Nhập môn Kỹ thuật Truyền thông

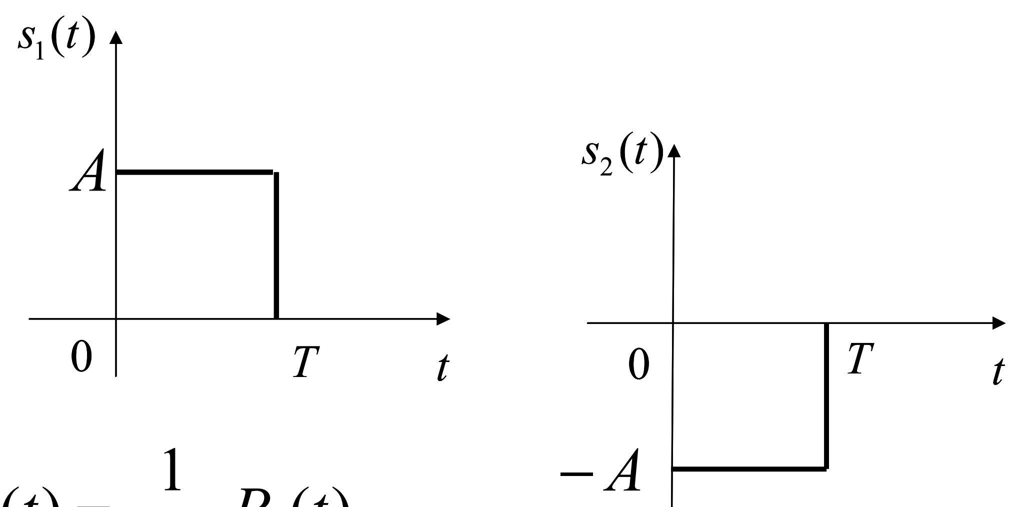
Phần 2: Các kỹ thuật điều chế số (Digital Modulations)

Bài 9: Không gian tín hiệu PAM (tiếp bài trước)

PGS. Tạ Hải Tùng

Bipolar NRZ (Non Return to Zero)

Signal set $$\boxed{M = \{s_1(t) = +AP_T(t)\,,\, s_2(t) = -AP_T(t)\,\}}$$



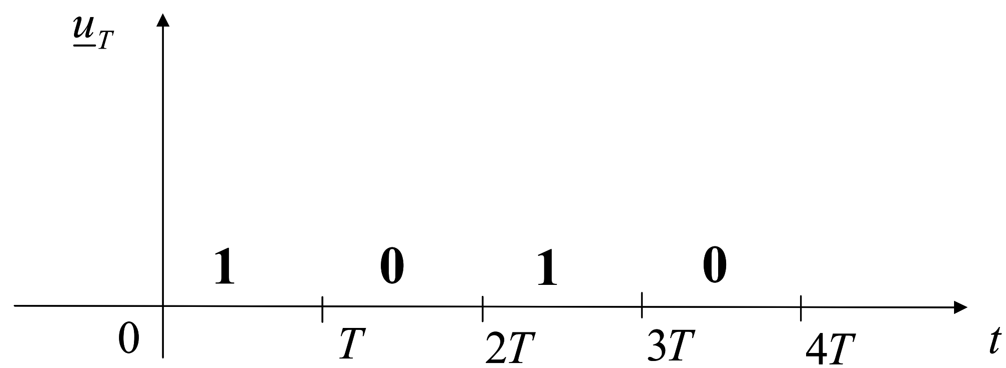
Versor $$b_1(t) = \frac{1}{\sqrt{T}} P_T(t)$$

Vector set $$M = \{\underline{s}_1 = (+\alpha)\,,\, \underline{s}_2 = (-\alpha)\,\}$$

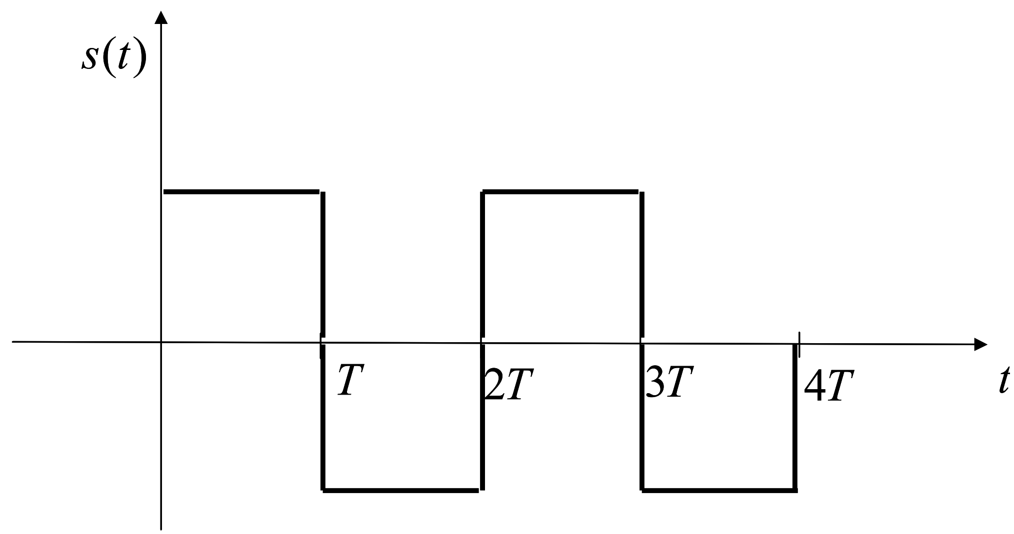
(it coincides with a <u>2-PAM with rectangular pulse)</u>

Bipolar NRZ

Transmitted waveform



$$s(t) = \sum_n a[n]p(t - nT)$$

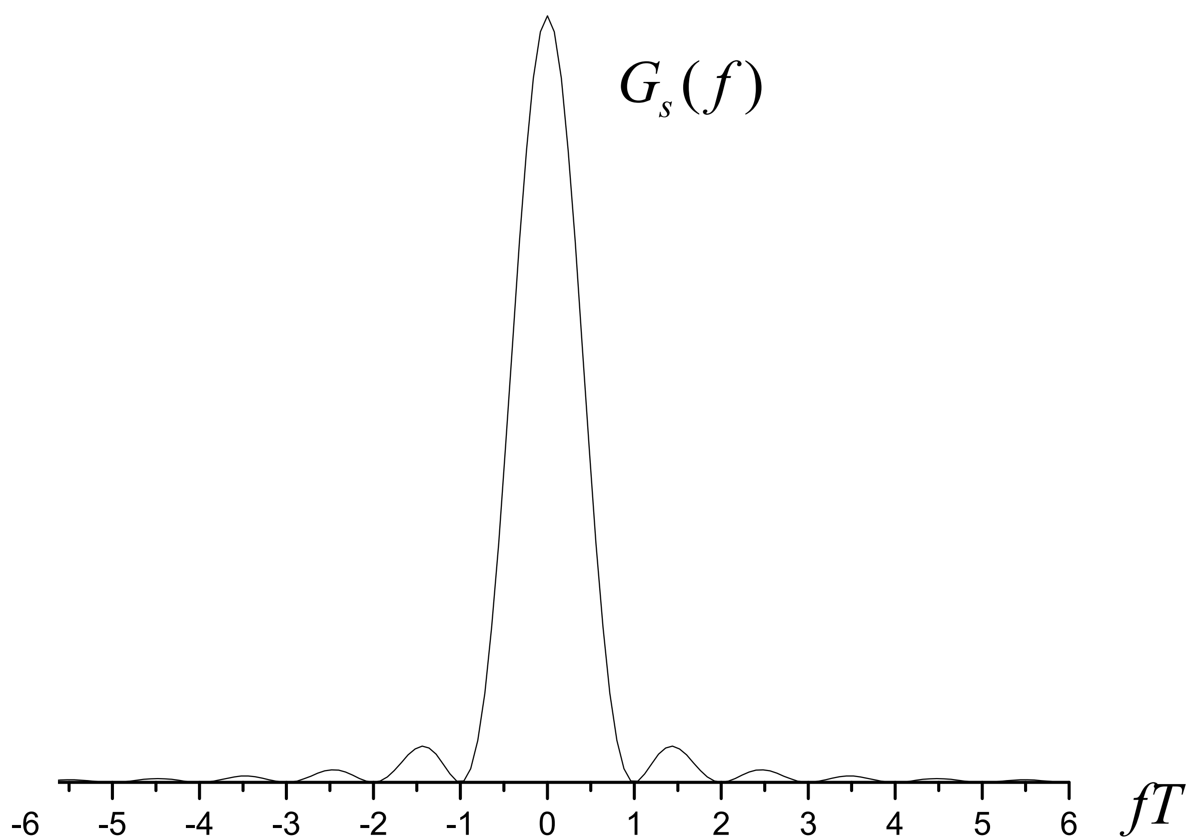


$$a[n] \in \{+\alpha, -\alpha\}$$

Bipolar NRZ

Signal spectrum

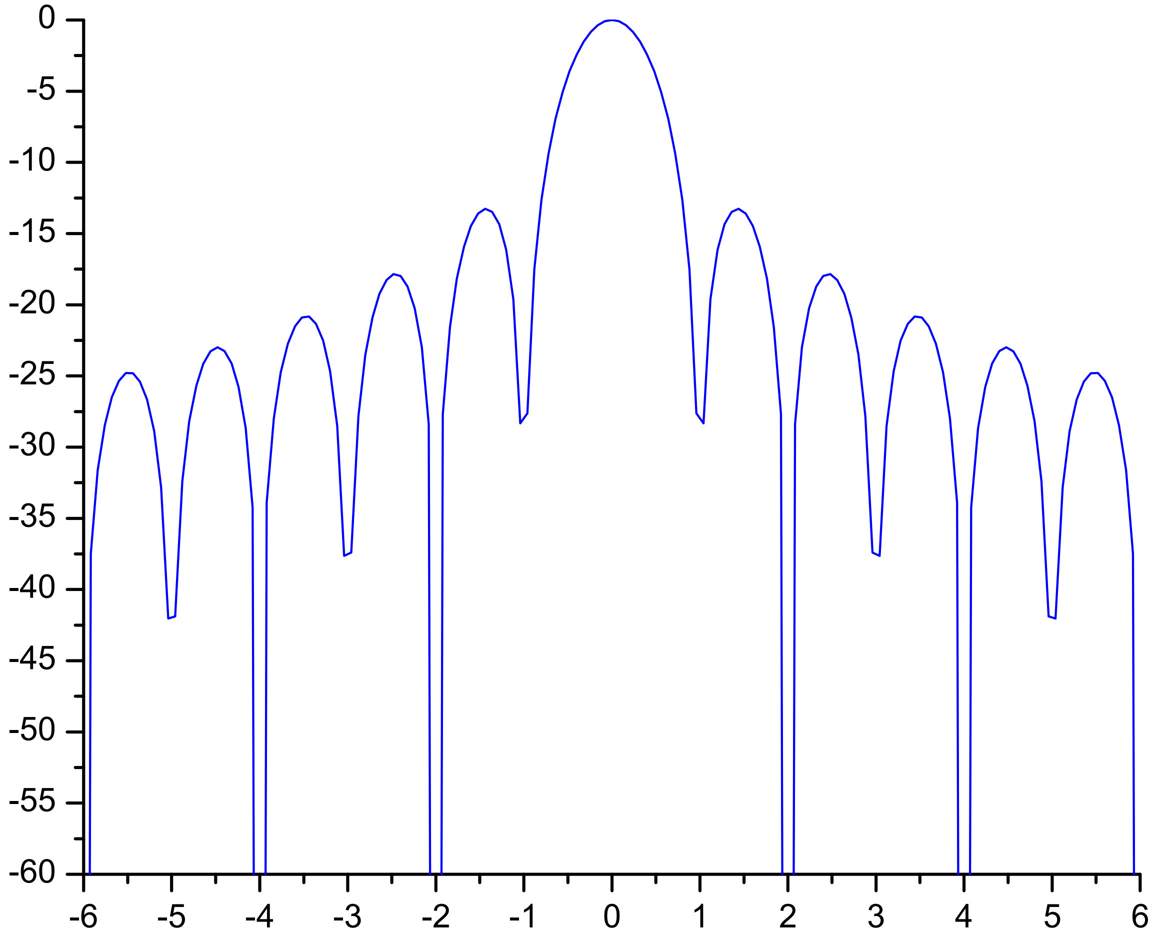
$$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = A^2 T \mathrm{sinc}^2(fT)$$



Bipolar NRZ

Signal spectrum

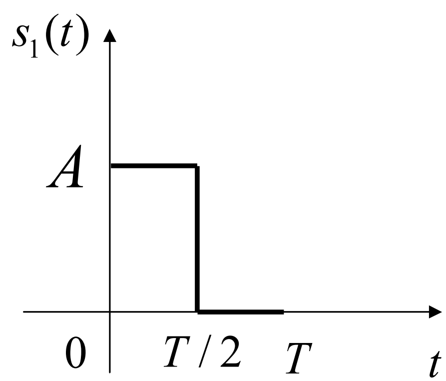
$$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = A^2 T \mathrm{sinc}^2(fT)$$

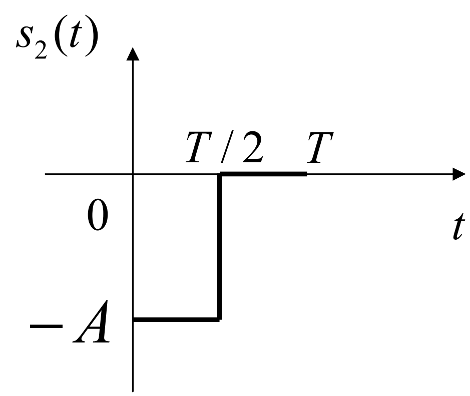


Bipolar RZ (Return to Zero)

Signal set

$$\boxed{M = \{ s_1(t) = +AP_{T/2}(t)\,,\, s_2(t) = -AP_{T/2}(t)\,\}}$$





Versor

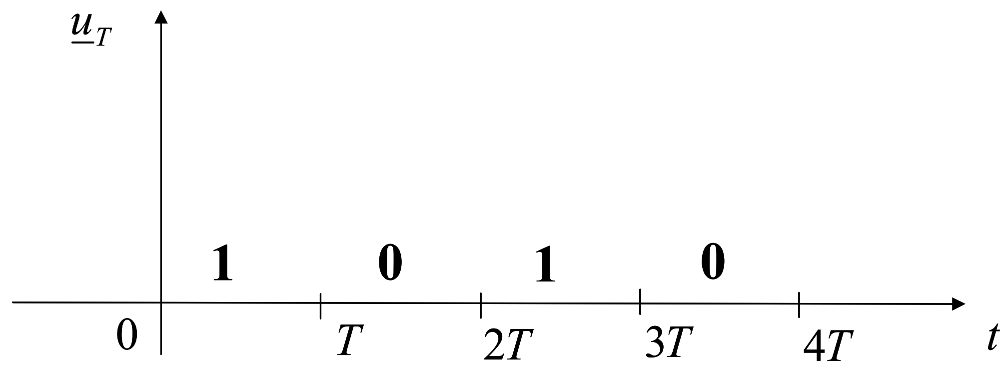
$$b_1(t) = \sqrt{\frac{2}{T}}P_{T/2}(t)$$

Vector set

$$M = \{ \underline{s_1} = (+\alpha)\,,\, \underline{s_2} = \left(-\alpha\right) \}$$

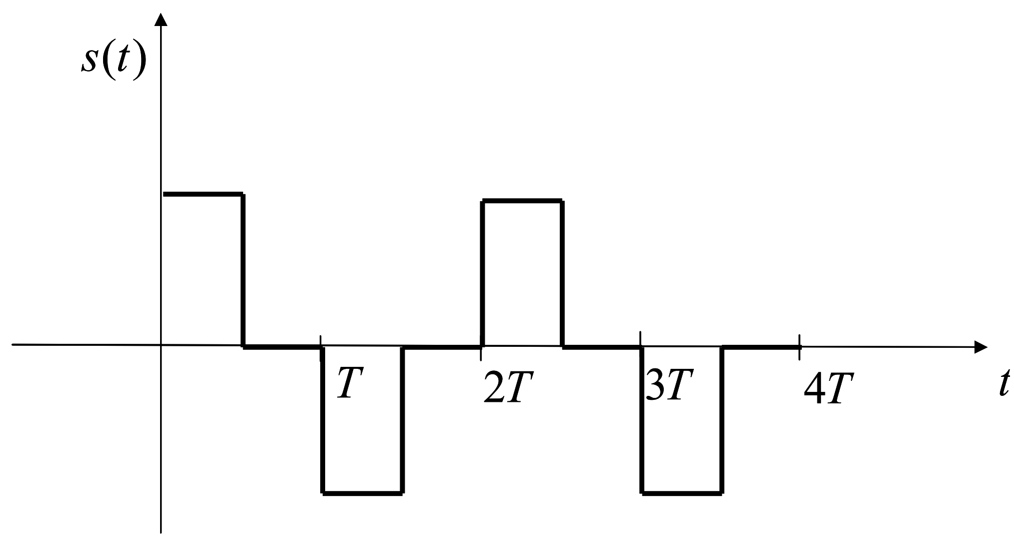
Bipolar RZ

Transmitted waveform



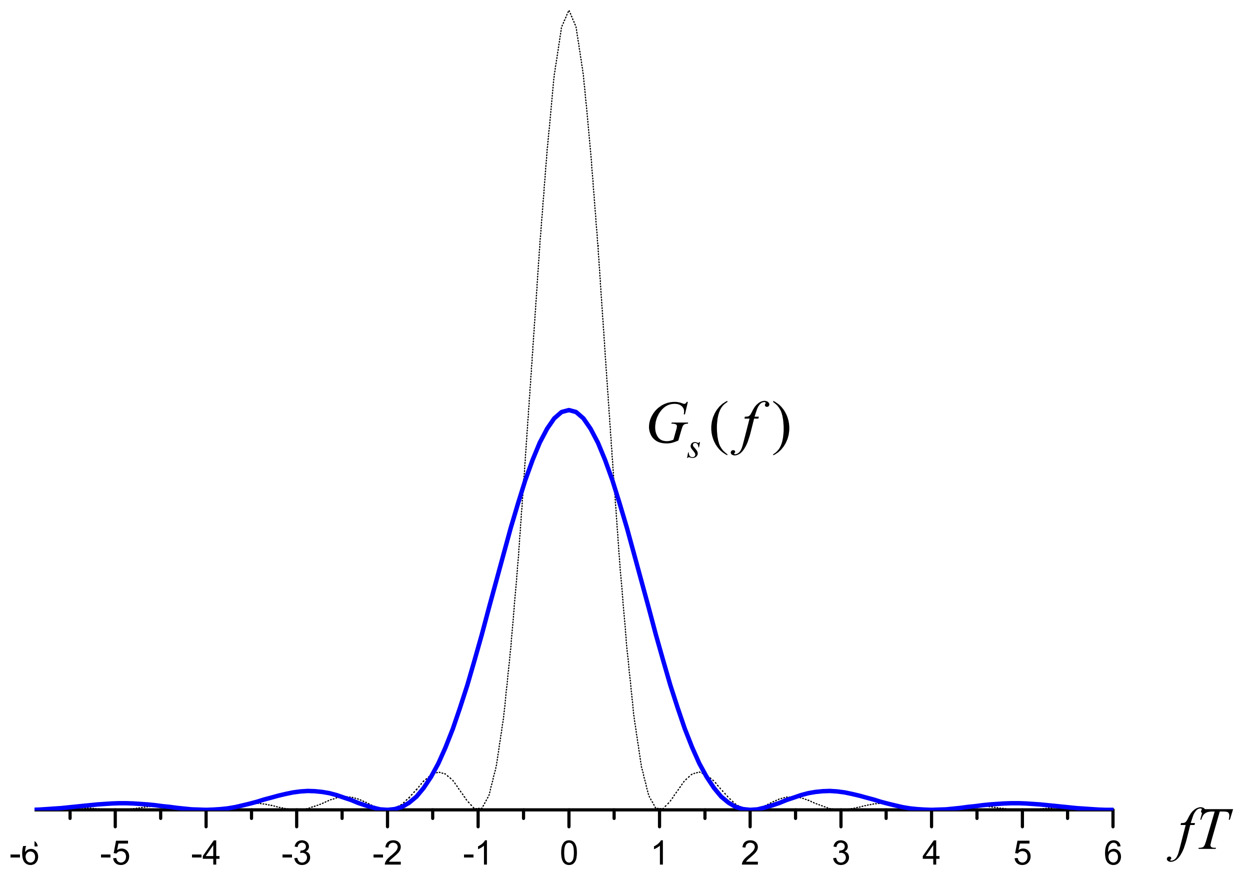
$$s(t) = \sum_n a[n]p(t - nT)$$

$$a[n] \in \{+\alpha, -\alpha\}$$



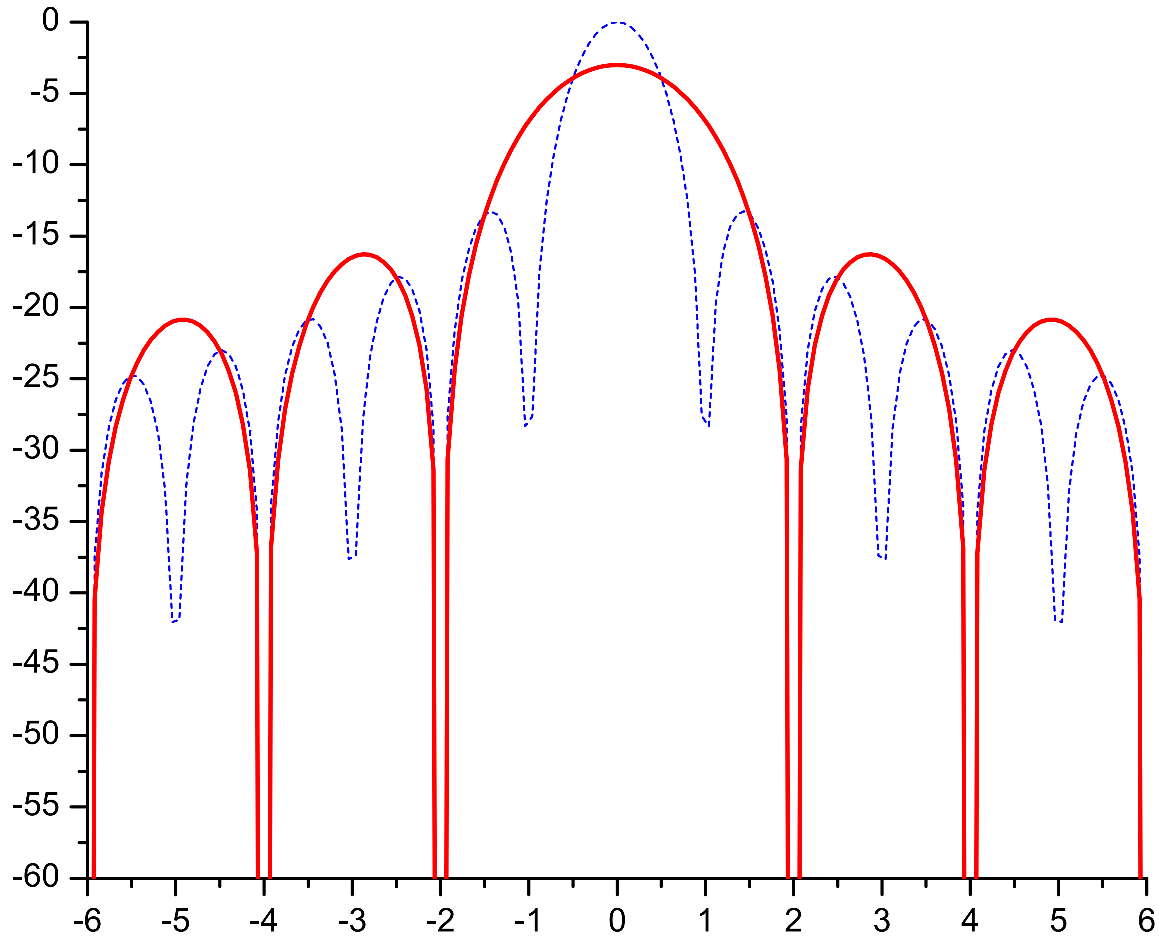
Bipolar RZ

Signal spectrum $$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = \frac{A^2 T}{4} \operatorname{sinc}^2(fT/2)$$



Bipolar RZ

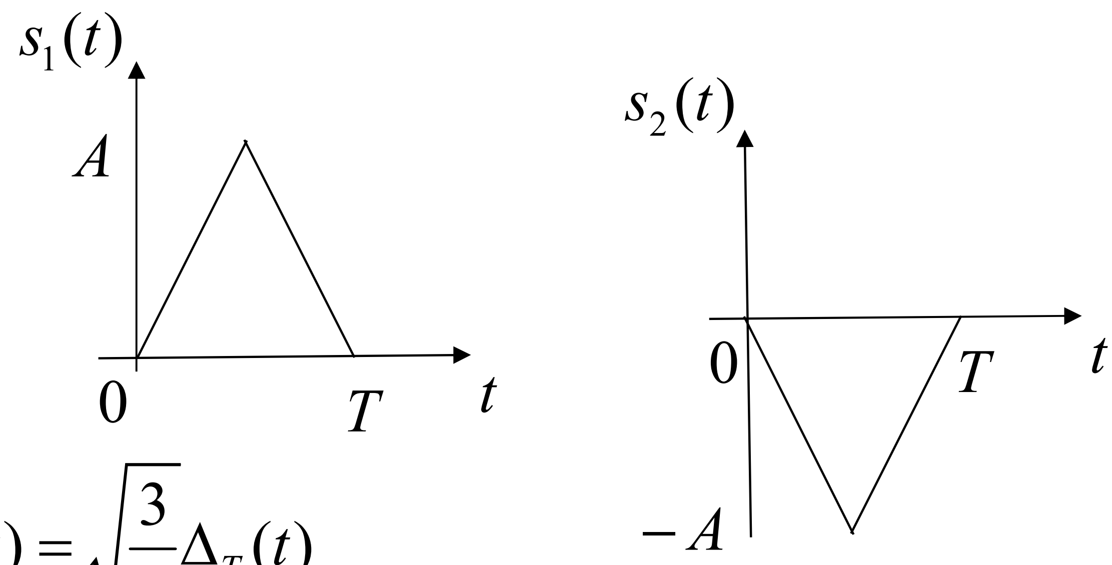
Signal spectrum $$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = \frac{A^2 T}{4} \mathrm{sinc}^2(fT/2)$$



Example: Bipolar triangular

Signal set

$$\boxed{M = \{s_1(t) = +A\Delta_T(t)\,,\, s_2(t) = -A\Delta_T(t)\,\}}$$



Versor

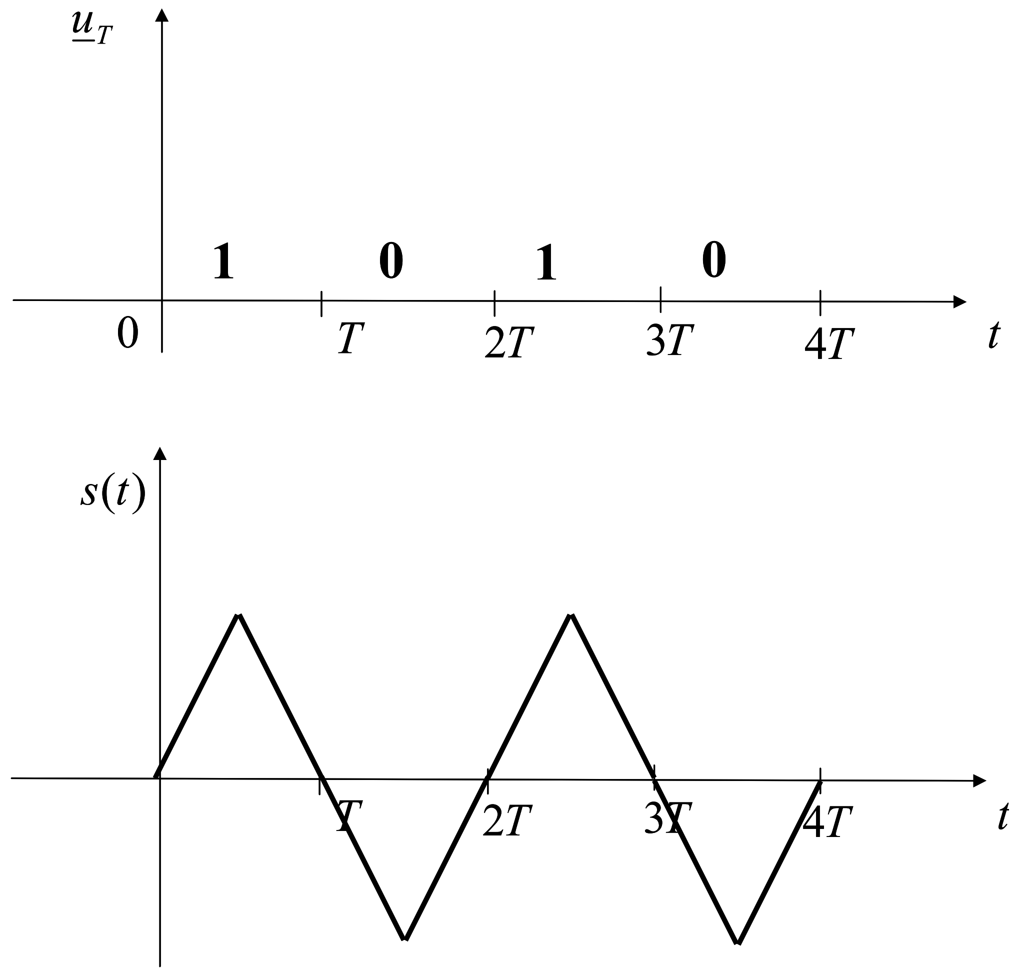
$$b_1(t) = \sqrt{\frac{3}{T}}\Delta_T(t)$$

Vector set

$$M = \{\underline{s_1} = (+\alpha)\,,\, \underline{s_2} = \left(-\alpha\right)\}$$

Example: Bipolar triangular

Transmitted waveform

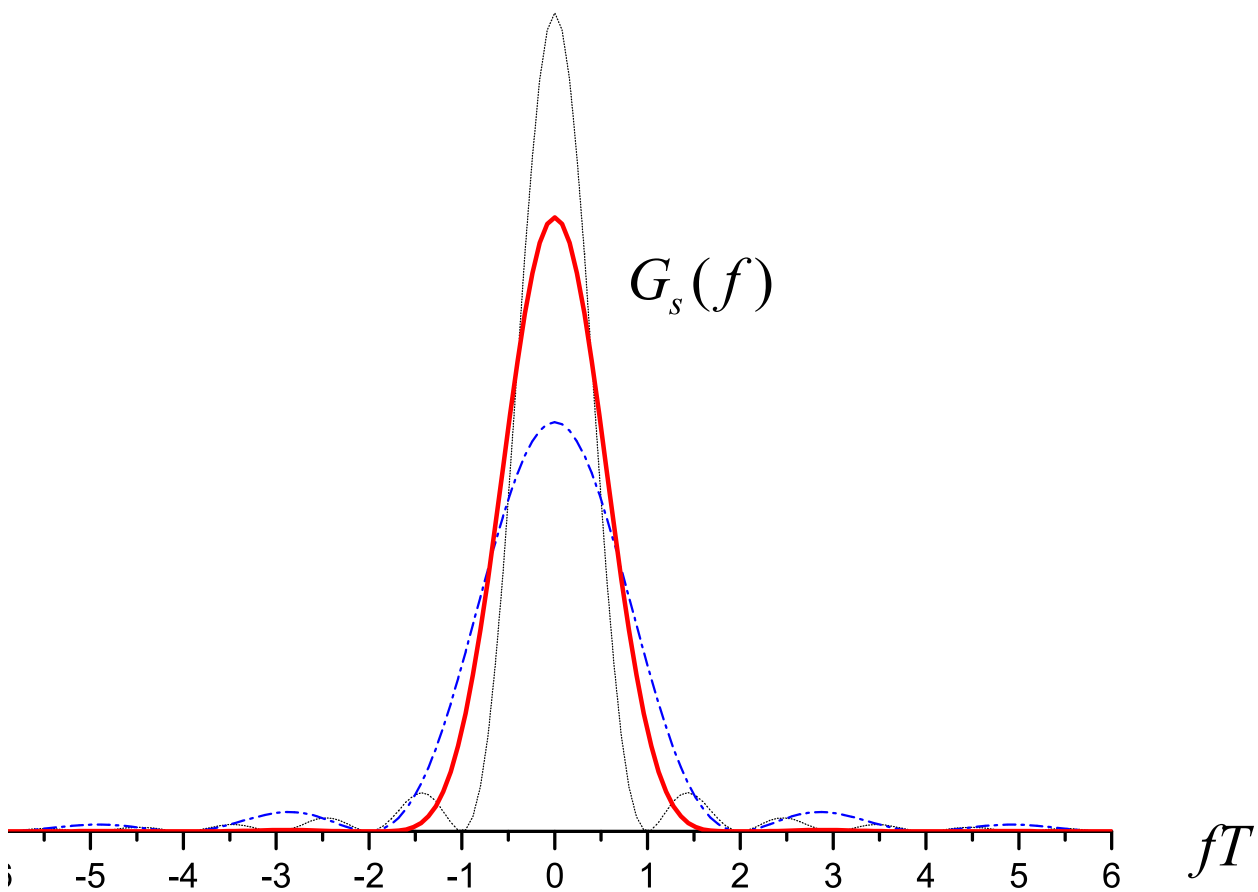


$$s(t) = \sum_n a[n] p(t - nT)$$

$$a[n] \in \{+\alpha, -\alpha\}$$

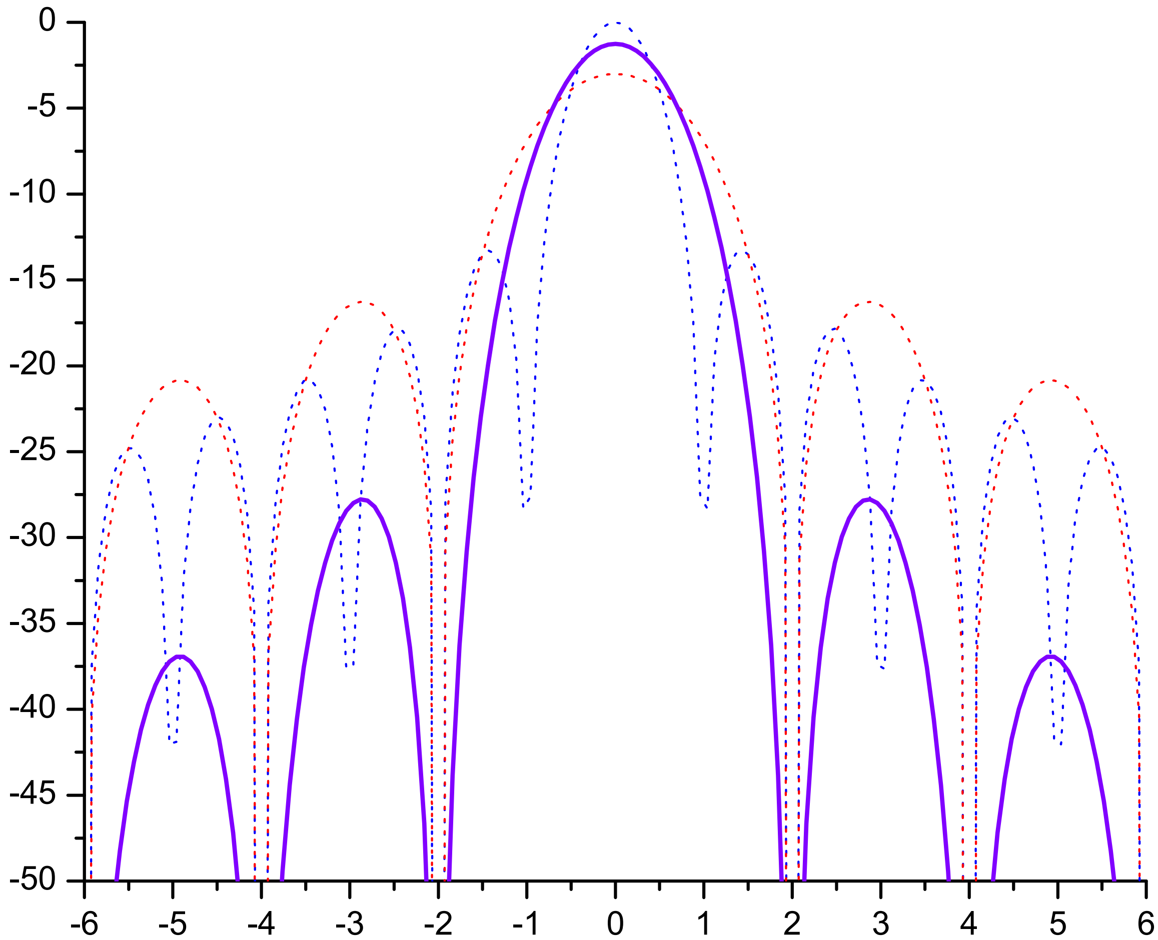
Example: Bipolar triangular

Signal spectrum $$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = \frac{A^2 T}{4} \operatorname{sinc}^4(fT/2)$$



Example: Bipolar triangular

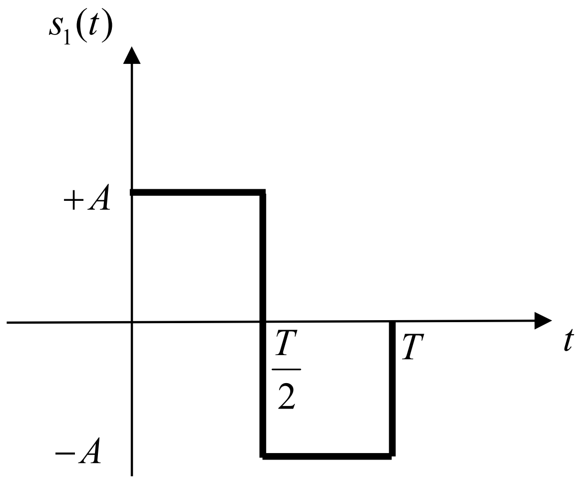
Signal spectrum
$$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = \frac{A^2 T}{4} \mathrm{sinc}^4(fT/2)$$

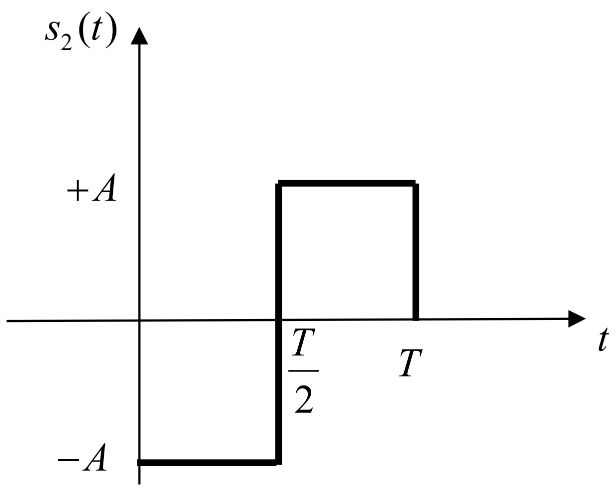


Manchester (biphase)

Signal set $$\boxed{M = \{s_1(t) = +Ax(t)\ ,\ s_2(t) = -Ax(t)\ \}}$$

$$x(t) = \left[+P_{T/2}(t) - P_{T/2}(t - T/2)\right]$$



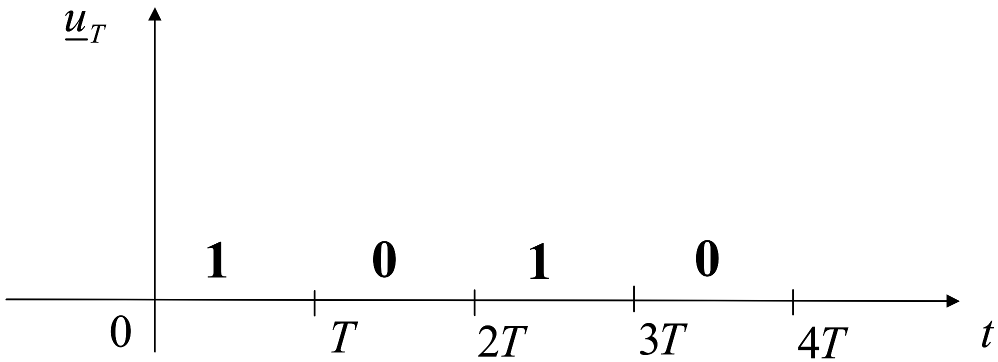


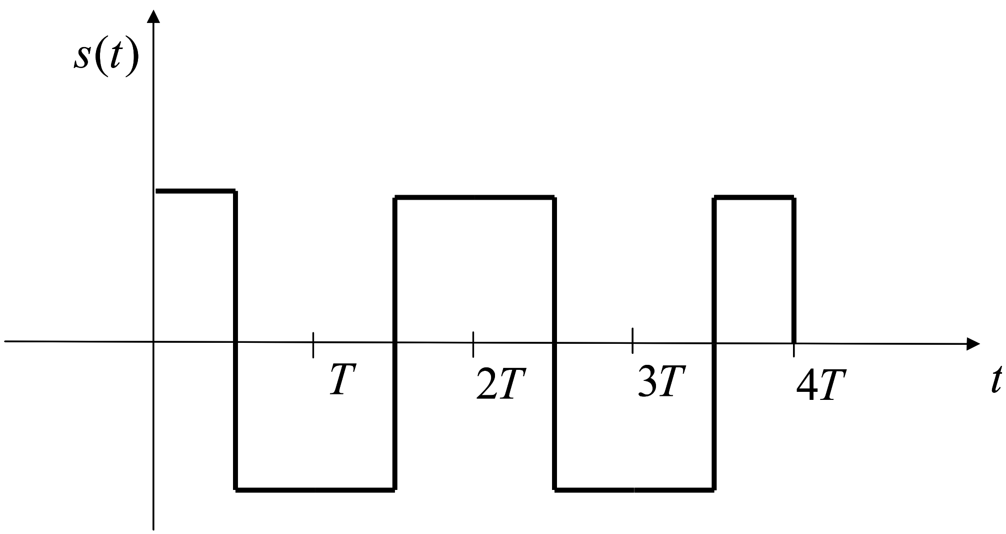
Versor $$b_1(t) = \frac{1}{\sqrt{T}}\left[+P_{T/2}(t) - P_{T/2}(t - T/2)\right]$$

Vector set $$M = \{\underline{s_1} = (+\alpha)\ ,\ \underline{s_2} = (-\alpha)\ \}$$

Manchester (biphase)

Transmitted waveform





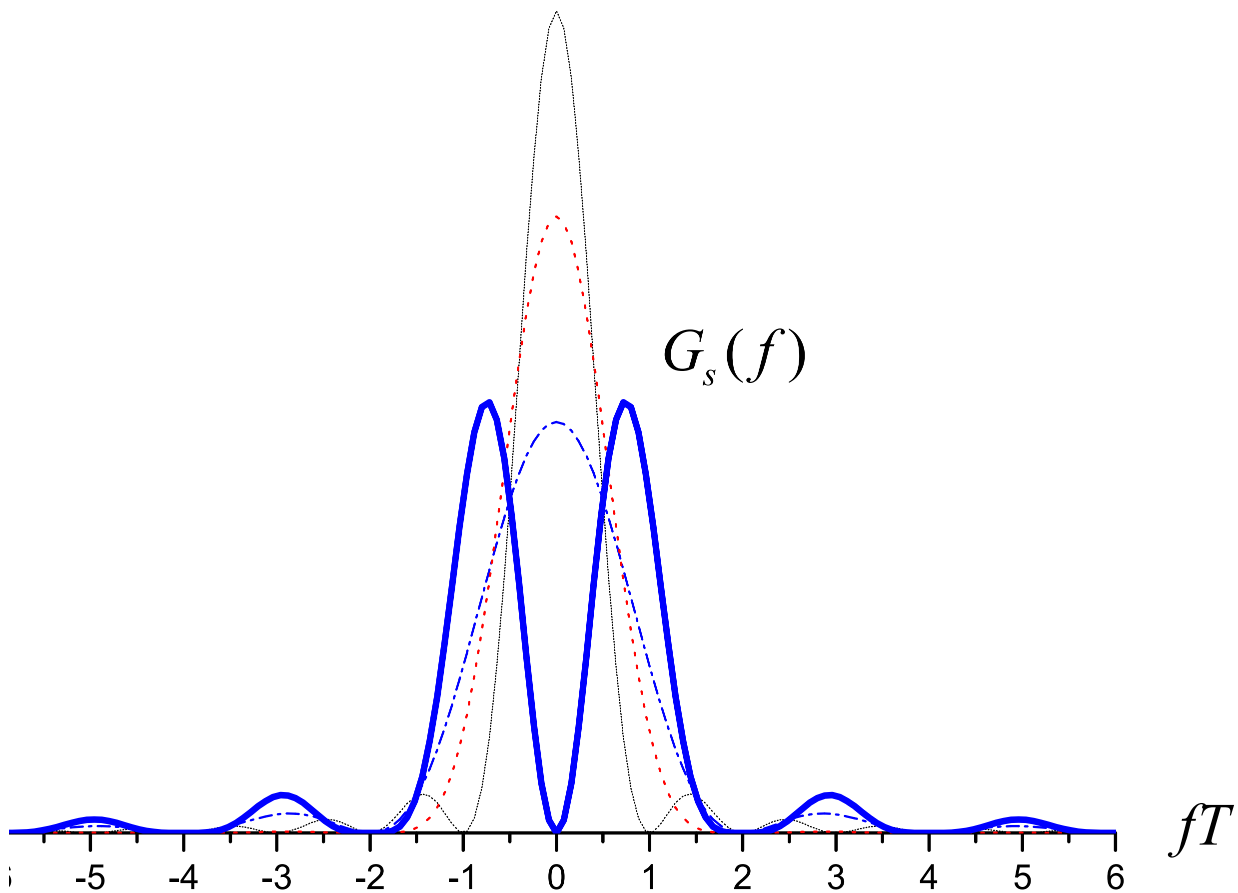
$$s(t) = \sum_n a[n]p(t - nT)$$

$$a[n] \in \{+\alpha, -\alpha\}$$

Manchester (biphase)

Signal spectrum $$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = A^2 T \frac{\sin^4(\pi f T/2)}{(\pi f T/2)^2}$$

(maximum at $f \approx 0.74/T$)

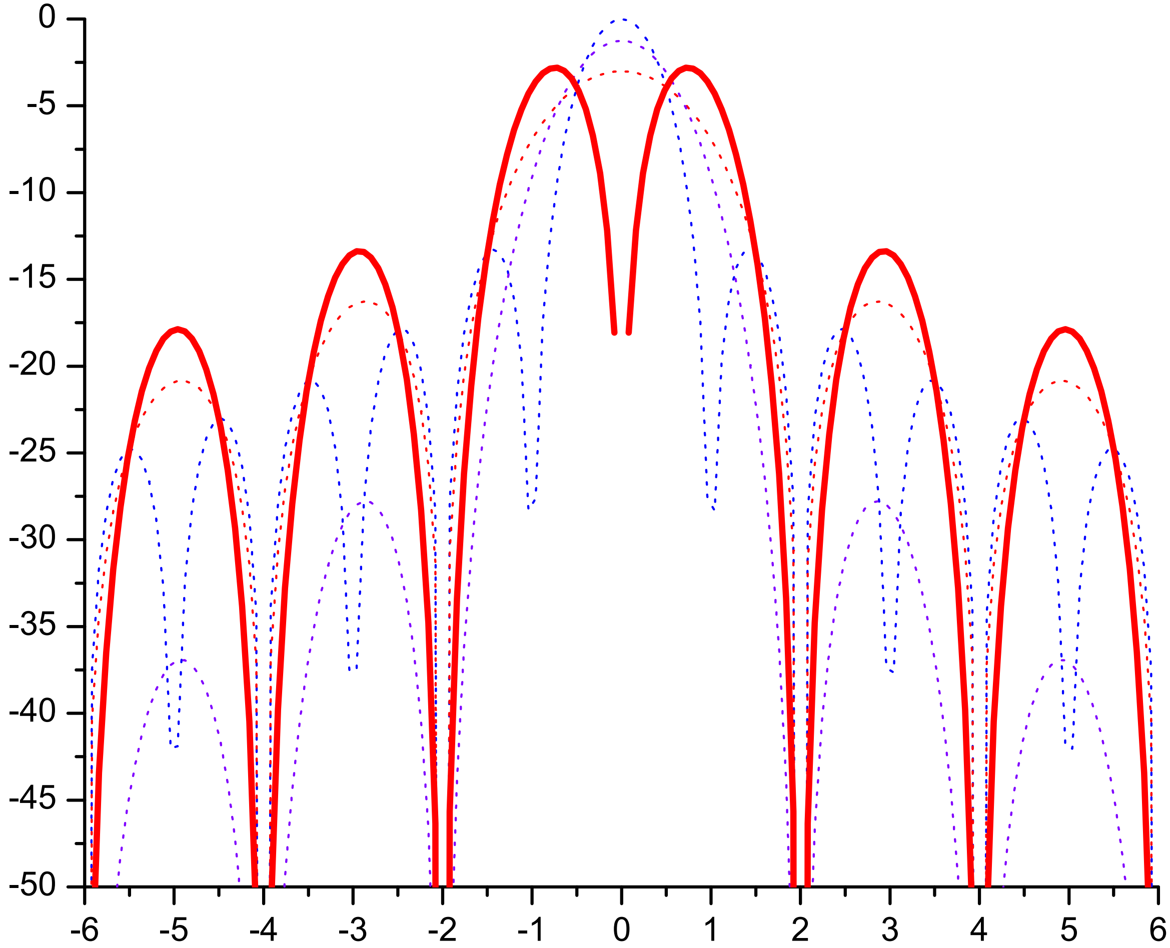


Manchester (biphase)

Signal spectrum

$$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} = A^2 T \frac{\sin^4(\pi f T/2)}{(\pi f T/2)^2}$$

(maximum at $f \approx 0.74/T$)



Manchester (biphase)

$$p(t) = b_1(t) = \frac{1}{\sqrt{T}}\left[+P_{T/2}(t) - P_{T/2}\left(t - \frac{T}{2}\right)\right]$$

$$P(f) = \frac{1}{\sqrt{T}}\left[+\frac{T}{2}\text{sinc}\left(f\frac{T}{2}\right)\exp\left(-j2\pi f\frac{T}{4}\right) - \frac{T}{2}\text{sinc}\left(f\frac{T}{2}\right)\exp\left(-j2\pi f\frac{3T}{4}\right)\right] =$$

$$= \left[+\frac{\sqrt{T}}{2}\text{sinc}\left(f\frac{T}{2}\right)\exp\left(-j2\pi f\frac{T}{4}\right)\right]\left[1 - \exp\left(-j\pi fT\right)\right]$$

$$\left|P(f)\right|^2 = \frac{T}{4}\text{sinc}^2\left(f\frac{T}{2}\right)\left|1 - \cos\left(-\pi fT\right) - j\sin\left(-\pi fT\right)\right|^2 =$$

$$= \frac{T}{4}\text{sinc}^2\left(f\frac{T}{2}\right)\left|1 - \cos\left(\pi fT\right) + j\sin\left(\pi fT\right)\right|^2 =$$

$$= \frac{T}{4}\text{sinc}^2\left(f\frac{T}{2}\right)\left[1 + \cos^2\left(\pi fT\right) - 2\cos\left(\pi fT\right) + \sin^2\left(\pi fT\right)\right] =$$

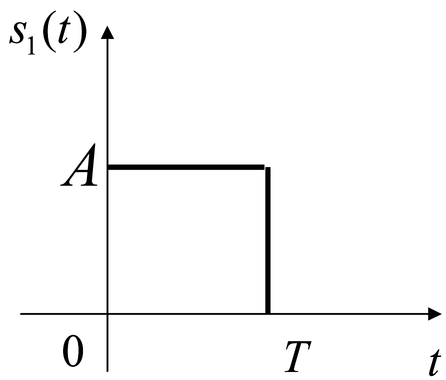
$$\boxed{\sin\left(\frac{A}{2}\right) = \sqrt{\frac{1 - \cos A}{2}}}$$

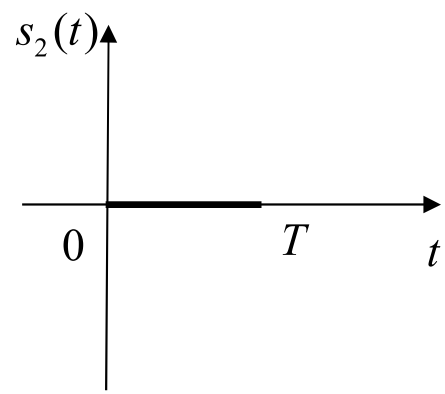
$$= \frac{T}{2}\text{sinc}^2\left(f\frac{T}{2}\right)\left[1 - \cos\left(\pi fT\right)\right] = T\text{sinc}^2\left(f\frac{T}{2}\right)\sin^2\left(\pi f\frac{T}{2}\right)$$

Unipolar NRZ

Signal set

$$\boxed{M = \{s_1(t) = +AP_T(t)\ ,\ s_2(t) = 0\ \}}$$





Versor

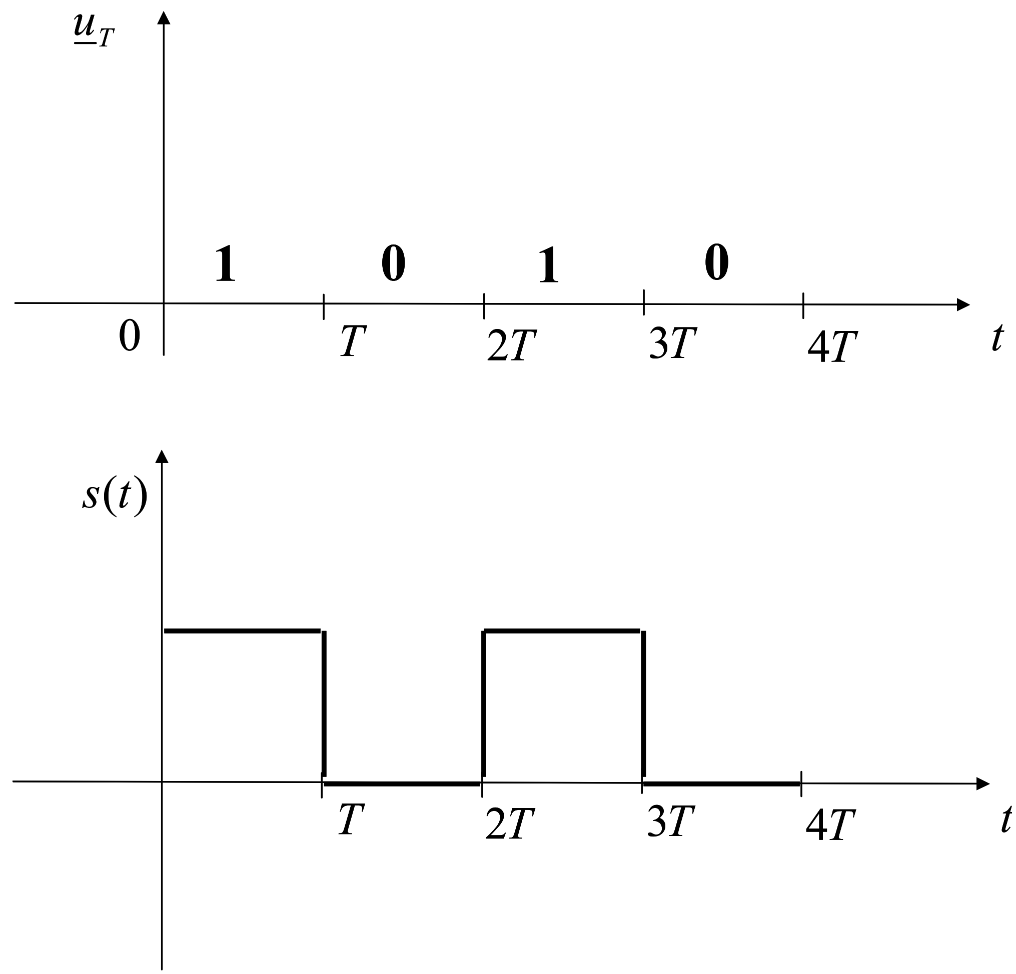
$$b_1(t) = \frac{1}{\sqrt{T}} P_T(t)$$

Vector set

$$M = \{\underline{s_1} = (+\alpha)\ ,\ \underline{s_2} = (0)\ \}$$

Unipolar NRZ

Transmitted waveform



$$s(t) = \sum_n a[n] p(t - nT)$$

$$a[n] \in \{+\alpha, 0\}$$

Unipolar NRZ

Signal spectrum

$$G_s(f) = \sigma_a^2 \frac{|P(f)|^2}{T} + \frac{\mu_a^2}{T^2} \sum_{n=-\infty}^{+\infty} \left| P\left(\frac{n}{T}\right) \right|^2 \delta\left(f - \frac{n}{T}\right)$$

$$|P(f)|^2 = x \operatorname{sinc}^2(\pi f T) \qquad x \in R$$

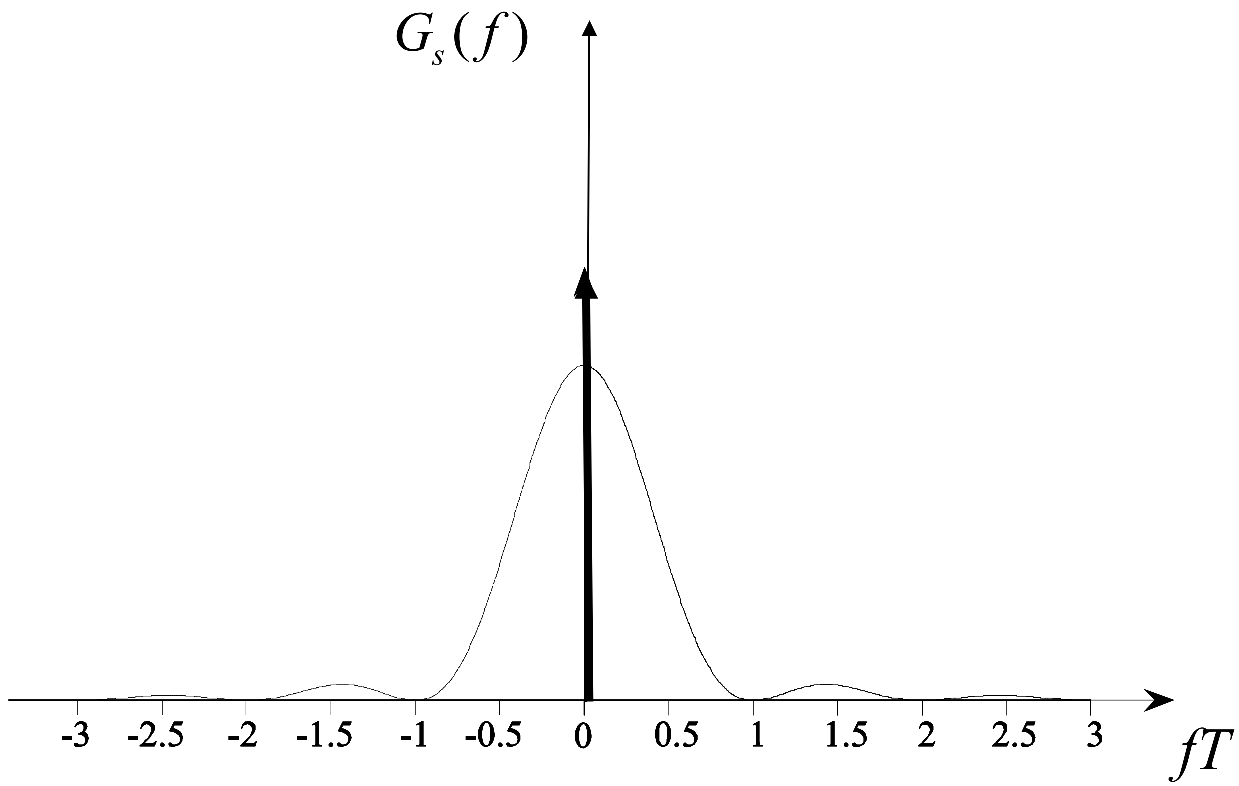
A Dirac delta at zero frequency

$$G_s(f) = \frac{A^2}{4} T \operatorname{sinc}^2(f\mathrm{T}) + \frac{A^2}{4} \delta(f)$$

Unipolar NRZ

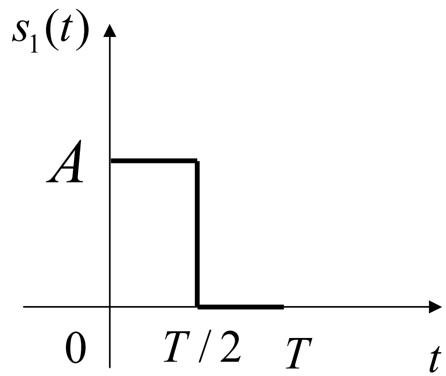
Signal spectrum

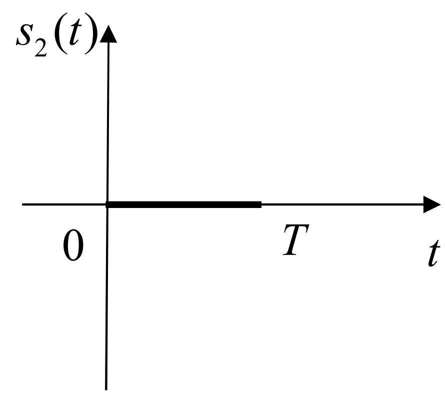
$$G_s(f) = \frac{A^2}{4} T \operatorname{sinc}^2(f\mathrm{T}) + \frac{A^2}{4}\delta\left(f\right)$$



Unipolar RZ

Signal set $\boxed{M=\{s_1(t)=+AP_{T/2}(t)\,,\,s_2(t)=0\,\}}$



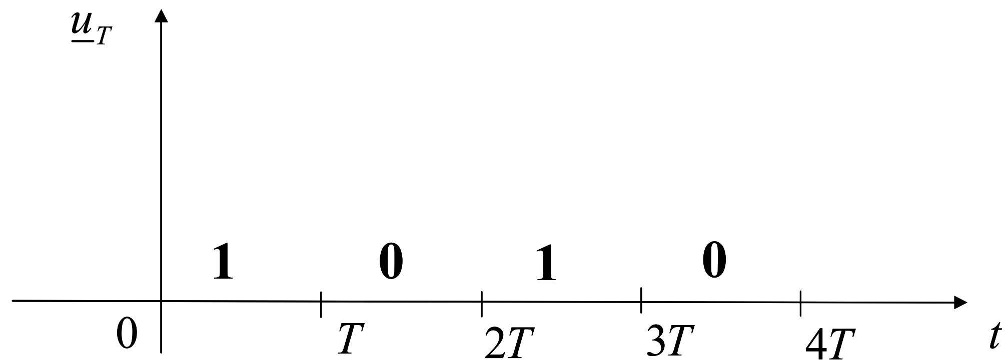


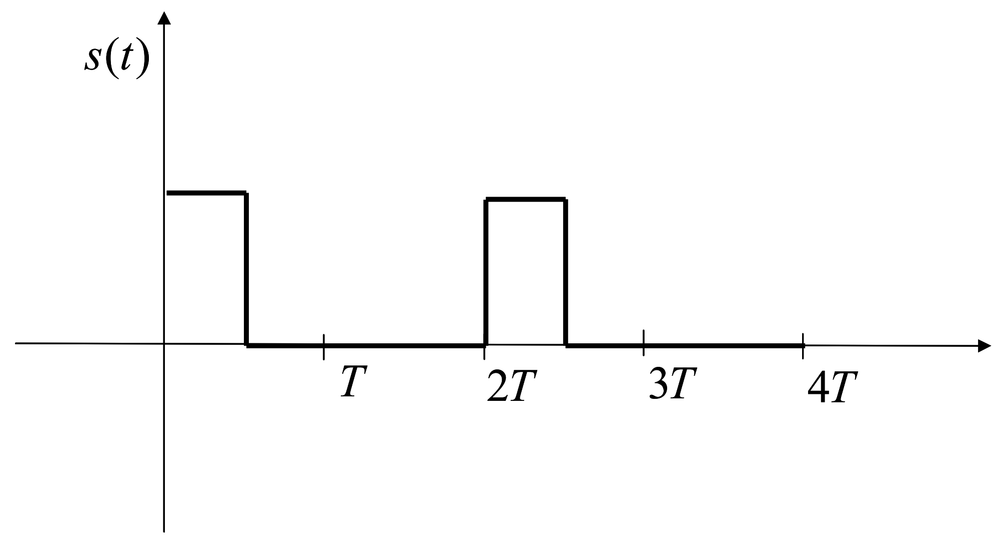
Versor $b_1(t)=\sqrt{\dfrac{2}{T}}P_{T/2}(t)$

Vector set $M=\{\underline{s_1}=(+\alpha)\,,\,\underline{s_2}=\left(0\right)\}$

Unipolar RZ

Transmitted waveform





$$s(t) = \sum_n a[n]p(t - nT)$$

$$a[n] \in \{+\alpha, 0\}$$

Unipolar RZ

Signal spectrum

$$G(f) = \sigma_a^2 \frac{|P(f)|^2}{T} + \frac{\mu_a^2}{T^2} \sum_{n=-\infty}^{+\infty} \left| P\left(\frac{n}{T}\right) \right|^2 \delta\left(f - \frac{n}{T}\right)$$

$$\left|P(f)\right|^2 = z\left[\frac{\sin(\pi fT/2)}{(\pi fT/2)}\right]^2 \qquad \left(z \in R\right)$$

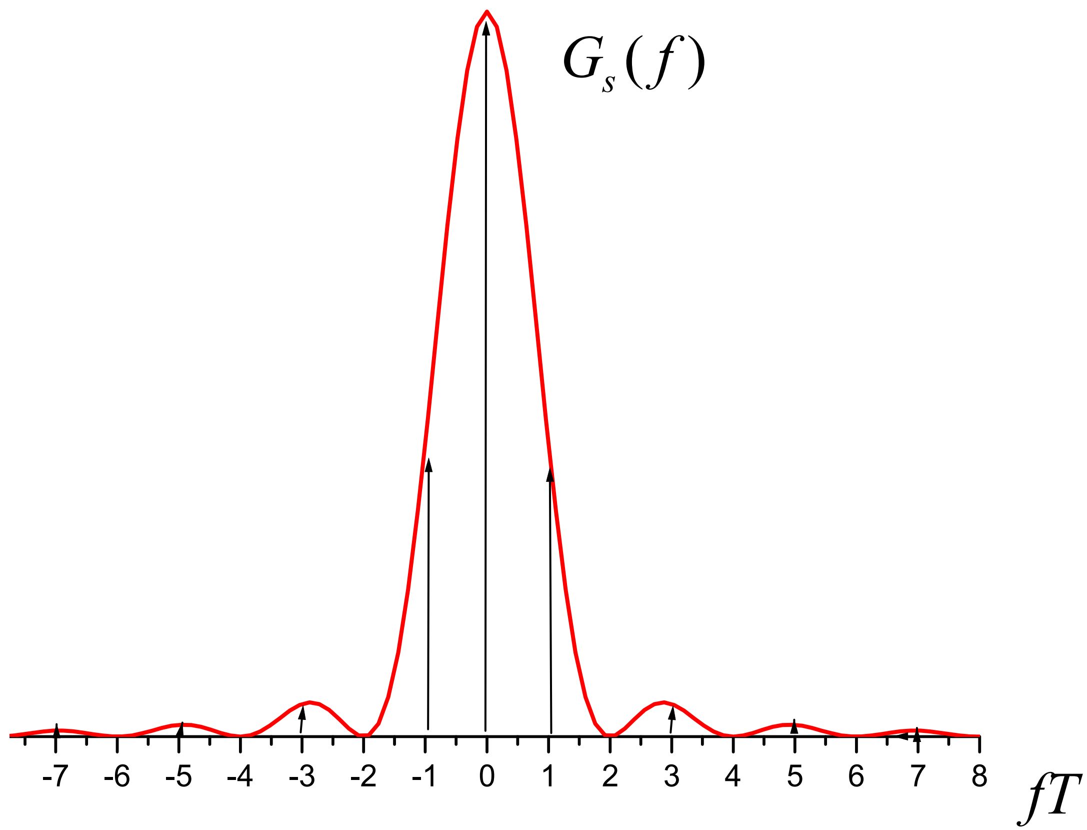
Dirac deltas at zero frequency and at odd multiples of $1/T$

$$G_s(f) = \frac{A^2}{16} T\mathrm{sinc}^2(fT/2) + \frac{A^2}{16} \sum_{i=-\infty}^{+\infty} \mathrm{sinc}^2\left(\frac{(2i+1)}{2}\right) \delta\left(f - \frac{(2i+1)}{T}\right)$$

Unipolar RZ

Signal spectrum

$$G_s(f) = \frac{A^2}{16} T \mathrm{sinc}^2(fT/2) + \frac{A^2}{16} \sum_{i=-\infty}^{+\infty} \mathrm{sinc}^2\left(\frac{(2i+1)}{2}\right) \delta\left(f - \frac{(2i+1)}{T}\right)$$



m-PAM constellation*: characteristics*

1. Base-band modulation
2. One-dimensional signal space
3. m signals, symmetrical with respect to the origin
4. Information associated to the impulse amplitude
   PAM=Pulse Amplitude Modulation

m-PAM constellation: *constellation*

SIGNAL SET $\boxed{M = \{s_i(t) = \alpha_i p(t)\}_{i=1}^{m}}$

Versor $b_1(t)=p(t) \quad (d=1)$

VECTOR SET

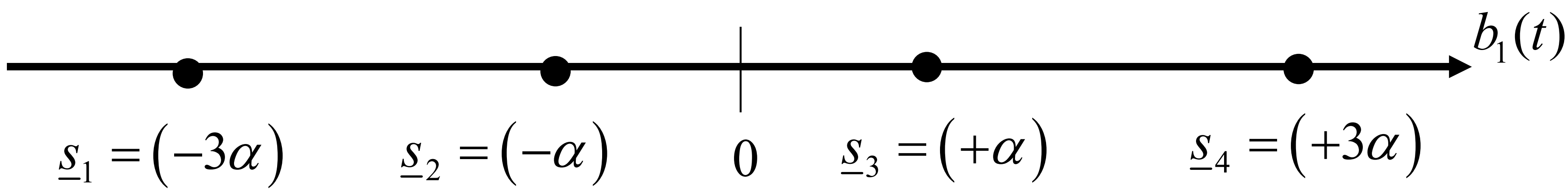
$$\boxed{M = \{\underline{s}_1 = (-(m-1)\alpha)\ ,\ \underline{s}_2 = (-(m-3)\alpha), \ldots, \underline{s}_{m-1} = (+(m-3)\alpha)\ ,\ \underline{s}_m = (+(m-1)\alpha)\} \subseteq R}$$

$$k = \log_2(m) \qquad \boxed{T = kT_b} \qquad \boxed{\boxed{R = \frac{R_b}{k}}}$$

m-PAM constellation*: constellation*

Example: 4-PAM constellation

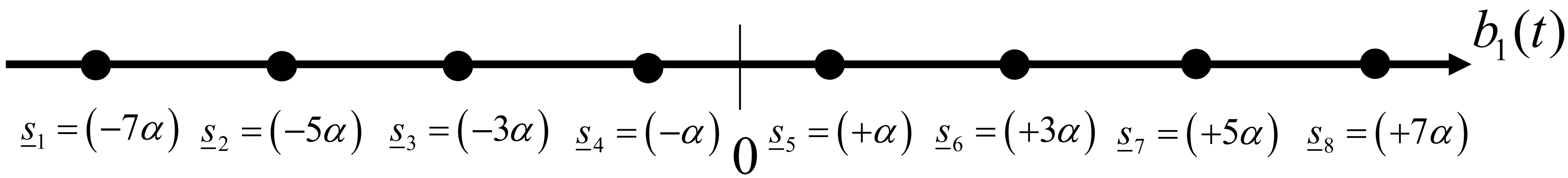
$$M = \{\underline{s}_1 = (\mathbf{-3\alpha}),\ \underline{s}_2 = (\mathbf{-\alpha}), \underline{s}_3 = (+\boldsymbol{\alpha}),\ \underline{s}_4 = (\mathbf{+3\alpha})\} \subseteq R$$



m-PAM constellation: constellation

Example: 8-PAM constellation

$$M=\{\underline{s}_1=(\mathbf{-7\alpha})\,,\,\underline{s}_2=(\mathbf{-5\alpha}),\underline{s}_3=(\mathbf{-3\alpha})\,,\,\underline{s}_4=(\mathbf{-\alpha}),\underline{s}_5=(+\boldsymbol{\alpha})\,,\,\underline{s}_6=(\mathbf{+3\alpha}),\underline{s}_7=(\mathbf{+5\alpha})\,,\,\underline{s}_8=(\mathbf{+7\alpha})\}\subseteq R$$

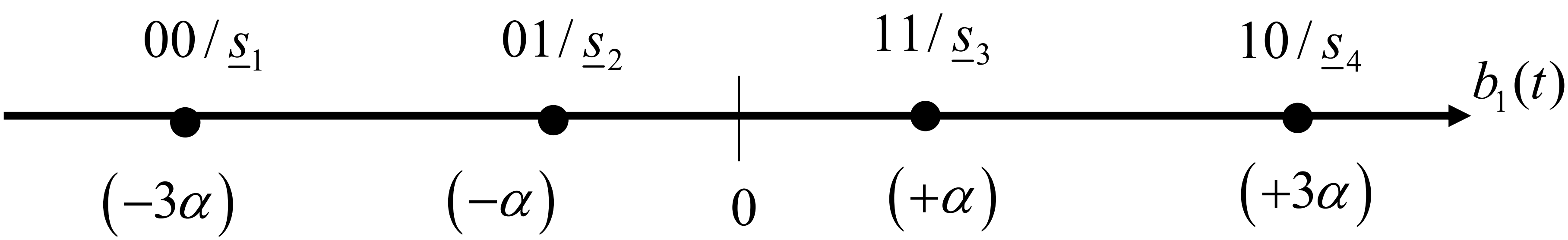


m-PAM constellation: *binary labelling*

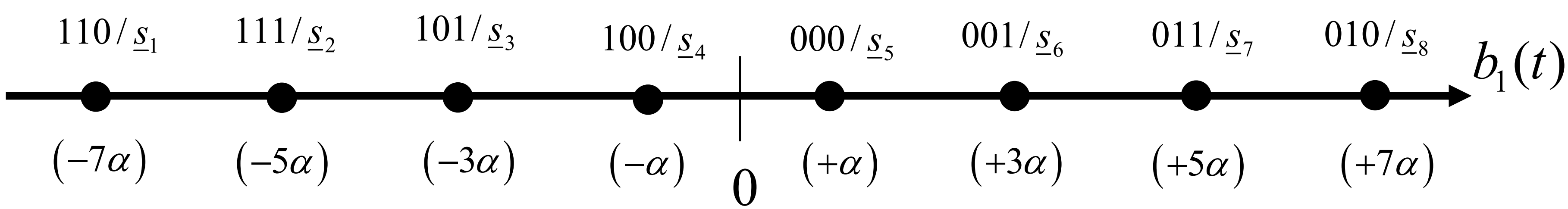
$$e : H_k \leftrightarrow M$$

It is always possible to build a Gray labeling

4-PAM:



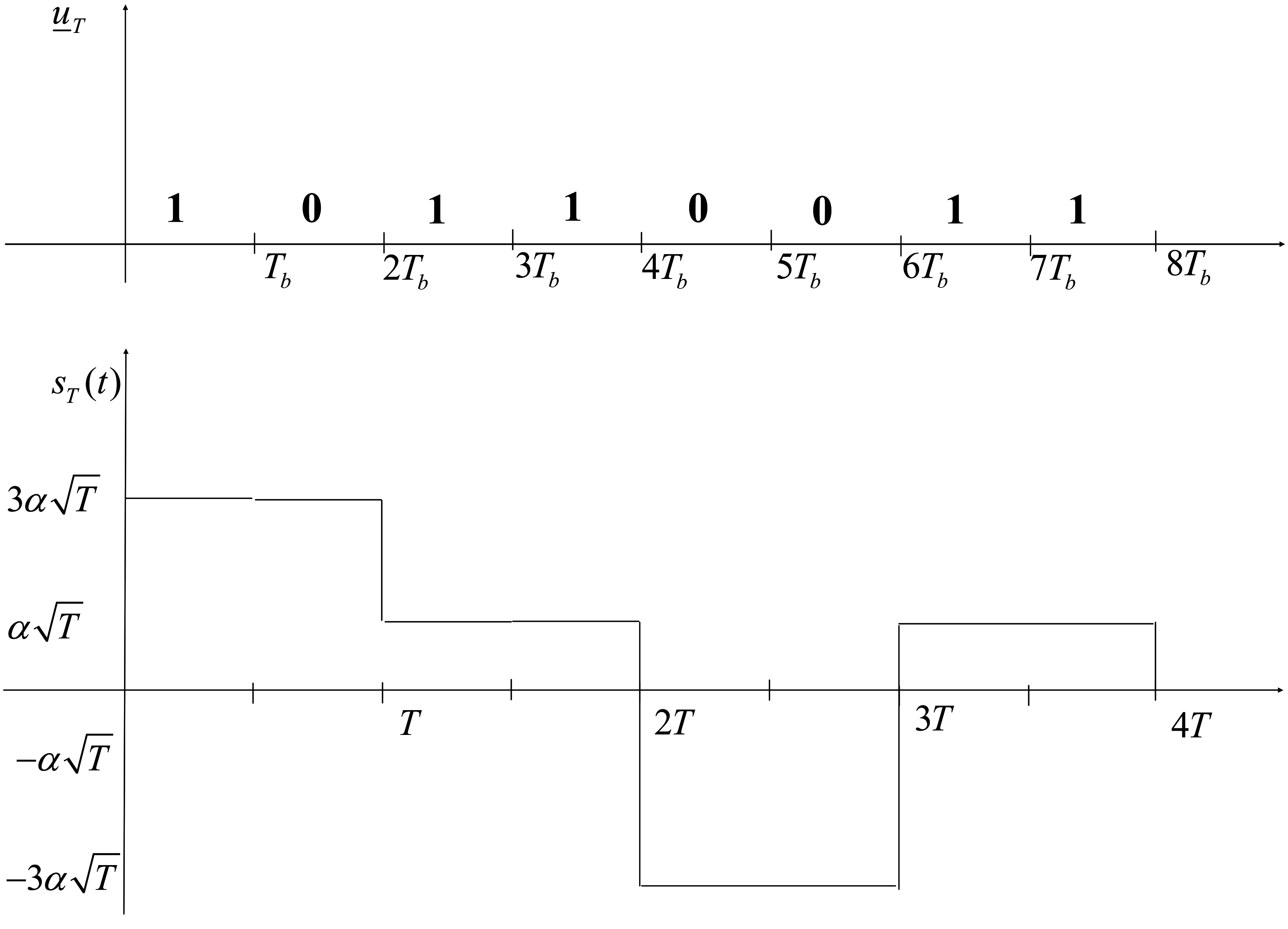
8-PAM:



m-PAM constellation: *transmitted waveform*

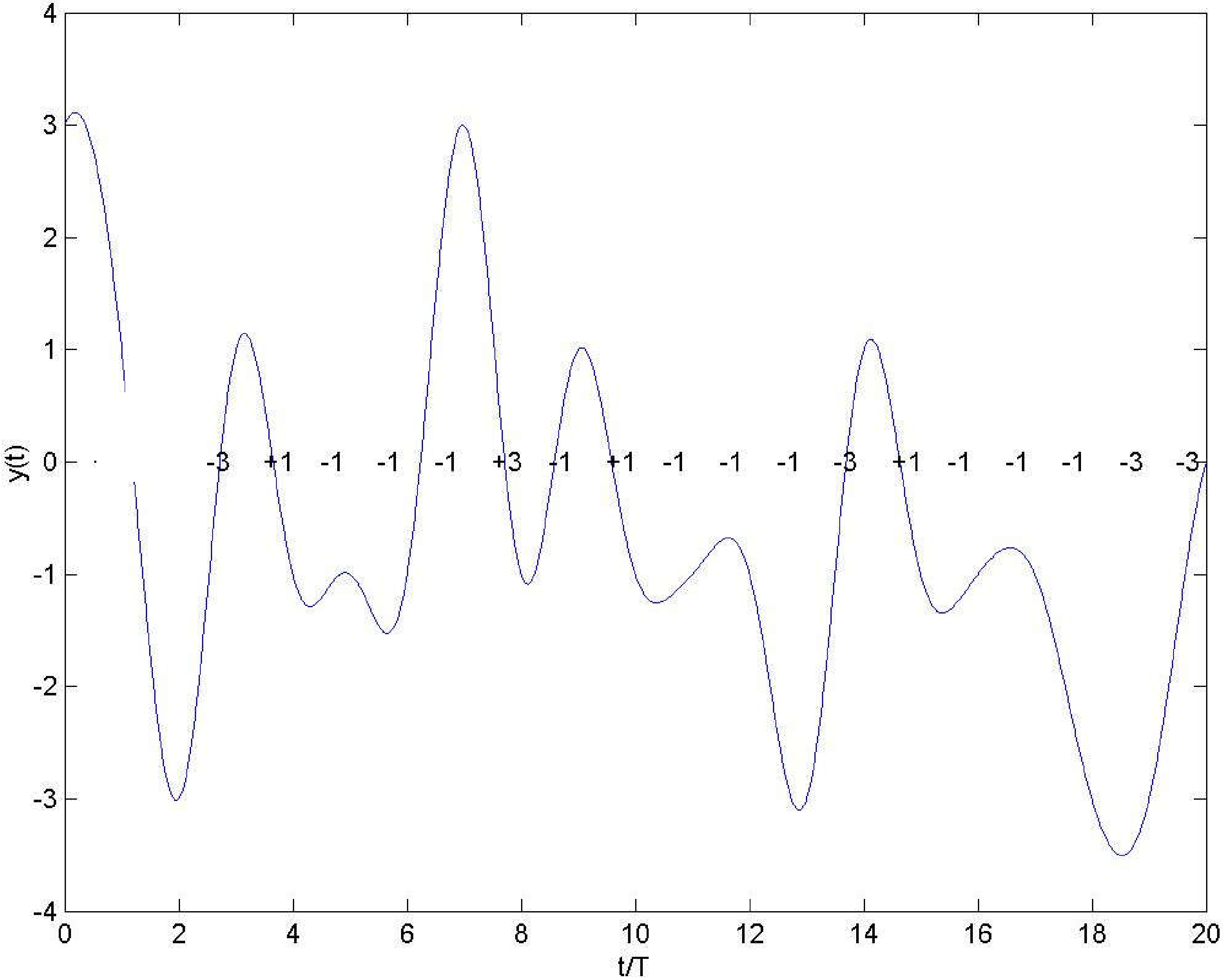
Example: 4-PAM

$$p(t) = \frac{1}{\sqrt{T}} P_T(t)$$



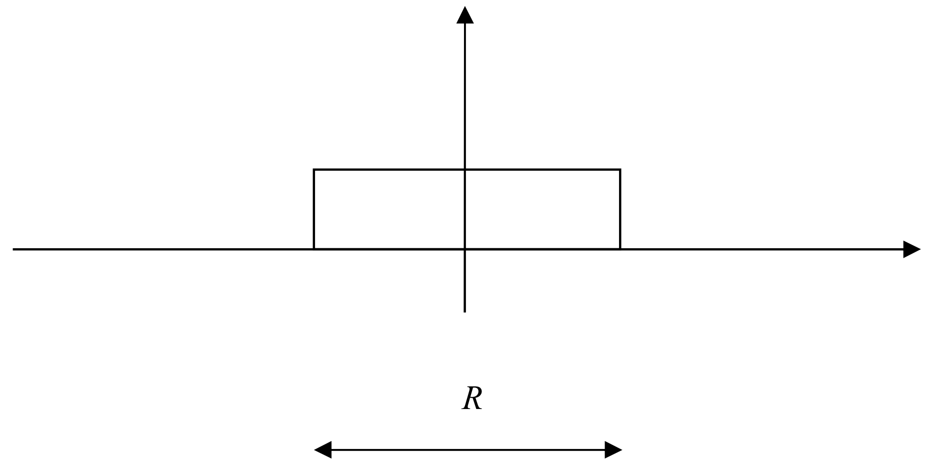
***m-PAM constellation**: transmitted waveform*

Example: 4-PAM $\quad p(t) = \text{ RRC } \alpha = 0.5$



m-PAM constellation: *bandwidth and spectral efficiency*

Case 1: $p(t)$ = ideal low pass filter



Total bandwidth (ideal case)

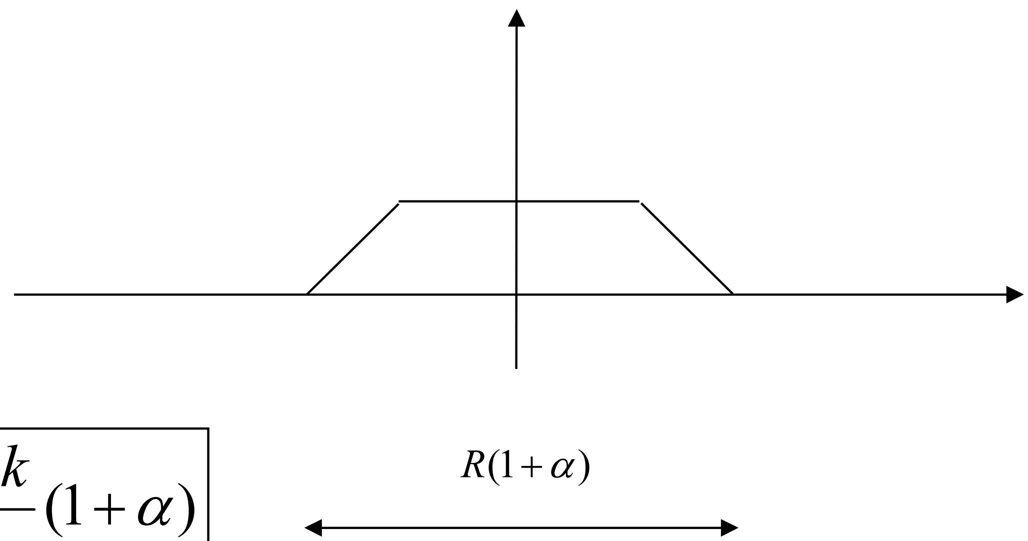
$$\boxed{B_{id} = \frac{R}{2} = \frac{R_b / k}{2}}$$

Spectral efficiency (ideal case)

$$\boxed{\eta_{id} = \frac{R_b}{B_{id}} = 2k \; bps / Hz}$$

m-PAM constellation:
bandwidth and spectral efficiency

Case 2: $p(t)$ = RRC filter roll off α



Total bandwidth $$\boxed{B = \frac{R}{2}(1+\alpha) = \frac{R_b/k}{2}(1+\alpha)}$$

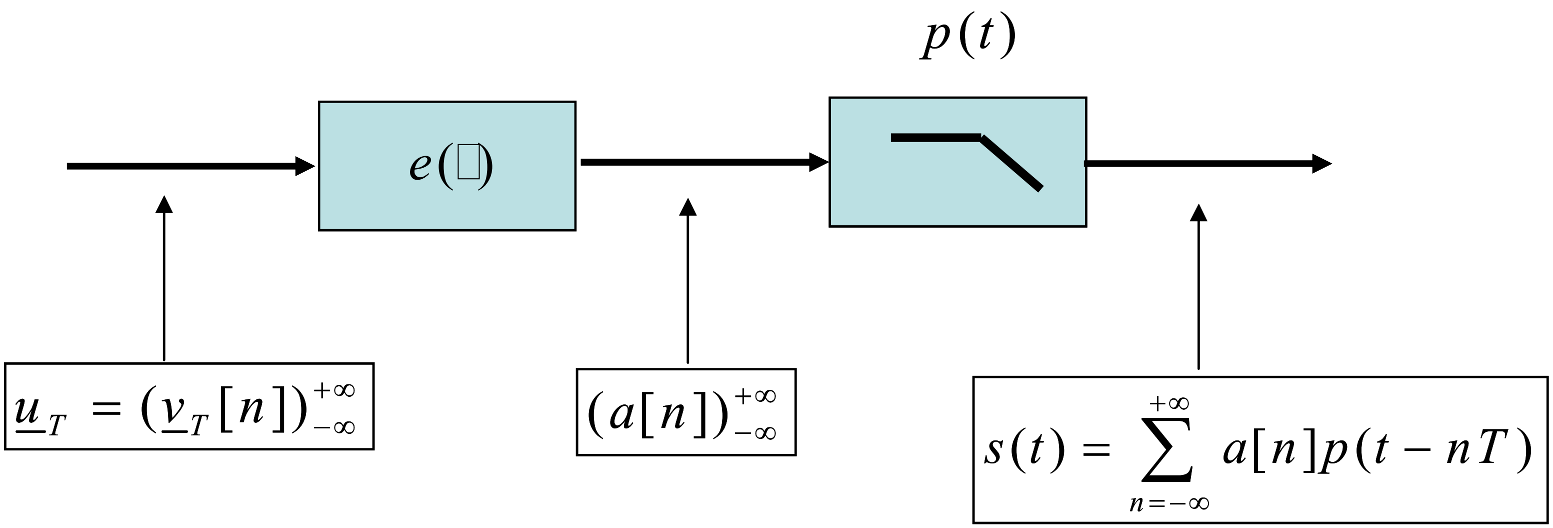
Spectral efficiency $$\boxed{\eta = \frac{R_b}{B} = \frac{2k}{(1+\alpha)}\ bps/Hz}$$

Exercize

Given a baseband channel with bandwidth B up to 4000 Hz, compute the maximum bit rate R_b we can transmit over it with a 256-PAM constellation in the two cases:

- Ideal low pass filter
- RRC filter with ?=0.25

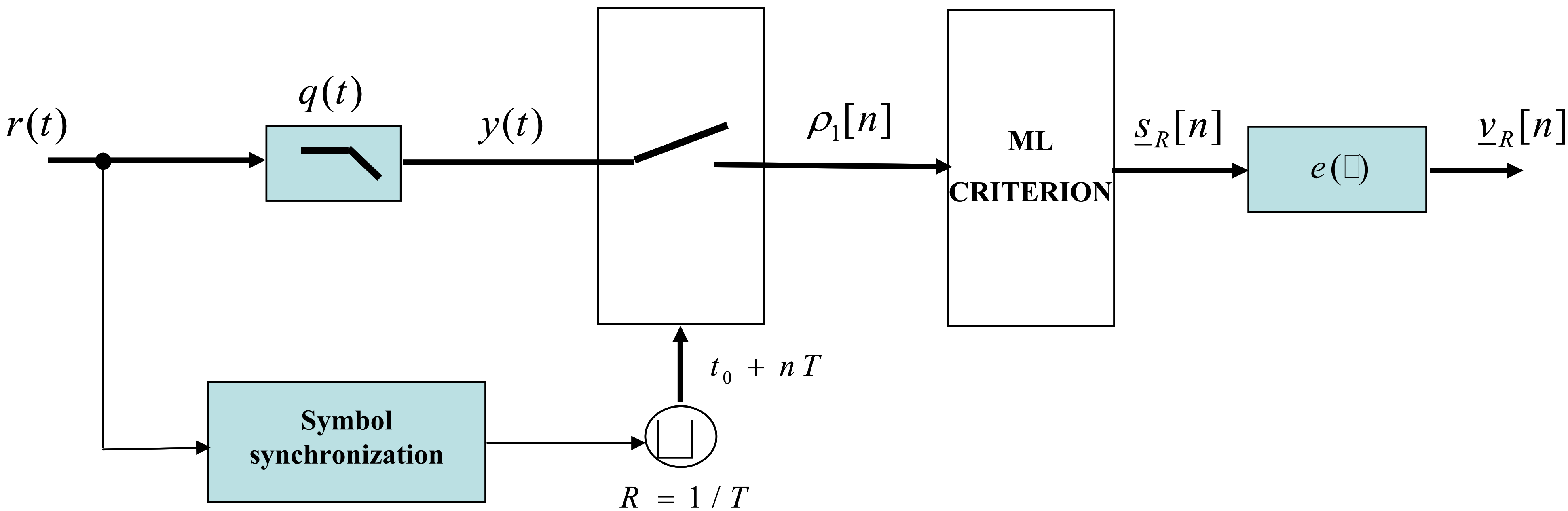
m-PAM constellation: *modulator*



Equal to 2-PAM, but we have m possible levels:

$$a[n] \in \{-(m-1)\alpha, -(m-3)\alpha, \ldots, +(m-3)\alpha, +(m-1)\alpha\}$$

m-PAM constellation: *demodulator*

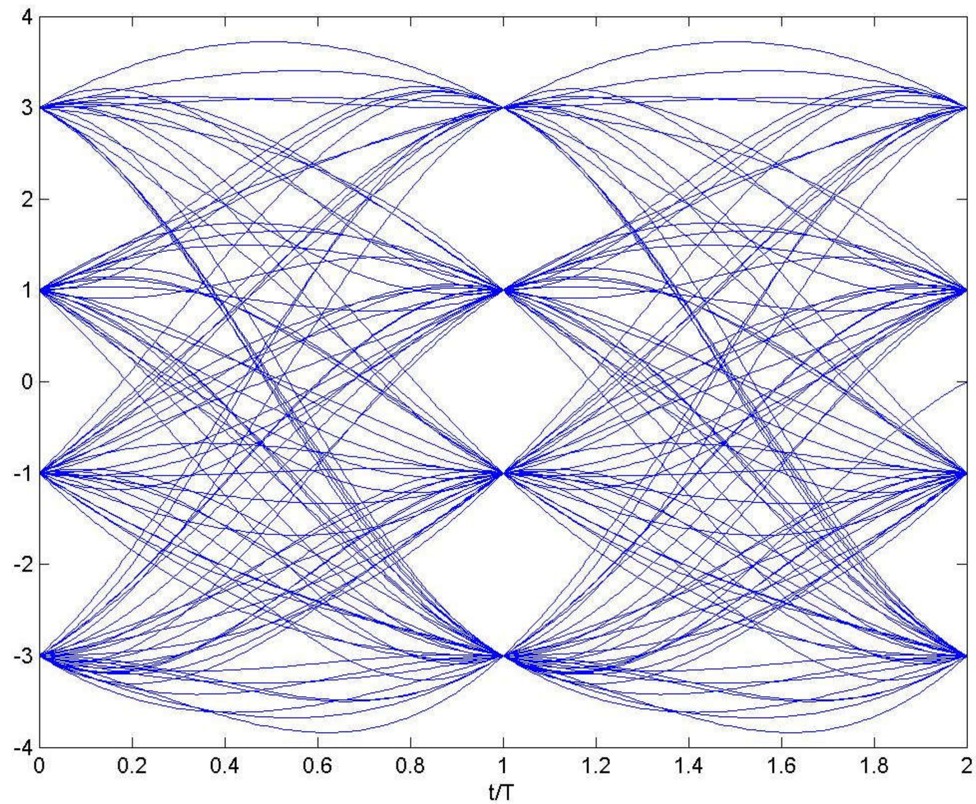


Equal to 2-PAM, but we have m possible levels:

$$a[n] \in \{-(m-1)\alpha, -(m-3)\alpha, \ldots, +(m-3)\alpha, +(m-1)\alpha\}$$

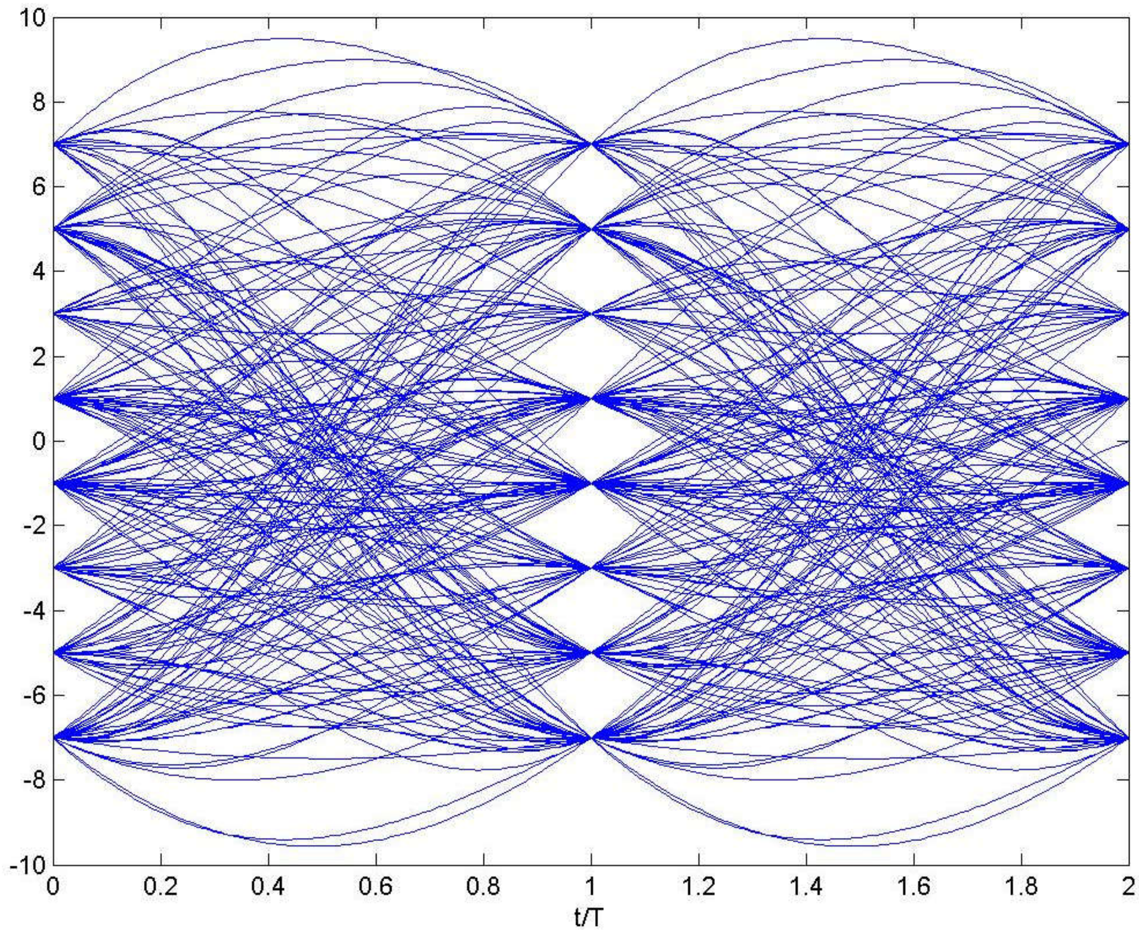
m-PAM constellation: *eye diagram*

4-PAM, $p(t)$ = RRC with ? =0.5



m-PAM constellation: *eye diagram*

8-PAM, $p(t)$ = RRC with ⍰ =0.5



***m-PAM constellation**: error probability*

By applying the asymptotic approximation we can obtain

$$P_b(e) \approx \frac{m-1}{mk}\, erfc\left(\sqrt{\frac{3k}{m^2-1}\frac{E_b}{N_0}}\right)$$

m-PAM constellation*: error probability*

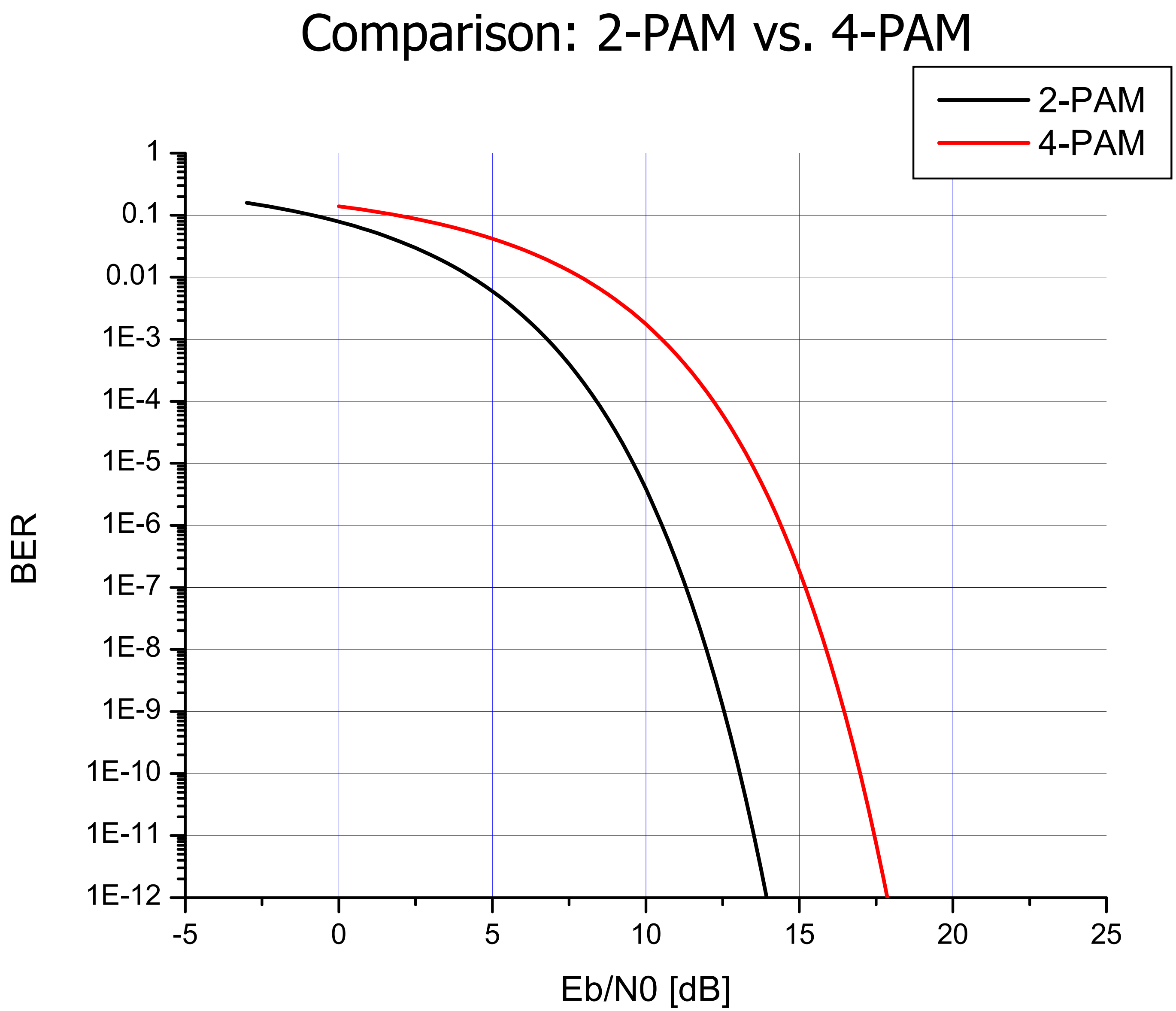
Comparison: 2-PAM vs. 4-PAM

$$2-\mathrm{PAM:} \qquad P_b(e)=\frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

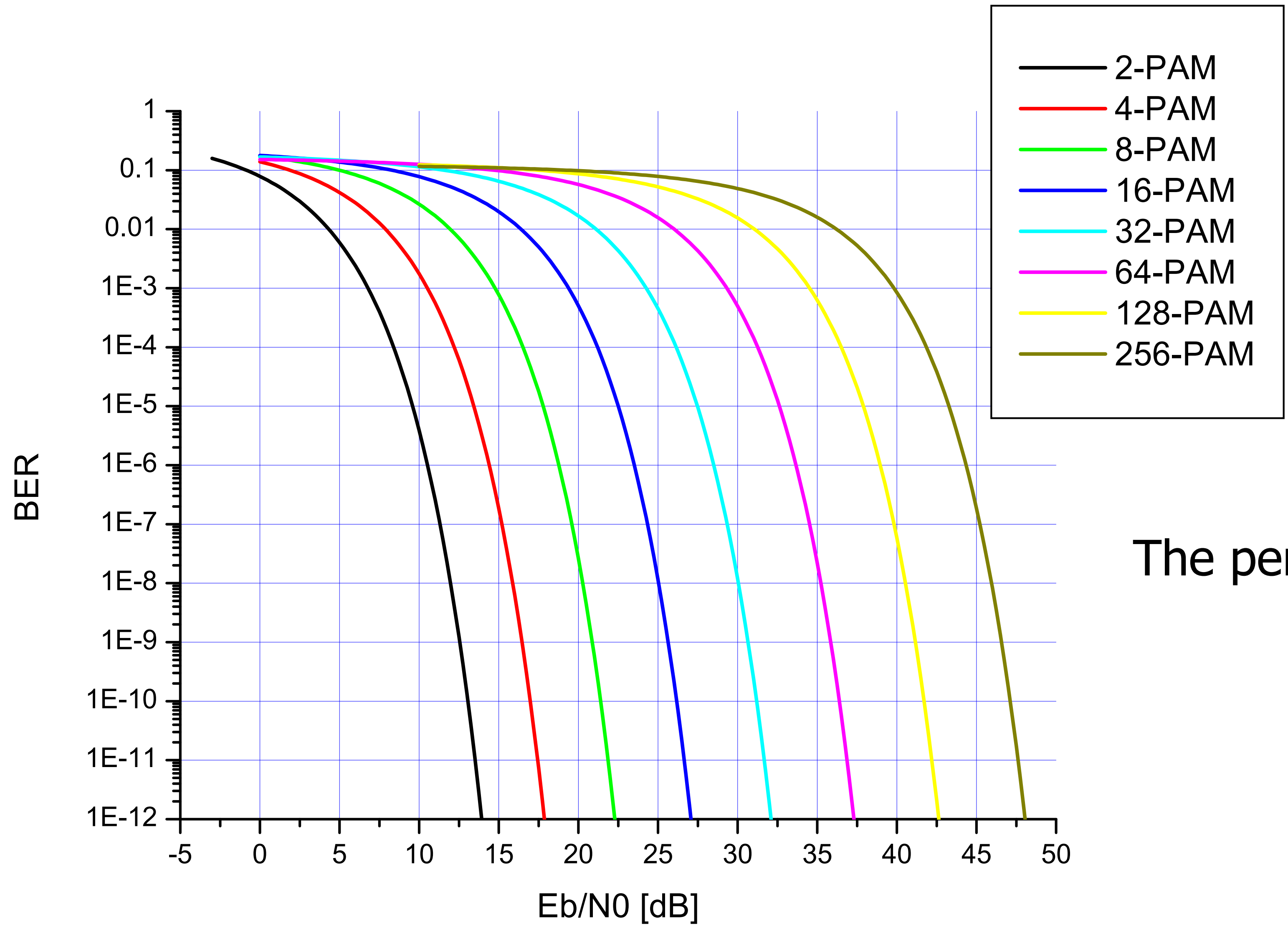
$$4-\mathrm{PAM:} \qquad P_b(e)\approx\frac{3}{8}erfc\left(\sqrt{\frac{2}{5}\frac{E_b}{N_0}}\right)$$

The 2-PAM constellation has better performance
The constellation gain is in the order of 10 log(5/2) = 4 dB

m-PAM constellation*: error probability*



***m-PAM constellation**: error probability*



The performance decrease

for increasing m

m-PAM constellation: performance/spectral efficiency trade-off

Given a baseband channel with bandwidth B and an m-PAM constellation, by increasing the number of signals $m=2^k$ we increase the spectral efficiency

$$\eta_{id} = R_b / B = 2k \quad bps / Hz$$

then we can transmit a higher bit rate R_b.

Unfortunately, the performance decrease:
fixed a BER value, the signal-to-noise ratio E_b/N_0 necessary to achieve it increases with m.

Example

Suppose B=4kHz.

With a (ideal) 2-PAM we transmit R_b = 8 kbps
With a (ideal) 256-PAM we transmit R_b = 64 kbps

However, fixed a target BER (e.g. BER=1e-10), a 256-PAM requires a larger ratio E_b/N_0 (34 dB of difference!).

As an example, at the parity of transmitted power, the link distance is very lower (by a factor of 50!)

Linear modulation

An m-PAM constellation is a base-band modulation characterized by a low pass TX filter $p(t)$.

Let us suppose to change this TX filter from $p(t)$ to $p(t)cos\,(2\pi f_0\,t)$

- The constellation stays unchanged → **the BER performance are the same**
- **The signal spectrum changes**

Linear modulation

$$s(t)=\sum_n a[n]p(t-nT)$$

$$G(f)=\sigma_a^2\frac{|P(f)|^2}{T}$$

$$\left.\begin{aligned} s'(t)&=\sum_n a[n]p'(t-nT)\\ p'(t)&=p(t)\cos(2\pi f_0 t)\end{aligned}\right\}$$

$$G'(f)=\frac{1}{4}[G(f-f_0)+G(f+f_0)]$$

The signal spectrum is translated around frequency f_0

Linear modulation

A linear modulation simply translates the spectrum around frequency f_0

(carrier frequency or Intermediate Frequency IF)

The modulation formats obtained by applying a linear modulation to m-PAM modulations are called m-ASK (Amplitude Shift Keying).

The only one really important is 2-ASK, which is always called 2-PSK (Phase Shift Keying).

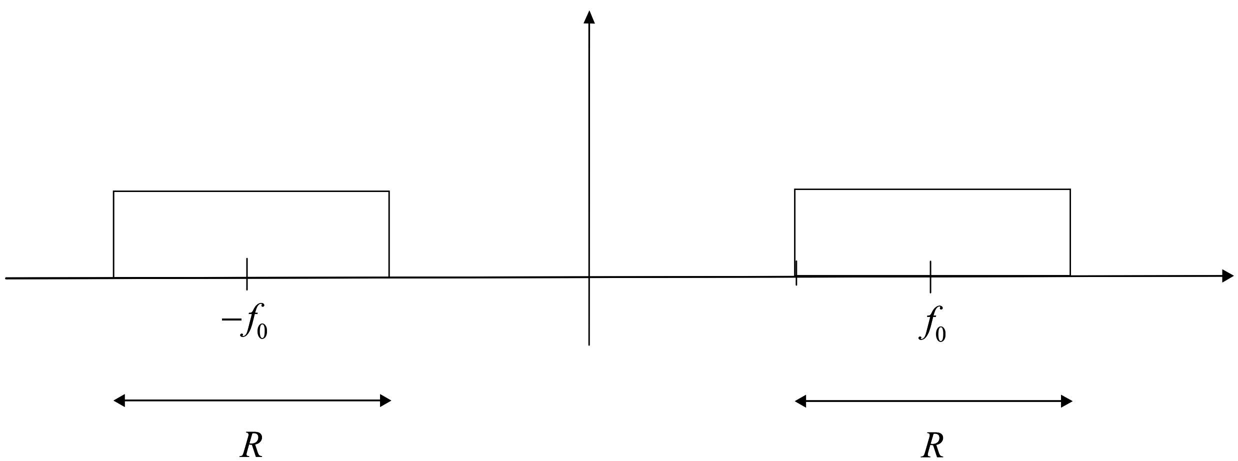
m-ASK constellation: *characteristics*

1. One-dimensional constellation identical to m-PAM
2. Versor $b_1(t) = p'(t) = p(t)\cos(2\pi f_0 t)$
3. Signal spectrum centred around f_0 → bandpass modulations
4. ASK (Amplitude Shift Keying)

m-ASK constellation: signal spectrum

$$G_s(f) = x\left[\left|P(f-f_0)\right|^2 + \left|P(f+f_0)\right|^2\right] \qquad x \in R$$

Example: $p(t)$ = ideal low pass filter



$$B_{id} = R = \frac{R_b}{k}$$

$$\boxed{\eta_{id} = \frac{R_b}{B_{id}} = k\ bps/Hz}$$

***m-ASK constellation**: properties*

Properties

- **Spectral efficiency halved with respect to m-PAM**
- BER performance identical to m-PAM
- No practical applications
  (only exception 2-ASK which is always called 2-PSK)