Fundamentals of AI and KR

Module 3: probabilistic and uncertain reasoning

Paolo Torroni

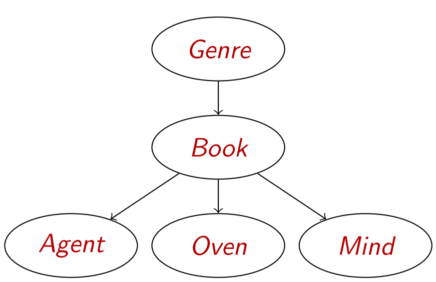
September 15, 2020 [25 minutes for solving + 10 for uploading]

Tinker, tailor, soldier, spy! Your book collection contains thousands of volumes, mostly spy fiction novels, but also a few cookbooks, and some artificial intelligence textbooks. You wish to organize them by genre. However, you don't have the time to read them all. And maybe, you don't need to, either ... of course, you can use a Bayesian network!

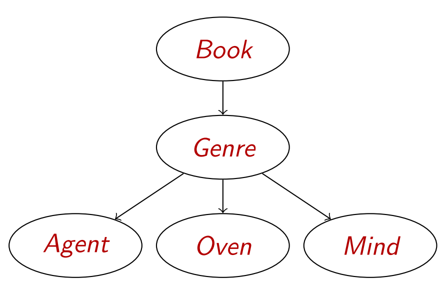
The idea is simple: the **words** written in a **book** can tell you a lot about the book's **genre**. So, if you open a book at a random page and you see words like *enemy*, *agent*, or *Langley*, that could indicate that you're more likely to be holding a spy novel rather than a recipe book or an AI textbook. Excited by this idea, you decide to build a small-scale prototype, with a handful of selected words, and see where this gets you. You begin to draw the following nodes:

- *Book*: the particular book you pick from the shelf (it could be a unique integer number representing the book)
- *Genre*: the book's genre (*spy*, *cooking*, or *ai*);
- *Agent*: the word *agent* is present in a random page of the book.
- *Oven*: the word *oven* is present in a random page of the book.
- *Mind*: the word *mind* is present in a random page of the book.

However, you are not sure how to connect the nodes. Here are two options:



(a)



(b)

Questions

1. Which option can more directly express the probabilities of words in genres and of genres in books?
2. Consider both options *(a)* and *(b)*: are *Agent* and *Mind* independent from one another? How about if you know the *Genre*?
3. Consider option *(a)*, and assume that you know which particular book you picked – for example, book number 482. Can the probabilities of words tell you anything about the book's genre?
4. Based on the above considerations, which option better serves the purpose of estimating a book's genre by looking at random words, *(a)* or *(b)*?
5. What independence assumption is implied by option *(b)*? (*Hint*: take two different books of the same genre, are the distributions of words in these two books the same?)
6. Again with reference to option *(b)*, consider the following query:

$$P(spy \mid mind, \neg oven).$$

   Evaluate that query using variable elimination. According to your statistics, 80% of your books are spy novels, whereas the percentage of cookbooks in your collection is equal to the number of the month in which you were born (for example, if you were born in March, 3% of your books are cookbooks).

7. After much tinkering with the network, you realize that a few books actually belong to two or more genres. Do you need to modify your network in order to model that fact, and if so, can you think of a suitable network?

Answers (including some extra comments)

1. The option that more directly expresses the probabilities of words in genres and of genres in books is *(b)*, because of the direct links from genre to word and from book to genre. Indeed, P(*Word*|*Genre*) is encoded in CPT_{Book} and P(*Genre*|*Book*) in CPT_{Genre}.

2. *Agent* and *Mind* are connected by an active trail in both *(a)* and *(b)*, so they're not independent.

   However, by option *(b)*, if *Genre* is known, the trail between *Agent* and *Mind* is broken. For that reason, *(b)* $\Rightarrow$ *P* $\models$ (*Agent* $\perp$ *Mind* | *Genre*).

3. No, because *(a)* $\Rightarrow$ *P* $\models$ ({*Agent*, *Oven*, *Mind* } $\perp$ *Genre* | *Book*)

4. *(b)*, since it has active trails that connect words and genre

5. The assumption is that given the genre, the distribution of words is independent of the specific book that was picked from the shelf. Any two books of the same genre are assumed to have the same word distribution.

6. Let's say I was born in March (month=3). Then, assuming books are located on the shelves at random — at least as far as their genre, considering that the whole point is to organize them by genre — P(*spy*)=0.8, P(*cooking*)=0.03, and P(*ai*)=1-0.83=0.17. We then have:

   $P(\textit{spy}|\textit{mind}, \neg\textit{oven}) =$

   $$= \frac{\sum_{b \in \textit{books}} P(\textit{spy}|b) \times P(\textit{mind}|\textit{spy}) \times P(\neg\textit{oven}|\textit{spy})}{\sum_{g \in \{\textit{spy},\textit{cooking},\textit{ai}\}} \sum_{b \in \textit{books}} P(g|b) \times P(\textit{mind}|g) \times P(\neg\textit{oven}|g)} =$$

   $$= \frac{P(\textit{spy}) \times P(\textit{mind}|\textit{spy}) \times P(\neg\textit{oven}|\textit{spy})}{\sum_{g \in \{\textit{spy},\textit{cooking},\textit{ai}\}} P(g) \times P(\textit{mind}|g) \times P(\neg\textit{oven}|g)} =$$

   $$= \frac{0.8 \times P(\textit{mind}|\textit{spy}) \times P(\neg\textit{oven}|\textit{spy})}{\begin{pmatrix} 0.8 \times P(\textit{mind}|\textit{spy}) \times P(\neg\textit{oven}|\textit{spy}) \\ +0.03 \times P(\textit{mind}|\textit{cooking}) \times P(\neg\textit{oven}|\textit{cooking}) \\ +0.17 \times P(\textit{mind}|\textit{ai}) \times P(\neg\textit{oven}|\textit{ai}) \end{pmatrix}}.$$

7. Neither network can model multiple genres, because in either case *Genre* is a single random variable. A possible solution consists in introducing a new random variable for each genre: