The Extended Functor Family

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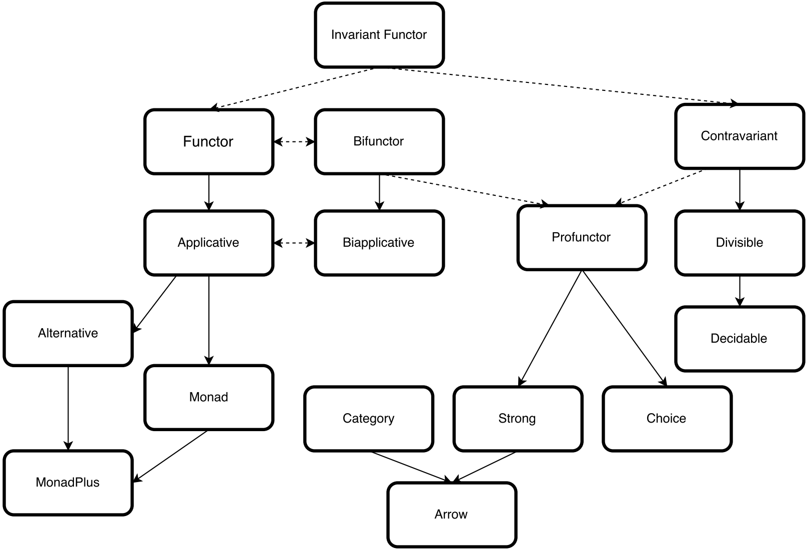
Fun with Functors | Good Math Bad Math
goodmath.scientopia.org/2011/10/25/**fun-with-functors**/
Oct 25, 2011 - So far, we've looked at the minimal basics of categories: what they are, and how to categorize the kinds of arrows that exist in categories in ...

Recitation 8: Functors
www.cs.cornell.edu/courses/cs312/2006fa/recitations/rec08.html
More **fun with functors**. Suppose you have a structure that makes use of not one, but two or more other structures. This seems to call for a functor that can take in ...

cbrad: Fun with functors
bradclow.blogspot.com/2009/02/**fun-with-functors**.html
Feb 15, 2009 - **Fun with functors**. I have been slowly working through some more Haskell with help from Tony. Quickly worked through the previous stuff we did ...

Tiusic: Fun with Functors
tiusic.blogspot.com/2013/06/**fun-with-functors**.html
Jun 4, 2013 - **Fun with Functors**. I just finished implementing the sound system for my game engine. It's a simple, fast, clean wrapper for OpenAL. There's a lot ...

GitHub - PawelPanasewicz/FunctorsAndFriends: Examples of using ...
https://github.com/PawelPanasewicz/FunctorsAndFriends
src/test/scala · using scala check from scala test, 2 months ago .gitattributes · **Fun with Functors**, 3 months ago .gitignore · **Fun with Functors**, 3 months ago .travis.


Invariant Functor
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Contravariant
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Biapplicative
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Functor

```haskell
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

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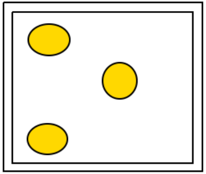
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

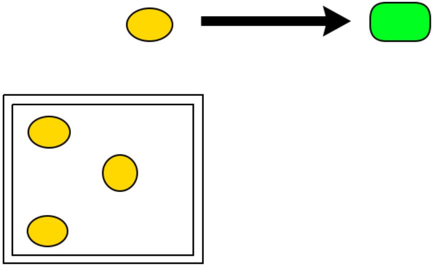
Laws:

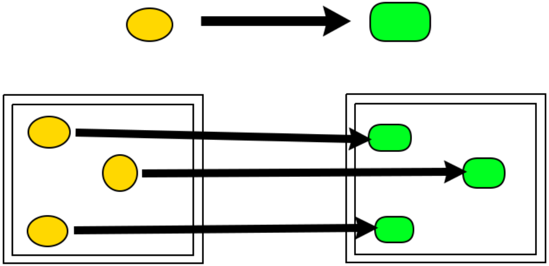
$$fmap\ id = id$$

$$fmap\ f\ .\ fmap\ g\ = fmap\ (f\ .\ g)$$

```haskell
instance Functor [] where
  fmap :: (a -> b) -> [a] -> [b]
  fmap f [] = []
  fmap f (x:xs) = f x : fmap f xs
```



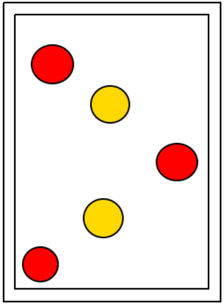


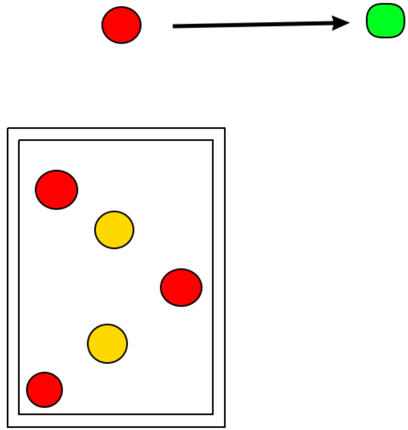


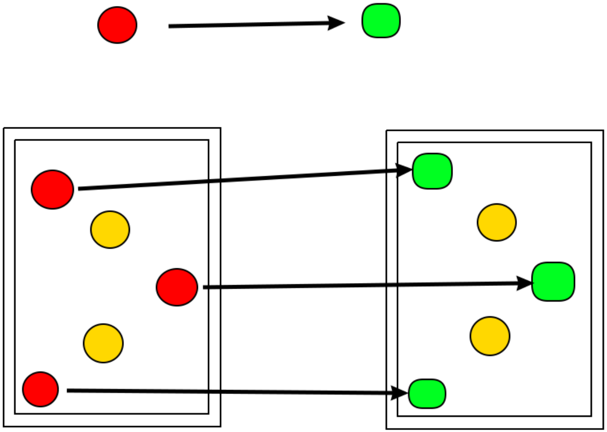
```haskell
instance Functor (x,) where
  fmap :: (a -> b) -> (x, a) -> (x, b)
  fmap f (x, a) = (x, f a)
```

```haskell
instance Functor (x,) where
  fmap :: (a -> b) -> (x, a) -> (x, b)
  fmap f (x, a) = (x, f a)


instance Functor (Either e) where
  fmap :: (a -> b) -> Either e a -> Either e b
  fmap f (Left e) = Left e
  fmap f (Right x) = Right (f x)
```







Bifunctor

```haskell
class Bifunctor p where
  bimap :: (a -> b) -> (x -> y) -> p a x -> p b y
```

```haskell
class Bifunctor p where
  bimap :: (a -> b) -> (x -> y) -> p a x -> p b y

  first :: (a -> b) -> p a x -> p b x
  second :: (x -> y) -> p a x -> p a y
```

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class Bifunctor p where
  bimap :: (a -> b) -> (x -> y) -> p a x -> p b y

  first :: (a -> b) -> p a x -> p b x
  second :: (x -> y) -> p a x -> p a y
```

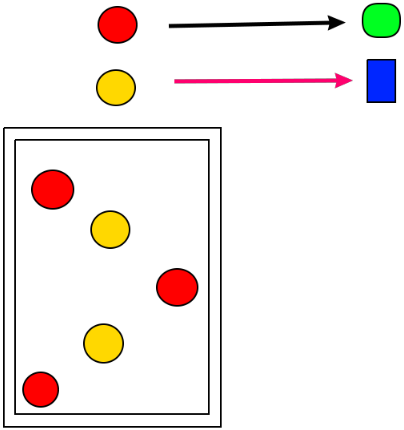
Laws:

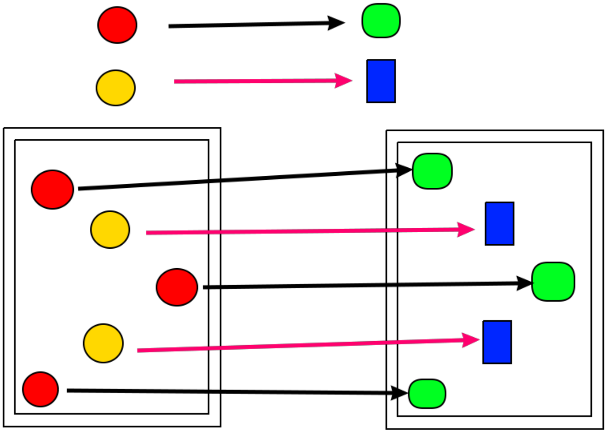
$$bimap\ id\ id\ =\ id$$

$$bimap\ f\ h\ .\ bimap\ g\ i\ =\ bimap\ (f\ .\ g)\ (h\ .\ i)$$

```haskell
instance Bifunctor (,) where
  bimap :: (a -> b) -> (x -> y) -> (a,x) -> (b,y)
  bimap f g (a,x) = (f a, g x)
```

```haskell
instance Bifunctor Either where
  bimap :: (a -> b) -> (x -> y) -> Either a x -> Either b y
  bimap f g (Left a) = Left (f a)
  bimap f g (Right x) = Right (g x)
```





Contravariant Functors

```haskell
newtype Predicate a =
  Predicate { runPredicate :: a -> Bool}
```

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  Predicate { runPredicate :: a -> Bool}
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Is `Predicate` a `Functor`?

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Is `Predicate` a `Functor`?

Can we write `fmap ::  (a -> b) -> Predicate a -> Predicate b`

```haskell
newtype Predicate a =
  Predicate { runPredicate :: a -> Bool}
```

Is `Predicate` a `Functor`?

Can we write `fmap ::  (a -> b) -> Predicate a -> Predicate b`

No!

Short diversion

Polarity

A type in a type signature can be in *positive* position or in *negative* position

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- A type on its own is in positive position, like
  - `i :: Int`
  - `result :: Maybe String`
  - `snacks :: [Banana]`

Polarity

A type in a type signature can be in *positive* position or in *negative* position

- A type on its own is in positive position, like
  - `i :: Int`
  - `result :: Maybe String`
  - `snacks :: [Banana]`
- Function return types are in positive position, but parameters are in negative position
  - `length :: [a] -> Int`
  - `buildRome :: Romulus -> Remus -> Rome`

Polarity

For `f` to be an instance of `Functor` every `a` in `f a` must be in positive position

We say that `f` is **covariant** in `a`

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```
data Maybe a = Nothing | Just a
```

Polarity

For `f` to be an instance of `Functor` every `a` in `f a` must be in positive position

We say that `f` is **covariant** in `a`

```
data Maybe a = Nothing | Just a

instance Functor Maybe where
  fmap :: (a -> b) -> Maybe a -> Maybe b
  fmap f Nothing  = Nothing
  fmap f (Just x) = Just (f x)
```

Polarity

```
newtype Predicate a =
  Predicate { runPredicate :: a -> Bool }
```

Polarity

```
newtype Predicate a =
  Predicate { runPredicate :: a -> Bool }
```

In `Predicate`, we only see `a` in negative position.
We say `Predicate` is **contravariant** in `a`.

```haskell
class Contravariant f where
  contramap :: (b -> a) -> f a -> f b
```

```haskell
class Contravariant f where
  contramap :: (b -> a) -> f a -> f b
```

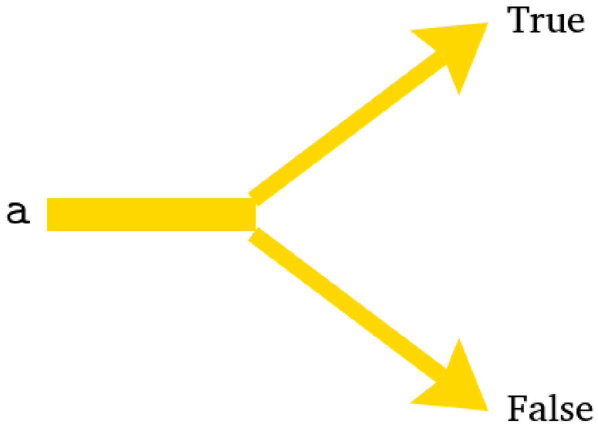
Laws:

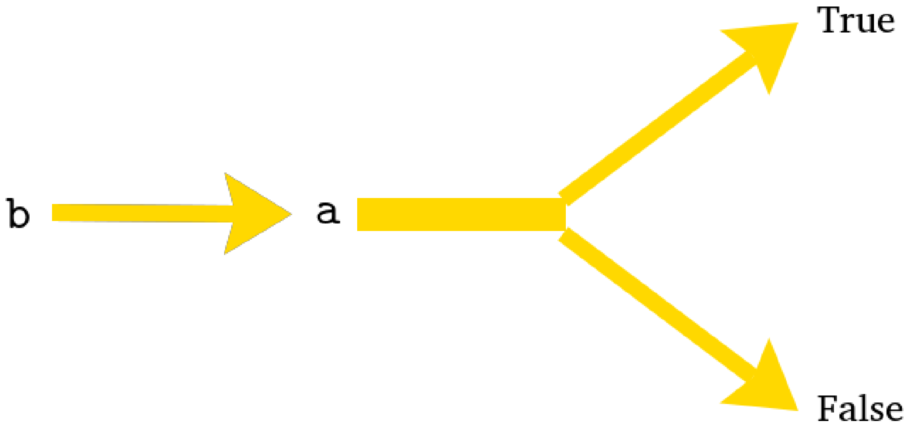
$$contramap\, id \;=\; id$$

$$contramap\ f\ .\ contramap\ g \;=\; contramap\ (g\ .\ f)$$

```haskell
newtype Predicate a =
  Predicate { runPredicate :: a -> Bool }
```

```haskell
newtype Predicate a =
  Predicate { runPredicate :: a -> Bool }


instance Contravariant Predicate where
  contramap :: (b -> a) -> Predicate a -> Predicate b
  contramap f (Predicate p) = Predicate (p . f)
```


a
True
False

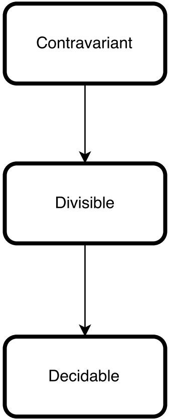

b
a
True
False

We think of a covariant `Functor` as being *full* of `a`'s.
A `Contravariant` functor can be thought of as *consuming* `a`'s.

```haskell
data Ordering = LT | EQ | GT

newtype Comparison a =
  Comparison { runComparison :: a -> a -> Ordering }
```

```haskell
data Ordering = LT | EQ | GT

newtype Comparison a =
  Comparison { runComparison :: a -> a -> Ordering }


instance Contravariant Comparison where
  contramap :: (b -> a) -> Comparison a -> Comparison b
  contramap f (Comparison c) = Comparison (\a b -> c (f a) (f b))
```

Corresponding to the more powerful forms of `Functor`, there are more powerful forms of `Contravariant`:



Why care about Contravariant?

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Discrimination (Linear-time sorting)

Now that we've talked about Bifunctor and Contravariant, we can finally talk about…

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Profunctor

```haskell
class Profunctor p where
  dimap :: (a -> b) -> (c -> d) -> p b c -> p a d
```

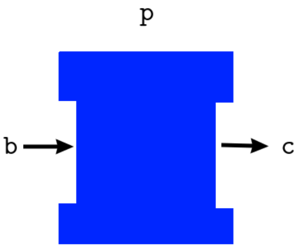
```haskell
class Profunctor p where
  dimap :: (a -> b) -> (c -> d) -> p b c -> p a d

  lmap :: (a -> b) -> p b c -> p a c
  rmap :: (c -> d) -> p b c -> p b d
```

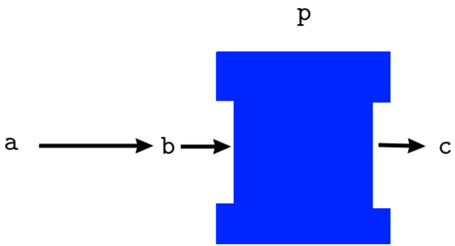
```haskell
class Profunctor p where
  dimap :: (a -> b) -> (c -> d) -> p b c -> p a d

  lmap :: (a -> b) -> p b c -> p a c
  rmap :: (c -> d) -> p b c -> p b d
```

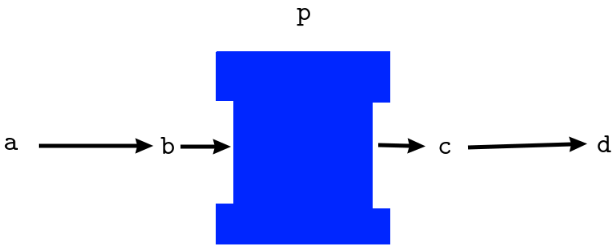
Laws:

$$dimap\ id\ id\ =\ id$$

$$dimap\ f\ g\ .\ dimap\ h\ k\ =\ dimap\ (h\ .\ f)\ (g\ .\ k)$$


p
b
c


p
a
b
c


p
a
b
c
d

```haskell
instance Profunctor (->) where
  dimap :: (a -> b) -> (c -> d) -> (b -> c) -> (a -> d)
  dimap ab cd bc = cd . bc . ab
```

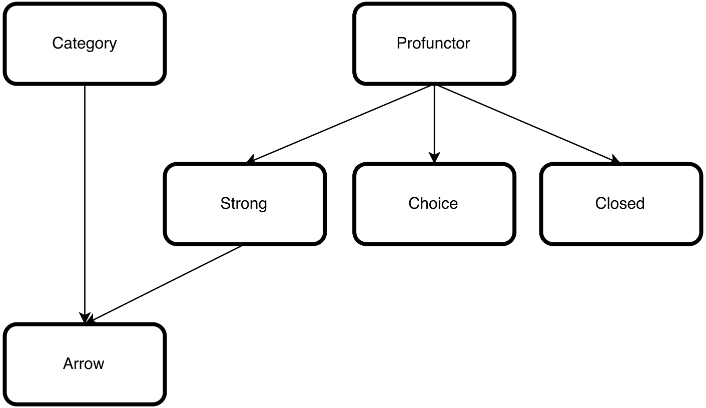
```haskell
-- import Control.Arrow

newtype Kleisli m b c = Kleisli { runKleisli :: b -> m c }
```

```haskell
-- import Control.Arrow

newtype Kleisli m b c = Kleisli { runKleisli :: b -> m c }


instance Monad m => Profunctor (Kleisli m) where
  dimap :: (a -> b) -> (c -> d) -> Kleisli m b c -> Kleisli m a d
  dimap ab cd (Kleisli bmc) =
    Kleisli (liftM cd . bmc . ab)
```

Every `Arrow` is a `Profunctor`!

Every `Arrow` is a `Profunctor`!

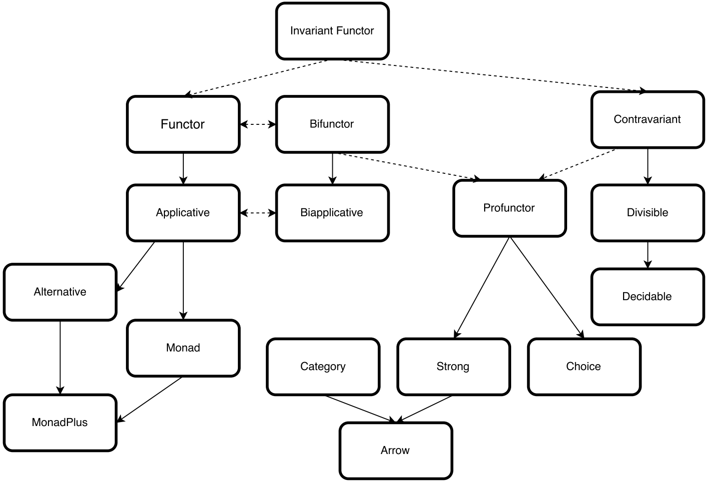
```
newtype WrappedArrow p b c = WrappedArrow { unwrap :: p b c }

instance Arrow p => Profunctor (WrappedArrow p) where
  dimap ab cd (WrappedArrow pbc) =
    WrappedArrow (arr ab . pbc . arr cd)
```


Category
Profunctor
Strong
Choice
Closed
Arrow

Why care about Profunctors?

Why care about Profunctors?

Lens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Category
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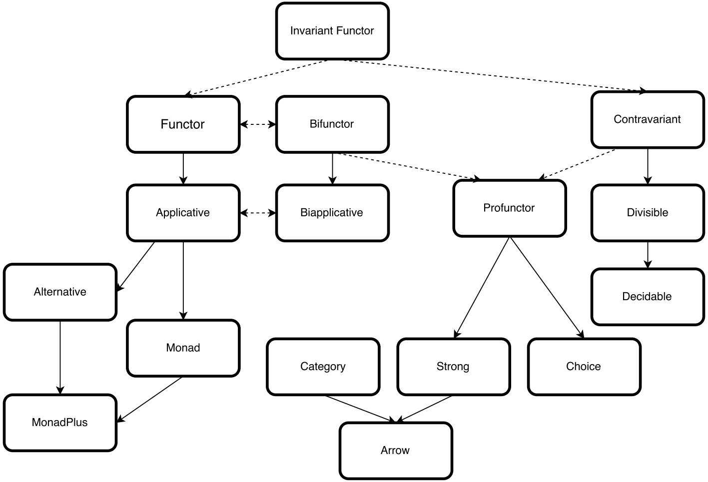
Thanks for listening!



References

- contravariant package
  https://hackage.haskell.org/package/contravariant
- profunctors package
  https://hackage.haskell.org/package/profunctors
- Discrimination is Wrong
  https://yow.eventer.com/yow-lambda-jam-2015-1305/discrimination-is-wrong-improving-productivity-by-edward-kmett-1890
- Fun with Profunctors
  https://www.youtube.com/watch?v=OJtGECfksds