

## I. Chuỗi

1. Xét sự hội tụ và tính tổng nếu có:

$$\text{a) } \sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{2^{n-1}}$$

$$\text{c) } \sum_{n=1}^{+\infty} \frac{2n-1}{2^n}$$

$$\text{e) } \sum_{n=1}^{+\infty} q^n \cos na \quad (|q| < 1)$$

$$\text{b) } \sum_{n=1}^{+\infty} \left( \frac{1}{2^n} + \frac{1}{3^n} \right)$$

$$\text{d) } \sum_{n=1}^{+\infty} q^n \sin na \quad (|q| < 1)$$

$$\text{f) } \sum_{n=1}^{+\infty} \frac{(-1)^n}{n^2 - 1}$$

2. Xét tính hội tụ của chuỗi:

$$\text{a) } \sum_{n=1}^{+\infty} \frac{1000^n}{n!}$$

$$\text{c) } \sum_{n=1}^{+\infty} \frac{n!}{n^n}$$

$$\text{e) } \sum_{n=1}^{+\infty} \frac{3^n n!}{n^n}$$

$$\text{g) } \sum_{n=1}^{+\infty} \frac{n^2}{\left(2 + \frac{1}{n}\right)^n}$$

$$\text{i) } \sum_{n=1}^{+\infty} \frac{1}{\sqrt[n]{\ln}}$$

$$\text{k) } \sum_{n=1}^{+\infty} \frac{n^5}{2^n + 3^n}$$

$$\text{m) } \sum_{n=1}^{+\infty} \sqrt[n]{2 - \underbrace{\sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}}}_n}$$

$$\text{o) } \sum_{n=1}^{+\infty} \int_0^{\pi/n} \frac{\sin^3 x}{1+x} dx$$

$$\text{q) } \sum_{n=1}^{+\infty} \frac{1}{n^p \ln^q n}$$

$$\text{b) } \sum_{n=1}^{+\infty} \frac{(n!)^2}{(2n)!}$$

$$\text{d) } \sum_{n=1}^{+\infty} \frac{2^n n!}{n^n}$$

$$\text{f) } \sum_{n=1}^{+\infty} \frac{(n!)^2}{2^{n^2}}$$

$$\text{h) } \sum_{n=1}^{+\infty} \frac{n^{n+1/n}}{\left(n + \frac{1}{n}\right)^n}$$

$$\text{j) } \sum_{n=1}^{+\infty} \frac{n^{n-1}}{(2n^2 + n + 1)^{n+1/2}}$$

$$\text{l) } \sum_{n=1}^{+\infty} \left( \frac{n-1}{n+1} \right)^{n(n-1)}$$

$$\text{n) } \sum_{n=1}^{+\infty} \left( n^{\frac{1}{n^2+1}} - 1 \right)$$

$$\text{p) } \sum_{n=1}^{+\infty} \frac{1}{\ln n!}$$

$$\text{r) } \sum_{n=1}^{+\infty} \frac{1}{n^{\sqrt[n]{n}}}$$

$$\text{s)} \sum_{n=1}^{+\infty} \frac{e^n n!}{n^n}$$

$$\text{t)} \sum_{n=1}^{+\infty} (e - (1 + \frac{1}{n})^n)^p$$

$$\text{u)} \sum_{n=1}^{+\infty} (\sqrt{n+1} - \sqrt{n})^p \ln \frac{n-1}{n+1}$$

$$\text{x)} \sum_{n=1}^{+\infty} \frac{1}{(\ln \ln n)^{\ln n}}$$

$$\text{y)} \sum_{n=1}^{+\infty} \frac{1}{(\ln n)^{\ln \ln n}}$$

$$\text{z)} \sum_{n=1}^{+\infty} \frac{1}{n(\ln n)^p (\ln \ln n)^q}$$

3. Xét tính hội tụ của chuỗi sau:

$$\text{a)} \sum_{n=1}^{+\infty} \frac{(-1)^{n(n-1)/2}}{2^n}$$

$$\text{b)} \sum_{n=1}^{+\infty} (-1)^n \left(\frac{2n+100}{3n+1}\right)^n$$

$$\text{c)} \sum_{n=1}^{+\infty} \frac{(-1)^n \sqrt{n}}{n+100}$$

$$\text{d)} \sum_{n=1}^{+\infty} \frac{(-1)^n}{\sqrt{n} + (-1)^n}$$

$$\text{e)} \sum_{n=1}^{+\infty} \frac{(-1)^n}{\sqrt[n]{n}}$$

$$\text{f)} \sum_{n=1}^{+\infty} \sin(\pi \sqrt{n^2 + k^2})$$

$$\text{g)} \sum_{n=1}^{+\infty} \frac{1}{\ln^2 n} \cos \frac{\pi n^2}{n+1}$$

$$\text{i)} \sum_{n=1}^{+\infty} \frac{\ln^{100} n}{n} \sin \frac{\pi n}{4}$$

4. Xét tính hội tụ tuyệt đối và bán hội tụ:

$$\text{a)} \sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{n^p}$$

$$\text{b)} \sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{n^{p+1/n}}$$

$$\text{c)} \sum_{n=1}^{+\infty} \ln\left(1 + \frac{(-1)^n}{n^p}\right)$$

$$\text{d)} \sum_{n=1}^{+\infty} \frac{(-1)^n}{x+n}$$

$$\text{e)} \sum_{n=1}^{+\infty} \frac{(-1)^n}{(n + (-1)^n)^p}$$

$$\text{f)} \sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{(\sqrt{n} + (-1)^{n-1})^p}$$

$$\text{g)} \sum_{n=1}^{+\infty} (-1)^n \frac{n-1}{n+1} \frac{1}{\sqrt[100]{n}}$$

$$\text{h)} \sum_{n=1}^{+\infty} \frac{(-1)^n}{n^2 \sqrt[n]{n}}$$

$$\text{i)} \sum_{n=1}^{+\infty} \sin n^2$$

$$\text{k)} \sum_{n=1}^{+\infty} \frac{(-1)^n \sqrt[n]{n}}{\ln n}$$

5. Xác định miền hội tụ và miền hội tụ tuyệt đối của các chuỗi hàm sau:

$$\text{a)} \sum_{n=1}^{+\infty} \frac{n}{x^n}$$

$$\text{b)} \sum_{n=1}^{+\infty} \frac{(-1)^n}{2n-1} \left(\frac{1-x}{1+x}\right)^n$$

$$\text{c) } \sum_{n=1}^{+\infty} \frac{n}{n+1} \left( \frac{x}{2x+1} \right)^n$$

$$\text{d) } \sum_{n=1}^{+\infty} \frac{n3^{2n}}{2n} x^n (1-x)^n$$

$$\text{e) } \sum_{n=1}^{+\infty} \frac{(-1)^n}{(x+n)^p}$$

$$\text{f) } \sum_{n=1}^{+\infty} \left( \frac{x(x+n)}{n} \right)^n$$

$$\text{g) } \sum_{n=1}^{+\infty} \frac{x^n}{1+x^{2n}}$$

$$\text{h) } \sum_{n=1}^{+\infty} n e^{-nx}$$

$$\text{i) } \sum_{n=1}^{+\infty} \frac{(n+x)^n}{n^{n+x}}$$

6. Xét sự hội tụ đều của các chuỗi hàm sau:

$$\text{a) } \sum_{n=1}^{+\infty} x^n \text{ trên tập } |x| < q < 1$$

$$\text{b) } \sum_{n=1}^{+\infty} x^n \text{ trên tập } |x| < 1$$

$$\text{c) } \sum_{n=1}^{+\infty} \frac{x^n}{n^2} \text{ trên tập } |x| \leq 1$$

$$\text{d) } \sum_{n=1}^{+\infty} \frac{x^n}{n!} \text{ trên } (0; +\infty)$$

$$\text{e) } \sum_{n=1}^{+\infty} \frac{1}{(x+n)(x+n+1)} \text{ trên } 0 < x < +\infty$$

$$\text{f) } \sum_{n=1}^{+\infty} \frac{1}{x^2 + n^2} \text{ trên } \mathbb{R}$$

$$\text{g) } \sum_{n=1}^{+\infty} \frac{x}{1+n^4 x^2} \text{ trên } [0; +\infty)$$

$$\text{h) } \sum_{n=1}^{+\infty} x^2 e^{-nx} \text{ trên } [0; +\infty)$$

$$\text{i) } \sum_{n=1}^{+\infty} \arctan \frac{2x}{x^2 + n^2} \text{ trên } \mathbb{R}$$

$$\text{j) } \sum_{n=1}^{+\infty} \frac{\sin nx}{n} \text{ trên } [\varepsilon; 2\pi - \varepsilon]$$

$$\text{k) } \sum_{n=1}^{+\infty} \frac{\sin nx}{n} \text{ trên } [0; 2\pi]$$

$$\text{l) } \sum_{n=1}^{+\infty} \frac{(-1)^n}{x+n} \text{ trên } (0; +\infty)$$

$$\text{m) } \sum_{n=1}^{+\infty} 2^n \sin \frac{1}{3^n x} \text{ trên } (0; +\infty)$$

7. Tìm miền hội tụ của chuỗi lũy thừa

$$\text{a) } \sum_{n=1}^{+\infty} \frac{x^n}{n^p}$$

$$\text{b) } \sum_{n=1}^{+\infty} \frac{3^n + (-2)^n}{n} (x+1)^n$$

$$\text{c) } \sum_{n=1}^{+\infty} \frac{(n!)^2}{(2n)!} x^n$$

$$\text{d) } \sum_{n=1}^{+\infty} \left( 1 + \frac{1}{n} \right)^{n^2} x^n$$

$$\text{e)} \sum_{n=1}^{+\infty} \frac{x^n}{a^n + b^n} \quad (a, b > 0)$$

$$\text{f)} \sum_{n=1}^{+\infty} \frac{n!}{a^{n^2}} x^n \quad (a > 1)$$

$$\text{g)} \sum_{n=1}^{+\infty} \frac{x^n}{a^{\sqrt{n}}} \quad (a > 0)$$

$$\text{h)} \sum_{n=1}^{+\infty} \left(1 + \frac{1}{n}\right)^{-n^2} e^{-nx}$$

$$\text{i)} \sum_{n=1}^{+\infty} \frac{x^n}{2^{n^2}}$$

$$\text{j)} \sum_{n=1}^{+\infty} \frac{3^{3n} (n!)^3}{(3n)!} \tan^n x$$

8. Tính tổng

$$\text{a)} \sum_{n=1}^{+\infty} \frac{x^{2n+1}}{2n+1}, \quad |x| < 1$$

$$\text{b)} \sum_{n=1}^{+\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \quad |x| < 1$$

$$\text{c)} \sum_{n=0}^{+\infty} \frac{x^{2n}}{(2n)!}$$

$$\text{d)} \sum_{n=1}^{+\infty} \frac{x^n}{n(n+1)}, \quad |x| < 1$$

$$\text{e)} \sum_{n=1}^{+\infty} n x^n, \quad |x| < 1$$

$$\text{f)} \sum_{n=1}^{+\infty} (-1)^{n-1} n^2 x^n, \quad |x| < 1$$

$$\text{g)} \sum_{n=1}^{+\infty} n(n+1)x^n, \quad |x| < 1$$

9. Phân tích thành chuỗi lũy thừa các hàm số sau:

$$\text{a)} e^{-x^2}$$

$$\text{b)} \cos^2 x$$

$$\text{c)} \frac{x^{10}}{1-x}$$

$$\text{d)} \frac{x}{\sqrt{1-2x}}$$

$$\text{e)} \ln \sqrt{\frac{1+x}{1-x}}$$

$$\text{f)} \frac{1}{1+x+x^2}$$

$$\text{g)} \frac{1}{1+x+x^2+x^3}$$

$$\text{h)*} \arctan x$$

$$\text{i)*} \arcsin x$$

$$\text{k)*} \ln(x + \sqrt{1+x^2})$$

(\* HD: sử dụng đạo hàm)

10. Khai triển thành chuỗi Fourier các hàm số sau trên các đoạn đã cho:

$$\text{a)} f(x) = \begin{cases} A, & 0 < x < l \\ 0, & l < x < 2l \end{cases} \quad \text{trên } (0, 2l)$$

$$\text{b)} f(x) = x \quad \text{trên } (-\pi, \pi)$$

$$\text{c)} f(x) = |x| \quad \text{trên } (-\pi, \pi)$$

$$\text{d) } f(x) = \begin{cases} ax, & -\pi < x < 0 \\ bx, & 0 < x < \pi \end{cases} \text{ trên } (-\pi, \pi)$$

$$\text{e) } f(x) = x \text{ trên } (a, a + 2l)$$

$$\text{f) } f(x) = x \sin x \text{ trên } (-\pi, \pi)$$