Stochastic Methods for Finance

6th Report

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1. Goals

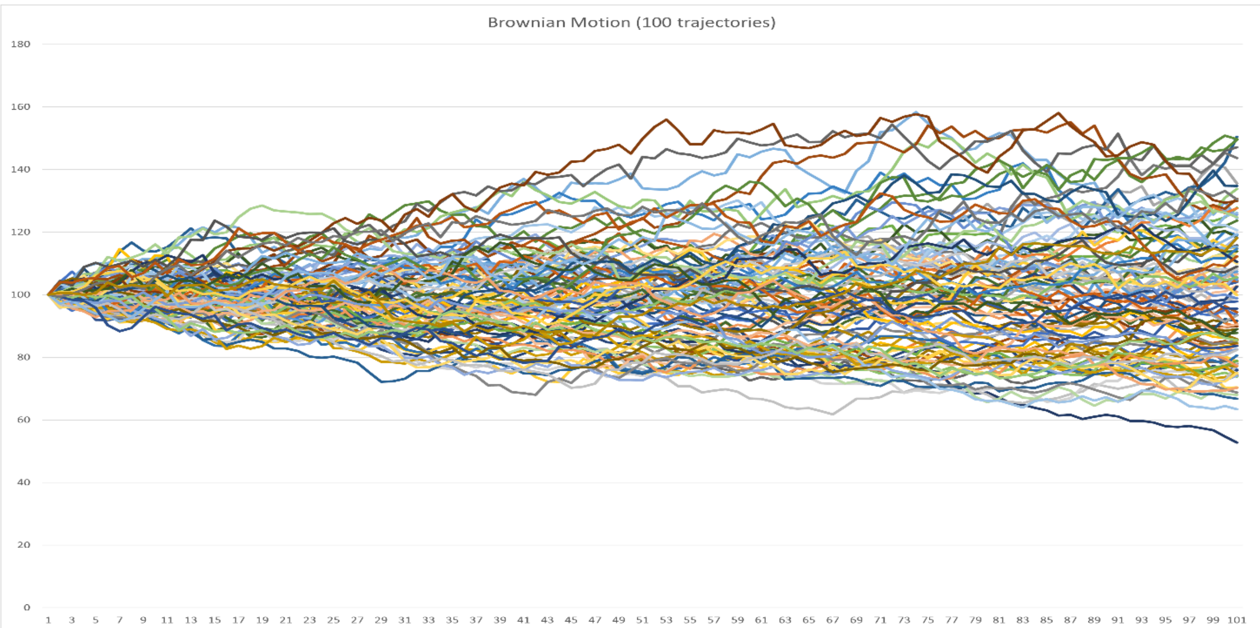
This report aims at:
- Building up a Dynamic Monte Carlo pricer for vanillas (Call/Put);
- Building up a Static Monte Carlo;
- Building up a pricer with the same procedure but using multiple step Euler-scheme based simulation;
- Building up another pricer this time applied to Asian Options.

2. Geometric Brownian Motion Simulation

First, I fix the following parameters for the BS market model:

S_0 = 100   r = 1%   K =99
σ = 20%   T = 1 y   dt (time step in the discretisation) = 1 day

Through a VBA code that takes the variables above as input, I simulate 100 trajectories, starting from S_0 = 100. This is the figure obtained.



3. Monte Carlo Dynamic and Static price with GBM

I first set the benchmark by computing the Black-Scholes prices for Call and Put

BS_Call	BS_Put
8.918504	6.93343796

For the MC Dynamic price, starting from the GBM generated, I take the 100th value (S_{100}) of each trajectory and calculate the payoff for Call and Put as follow:

$$\text{Payoff (Call)} = (S_T - K)^+ \qquad \text{Payoff (Put)} = (K - S_T)^+$$

I then discount the payoff obtained by e^{-rT}.
The Call and Put prices are given by the mean of the correspondent discounted payoffs.

MC dynamic price	
CALL	PUT
10.04309	8.24934398

For the MC static case I consider 501 values simulated with a single step (S_1) and I calculate the Call and Put prices in the same manner as before.

MC static price	
CALL	PUT
8.402078	5.70197746

4. Euler-scheme based Simulation

In this case I compute the Put and Call prices in the same way as in the previous point but using the multiple step Euler-scheme based simulation with N = 501 trajectories.
To generate the values for the Brownian Motion I use the formula:

$$S_t = S_{t-1} + S_{t-1} * \sigma * \sqrt{dt} * (Z_k / * \sqrt{100})$$

Where σ is the volatility, dt is the time-step for the discretisation, Z_1, Z_2... is a sequence of independent standard normal random variables and 100 is the number of steps = k

I calculate the Call and Put prices with the same procedure, and these are the prices obtained respectively for the multiple and single step case:

Prices with Euler (D)	
Call price	Put price
8.594838	6.913903

Prices with Euler (S)	
Call price	Put price
8.350058	7.643325

5. Asian Options

I calculate the Call and Put prices for Asian Options where the payoff is given by:

Payoff (Call) $=(\sum S_t/N - K)^+$ Payoff (Put) $=(K - \sum S_t/N)^+$

Asian Options (D)	
Call price	Put price
9.696638	8.270599

6. Conclusions

To better appreciate the comparison between the prices I compute the percent error of each one with respect to the benchmark, the Black-Scholes price.

$$\delta = \left| \frac{v_A - v_E}{v_E} \right| \cdot 100\%$$

δ = percent error

v_A = actual value observed

v_E = expected value

BS_Call	8.918504
BS_Put	6.933438

	Price	Percent error
MC dynamic Call price	10.04309	12.61%
MC dynamic Put price	8.249344	18.98%
MC static Call price	8.402078	5.79%
MC static Put price	5.701977	17.76%
Call price with Euler (D)	8.594838	3.63%
Put price with Euler(D)	6.913903	0.28%
Call price with Euler (S)	8.350058	6.37%
Put price with Euler(S)	7.643325	10.24%
Asian Call Option price	9.696638	8.72%
Asian Put Option price	8.270599	19.29%