Stochastic Methods for Finance

4th Report

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BLOCK

1. Goal

The following report is divided into three parts:

-Part 1 aims at displaying the surfaces of the Greeks for different underlying S, maturity T and volatility sd, with fixed strike K = 100 and interest rate r = 0.01;
-In Part 2 we apply the same procedure to a specific asset which does not pay dividends;
-In Part 3 we choose the same asset, estimate the historical volatility and we price with the Black-Scholes formula the ATM calls for the maturity chosen in Part 2.

2. Overview and statistics on the company: Block Inc.

Block Inc. (SQ), formerly Square, is a financial services and online payments company that provides hardware and software tools to help businesses run and grow their operations. Among Block's primary products are a point-of-sale hardware system, software for payments processing and analytics, Square Debit Card (a free business debit card), and Cash App (a peer-to-peer (P2P) payment service). The company also owns a majority in music-streaming service TIDAL and has recently launched initiatives to promote access to Bitcoin and other blockchain technologies

Block faces a broad array of small and large competitors such as business software companies, payroll processors, payment terminal vendors, and banks. They include PayPal Holdings Inc. (PYPL), Intuit Inc. (INTU), Shopify Inc. (SHOP), and venture capital startup ShopKeep.

Block, Inc. was incorporated in 2009 and is based in San Francisco, California.

The corporation counts 8.52K employees. Looking at the major holders, of the shares are held by All Insider, while 61.53% are held by Institutions. Morgan Stanley with 26,652,816 shares and Vanguard Group, Inc. (The) 25,189,188 are the top institutional holders.
Block has a Market Cap of $84.22B and it has an Enterprise Value of $81.59B. As regard the income statement the Revenue (ttm) is $17.66B, the Gross Profit (ttm) $4.42B and the Net Income $166.28M.
Analysing the profitability and management effectiveness of the company there is a Profit Margin of 0.94% and a Return on Assets and Return on Equity of 1.22% and 5.30% respectively. The Beta (5Y Monthly) value is 2.31.
Today, April 7, 2022- 6:00 pm Rome GMT+01:00, the price of an option is $128.93.
At the open it was $128.00 and at the previous close the price was $128.77.
The day's range settles between $120.12 and $132.80, while the 52 week's one ranges between $82.72 and $289.23. The Forward Dividend & Yeld and the Ex-Dividend are not provided.

3. Part 1

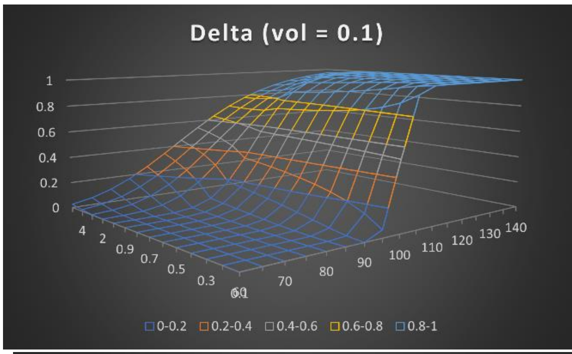
In order to compute the different Greeks, I use in the correspondent formula the following variables:

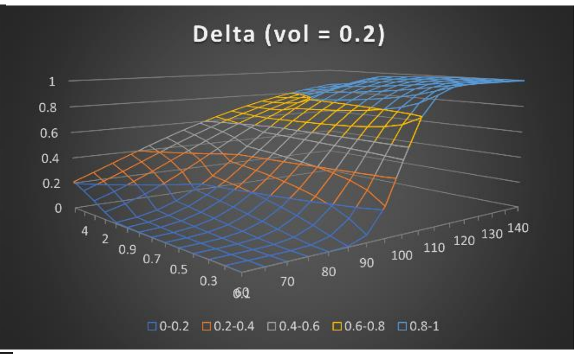
S = 60, 65, 70…140
K = 100
T = 0.1, 0.2, 0.3, 0.4…1, 2, 3, 4, 5
sd = 0.2, 0.3, 0.1
r = 0.01
d (Dividend Yield) = 0 (since the asset does not pay dividends)

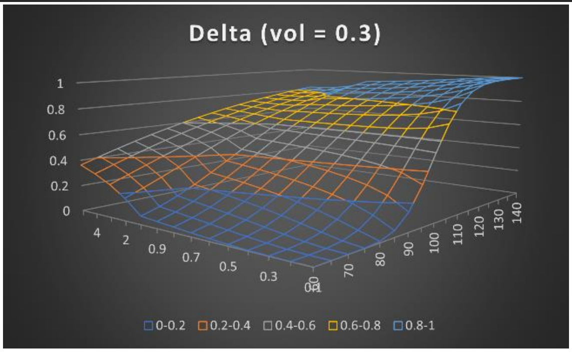
3.1 Delta

Delta is a measure of the change in an option's price (the premium of an option) resulting from a change in the underlying security. The value of delta ranges from -100 to 0 for puts and 0 to 100 for calls (-1.00 and 1.00 without the decimal shift, respectively). Call options have a positive relationship with the price of the underlying asset. If the underlying asset's price rises, so does the call premium, provided there are no changes in other variables such as implied volatility or time remaining until expiration. We will assess how changes in implied volatility and maturity will affect the Delta.
This Greek is commonly used when determining the likelihood of an option being in-the-money at expiration.
Traders can use delta to measure the directional risk of a given option or options strategy. Higher deltas may be suitable for higher-risk, higher-reward strategies that are more speculative, while lower deltas may be ideally suited for lower-risk strategies with high win rates.

Positive deltas are long (buy) market assumptions, negative deltas are short (sell) market assumptions, and neutral deltas are neutral market assumptions. When you buy a call option, you want a positive delta since the price will increase along with the underlying asset price. When you buy a put option, you want a negative delta where the price will decrease if the underlying asset price increases







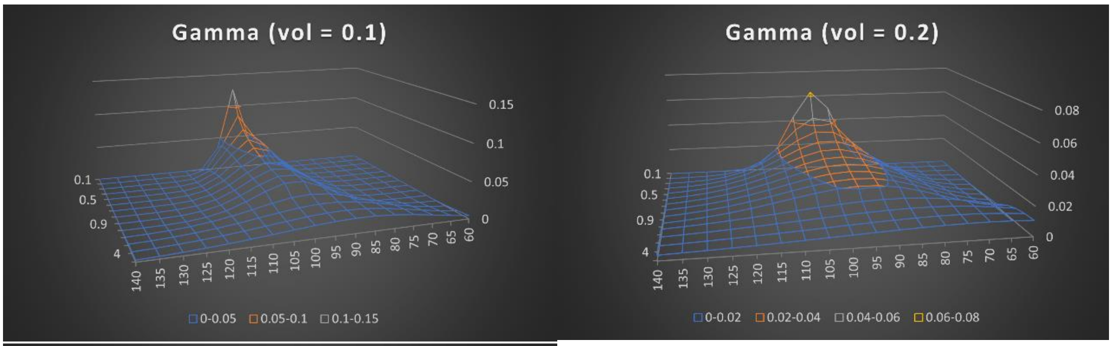
(Fig 3.1 a-b-c)

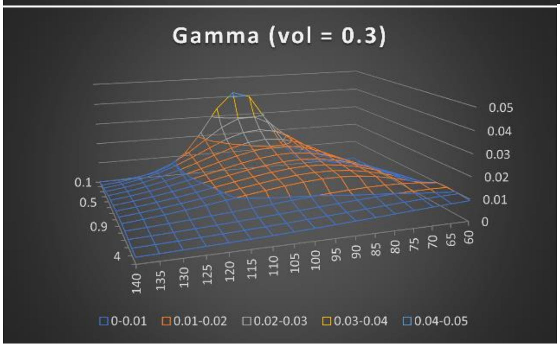
An ATM option (in our case 100) has a delta value of approximately 50 (0.5). That means the price will rise or fall by half a point with a one-point move up or down in the underlying. Delta changes as the options become more profitable or in-the-money. In-the-money means that a profit exists due to the option's strike price being more favorable to the underlying's price. As the option gets further in the money, delta approaches 1.00 on a call. We can see that clearly in the fig 3.1a-b-c , where we also notice that a smaller volatility results in a steeper increase of the delta for S greater than 100. A change in maturity does not affect too much the value of delta.

3.2 Gamma

Gamma measures the rate of changes in delta over time. Since delta values are constantly changing with the underlying asset's price, gamma is used to measure the rate of change and provide traders with an idea of what to expect in the future. Gamma values are highest for at-the-money options and lowest for those deep in- or out-of-the-money. When call options are deep out-of-the-money, they generally have a small delta because changes in the underlying generate tiny changes in pricing. While Delta changes based on the underlying asset price, gamma is a constant that represents the rate of change of delta. This makes gamma useful for determining the stability of delta, which can be used to determine the likelihood of an option reaching the strike price at expiration.

The option with the higher gamma will have a higher risk since an unfavourable move in the underlying asset will have an oversized impact. High gamma values mean that the option tends to experience volatile swings, which is a bad thing for most traders looking for predictable opportunities. Gamma is positive for long options and negative for short options.





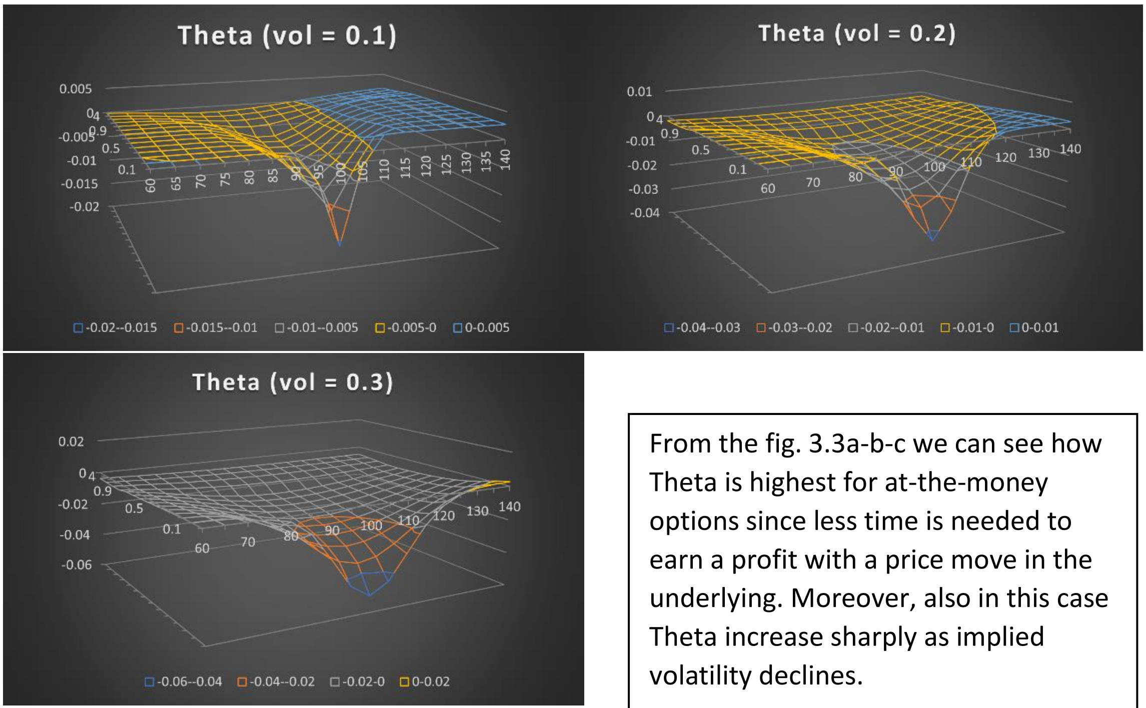
From the fig 3.2a-b-c we can appreciate how Gamma values are the highest for at-the-money options and lower for in- or out-of-the-money. Moreover, a smaller volatility results in a steeper increase of Gamma for ATM options.

(Fig. 3.2 a-b-c)

3.3 Theta

Theta measures the rate of time decay in the value of an option or its premium. Time decay represents the erosion of an option's value due to the passage of time. As time passes, the chance of an option being profitable or in-the-money lessens. Time decay tends to accelerate as the expiration date of an option draws closer because there's less time left to earn a profit from the trade.

Theta is always negative for a single option since time moves in the same direction. As soon as an option is purchased by a trader, the clock starts ticking, and the value of the option immediately begins to diminish until it expires. For this reason, Theta values are always negative for long options and will always have a zero time value at expiration. Theta is good for sellers and bad for buyers. A good way to visualize it is to imagine an hourglass in which one side is the buyer, and the other is the seller. The buyer must decide whether to exercise the option before time runs out. But in the meantime, the value is flowing from the buyer's side to the seller's side of the hourglass. The movement may not be extremely rapid, but it's a continuous loss of value for the buyer.

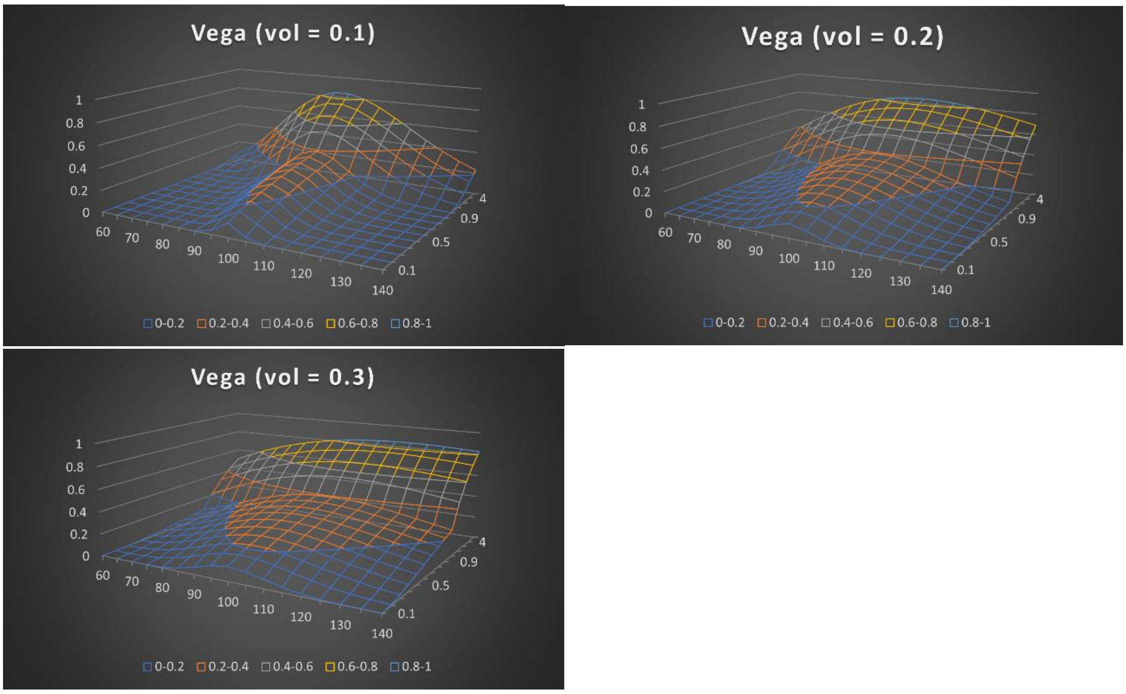


(Fig 3.3 a-b-c)

From the fig. 3.3a-b-c we can see how Theta is highest for at-the-money options since less time is needed to earn a profit with a price move in the underlying. Moreover, also in this case Theta increase sharply as implied volatility declines.

3.4 Vega

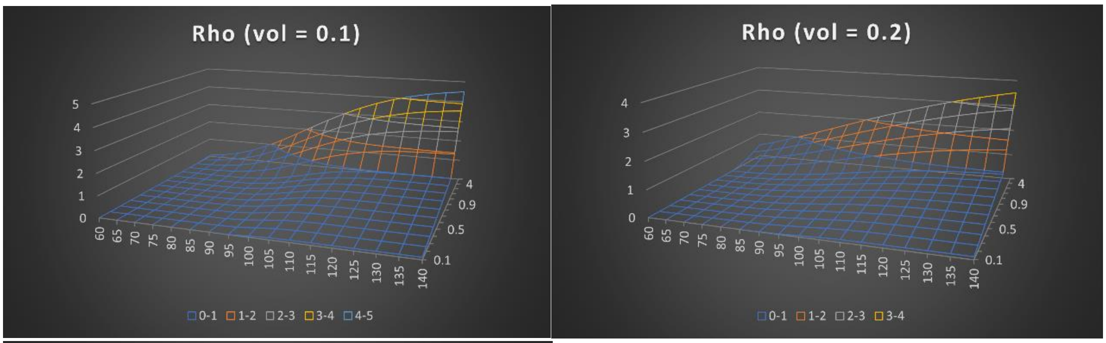
Vega measures the risk of changes in implied volatility or the forward-looking expected volatility of the underlying asset price. While delta measures actual price changes, Vega is focused on changes in expectations for future volatility.
Higher volatility makes options more expensive since there's a greater likelihood of hitting the strike price at some point.
Vega tells us approximately how much an option price will increase or decrease given an increase or decrease in the level of implied volatility. Option sellers benefit from a fall in implied volatility, but it is just the reverse for option buyers.
It's important to remember that implied volatility reflects price action in the options market. When option prices are bid up because there are more buyers, implied volatility will increase.
Long option traders benefit from pricing being bid up, and short option traders benefit from prices being bid down. This is why long options have a positive Vega, and short options have a negative Vega.
Vega can increase or decrease without price changes of the underlying asset, due to changes in implied volatility; it can increase in reaction to quick moves in the underlying asset. Vega falls as the option gets closer to expiration.

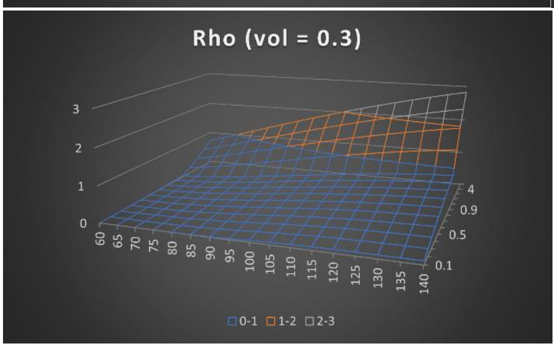


(Fig 3.4 a-b-c)

3.5 Rho

Rho represents the rate of change between an option's value and a 1% change in the interest rates. This measures sensitivity to interest rates. Assume a call option has a rho of 0.05 and a price of $1.25. If interest rates rise by 1%, the value of the call option will increase to $1.30, all else being equal. The opposite is true for put options. Rho is greatest for at-the-money options with long times until expiration.





In the Fig.3.5 a-b-c we can appreciate how Rho increases with maturity. As implied volatility increases, we can notice a slight sharpening in the growth of Rho with respect to S.

(Fig.3.5 a-b-c)

4. Part 2

For Part 2 we apply the same procedure to an asset which does not pay dividends, I chose Block Inc.

K/T	05.06.22	07.15.22	09.16.22	01.20.23	06.16.23
70	167.38%	100.57%	87.51%	82.04%	76.48%
90	124.24%	85.83%	78.22%	73.79%	70.22%
95	116.02%	85.22%	77.01%	71.84%	67.98%
100	111.06%	83.12%	74.84%	70.55%	68.16%
105	104.24%	80.54%	73.49%	69.64%	66.37%
110	100.95%	77.42%	69.92%	67.69%	65.72%
115	95.85%	74.00%	69.55%	67.25%	64.81%
120	92.87%	72.88%	68.46%	65.89%	64.11%
125	89.66%	72.14%	67.25%	65.28%	63.31%
130	87.02%	70.53%	66.34%	64.44%	62.44%
135	84.02%	70.31%	65.39%	63.68%	62.01%
140	84.28%	68.48%	65.20%	63.02%	61.32%
145	82.58%	68.21%	63.87%	62.15%	60.61%
150	81.18%	67.68%	63.48%	61.74%	59.97%
155	80.62%	66.70%	62.68%	61.20%	59.60%
160	80.52%	66.17%	62.20%	60.26%	58.94%
165	79.39%	65.00%	61.57%	60.05%	58.47%
170	79.59%	65.09%	61.35%	59.57%	58.15%
175	78.91%	64.53%	60.88%	58.94%	57.69%
180	79.15%	64.12%	60.53%	58.56%	57.25%
185	79.35%	63.94%	60.26%	58.25%	56.98%
190	79.69%	63.78%	60.11%	57.97%	56.65%
195	79.93%	63.48%	59.67%	57.64%	56.20%
200	80.37%	63.38%	59.29%	57.45%	56.01%

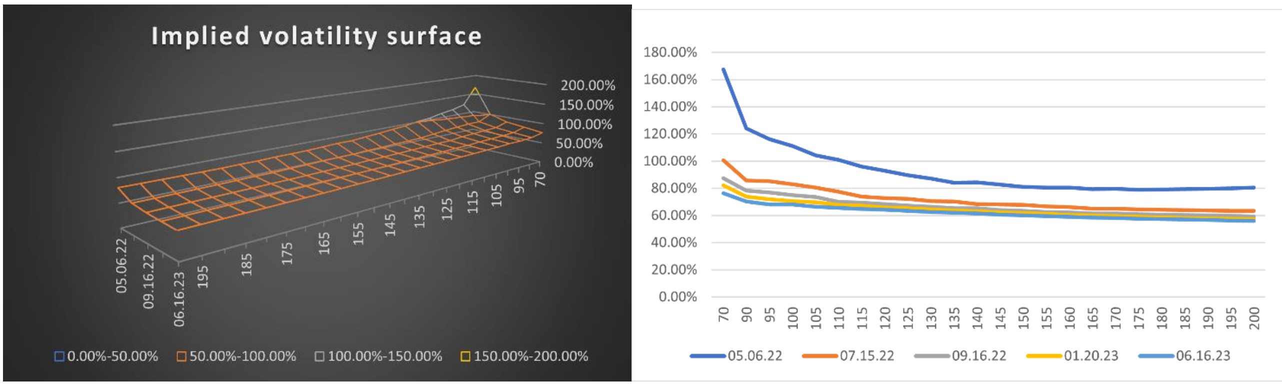
This is the table of the implied volatility as function of the strikes K and maturity T.

In fig.5a is possible to observe the implied volatility surface, where we can appreciate a slight increase in the volatility for strikes under 130 (K ATM). In the fig. 5b, this small increase translates in a mild **smile**, more pronounced on the left side.

CallRho(StockPrice As Double, StrikePrice As Double, TimeToExpiration As Double, Volatility As Double, RiskFreeRate As Double, DividendYield As Double)

I know calculate the Greeks, using as input variables:

S = 70, 90, 70…200
K(ATM) = 130
T = 1/12, 3/12, 1/2, 9/12, 1
Vol (from the table, as function of K and T)
r (Risk free rate, from the Spot rate US)
d (Dividend Yield) = 0 (since the asset does not pay dividends)



(Fig 5a-b)

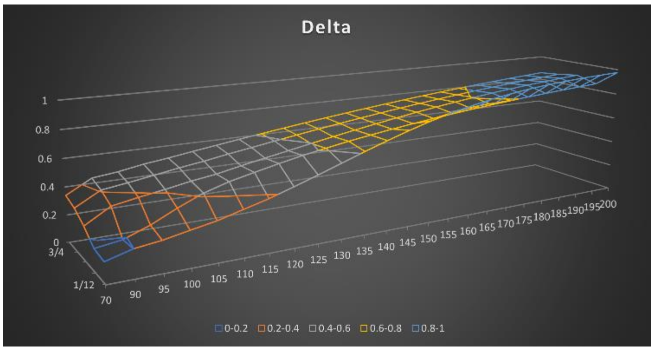


Fig. 5c

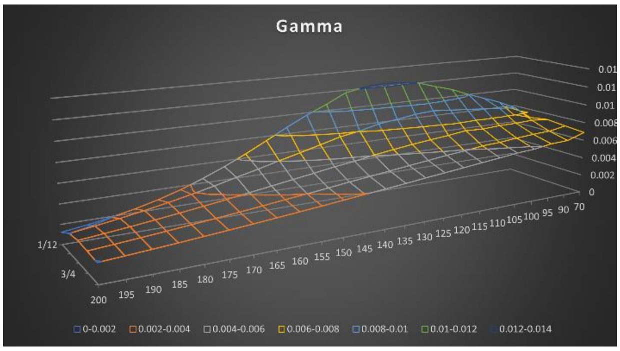


Fig. 5d

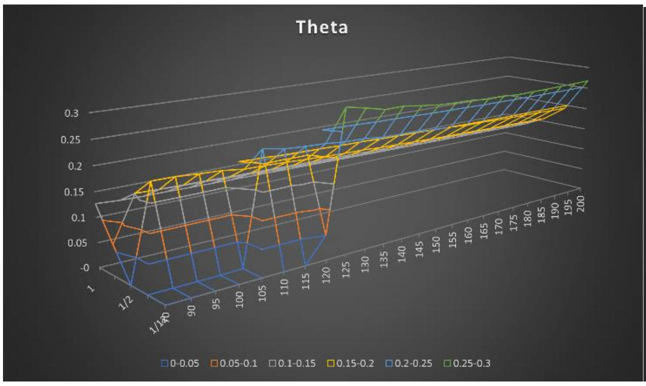


Fig. 5e

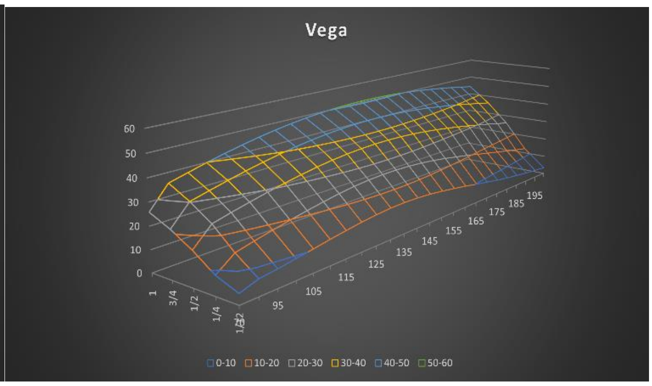


Fig. 5f

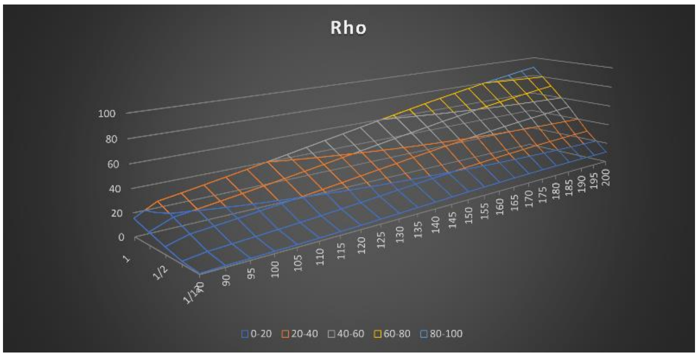


Fig. 5g

From fig.5c is possible to appreciate how as the call option goes further in the money (S > K ATM) **Delta** approaches 1. The surface is also coherent with the fact that higher volatilities (such the ones obtained) correlate with less steeper increments of the Greek, which is in fact quite smooth.

From **Gamma** 3D surface (fig. 5d) we can see the increase of the Greek value as the underlying gets closer to the strike ATM (130).

In the fig. 5f and fig. 5g we can see how both **Vega** and **Rho** grows with maturity.

5. Part 3

In the third part we choose the same asset as before and we estimate the historical volatility. To do so I download the historical data from Yahoo Finance and I then compute the daily sigma, namely the standard deviation of the daily returns, calculated for each maturity. These are the daily volatilities obtained:

05.06.22	07.15.22	09.16.22	01.20.23	06.16.23
sig_day_1m	sig_day_3m	sig_day_6m	sig_day_9m	simga_day_1y
0.025354696	0.027552318	0.02717519	0.026904172	0.041946016

From these values I calculate the correspondent yearly volatilities for each maturity T, by multiply each sigma daily for $\sqrt{252}$ (the opening days in the exchange for one year).

	05.06.22	07.15.22	09.16.22	01.20.23	06.16.23
sigma_yearly	0.402493323	0.437379482	0.431392771	0.427090485	0.665872369

I can now use these yearly volatilities in the Black-Scholes formula, in order to price the ATM call options for the maturities chosen in the second part.

BSCall(Stock As Double, Exercise As Double, Rate As Double, Sigma As Double, Time As Double) As Double

Where the following variables are used as input:

S = 128.93
K (ATM) = 130
r is the *risk-free rate* used before (different for every maturity)

	05.06.22	07.15.22	09.16.22	01.20.23	06.16.23
risk	0.9617%	0.9724%	1.3680%	1.7329%	2.0182%

σ_y from the table above
T = 1/12, 3/12, 1/2, 3/4, 1

These are the prices we estimated, compare to the Call Mid Prices quoted in the market at each maturity:

	05.06.22	07.15.22	09.16.22	01.20.23	06.16.23
BS Call Price	5.524728848	10.88571775	15.5487493	19.17787589	34.19684972
Market Call Price	8.58	15.4	19.25	24.75	31.78

Overall, the estimations are not too satisfying, except price for T = 1, which is quite close.
The imprecisions may be due to a not very accurate correspondence between the maturity T and the period of time taken effectively into consideration, and other factors not taken into account.

Finally, we analyse the difference between the quoted ATM implied volatility and the historical volatility, by visualising them:

	05.06.22	07.15.22	09.16.22	01.20.23	06.16.23
Quoted ATM implied volatilty (130)	0.87	0.71	0.66	0.64	0.62
sigma_year	0.402493323	0.437379482	0.431392771	0.427090485	0.665872369