# Nhập môn Kỹ thuật Truyền thông

## Phần 2: Các kỹ thuật điều chế số
## (Digital Modulations)

## Bài 10: Không gian tín hiệu PSK
## (Phase Shift Keying)

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## ***2-PSK**: characteristics*

1. Bandpass modulation
2. One-dimensional signal space and antipodal binary constellation (equal to 2-PAM)
3. TX filter $p(t)cos(2\pi f_0 t)$
4. Information associated to the carrier phase = Phase Shift Keying

# ***2-PSK**: constellation*

SIGNAL SET $$M = \{s_1(t) = +\alpha\, p(t)\cos(2\pi f_0 t)\ ,\ s_2(t) = -\alpha\, p(t)\cos(2\pi f_0 t)\ \}$$

Information associated to the impulse amplitude
BUT
we can also write

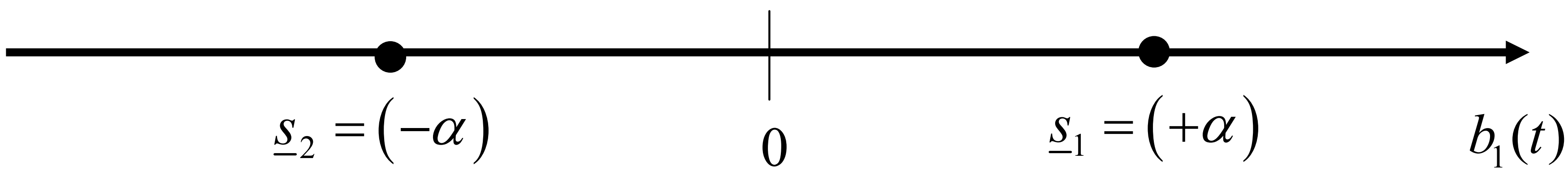
SIGNAL SET $$\boxed{M = \{s_1(t) = +\alpha\, p(t)\cos(2\pi f_0 t)\ ,\ s_2(t) = +\alpha\, p(t)\cos(2\pi f_0 t - \pi)\ \}}$$

**Information associated to the carrier phase**

# ***2-PSK**: constellation*

Versor $b_1(t)=p(t)\cos(2\pi f_0 t) \quad (d=1)$

VECTOR SET $M = \{s_1 = (+\alpha)\,,\, s_2 = (-\alpha)\} \subseteq R$



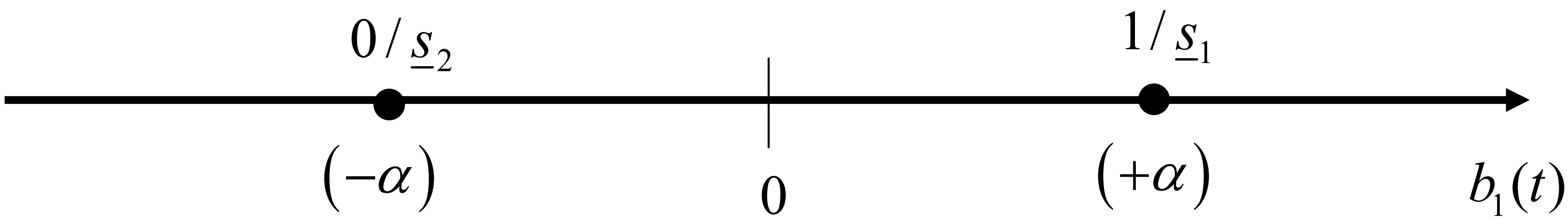
# ***2-PSK**: binary labeling*

(example)

$$e : H_1 \leftrightarrow M$$

$$e(1) = \underline{s}_1$$

$$e(0) = \underline{s}_2$$



# **2-PSK**: *transmitted waveform*

$m = 2 \rightarrow k = 1$ $\boxed{R = R_b}$ $\boxed{T = T_b}$

Transmitted waveform $$s(t) = \sum_{n=-\infty}^{+\infty} a[n] b_1(t - nT)$$

where $$a[n] \in \{+\alpha, -\alpha\}$$

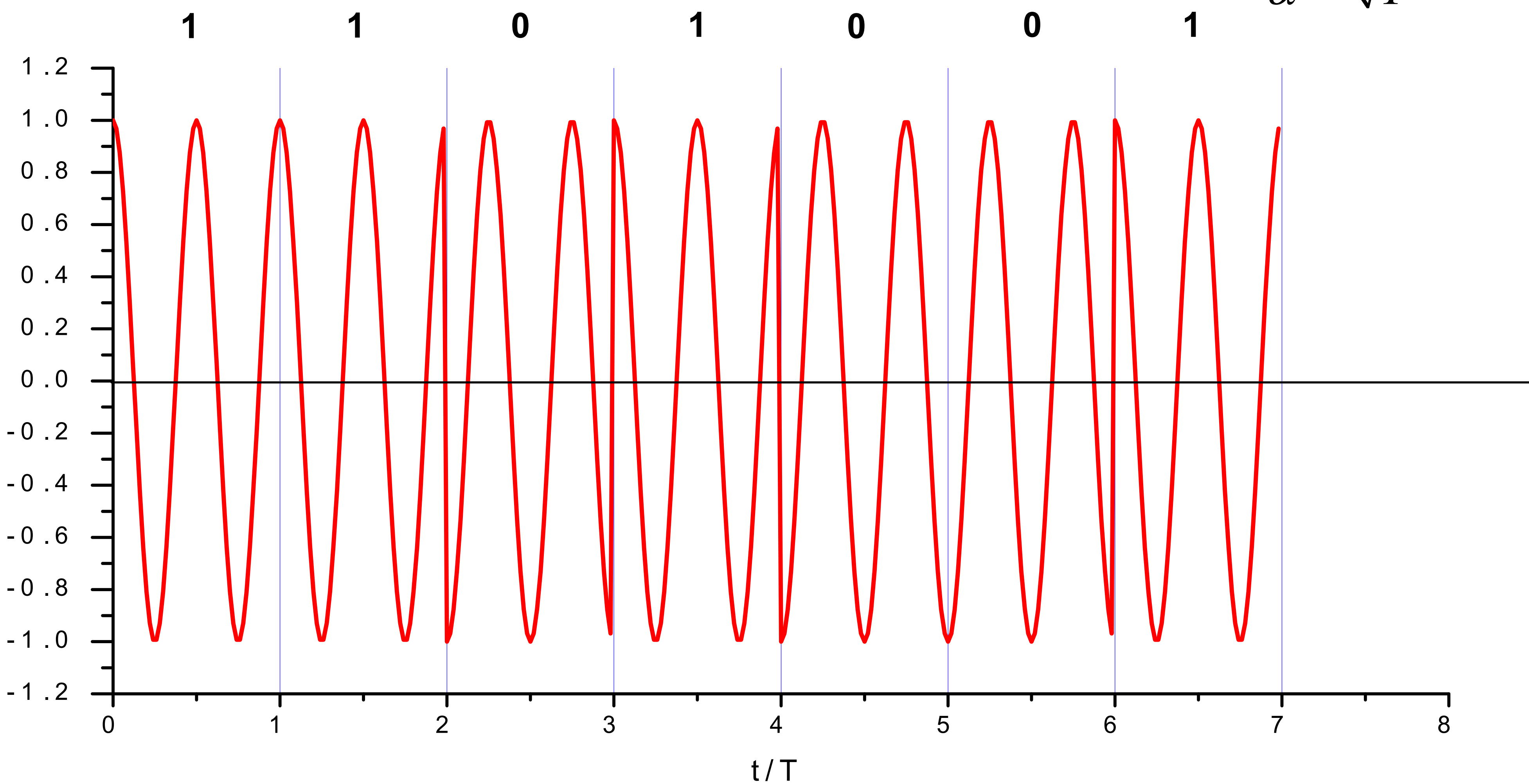
$$b_1(t) = p(t) \cos(2\pi f_0 t)$$

# **2-PSK**: *transmitted waveform*

example for $p(t) = \frac{1}{\sqrt{T}} P_T(t)$

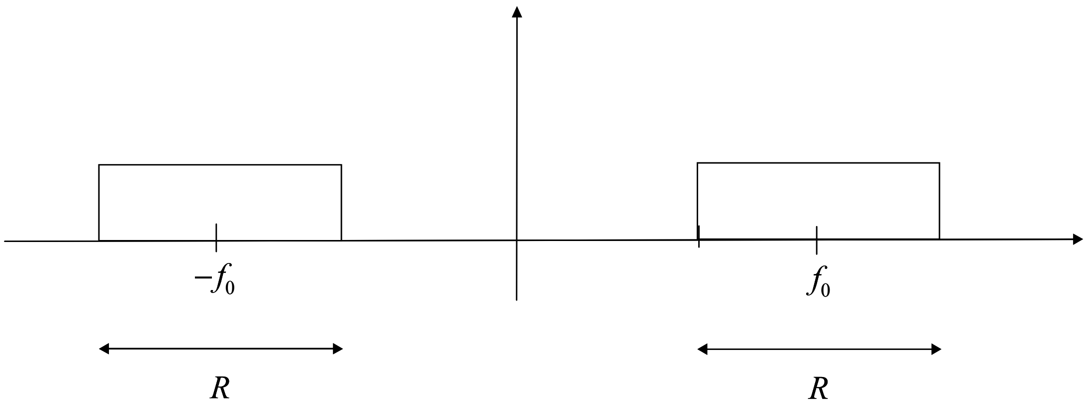
$f_0 = 2R_b$

$\alpha = \sqrt{T}$



# **2-PSK**: *bandwidth and spectral efficiency*

Case 1: $p(t)$ = ideal low pass filter




Total bandwidth
(ideal case)

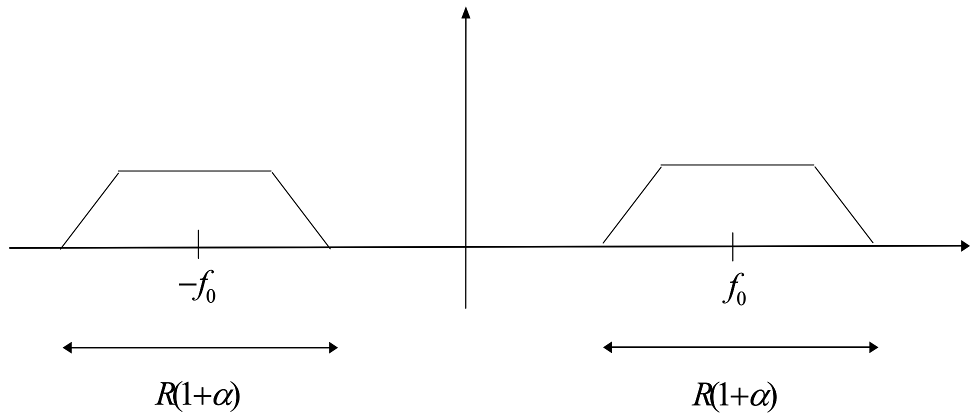
$$\boxed{B_{id} = R = R_b}$$

Spectral efficiency
(ideal case)

$$\boxed{\eta_{id} = \frac{R_b}{B_{id}} = 1\ bps/Hz}$$

# **2-PSK**: *bandwidth and spectral efficiency*

Case 2: $p(t)$ = RRC filter with roll off $\alpha$




Total bandwidth $$B = R(1+\alpha) = R_b(1+\alpha)$$

Spectral efficiency $$\boxed{\eta = \frac{R_b}{B} = \frac{1}{(1+\alpha)}\ bps/Hz}$$

# *Exercize*

Given a bandpass channel with bandwidth $B$ = 4000 Hz, centred around $f_0$=2 GHz, compute the maximum bit rate $R_b$ we can transmit over it with a 2-PSK constellation in the two cases:

- Ideal low pass filter
- RRC filter with ⍰=0.25

# ***2-PSK***: *modulator*

The transmitted waveform is given by $$s(t) = \sum_n a[n] b_1(t - nT)$$

Where $$b_1(t) = p(t)\cos(2\pi f_0 t)$$

Then we must generate $$s(t) = \sum_n a[n] p(t - nT)\cos(2\pi f_0 (t - nT))$$

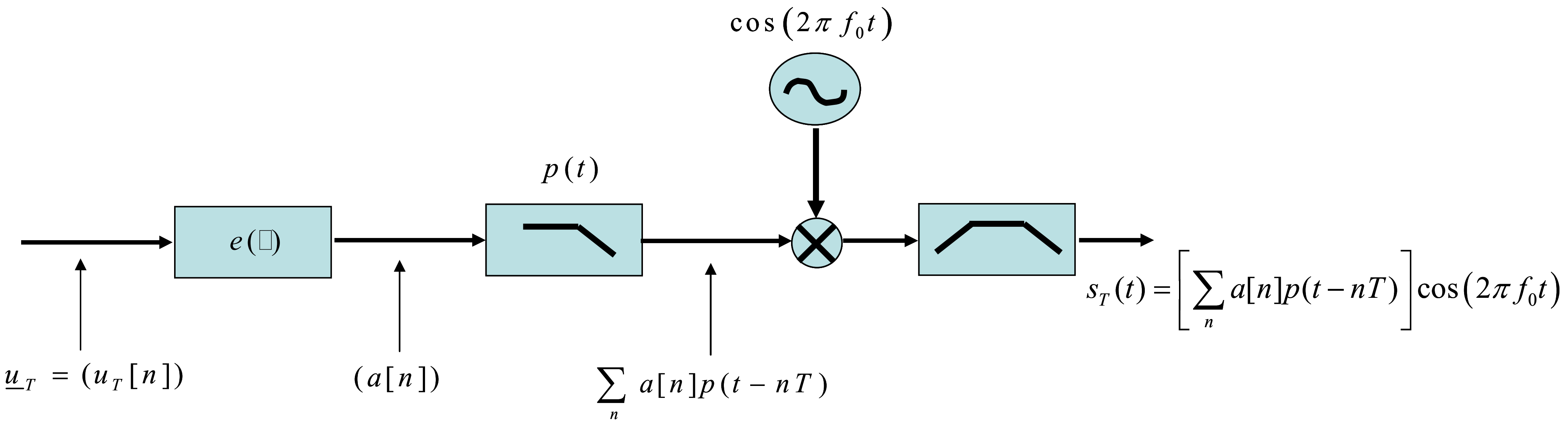
**We choose $f_0$ multiple of $R=1/T$**

It follows $$\cos(2\pi f_0 (t - nT)) = \cos(2\pi f_0 t - 2\pi f_0 nT) = \cos(2\pi f_0 t)$$

Then we can generate

$$\boxed{s(t) = \left[\sum_n a[n] p(t - nT)\right]\cos(2\pi f_0 t)}$$

# **2-PSK**: *modulator*



# 2-PSK: demodulator

Given the received signal $\rho(t)$

the received symbol is obtained by projecting it on the versor $b_1(t) = p(t)\cos(2\pi f_0 t)$

$$\rho[0] = \int_{-\infty}^{+\infty} \rho(t) b_1(t) dt = \boxed{\int_{-\infty}^{+\infty} \rho(t) p(t) \cos(2\pi f_0 t) dt}$$

This projection could be computed by using a matched filter

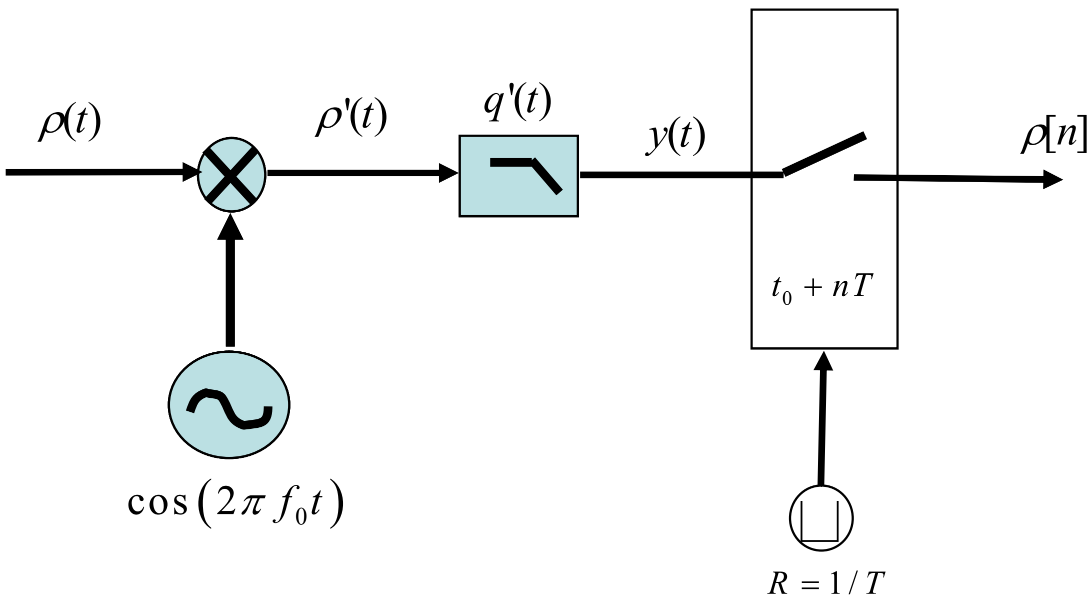
$$q(t) = b_1(T-t) = p(T-t)\cos(2\pi f_0 (T-t))$$

# *2-PSK: demodulator*

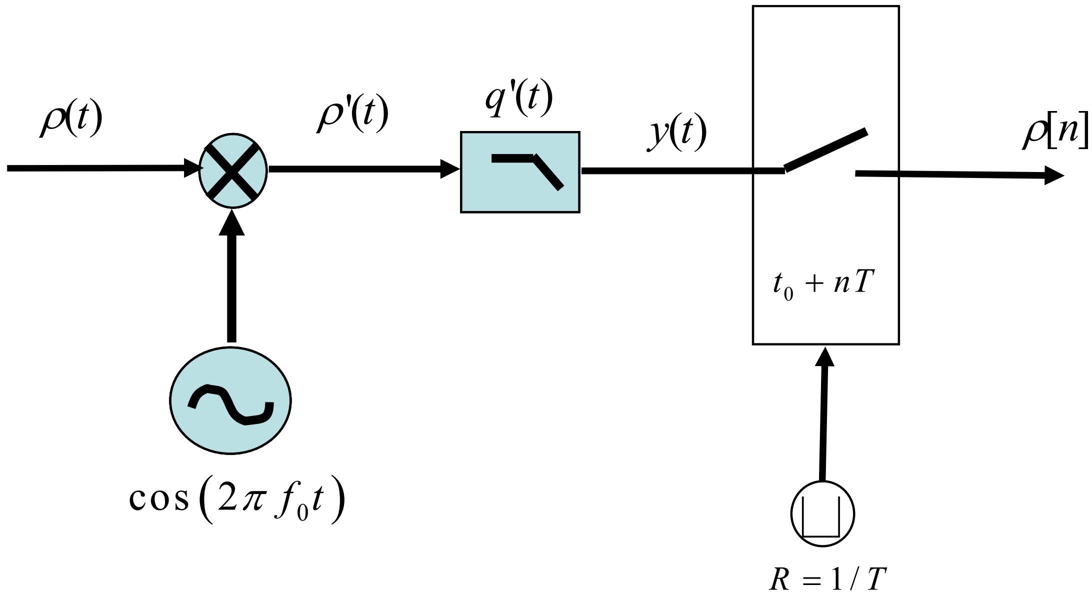
As an alternative, we can work as follows:

1. Given the received signal $\rho(t)$ multiply it by $\cos(2\pi f_0 t)$

2. Use a filter matched to $p(t)$: $q'(t) = p(T - t)$



# **2-PSK**: *demodulator*

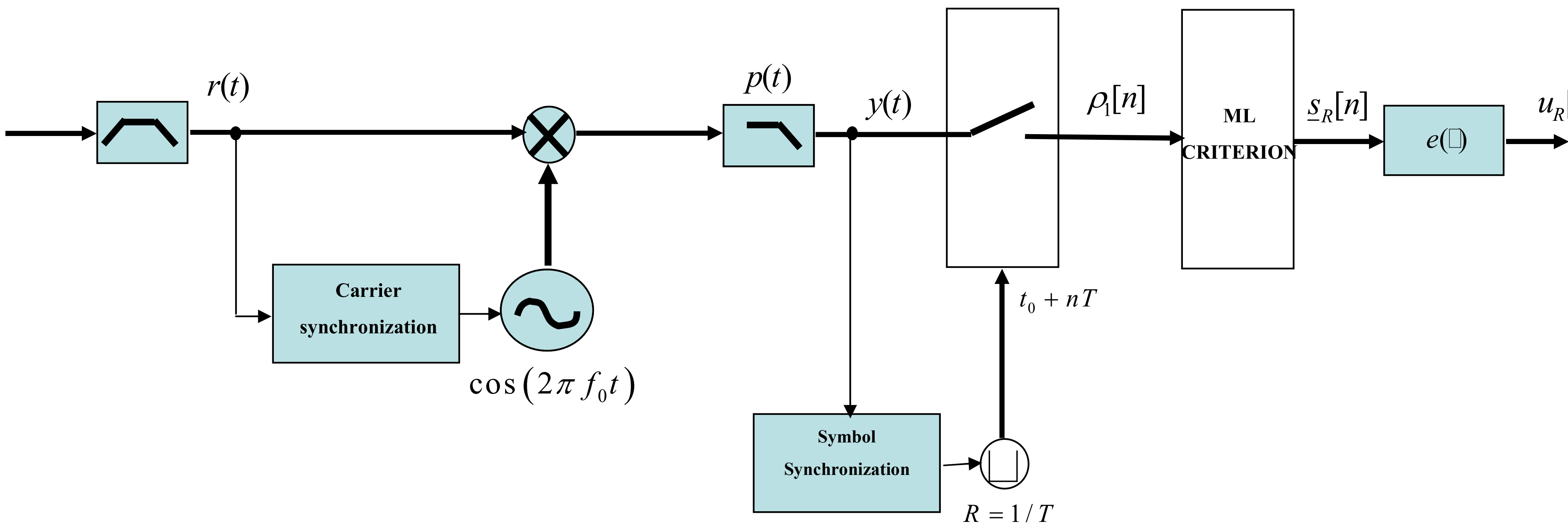



By sampling the matched filter output waveform we obtain

$$y(t) = \int_{-\infty}^{+\infty} \rho'(\tau) q'(t-\tau) d\tau = \int_{-\infty}^{+\infty} \rho(\tau) \cos(2\pi f_0 \tau) p(T - t + \tau) d\tau$$

$$y(t=T) = \boxed{\int_{-\infty}^{+\infty} \rho(\tau) \cos(2\pi f_0 \tau) p(\tau) d\tau} = \rho[0]$$

# 2-PSK: demodulator



# **2-PSK**: *interpretation*

We generate a baseband signal

$$v(t) = \sum_n a[n] p(t - nT)$$

Multiplication by cosine shifts the spectrum around $f_0$

$$s(t) = v(t)\cos(2\pi f_0 t)$$

# 2-PSK: *interpretation*

At the receiver side, multiplication by cosine generates

$$s(t)\cos(2\pi f_0 t) = v(t)\cos(2\pi f_0 t)\cos(2\pi f_0 t) = v(t)\cos^2(2\pi f_0 t) = v(t)\left[\frac{1+\cos(2\pi\left(2f_0\right)t)}{2}\right]$$

This signal enters the matched filter $q(t)=p(T-t)$.

It is a low pass filter: the high frequency component around $2f_0$ is eliminated

Only the baseband component $v(t) = \sum_n a[n]p(t-nT)$ survives

The matched filter output is then equal to $a[n]$ when sampled at $t_0 + nT$

# *2-PSK: analytic signal*

The 2-PSK transmitted waveform

$$s(t) = \left[ \sum_n a[n] p(t - nT) \right] \cos(2\pi f_0 t)$$

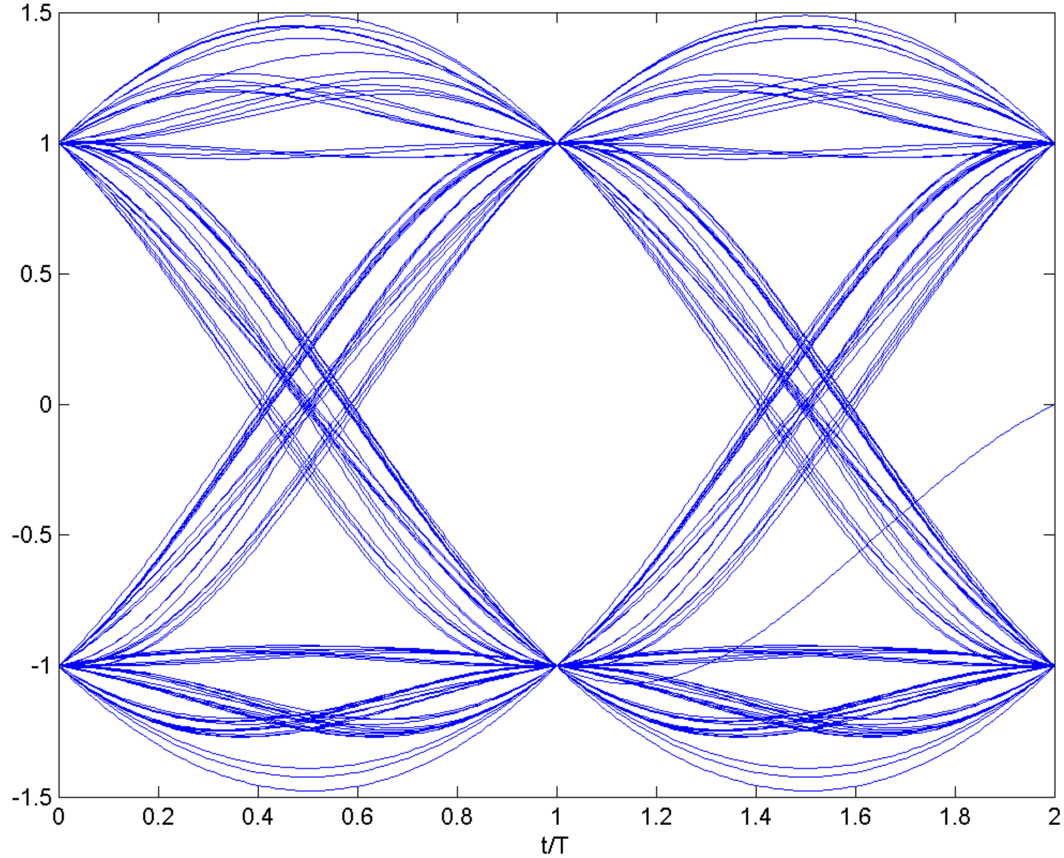
can be written as

$$s(t) = \mathrm{Re}\left[ \dot{s}(t) \right] = \mathrm{Re}\left[ \sum_n a[n] p(t - nT)\, \mathrm{e}^{j2\pi f_0 t} \right]$$

Where $\dot{s}(t)$ is called the **analytic signal** associated with $s(t)$

# ***2-PSK**: Eye diagram*

2-PSK constellation with RRC filter (=0.5)



# ***2-PSK**: error probability*

ERROR PROBABILITY

$$BER = \frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}}$$