

# Recent Developments in Rocket Nozzle Configurations

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THE THRUST of a rocket engine is primarily dependent upon the momentum imparted to the products of combustion by discharging them through an exhaust nozzle. During their passage through the nozzle, the exhaust gases are continuously accelerated from low subsonic velocities to high supersonic velocities at the nozzle exit. For the purpose of understanding the flow process, the nozzle can be divided into three portions: the convergent subsonic section, the throat section, and the divergent supersonic section. This division is appropriate because of the different effects each part has in determining the thrust developed by the rocket, and because different methods of analysis have to be used in determining the flow field in these three regions.

The mass flow rate through a rocket nozzle is determined by the area of the throat section and operating conditions of the combustion chamber. Design changes in the configuration of the convergent portion of the nozzle would influence the mass flow of the exhaust gases and also, to some extent, the combustion efficiency achieved in the chamber. The sonic velocity attained by the exhaust gases is fixed by the combustion chamber conditions and can be further increased by the expansion occurring in the divergent supersonic portion of the nozzle. This additional velocity, leading to thrust increment, is dependent only on the configuration of the diverging nozzle walls and the exit area. In recent years the design of the divergent supersonic portion of the rocket nozzle has received considerable attention. The scope of this review will be limited to this portion of the nozzle and improvement thereof.

Analysis of rocket nozzle flows in any real case should include radiative heat loss, chemical reactions due to incomplete combustion, and chemical properties of the exhaust gases. However, gross performance comparisons can be made by assuming that the exhaust gases expand adiabatically and behave like an ideal gas with a constant ratio of specific heat capacities.

The appropriate choice of nozzle configuration for a specific rocket engine may, to a great extent, depend on the fabrication methods of nozzle walls, cooling requirements, allowable limits on dimensions, influence of nozzle weight on overall rocket performance, etc. Detailed examination of all these features involves various special engineering fields aside from the knowledge of supersonic expansion process of the

exhaust gases, and, as such, is not considered within the scope of this paper.

The thrust performance of uniform exit flow nozzles and conical nozzles serves as reference in comparing the performance of other types of nozzles and is briefly examined prior to the discussion of recent developments in nozzle configurations.

## Uniform Exit Flow Nozzle

The thrust of a nozzle can be shown to reach a maximum when the nozzle is designed for parallel uniform exit flow, with exit plane pressure equal to ambient pressure. Such a nozzle is generally known as the *ideal nozzle* for expanding the gases from the chamber pressure  $P_c$  to ambient pressure  $p_a$ . The ratio of exit area to throat area of such a nozzle is given by

$$\frac{A_e}{A_t} = \left( \frac{\gamma - 1}{2} \right)^{1/2} \left( \frac{2}{\gamma + 1} \right)^{(\gamma+1)/2(\gamma-1)} \left( \frac{p_a}{P_c} \right)^{-(1/\gamma)} \left[ 1 - \left( \frac{p_a}{P_c} \right)^{(\gamma-1)/\gamma} \right]^{-1/2} \quad [1]$$

where  $\gamma$  is the isentropic exponent governing the expansion process. For rocket engines operating at high altitudes, the ratio  $p_a/P_c$  will be very low, and large area ratios for the ideal nozzles would be required.

To obtain optimum performance from a rocket engine operating over a wide range of altitudes, variations in the nozzle exit area as indicated by Eq. 1 would be required. Mechanical variation of rocket nozzle area ratio is very difficult because of the extremely high temperatures involved and the attendant complexity. Thus taking into consideration the performance requirements over the entire trajectory of the rocket vehicle, an appropriate fixed exit area for the nozzle is chosen. The performance merit of the divergent supersonic portion of a nozzle can be measured in terms of its vacuum thrust coefficient  $C_{fv}$ , defined as

$$C_{fv} = \frac{\text{Thrust when discharging to vacuum}}{P_c A_t}$$

where  $P_c$  and  $A_t$  denote the chamber pressure and throat area

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respectively. The vacuum thrust coefficient of an ideal nozzle is an explicit function of nozzle area ratio  $A_e/A_t$  and the isentropic exponent  $\gamma$ .

When the ideal nozzle is operated in the presence of an ambient pressure  $p_a$ , the thrust coefficient is written

$$C_f = C_{f,ideal} - \frac{p_a}{P_c} \cdot \frac{A_e}{A_t} \quad [2]$$

The above equation is often referred to as one-dimensional thrust coefficient equation. One-dimensional thrust coefficients for various values of isentropic exponent  $\gamma$ , area ratio  $A_e/A_t$ , and operating pressure ratio  $p_a/P_c$  can be evaluated from flow tables such as are given in (1),<sup>1</sup> or more exactly, from tabulations of thermodynamic data for the particular combustion gas.

The design of nozzle wall configuration to produce the desired uniform exit flow can be accomplished by the well-known method of characteristics (2). The computation for the wall contour generally requires high speed computing machines; several such contours are described in (3). Foelsch (4) developed an elegant approximation to obtain the nozzle contour by hand computations. The length of a uniform exit flow nozzle can be reduced to some extent by allowing all the expansion to occur immediately downstream of the throat, and then constructing the nozzle contour to turn the flow suitably so as to achieve uniform axial flow at the exit. Minimum lengths of nozzles required to obtain uniform exit flows for different values of exit to throat area ratio are computed for  $\gamma = 1.23$  (see Fig. 1). In computing these nozzle lengths it was felt that a sharp corner downstream of the throat should be avoided, and thus a wall contour having a radius of curvature equal to 0.4 times the throat section radius was used.

As is evident from Fig. 1, the ideal nozzles that give maximum thrust performance are long and, consequently, heavy. The following sections will deal with methods of reducing nozzle length without appreciable loss in thrust performance.

## Conical Nozzles

The vacuum thrust coefficient of any nozzle shorter than the uniform flow nozzle will naturally be less than the one-dimensional value. The thrust coefficient  $C_{f_v}$ , used in Eq. 2, will have to be adjusted accordingly. The simplest geometric shape for the divergent nozzle is a truncated cone, as shown in Fig. 2a. Even though the exit velocity of a conical nozzle is essentially equal to the one-dimensional value corresponding to the area ratio, the flow directions are not all axial, and hence there is a thrust decrement due to the flow divergence. Assuming a conical type flow at the nozzle exit, Malina (5) has shown that the exit momentum is equal to the value computed from one-dimensional theory multiplied by a factor

$$\lambda = (1 + \cos \alpha)/2 \quad [3]$$

where  $\alpha$  denotes the half angle of the cone. The thrust coefficient of a conical nozzle discharging into vacuum can be written

$$C_{f_{conical}} = \frac{p_e}{P_c} \cdot \frac{A_e}{A_t} + \left( \frac{1 + \cos \alpha}{2} \right) \frac{\rho_e V_e^2 A_e}{P_c A_t} \quad [4]$$

where  $p_e$ ,  $\rho_e$ ,  $V_e$  are the one-dimensional pressure, density, and velocity computed at the exit of the nozzle. Experimental results (6) on conical nozzles of various cone half angles bear out the validity of the above relationship.

In using Eq. 4, the exit flow parameters based on the ratio of throat to exit areas measured in plane sections normal to the axis are usually computed. Landsbaum (7) suggested

<sup>1</sup> Numbers in parentheses indicate References at end of paper.

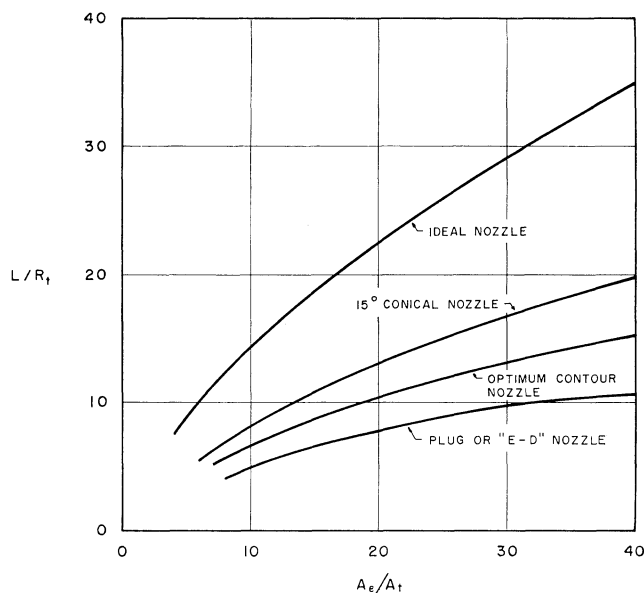


Fig. 1 Length comparison of various types of nozzles

that, to be consistent with the concept of conical exit flow, the exit area preferably should be computed over a spherical surface. This spherical surface area will be slightly larger than the plane section area  $A_e$ . Strictly speaking, the exit flow parameters are functions of conical flow expanding from a critical section to the exit section, both measured along spherical surfaces normal to the conical flow field. The critical section area may differ from the geometrical throat area  $A_t$  due to the effect of nozzle configuration as well as the nonideal nature of the flowing medium. Even for inviscid flows, the nonuniformity of flow in the throat region caused by configuration geometry yields critical section area significantly less than the plane cross-sectional area  $A_t$ , as shown in (8).

In the case of small rocket engines fitted with small area ratio nozzles, simple fabrication methods are preferred, and it is a common practice to use conical nozzles. Usually a half cone angle of  $15^\circ$  is used for the divergent section and, as can be seen from Fig. 1, the length of  $15^\circ$  conical nozzle is considerably less than that of a uniform exit nozzle. The decrement in thrust coefficient of a  $15^\circ$  conical nozzle compared to one-dimensional is only about 1.7% and varies slightly with altitude conditions, as can be seen from Eq. 4. Thus, a  $15^\circ$  conical nozzle is often used as a reference in comparing lengths and thrust performance of new types of nozzles.

## Contoured Nozzles

The loss in thrust of a conical nozzle due to flow divergence becomes large with increasing cone angles associated with reduction in nozzle length. By contouring the nozzle wall, as shown in Fig. 2b, the flow can be turned closer to axial direction, and the loss in thrust due to flow divergence can be reduced to some extent. Several methods have been suggested for contouring the nozzle walls. Sinyarev and Doborovolskii, in their textbook on liquid rocket engines (9), have suggested the use of a suitable circular arc for the nozzle contour. Dillaway (10) computed nozzle contours by a gradual reduction of the nozzle wall slope as one approaches the nozzle exit. Instead of summing up the flow divergence loss across the entire exit plane, Overall (11) incorrectly concluded that the loss in thrust depends only on the nozzle wall slope at the exit. The exit flow of a contoured nozzle is strongly affected by the nozzle contour, and no simple relation such as Eq. 4 can be

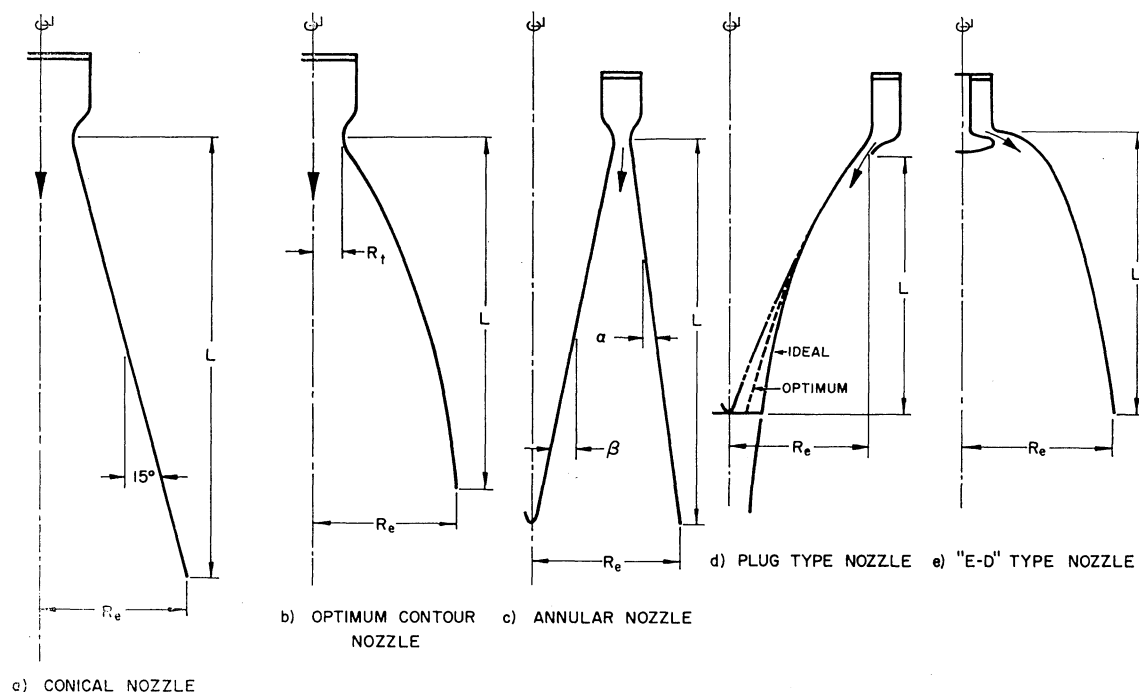


Fig. 2a-e Main features of various types of rocket nozzles

derived. The vacuum thrust coefficient of a contoured nozzle can be evaluated as

$$C_{fv} = \int_{A_e} \frac{p}{P_c A_t} 2\pi r dr + \int_{A_e} \frac{\rho V^2 \cos \theta}{P_c A_t} 2\pi r dr \quad [5]$$

where the integration is carried out across the entire area  $A_e$  of the exit plane normal to the nozzle axis. The pressure  $p$ , density  $\rho$ , velocity  $V$ , and its direction  $\theta$  are evaluated in the exit plane at a distance  $r$  from the nozzle axis. Landsbaum (12) suggested that one can examine truncated portions of several uniform exit flow nozzles of different area ratios and choose the one contour that yields the best performance. A similar approach was suggested by Demuth and Ditore (13). Experimental evaluation of such cutoff portions of uniform exit flow nozzles was reported by Farley and Campbell (14) and showed close agreement with theoretical estimates. Recently Ahlberg, Hamilton, Migdal, and Nilson (15) made a study of nozzle optimization by considering truncated portions of uniform exit flow nozzles.

In contrast to all the forementioned semiempirical methods, a direct and very elegant approach to nozzle contour design was made by Guderley and Hantsch (16). They formulated the problem of finding exit area and nozzle contour to yield optimum thrust when the nozzle length and ambient pressure have prescribed values. Using the calculus of variations, they solved the problem for the exit flow field required to deliver optimum thrust under the prescribed restrictions. After the required exit flow is found, the method of characteristics in supersonic flow is used to develop the nozzle contour between the throat and the nozzle exit. The nozzle contour thus obtained can also be interpreted as the optimum thrust contour for the particular values of exit area and length.

Because of the complicated nature of the solution presented by Guderley and Hantsch, the optimum thrust contours for rocket nozzles were completely ignored until recent years. In reformulating the problem, Rao (17) found a simplified approach for the contour calculations. In a recent paper Guderley (18) discussed the difference between his and Rao's method of reaching the same solution. Several optimum thrust nozzles were computed for  $\gamma = 1.23$ , and a systematic study of their parameters was given in (19). The length of

a contoured nozzle required to yield the same vacuum thrust coefficient as that of a  $15^\circ$  conical nozzle is shown in Fig. 1 plotted against nozzle area ratio.

Computing nozzle contours by the method of characteristics can also be extended to the case of variable value of  $\gamma$ , as shown in (20). The method of optimizing nozzle contours for the case of nonconstant  $\gamma$  was treated in (21). Examination of contour calculations at different values of  $\gamma$ , given in (19), has shown that when the exit area ratio and length are both prescribed, the differences in nozzle contours are negligible. However, it should be remembered that the flow process in a nozzle of a given configuration is strongly governed by the isentropic exponent  $\gamma$ , and hence the nozzle thrust coefficient would greatly depend upon the value of  $\gamma$ . It was also observed that the optimum thrust nozzle contour can be closely approximated by a parabola. The parabola has to fit the maximum wall inclination  $\theta_m$  in the throat region and the wall inclination  $\theta_e$  at the nozzle exit. Parameters  $\theta_m$  and  $\theta_e$  are implicit functions of the nozzle exit area and length. Charts showing the values of these parameters and a simple geometric construction of the parabolic arc are given in (22).

An optimum thrust nozzle contour (16, 17) designed for an ambient pressure  $p_a$  and prescribed length would require the flow conditions on the nozzle wall at the exit to satisfy the following relationship

$$\frac{p - p_a}{(1/2)\rho V^2} \sqrt{M^2 - 1} = \sin 2\theta \quad [6]$$

where  $p$ ,  $\rho$ ,  $V$ ,  $M$  and  $\theta$  denote the pressure, density, velocity, Mach number, and flow direction, respectively. For the case of small divergence angles, Knuth (23) suggests that the flow conditions at any individual element of nozzle wall can be assumed to be independent of the rest of the wall contour. He shows that by applying Eq. 6 at each of the individual wall elements, a relationship between the local flow parameters is obtained. The nozzle contour is then constructed through stepwise integration.

In general, the rocket nozzle contours chosen for optimum thrust performance require wall angles of about  $28^\circ$  to  $30^\circ$

downstream of the throat. The required wall inclination at the nozzle exit is about  $10^\circ$  to  $14^\circ$  depending upon the area ratio and length of the nozzle. Nozzle contours having such parameters are widely used for liquid rocket engines, and no specific difficulties attributable to the contour are experienced with hot exhaust gases. However, the exhaust products of solid rocket engines often contain particles of metal oxides. The dynamics of hot gases laden with small particles up to as much as 30% by weight has to be considered. Optimization of nozzle contours for such flows must then consider the velocity lag of particles and hence becomes very difficult. Kliegel (24) analyzed flows of such gas-particle mixtures through several contoured nozzles. He concluded that in the case of highly curved nozzle walls, the gas particles impinge on the wall at the exit, produce a decrement in thrust, and aggravate problems of erosion. Consequently, for solid rocket engine applications, he suggests nozzle contours with less amount of turning than the optimum thrust contours.

### Annular Nozzles

In conventional nozzles we generally have the throat and exit areas formed by circular cross sections normal to the nozzle axis. There is another type of nozzle, widely used in jet engines, called an *annular nozzle*. The throat section is an annulus formed between a central plug and an outer tail pipe. Downstream of the throat section the exhaust gases are made to expand in a diverging annulus formed between the converging central plug and the diverging tailpipe, as shown in Fig. 2c. If the central plug vertex lies in the exit plane and the diverging walls are formed of conical surfaces, an expression for thrust lost due to flow divergence can be derived.

$$\lambda = \frac{(1/2) (\sin \alpha + \sin \beta)^2}{(\alpha + \beta) \sin \beta + \cos \beta - \cos \alpha} \quad [7]$$

where  $\alpha$  is the angle between the outer wall and the nozzle axis, and  $\beta$  is the angle between the inner plug wall and the nozzle axis. For the same exit to throat area ratio and nozzle length, the throat annulus can be located at different radial distance from the nozzle axis, leading to corresponding changes in the values of  $\alpha$  and  $\beta$ . In the case of a divergent annular nozzle formed between a straight tailpipe and a conical central plug, the thrust decrement due to flow divergence can be obtained by substituting  $\alpha$  equal to zero in Eq. 7. As  $\beta$  approaches zero the inner plug vanishes, and only the diverging outer wall remains. In this case Eq. 7 reduces to the flow divergence loss factor discussed in connection with the conical nozzle thrust performance.

The thrust loss due to flow divergence in an annular nozzle, as indicated by Eq. 7, would be less than that in a conical nozzle of the same area ratio and length. The flow divergence loss in an annular nozzle can be reduced further by contouring the nozzle walls. However, in this case the contour optimization becomes more complicated than in a conventional nozzle, since the inner wall and outer wall contours have to be optimized simultaneously.

### Self-Adjusting Type Nozzles

In the several types of nozzles discussed above, the expansion of the exhaust gases is governed solely by the diverging walls of the nozzle; this expansion process can be optimized by suitably contouring the divergent walls. The ambient pressure has no influence on the expansion process occurring inside the nozzle, and the exhaust gases would continue to expand along the diverging nozzle walls to pressures considerably below the ambient pressure. This overexpansion of the gases would proceed until flow separates from nozzle walls (25, 26). The nozzle wall pressure would then

rise abruptly to the ambient pressure across an oblique shock (27, 28). The occurrence of these subambient pressures on some portions of the nozzle walls reflects a loss in thrust performance in rocket engines operating below the design altitudes. In recent years, considerable attention has been given to the development of new types of nozzles in which the expansion process is directly or indirectly regulated by the ambient pressure. Since the exhaust flow in these types of nozzles adjusts itself to conform to the external conditions, this author prefers to denote them as *self-adjusting type*.

These self-adjusting type nozzles can be classified in two categories: 1) plug-type nozzles and 2) expansion-deflection type nozzles. The essential features of these nozzle types

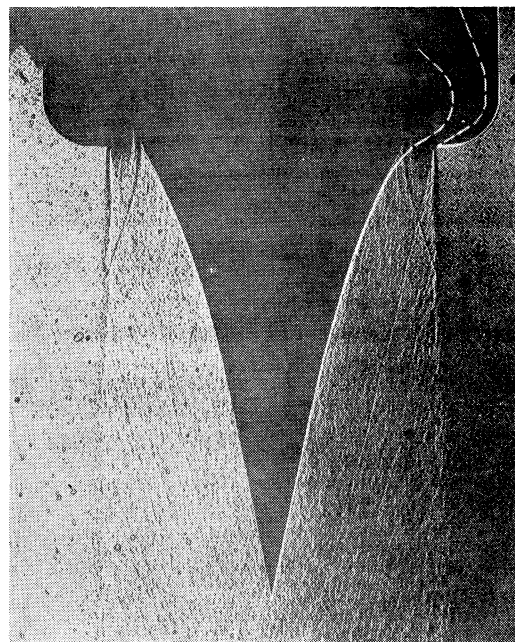


Fig. 3a Shadowgraph of flow around a plug nozzle at design pressure ratio

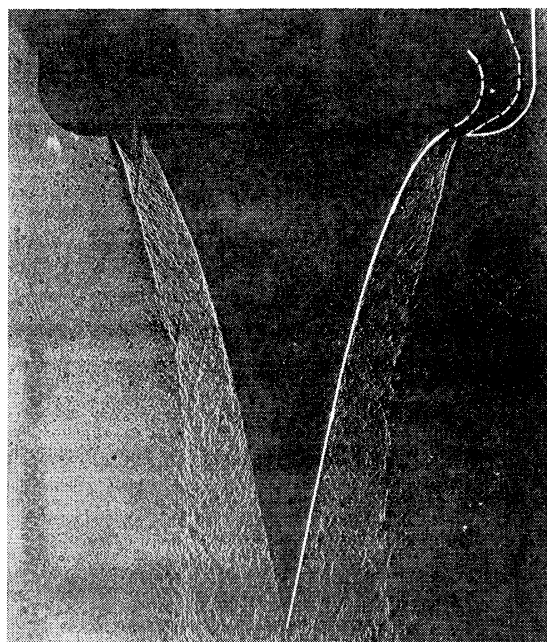


Fig. 3b Shadowgraph of flow around a plug nozzle at below design pressure ratio

will be briefly described here, and the reader is advised to refer to cited literature for detailed information.

### Plug Type Nozzles

An all external expansion ideal plug nozzle is schematically shown in Fig. 2d. The nozzle throat is located as an annulus at the outer diameter, with the exhaust gases issuing in an inward direction. At the outer edge of the annulus, often denoted *cowl-lip*, following jet engine terminology, the exhaust gases expand abruptly to the ambient pressure. The flow of the exhaust gases is controlled by the expansion waves originating from the cowl-lip, and the flow turning is affected by the plug surface. One can design the plug surface so as to expand the gases from a chamber pressure  $P_c$  to an ambient pressure  $p_a$ , producing uniform exit flow parallel to the nozzle axis. The external diameter, i.e., cowl-lip diameter, of such an ideal plug would be the same as the exit diameter of a uniform flow convergent-divergent nozzle expanding the gases to the same ambient pressure  $p_a$ . However, the plug type nozzle is much shorter than the equivalent convergent-divergent nozzle as shown in Fig. 2d. Simple approximate methods suggested by Krase (29) or Greer (30) can be used to design ideal plug nozzle contours by hand calculations. Exact computations generally require high speed computing machines.

The thrust performance of the ideal plug nozzle shown in Fig. 2d at design and above design pressure ratios, i.e.,  $P_c/p_a$ , would be identical to that of a convergent-divergent ideal nozzle. Any ballooning out of the exhaust gases that occurs at above design pressure ratios does not affect the flow along the wall in either type of nozzle. However, at below design pressure ratio, the flow in the plug nozzle is radically different from that in a conventional nozzle. The expansion occurring at the cowl-lip would proceed only up to the ambient pressure  $p_a$  and not all the way down to the design exit pressure  $p_e$ . Shadowgraphs of flow around a plug nozzle operating at two different pressure ratios are reproduced from (33) and shown in Figs. 3a and 3b. The self-adjusting nature of the flow at below design operating pressure ratio can be seen clearly in this shadowgraph. This terminated expansion wave would intersect with plug wall at a point where wall pressure  $p_w$  is nearly equal to  $p_a$ ; the difference between  $p_w$  and  $p_a$  is only due to the axisymmetric nature of flow. If the expansion were to proceed uninterrupted, the wall pressures would continue to fall to reach  $p_e$  at plug vertex. Now, however, the expansion is interrupted at  $p_w \simeq p_a$ , and the convex surface of the plug downstream of this point causes a gradual compressive turning of the exhaust gases. Consequently, the wall pressure downstream of this location would rise to values higher than  $p_a$ . Beale and Povolny (31) measured wall pressure distribution on an external plug operating at below design condition, and their data shows this increase in wall pressures due to the compression. These increased

pressures on the downstream portions of the plug wall give rise to a thrust increment over that of a convergent-divergent nozzle. This behavior, briefly discussed in (29), is also experimentally indicated by data given in (32). A comparison between the performance of a conventional nozzle and a plug type nozzle was given in a recent paper by Berman (33). Due to the self-adjusting nature of the flow, a plug nozzle operating at below design pressure ratio shows a thrust advantage over a conventional nozzle, as indicated in Fig. 4, which is reproduced from (33). It should be borne in mind that the amount of recompression occurring on the plug surface is intimately connected with the plug wall contour and consequently affects the off-design performance of the plug nozzle. Thus, general comparisons regarding the thrust advantage of plug nozzles cannot be made.

Plug nozzle concept applied to rocket engines presents a great advantage in the development of large engines. The annular combustion chamber can be built up from segments of nearly rectangular chambers; the consequent design advantages are discussed in (33).

### Plug Contours

Just as in the case of conventional convergent-divergent nozzles, one may ask whether the plug length can be shortened without undue loss in thrust. The wall pressures on an ideal plug nozzle and their contribution to axial thrust fall off considerably as the vertex is approached. Hence, the plug end portion can be cut off to some extent without much sacrifice in thrust. The plug end may further be modified by using a large cone angle at the plug vertex so as to avoid a flattened base. Both these plug modifications are shown in Fig. 2d. Experimental data taken on low expansion ratio plug nozzles, reported in (34), indicate that cone half angles as large as  $40^\circ$  can be used without undue loss in thrust performance. Berman and Crimp (35) also indicated that experimental data on plug type rocket nozzles showed performance reduction of only 1% by using cone half angles even up to  $30^\circ$ .

Similar to the approach taken in optimizing the conventional nozzle contour, optimum plug contour for a given length may be sought. Rao discussed the solution of such problems and presented typical optimum plug contours in (36). The reduction in the nozzle length that can be achieved by the use of plug type nozzle is indicated in Fig. 1. For comparison, the equivalent area ratio of a plug nozzle is defined as  $\pi R_c^2/A_t$ , where  $R_c$  is the radius of the cowl-lip and  $A_t$  denotes the area of the throat annulus. The length required for a plug nozzle to yield 99.5% of vacuum thrust coefficient of ideal nozzle of the same area ratio is plotted in Fig. 1 against nozzle area ratio. The plug contour solution as given by Rao yields a blunt base diameter less than that of a truncated ideal plug contour as sketched in Fig. 2d. It should be noted that, due to the convergence of flow along the plug surface, boundary layer thickness increases rapidly towards the plug vertex. The plug contours (36) computed for inviscid flow can be further corrected to accommodate this increasing boundary layer thickness. Such corrections would reduce the size of the plug base, bringing it closer to a conical vertex. Thus, a conical plug vertex with optimized flow direction at the throat may be a suitable approach. The method indicated in (36) can be used to obtain the optimum flow direction at the throat applicable to the particular plug parameters.

In evaluating the thrust performance of a plug type nozzle, it should be remembered that the plug wall pressures are affected by the external pressure at the cowl and also by the flow behavior near the plug vertex. If the missile flight conditions are such that the pressure downstream of the cowl is different from ambient pressure, necessary corrections to the performance should be made. It is difficult to predict analytically the flow at the plug vertex at subdesign pressure

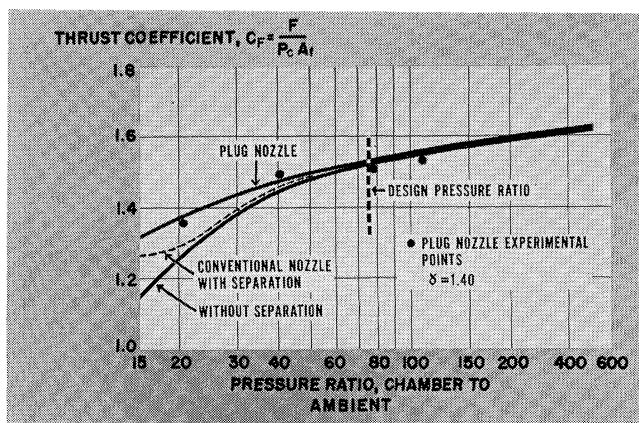


Fig. 4 Thrust performance of plug and conventional nozzles



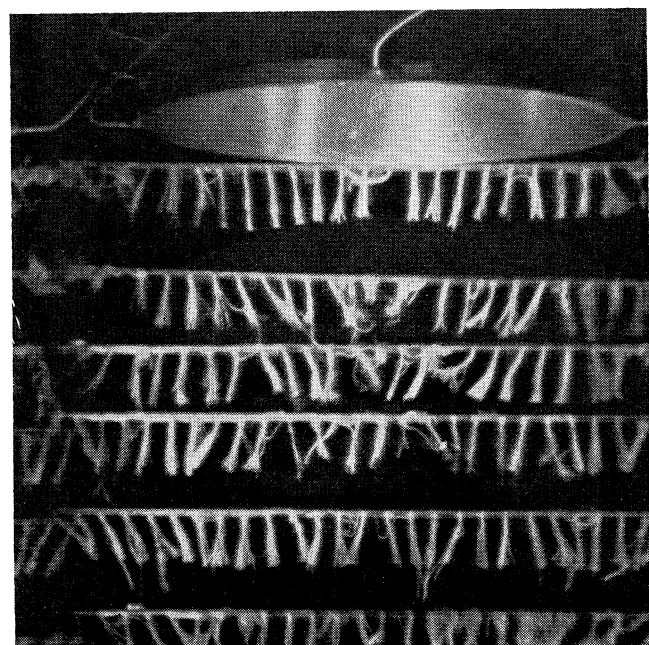
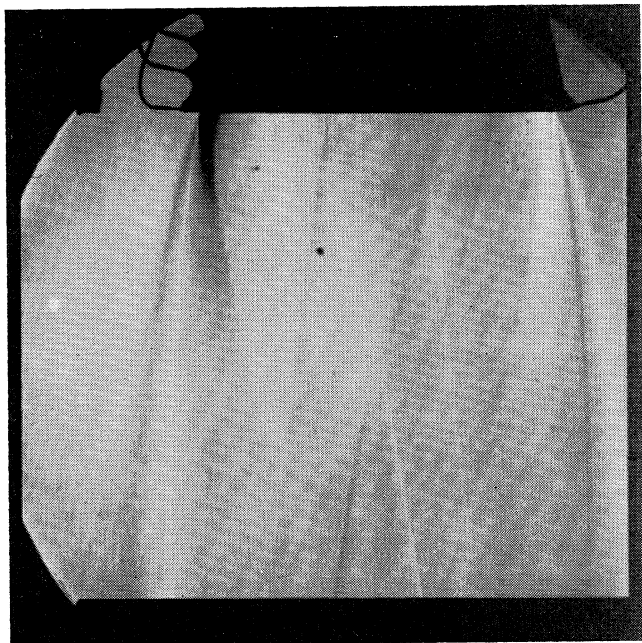


Fig. 5a Schlieren and tuft photographs of flow in an E-D nozzle at high pressure ratio

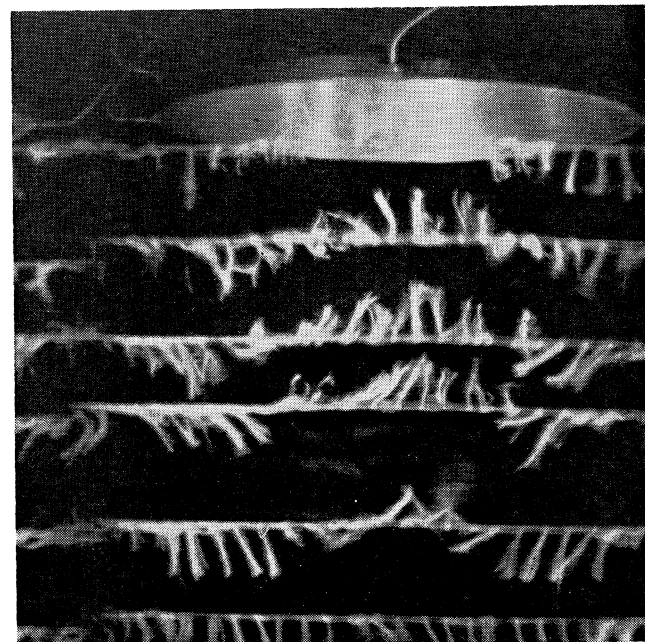
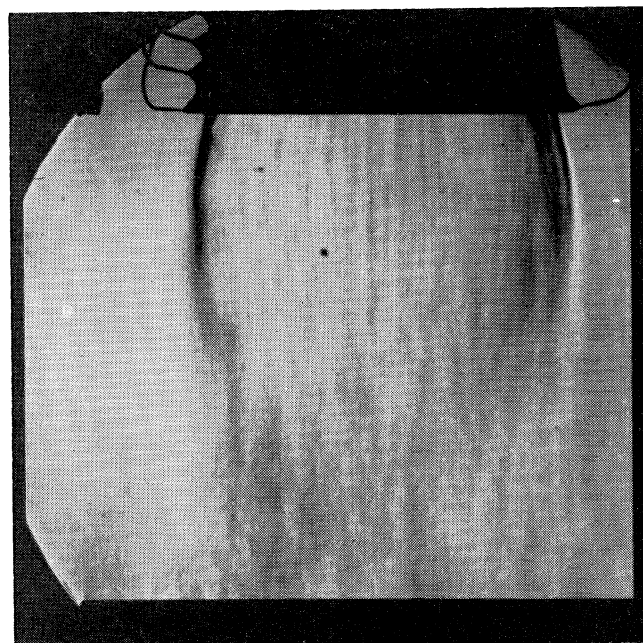


Fig. 5b Schlieren and tuft photograph of flow in an E-D nozzle at low pressure ratio

ratio operation, and the thrust performance can best be ascertained through tests simulating the rocket flight conditions.

In a recent paper, Berman and Neuffer (37) discussed modifications to the plug nozzle concept by incorporating a certain amount of internal expansion preceding the external plug. Since the effect of the external pressure on the flow is thus partly restricted, one would find that in this case the self-adjusting property of the flow is inhibited.

#### E-D Type Nozzle

Instead of locating the throat of the rocket motor at the outer diameter, as in the case of a plug nozzle, a compact combustion chamber with the throat section annulus located close to the nozzle centerline can be constructed. In this case, the exhaust gases issue forth from the throat in an outward direction as shown in Fig. 2e and expand around the shoulder of the central plug. The nozzle wall contour in the form of a shroud would turn the expanding exhaust gases

into a nearly axial direction. This type of nozzle is denoted the *Expansion-Deflection* or *E-D* type, and its essential features are discussed in (38).

The self-adjustment of flow in an E-D type nozzle occurs because the pressure at the base of the central plug limits the amount of expansion of the exhaust gases around the shoulder at the throat. Hence a certain amount of compressive turning takes place along the nozzle wall in a manner similar to that occurring in a plug nozzle. Even though the pressure at the base of the central plug cannot be directly related to the ambient pressure, it was found to produce sufficient self-adjustment of the flow during low pressure ratio operation. Schlieren and tuft photographs of flow in an E-D nozzle are reproduced from (38) and presented here in Figs. 5a and 5b. The adjustment of flow at low pressure ratios in an E-D nozzle causes an annular flow attached to the nozzle walls. Hence, the nature of flow in the central region cannot be deciphered from the schlieren photograph. The accompanying tuft photograph shows the main flow

annulus near the wall and eddies, some even in the reverse direction, in the central region. The wall pressure distribution reported in (39) indicates that the flow along the wall of an E-D nozzle, operating at below design pressure ratio, undergoes a gradual compression just as in the case of the plug nozzle.

Considering an annular throat section with radially outward flow, the method of optimizing the wall contour of E-D type nozzle was presented in (39). It was observed that the length of an E-D type nozzle is nearly the same as that of a plug nozzle for equivalent thrust performance.

When the ratio of chamber pressure to ambient pressure is sufficiently high, the thrust performance of an E-D nozzle would be the same as that of a plug nozzle designed for comparable parameters. However, when this operating pressure ratio is low, i.e., at low altitudes, the flow in a plug nozzle adjusts itself to the pressure at the cowl-lip, whereas in an E-D nozzle the flow adjusts itself to the pressure occurring behind the central plug. If the secondary effects caused by flight velocity of the missile are ignored, it can be said that the plug nozzle flow adjusts itself to the ambient pressure. In the case of an E-D nozzle, however, the pressure at the base of the central plug can be lower than the ambient because of the ejector action of the surrounding supersonic stream. Since the flow in an E-D nozzle adjusts itself to a lower pressure, the compression on the nozzle wall would be retarded to some extent compared to a plug nozzle. Thus, under similar subdesign conditions, an E-D nozzle may show lower thrust performance than a plug type nozzle. It appears feasible that a certain amount of bleed could spoil the ejector action in an E-D nozzle so as to bring the pressure behind the central base up to the ambient. Only by such means can the performance of an E-D nozzle be brought up to that of an all external plug nozzle at the subdesign pressure ratios. Since the E-D type nozzle is built around a compact combustion chamber, it may present certain advantages regarding weight and cooling requirements, the consideration of which is beyond the scope of the present discussions. If nozzles of very high area ratios of the order of 100 and above are required for rocket engines operating in space, the E-D type nozzle may be chosen in view of the compact combustion chamber and reduced length.

## Concluding Remarks

In retrospect, the subject of this review has been limited to different types of nozzles and nozzle contours with a view toward improving the nozzle thrust performance. The rocket nozzle, however, must always be considered as a part of the total system, which sometimes leads to a set of confining restrictions overweighing the problem of thrust optimization. In order to provide the system design engineer with a greater choice of nozzle parameters, a large variety of novel nozzle configurations, such as the toroidal, horizontal flow, and reverse flow, are currently being studied by several rocket engine manufacturers. A problem that in the writer's knowledge has not been approached is that of thrust optimization including irreversibility in the nozzle flow.

In addition to providing forward thrust, the propulsion nozzle may be called on to provide side forces for vehicle guidance. These side forces may come either from jet vanes, nozzle rotation, or secondary fluid injection. The gas dynamic design of such nozzles would depend upon a new set of performance parameters in addition to optimization of thrust. These additional features of nozzle performance might be the decisive factors in the choice of the type of nozzle and its contour.

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