

Sample Exam Logic

1. Inspector Craig is confronted with a nasty case, in which there are four suspects: Albert, Bernard, Colin, and Dave. Craig has established the following four facts:
 - i. If Albert is guilty, then Bernard is guilty too.
 - ii. If Bernard is guilty, then Colin is guilty or Albert is innocent.
 - iii. If Dave is innocent, then Albert is guilty and Colin is innocent.
 - iv. If Dave is guilty, then so is Albert.

Which of the suspects are certainly guilty? Determine the answer with the aid of a truth table.

2. Semantic tableaux propositional logic:
 - a. Investigate by means of a semantic tableau whether the formula $(\neg p \wedge q) \rightarrow (p \rightarrow q)$ is a tautology. If it is not a tautology, then give all counterexamples.
 - b. Given is the consequence $p \rightarrow q, (q \rightarrow p) \rightarrow \neg r \ / \ q \vee r$. Investigate the validity of this consequence with the aid of a semantic tableau. If this consequence is not valid, give all the counterexamples.
3. Prove by means of natural deduction:
 - a. from premises $p \rightarrow q$ and $(q \wedge r) \rightarrow s$, derive $(p \wedge r) \rightarrow s$
 - b. from premises $(p \vee s) \rightarrow (q \vee r)$ and $(q \vee s) \rightarrow r$, derive $p \rightarrow r$

4. Using the method with Venn diagrams decide whether the following syllogisms are valid or not:

(a) All students are clever
 No animals are clever

 No student is an animal

(b) No human is a monster
 All animals are monsters

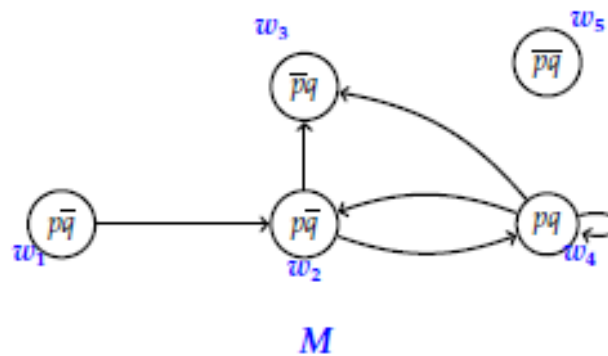
 No human is an animal

(c) All numbers are green
 Some green things are not salty

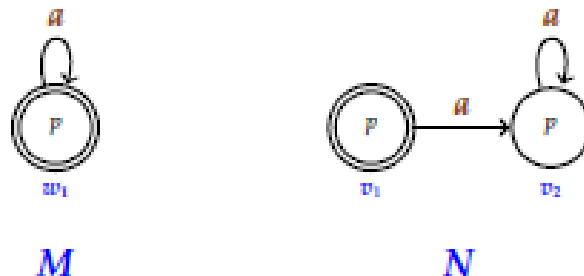
 Some numbers are not salty

5. Consider the domain of people. Some people are prophets, who may have followers. Px means that x is a prophet. Fxy means that x is a follower of y . The predicate $=$ may also be used. Now write each of the following sentences as a formula of predicate logic:
 - a. All prophets have at least one follower.
 - b. There is a prophet who has at least two followers.
 - c. There is a prophet who has precisely two followers.
 - d. A follower of a prophet also follows any prophet that the first prophet follows.

6. Investigate the validity of the following consequences by means of a semantic tableau. If a consequence is not valid, give at least one counterexample.
- $\forall x(Ax \rightarrow Bx), \neg\exists xBx / \neg\exists xAx$
 - $\forall x(Ax \vee Bx), \exists x(Cx \rightarrow Ax), \exists xBx / \exists xAx$
7. Prove by means of natural deduction:
- from premise $\neg\exists xAx$, derive $\forall x\neg Ax$
 - from premises $\exists x(Ax \vee \neg Bx)$, $\forall x(Ax \rightarrow Cx)$, and $\forall x(\neg Cx \rightarrow Bx)$, derive $\exists xCx$
8. Translate each one of the following sentences into a formula of epistemic logic, using appropriate translation keys.
- Natalia knows that dragons fly, but she does not know that she knows it.
 - Natalia considers it possible that she considers it possible that her hat is pink, and she does not consider it possible that her hat is pink.
 - Natalia knows that if Daffy is a bird, then he flies, and she also knows that Daffy is a bird, but she does not know that Daffy flies.
9. Models in epistemic logic:
- For each world in the following model, provide a formula that is true only in that world and false in all the others.



- Is there an epistemic formula that distinguishes the following two models (i.e., it is true in one but false in the other)? Give arguments for your answer.



10. Using a semantic tableau, give a proof that the following “distribution” principle for epistemic logic is a valid inference:

$$\Box(p \rightarrow q) \models \Box p \rightarrow \Box q$$