

Chapter 2 - Probability: Univariate Models

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Motivation

This document contains a summary of the main topics of the second chapter of the book *Probabilistic Machine Learning: An Introduction* from Kevin P. Murphy.

1 Probability Functions

When modeling a random variable (rv) X , there are some functions and notations to memorize and interpret:

- **Probability Mass Function (pmf)**: it is defined for a **discrete** rv and corresponds to the function that maps a value $X = x$ to a probability. It is usually written in lowercase as $0 \leq p(X = x) \leq 1$
- **Cumulative Distribution Function (cdf)**: it is defined for both **discrete and continuous** rvs and corresponds to the function that maps a value $X = x$ to a value that computes $p(X \leq x)$
- **Probability Density Function (pdf)**: it is defined for **continuous** rvs and corresponds to the derivative of its cdf.

1.0.1 Probability Density Function

Definition 1.1 (Probability Density Function). Given a continuous random variable X , its probability density function $P(X = x)$ is given by:

$$p(x) = \frac{d}{dx} P(x) \quad (1)$$

such that its cumulative distribution function is given by (2)

$$P(a \leq x \leq b) = \int_a^b p(x) dx \quad (3)$$

Two major remarks must be notice regarding the pdf defined above:

- The pdf is basically the **derivative** of the cdf
- The pdf is not defined when the cdf does not have a derivative

1.0.2 Quantiles/ Inverse CDF/ Percent Point Function (ppf)

The *quantile* function (also called the Percent Point Function or simply *ppf*) is defined as the inverse function of the *cdf*. This means that given a random variable X 's cumulative distribution function $CDF(X \leq x)$, its PPF is given by $PPF(q) = CDF^{-1}(X \leq x_q)$. The output of the *PPF* function is interpreted as the value $X = x_q$ such that $P(X \leq x_q) = q$.

Let's consider the standard normal random variable $X \sim \mathcal{N}(0, 1)$. Suppose we want to calculate $PPF(0.025) = x_q$, i.e., we want to find x_q such that $CDF(X \leq x_q) = 0.025$. We know that *CDF* is the integral of the *PDF*, i.e., the area under the curve representing the probability density of X . Hence, we're

looking for find x_q such that $CDF(X \leq x_q) = \int_{-\infty}^{x_q} p(x) dx = 0.025$. This means that we're looking for the value in the x-axis for which the integral of the plot adds up to 0.05, which represents 2.5% of the total area under the curve.

For $X \sim \mathcal{N}(0, 1)$, $CDF(X \leq x_q) = \int_{-\infty}^{x_q} p(x) dx = 0.025 \implies x_q = -1.96$, i.e., $PPF(0.025) = -1.96$, the red value in the plot below. The blue value is inferred because of the normal distribution symmetry. Hence, if we are interpreting the plot from a Bayesian perspective, for a standard normal distributed rv, there is a 2.5% chance that the observed value of it will be less than -1.96 . On the other hand, the frequentist approach would say that in a long run, 2.5% of the total observations will be of values less or equal to -1.96 .

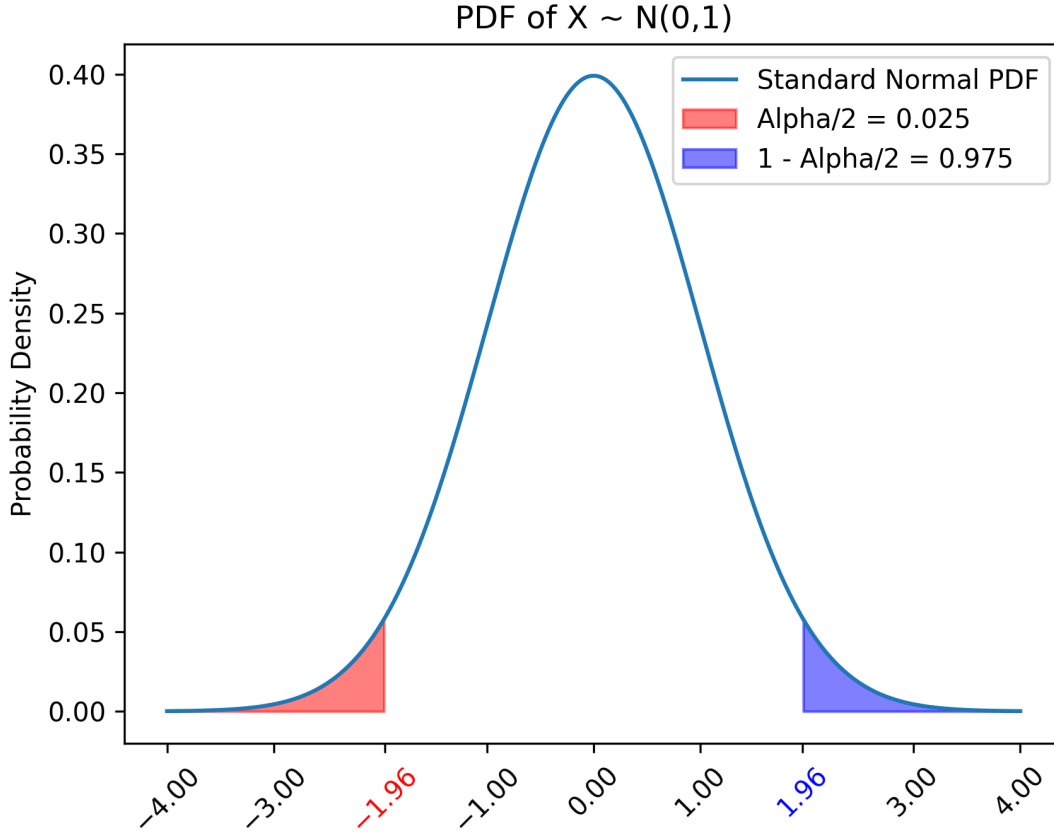


Figure 1: The sinusoidal embeddings difference vector's modules decrease linearly with the distance between the subtracted vectors

1.1 What Properties Should Sinusoidal Embeddings (SE) Should Apply?

1. **Periodicity:** the model trained with sinusoidal embeddings should be able to capture the relative positions of tokens effectively, which means that the distances between the embeddings of a pair of clauses should not depend on their absolute position within the longer sentence. This is achieved by making the sinusoidal embeddings functions periodic, which was to be expected from the name of the function.
2. **Unique Representation (Injective Function):** this one is straight forward from the fact that sinusoidal embeddings must represent the positions of tokens within a sentence. This means that sets of tokens in different positions should not be mapped to the same output, otherwise different positions would have the same representation.
3. **Scale Invariance:** sinusoidal embeddings should represent tokens positions consistently regardless of the sequence length. This property is crucial for handling sequences of varying lengths in a trans-

former model including those that are longer than the ones the model were trained on. Essentially, the scale invariance property says that the distance of two inputs x_t and x_{t-k} should be similar among different values of t . In other words, $x_t - x_{t-k}$ should not depend on t .

4. **Linearity:** this property is somewhat related to the scale invariance property. For the Sinusoidal Embedding function to be linear is good, because functions having such property are more easily learned by neural networks. Moreover, if the Sinusoidal Embedding Function is linear we also achieve the scale invariance property. This is true, because if such Sinusoidal Embedding function (SE) is linear, for any pair of sets of tokens separated by a distance of k , say X_t, X_{t+k} , there is a linear transformation M such that $M \times SE(X_t) = SE(X_{t+k})$. Hence, in order to represent X_{t+k} , the model must only learn the linear transformation M regardless of the value of k .

1.2 How are Sinusoidal Embeddings Defined?

The author's of the *Attention is All You Need* paper define the Sinusoidal Embeddings Function (SE) like the following.

Definition 1.2 (Sinusoidal Embeddings). *For a position pos in the sequence and a dimension i (where i ranges from 0 to $\frac{d}{2} - 1$), the embedding is given by:*

$$SE_{(pos, 2i)} = \sin\left(\frac{pos}{10000^{\frac{2i}{d}}}\right) \quad (4)$$

$$SE_{(pos, 2i+1)} = \cos\left(\frac{pos}{10000^{\frac{2i}{d}}}\right) \quad (5)$$

$$(6)$$

This function can be vectorially expressed as the following:

$$\mathbf{SE}(pos) = \left[\sin\left(pos \cdot e^{-\frac{2i \ln(10000)}{d}}\right), \cos\left(pos \cdot e^{-\frac{2i \ln(10000)}{d}}\right) \right]_{i=0}^{\frac{d}{2}-1} \quad (7)$$

where:

- pos is the position in the sequence.
- i is the dimension index.
- d is the dimensionality of the embeddings.

Notice that SE is essentially a function that outputs \sin values to the even dimensions of an input x_t and \cos for the odd ones with exponentially decreasing frequencies.

In the next section we will use the definition of SE function to verify the validity of the previous 4 properties it should have. Some verification will be done analitically and others will be done using *Python*.

2 Properties Verification

In this section, we will be checking each of the 4 presented properties of the Sinusoidal Embeddings using analytical techniques.

2.1 Peridiocity

This peridiocity of the SE function is straight-forward. Different dimensions i and $i+k$ of an input $x_t \in \mathbb{R}^d$ with position pos will have the same values output by the \sin and \cos because such functions are periodic.

i

Info: One question that a reader might have is whether the peridiocity property does not contradict the injectivity one. The answer for that is no. The peridiocity property is observed at a single dimension level, which means $SE_{(pos, 2i)}$ that outputs a single value, is be periodic. The injectivity, on the other hand, is observed at the entire embedding level, which means $SE_{(pos)}$, that outputs a vector (embedding) of dimension d is injective.

2.2 Unique Representation (Injective Function)

We are gonna prove that two sequences from different positions are **not** mapped to the same vector/output, using a *proof by contradiction*. Let's assume that two sequences x_{t_1} and x_{t_2} , with different positions t_1 and t_2 , respectively, are mapped to the same output using the previously SE function. This assumption implies the following **for any dimension i**

$$SE(x_{t_1}) = SE(x_{t_2}) \implies SE(t_1, 2i) = SE(t_2, 2i) \text{ and } SE(t_1, 2i + 1) = SE(t_2, 2i + 1) \quad (8)$$

$$\implies \sin\left(\frac{t_1}{10000^{\frac{2i}{d}}}\right) = \sin\left(\frac{t_2}{10000^{\frac{2i}{d}}}\right) \text{ and } \cos\left(\frac{t_1}{10000^{\frac{2i}{d}}}\right) = \cos\left(\frac{t_2}{10000^{\frac{2i}{d}}}\right) \quad (9)$$

For the last implication to be true, either the arguments of the \sin and \cos functions are the exact same or they are separated by $2\pi k$. We know they are not the same, because $t_1 \neq t_2$. Therefore, we're left with the condition:

$$\left| \frac{t_1}{10000^{\frac{2i}{d}}} - \frac{t_2}{10000^{\frac{2i}{d}}} \right| = 2\pi k \quad (10)$$

$$\implies \left| \frac{t_1 - t_2}{10000^{\frac{2i}{d}}} \right| = 2\pi k \quad (11)$$

$$\implies |t_1 - t_2| = 2\pi k 10000^{\frac{2i}{d}} \quad (12)$$

$$(13)$$

Since t_1 and t_2 are integers that represent the sequences positions, and $10000^{\frac{2i}{d}}$ is a positive real number, the right side of the equation $|t_1 - t_2| = 2\pi k \cdot 10000^{\frac{2i}{d}}$ must also be an integer. However, $2\pi k \cdot 10000^{\frac{2i}{d}}$ is generally not an integer because 2π is an irrational number, which leads us to a contradiction.

Hence, the initial assumption $SE(x_{t_1}) = SE(x_{t_2})$ must be false, which let's us say that **two sequences x_{t_1} and x_{t_2} , with different positions t_1 and t_2 , respectively, are not mapped to the same output using the previously SE function.**

2.3 Linearity & Scale Invariance

As previously mentioned, the scale invariance property is a consequence of $SE(pos)$'s linearity. Hence, by proving that $SE(pos)$ is linear we also prove that such function is scale invariant.

We need to find $M \in \mathbb{R}^{d \times d}$ such that $M \times SE(x_t) = SE(x_{t+k})$. We have the following system of equations:

$$\begin{bmatrix} m_{00} & m_{01} & \cdots & m_{0(d-1)} \\ m_{10} & m_{11} & \cdots & m_{1(d-1)} \\ \vdots & \vdots & \ddots & \vdots \\ m_{(d-1)0} & m_{(d-1)1} & \cdots & m_{(d-1)(d-1)} \end{bmatrix} \begin{bmatrix} \sin(\omega_0 t) \\ \cos(\omega_0 t) \\ \vdots \\ \sin(\omega_{(d-2)/2} t) \\ \cos(\omega_{(d-2)/2} t) \end{bmatrix} = \begin{bmatrix} \sin(\omega_0(t+k)) \\ \cos(\omega_0(t+k)) \\ \vdots \\ \sin(\omega_{(d-2)/2}(t+k)) \\ \cos(\omega_{(d-2)/2}(t+k)) \end{bmatrix} \quad (14)$$

$$m_{00} \sin(\omega_0 t) + m_{01} \cos(\omega_0 t) + \cdots + m_{0(d-1)} \cos(\omega_{d-1} t) = \sin(\omega_0 t) \cos(\omega_0 k) + \sin(\omega_0 k) \cos(\omega_0 t) \quad (15)$$

$$m_{10} \sin(\omega_0 t) + m_{11} \cos(\omega_0 t) + \cdots + m_{1(d-1)} \cos(\omega_{d-1} t) = \cos(\omega_0 t) \cos(\omega_0 k) - \sin(\omega_0 k) \sin(\omega_0 t) \quad (16)$$

\vdots

$$m_{(d-1)0} \sin(\omega_0 t) + m_{(d-1)1} \cos(\omega_0 t) + \cdots + m_{(d-1)(d-1)} \cos(\omega_{(d-2)/2} t) = \cos(\omega_{(d-2)/2} t) \cos(\omega_{(d-2)/2} k) - \sin(\omega_{(d-2)/2} k) \sin(\omega_{(d-2)/2} t) \quad (17)$$

This system has a solution:

$$m_{00} = \cos(\omega_0 k) \quad (18)$$

$$m_{01} = \sin(\omega_0 k) \quad (19)$$

$$m_{02} = m_{03} = \dots = m_{0(d-1)} = 0 \quad (20)$$

$$m_{10} = -\sin(\omega_0 k) \quad (21)$$

$$m_{11} = \cos(\omega_0 k) \quad (22)$$

$$m_{12} = m_{13} = \dots = m_{1(d-1)} = 0 \quad (23)$$

$$m_{22} = \cos(\omega_1 k) \quad (24)$$

$$m_{23} = \sin(\omega_1 k) \quad (25)$$

$$m_{20} = m_{21} = m_{24} = \dots = m_{2(d-1)} = 0 \quad (26)$$

$$m_{32} = -\sin(\omega_1 k) \quad (27)$$

$$m_{33} = \cos(\omega_1 k) \quad (28)$$

$$m_{30} = m_{31} = m_{34} = \dots = m_{3(d-1)} = 0 \quad (29)$$

\vdots

$$m_{(d-2)(d-2)} = \cos(\omega_{(d-2)/2} k) \quad (30)$$

$$m_{(d-2)(d-1)} = \sin(\omega_{(d-2)/2} k) \quad (31)$$

$$m_{(d-2)0} = m_{(d-2)1} = m_{(d-2)2} = \dots = m_{(d-2)(d-3)} = 0 \quad (32)$$

$$m_{(d-1)(d-2)} = -\sin(\omega_{(d-2)/2} k) \quad (33)$$

$$m_{(d-1)(d-1)} = \cos(\omega_{(d-2)/2} k) \quad (34)$$

$$m_{(d-1)0} = m_{(d-1)1} = m_{(d-1)2} = \dots = m_{(d-1)(d-3)} = 0 \quad (35)$$

$$(36)$$

Which lets us define M as:

$$M = \begin{bmatrix} \cos(\omega_0 k) & \sin(\omega_0 k) & 0 & 0 & \dots & 0 & 0 \\ -\sin(\omega_0 k) & \cos(\omega_0 k) & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \cos(\omega_1 k) & \sin(\omega_1 k) & \dots & 0 & 0 \\ 0 & 0 & -\sin(\omega_1 k) & \cos(\omega_1 k) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \cos(\omega_{(d-2)/2} k) & \sin(\omega_{(d-2)/2} k) \\ 0 & 0 & 0 & 0 & \dots & -\sin(\omega_{(d-2)/2} k) & \cos(\omega_{(d-2)/2} k) \end{bmatrix} \quad (37)$$

We found a matrix $M \in \mathbb{R}^{d \times d}$ such that $M \times SE(x_t) = SE(x_{t+k})$. Hence, SE is a linear function. Moreover, notice how \mathbf{M} **does not depend on t** , only on k . This is what gives us the scale invariability property.

3 Visualizing the Sinusoidal Embeddings Properties With *Python*

In this section, we will write some *Python* functions and classes to visualize the 4 cited properties of Sinusoidal Embeddings. Visualization is a great way to grasp such function's behaviour without having to necessarily prove it (even though you can always come back to this post with you're interested in the proofs).

3.1 Sinusoidal Embedding Definition With Pytorch

The sinusoidal embeddings module will store a multi-embedding tensor with $shape = (max_pos, embed_dim)$, where max_pos represents the maximum position we are interested in representing. This way, each of such tensor's row represents a the sinusoidal embedding for a position pos such that $SE(pos), 0 \leq pos < max_pos$.

```

1 import torch.nn as nn
2
3
4 class SinusoidalEmbeddings(nn.Module):
5
6     def __init__(self, max_pos:int, embed_dim: int):
7         super().__init__()
8         # Returns a tensor with shape (time_steps, 1).
9         positions = torch.arange(max_pos).unsqueeze(1).float()
10
11         # Creates a tensor with shape (embed_dim //2,). We just
12         # need
13         # half of the dimensions of the input embeddings to
14         # compute all
15         # of the sinusoidal embeddings frequencies
16         dimensions = torch.arange(start = 0, end = embed_dim,
17                                   step = 2).float()
18
19         # Compute the frequencies vector
20         frequencies = torch.exp(dimensions * -(math.log(10000.0)
21                                           / embed_dim))
22
23         # Initialize the embeddings tensor with shape (
24         # time_steps, embed_dim)
25         embeddings = torch.zeros(time_steps, embed_dim,
26                                   requires_grad=False)
27
28         # Apply sin to even indices (0, 2, 4, ...) of the input
29         # embeddings
30         embeddings[:, 0::2] = torch.sin(positions * frequencies)
31
32         # Apply cos to odd indices (1, 3, 5, ...) of the input
33         # embeddings
34         embeddings[:, 1::2] = torch.cos(positions * frequencies)
35
36         self.embeddings = embeddings
37
38     def forward(self, x, t):
39         embeds = self.embeddings[t].to(x.device)
40         return embeds[:, :, None, None]

```

Listing 1: Sinusoidal Embedding Module Definition

3.2 Sinusoidal Embeddings Periodicity

As previously mentioned, the periodicity of Sinusoidal Embeddings is observed in each of its dimensions. Hence, we need to plot its dimensions values for different positions to see their periodic behavior.

```

1 max_pos = 100
2
3
4 # Our embeddings will only have 4 dimension
5 embed_dim = 4
6 sinusoidal_embeddings = SinusoidalEmbeddings(max_pos, embed_dim)
7
8 # Generate embeddings for a range of time steps
9 embeddings = sinusoidal_embeddings.embeddings

```

```

10
11 # Convert embeddings to numpy for plotting
12 embeddings_np = embeddings.numpy()
13
14 # Plot the sinusoidal embeddings for different time steps
15 plt.figure(figsize=(14, 8))
16 for i in range(embed_dim):
17     plt.plot(embeddings_np[:, i], label=f"Dim {i} (i = {i//2})")
18
19 plt.title("SE(pos, 2i)")
20 plt.xlabel("pos")
21 plt.ylabel("Value")
22 plt.legend(loc="upper right", bbox_to_anchor=(1.15, 1))

```

Listing 2: Generating the plot of the embedding's dimensions for in different positions

As noticed in the plot above, the value of the function $SE(pos, 2i)$ defined in 1.2 repeats itself in a frequency that is inversely proportional to i , which is the dimension being represented (an index in the multidimensional embedding). As a consequence, **the higher the dimension of our model's embeddings (previously called d and called $embed_dim$ in the Sinusoidal Embedding module), the less SE values vary.**

Intuitively, such consequence means that embeddings close to each other (they represent sentences in not very distant positions), will have their differences captured in lower dimensions, because their higher dimensions are likely to be very similar. The exact opposite is true for embeddings that represent sentences that are far away from each other. Let's visualize that by plotting different embeddings' heatmaps.

```

1
2 import torch
3
4 # We'll create embeddings with many dimensions to better see the
5 # frequency decay effect
6 max_pos = 100
7 embed_dim = 128
8 sinusoidal_embeddings = SinusoidalEmbeddings(max_pos, embed_dim)
9
10 x = torch.zeros(embed_dim)
11 results = torch.zeros(max_pos, embed_dim)
12 for pos in range(max_pos):
13     results[pos] = x + sinusoidal_embeddings.embeddings[pos]
14
15 tensor_np = results.numpy()
16 # Plot the heatmap using matplotlib
17 plt.figure(figsize=(16, 6))
18 plt.imshow(tensor_np, aspect='auto', cmap='RdBu')
19 plt.colorbar(label='')
20 plt.xlabel('2i')
21 plt.ylabel('pos')
22 plt.title('SE(pos, 2i)')
23 plt.show()

```

Listing 3: Generating the plot of the sinusoidal embeddings for different positions

In the plot above, we see that embeddings with close positions (two close values in the pos axis) have different values of $SE(pos, 2i)$ for very small $2i$ (lower dimensions) while their values for higher dimensions are similar. On the other hand, if we pick two embeddings with very distant pos , they might differ from each other only in higher dimension. What this means is that in order to represent longer sentences (that contain positions very distant from each other), our model needs to have more dimensions. **The longer the input sentences, the higher the model's dimensions need to be.**

3.3 Unique Representativeness (SE is an injective function)

Figure ?? shows us that no two rows are the same because of the exponential decay of the frequencies with the increase of the dimension (increase of $2i$). As each row represents embeddings with different positions, what the figure is essentially showing us is that two embeddings with different positions will never have the same representation, i.e., SE is injective.

3.4 Linearity & Scale Invariance

As previously mentioned, to be scale invariant, SE must be such that the distance between $SE(pos)$ and $SE(pos+k)$ must be the same as the one between $SE(0)$ and $SE(k)$. In order to check that, we'll calculate the distance between every two sinusoidal embeddings of our module and see the linear property of such distance. Hence, all the rows will have to be subtracted from the first one, from the second one and so on up until the last one.

```
1 import torch
2
3
4 # We'll create embeddings with many dimensions to better see the
5 # frequency decay effect
6 max_pos = 1000
7 embed_dim = 1000
8
9
10 sinusoidal_embeddings = SinusoidalEmbeddings(max_pos, embed_dim).
11     embeddings
12
13 # A single column tensor where each row's single element contains an
14 # entire sinusoidal embeddings tensor
15 T_2 = sinusoidal_embeddings[:, None, :]
16
17 # A single row tensor where column's single element contains an
18 # entire sinusoidal embeddings tensor
19 T_1 = sinusoidal_embeddings[None, :, :]
20
21 # By broadcasting, this operation will save the desired differences
22 # in the tensor's last dimension
23 differences = T_2 - T_1
24
25 # As the differences were saved in the last dimension, we need
26 # to calculate the norm with respect to it, which is indicated
27 # by dim=-1
28 distances = torch.norm(differences, p=2, dim=-1)
29
30 # Plot the resulting 2D tensor as a heatmap
31 plt.figure(figsize=(8, 6))
32 plt.imshow(distances.numpy(), aspect="auto", cmap="PuRd")
33 plt.colorbar(label="L2 Norm (Euclidean Distance)")
34 plt.xlabel("pos")
35 plt.ylabel("pos")
36 plt.title("Euclidean Distances Between Different Position Sinusoidal
37     Embeddings")
38 plt.show()
```

Listing 4: Generating the plot of the module of the difference between every two different positions sinusoidal embeddings

Figure ?? above shows us how the distance between sinusoidal embeddings of different positions decreases smoothly and linearly for embedding with $d = 1000$.

Question 1

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- (a) Do this.
- (b) Do that.
- (c) Do something else.

3.5 Algorithmic issues

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Algorithm 1: FastTwoSum

Input: (a, b) , two floating-point numbers

Result: (c, d) , such that $a + b = c + d$

```
if  $|b| > |a|$  then
  | exchange  $a$  and  $b$  ;
end
 $c \leftarrow a + b$  ;
 $z \leftarrow c - a$  ;
 $d \leftarrow b - z$  ;
return  $(c, d)$  ;
```

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Question 2 (with optional title)

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4 Implementation

Proin lobortis efficitur dictum. Pellentesque vitae pharetra eros, quis dignissim magna. Sed tellus leo, semper non vestibulum vel, tincidunt eu mi. Aenean pretium ut velit sed facilisis. Ut placerat urna facilisis dolor suscipit vehicula. Ut ut auctor nunc. Nulla non massa eros. Proin rhoncus arcu odio, eu lobortis metus sollicitudin eu. Duis maximus ex dui, id bibendum diam dignissim id. Aliquam quis lorem lorem. Phasellus sagittis aliquet dolor, vulputate cursus dolor convallis vel. Suspendisse eu tellus feugiat, bibendum lectus quis, fermentum nunc. Nunc euismod condimentum magna nec bibendum. Curabitur elementum nibh eu sem cursus, eu aliquam leo rutrum. Sed bibendum augue sit amet pharetra ullamcorper. Aenean congue sit amet tortor vitae feugiat.

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```
hello.py
1  #! /usr/bin/python
2
3  import sys
4  sys.stdout.write("Hello World!\n")
```

Fusce eleifend porttitor arcu, id accumsan elit pharetra eget. Mauris luctus velit sit amet est sodales rhoncus. Donec cursus suscipit justo, sed tristique ipsum fermentum nec. Ut tortor ex, ullamcorper varius congue in, efficitur a tellus. Vivamus ut rutrum nisi. Phasellus sit amet enim efficitur, aliquam nulla id, lacinia mauris. Quisque viverra libero ac magna maximus efficitur. Interdum et malesuada fames ac ante ipsum primis in faucibus. Vestibulum mollis eros in tellus fermentum, vitae tristique justo finibus. Sed quis vehicula nibh. Etiam nulla justo, pellentesque id sapien at, semper aliquam arcu. Integer at commodo arcu. Quisque dapibus ut lacus eget vulputate.

Command Line

```
$ chmod +x hello.py
$ ./hello.py

Hello World!
```

Vestibulum sodales orci a nisi interdum tristique. In dictum vehicula dui, eget bibendum purus elementum eu. Pellentesque lobortis mattis mauris, non feugiat dolor vulputate a. Cras porttitor dapibus lacus at pulvinar. Praesent eu nunc et libero porttitor malesuada tempus quis massa. Aenean cursus ipsum a velit ultricies sagittis. Sed non leo ullamcorper, suscipit massa ut, pulvinar erat. Aliquam erat volutpat. Nulla non lacus vitae mi placerat tincidunt et ac diam. Aliquam tincidunt augue sem, ut vestibulum est volutpat eget. Suspendisse potenti. Integer condimentum, risus nec maximus elementum, lacus purus porta arcu, at ultrices diam nisl eget urna. Curabitur sollicitudin diam quis sollicitudin varius. Ut porta erat ornare laoreet euismod. In tincidunt purus dui, nec egestas dui convallis non. In vestibulum ipsum in dictum scelerisque.



Notice: In congue risus leo, in gravida enim viverra id. Donec eros mauris, bibendum vel dui at, tempor commodo augue. In vel lobortis lacus. Nam ornare ullamcorper mauris vel molestie. Maecenas vehicula ornare turpis, vitae fringilla orci consectetur vel. Nam pulvinar justo nec neque egestas tristique. Donec ac dolor at libero congue varius sed vitae lectus. Donec et tristique nulla, sit amet scelerisque orci. Maecenas a vestibulum lectus, vitae gravida nulla. Proin eget volutpat orci. Morbi eu aliquet turpis. Vivamus molestie urna quis tempor tristique. Proin hendrerit sem nec tempor sollicitudin.