = case f a of (bc,d) -> case g bc of (b,c) -> (a,(b,c))
```

```haskell
class Contravariant f => Divisible f where
  divide  :: (c -> (a, b)) -> f a -> f b -> f c
  conquer :: f a


instance Divisible Predicate where
  divide cab (Predicate pa) (Predicate pb) =
    Predicate $ \c ->
      case cab c of
        (a,b) -> pa a && pb b

  conquer = Predicate (\_ -> True)
```


Peters
light & Creamy
french vanilla
1.0L
Peters
light & Creamy
french vanilla
1.8L

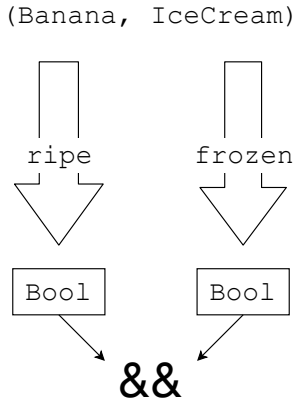
```
ingredients :: (Banana, IceCream)
```

```
ingredients :: (Banana, IceCream)


ripe :: Predicate Banana
frozen :: Predicate IceCream
```

```
ingredients :: (Banana, IceCream)


ripe :: Predicate Banana
frozen :: Predicate IceCream


divide    :: Divisible f => (c -> (a,b)) -> f a -> f b -> f c
divide id :: Divisible f =>                 f a -> f b -> f (a,b)
```

```
ingredients :: (Banana, IceCream)


ripe :: Predicate Banana
frozen :: Predicate IceCream


divide    :: Divisible f => (c -> (a,b)) -> f a -> f b -> f c
divide id :: Divisible f =>                 f a -> f b -> f (a,b)


divide id ripe frozen :: Predicate (Banana, IceCream)
```

```
ingredients :: (Banana, IceCream)


ripe :: Predicate Banana
frozen :: Predicate IceCream


divide    :: Divisible f => (c -> (a,b)) -> f a -> f b -> f c
divide id :: Divisible f =>                 f a -> f b -> f (a,b)


divide id ripe frozen :: Predicate (Banana, IceCream)


runPredicate (divide id ripe frozen) ingredients :: Bool
```


(Banana, IceCream)
ripe
frozen
Bool
Bool
&&

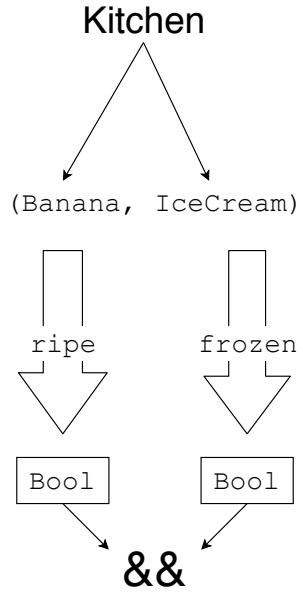
```
data Kitchen = Kitchen Rice Curry Banana Apple IceCream
```

```
data Kitchen = Kitchen Rice Curry Banana Apple IceCream


ripe :: Predicate Banana
frozen :: Predicate IceCream
```

```haskell
data Kitchen = Kitchen Rice Curry Banana Apple IceCream


ripe :: Predicate Banana
frozen :: Predicate IceCream


getIngredients :: Kitchen -> (Banana, IceCream)
getIngredients (Kitchen _ _ b _ i) = (b,i)
```

```haskell
data Kitchen = Kitchen Rice Curry Banana Apple IceCream


ripe :: Predicate Banana
frozen :: Predicate IceCream


getIngredients :: Kitchen -> (Banana, IceCream)
getIngredients (Kitchen _ _ b _ i) = (b,i)


divide :: Divisible f => (c -> (a,b)) -> f a -> f b -> f c
```

```haskell
data Kitchen = Kitchen Rice Curry Banana Apple IceCream


ripe :: Predicate Banana
frozen :: Predicate IceCream


getIngredients :: Kitchen -> (Banana, IceCream)
getIngredients (Kitchen _ _ b _ i) = (b,i)


divide :: Divisible f => (c -> (a,b)) -> f a -> f b -> f c


divide getIngredients ripe frozen :: Predicate Kitchen
```


Kitchen
(Banana, IceCream)
ripe
frozen
Bool
Bool
&&

What about `Alternative`?

```haskell
class Contravariant f => Divisible f where
  divide  :: (c -> (a, b)) -> f a -> f b -> f c
  conquer :: f a
```

```haskell
class Contravariant f => Divisible f where
  divide  :: (c -> (a, b)) -> f a -> f b -> f c
  conquer :: f a

class Divisible f => Decidable f where
  choose :: (c -> Either a b) -> f a -> f b -> f c
  lose :: (a -> Void) -> f a
```

```haskell
data Void

absurd :: Void -> a
absurd v = case v of {}
```

```haskell
data Void

absurd :: Void -> a
absurd v = case v of {}


left :: Either a Void -> a
left = either id absurd

right :: Either Void b -> b
right = either absurd id
```

```haskell
class Divisible f => Decidable f where
  choose :: (c -> Either a b) -> f a -> f b -> f c
  lose :: (a -> Void) -> f a
```

```haskell
class Divisible f => Decidable f where
  choose :: (c -> Either a b) -> f a -> f b -> f c
  lose :: (a -> Void) -> f a
```

Laws:

```haskell
choose Left m (lose f)  = m

choose Right (lose f) m = m

choose f (choose g m n) o = choose f' m (choose id n o) where
  f' bcd = either (either id (Right . Left) . g) (Right . Right) . f
```

```haskell
class Divisible f => Decidable f where
  choose :: (c -> Either a b) -> f a -> f b -> f c
  lose :: (a -> Void) -> f a


instance Decidable Predicate where
  choose cab (Predicate pa) (Predicate pb) =
    Predicate $ \c ->
      case cab c of
        Left a -> pa a
        Right b -> pb b

  lose av = Predicate (\a -> absurd (av a))
```

Predicates are boring

```haskell
newtype Printer a = Printer {
  runPrinter :: a -> String
}
```

```haskell
string :: Printer String
string = Printer id

konst :: String -> Printer a
konst s = Printer (const s)

showP :: Show a => Printer a
showP = Printer show

int :: Printer Int
int = showP

newline :: Printer ()
newline = konst "\n"
```

```haskell
instance Contravariant Printer where
  contramap ba (Printer as) = Printer (as . ba)
```

```haskell
instance Contravariant Printer where
  contramap ba (Printer as) = Printer (as . ba)


instance Divisible Printer where
  divide cab (Printer as) (Printer bs) = Printer $ \c ->
    case cab c of
      (a,b) -> as a <> bs b

  conquer = Printer (const "")
```

```haskell
instance Decidable Printer where
  choose cab (Printer as) (Printer bs) = Printer $ \c ->
    case cab c of
      Left  a -> as a
      Right b -> bs b

  lose av = Printer (\a -> absurd (av a))
```

```
(>$<) :: Contravariant f => (b -> a) -> f a -> f b
(>$<) = contramap
```

```
(>$<) :: Contravariant f => (b -> a) -> f a -> f b
(>$<) = contramap

(>*<) :: Divisible f => f a -> f b -> f (a,b)
(>*<) = divide id
```

```
(>$<) :: Contravariant f => (b -> a) -> f a -> f b
(>$<) = contramap

(>*<) :: Divisible f => f a -> f b -> f (a,b)
(>*<) = divide id

(>|<) :: Decidable f => f a -> f b -> f (Either a b)
(>|<) = choose id
```

```
(>$<) :: Contravariant f => (b -> a) -> f a -> f b
(>$<) = contramap

(>*<) :: Divisible f => f a -> f b -> f (a,b)
(>*<) = divide id

(>|<) :: Decidable f => f a -> f b -> f (Either a b)
(>|<) = choose id

(>*) :: Divisible f => f a -> f () -> f a
(>*) = divide (\a -> (a,()))

(*<) :: Divisible f => f () -> f a -> f a
(*<) = divide (\a -> ((),a))
```

```
infixr 3 >$<

infixr 4 >*<

infixr 3 >|<

infixr 4 >*

infixr 4 *<
```

```haskell
data Car = Car
  { make   :: String
  , model  :: String
  , engine :: Engine
  }

data Engine = Pistons Int | Rocket
```

```haskell
data Car = Car
  { make   :: String
  , model  :: String
  , engine :: Engine
  }

data Engine = Pistons Int | Rocket


car :: Car
car = Car "Toyota" "Corolla" (Pistons 4)
```

```haskell
engineToEither :: Engine -> Either Int ()
engineToEither e = case e of
  Pistons i -> Left i
  Rocket    -> Right ()

enginePrint :: Printer Engine
enginePrint =
  engineToEither
    >$< konst "Pistons: " *< int
    >|< konst "Rocket"
```

```haskell
carToTuple :: Car -> (String, (String, Engine))
carToTuple (Car ma mo e) = (ma, (mo, e))

carPrint :: Printer Car
carPrint =
  carToTuple
    >$< (konst "Make:  " >*< string >* newline)
    >*< (konst "Model: " >*< string >* newline)
    >*< enginePrint
```

```
putStrLn $ runPrinter carPrint car
```

```
Make:  Toyota
Model: Corolla
Pistons: 4
```

- `Applicative` and `Alternative` let us talk about how to *combine* multiple *results*
- `Divisible` and `Decidable` let us talk about how to *consume* multiple *inputs*

Thanks for listening!

Questions?

References

- https://hackage.haskell.org/package/contravariant
- https://github.com/qfpl/invariant-extras
- https://hackage.haskell.org/package/generics-eot
- Discrimination is Wrong
  https://www.youtube.com/watch?v=eXDJ5Jcbgk8

"Is there a contravariant `Monad`?" No

```
class Applicative f => MonadJ f where
  join :: f (f a) -> f a
```

but

```
newtype Compose f g a = Compose (f (g a))

instance (Contravariant f, Contravariant g) =>
  Functor (Compose f g) where
  fmap z (Compose fga) = Compose $ contramap (contramap z) fga
```

"Is anything both contravariant and covariant?"

```haskell
data Const c a = Const c

instance Functor (Const c) where
  fmap f (Const c) = Const c

instance Contravariant (Const c) where
  contramap f (Const c) = Const c
```

Every `a` is in positive position
Every `a` is in negative position