Stochastic Methods for Finance

3rd Report

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March 31st, 2022

1. Goal

The aim of this report is comparing three different models, namely the **Binomial model**, the **Black-Scholes model**, and the **Leisen-Reimer model**. All three are used to price Call options.
For the purpose of this project, we set an asset *S = 100*, an interest rate *r = 1%*, a volatility *sd = 20%*, a maturity *T = 1* and a *strike ATM = 100*

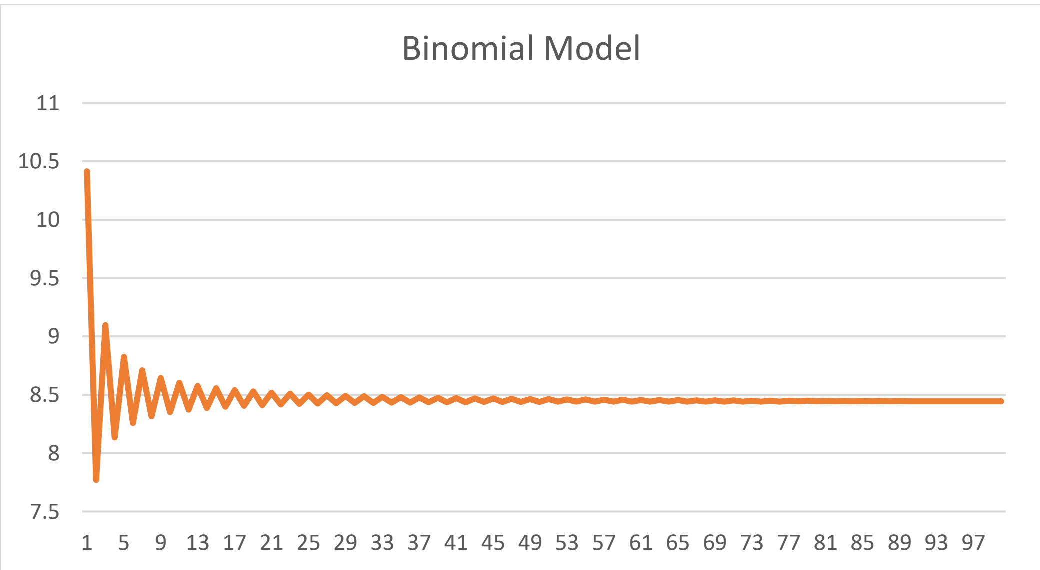
2. Binomial Model

The Binomial Model function:

Bi_Call_Eur(s, x, t, r, sd, n As Integer)

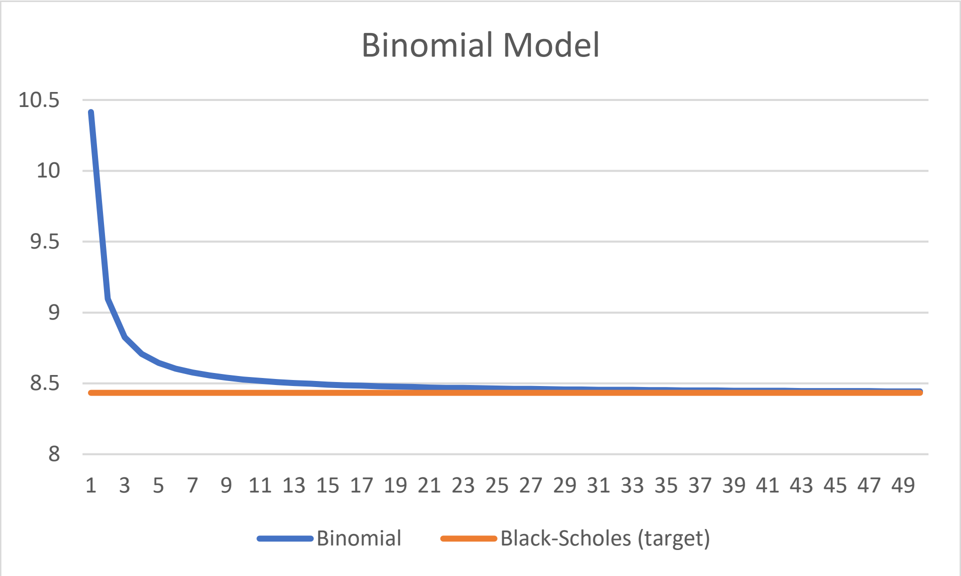
takes as input S = 100, the strike x = 100, T = 1, r = 0.01, sd = 0.2
and a number of steps n = 100

The Call price obtained is 8.44305.



(2a)

From the plot we can see how the curve oscillates, with value quite inaccurate during the first 10 steps, before stabilizing on the value 8,44 around the 74^{Th} iteration.
Since the curve is oscillating, I decide to isolate the upper part, taking only the odd steps, to better appreciate the convergence to the value.



(2b)

3. Black-Scholes Model

The Black-Scholes Model is a mathematical go-to model, that presents a theoretical estimate of the price of European-style options independently of the risk of the underlying security. The price obtained through this model will be the target value to perform our convergence analysis.
The Black-Scholes formula:

BSCall(Stock As Double, Exercise As Double, Rate As Double, Sigma As Double, Time As Double)

takes as input the same variables as before: Stock = 100, Exercise = 100, Rate = 0.01, Sigma (volatility) = 0.20 and Time = 1

The Call price estimated by the model is 8.43331869 and it will be the target price for the next comparisons.

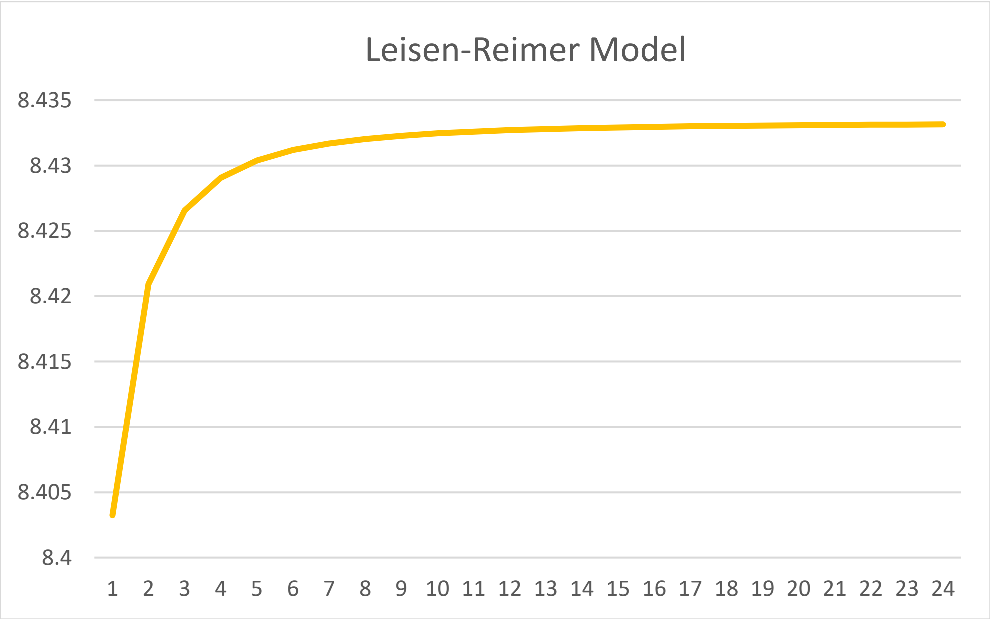
4. Leisen-Reimer Model

The Leisen Reimer model was introduced by Dietmar Leisen and Matthias Reimer in 1995. Its main benefit is greater precision with smaller number of steps. Generally, all binomial models become more precise (converge to continuous model solutions like Black-Scholes) with growing number of steps, as the duration of each step becomes shorter. This convergence is not always smooth. In practical use, where higher number of steps requires more computing resources, desired qualities of a binomial model are for the convergence (precision improvement) to be fast and smooth. This is where Leisen-Reimer model provides a good solution

The formula:

LeisenReimerTrunc(AmeEurFlag As String, CallPutFlag As String, s As Double, x As Double, t As Double, r As Double, b As Double, v As Double, n As Integer) As Variant

takes as input the following variables:
AmeEurFlag = “e” (since we are considering European options)
CallPutFlag = “c” (since we are pricing a Call option)
S = 100, x = 100, t = 1, r = 0.01, b (cost of carry, that in this case I can replace with r) = 0.01
V (volatility) = 0.20
Since the method takes only odd values for the number of steps (‘n-1’), for n = 1, the method is not applicable, therefore I start from n=3, up to n = 99.
The price obtained with the Leisen-Reimer model is 8.433278254 and it is very close (equal up to the third decimal place) to our target value provided by the Black-Scholes (8.43331869).



(4a)

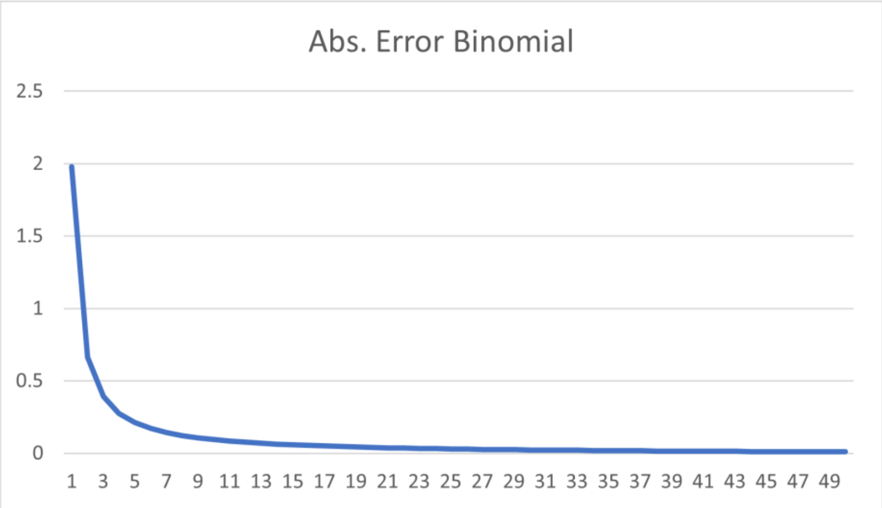
5. Comparison and Conclusion

5.1 Binomial Model vs Black-Scholes Model

To better appreciate the comparison between these two models I compute the *absolute error* between them, with the formula:

$$\text{Abs. error} = | X_i - X|$$

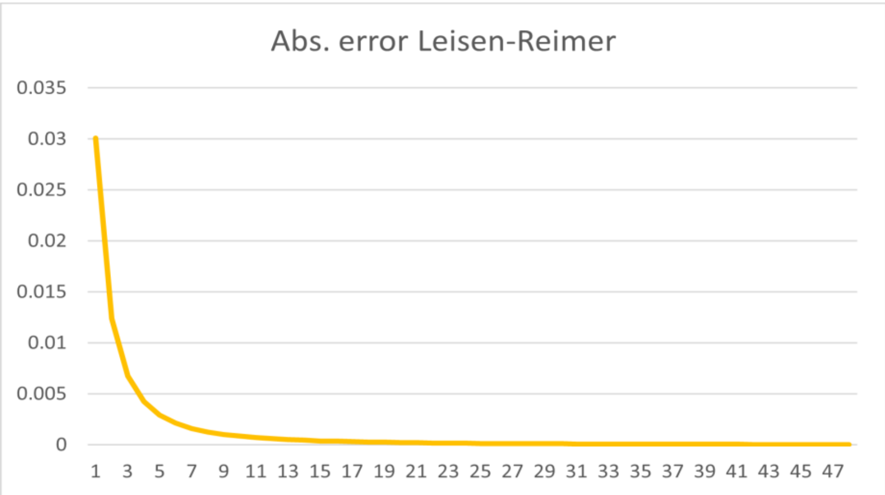
Where X_i is the target price from the Black-Scholes model (8.43331869) and X is the price from the Binomial Model (considering only the odd steps as we did before, up to n = 99).



After 99 iterations the Absolute error settles to 0.010038611.
The figure 5a shows the behaviour of the curve for the first 49 steps.

(5a)

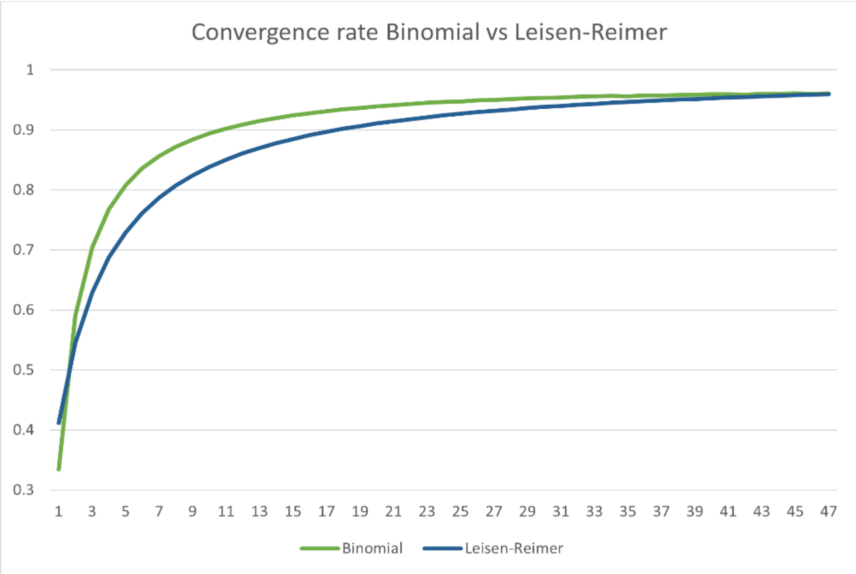
5.2 Leisen-Reimer Model vs Black-Scholes Model



I calculate in the same way as before the absolute error for the Leisen-Reimer estimation with respect to the target price given by the Black-Scholes. After 99 iterations the error settles to 0.000038862

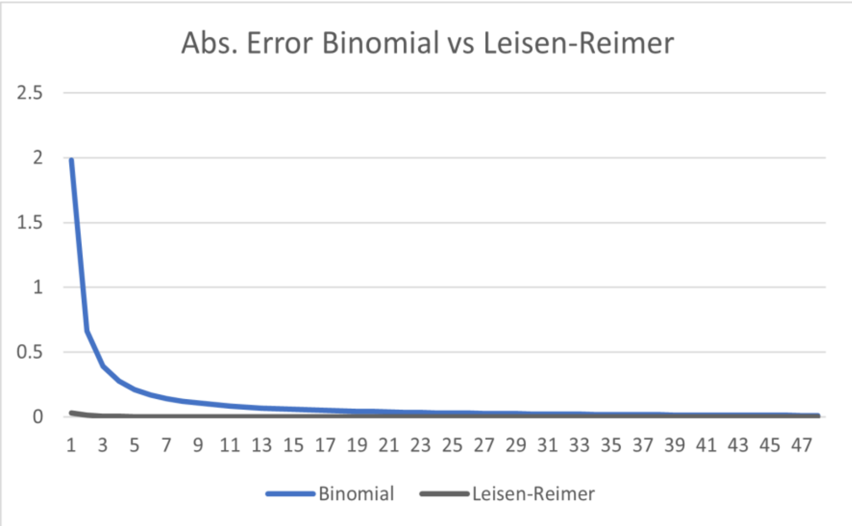
(5b)

5.3 Binomial Model vs Leisen-Reimer Model

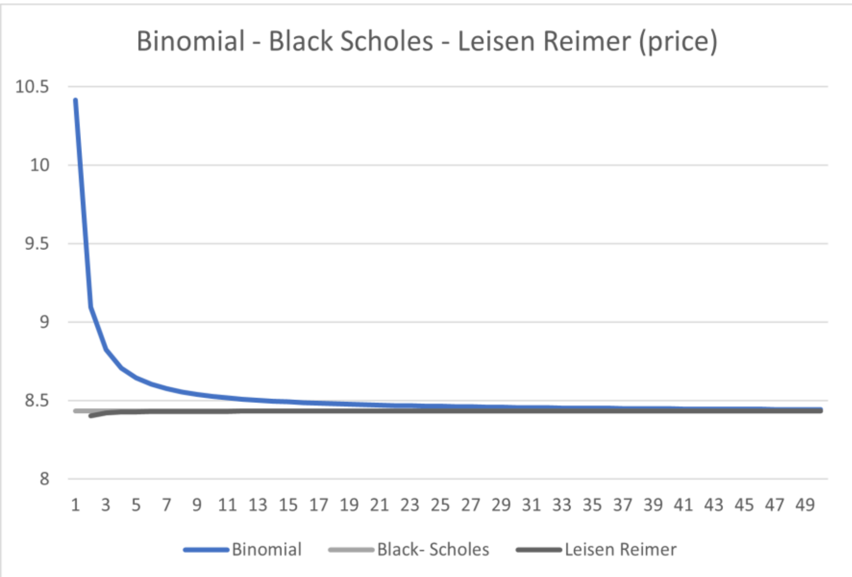


(5c)

The figure 5c shows the curve of the Binomial and Leisen-Reimer *convergence rate*, the measure of how fast the difference between the target (B-S price) and the estimate goes to zero. The convergence rate is the ratio between the error at a given step and the error at the previous one.



(5d)



(5e)

Comparing the absolute errors, the Leisen-Reimer model provides a substantially better error than the simple Binomial with the same number of steps (0.000039 against 0.010, 258 times better!). Nonetheless, the result that strikes the most is that to outperform the precision of the Binomial after 99 steps, the Leisen-Reimer only needs 5.

This is also shown in the fig.5e where it is clear how faster the Leisen-Reimer model converges to the target value.

In the lights of these results, we can safely affirm the superiority of the Leisen-Reimer performance over the simple Binomial model.