# Bài tập phương trình vi phân: Thầy Trinh Ngọc Hải

# A. Phương trình cấp 1

# 1. Phương trình phân ly biến số: Bài tập:51-65

**51.** 
$$xy dx + (x+1) dy = 0$$
.

**52.** 
$$\sqrt{y^2+1} \, dx = xy \, dy$$
.

**53.** 
$$(x^2 - 1)y' + 2xy^2 = 0$$
;  $y(0) = 1$ .

**54.** 
$$y' \operatorname{ctg} x + y = 2; \ y(x) \to -1$$
 при  $x \to 0$ .

**55.** 
$$y' = 3\sqrt[3]{y^2}$$
;  $y(2) = 0$ .

**56.** 
$$xy' + y = y^2$$
;  $y(1) = 0.5$ .

**57.** 
$$2x^2yy' + y^2 = 2$$
. **58.**  $y' - xy^2 = 2xy$ .

**58.** 
$$y' - xy^2 = 2xy$$
.

**59.** 
$$e^{-s} \left( 1 + \frac{ds}{dt} \right) = 1.$$
 **60.**  $z' = 10^{x+z}.$ 

**60.** 
$$z' = 10^{x+z}$$
.

**61.** 
$$x \frac{dx}{dt} + t = 1$$

**61.** 
$$x \frac{dx}{dt} + t = 1$$
. **62.**  $y' = \cos(y - x)$ .

**63.** 
$$y' - y = 2x - 3$$
.

**64.** 
$$(x+2y)y'=1$$
;  $y(0)=-1$ .

**65.** 
$$y' = \sqrt{4x + 2y - 1}$$
.

#### 2. Phương trình đẳng cấp: Bài tâp:101-129

**101.** 
$$(x+2y) dx - x dy = 0$$
.

**102.** 
$$(x-y) dx + (x+y) dy = 0.$$

**103.** 
$$(y^2 - 2xy) dx + x^2 dy = 0.$$

**104.** 
$$2x^3y' = y(2x^2 - y^2)$$
.

**105.** 
$$y^2 + x^2y' = xyy'$$
.

**106.** 
$$(x^2 + y^2)y' = 2xy$$
.

**120.** 
$$y' = \frac{y+2}{x+1} + \operatorname{tg} \frac{y-2x}{x+1}$$
.

121. 
$$x^3(y'-x)=y^2$$
.

122. 
$$2x^2y' = y^3 + xy$$
.

**123.** 
$$2x dy + (x^2y^4 + 1)y dx = 0.$$

**124.** 
$$y dx + x(2xy + 1) dy = 0$$
.

**125.** 
$$2y' + x = 4\sqrt{y}$$
.

126. 
$$y' = y^2 - \frac{2}{x^2}$$
.

**127.** 
$$2xy' + y = y^2\sqrt{x - x^2y^2}$$
.

**128.** 
$$\frac{2}{3}xyy' = \sqrt{x^6 - y^4} + y^2$$
.

**129.** 
$$2y + (x^2y + 1)xy' = 0$$
.

**107.** 
$$xy' - y = x \operatorname{tg} \frac{y}{x}$$
.

108. 
$$xy' = y - xe^{y/x}$$
.

**109.** 
$$xy' - y = (x+y) \ln \frac{x+y}{x}$$
.

**110.** 
$$xy' = y \cos \ln \frac{y}{x}$$
.

111. 
$$(y + \sqrt{xy}) dx = x dy$$
.

**112.** 
$$xy' = \sqrt{x^2 - y^2} + y$$
.

**113.** 
$$(2x - 4y + 6) dx + (x + y - 3) dy = 0$$
.

**114.** 
$$(2x + y + 1) dx - (4x + 2y - 3) dy = 0.$$

**115.** 
$$x - y - 1 + (y - x + 2)y' = 0$$
.

**116.** 
$$(x+4y)y'=2x+3y-5$$
.

**117.** 
$$(y+2) dx = (2x + y - 4) dy$$
.

**118.** 
$$y' = 2\left(\frac{y+2}{x+y-1}\right)^2$$
.

**119.** 
$$(y'+1)\ln\frac{y+x}{x+3} = \frac{y+x}{x+3}$$

3. Phương trình tuyến tính cấp 1: Bài tập:137-171

**137.** 
$$(2x+1)y'=4x+2y$$
.

138. 
$$y' + y \operatorname{tg} x = \sec x$$
.

**139.** 
$$(xy + e^x) dx - x dy = 0.$$

**140.** 
$$x^2y' + xy + 1 = 0$$
.

**141.** 
$$y = x(y' - x \cos x)$$
.

**142.** 
$$2x(x^2+y) dx = dy$$
.

**143.** 
$$(xy'-1)\ln x=2y$$
.

**144.** 
$$xy' + (x+1)y = 3x^2e^{-x}$$
.

**145.** 
$$(x+y^2) dy = y dx$$
.

**146.** 
$$(2e^y - x)y' = 1$$
.

**147.** 
$$(\sin^2 y + x \operatorname{ctg} y)y' = 1.$$

**148.** 
$$(2x + y) dy = y dx + 4 \ln y dy$$
.

149. 
$$y' = \frac{y}{3x-y^2}$$
.

**161.** 
$$x dx = (x^2 - 2y + 1) dy$$
.

**162.** 
$$(x+1)(yy'-1)=y^2$$
.

**163.** 
$$x(e^y - y') = 2$$
.

**164.** 
$$(x^2-1)y'\sin y + 2x\cos y = 2x-2x^3$$
.

**165.** 
$$y(x) = \int_{0}^{x} y(t) dt + x + 1.$$

**166.** 
$$\int_{0}^{x} (x-t)y(t) dt = 2x + \int_{0}^{x} y(t) dt.$$

**150.** 
$$(1-2xy)y'=y(y-1)$$
.

151. 
$$y' + 2y = y^2 e^x$$
.

**152.** 
$$(x+1)(y'+y^2)=-y$$
.

**153.** 
$$y' = y^4 \cos x + y \operatorname{tg} x$$
.

154. 
$$xy^2y' = x^2 + y^3$$
.

**155.** 
$$xy \, dy = (y^2 + x) \, dx$$
.

**156.** 
$$xy' - 2x^2\sqrt{y} = 4y$$
.

**157.** 
$$xy' + 2y + x^5y^3e^x = 0.$$

158. 
$$2y' - \frac{x}{y} = \frac{xy}{x^2-1}$$
.

**159.** 
$$y'x^3 \sin y = xy' - 2y$$
.

**160.** 
$$(2x^2y \ln y - x)y' = y$$
.

**167.** 
$$x^2y' + xy + x^2y^2 = 4$$
.

**168.** 
$$3y' + y^2 + \frac{2}{x^2} = 0$$
.

**169.** 
$$xy' - (2x+1)y + y^2 = -x^2$$
.

170. 
$$y' - 2xy + y^2 = 5 - x^2$$
.

171. 
$$y' + 2ye^x - y^2 = e^{2x} + e^x$$
.

# 4. Phương trình vi phân toàn phần: Bài tập:186-220

**186.** 
$$2xy dx + (x^2 - y^2) dy = 0.$$

**187.** 
$$(2 - 9xy^2)x dx + (4y^2 - 6x^3)y dy = 0.$$

**188.** 
$$e^{-y} dx - (2y + xe^{-y}) dy = 0.$$

**189.** 
$$\frac{y}{x} dx + (y^3 + \ln x) dy = 0.$$

**190.** 
$$\frac{3x^2+y^2}{y^2} dx - \frac{2x^3+5y}{y^3} dy = 0.$$

**191.** 
$$2x\left(1+\sqrt{x^2-y}\right)\,\mathrm{d}x-\sqrt{x^2-y}\,\mathrm{d}y=0.$$

**192.** 
$$(1 + y^2 \sin 2x) dx - 2y \cos^2 x dy = 0.$$

**193.** 
$$3x^2(1+\ln y)\,\mathrm{d}x = \left(2y-\frac{x^3}{y}\right)\mathrm{d}y.$$

**194.** 
$$\left(\frac{x}{\sin y} + 2\right) dx + \frac{(x^2 + 1)\cos y}{\cos 2y - 1} dy = 0.$$

**195.** 
$$(x^2 + y^2 + x) dx + y dy = 0.$$

**196.** 
$$(x^2 + y^2 + y) dx - x dy = 0.$$

**197.** 
$$y dy = (x dy + y dx)\sqrt{1 + y^2}$$
.

**198.** 
$$xy^2(xy'+y)=1$$
.

**199.** 
$$y^2 dx - (xy + x^3) dy = 0.$$

**200.** 
$$\left(y - \frac{1}{x}\right) dx + \frac{dy}{y} = 0.$$

**201.** 
$$(x^2 + 3 \ln y)y \, dx = x \, dy$$
.

**202.** 
$$y^2 dx + (xy + tg xy) dy = 0.$$

**203.** 
$$y(x+y) dx + (xy+1) dy = 0$$
.

**204.** 
$$y(y^2+1) dx + x(y^2-x+1) dy = 0$$
.

**205.** 
$$(x^2 + 2x + y) dx = (x - 3x^2y) dy$$
.

**206.** 
$$y dx - x dy = 2x^3 tg \frac{y}{x} dx$$
.

**207.** 
$$y^2 dx + (e^x - y) dy = 0$$
.

**208.** 
$$xy dx = (y^3 + x^2y + x^2) dy$$
.

**209.** 
$$x^2y(y\,dx + x\,dy) = 2y\,dx + x\,dy$$
.

**210.** 
$$(x^2 - y^2 + y) dx + x(2y - 1) dy = 0.$$

**211.** 
$$(2x^2y^2 + y) dx + (x^3y - x) dy = 0.$$

**212.** 
$$(2x^2y^3 - 1)y dx + (4x^2y^3 - 1)x dy = 0.$$

**213.** 
$$y(x+y^2) dx + x^2(y-1) dy = 0.$$

**214.** 
$$(x^2 - \sin^2 y) dx + x \sin 2y dy = 0.$$

**215.** 
$$x(\ln y + 2 \ln x - 1) dy = 2y dx$$
.

**216.** 
$$(x^2 + 1)(2x dx + \cos y dy) = 2x \sin y dx$$
.

**217.** 
$$(2x^3y^2 - y) dx + (2x^2y^3 - x) dy = 0.$$

**218.** 
$$x^2y^3 + y + (x^3y^2 - x)y' = 0$$
.

**219.** 
$$(x^2 - y) dx + x(y + 1) dy = 0.$$

**220.** 
$$y^2(y dx - 2x dy) = x^3(x dy - 2y dx).$$

#### 5. Phương trình không giải được đối với đạo hàm: Bài tập:241-296

**241.** 
$$y'^2 - y^2 = 0$$
.

**242.** 
$$8y'^3 = 27y$$
.

**243.** 
$$(y'+1)^3 = 27(x+y)^2$$
.

**245.** 
$$y^2(y'^2+1)=1$$
.

**245.** 
$$y'^2 - 4y^3 = 0$$
.

**245.** 
$$y'^2 - 4y^3 = 0$$
. **246.**  $y'^2 = 4y^3(1-y)$ .

**247.** 
$$xy'^2 = y$$
.

**245.** 
$$yy'^3 + x = 1$$
.

**249.** 
$$y'^3 + y^2 = yy'(y'+1)$$
.

**250.** 
$$4(1-y) = (3y-2)^2 y'^2$$
.

**251.** 
$$y'^2 + xy = y^2 + xy'$$
. **252.**  $xy'(xy' + y) = 2y^2$ .

**252.** 
$$xy'(xy'+y)=2y^2$$
.

**253.** 
$$xy'^2 - 2yy' + x = 0$$
. **254.**  $xy'^2 = y(2y' - 1)$ .

**254.** 
$$xy'^2 = y(2y'-1)$$
.

**255.** 
$$y'^2 + x = 2y$$
.

**256.** 
$$y'^3 + (x+2)e^y = 0$$
.

**257.** 
$$y'^2 - 2xy' = 8x^2$$
. **258.**  $(xy' + 3y)^2 = 7x$ .

**258.** 
$$(xy'+3y)^2=7x$$
.

**259.** 
$$y'^2 - 2yy' = y^2(e^x - 1)$$
.

**260.** 
$$y'(2y - y') = y^2 \sin^2 x$$
.

**261.** 
$$y'^4 + y^2 = y^4$$
.

**262.** 
$$x(y-xy')^2=x{y'}^2-2yy'$$
.

**263.** 
$$y(xy'-y)^2 = y - 2xy'$$
.

**264.** 
$$yy'(yy'-2x)=x^2-2y^2$$
.

**265.** 
$$y'^2 + 4xy' - y^2 - 2x^2y = x^4 - 4x^2$$
.

**266.** 
$$y(y-2xy')^2=2y'$$
.

**267.** 
$$x = y'^3 + y'$$
.

**268.** 
$$x(y'^2-1)=2y'$$
.

**269.** 
$$x = y'\sqrt{{y'}^2 + 1}$$
.

**270.** 
$$y'(x - \ln y') = 1$$
.

**271.** 
$$y = {y'}^2 + 2{y'}^3$$
.

**272.** 
$$y = \ln(1 + {y'}^2)$$
.

**273.** 
$$(y'+1)^3 = (y'-y)^2$$
.

**274.** 
$$y = (y'-1)e^{y'}$$
.

**275.** 
$$y'^4 - y'^2 = y^2$$
.

**276.** 
$$y'^2 - y'^3 = y^2$$
.

**277.** 
$${v'}^4 = 2uv' + v^2$$
.

**278.** 
$$y'^2 - 2xy' = x^2 - 4y$$
.

**279.** 
$$5y + {y'}^2 = x(x + y')$$
.

**280.** 
$$x^2y'^2 = xyy' + 1$$
.

**281.** 
$$y'^3 + y^2 = xyy'$$
.

**282.** 
$$2xy' - y = y' \ln yy'$$
.

**283.** 
$$y' = e^{xy'/y}$$
.

**284.** 
$$y = xy' - x^2y'^3$$
.

**285.** 
$$y = 2xy' + y^2y'^3$$
.

**286.** 
$$y(y-2xy')^3={y'}^2$$
.

Решить уравнения Лагранжа и Клеро (задачи 287—296).

**287.** 
$$y = xy' - y'^2$$
.

**288.** 
$$y + xy' = 4\sqrt{y'}$$
.

**289.** 
$$y = 2xy' - 4y'^3$$
.

**290.** 
$$y = xy' - (2 + y')$$
.

**291.** 
$$y'^3 = 3(xy' - y)$$
.

**292.** 
$$y = xy'^2 - 2y'^3$$
.

**293.** 
$$xy' - y = \ln y'$$
.

**294.** 
$$xy'(y'+2) = y$$
.

**295.** 
$$2y'^2(y-xy')=1$$
.

**296.** 
$$2xy' - y = \ln y'$$
.

#### 6. Các dang phương trình bậc nhất khác: Bài tập:301-420

**301.** 
$$xy' + x^2 + xy - y = 0$$
.

**302.** 
$$2xy' + y^2 = 1$$
.

**303.** 
$$(2xy^2 - y) dx + x dy = 0.$$

**304.** 
$$(xy'+y)^2=x^2y'$$
.

**305.** 
$$y - y' = y^2 + xy'$$
.

**306.** 
$$(x+2y^3)y'=y$$
.

**307.** 
$$y'^3 - y'e^{2x} = 0$$
.

**308.** 
$$x^2y' = y(x+y)$$
.

**309.** 
$$(1-x^2) dy + xy dx = 0.$$

**310.** 
$$y'^2 + 2(x-1)y' - 2y = 0$$
.

**311.** 
$$y + y' \ln^2 y = (x + 2 \ln y)y'$$
.

**312.** 
$$x^2y' - 2xy = 3y$$
.

**313.** 
$$x + yy' = y^2(1 + {y'}^2)$$
.

**314.** 
$$y = (xy' + 2y)^2$$
.

**315.** 
$$y' = \frac{1}{x - y^2}$$
.

**316.** 
$$y'^3 + (3x - 6)y' = 3y$$
. **317.**  $x - \frac{y}{y'} = \frac{2}{y}$ .

**317.** 
$$x - \frac{y}{y'} = \frac{2}{y}$$

**318.** 
$$2y'^3 - 3y'^2 + x = y$$
.

**319.** 
$$(x+y)^2y'=1$$
.

**320.** 
$$2x^3yy' + 3x^2y^2 + 7 = 0$$
.

$$321. \ \frac{\mathrm{d}x}{x} = \left(\frac{1}{y} - 2x\right) \, \mathrm{d}y.$$

**322.** 
$$xy' = e^y + 2y'$$
.

**323.** 
$$2(x-y^2) dy = y dx$$
.

**323.** 
$$2(x-y^2) dy = y dx$$
.

**324.** 
$$x^2y'^2 + y^2 = 2x(2 - yy')$$
.

**325.** 
$$dy + (xy - xy^3) dx = 0.$$

**326.** 
$$2x^2y' = y^2(2xy' - y)$$
.

**327.** 
$$\frac{y-xy'}{x+yy'}=2.$$

**328.** 
$$x(x-1)y' + 2xy = 1$$
.

**329.** 
$$xy(xy'-y)^2+2y'=0$$
.

**330.** 
$$(1-x^2)y'-2xy^2=xy$$
.

Решить уравнения 331-420.

**331.** 
$$y' + y = xy^3$$
.

**332.** 
$$(xy^4 - x) dx + (y + xy) dy = 0.$$

**333.** 
$$(\sin x + y) dy + (y \cos x - x^2) dx = 0.$$

**347.** 
$$(4xy-3)y'+y^2=1$$
.

**348.** 
$$y'\sqrt{x} = \sqrt{y-x} + \sqrt{x}$$
.

**349.** 
$$xy' = 2\sqrt{y}\cos x - 2y$$
.

**350.** 
$$3y'^4 = y' + y$$
.

**351.** 
$$y^2(y-xy')=x^3y'$$
.

**352.** 
$$y' = (4x + y - 3)^2$$
.

**353.** 
$$(\cos x - x \sin x)y dx + (x \cos x - 2y) dy = 0.$$

**354.** 
$$x^2y'^2 - 2xyy' = x^2 + 3y^2$$
.

**355.** 
$$\frac{xy'}{y} + 2xy \ln x + 1 = 0.$$

**356.** 
$$xy' = x\sqrt{y - x^2} + 2y$$
.

**357.** 
$$(1 - x^2y) dx + x^2(y - x) dy = 0.$$

**358.** 
$$(2xe^y + y^4)y' = ye^y$$
.

**334.** 
$$3y'^3 - xy' + 1 = 0$$
.

**335.** 
$$yy' + y^2 \operatorname{ctg} x = \cos x$$
.

**336.** 
$$(e^y + 2xy) dx + (e^y + x)x dy = 0.$$

**337.** 
$$xy'^2 = y - y'$$
.

**338.** 
$$x(x+1)(y'-1) = y$$
.

**339.** 
$$y(y-xy')=\sqrt{x^4+y^4}$$
.

**340.** 
$$xy' + y = \ln y'$$
.

**341.** 
$$x^2(dy - dx) = (x + y)y dx$$
.

**342.** 
$$y' + x\sqrt[3]{y} = 3y$$
.

**343.** 
$$(x\cos y + \sin 2y)y' = 1$$
.

**344.** 
$$y'^2 - yy' + e^x = 0$$
.

**345.** 
$$y' = \frac{x}{y}e^{2x} + y$$
.

**346.** 
$$(xy'-y)^3={y'}^3-1$$
.

**359.** 
$$xy'(\ln y - \ln x) = y$$
.

**360.** 
$$2y' = x + \ln y'$$
.

**361.** 
$$(2x^2y - 3y^2)y' = 6x^2 - 2xy^2 + 1$$
.

**362.** 
$$yy' = 4x + 3y - 2$$
.

**363.** 
$$y^2y' + x^2\sin^3 x = y^3\operatorname{ctg} x$$
.

**364.** 
$$2xy' - y = \sin y'$$
.

**365.** 
$$(x^2y^2+1)y+(xy-1)^2xy'=0$$
.

**366.** 
$$y \sin x + y' \cos x = 1$$
.

**367.** 
$$x dy - y dx = x\sqrt{x^2 + y^2} dx$$
.

**368.** 
$$y^2 + x^2 y'^5 = xy(y'^2 + y'^3)$$
.

**369.** 
$$y' = \sqrt[3]{2x-y} + 2$$
.

**370.** 
$$(x - y \cos \frac{y}{x}) dx + x \cos \frac{y}{x} dy = 0.$$

**371.** 
$$2\left(x^2y + \sqrt{1 + x^4y^2}\right) dx + x^3 dy = 0.$$

**372.** 
$$(y' - x\sqrt{y})(x^2 - 1) = xy$$
.

**373.** 
$$y'^3 + (y'^2 - 2y')x = 3y' - y$$
.

**374.** 
$$(2x+3y-1) dx + (4x+6y-5) dy = 0$$
.

**375.** 
$$(2xy^2 - y) dx + (y^2 + x + y) dy = 0.$$

**376.** 
$$y = y'\sqrt{1 + {y'}^2}$$
.

**377.** 
$$y^2 = (xyy' + 1) \ln x$$
.

378. 
$$4y = x^2 + {y'}^2$$
.

**379.** 
$$2x dy + y dx + xy^2(x dy + y dx) = 0.$$

**380.** 
$$x dx + (x^2 \operatorname{ctg} y - 3 \cos y) dy = 0.$$

**381.** 
$$x^2y'^2 - 2(xy - 2)y' + y^2 = 0$$
.

**382.** 
$$xy' + 1 = e^{x-y}$$
.

**383.** 
$$y' = \operatorname{tg}(y - 2x)$$
.

**384.** 
$$3x^2 - y = y'\sqrt{x^2 + 1}$$
.

**397.** 
$$y' - 8x\sqrt{y} = \frac{4xy}{x^2 - 1}$$
.

**398.** 
$$[2x - \ln(y+1)] dx - \frac{x+y}{y+1} dy = 0.$$

**399.** 
$$xy' = (x^2 + \operatorname{tg} y) \cos^2 y$$
.

**400.** 
$$x^2(y - xy') = yy'^2$$
.

**401.** 
$$y' = \frac{3x^2}{x^3 + y + 1}$$
.

**402.** 
$$y' = \frac{(1+y)^2}{x(y+1)-x^2}$$
.

**403.** 
$$(y-2xy')^2=4yy'^3$$
.

**404.** 
$$6x^5y dx + (y^4 \ln y - 3x^6) dy = 0.$$

**405.** 
$$y' = \frac{1}{2}\sqrt{x} + \sqrt[3]{y}$$
.

**406.** 
$$2xy' + 1 = y + \frac{x^2}{y-1}$$
.

**385.** 
$$yy' + xy = x^3$$
.

**386.** 
$$x(x-1)y' + y^3 = xy$$
.

**387.** 
$$xy' = 2y + \sqrt{1 + {y'}^2}$$
.

**388.** 
$$(2x + y + 5)y' = 3x + 6$$
.

**389.** 
$$y' + \operatorname{tg} y = x \sec y$$
.

**390.** 
$$y'^4 = 4y(xy'-2y)^2$$
.

**391.** 
$$y' = \frac{y^2 - x}{2y(x+1)}$$
.

**392.** 
$$xy' = x^2e^{-y} + 2$$
.

**393.** 
$$y' = 3x + \sqrt{y - x^2}$$
.

**394.** 
$$x dy - 2y dx + xy^2 (2x dy + y dx) = 0.$$

**395.** 
$$(x^3 - 2xy^2) dx + 3x^2y dy = x dy - y dx$$
.

**396.** 
$$(yy')^3 = 27x(y^2 - 2x^2)$$
.

**407.** 
$$yy' + x = \frac{1}{2} \left( \frac{x^2 + y^2}{x} \right)^2$$
.

**408.** 
$$y' = \left(\frac{3x + y^3 - 1}{y}\right)^2$$
.

**409.** 
$$\left(x\sqrt{y^2+1}+1\right)(y^2+1)\,\mathrm{d}x = xy\,\mathrm{d}y.$$

**410.** 
$$(x^2 + y^2 + 1)yy' + (x^2 + y^2 - 1)x = 0.$$

**411.** 
$$y^2(x-1) dx = x(xy+x-2y) dy$$
.

**412.** 
$$(xy'-y)^2 = x^2y^2 - x^4$$
.

**413.** 
$$xyy' - x^2\sqrt{y^2 + 1} = (x+1)(y^2 + 1).$$

**414.** 
$$(x^2-1)y'+y^2-2xy+1=0$$
.

**415.** 
$$y' \operatorname{tg} y + 4x^3 \cos y = 2x$$
.

**416.** 
$$(xy'-y)^2 = {y'}^2 - \frac{2yy'}{x} + 1$$
.

**417.** 
$$(x+y)(1-xy) dx + (x+2y) dy = 0.$$

**418.** 
$$(3xy + x + y)y dx + (4xy + x + 2y)x dy = 0.$$

**419.** 
$$(x^2 - 1) dx + (x^2y^2 + x^3 + x) dy = 0.$$

**420.** 
$$x(y'^2 + e^{2y}) = -2y'$$
.

# B. Phương trình cấp 2

#### 1. Phương trình bậc 2 hạ cấp được: Bài tập:423-505

**423.** 
$$y^3y''=1$$
.

**425.** 
$$y'' = 2yy'$$
.

**427.** 
$$y''(e^x + 1) + y' = 0$$
. **428.**  $y''' = y''^2$ .

**429.** 
$$yy'' = y'^2 - y'^3$$
.

**431.** 
$$2yy'' = y^2 + {y'}^2$$
.

**433.** 
$$y''^2 + y' = xy''$$
.

**435.** 
$$xy''' = y'' - xy''$$
.

**437.** 
$$y'' = e^y$$
.

**439.** 
$$2y'(y''+2) = xy''^2$$
. **440.**  $y^4 - y^3y'' = 1$ .

**441.** 
$$y'^2 = (3y - 2y')y''$$
. **442.**  $y''(2y' + x) = 1$ .

**443.** 
$$y''^2 - 2y'y''' + 1 = 0$$
.

**444.** 
$$(1-x^2)y'' + xy' = 2$$
.

**445.** 
$$yy'' - 2yy' \ln y = {y'}^2$$
.

**446.** 
$$(y'+2y)y''={y'}^2$$
. **447.**  $xy''=y'+x\sin\frac{y'}{x}$ .

**448.** 
$$v'''v'^2 = v''^3$$
.

**450.** 
$$xy'' = y' + x(y'^2 + x^2)$$
.

**451.** 
$$xy^{IV} = 1$$
.

**452.** 
$$xy'' = \sin x$$
.

**453.** 
$$y''' = 2xy''$$
.

**454.**  $xu^{IV} + v''' = e^x$ .

**449.**  $yy'' + y = {y'}^2$ .

**461.** 
$$xy'' = 2yy' - y'$$
. **462.**  $xy'' - y' = x^2yy'$ .

**463.** 
$$xyy'' - xy'^2 = yy'$$
. **464.**  $yy'' = y'^2 + 15y^2\sqrt{x}$ .

**465.** 
$$(x^2+1)(y'^2-yy'')=xyy'.$$

**466.** 
$$xyy'' + xy'^2 = 2yy'$$
.

**467.** 
$$x^2yy'' = (y - xy')^2$$
.

**468.** 
$$y'' + \frac{y'}{x} + \frac{y}{x^2} = \frac{{y'}^2}{y}$$
.

**469.** 
$$y(xy'' + y') = xy'^2(1-x)$$
.

**470.** 
$$x^2yy'' + y'^2 = 0$$
.

**471.** 
$$x^2(y'^2 - 2yy'') = y^2$$
.

**472.** 
$$xyy'' = y'(y + y')$$
.

**473.** 
$$4x^2y^3y'' = x^2 - y^4$$
.

**474.** 
$$x^3y'' = (y - xy')(y - xy' - x)$$
.

**475.** 
$$\frac{y^2}{x^2} + {y'}^2 = 3xy'' + \frac{2yy'}{x}$$
.

**476.** 
$$y'' = \left(2xy - \frac{5}{x}\right)y' + 4y^2 - \frac{4y}{x^2}$$
.

**424.** 
$$y'^2 + 2yy'' = 0$$
.  
**426.**  $yy'' + 1 = y'^2$ .

**430.** 
$$y''' = 2(y'' - 1) \operatorname{ctg} x$$
.

**432.** 
$$y''^3 + xy'' = 2y'$$
.

**434.** 
$$y'' + y'^2 = 2e^{-y}$$
.

**436.** 
$$y''^2 = y'^2 + 1$$
.

**438.** 
$$y'' - xy''' + y'''^3 = 0$$
.

**440.** 
$$y^4 - y^3y'' = 1$$

**442.** 
$$y''(2y'+x)=1$$
.

**456.** 
$$y'y''' = 2y''^2$$
.  
**458.**  $5y'''^2 - 3y''y^{IV} = 0$ .  
**460.**  $y'' = xy' + y + 1$ .

**455.** 
$$yy''' + 3y'y'' = 0$$
.  
**457.**  $yy'' = y'(y' + 1)$ .  
**459.**  $yy'' + y'^2 = 1$ .

**477.** 
$$x^2(2yy'' - y'^2) = 1 - 2xyy'$$
.

**478.** 
$$x^2(yy''-y'^2)+xyy'=(2xy'-3y)\sqrt{x^3}$$
.

**479.** 
$$x^4(y'^2 - 2yy'') = 4x^3yy' + 1$$
.

**480.** 
$$yy' + xyy'' - xy'^2 = x^3$$
.

**481.** 
$$y''(3 + yy'^2) = y'^4$$
.

**481.** 
$$y''(3+y{y'}^2)={y'}^4$$
. **482.**  $y''^2-y'y'''=\left(\frac{y'}{x}\right)^2$ .

**483.** 
$$yy' + 2x^2y'' = xy'^2$$
.

**484.** 
$$y'^2 + 2xyy'' = 0$$
.

**485.** 
$$2xy^2(xy'' + y') + 1 = 0$$
.

**486.** 
$$x(y'' + y'^2) = y'^2 + y'$$
.

**487.** 
$$y^2(y'y'''-2y''^2)=y'^4$$
.

**488.** 
$$y(2xy'' + y') = xy'^2 + 1$$
.

**489.** 
$$y'' + 2yy'^2 = (2x + \frac{1}{x})y'$$
.

**490.** 
$$y'y''' = y''^2 + y'^2y''$$
. **491.**  $yy'' = y'^2 + 2xy^2$ .

**491.** 
$$yy'' = {y'}^2 + 2xy^2$$
.

**492.** 
$$y''^4 = y'^5 - yy'^3y''$$
.

**493.** 
$$2yy''' = y'$$
.

**494.** 
$$y'''y'^2 = 1$$
.

**495.** 
$$y^2y''' = y'^3$$
.

**496.** 
$$x^2yy'' + 1 = (1 - y)xy'$$
.

**497.** 
$$yy'y''' + 2y'^2y'' = 3yy''^2$$
.

**498.** 
$$(y'y''' - 3y''^2)y = y'^5$$
.

**499.** 
$$y^2(y'y'''-2y''^2)=yy'^2y''+2y'^4$$
.

**500.** 
$$x^2(y^2y''' - {y'}^3) = 2y^2y' - 3xyy'^2$$
.

**501.** 
$$yy'' = 2xy'^2$$
;  $y(2) = 2$ ,  $y'(2) = 0.5$ .

**502.** 
$$2y''' - 3y'^2 = 0$$
;  $y(0) = -3$ ,  $y'(0) = 1$ ,  $y''(0) = -1$ .

**503.** 
$$x^2y'' - 3xy' = \frac{6y^2}{x^2} - 4y$$
;  $y(1) = 1$ ,  $y'(1) = 4$ .

**504.** 
$$y''' = 3yy'; \ y(0) = -2, \ y'(0) = 0, \ y''(0) = 4,5.$$

**505.** 
$$y'' \cos y + {y'}^2 \sin y = y'$$
;  $y(-1) = \frac{\pi}{6}$ ,  $y'(-1) = 2$ .

# 2. Phương trình tuyến tính cấp 2 hệ số biến thiên: Bài tập:511-611

**511.** 
$$y'' + y' - 2y = 0$$
.

**512.** 
$$y'' + 4y' + 3y = 0$$
.

**513.** 
$$y'' - 2y' = 0$$
.

**514.** 
$$2y'' - 5y' + 2y = 0$$
.

**515.** 
$$y'' - 4y' + 5y = 0$$
.

**516.** 
$$y'' + 2y' + 10y = 0$$
.

**517.** 
$$u'' + 4y = 0$$
.

**518.** 
$$y''' - 8y = 0$$
.

**519.** 
$$y^{IV} - y = 0$$
.

**520.** 
$$y^{IV} + 4y = 0$$
.

**521.** 
$$y^{VI} + 64y = 0$$
.

**522.** 
$$y'' - 2y' + y = 0$$
.

**523.** 
$$4y'' + 4y' + y = 0$$
.

**524.** 
$$y^{V} - 6y^{IV} + 9y''' = 0$$
.

**525.** 
$$y^{V} - 10y''' + 9y' = 0$$
.

**526.** 
$$y^{IV} + 2y'' + y = 0$$
.

**527.** 
$$y''' - 3y'' + 3y' - y = 0$$
.

**528.** 
$$y''' - y'' - y' + y = 0$$
.

**529** 
$$u^{IV} = 5u'' + 4u = 0$$

**529.** 
$$y^{IV} - 5y'' + 4y = 0$$
. **530.**  $y^{V} + 8y''' + 16y' = 0$ .

**531.** 
$$y''' - 3y' + 2y = 0$$
.

**532.** 
$$y^{IV} + 4y'' + 3y = 0$$
.

**540.** 
$$y'' - 3y' + 2y = x \cos x$$
.

**541.** 
$$y'' + 3y' - 4y = e^{-4x} + xe^{-x}$$
.

**542.** 
$$y'' + 2y' - 3y = x^2 e^x$$
.

**543.** 
$$y'' - 4y' + 8y = e^{2x} + \sin 2x$$
.

**544.** 
$$y'' - 9y = e^{3x} \cos x$$
.

**545.** 
$$y'' - 2y' + y = 6xe^x$$
.

**546.** 
$$y'' + y = x \sin x$$
.

**547.** 
$$y'' + 4y' + 4y = xe^{2x}$$
.

**548.** 
$$y'' - 5y' = 3x^2 + \sin 5x$$
.

**533.** 
$$y'' - 2y' - 3y = e^{4x}$$
.

**534.** 
$$y'' + y = 4xe^x$$
.

**535.** 
$$y'' - y = 2e^x - x^2$$
.

**536.** 
$$y'' + y' - 2y = 3xe^x$$
.

**537.** 
$$y'' - 3y' + 2y = \sin x$$
.

**538.** 
$$y'' + y = 4 \sin x$$
.

**539.** 
$$y'' - 5y' + 4y = 4x^2e^{2x}$$
.

**549.** 
$$y'' - 2y' + 2y = e^x + x \cos x$$
.

**550.** 
$$y'' + 6y' + 10y = 3xe^{-3x} - 2e^{3x}\cos x$$
.

**551.** 
$$y'' - 8y' + 20y = 5xe^{4x}\sin 2x$$
.

**552.** 
$$y'' + 7y' + 10y = xe^{-2x}\cos 5x$$
.

**553.** 
$$y'' - 2y' + 5y = 2xe^x + e^x \sin 2x$$
.

**554.** 
$$y'' - 2y' + y = 2xe^x + e^x \sin 2x$$
.

**555.** 
$$y'' - 8y' + 17y = e^{4x}(x^2 - 3x\sin x)$$
.

**556.** 
$$y''' + y' = \sin x + x \cos x$$
.

**557.** 
$$y''' - 2y'' + 4y' - 8y = e^{2x} \sin 2x + 2x^2$$
.

**558.** 
$$y'' - 6y' + 8y = 5xe^{2x} + 2e^{4x}\sin x$$
.

**559.** 
$$y'' + 2y' + y = x(e^{-x} - \cos x)$$
.

**560.** 
$$y''' - y'' - y' + y = 3e^x + 5x \sin x$$
.

**561.** 
$$y'' - 6y' + 13y = x^2 e^{3x} - 3\cos 2x$$
.

**562.** 
$$y'' - 9y = e^{-3x}(x^2 + \sin 3x)$$
.

**563.** 
$$y^{IV} + y'' = 7x - 3\cos x$$
.

**564.** 
$$y'' + 4y = \cos x \cdot \cos 3x$$
.

**565.** 
$$y''' - 4y'' + 3y' = x^2 + xe^{2x}$$
.

**566.** 
$$y'' - 4y' + 5y = e^{2x} \sin^2 x$$
.

**567.** 
$$y'' + 3y' + 2y = e^{-x} \cos^2 x$$
.

**568.** 
$$y'' - 2y' + 2y = (x + e^x) \sin x$$
.

**569.** 
$$y^{\text{IV}} + 5y'' + 4y = \sin x \cdot \cos 2x$$
.

**570.** 
$$y'' - 3y' + 2y = 2^x$$
.

**571.** 
$$y'' - y = 4 \operatorname{sh} x$$
.

**572.** 
$$y'' + 4y' + 3y = \operatorname{ch} x$$
.

**573.** 
$$y'' + 4y = \sin x \cdot \sin 2x$$
.

**574.** 
$$y'' + 2y' + 2y = \operatorname{ch} x \cdot \sin x$$
.

**575.** 
$$y'' - 2y' + y = \frac{e^x}{x}$$
.

**576.** 
$$y'' + 3y' + 2y = \frac{1}{e^x + 1}$$
.

**577.** 
$$y'' + y = \frac{1}{\sin x}$$
.

**578.** 
$$y'' + 4y = 2 \operatorname{tg} x$$
.

**579.** 
$$y'' + 2y' + y = 3e^{-x}\sqrt{x+1}$$
.

**580.** 
$$y'' + y = 2 \sec^3 x$$
.

**581\*.** 
$$x^3(y''-y)=x^2-2$$
.

**582.** 
$$y'' - 2y' + y = 0$$
;  $y(2) = 1$ ,  $y'(2) = -2$ .

**583.** 
$$y'' + y = 4e^x$$
;  $y(0) = 4$ ,  $y'(0) = -3$ .

**584.** 
$$y'' - 2y' = 2e^x$$
;  $y(1) = -1$ ,  $y'(1) = 0$ .

**585.** 
$$y'' + 2y' + 2y = xe^{-x}$$
;  $y(0) = y'(0) = 0$ .

**586.** 
$$y''' - y' = 0$$
;  $y(0) = 3$ ,  $y'(0) = -1$ ,  $y''(0) = 1$ .

**587.** 
$$y''' - 3y' - 2y = 9e^{2x}$$
;  $y(0) = 0$ ,  $y'(0) = -3$ ,  $y''(0) = 3$ .

**588.** 
$$y^{\text{IV}} + y'' = 2\cos x$$
;  $y(0) = -2$ ,  $y'(0) = 1$ ,  $y''(0) = y'''(0) = 0$ .

В задачах 589—600 решить уравнения Эйлера

**589.** 
$$x^2y'' - 4xy' + 6y = 0$$
.

**590.** 
$$x^2y'' - xy' - 3y = 0$$
.

**591.** 
$$x^3y''' + xy' - y = 0$$
.

**592.** 
$$x^2y''' = 2y'$$
.

**593.** 
$$x^2y'' - xy' + y = 8x^3$$
.

**594.** 
$$x^2y'' + xy' + 4y = 10x$$
.

**595.** 
$$x^3y'' - 2xy = 6 \ln x$$
.

**596.** 
$$x^2y'' - 3xy' + 5y = 3x^2$$
.

**597.** 
$$x^2y'' - 6y = 5x^3 + 8x^2$$
.

**598.** 
$$x^2y'' - 2y = \sin \ln x$$
.

**598.** 
$$x^2y'' - 2y = \sin \ln x$$
.

**599.** 
$$(x-2)^2y''-3(x-2)y'+4y=x$$
.

**600.** 
$$(2x+3)^3y'''+3(2x+3)y'-6y=0.$$

**601.** 
$$y'' + 2y' + y = \cos ix$$
.

**602.** 
$$y'' - 2y' + y = xe^x \sin^2 ix$$
.

**603.** 
$$y'' + 2iy = 8e^x \sin x$$
.

**604.** 
$$y'' + 2iy' - y = 8\cos x$$
.

**605.** 
$$y''' - 8iy = \cos 2x$$
.

**606.** 
$$y'' - \frac{2y}{x^2} = 3\ln(-x)$$
.

**607.** 
$$y'' + 2y' + y = xe^x + \frac{1}{xe^x}$$
.

**608.** 
$$y'' + 2y' + 5y = e^{-x}(\cos^2 x + \operatorname{tg} x)$$
.

**609.** 
$$x^2y'' - 2y = \frac{3x^2}{x+1}$$
.

**610.** 
$$x^2y'' - xy' + y = \frac{\ln x}{x} + \frac{x}{\ln x}$$
.

**611\*.** 
$$y'' + y = f(x)$$
.

#### 3. Phương trình tuyến tính cấp 2 hệ số hằng số: Bài tập:681-700

**681.** 
$$(2x+1)y'' + 4xy' - 4y = 0$$
.

**682.** 
$$x^2(x+1)y''-2y=0$$
;  $y_1=1+\frac{1}{x}$ .

**683.** 
$$xy'' - (2x+1)y' + (x+1)y = 0.$$

**684.** 
$$xy'' + 2y' - xy = 0$$
;  $y_1 = \frac{e^x}{x}$ .

**685.** 
$$y'' - 2(1 + tg^2 x)y = 0$$
;  $y_1 = tg x$ .

**686.** 
$$x(x-1)y'' - xy' + y = 0$$
.

**687.** 
$$(e^x + 1)y'' - 2y' - e^x y = 0$$
;  $y_1 = e^x - 1$ .

**688.** 
$$x^2y'' \ln x - xy' + y = 0$$
.

**689.** 
$$y'' - y' \operatorname{tg} x + 2y = 0$$
;  $y_1 = \sin x$ .

**690.** 
$$(x^2-1)y''+(x-3)y'-y=0.$$

**691.** 
$$xy'' - (x+1)y' - 2(x-1)y = 0.$$

**692.** 
$$y'' + 4xy' + (4x^2 + 2)y = 0$$
;  $y_1 = e^{ax^2}$ .

**693.** 
$$xy'' - (2x+1)y' + 2y = 0.$$

**694.** 
$$x(2x+1)y'' + 2(x+1)y' - 2y = 0$$
.

**695.** 
$$x(x+4)y'' - (2x+4)y' + 2y = 0.$$

**696.** 
$$x(x^2+6)y''-4(x^2+3)y'+6xy=0.$$

**697.** 
$$(x^2+1)y''-2y=0$$
.

**698.** 
$$2x(x+2)y'' + (2-x)y' + y = 0.$$

**699.** 
$$xy''' - y'' - xy' + y = 0$$
;  $y_1 = x$ ,  $y_2 = e^x$ .

**700.** 
$$x^2(2x-1)y''' + (4x-3)xy'' - 2xy' + 2y = 0;$$
  
 $y_1 = x, y_2 = 1/x.$ 

## C. Hệ phương trình

#### 4. Hệ phương trình tuyến tính với hệ số hằng số: Bài tập:786-873

**786.** 
$$\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = 3x + 4y. \end{cases}$$
 **787.**  $\begin{cases} \dot{x} = x - y, \\ \dot{y} = y - 4x. \end{cases}$ 

**788.** 
$$\begin{cases} \dot{x} + x - 8y = 0, \\ \dot{y} - x - y = 0. \end{cases}$$
 **789.** 
$$\begin{cases} \dot{x} = x + y, \\ \dot{y} = 3y - 2x. \end{cases}$$

**790.** 
$$\begin{cases} \dot{x} = x - 3y, \\ \dot{y} = 3x + y. \end{cases}$$
 **791.** 
$$\begin{cases} \dot{x} + x + 5y = 0, \\ \dot{y} - x - y = 0. \end{cases}$$

**792.** 
$$\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = 4y - x. \end{cases}$$
 **793.** 
$$\begin{cases} \dot{x} = 3x - y, \\ \dot{y} = 4x - y. \end{cases}$$

**794.** 
$$\begin{cases} \dot{x} = 2y - 3x, \\ \dot{y} = y - 2x. \end{cases}$$
 **795.** 
$$\begin{cases} \dot{x} - 5x - 3y = 0, \\ \dot{y} + 3x + y = 0. \end{cases}$$

796. 
$$\begin{cases} \dot{x} = x + z - y, \\ \dot{y} = x + y - z, \\ \dot{z} = 2x - y \end{cases}$$
 797. 
$$\begin{cases} \dot{x} = x - 2y - z, \\ \dot{y} = y - x + z, \\ \dot{z} = x - z, \end{cases}$$
 ( $\lambda_1 = 1, \ \lambda_2 = 2, \ \lambda_3 = -1$ ). ( $\lambda_1 = 0, \ \lambda_2 = 2, \ \lambda_3 = -1$ ).

798. 
$$\begin{cases} \dot{x} = 2x - y + z, \\ \dot{y} = x + 2y - z, \\ \dot{z} = x - y + 2z \end{cases}$$
 799. 
$$\begin{cases} \dot{x} = 3x - y + z, \\ \dot{y} = x + y + z, \\ \dot{z} = 4x - y + 4z \end{cases}$$
 ( $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$ ). ( $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 5$ ).

800. 
$$\begin{cases} \dot{x} = 4y - 2z - 3x, \\ \dot{y} = z + x, \\ \dot{z} = 6x - 6y + 5z \\ (\lambda_1 = 1, \ \lambda_2 = 2, \ \lambda_3 = -1). \end{cases}$$
801. 
$$\begin{cases} \dot{x} = x - y - z, \\ \dot{y} = x + y, \\ \dot{z} = 3x + z \\ (\lambda_1 = 1, \ \lambda_{2,3} = 1 \pm 2i). \end{cases}$$

802. 
$$\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = x + 3y - z, \\ \dot{z} = 2y + 3z - x \\ (\lambda_1 = 2, \ \lambda_{2, 3} = 3 \pm i). \end{cases}$$
803. 
$$\begin{cases} \dot{x} = 2x + 2z - y, \\ \dot{y} = x + 2z, \\ \dot{z} = y - 2x - z \\ (\lambda_1 = 1, \ \lambda_{2, 3} = \pm i). \end{cases}$$

$$(\lambda_{1} = 1, \lambda_{2} = 2, \lambda_{3} = -1). \qquad (\lambda_{1} = 1, \lambda_{2,3} = 1 \pm 2i).$$

$$802. \begin{cases} \dot{x} = 2x + y, \\ \dot{y} = x + 3y - z, \\ \dot{z} = 2y + 3z - x \end{cases} \qquad 803. \begin{cases} \dot{x} = 2x + 2z - y, \\ \dot{y} = x + 2z, \\ \dot{z} = y - 2x - z \end{cases}$$

$$(\lambda_{1} = 1, \lambda_{2,3} = \pm i). \qquad (\lambda_{1} = 1, \lambda_{2,3} = \pm i).$$

$$804. \begin{cases} \dot{x} = 4x - y - z, \\ \dot{y} = x + 2y - z, \\ \dot{z} = x - y + 2z \end{cases} \qquad 805. \begin{cases} \dot{x} = 2x - y - z, \\ \dot{y} = 3x - 2y - 3z, \\ \dot{z} = 2z - x + y \end{cases}$$

$$(\lambda_{1} = 0, \lambda_{2} = \lambda_{3} = 1).$$

806. 
$$\begin{cases} \dot{x} = y - 2x - 2z, \\ \dot{y} = x - 2y + 2z, \\ \dot{z} = 3x - 3y + 5z \end{cases}$$

$$(\lambda_1 = 0, \lambda_2 = \lambda_3 = 1).$$

$$\begin{cases} \dot{x} = 3x - 2y - z, \\ \dot{y} = 3x - 4y - 3z, \\ \dot{z} = 2x - 4y \end{cases}$$

$$(\lambda_1 = 3, \lambda_2 = \lambda_3 = -1).$$

$$(\lambda_1 = 0, \lambda_2 = \lambda_3 = 1).$$

808. 
$$\begin{cases} \dot{x} = x - y + z, \\ \dot{y} = x + y - z, \\ \dot{z} = 2z - y \\ (\lambda_1 = \lambda_2 = 1, \lambda_3 = 2). \end{cases}$$
809. 
$$\begin{cases} \dot{x} = y - 2z - x, \\ \dot{y} = 4x + y, \\ \dot{z} = 2x + y - z \\ (\lambda_1 = 1, \lambda_2 = \lambda_3 = -1). \end{cases}$$
810. 
$$\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = 2y + 4z, \\ \dot{z} = x - z \\ (\lambda_1 = \lambda_2 = 0, \lambda_3 = 3). \end{cases}$$
811. 
$$\begin{cases} \dot{x} = 2x - y - z, \\ \dot{y} = 2x - y - 2z, \\ \dot{z} = 2z - x + y \\ (\lambda_1 = \lambda_2 = \lambda_3 = 1). \end{cases}$$

810. 
$$\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = 2y + 4z, \\ \dot{z} = x - z \end{cases}$$
 811. 
$$\begin{cases} \dot{x} = 2x - y - z, \\ \dot{y} = 2x - y - 2z, \\ \dot{z} = 2z - x + y \end{cases}$$
 
$$(\lambda_1 = \lambda_2 = 0, \ \lambda_3 = 3).$$
 
$$(\lambda_1 = \lambda_2 = \lambda_3 = 1).$$

812. 
$$\begin{cases} \dot{x} = 4x - y, \\ \dot{y} = 3x + y - z, \\ \dot{z} = x + z \end{cases}$$
 ( $\lambda_1 = \lambda_2 = \lambda_3 = 2$ ). 
$$\dot{z} = x + z$$
 813. 
$$\begin{cases} \ddot{x} = 2x - 3y, \\ \ddot{y} = x - 2y. \end{cases}$$
 814. 
$$\begin{cases} \ddot{x} = 3x + 4y, \\ \ddot{y} = -x - y. \end{cases}$$

**813.** 
$$\begin{cases} \ddot{x} = 2x - 3y, \\ \ddot{y} = x - 2y. \end{cases}$$
 **814.** 
$$\begin{cases} \ddot{x} = 3x + 4y, \\ \ddot{y} = -x - y \end{cases}$$

815. 
$$\begin{cases} \ddot{x} = 2y, \\ \ddot{y} = -2x. \end{cases}$$

816. 
$$\begin{cases} \ddot{x} = 3x - y - z, \\ \ddot{y} = -x + 3y - z, \\ \ddot{z} = -x - y + 3z. \end{cases}$$

817. 
$$\begin{cases} 2\dot{x} - 5\dot{y} = 4y - x, \\ 3\dot{x} - 4\dot{y} = 2x - y. \end{cases}$$

817. 
$$\begin{cases} 2\dot{x} - 5\dot{y} = 4y - x, \\ 3\dot{x} - 4\dot{y} = 2x - y. \end{cases}$$
 818. 
$$\begin{cases} \ddot{x} + \dot{x} + \dot{y} - 2y = 0, \\ \dot{x} - \dot{y} + x = 0. \end{cases}$$

819. 
$$\begin{cases} \ddot{x} - 2\ddot{y} + \dot{y} + x - 3y = 0, \\ 4\ddot{y} - 2\ddot{x} - \dot{x} - 2x + 5y = 0. \end{cases}$$

820. 
$$\begin{cases} \ddot{x} - x + 2\ddot{y} - 2y = 0, \\ \dot{x} - x + \dot{y} + y = 0. \end{cases}$$

821. 
$$\begin{cases} \ddot{x} - 2\dot{y} + 2x = 0, \\ 3\dot{x} + \ddot{y} - 8y = 0. \end{cases}$$

821. 
$$\begin{cases} \ddot{x} - 2\dot{y} + 2x = 0, \\ 3\dot{x} + \ddot{y} - 8y = 0. \end{cases}$$
 822. 
$$\begin{cases} \ddot{x} + 3\ddot{y} - x = 0, \\ \dot{x} + 3\dot{y} - 2y = 0. \end{cases}$$

823. 
$$\begin{cases} \ddot{x} + 5\dot{x} + 2\dot{y} + y = 0, \\ 3\ddot{x} + 5x + \dot{y} + 3y = 0. \end{cases}$$

**826.** 
$$\begin{cases} \dot{x} = y + 2e^t, \\ \dot{y} = x + t^2. \end{cases}$$

827. 
$$\begin{cases} \dot{x} = y - 5\cos t, \\ \dot{y} = 2x + y. \end{cases}$$

824. 
$$\begin{cases} \ddot{x} + 4\dot{x} - 2x - 2\dot{y} - y = 0, \\ \ddot{x} - 4\dot{x} - \ddot{y} + 2\dot{y} + 2y = 0. \end{cases}$$

828. 
$$\begin{cases} \dot{x} = 3x + 2y + 4e^{5t}, \\ \dot{y} = x + 2y. \end{cases}$$

828. 
$$\begin{cases} \dot{x} = 3x + 2y + 4 e^{5t}, \\ \dot{y} = x + 2y. \end{cases}$$
829. 
$$\begin{cases} \dot{x} = 2x - 4y + 4 e^{-2t}, \\ \dot{y} = 2x - 2y. \end{cases}$$

825. 
$$\begin{cases} 2\ddot{x} + 2\dot{x} + x + 3\ddot{y} + \dot{y} + y = 0, \\ \ddot{x} + 4\dot{x} - x + 3\ddot{y} + 2\dot{y} - y = 0. \end{cases}$$

830. 
$$\begin{cases} \dot{x} = 4x + y - e^{2t}, \\ \dot{y} = y - 2x. \end{cases}$$

831. 
$$\begin{cases} \dot{x} = 2y - x + 1, \\ \dot{y} = 3y - 2x. \end{cases}$$

834. 
$$\begin{cases} \dot{x} = x + 2y, \\ \dot{y} = x - 5\sin t. \end{cases}$$

835. 
$$\begin{cases} \dot{x} = 2x - 4y, \\ \dot{y} = x - 3y + 3e^{t}. \end{cases} = 5x - 3y + 2e^{3t}, \\ = x + y + 5e^{-t}. \end{cases}$$
833. 
$$\begin{cases} \dot{x} = 2x + y + e^{t}, \\ \dot{y} = -2x + 2t. \end{cases}$$

833. 
$$\begin{cases} \dot{x} = 2x + y + e^t, \\ \dot{y} = -2x + 2t. \end{cases}$$

836. 
$$\begin{cases} \dot{x} = 2x - y, \\ \dot{y} = y - 2x + 18t. \end{cases}$$

837. 
$$\begin{cases} \dot{x} = x + 2y + 16t e^t, \\ \dot{y} = 2x - 2y. \end{cases}$$

**838.** 
$$\begin{cases} \dot{x} = 2x + 4y - 8, \\ \dot{y} = 3x + 6y. \end{cases}$$

839. 
$$\begin{cases} \dot{x} = 2x - 3y, \\ \dot{y} = x - 2y + 2\sin t. \end{cases}$$

840. 
$$\begin{cases} \dot{x} = x - y + 2\sin t, \\ \dot{y} = 2x - y. \end{cases}$$

**841.** 
$$\begin{cases} \dot{x} = 2x - y, \\ \dot{y} = x + 2e^t. \end{cases}$$

**842.** 
$$\begin{cases} \dot{x} = 4x - 3y + \sin t, \\ \dot{y} = 2x - y - 2\cos t. \end{cases}$$

843. 
$$\begin{cases} \dot{x} = 2x + y + 2e^t, \\ \dot{y} = x + 2y - 3e^{4t}. \end{cases}$$

844. 
$$\begin{cases} \dot{x} = x - y + 8t, \\ \dot{y} = 5x - y. \end{cases}$$

**845.** 
$$\begin{cases} \dot{x} = 2x - y, \\ \dot{y} = 2y - x - 5 e^t \sin t. \end{cases}$$

846. 
$$\begin{cases} \dot{x} = y + \operatorname{tg}^2 t - 1, \\ \dot{y} = -x + \operatorname{tg} t. \end{cases}$$
 847. 
$$\begin{cases} \dot{x} = 2y - x, \\ \dot{y} = 4y - 3x + \frac{e^{3t}}{e^{2t} + 1}. \end{cases}$$

848. 
$$\begin{cases} \dot{x} = -4x - 2y + \frac{2}{e^t - 1}, \\ \dot{y} = 6x + 3y - \frac{3}{e^t - 1}. \end{cases}$$

849. 
$$\begin{cases} \dot{x} = x - y + \frac{1}{\cos t}, \\ \dot{y} = 2x - y. \end{cases}$$
 850. 
$$\begin{cases} \dot{x} = 3x - 2y, \\ \dot{y} = 2x - y + 15 e^t \sqrt{t}. \end{cases}$$

**851.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ .  
**852.**  $\dot{x} = Ax$ ,  $A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$ .

**853.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}$ .

**854.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$ 

**855.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 3 & -1 & -2 \end{pmatrix}$ 

**856.** 
$$\dot{x} = Ax, \ A = \begin{pmatrix} 1 & -2 & 2 \\ 1 & 4 & -2 \\ 1 & 5 & -3 \end{pmatrix}.$$

**857.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} -1 & -2 & 2 \\ -2 & -1 & 2 \\ -3 & -2 & 3 \end{pmatrix}$ .

855. 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 3 & -1 & -2 \end{pmatrix}$ .  
856.  $\dot{x} = Ax$ ,  $A = \begin{pmatrix} 1 & -2 & 2 \\ 1 & 4 & -2 \\ 1 & 5 & -3 \end{pmatrix}$ .  
857.  $\dot{x} = Ax$ ,  $A = \begin{pmatrix} -1 & -2 & 2 \\ -2 & -1 & 2 \\ -3 & -2 & 3 \end{pmatrix}$ .  
858.  $\dot{x} = Ax$ ,  $A = \begin{pmatrix} -3 & 2 & 2 \\ -3 & -1 & 1 \\ -1 & 2 & 0 \end{pmatrix}$ .

**859.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 3 & -3 & 1 \\ 3 & -2 & 2 \\ -1 & 2 & 0 \end{pmatrix}$ .

**860.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ .

**861.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{pmatrix}$ .

**862.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ .  
**863.**  $\dot{x} = Ax$ ,  $A = \begin{pmatrix} -2 & 1 & 2 \\ -1 & 0 & 2 \\ -2 & 0 & 3 \end{pmatrix}$ .

**863.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} -2 & 1 & 2 \\ -1 & 0 & 2 \\ -2 & 0 & 3 \end{pmatrix}$ .

**864.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 2 & 2 & -3 \end{pmatrix}$ .

**865.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 4 & 2 & -2 \\ 1 & 3 & -1 \\ 3 & 3 & -1 \end{pmatrix}$ 

**866.** 
$$\dot{x} = Ax$$
,  $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & -1 & 0 \\ 3 & -1 & -1 \end{pmatrix}$ 

**867.** 
$$A = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$$
. **868.**  $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ . **869.**  $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$ . **870.**  $A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$ .

**869.** 
$$A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$
. **870.**  $A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$ 

**871.** 
$$A = \begin{pmatrix} -2 & -4 \\ 1 & 2 \end{pmatrix}$$
. **872.**  $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ .

**873.** 
$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

### 5. Hệ phương trình phi tuyến: Bài tập:1141-1160

1141. 
$$y' = \frac{x}{z}, \ z' = -\frac{x}{y}.$$

**1142.** 
$$y' = \frac{y^2}{z-x}, \ z' = y+1.$$

**1143.** 
$$y' = \frac{z}{x}, \ z' = \frac{z(y+2z-1)}{x(y-1)}.$$

**1144.** 
$$y' = y^2 z$$
,  $z' = \frac{z}{x} - yz^2$ .

**1145.** 
$$2zy' = y^2 - z^2 + 1$$
,  $z' = z + y$ .

1146. 
$$\frac{dx}{2y-z} = \frac{dy}{y} = \frac{dz}{z}$$
.

1147. 
$$\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{z}$$
.

1148. 
$$\frac{dx}{y+z} = \frac{dy}{x+z} = \frac{dz}{x+y}$$
.

1149. 
$$\frac{dx}{y-x} = \frac{dy}{x+y+z} = \frac{dz}{x-y}$$
.

1150. 
$$\frac{\mathrm{d}x}{z} = \frac{\mathrm{d}y}{u} = \frac{\mathrm{d}z}{x} = \frac{\mathrm{d}u}{y}.$$

1151. 
$$\frac{dx}{y-u} = \frac{dy}{z-x} = \frac{dz}{u-y} = \frac{du}{x-z}$$
.

1152. 
$$\frac{dx}{z} = \frac{dy}{xz} = \frac{dz}{y}$$
.

1153. 
$$\frac{dx}{z^2-y^2} = \frac{dy}{z} = -\frac{dz}{y}$$
.

1154. 
$$\frac{\mathrm{d}x}{x} = \frac{\mathrm{d}y}{y} = \frac{\mathrm{d}z}{xy+z}$$
.

1155. 
$$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy\sqrt{z^2+1}}$$
.

1156. 
$$\frac{dx}{x+y^2+z^2} = \frac{dy}{y} = \frac{dz}{z}$$
.

1157. 
$$\frac{dx}{x(y+z)} = \frac{dy}{z(z-y)} = \frac{dz}{y(y-z)}$$
.

1158. 
$$-\frac{\mathrm{d}x}{x^2} = \frac{\mathrm{d}y}{xy - 2z^2} = \frac{\mathrm{d}z}{xz}$$
.

1159. 
$$\frac{\mathrm{d}x}{x(z-y)} = \frac{\mathrm{d}y}{y(y-x)} = \frac{\mathrm{d}z}{y^2-xz}$$
.

1160. 
$$\frac{\mathrm{d}x}{x(y^2-z^2)} = -\frac{\mathrm{d}y}{y(z^2+x^2)} = \frac{\mathrm{d}z}{z(x^2+y^2)}$$
.