

Category theory and life

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Plan

1. What is category theory?

Plan

1. What is category theory?
2. Objects and morphisms

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2. Objects and morphisms
3. The principle of sameness

Plan

1. What is category theory?
2. Objects and morphisms
3. The principle of sameness
4. Universal properties

Plan

1. What is category theory?
2. Objects and morphisms
3. The principle of sameness
4. Universal properties
5. Visual representation.

1. What is category theory?

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Category theory is the mathematics of mathematics.

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Mathematics is the study of how things work.

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Category theory is ...

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Category theory is ...

—the logical study of
the logical study of
how logical things work.

1. What is category theory?



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1. What is category theory?

The Imperial Severan Dynasty

source: www.roman-emperors.org

Note: Dates are birth and death; intermediate date is accession to the throne

| son/daughter

--- affair

— marriage

Roman Emperor

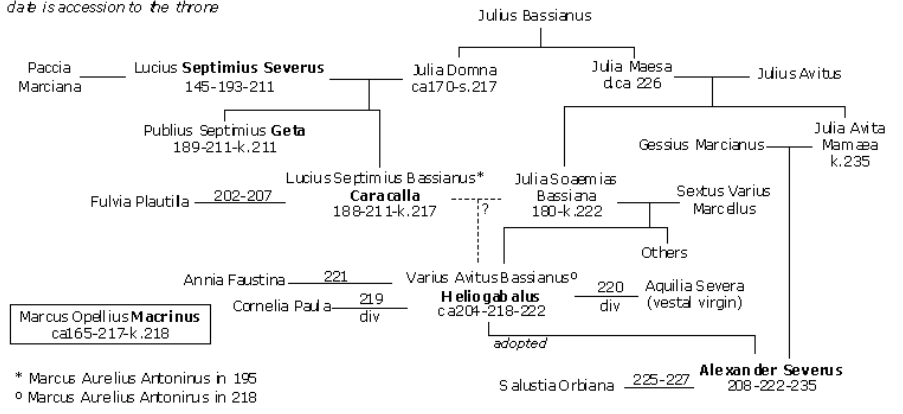
k. - killed

ca. - about

d. - died

s. - suicide

div - divorce



2. Objects and morphisms

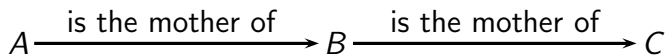
2. Objects and morphisms

$$A \longrightarrow B$$

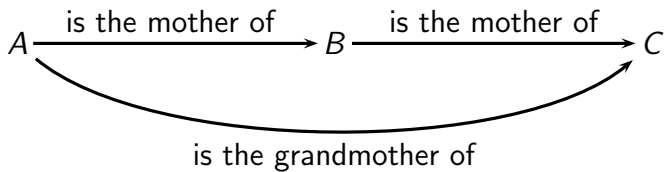
2. Objects and morphisms

$A \xrightarrow{\text{is the mother of}} B$

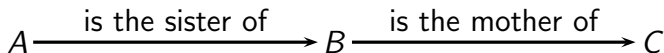
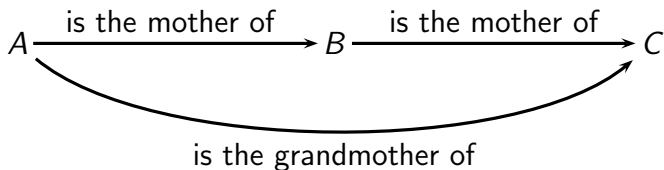
2. Objects and morphisms



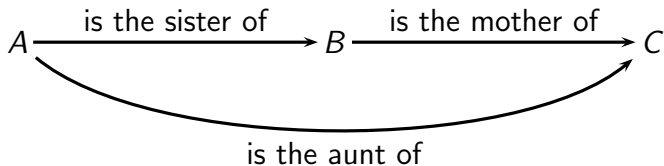
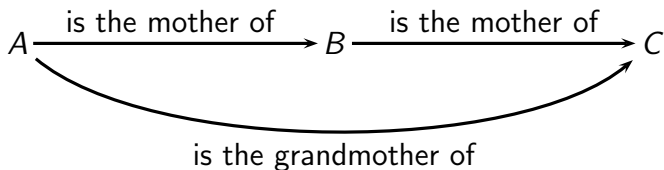
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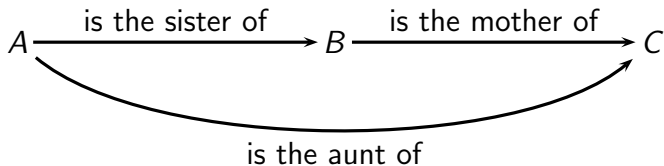
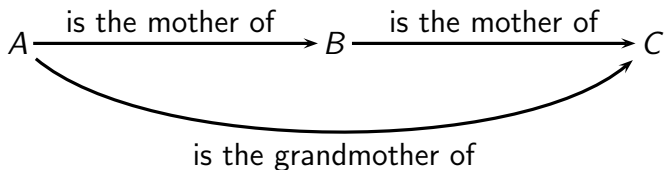
2. Objects and morphisms



2. Objects and morphisms

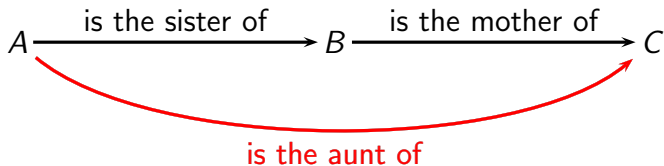
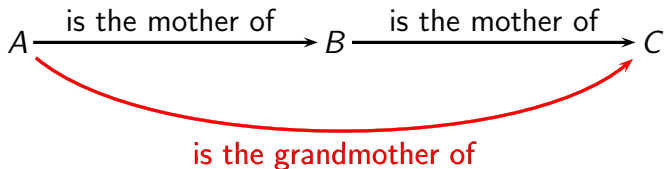


2. Objects and morphisms



Composition means we can omit arrows that can be deduced.

2. Objects and morphisms



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2. Objects and morphisms

A category consists of

- some objects A, B, C, \dots
- some morphisms (arrows) $A \longrightarrow B$
- composition and identities
- associativity and unit laws.

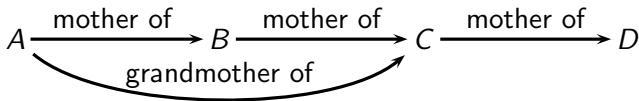
2. Objects and morphisms

Associativity:

$$A \xrightarrow{\text{mother of}} B \xrightarrow{\text{mother of}} C \xrightarrow{\text{mother of}} D$$

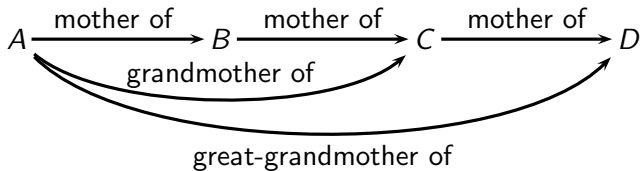
2. Objects and morphisms

Associativity:



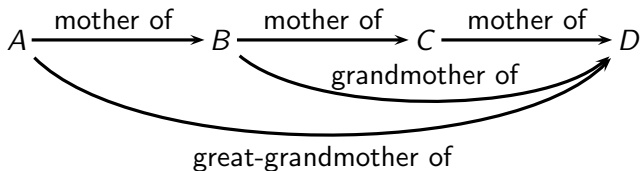
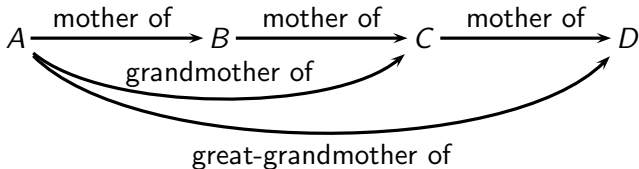
2. Objects and morphisms

Associativity:



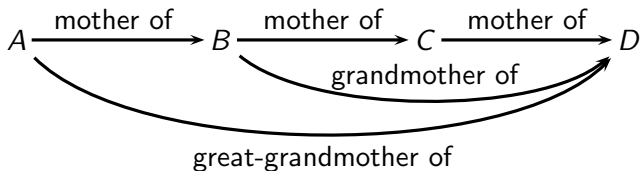
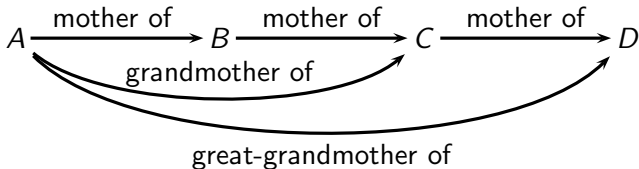
2. Objects and morphisms

Associativity:



2. Objects and morphisms

Associativity:



“The grandmother of my mother
is the mother of my grandmother.”

2. Objects and morphisms

Commutative squares

2. Objects and morphisms

Commutative squares

$$A \xrightarrow{\text{is the brother of}} B$$

2. Objects and morphisms

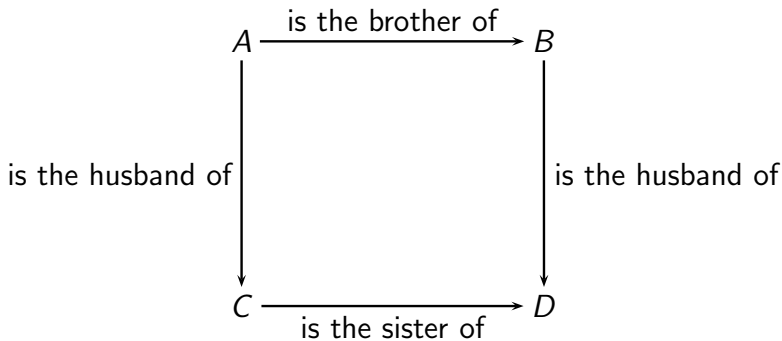
Commutative squares

$$A \xrightarrow{\text{is the brother of}} B$$

$$C \xrightarrow{\text{is the sister of}} D$$

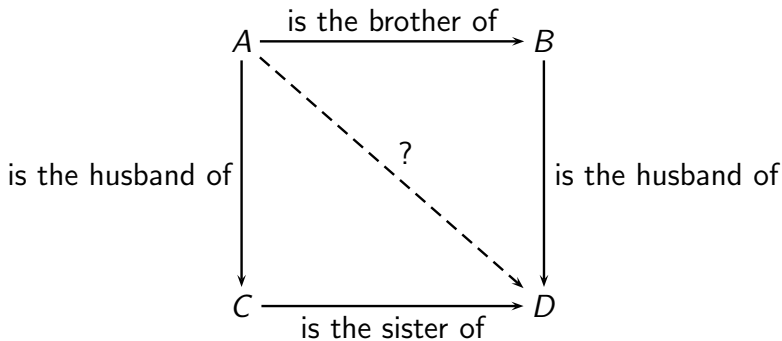
2. Objects and morphisms

Commutative squares



2. Objects and morphisms

Commutative squares



2. Objects and morphisms

Commutative squares: Super Smash Bros

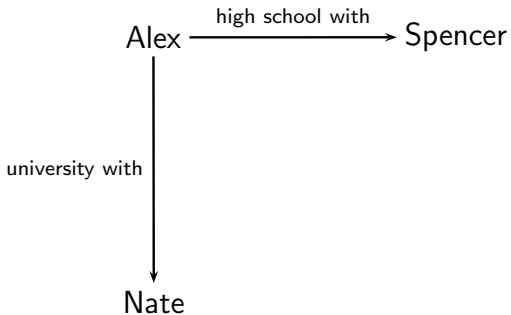
2. Objects and morphisms

Commutative squares: **Super Smash Bros**

Alex $\xrightarrow{\text{high school with}}$ Spencer

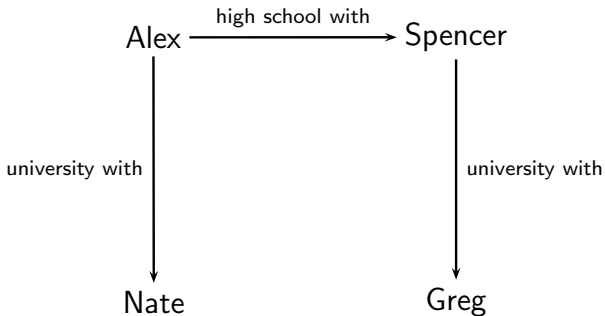
2. Objects and morphisms

Commutative squares: Super Smash Bros



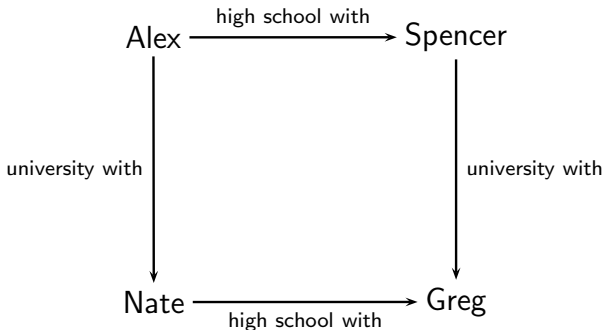
2. Objects and morphisms

Commutative squares: Super Smash Bros



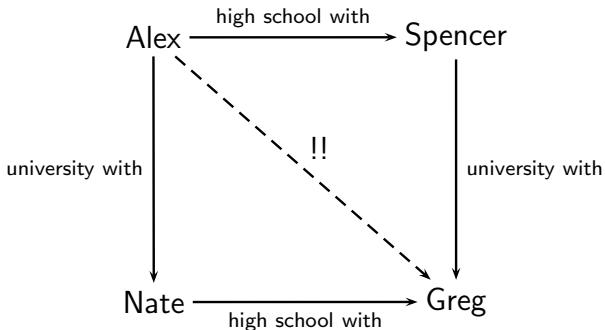
2. Objects and morphisms

Commutative squares: Super Smash Bros



2. Objects and morphisms

Commutative squares: Super Smash Bros



2. Objects and morphisms

Factors of 30

2. Objects and morphisms

Factors of 30

1,

2. Objects and morphisms

Factors of 30

1, 2,

2. Objects and morphisms

Factors of 30

1, 2, 3,

2. Objects and morphisms

Factors of 30

1, 2, 3, 5,

2. Objects and morphisms

Factors of 30

1, 2, 3, 5, 6,

2. Objects and morphisms

Factors of 30

1, 2, 3, 5, 6, 10,

2. Objects and morphisms

Factors of 30

1, 2, 3, 5, 6, 10, 15,

2. Objects and morphisms

Factors of 30

1, 2, 3, 5, 6, 10, 15, 30

2. Objects and morphisms

Factors of 30

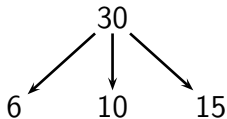
1, 2, 3, 5, 6, 10, 15, 30

30

2. Objects and morphisms

Factors of 30

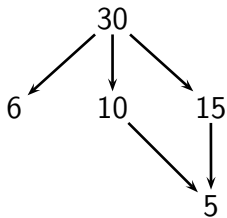
1, 2, 3, 5, 6, 10, 15, 30



2. Objects and morphisms

Factors of 30

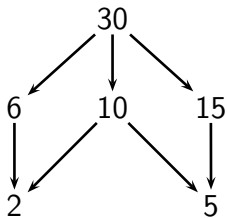
1, 2, 3, 5, 6, 10, 15, 30



2. Objects and morphisms

Factors of 30

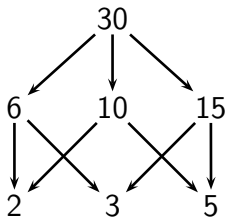
1, 2, 3, 5, 6, 10, 15, 30



2. Objects and morphisms

Factors of 30

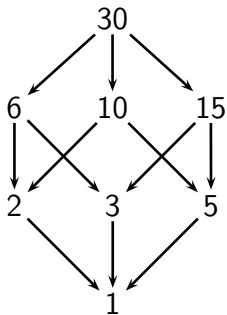
1, 2, 3, 5, 6, 10, 15, 30



2. Objects and morphisms

Factors of 30

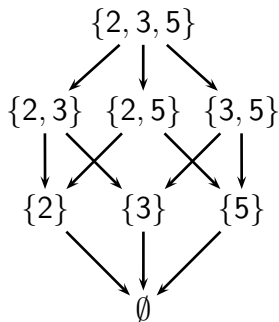
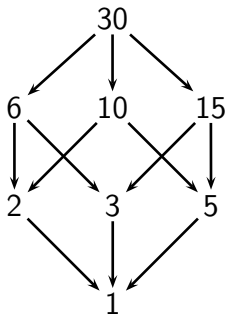
1, 2, 3, 5, 6, 10, 15, 30



2. Objects and morphisms

Factors of 30

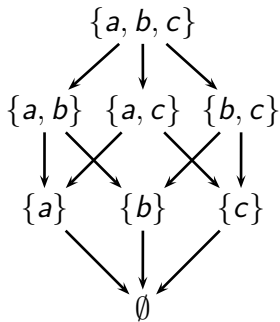
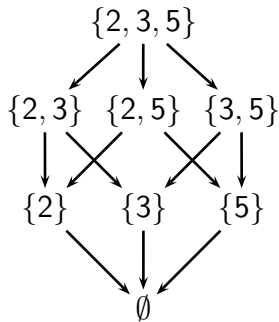
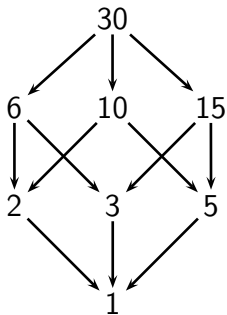
1, 2, 3, 5, 6, 10, 15, 30



2. Objects and morphisms

Factors of 30

1, 2, 3, 5, 6, 10, 15, 30



2. Objects and morphisms

Factors of 42

2. Objects and morphisms

Factors of 42

1,

2. Objects and morphisms

Factors of 42

1, 2,

2. Objects and morphisms

Factors of 42

1, 2, 3,

2. Objects and morphisms

Factors of 42

1, 2, 3, 6,

2. Objects and morphisms

Factors of 42

1, 2, 3, 6, 7,

2. Objects and morphisms

Factors of 42

1, 2, 3, 6, 7, 14,

2. Objects and morphisms

Factors of 42

1, 2, 3, 6, 7, 14, 21,

2. Objects and morphisms

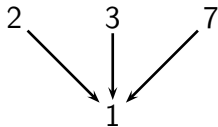
Factors of 42

1, 2, 3, 6, 7, 14, 21, 42

2. Objects and morphisms

Factors of 42

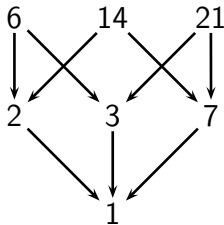
1, 2, 3, 6, 7, 14, 21, 42



2. Objects and morphisms

Factors of 42

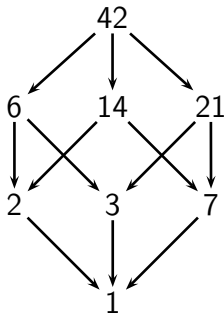
1, 2, 3, 6, 7, 14, 21, 42



2. Objects and morphisms

Factors of 42

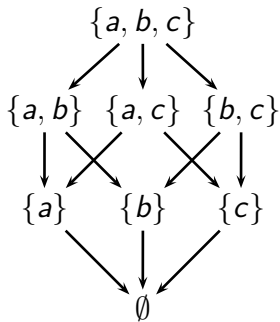
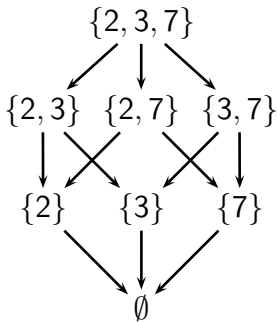
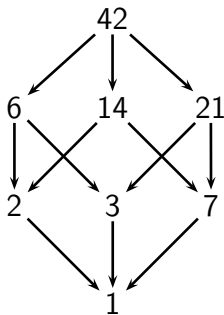
1, 2, 3, 6, 7, 14, 21, 42



2. Objects and morphisms

Factors of 42

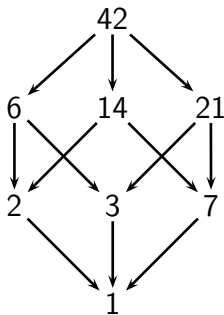
1, 2, 3, 6, 7, 14, 21, 42



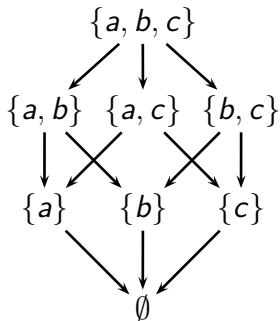
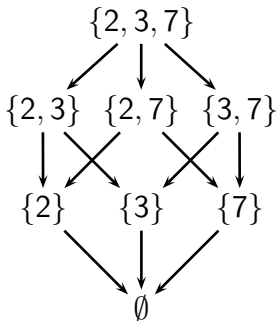
2. Objects and morphisms

Factors of 42

1, 2, 3, 6, 7, 14, 21, 42



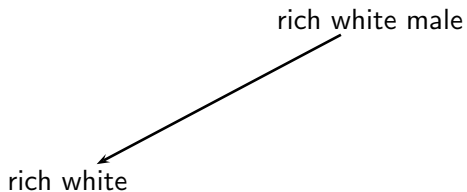
$6 < 7$



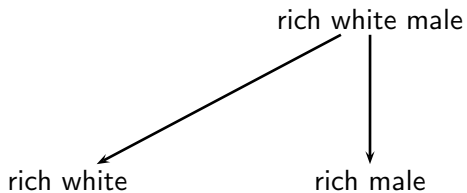
2. Objects and morphisms

rich white male

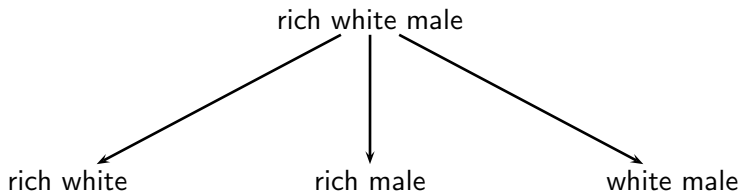
2. Objects and morphisms



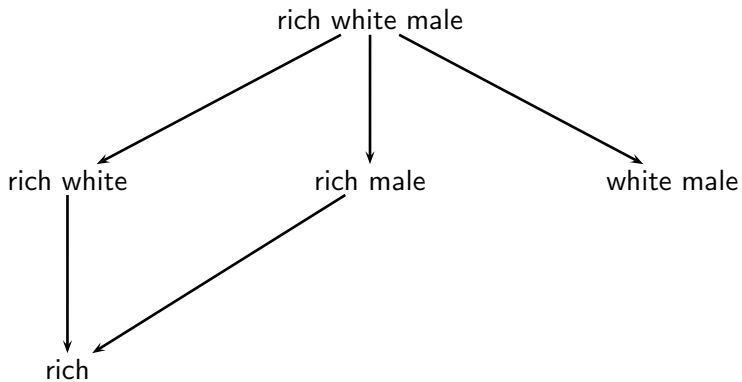
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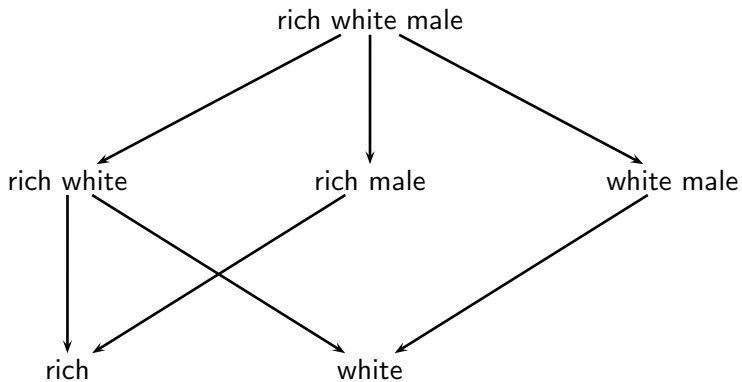
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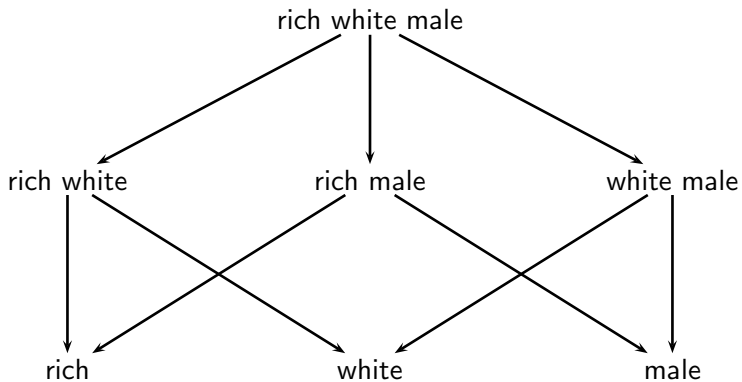
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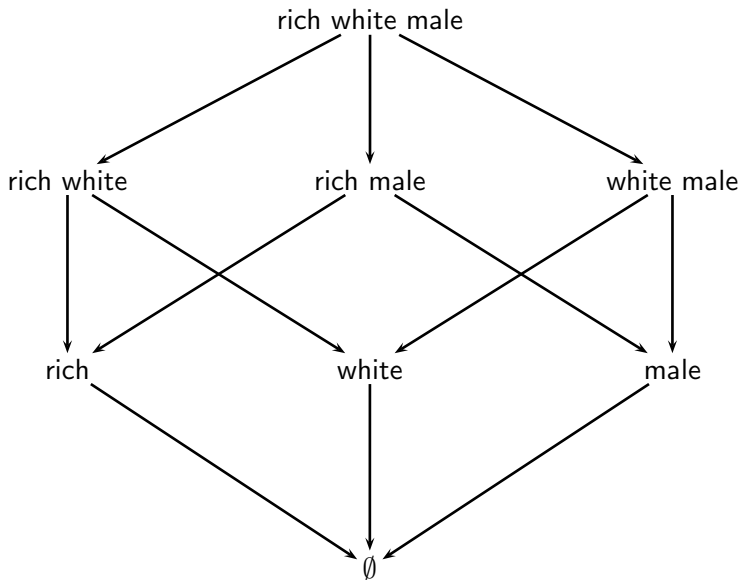
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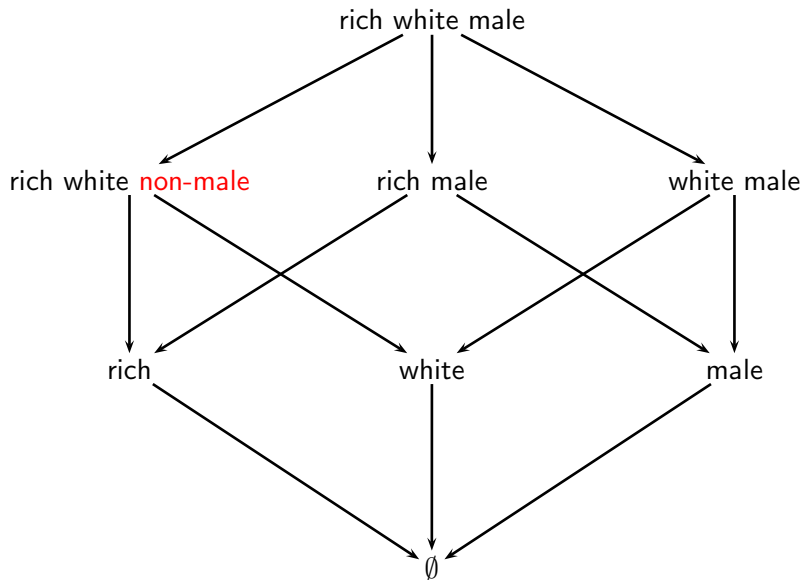
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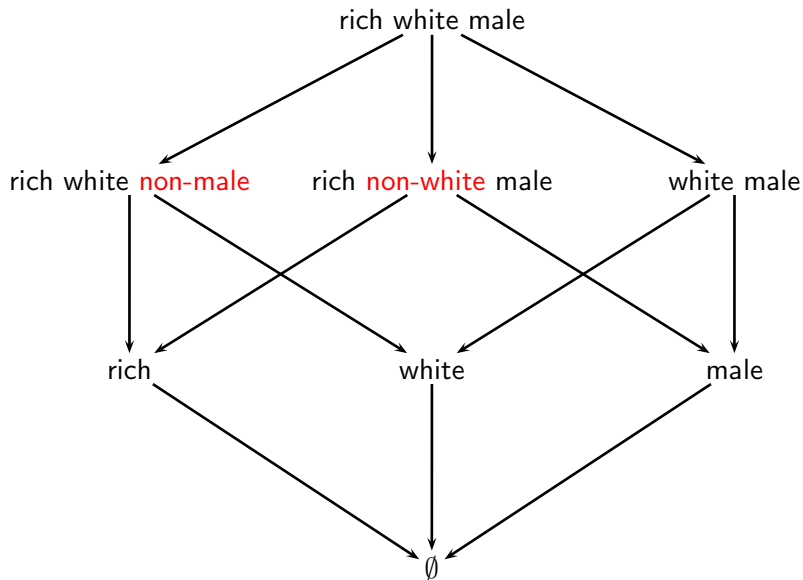
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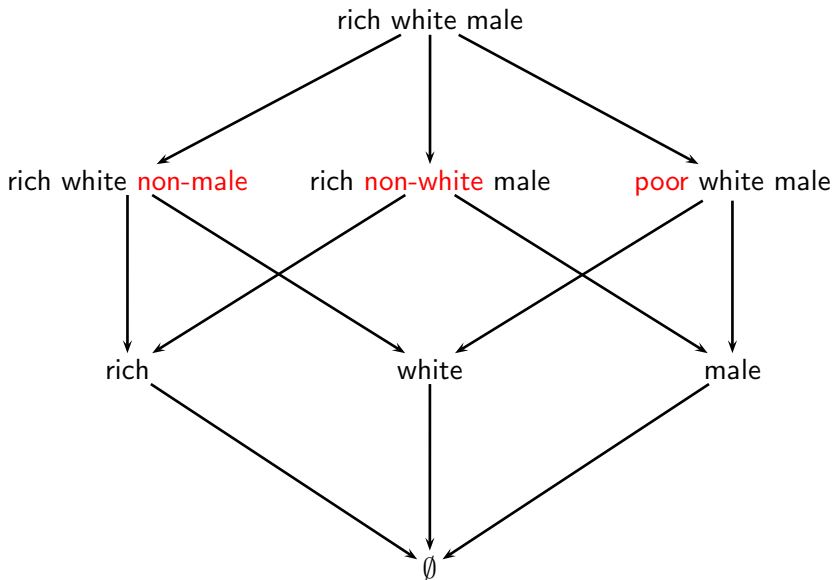
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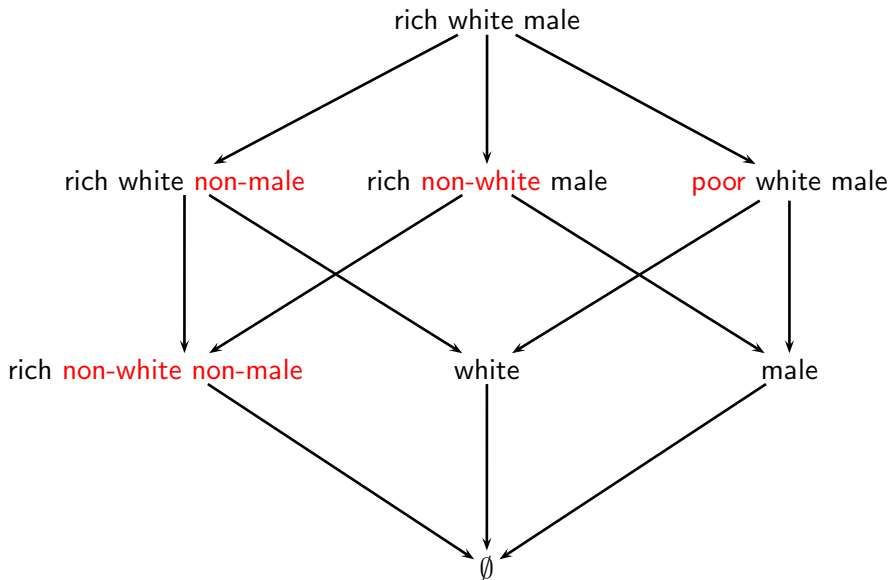
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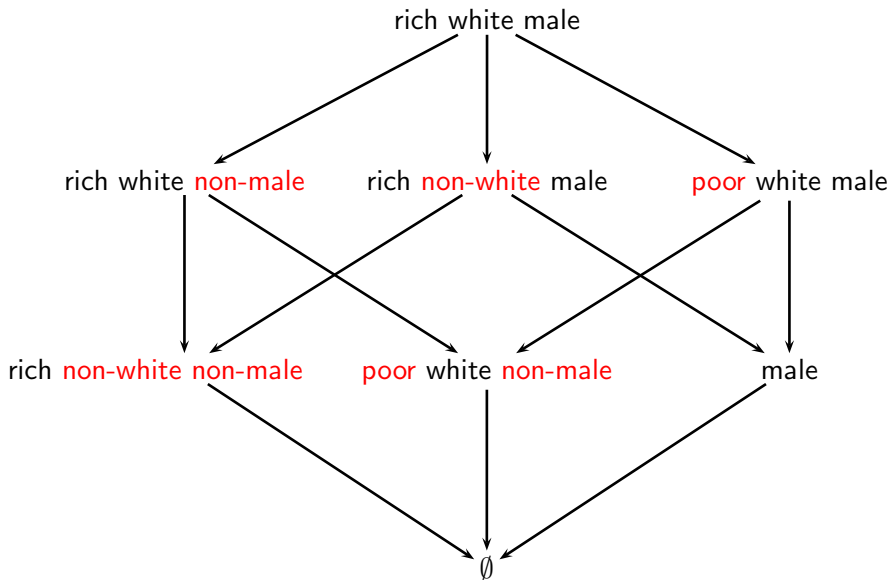
2. Objects and morphisms



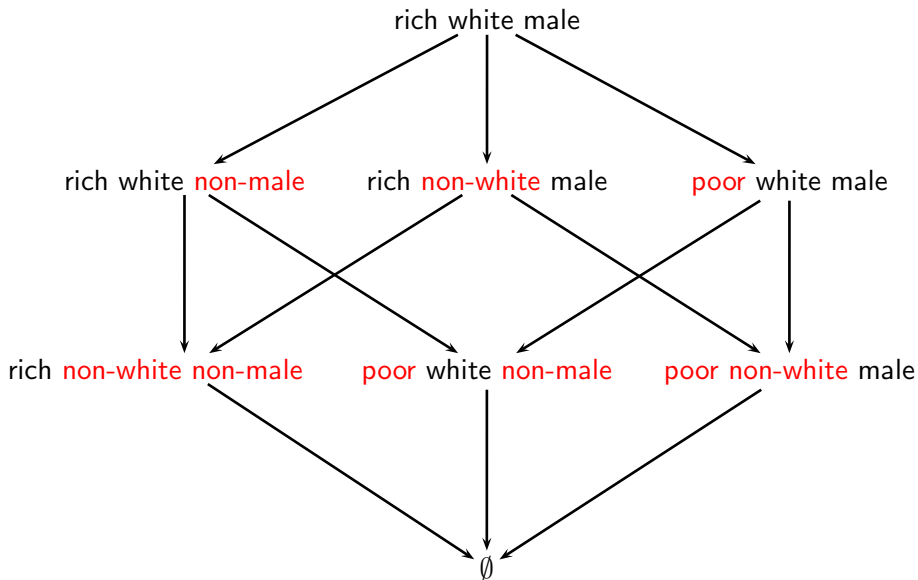
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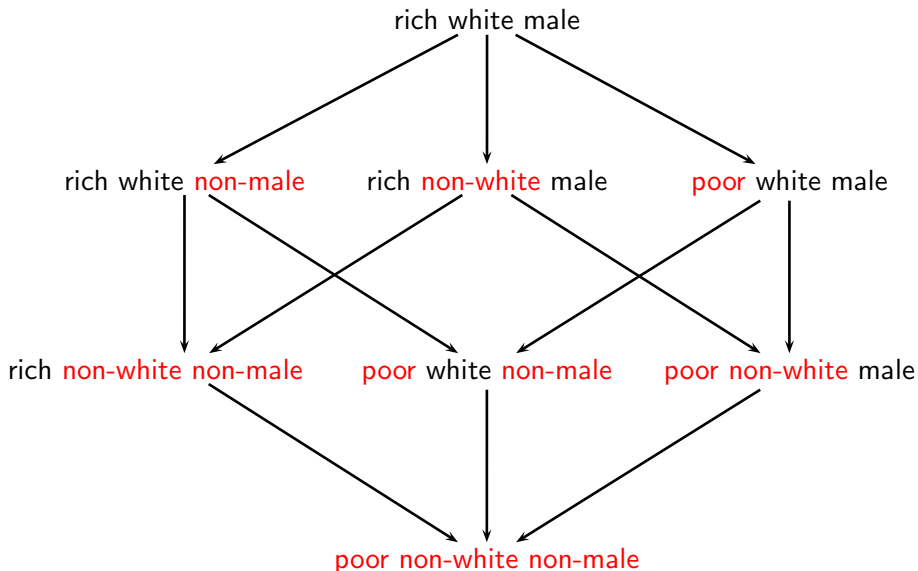
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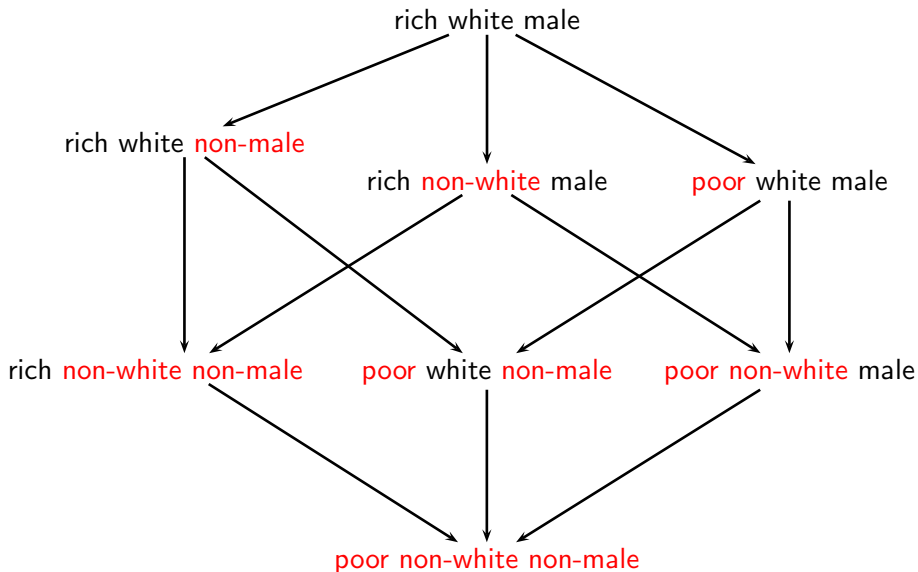
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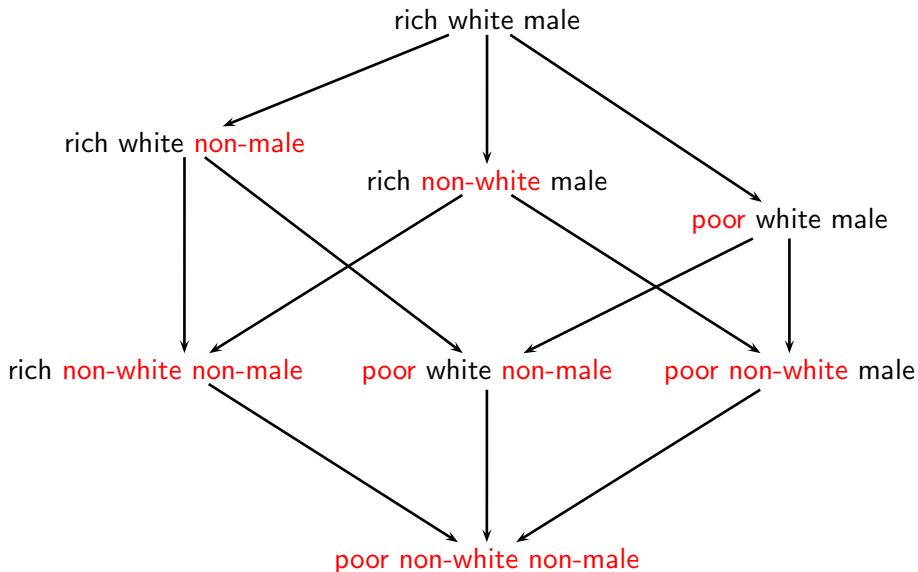
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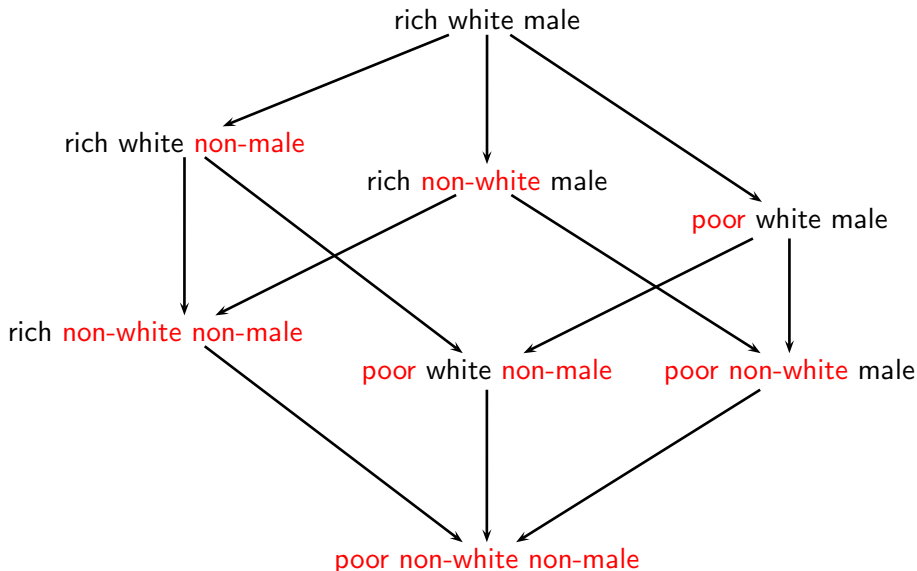
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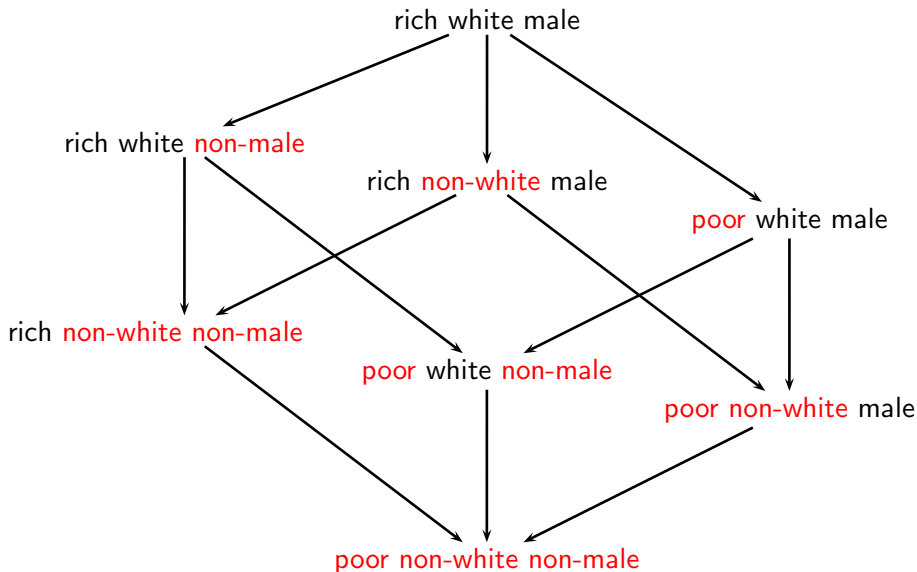
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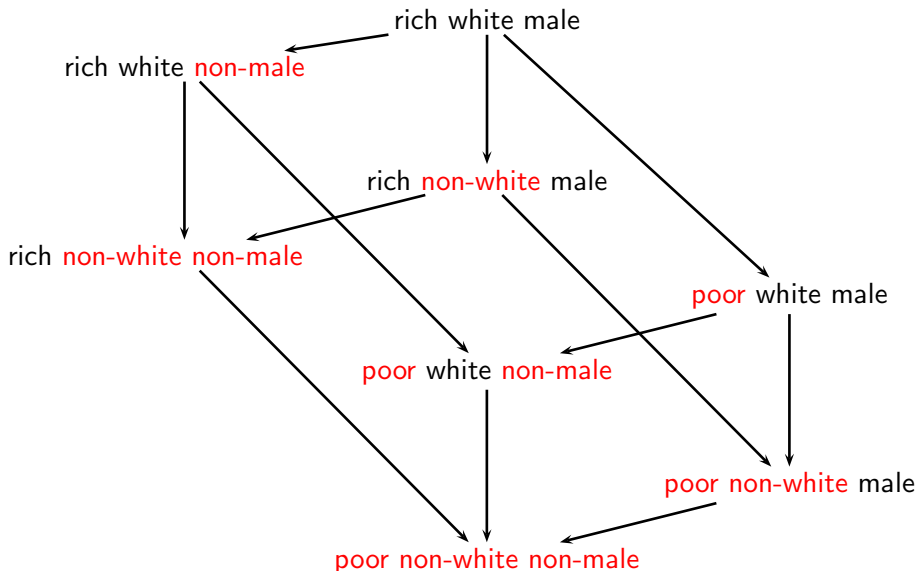
2. Objects and morphisms



2. Objects and morphisms



2. Objects and morphisms



3. Sameness

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All equations are lies.

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$$8 + 1 = 1 + 8$$

3. Sameness

All equations are lies.

$$8 + 1 = 1 + 8$$

$$2 \times 5 = 5 \times 2$$

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$$(8 + 5) + 5 = 8 + (5 + 5)$$

3. Sameness

All equations are lies.

$$8 + 1 = 1 + 8$$

$$2 \times 5 = 5 \times 2$$

$$(8 + 5) + 5 = 8 + (5 + 5)$$

The only equation that is not a lie is

$$x = x.$$

3. Sameness

All equations are lies. . . or useless.

$$8 + 1 = 1 + 8$$

$$2 \times 5 = 5 \times 2$$

$$(8 + 5) + 5 = 8 + (5 + 5)$$

The only equation that is not a lie is

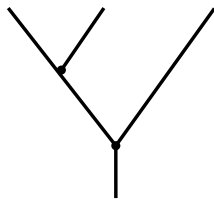
$$x = x.$$

3. Sameness

Example: associativity

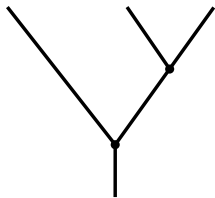
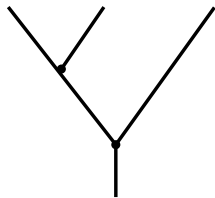
3. Sameness

Example: associativity



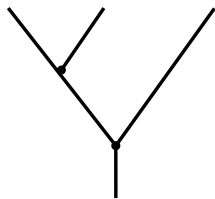
3. Sameness

Example: associativity



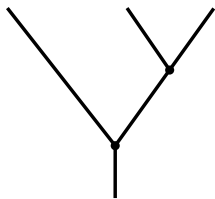
3. Sameness

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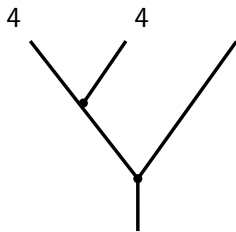
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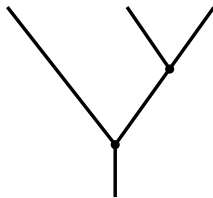
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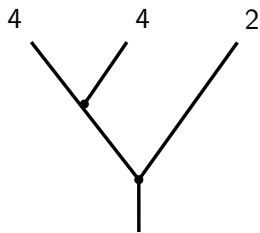
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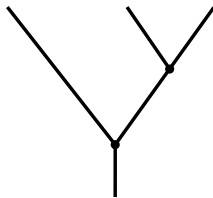
3. Sameness

Example: associativity



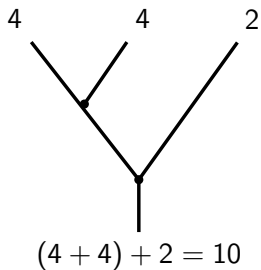
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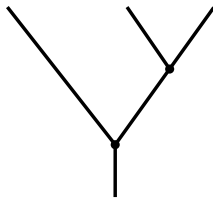
3. Sameness

Example: associativity



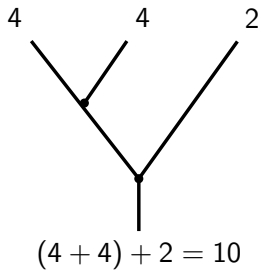
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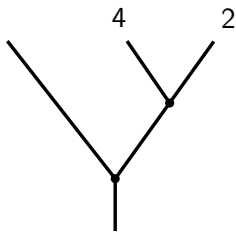
3. Sameness

Example: associativity



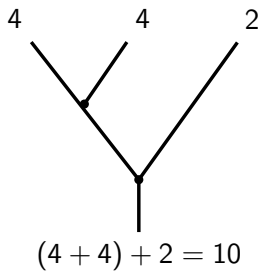
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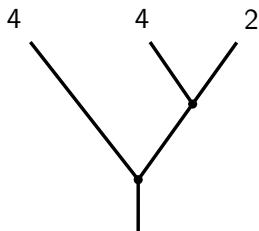
3. Sameness

Example: associativity



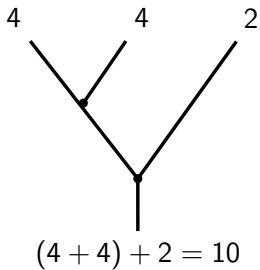
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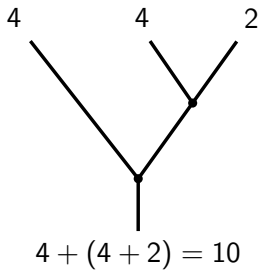
3. Sameness

Example: associativity



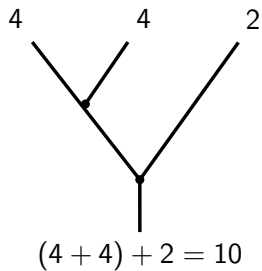
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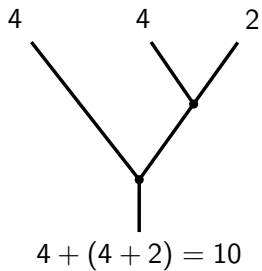


3. Sameness

Example: associativity

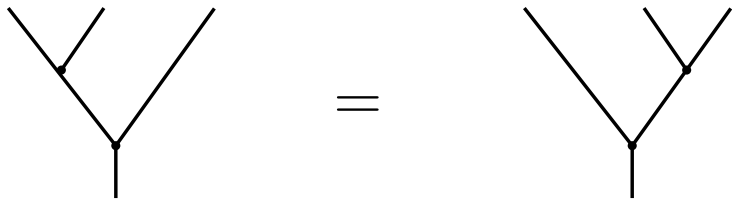


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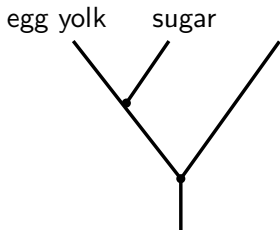
3. Sameness

Example: associativity



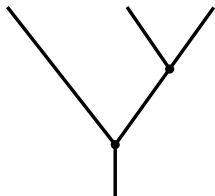
3. Sameness

Example: associativity



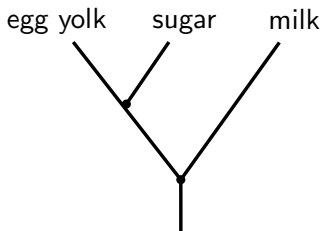
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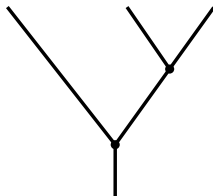
3. Sameness

Example: associativity



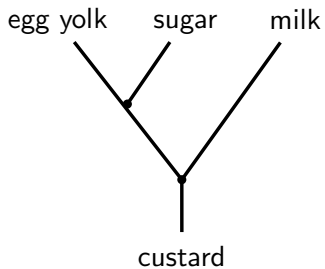
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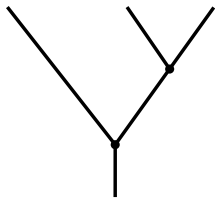
3. Sameness

Example: associativity



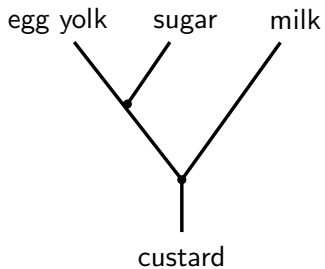
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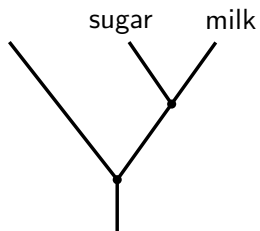
3. Sameness

Example: associativity



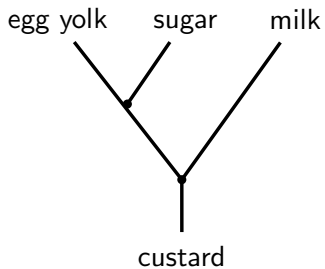
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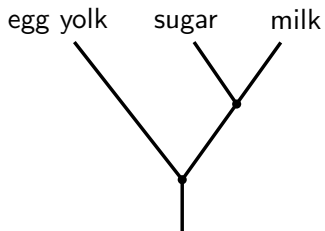
3. Sameness

Example: associativity



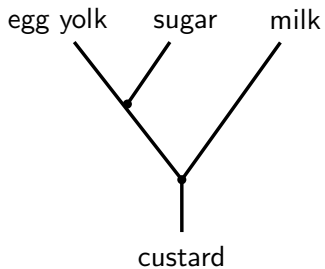
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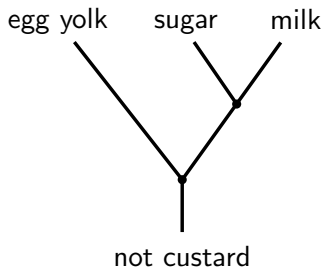
3. Sameness

Example: associativity



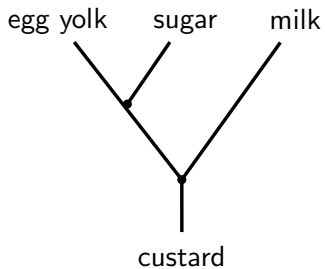
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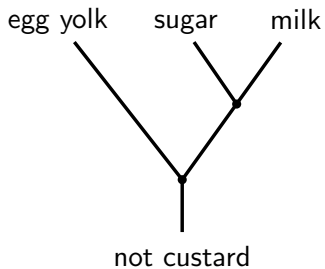


3. Sameness

Example: associativity



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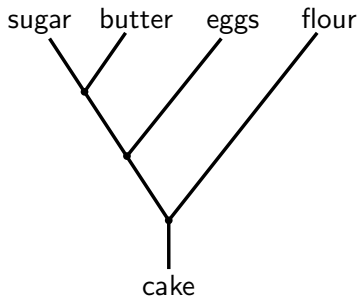


3. Sameness

For cake, it doesn't really matter.

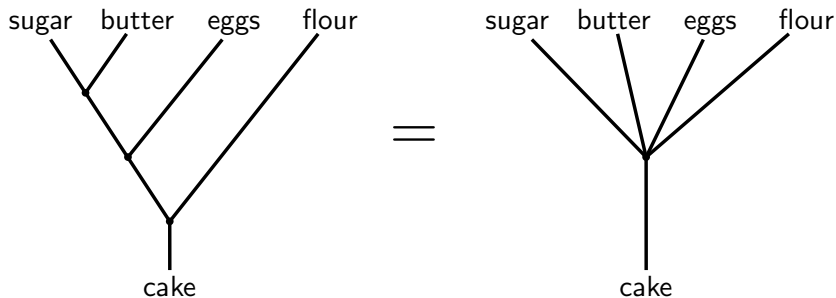
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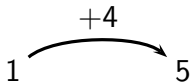


3. Sameness

Invertibility

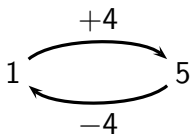
3. Sameness

Invertibility



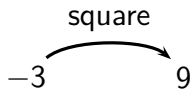
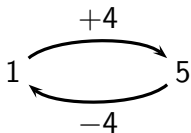
3. Sameness

Invertibility



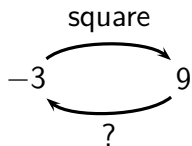
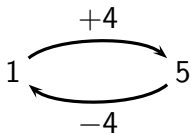
3. Sameness

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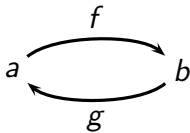
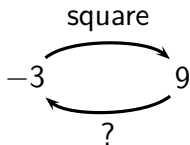
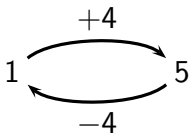
3. Sameness

Invertibility



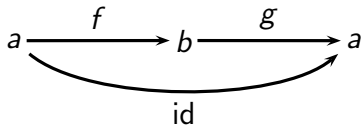
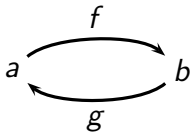
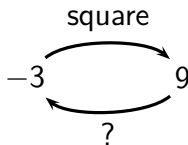
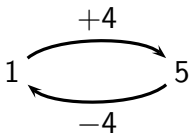
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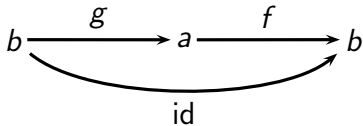
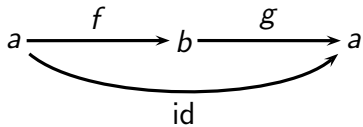
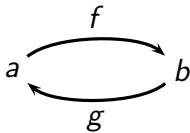
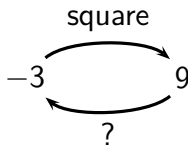
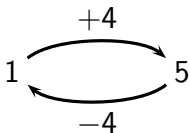
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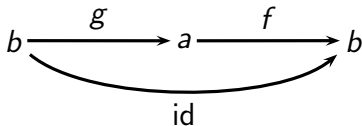
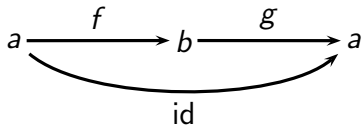
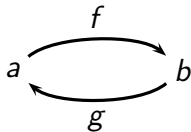
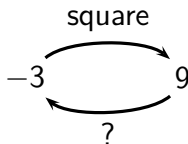
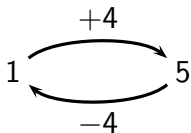
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Invertibility



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Invertibility



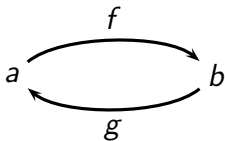
Then a and b are called isomorphic.

3. Sameness

Isomorphic objects are treated as the same
by the rest of the category.

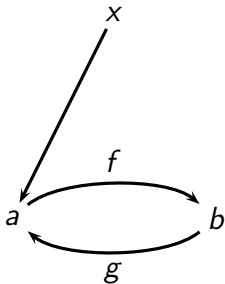
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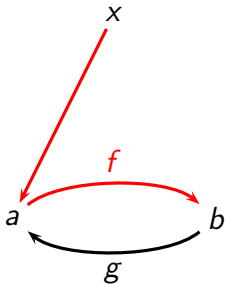
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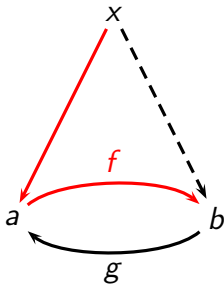
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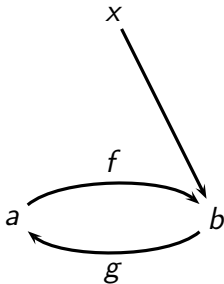
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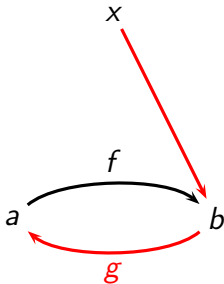
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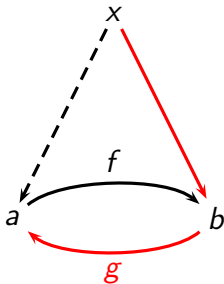
3. Sameness

Isomorphic objects are treated as the same
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3. Sameness

Isomorphic objects are treated as the same by the rest of the category.



3. Visual representation

Things can be isomorphic in one category but not another.

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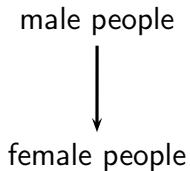


3. Sameness

Whole categories can be isomorphic.

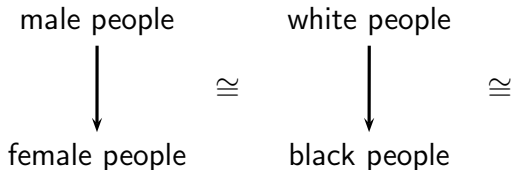
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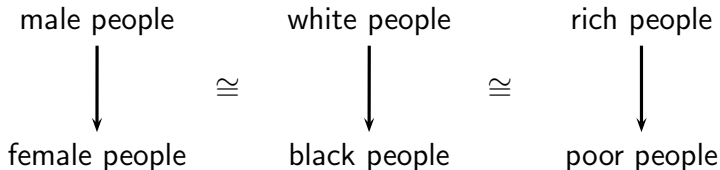
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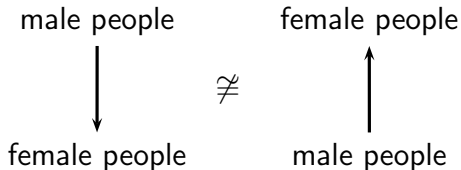
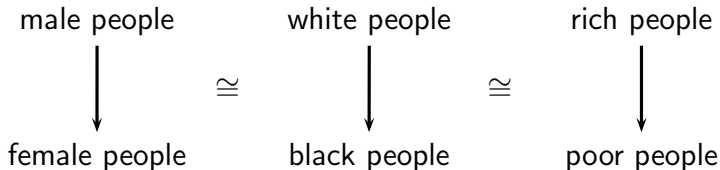
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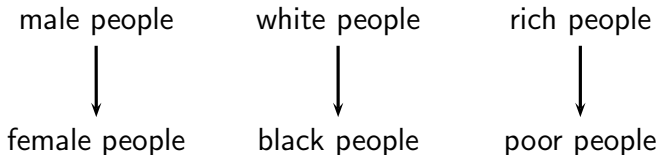
Equality doesn't mean men and women are the same —
it is about when the world should treat them the same.

4. Universal properties

Characterising things by the role they play in that context

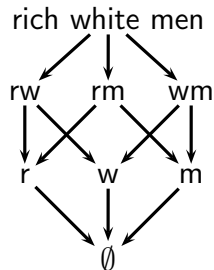
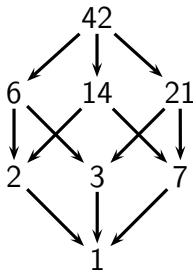
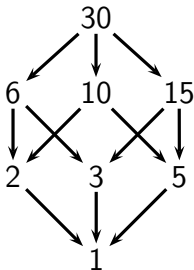
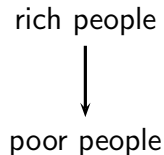
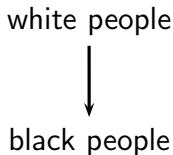
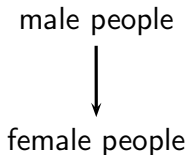
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An initial object in a category is:

An object I such that for all objects X
there is a unique morphism $I \longrightarrow X$.

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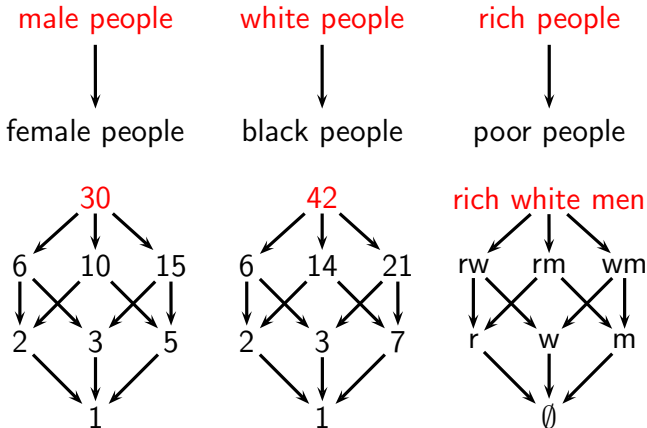
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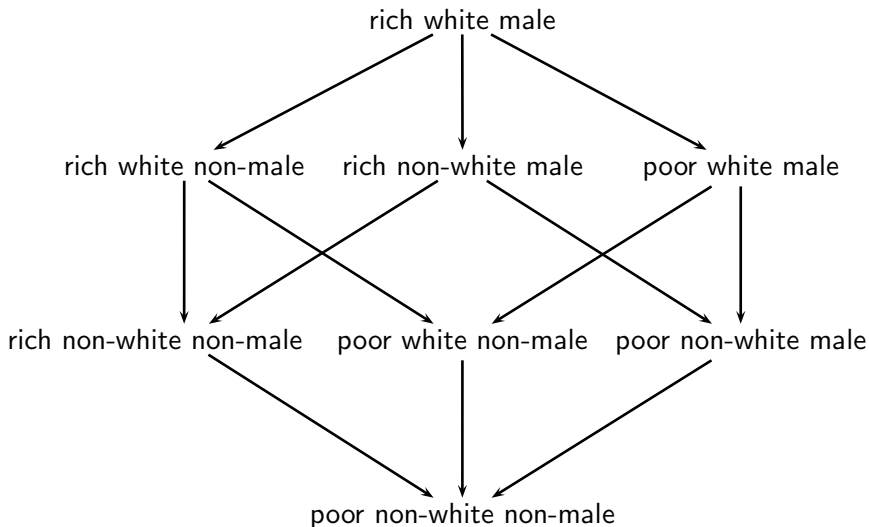


4. Universal properties

Something not universal in one place can be universal in another.

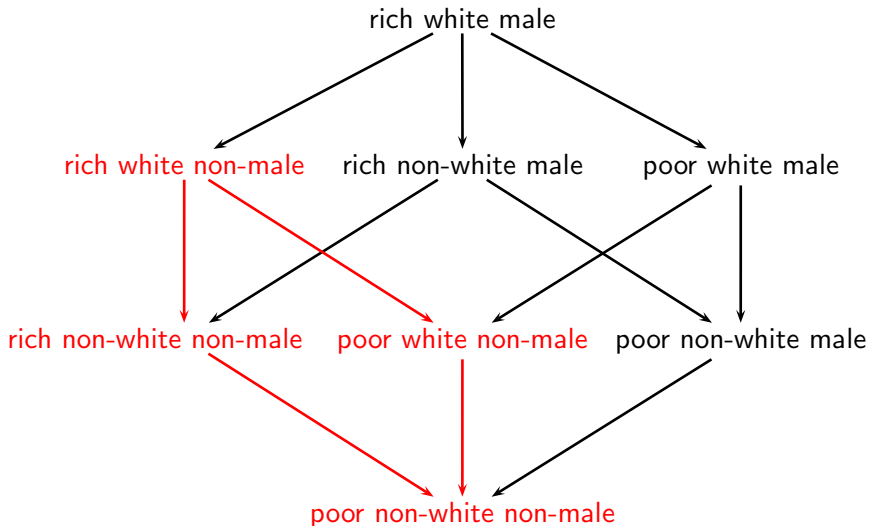
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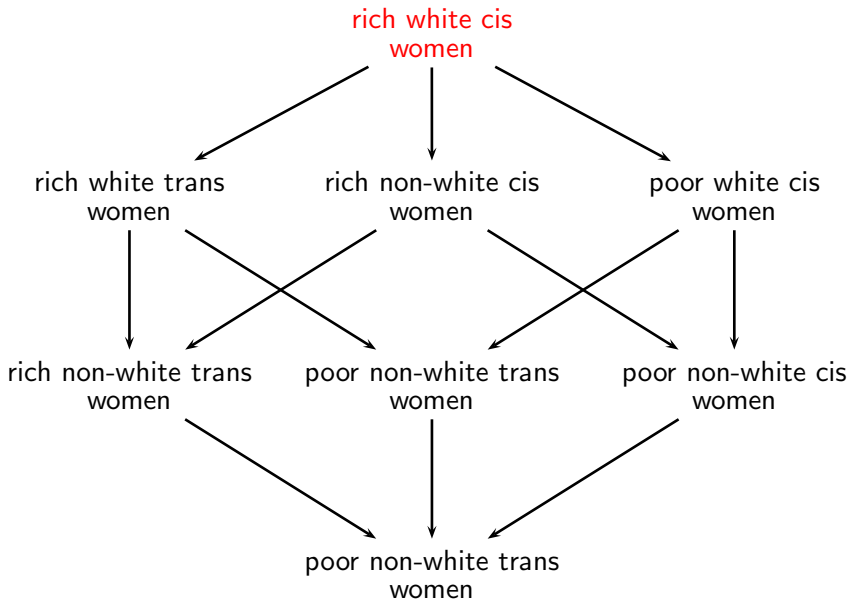


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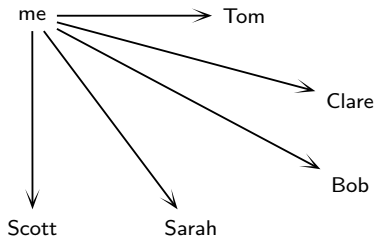
5. Visual representation

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Friend maps

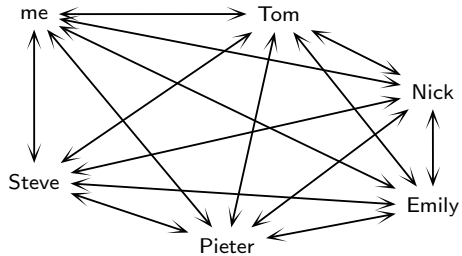
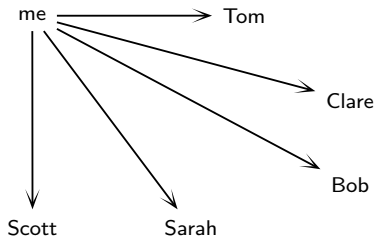
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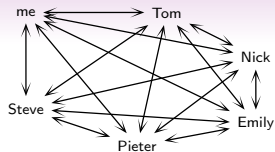
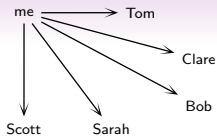


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Friend maps



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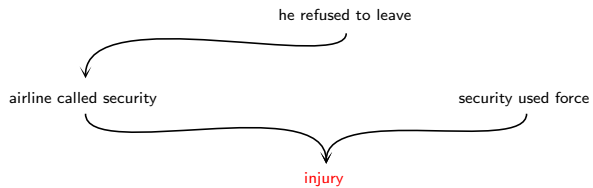
The United Incident

he refused to leave

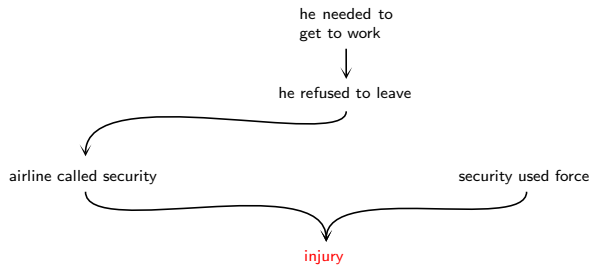
security used force

injury

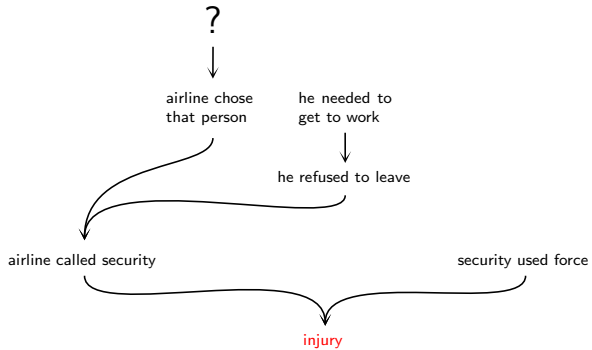
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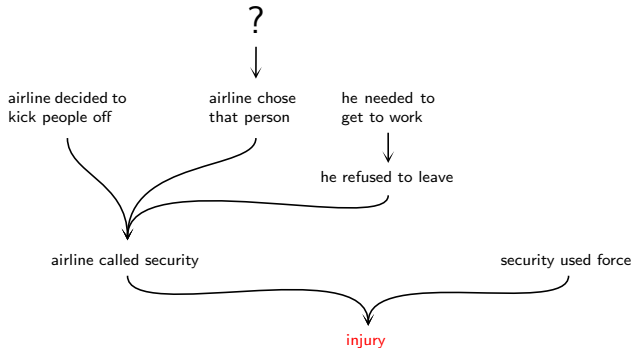
The United Incident



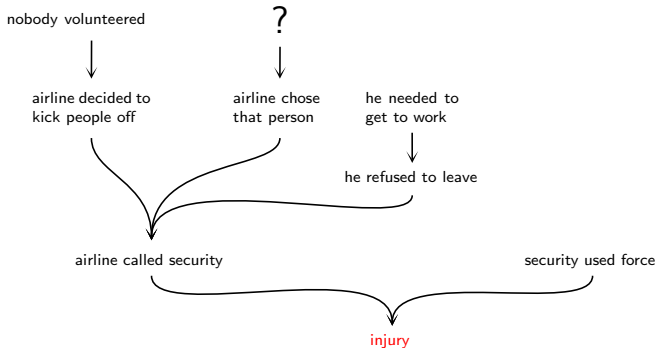
The United Incident



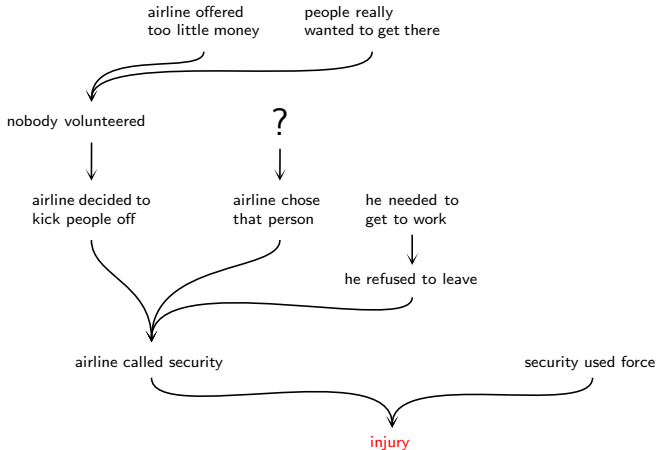
The United Incident



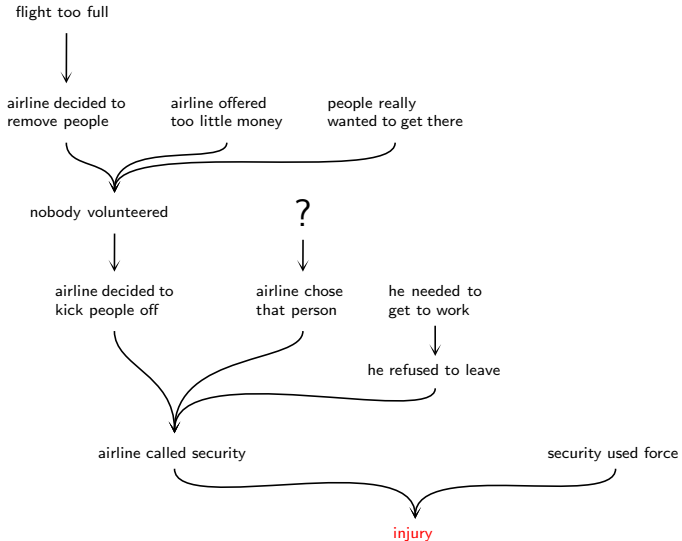
The United Incident



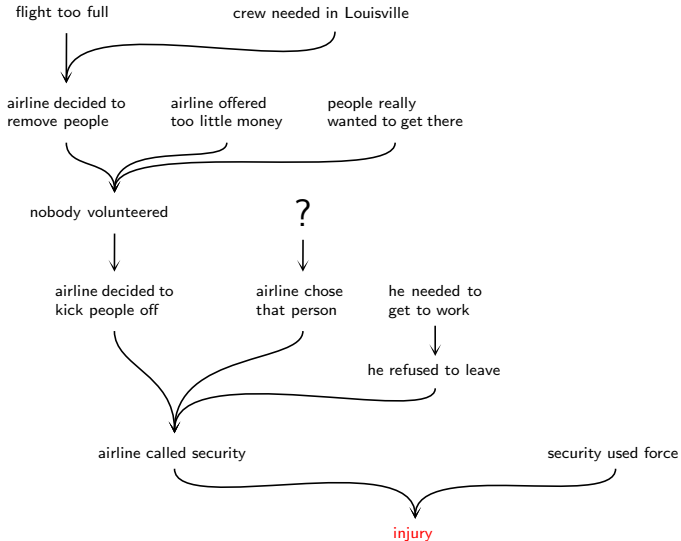
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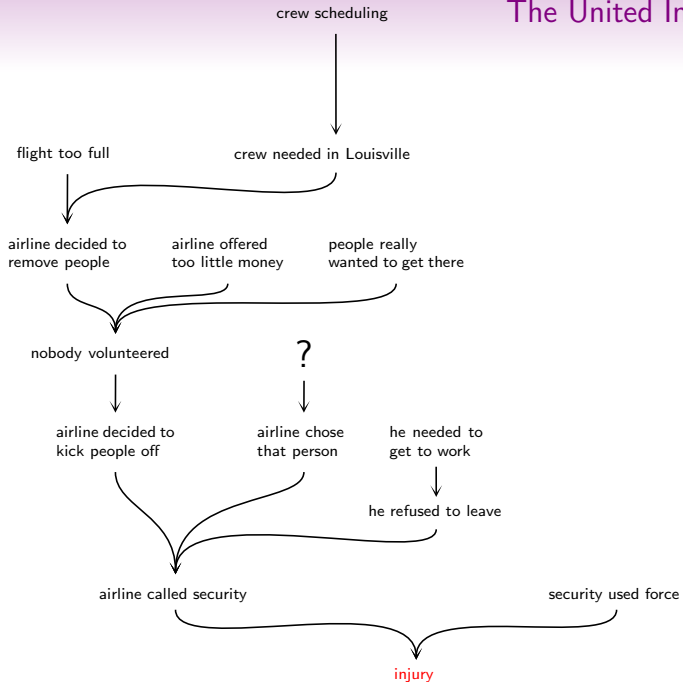
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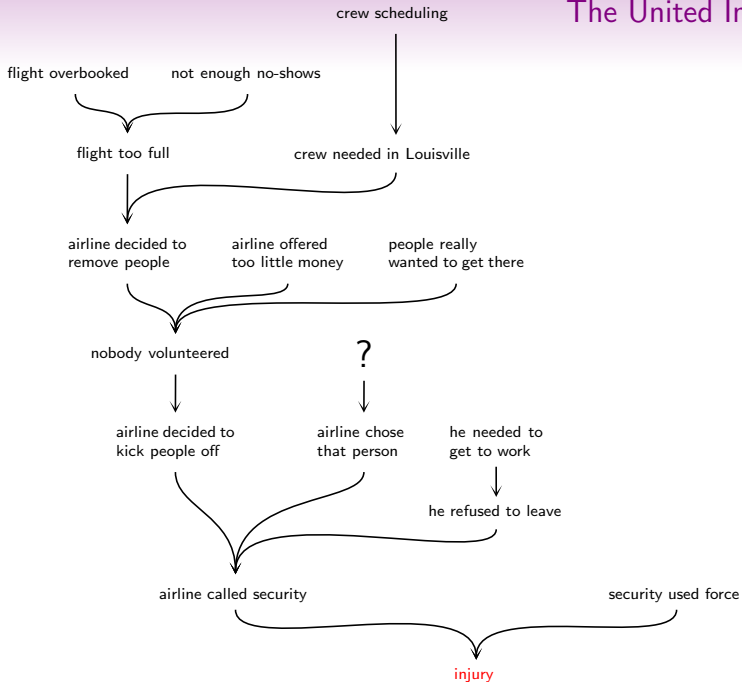
The United Incident



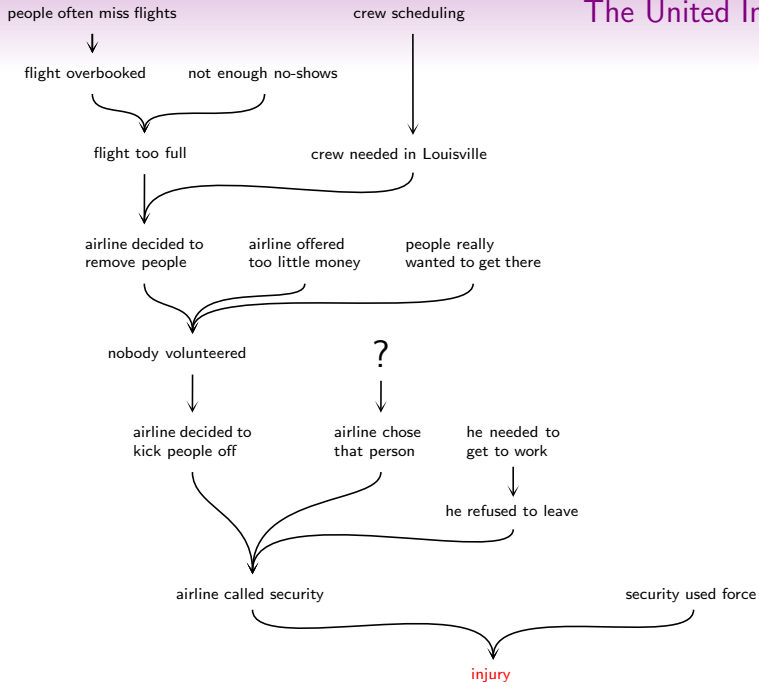
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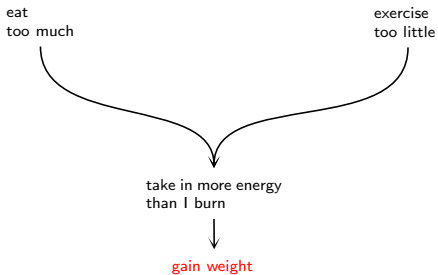


gain weight

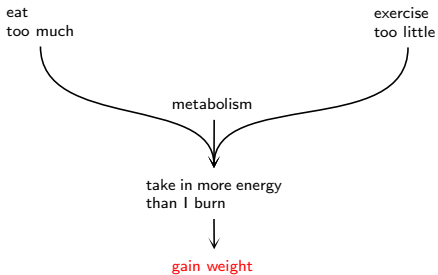
take in more energy
than I burn



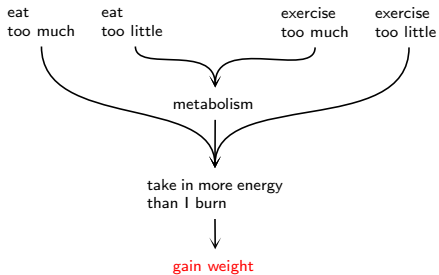
gain weight



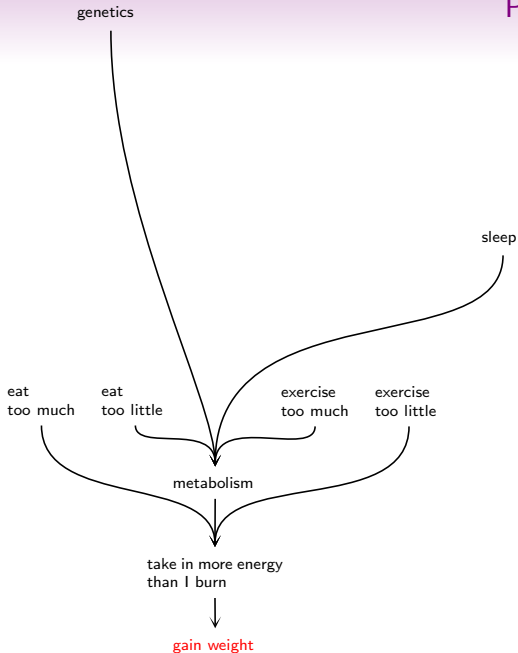
Putting On Weight



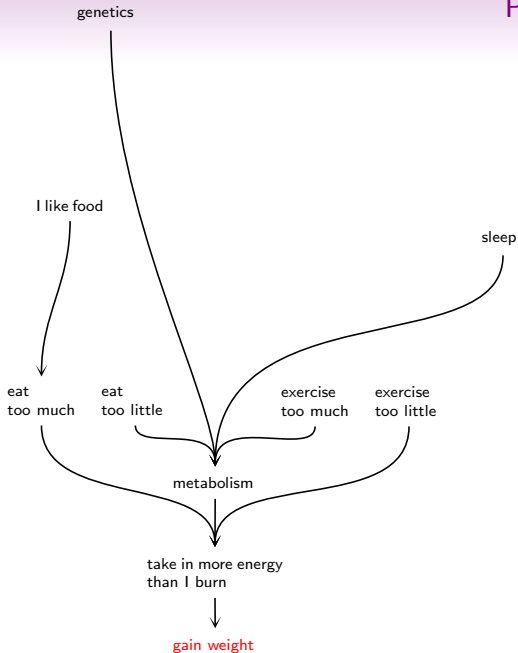
Putting On Weight



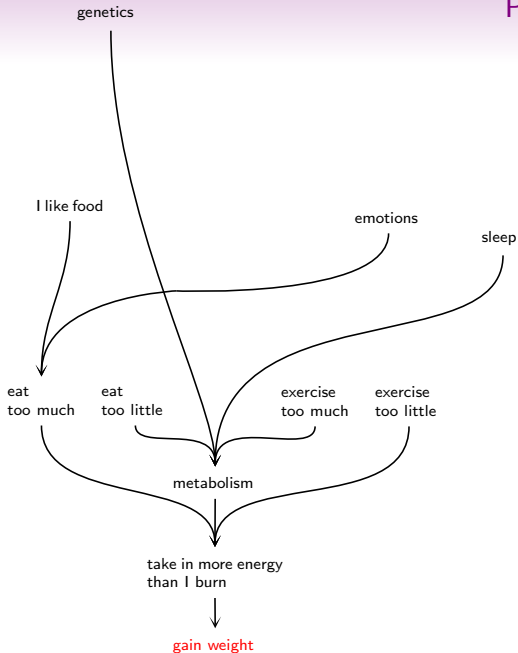
Putting On Weight



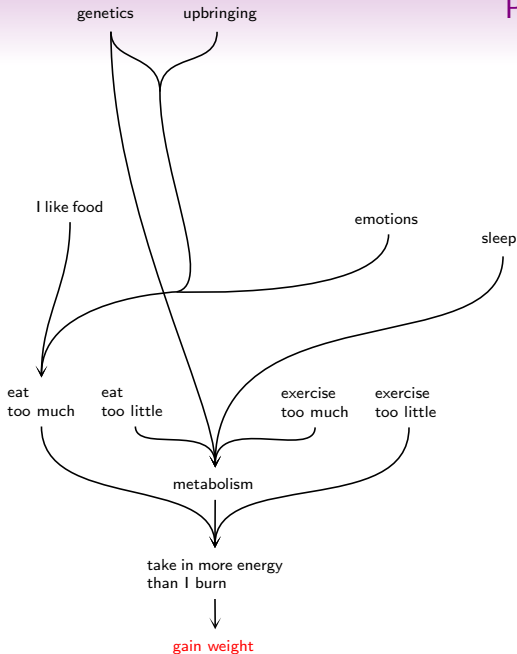
Putting On Weight



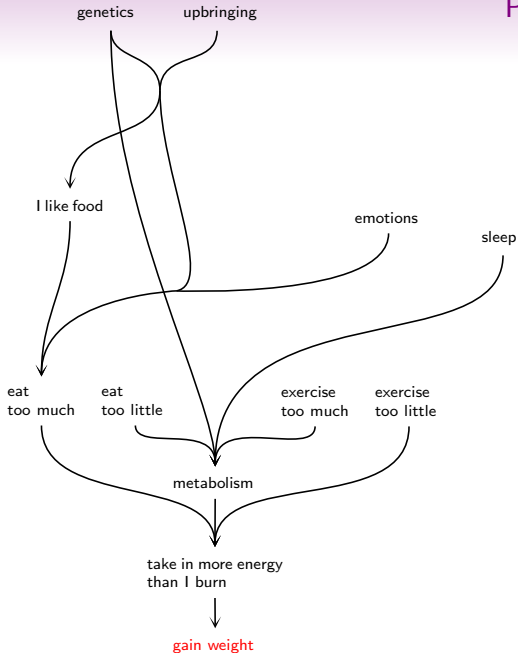
Putting On Weight



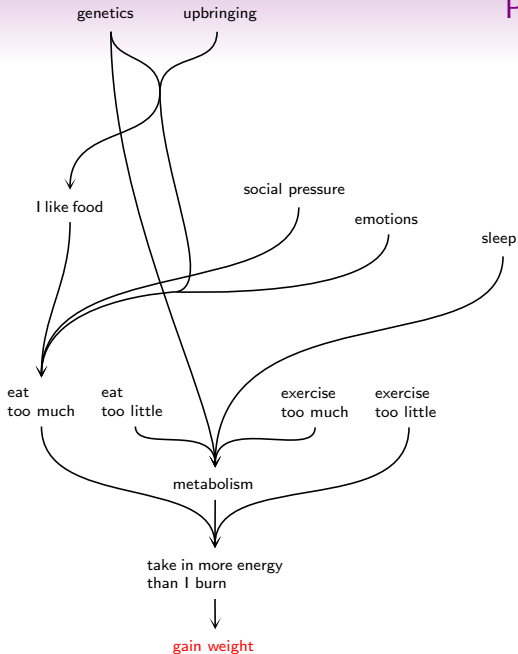
Putting On Weight



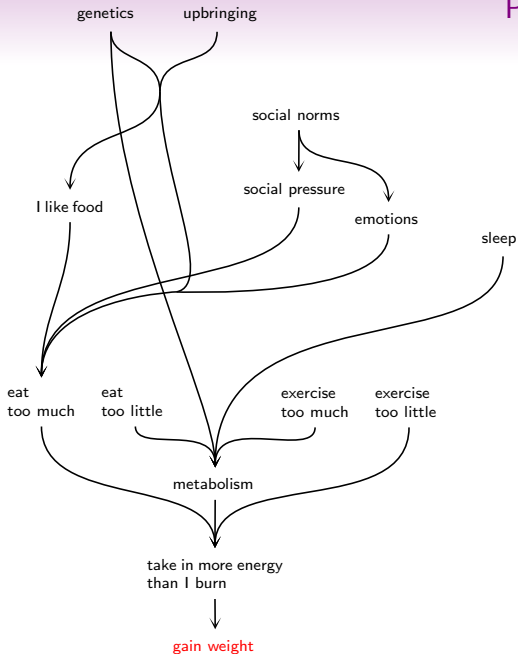
Putting On Weight



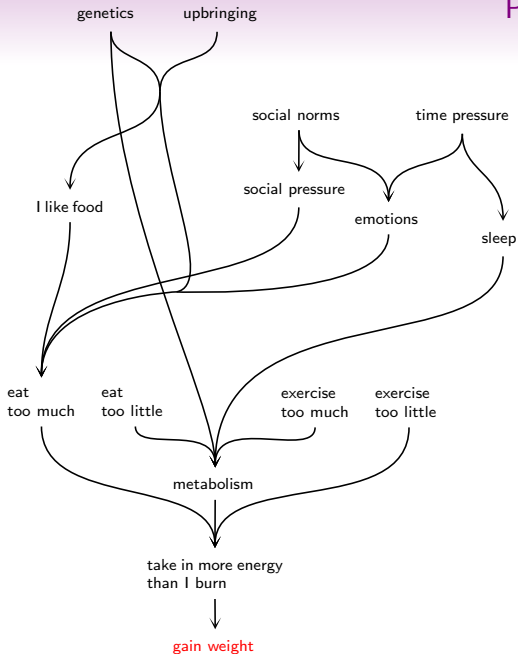
Putting On Weight



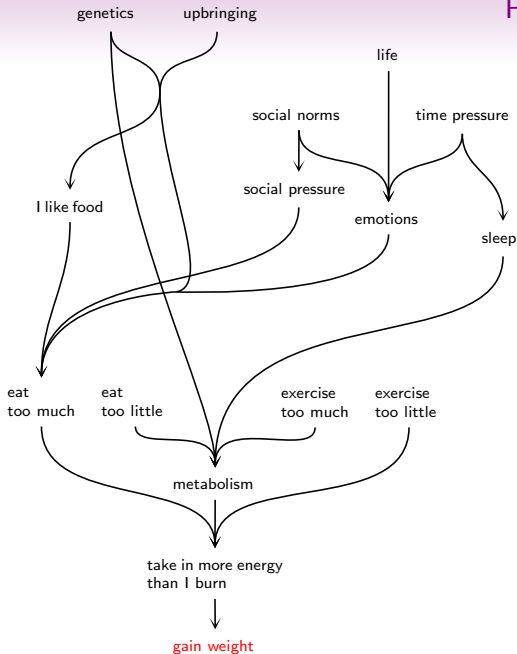
Putting On Weight



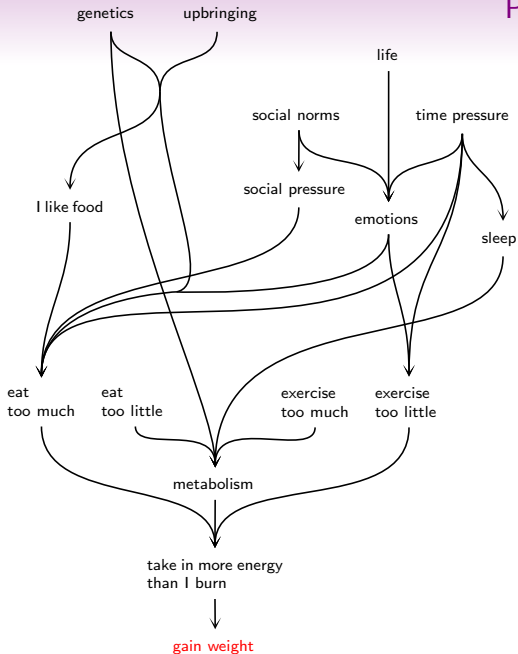
Putting On Weight



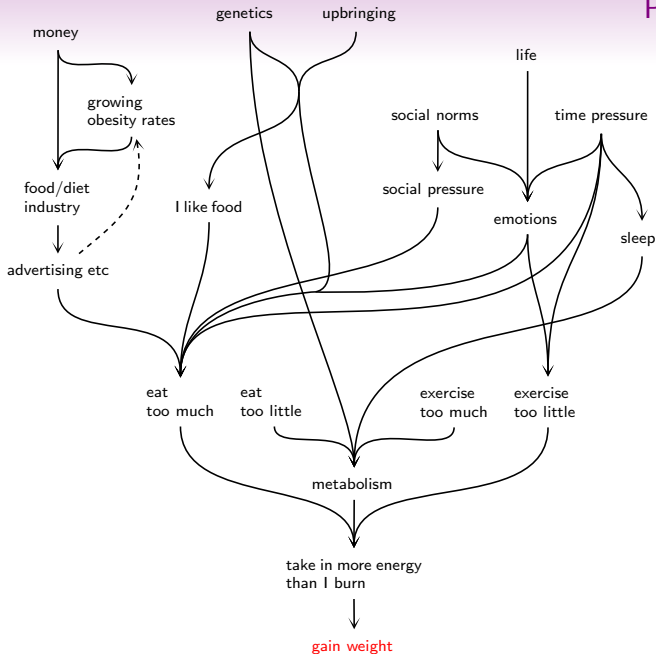
Putting On Weight



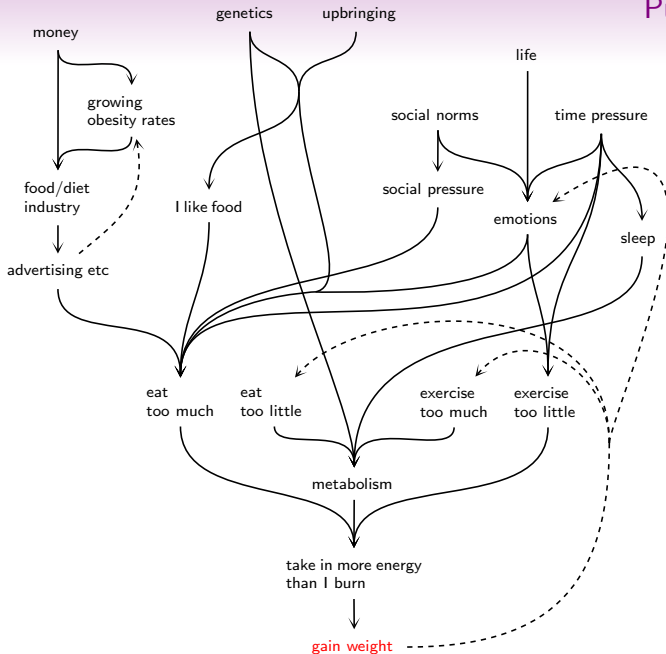
Putting On Weight



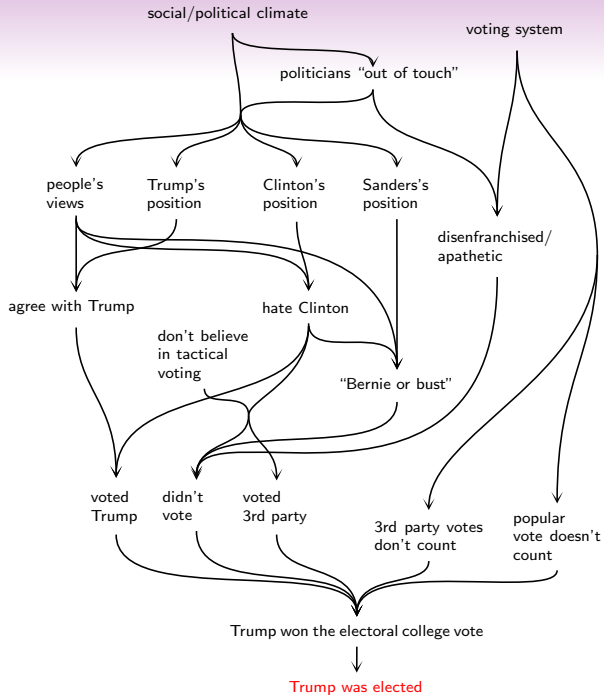
Putting On Weight



Putting On Weight



The 2016 US Election



Lessons from category theory

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 . . . but casts a wider light on it.**