Appendix B

Finding identities between hypergeometric functions: Reddot

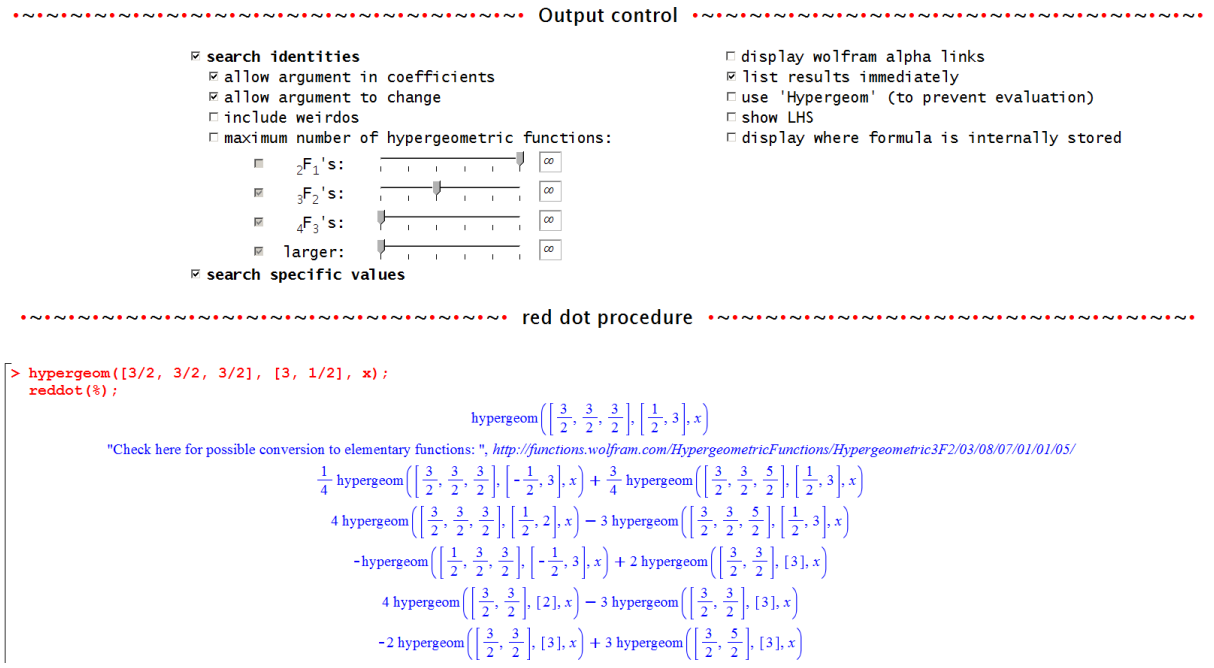

Figure B.1: Interface of the reddot procedure.

A program for finding hypergeometric identities is provided in the file `maple.mw`. It tests a total of 988 equalities as to whether they are applicable to a given hypergeometric function, applies them in all possible ways, and returns a list of expressions equivalent to the input. The equalities known to this program have been collected from `https://functions.wolfram.com/HypergeometricFunctions/` for identities and special values of ${}_0F_1$, ${}_1F_0$, ${}_1F_2$, ${}_2F_1$, ${}_2F_2$, ${}_2F_3$, ${}_3F_2$, ${}_4F_3$, ${}_5F_4$ and ${}_6F_5$. For roughly 40000 further conversions to elementary functions for hypergeometrics with specific numerical values of the parameters, the program provides a link to the relevant wolfram alpha site.

Usage

After opening the file, if Maple asks whether to execute content automatically, click "Yes". Input any hypergeometric function, and apply the procedure `reddot` to it.

Options

The following options can be activated by ticking the corresponding boxes:

search identities

Search for identities between different hypergeometric functions. There are multiple sub-options to narrow down the often very long list of results:

allow arguments in coefficients

If true, includes results in which the argument appears outside of hypergeometric functions, such as the argument x in the identity

$$ {}_3F_2\left[\begin{matrix}1, \frac{3}{2}, \frac{3}{2} \\ \frac{1}{2}, 3\end{matrix}; x\right] = \frac{24}{x^2}\,{}_2F_1\left[\begin{matrix}-\frac{1}{2}, -\frac{1}{2} \\ \frac{-3}{2}\end{matrix}; x\right] + \frac{4\,(x-6)}{x^2} $$

allow argument to change

If true, includes results where the argument of the hypergeometric function is transformed in some way, such as

$$ {}_2F_1\left[\begin{matrix}\frac{1}{2}, 1 \\ \frac{3}{2}\end{matrix}; x\right] = \frac{1}{1+\sqrt{x}}\,{}_2F_1\left[\begin{matrix}1, 1 \\ 2\end{matrix}; \frac{2\sqrt{x}}{1+\sqrt{x}}\right] $$

include weirdos

Includes a small number of especially complicated identities.

maximum number of hypergeometric functions

One may wish to limit the total number of hypergeometric functions of each size; adjust the sliders to filter out results containing too many such functions.

search specific values

Search for non-hypergeometric representations of the input hypergeometric function, such as

$$ {}_2\mathrm{F}_1\left[\begin{matrix}\frac{1}{2}, 1\\ \frac{3}{2}\end{matrix}; x\right] = \frac{1}{\sqrt{x}}\,\mathrm{arctanh}\left(\sqrt{x}\right) $$

The remaining options control the display of the output.

display wolfram alpha links

Toggles the line with the link to the relevant wolfram alpha site:

"Check here for possible conversion to elementary functions: ", *http://functions.wolfram.com/HypergeometricFunctions/Hypergeometric3F2/03/08/06/02/01/05/*

list results immediately

If ticked, any detected results are immediately printed on screen. If not, the screen remains blank until the `reddot` function returns the set of results.

use 'Hypergeom'

Sometimes, maple automatically replaces simple hypergeometric functions by their elementary representation. Returning the output using the capitalized `Hypergeom` function name prevents this.

show LHS

Display LHS (the input hypergeometric function) together with the found RHS.

display where formula is internally stored

This is merely a debug option and can be ignored.