Fundamentals of AI and KR

Module 3: probabilistic and uncertain reasoning

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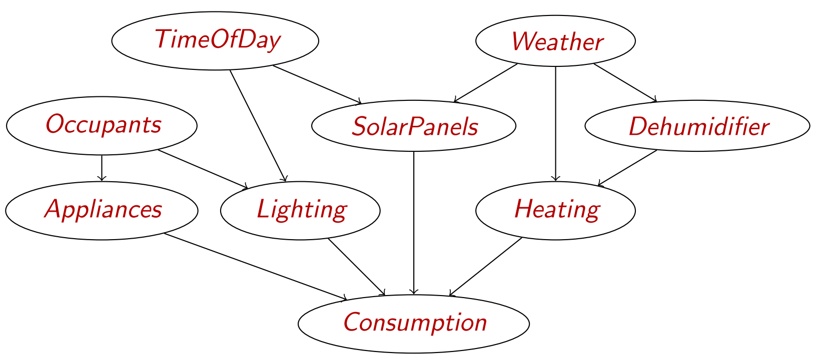
A study about energy efficiency of buildings showed that energy *Consumption* is determined by a number of factors as illustrated by the following network:



In particular, *Appliances*, *Lighting* system, *SolarPanels* and *Heating* system have a direct impact on energy consumption.

To keep the model simple, the variables were defined as all binary, except for:

- *TimeOfDay* $\in \{morning, afternoon, evening\}$
- *SolarPanels* $\in \{low, medium, high\}$
- *Consumption* $\in \{low, medium, high\}$

SolarPanels indicates the amount of energy production from solar panels. The other variables' meaning is self-evident:

- *Weather* $\in \{clear, overcast\}$
- *Occupants* $\in \{yes, no\}$, whether there are people in the building
- *Appliances* $\in \{high, low\}$, high/low energy demand from appliances
- *Lighting* $\in \{high, low\}$, high/low energy demand from lighting system
- *Heating* $\in \{high, low\}$, heating system set to high/low
- *Dehumidifier* $\in \{high, low\}$, dehumidifier set to high/low

Questions

1. *[1 pt]* How many independent parameters are needed to define the CPT for *SolarPanels*?

2. *[1 pt]* Show the Markov blanket of *SolarPanels*, and express, as a product of conditional probabilities, the probability of *SolarPanels* given its Markov blanket.

3. *[1 pt]* Suppose you know that the energy demand from the *Lighting* system is low. If that's all you know about this system, how is the energy demand of *Appliances* related to the *Weather*?

   - if it is related, show how it is;
   - if it is independent, explain why.

4. *[3.5 pt]* Use **variable elimination** to calculate the probability distribution of *Heating*, for a house with *SolarPanels* producing a *high* amount of energy, and *Dehumidifiers* set to *low*.

   → In doing that, (1) for simplicity, assume all **uniform** prior distributions for *TimeOfDay*, *Weather*, and *Occupants*, and (2) **assign values** to the relevant cells of the CPTs as you see fit, but preferably, in a way that makes sense. For example, the probability of *SolarPanels* producing a $high$ amount of energy in the $morning$ should be higher than the probability of *SolarPanels* producing a $high$ amount of energy during the $night$.

   → You don't have to do all the math: you can stop after summing out one variable.

5. *[1.5 pt]* Assume you would estimate the same probability distribution as above, by **likelihood weighting**. Illustrate how to draw a sample, and what weight you would attribute to such a sample. Base your answer on the same CPTs you defined to answer the previous point, and on the following random sequence: .11, .93, .28, .53, .05, . . .

6. *[3 pt]* Explain the conceptual and methodological differences between two types of analysis you could carry out about energy *Consumption* in relation with *SolarPanels*' energy production:

   a) estimating the probability distribution of energy *Consumption* when the **measured** energy production from *SolarPanels* is $medium$
   b) estimating the probability distribution of energy *Consumption* when *SolarPanels* **are set to** have a $medium$ energy production (even when they could have, say, a $high$ production)

Answers (including some extra comments)

1. *SolarPanels* is a discrete variable with 3 possible values and 2 parents, with arity 3 and 2, hence: (3-1)×3×2=**12** independent parameters.

2. The Markov blanket of *SolarPanels*, mb(*S*), is { *T*, *W*, *C*, *A*, *L*, *H* }.

   P(*S*|*t*, *w*, *c*, *a*, *l*, *h*))=P(*S*|*t*,*w*)× P(*c*|*a*, *l*, *S*, *h*)

3. *Appliances* is independent of *Weather* given *Lighting* because they're d-separated. (They would be independent, even unconditionally)

4. **P**(*H*|*S=high*, *D=low*)'s **ancestral tree** is composed by {*T*, *W* }. Hence,

$$
\begin{gathered}
\mathbf{P}(H|S{=}high,\ D{=}low) = \\
\alpha\ \mathbf{P}(T) \times \mathbf{P}(W) \times \mathbf{P}(S{=}high|T,W) \times \mathbf{P}(D{=}low|W) \times \mathbf{P}(H|W,D{=}low) = \\
\alpha \sum_{w\in\{c,o\}} \mathbf{P}(H|w,D{=}low) \times \mathbf{P}(w) \times \mathbf{P}(D{=}low|w) \times \\
\sum_{t\in\{n,m,a,e\}} \mathbf{P}(S{=}high|t,w) \times \mathbf{P}(t) = \\
\alpha \sum_{w\in\{c,o\}} \mathbf{P}(H|w,D{=}low) \times \mathbf{P}(D{=}low|w) \times \sum_{t\in\{m,a,e\}} \mathbf{P}(S{=}high|t,w)
\end{gathered}
$$

P(*t*) and **P**(*w*) can be ignored because we assume a uniform distribution.

A possible CPT for **P**(*H*|*w*,*D=low*) is:

w	**P**(*H=high*|*w*,*D=low*)	**P**(*H=low*|*w*,*D=low*)
c	0.1 (a)	0.9 (1-a)
o	0.2 (b)	0.8 (1-b)

A possible CPT for **P**(*S=high*|*t*,*w*) is:

t,*w*	**P**(*S=high*|*t*,*w*)
c, m	0.8 (c)
c, a	0.7 (d)
c, e	0.01 (e)
o, m	0.4 (f)
o, a	0.2 (g)
o, e	0.001 (h)

A possible CPT for **P**(*D=low*|*w*) is:

w	**P**(*D=low*|*w*)
c	0.8 (j)
o	0.4 (k)

With these values, we obtain:

$$f_1(w) = \sum_{t\in\{m,a,e\}} \mathbf{P}(S{=}high|t,w) =$$

w	$f_1(w) = \sum_{t\in\{m,a,e\}}$ **P**(*S=high*|*w*)
c	c+d+e = 1.51
o	f+g+h = 0.601

$$f_2(H) = \sum_{w \in \{c,o\}} \mathbf{P}(H|w, D{=}low) \times \mathbf{P}(D{=}low|w) \times f_1(w) =$$

H	$f_2(H)$
high	a $\times$ j $\times f_1(w{=}\text{c})$ + b $\times$ k $\times f_1(w{=}\text{o})$ = 0.1 $\times$ 0.8 $\times$ 1.51 + 0.2 $\times$ 0.4 $\times$ 0.601 = 0.16888
low	(1-a) $\times$ j $\times f_1(w{=}\text{c})$ + (1-b) $\times$ k $\times f_1(w{=}\text{o})$ = 0.9 $\times$ 0.8 $\times$ 1.51 + 0.8 $\times$ 0.4 $\times$ 0.601 = 1.27952

Hence, $\mathbf{P}(H|S{=}high,\ D{=}low) = \langle \frac{f_2(high)}{f_2(high)+f_2(low)}, \frac{f_2(low)}{f_2(high)+f_2(low)} \rangle =$ $\langle 0.1166, 0.8834 \rangle$

5. Using the sequence .11, .93, .28, .53, .05, I need to draw a sample for T, W, and H|W,D=low, and assign it the correct weight.

   - T has a uniform distribution, so we have $.11 \in [0, .33) \rightarrow T = morning$
   - W has a uniform distribution, so we have $.93 \in (.5, 1) \rightarrow W = overcast$
   - P(H|W=overcast,D=low)=$\langle b, 1{-}b \rangle$=$\langle .2, .8 \rangle$, so we have $.28 \in [.2, 1) \rightarrow H = low$
   - P(S=high|T=morning,W=overcast) = f = 0.4
   - P(D=low|W=overcast) = k = 0.4

   Therefore, we have a sample with H=low and weight = 0.4 $\times$ 0.4 = 0.16