Stochastic Methods for Finance

5th Report

Giovanni Dal Mas

2053346

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1. Goal

This report aims at calculating the Value at Risk (VaR) using three different approaches:

-The **Variance-Covariance Method**
-The **Monte Carlo Simulation**
-The **Historical Simulation**
And checking in all cases the **additivity** of the VaR

2. Overview and statistics of the companies: Booking Holdings Inc. and Edwards Lifesciences Corporation

Booking Holdings Inc. provides travel and restaurant online reservation and related services worldwide. The company operates Booking.com, which offers online accommodation reservations, Rentalcars.com that provides online rental car reservation services and Priceline, Agoda, KAYAK, OpenTable among others. The company, formerly known as The Priceline Group Inc, was founded in 1997 and is headquartered in Norwalk, Connecticut.
The corporation counts 20.097K employees. Looking at the major holders, 0.23% of the shares are held by All Insider, while 93.84% are held by Institutions. Vanguard Group, Inc. (The) with 3,143,232 shares and Blackrock Inc. with 2,608,920 are the top institutional holders.

Booking has a Market Cap of $85.776B and it has an Enterprise Value of $90.26B. As regard the income statement the Revenue (ttm) is $10.96B, the Gross Profit (ttm) $8.78B and the Net Income $1.16B.
Analysing the profitability and management effectiveness of the company there is a Profit Margin of 10.63% and a Return on Assets and Return on Equity of 7.26% and 21.05% respectively. The Beta (5Y Monthly) value is 1.22.
Today, May 3, 2022- 6:00 pm Rome GMT+01:00, the price of an option is $2103.33.
At the open it was $2088.00 and at the previous close the price was $2,105.93.
The day’s range settles between $1,998.07 and $2,109.47, while the 52 week’s one ranges between $1,796.45 and $2,715.66. The Forward Dividend & Yeld and the Ex-Dividend are not provided.

Edwards Lifesciences Corporation provides products and technologies for structural heart disease, and critical care and surgical monitoring in the United States, Europe, Japan, and internationally. The company distributes its products through a direct sales force and independent distributors. The corporation was founded in 1958 and is headquartered in Irvine, California.

Edwards Lifesciences Corporation counts 15.70K employees. Looking at the major holders, 0.76% of the shares are held by All Insider, while 86.03% are held by Institutions. Blackrock Inc. with 57,040,983 shares and Vanguard Group, Inc. (The) with 49,076,649 are the top institutional holders.
The company has a Market Cap of $67.55B and it has an Enterprise Value of $67.55B. As regard the income statement the Revenue (ttm) is $5.23B, the Gross Profit (ttm) $3.98B and the Net Income $1.5B.
Analysing the profitability and management effectiveness of the company there is a Profit Margin of 28.73% and a Return on Assets and Return on Equity of 12.60% and 28.88% respectively. The Beta (5Y Monthly) value is 1.15.
Today, May 3, 2022- 6:00 pm Rome GMT+01:00, the price of an option is $104.54.
At the open it was $105.91 and at the previous close the price was $105.93.
The day’s range settles between $103.62 and $109.18, while the 52 week’s one ranges between $87.32 and $131.73. The Forward Dividend & Yeld and the Ex-Dividend are not provided.

3. VaR

Value at Risk is a special type of downside risk measure. It has three components: a *time period*, a *confidence level* (typically either 95% or 99%), and *an estimate of investment loss* (expressed either in dollar or percentage terms). VaR makes a probabilistic estimate. With a given confidence level, it asks, "What is our maximum expected loss over a specified time period?", for example, “What is the most I can—with a 95% or 99% level of confidence—expect to lose in dollars over the next month?”. There are three methods by which VAR can be calculated: the Variance-Covariance Method, the Monte Carlo Simulation and the Historical Simulation.

4. The Variance-Covariance Method

This method assumes that stock returns are normally distributed. It only requires that we estimate two factors: an expected (or *average*) *return* and a *standard deviation* (volatility), which allow us to construct our normal distribution.

I start with a portfolio of $1 000 000 dollars equally balanced between my two assets. From Yahoo Finance I download the historical series of the past 6 months. I then divide the initial amount invested ($ 500 000 per asset) by the Adjust Close at T=0 ($2437.01 for Booking and $117.37 for Edwards Lifesciences) obtaining the number of shares per asset, respectively 205.17 and 4260.03. Knowing the number of shares, I can now construct the column of ending values of my portfolio, from which I deduce the daily returns:

$$(V_{n+1}-V_n)/V_n$$

V_{n+1} = value of the portfolio today
V_n = value of portfolio yesterday

The standard deviation of the daily returns of the past 6 months constitutes the *daily volatility* of my portfolio: $\sigma_p = 0.018352335$

I compute the daily volatility of the two assets in the same way from the daily returns of each stock during the same time period:

$$\sigma_{bkng} = 0.02825704 \quad \sigma_{ew} = 0.022882491$$

I can now calculate the Global VaR on the portfolio and the Joint VaR (the combination of the two single VaRs on the assets). I choose three confidence levels: 95%, 99%, 99.5%
At CL 95%:

$$\text{Global VaR} = \sigma_p * \sqrt{T} * 1000000 * 1.65$$

Where:
T (days horizon) = 1,...,100
1000000 is the notional
1.65 is the quantile for 95% CL
The other quantiles are 2.33 and 2.58, respectively for CL 99% and 99.5%

The single parametric VaRs at 95% CL for the two assets are computed through the formula:

$$\text{bkng\_VaR} = \sigma_{bkng} * \sqrt{T} * 500000 * 1.65$$
$$\text{ew\_VaR} = \sigma_{bkng} * \sqrt{T} * 500000 * 1.65$$

For CL 99% and 99.5% the quantiles are always 2.33 and 2.58

Below the tables of the VaR at (95%, 99%, 99.5%) with T= 1,...,100 days horizon

Global VaR	1	2	3	4	5	50	100
95%	30281.35	42824.3	52448.84	60562.71	67711.16	214121.5	302813.5
99%	42760.94	60473.1	74064.12	85521.88	95616.37	302365.5	427609.4
99.50%	47349.02	66961.63	82010.92	94698.05	105875.6	334808.2	473490.2

bkng_VaR	1	2	3	4	5	50	100
95%	23312.06	32968.23	40377.67	46624.12	52127.35	164841.1	233120.6
99%	32919.45	46555.14	57018.16	65838.9	73610.13	232775.7	329194.5
99.50%	36451.58	51550.32	63135.99	72903.16	81508.22	257751.6	364515.8

ew_VaR	1	2	3	4	5	50	100
95%	18878.05	26697.6	32697.75	37756.11	42212.61	133488	188780.5
99%	26658.1	37700.25	46173.19	53316.2	59609.33	188501.2	266581
99.50%	29518.41	41745.34	51127.39	59036.83	66005.18	208726.7	295184.1

joint N VaR	1	2	3	4	5	50	100
95%	42190.11	59665.83	73075.42	84380.23	94339.96	298329.2	421901.1
99%	59577.55	84255.38	103191.3	119155.1	133219.5	421276.9	595775.5
99.50%	65969.99	93295.66	114263.4	131940	147513.4	466478.3	659699.9

The Joint VaR is the sum of the two single VaRs on the assets and in this case, we say that the VaR is *subadditive*, since the VaR of the combination of the assets (Global VaR) is lower than the combination of VaRs (Joint VaR). For example, for a confidence level of 99% at T=1:

$$42760.94 < (32919.45 + 26658.1 = 59577.55)$$

Concretely this result tells us that it is better to consider our portfolio as a whole. In fact, we can exploit the *diversification effect*, due to the correlation between the two assets (*corr* = 0.374011519).

5. The Var-Cov Method using the Riskless Exponentially Weighted Moving Average (EWMA)

The main idea of the EWMA is that the volatility shouldn't be flat for all the daily returns within an historical window. The daily returns closer in time should have more weight and influence more the volatility and the variations more distant in time should have weights closer and closer to zero.

We first compute the daily volatility for the two assets, that we will then use to calculate the Joint VaR with the previous method.

We take the squared daily returns, and we multiply them for the column of the weights, computed as follow:

$W_{i-1} = 1 - \lambda = 1 - 0.94 = 0.06$ $W_{i-2} = 0.06 * 0.94 = 0.0564$ $W_{i-3} = 0.0564 * 0.94 = 0.053016$ and so on.

		BKNG	$U_i = (S_i - (S_{i-1}))/S_{i-1}$	Ui^2	λ=0.94	
Date		**Adj Close**	**Daily Returns**	**Squared Returns**	**Weights**	**λ*Ui^2**
02/05/2022	0	2192.91992				
29/04/2022	1	2210.31006	-0.007867736	6.19013E-05	0.06	3.71408E-06
28/04/2022	2	2317.80005	-0.046375868	0.002150721	0.0564	0.000121301
04/11/2021	122	2437.01001	0.07466525	0.0055749	3.36218E-05	1.87438E-07
		sum		0.097438556	0.999473258	
		avg	-0.000465142	0.000798677		0.00075686
		daily vol		0.001315677		0.027511081

The sum of the *λ*Ui^2* is an estimate of the daily variance and its squared root constitutes the daily volatility we will use. σ_{bkng_ewma} =0.027511081

We calculate the daily volatility for the second asset and for the portfolio with the same procedure.
σ_{ew} = 0.027755486 σ_p = 0.022117525

With the new daily volatily estimated following the riskless EWMA we calculate as before the Global and Joint VaR.

Global VaR	1	2	3	4	5	50	100
95%	36493.92	51610.19	63209.32	72987.83	81602.88	258051	364939.2
99%	51533.83	72879.85	89259.22	103067.7	115233.2	364399.2	515338.3
99.50%	57063.21	80699.57	98836.39	114126.4	127597.2	403497.9	570632.1

bkng_VaR	1	2	3	4	5	50	100
95%	22696.64	32097.9	39311.74	45393.28	50751.23	160489.5	226966.4
99%	32050.41	45326.12	55512.94	64100.82	71666.9	226630.6	320504.1
99.50%	35489.29	50189.44	61469.26	70978.59	79356.48	250947.2	354892.9

ew_VaR	1	2	3	4	5	50	100
95%	22898.28	32383.05	39660.98	45796.55	51202.1	161915.3	228982.8
99%	32335.14	45728.8	56006.11	64670.28	72303.57	228644	323351.4
99.50%	35804.58	50635.32	62015.35	71609.15	80061.47	253176.6	358045.8

joint N VaR	1	2	3	4	5	50	100
95%	45594.92	64480.95	78972.71	91189.84	101953.3	322404.8	455949.2
99%	64385.55	91054.92	111519	128771.1	143970.5	455274.6	643855.5
99.50%	71293.87	100824.8	123484.6	142587.7	159417.9	504123.8	712938.7

The sub-additivity is still preserved.

In percentage the Global and Joint VaR are respectively:

Global VaR	1
95%	3.65%
99%	5.15%
99.50%	5.71%

joint N VaR	1
95%	4.56%
99%	6.44%
99.50%	7.13%

6. The Monte Carlo Simulation

A Monte Carlo simulation is a model used to predict the probability of different outcomes when the intervention of random variables is present. It helps to explain the impact of risk and uncertainty in prediction and forecasting. The Monte Carlo simulation is complex but has the advantage of allowing users to tailor ideas about future patterns that depart from historical patterns.

Using the following parameters:

initial portfolio value = $1 000 000

historical daily vol σ_p = 0.018352335 (calculated at point 4)

time T= 126 (trading days within 6 months)

I create the Seed Value table which gives us an estimate of the possible ending value for our portfolio. The number of iterations is 500.

I follow the same procedure for the single assets to compute the Joint VaR. In this case I use as initial value $ 500 000 and as volatility σ_{bkng} = 0.02825704 and σ_{ew} = 0.022882491 respectively.

I calculate the table of *Percent Loss* with the *percentile* function implemented on Excel over the Seed Value table, with percentile 1-0.95, 1-0.99, 1-0.995

The *Absolute Loss* is simply the Percent Loss multiplied for the initial value of the asset/portfolio and it constitutes the value that our maximum downside risk shouldn't exceed over six trading months.

Global Monte Carlo VaR	Percent Loss	Absolute Loss
0.95	3.72%	37216.74228
0.99	5.06%	50590.72017
0.995	5.45%	54531.14525

Monte Carlo VaR (BKNG)	Percent Loss	Absolute Loss
0.95	2.60%	13021.54019
0.99	3.55%	35495.57033
0.995	3.60%	36046.71253

Monte Carlo VaR (EW)	Percent Loss	Absolute Loss
0.95	3.05%	15225.26593
0.99	4.23%	21130.49867
0.995	4.51%	22532.34098

Joint MC Var	Percent Loss	Absolute Loss
0.95	5.65%	28246.80612
0.99	7.78%	56626.06899
0.995	8.11%	58579.05351

In this case for CL 95% the VaR is *not subadditive* since the Global VaR of the portfolio is larger than the sum of the VaRs of its components. For CL 99% and 99.5% the VaR is *subadditive*.

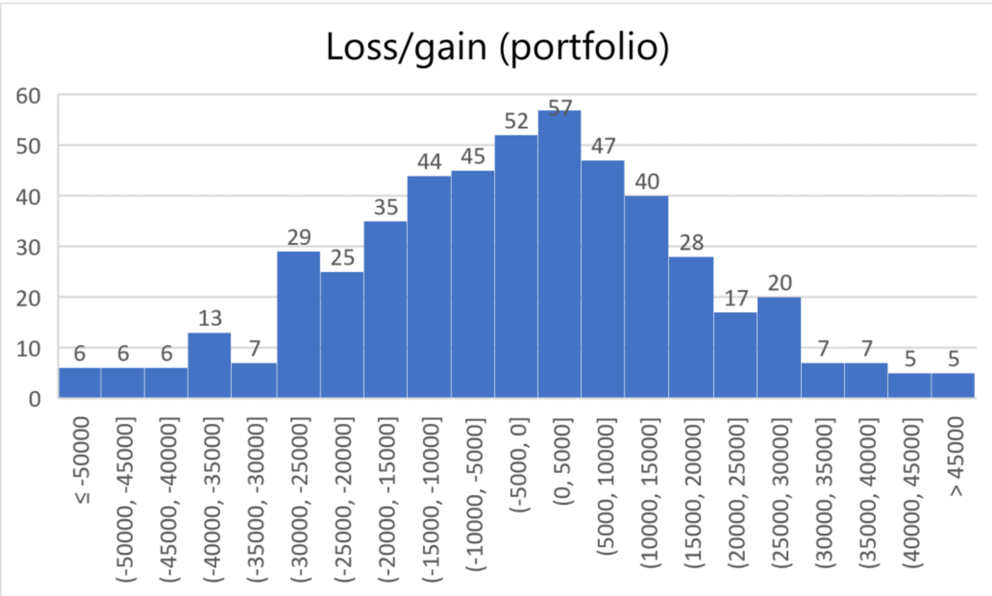


Fig.6a

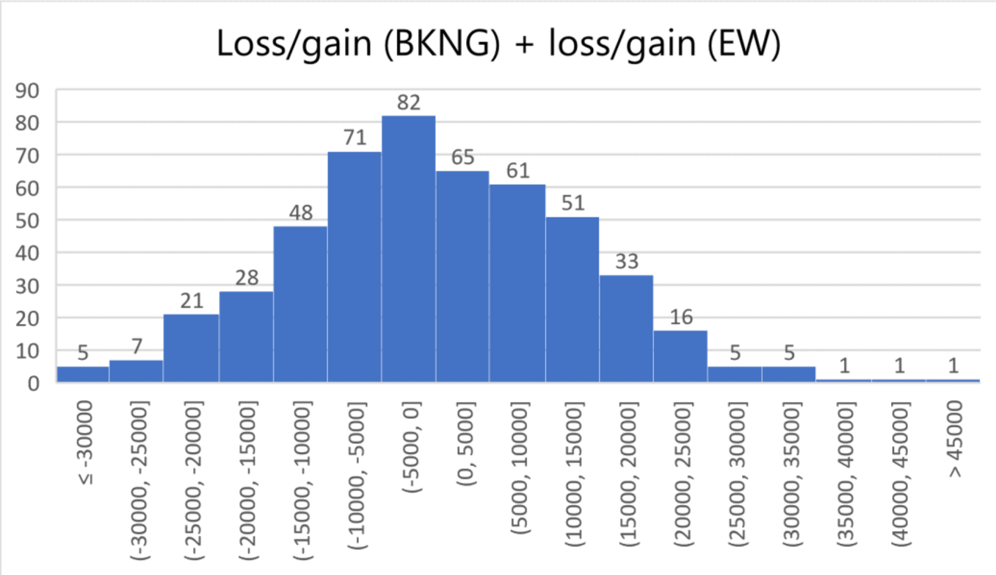


Fig.6b

The figures above represent the histograms of the loss/gain on the portfolio and on the combination of the two assets. As we would expect the centre of the distribution is around zero.

7. The Historical Simulation

The historical method simply re-organizes actual historical returns, putting them in order from worst to best. It then assumes that history will repeat itself, from a risk perspective.
I sort the daily returns of the global portfolio from worst to best and I do the same with the sum of the daily returns on the two assets (to calculate the Joint VaR). For VaR(95%) I find the daily return correspondent to the 1-95% percentile of the 122 elements time series (number of trading days for the 6 months window I chose). I do the same for the percentile 1-99% and 1-99.5%.
Accordingly, I also compute the *Conditional VaR*, the expected loss given that the loss is greater than then VaR level. In this case I sum the value from the position of the percentile (1-95%, 1-99%, 1-99.5%) up to the first (worst).

Global	
VaR(95)	-3.76%
VaR(99)	-5.01%
VaR(99.5)	-5.13%

Global	
C-VaR(95)	-4.34%
C-VaR(99)	-8.31%
C-VaR(99.5)	-8.41%

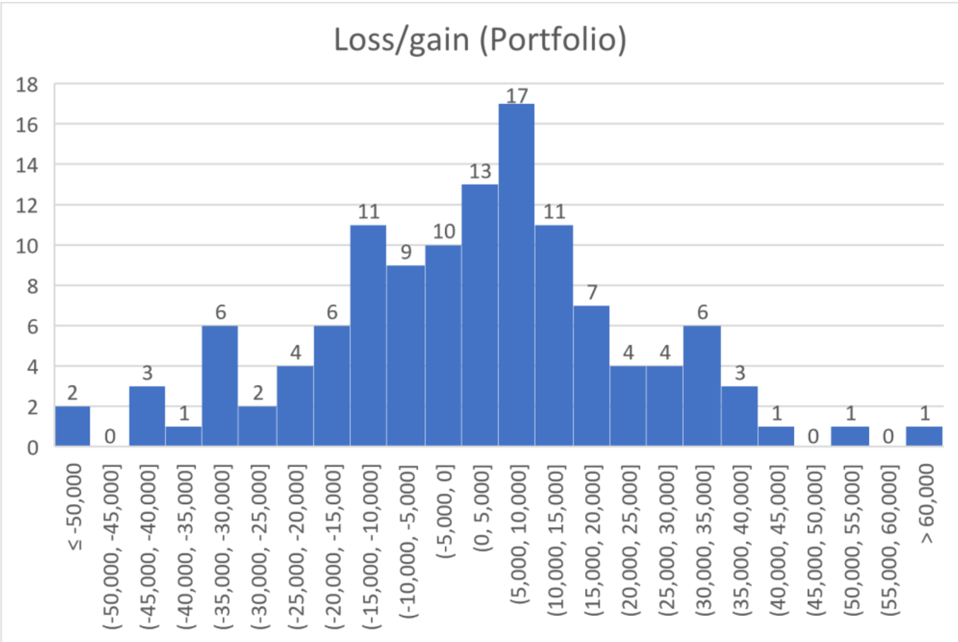
Joint	
VaR(95)	-6.89%
VaR(99)	-10.53%
VaR(99.5)	-11.55%

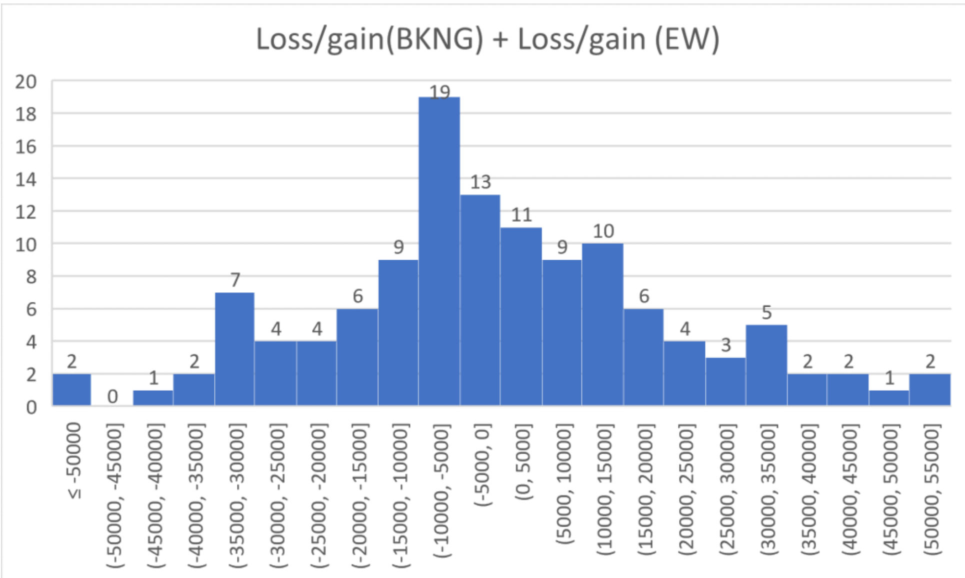
Joint	
C-VaR(95)	-8.45%
C-VaR(99)	-18.10%
C-VaR(99.5)	-18.94%

The VaR computed is subadditive.

8. Historical VaR

To calculate the Historical VaR according to the historical value of the returns I use the following procedure. I first calculate the daily returns for the portfolio and for the single assets. Then, in the case of the portfolio, I multiply the daily returns for the initial value \$1 000 000 and I obtain the gain/loss table of the portfolio. I do the same for the combination of the two assets daily returns, multiplying each return for \$500 000. The sum of the two gain/loss columns constitutes the table I will use to compute the Joint VaR.
I plot the two tables obtained:





I calculate the VaRs expressed in term of *loss amount* ($) with the *percentile* function implemented on Excel over the Loss/gain table, with percentile 1-0.95, 1-0.99, 1-0.995. To obtain the *percentage loss* I simply divide the loss amount by $1 000 000.

Global VaR @		
95%	-34621.2	-3.46%
99%	-48817.3	-4.88%
99.50%	-50540.3	-5.05%

Joint VaR @		
95%	-33891.7	-3.39%
99%	-50267.9	-5.03%
99.50%	-54662.9	-5.47%

As in the case of Monte Carlo Simulation, for CL 95% the VaR is *not subadditive*. It is *subadditive* for CL 99% and 99.5%

9. Conclusion

Comparing all the VaR obtained, the largest Global VaR @ 95% was computed with the Historical Simulation (-3.76%). The largest Global VaR @ 99% was obtained with the Variance-Covaraince Method (-5.15%) and also the one at 99.5% (-5.71%).
The lowest Global VaR at 95% is the Historical VaR (-3.46%) and also at 99% and 99.5% with -4.88% and -5.05%.
The highest Joint VaR at 95% is -6.89%, obtained with the Historical Simulation. The highest at 99% and 99.5% are still the ones obtained with the same model (-10.53% and -11.55%).
The lowest Joint VaR at 95%, 99% and 99.5% were computed with the Historical VaR (-3.39%, -5.03%, -5.47%).