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Flight gate scheduling: State-of-the-art and recent developments☆

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Abstract

This paper surveys a large variety of mathematical models and up-to-date solution techniques developed for solving a general flight gate scheduling problem that deals with assigning different aircraft activities (arrival, departure and intermediate parking) to distinct aircraft stands or gates. The aim of the work is both to present various models and solution techniques which are available in nowadays literature and to give a general idea about new open problems that arise in practise. We restrict the scope of the paper to flight gate management without touching scheduling of ground handling operations.
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1. Introduction

Due to the growth of air transport traffic (it has roughly doubled since the early 1980s) techniques for managing and allocating airport and airline resources in a dynamic operational environment effectively and efficiently have gained an ever-increasing interest. Strong competition between airlines and the demand of passengers for more comfort have lead to complex planning problems that require new models and methods. The scheduling problems nowadays faced by airport and air-line managers are even more complicated than most other traditional scheduling problems. This fact can be easily explained. First of all, a wide range of resource modules apparently have to be considered: flights, terminals, crews, baggage etc. Moreover, decisions about the usage of these resources influence each other, that is, the resources are highly interdependent. As a consequence these modules set up the basis of a complex resource management system for airports and airlines of any size.

This paper surveys a large variety of mathematical models and techniques developed for solving a general flight gate scheduling problem. In doing so we do not restrict ourself to collecting various models and solution techniques which are available in the open literature. We also intend to give an idea about new open problems arising in practice. In particular, we concentrate on issues concerning robust scheduling and generating stable assignments.

The paper is organized as follows: In Section 2 we detail the problem setting under consideration. A rough classification of flight gate scheduling problems is proposed in Section 3 while a brief literature review is presented in Section 4. Section 5 presents two particularly interesting approaches that have recently been proposed, based on quadratic assignment model and multi-mode scheduling formulation, respectively. A short description of new research directions is given in Section 6, along with some concluding remarks.

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2. Problem setting

There are several major classes of decisions for which airline and airport management is responsible: crew scheduling, disruption management, airline fleet assignment, aircraft scheduling and rotation, ground operations scheduling and some others that can be modeled as traditional machine scheduling problems. Nevertheless, one of the most important and most complicated airport management topics is flight gate scheduling.

The main purpose of gate scheduling is to find an assignment of flights, or rather of the aircraft serving a flight, to aircraft stands, as well as start and completion times for processing an aircraft at the position it has been assigned to. Aircraft stands at the terminal and off-pier stands on the apron are often simply referred to as "gates".

Of course, a gate assignment must be suitable for airport services and convenient for passengers.

A well-constructed schedule must satisfy a set of strict rules and constraints:

1. one gate can process only one aircraft at the same time,
2. service requirements and space restrictions with respect to adjacent gates must be fulfilled,
3. minimum ground time and minimum time between subsequent aircraft have to be assured.

Typical objectives are:

1. the number of un-gated (open) aircraft activities has to be minimized,
2. preferences of certain aircraft for particular gates have to be maximized,
3. the total walking distance for passengers has to be minimized,
4. the deviation of the current schedule from a reference schedule has to be minimized in order to increase schedule attractiveness and passenger comfort,
5. the number of expensive aircraft towing procedures (that otherwise decrease the available time for some ground service operations on the ramp as well as in the terminal) has to be minimized.

Some special soft or strict constraints may also be introduced (see e.g. [1–4]). For example, the assignment of a large aircraft to a particular gate may imply that neighboring gates can only accept aircraft of a certain size or are even completely blocked.

All these requirements make a gate scheduling problem very complicated both from a theoretical and a practical point of view. In fact, the multiple criteria and multiple constraints nature of the problem make it very unlikely that an optimal solution can be found and verified. Therefore, one has to determine a solution that provides an appropriate compromise between all the different objectives while assuring a set of hard constraints. Moreover, any practical gate scheduling instance of a big international airport usually has to deal with a large number of daily aircraft activities (around 1000) which have to be assigned to a pretty large number (around 100) of different flight gates.

The basic input data for gate scheduling is a flight timetable with arrival and departure times and additional specifications of flights: the origin and destination of a flight, the type of aircraft, the number of passengers, the cargo volume, the type of flight (domestic or international) as well as gate preferences, required airport services and inspection facilities.

It is worth pointing to—from a practical point of view—one of the most important issues of gate scheduling: a gate schedule should be insensitive to small changes of input data; in other words schedule flexibility is required. Obviously, the input data of any flight gate scheduling problem are subject to uncertainty and may change over time.

Input data uncertainty in gate scheduling may have a couple of reasons: (1) flight or gate breakdown, (2) flight earliness or tardiness, (3) emergency flights, (4) severe weather conditions, (5) errors made by staff and many others. For example, a tardy arrival of one aircraft may generate a chain of delayed arrivals for other aircraft which have been assigned to the same gate. In the worst case, this may lead to a "domino effect" and finally require a complete rescheduling, a fact which is absolutely undesirable.

Obviously, reliability of input data in the complex system of a modern airport cannot be guaranteed. Hence, new gate scheduling techniques try to find a schedule (being non-optimal but as close as possible to an optimal one), which has the property of flexibility to changes of input data. Flexible gate scheduling gives terminal operators the possibility to react quickly and properly to accommodate necessary changes or updates in the flight schedule. Finally, an appropriate flexible gate assignment is supposed also to have an impact on efficiency of airlines and airports business activities as well as on passenger service satisfaction.

3. Classification

Since the early seventies a large number of papers has been written on different topics which have to be addressed by airport and airline managers. We refer the reader to the survey [5], where one can find a comprehensive description of the scheduling problems arising in the airline industry (e.g. aircraft rotation, fleet assignment, crew scheduling). In turn, here we focus on a review of the literature concerning gate scheduling issues, a key activity in any airport.

Flight gates are scarce and expensive resources. Therefore, it is very important to use the available gates in the best possible way. Flight gates are the major items addressed in the gate assignment problem (GAP). The basic constraints of the GAP are that one gate can only accommodate a single aircraft at a time and that two flights must therefore not be assigned to the same gate if they overlap in time. In this section we roughly classify some of the research directions in this area.

Single or multiple time slot models. Gate assignment optimization models can be classified as single or multiple time slot models. Single time slot models consider the assignment of a batch of flights that arrive within a single given time period (slot) at gates. In this case only one flight can be assigned to each gate. In multiple time slot models the entire time interval is divided into a fixed number of time slots. The width of the time slots must be carefully selected because it influences the problem size as well as possible gate utilization.

Types of objectives. Gate assignment optimization models can be classified with respect to the main objectives considered. For example, passenger walking distance minimization is the most frequently used goal, present not only in gate assignment, but also in the design of airport terminals. This objective is easily motivated and clearly understood, but it leads to models which can hardly be solved. At the same time, there exists a large variety of different objectives, the consideration of which is at least as important as total walking distance minimization. All these objectives can be divided into two big classes: passenger-oriented and airport-oriented objectives. For example, Teodorovic et al. focus on total passenger delay and the number of flights cancellations in the case of irregularity of flights (see [6,7]). In turn, Chang [8] considers the distance passengers have to carry their baggage as an objective in addition to passenger walking distance. In contrast to previous ones, airport-oriented objectives like total gate preferences, number of aircraft towing procedures and others can be addressed.

Mathematical models. It is well-known, that the single-slot GAP can be modeled in analogy to the NP-hard quadratic assignment problem [9–11] which is a facility location problem where the cost of placing a flight at a gate depends on the placement of other facilities and transport volume between two facilities (see also [12]).

Additionally, the single time slot GAP can be stated as a linear integer program [13] with the objective of minimizing the total walking distance for arriving and departing passengers. In [14] an integer program with an extended objective function that takes into account transfer of passengers is proposed.

Haghani and Chen [4] formulate a multiple time slot version of the GAP with the objective of passenger walking and baggage transport distance minimization as an integer program. They introduce time-indexed binary variables that indicate the assignment of a particular flight to some gate in a given time slot.

4. State-of-the-art algorithms

There are two main research streams actively developed in flight gate scheduling: the first is based on mathematical programming techniques and the second is based on rule based expert systems. We start the review with mathematical programming techniques.

Babic et al. [13] use branch and bound, with some enhancements to accelerate computation, in order to determine an optimal solution of the GAP. The objective is to reduce the number of passengers who have to walk maximum distances—at the price that more passengers have to walk the minimum distances, compared to random aircraft position assignment. Contrary to [13] Mangoubi and Mathaisel [14] take into account transfer passengers. Moreover, they use the LP relaxation and greedy heuristics to solve the GAP. Bihr [15] uses 0–1 integer programming to solve the minimum walking distance gate assignment problem for fixed arrivals in a hub using a simplified formulation as an assignment problem.

The aforementioned papers (as well as the approaches of [16,17]), head towards improved passenger satisfaction mainly by reducing passenger walking distance inside the terminal building. Unfortunately, the assignment is very sensitive with respect to small changes of the flight schedule. In turn, Wirasinghe and Bandara (see e.g. [18,19]) additionally integrate the cost of delays to minimize intra-terminal travel in terminal design process. Furthermore, they employ an approximation algorithm in their analysis.

Xu an Bailey [20] propose a tabu search algorithm for a single slot GAP with the objective function of minimizing the overall distances, that passengers have to walk in order to get connecting flights. The problem is formulated as a quadratic assignment problem and reformulated as a mixed 0–1 integer linear program. A simple tabu search metaheuristic to solve the problem is developed. The algorithm exploits the special properties of different types of neighborhood moves, and creates effective candidate list strategies. Some computational experiments are presented and analyzed.

Some models try to improve the performance of static gate assignment by taking into account stochastic flight delays (including early or late arrivals and late departures). For example, Hassounah and Steuart [21] show that planned buffer times could improve schedule punctuality. Yan and Chang [22] and Yan and Huo [23] use in their static gate assignment problems a fixed buffer time between two continuous flights assigned to the same gate in order to absorb the stochastic flight delays. Yan and Chang [22] develop a multi-commodity network flow model. Moreover, they use Lagrangian relaxation with sub-gradient optimization and some heuristics to solve the GAP. Yan and Huo [23] formulate a dual objective 0–1 integer programming model for the aircraft position allocation. The first objective tries to minimize passenger walking time while the second objective aims at minimizing passenger waiting times. The authors argue that, e.g. during peak hours, an aircraft might have to wait for an available gate, and hence passengers have to wait on the aircraft until a gate is available. In [24] the authors propose a simulation framework, that is not only able to analyze the effects of stochastic flight delays on static gate assignments (cf. [22,23,25]), but can also evaluate flexible buffer times and real-time gate assignment rules.

In [26–28] a GAP where the number of flights exceeds the number of available gates is studied. The primary goals are to minimize the number of open (non-assigned) flights and the total connection times. A two-stage algorithm, which exploits both a greedy strategy to minimize the number of open flights and a tabu search metaheuristic improved by a new neighborhood search technique to minimize the total connection times, is proposed to solve the problem. We will consider this model in the next section in more details.

Recently, some authors try to take into account the dynamic character of the GAP. A delayed departure may delay the arrival of another aircraft scheduled to the same gate, or require the flight to be reassigned. When gate idle times are distributed uniformly among the gates, the probability that the delayed departure time will still be earlier than the arrival of the next flight is maximized. One of the first attempts to realize an approach aiming at robust schedules is due to [1–3,29] where the authors propose to utilize gates as uniformly as possible to provide schedule robustness to small changes of input data. In [3] mathematical models and (optimal and heuristic) procedures are proposed to provide solutions with minimum dispersion of idle time periods for the GAP.

The aircraft gate reassignment problem occurs when the departure of an incoming aircraft is delayed. If the delay is significant enough to delay the arrival of subsequent incoming aircraft at the assigned gate, the management must revise the gate assignment to minimize extra delay times. Two papers describe approaches for solving the gate reassignment problem. In [30] a genetic algorithm is proposed which efficiently calculates minimum extra delayed time schedules that are at least as effective as solutions generated by experienced gate managers. In [31] an integral minimum cost network flow model is introduced. This model aims at reconstructing airlines schedules in response to delays by transforming the routing problem into a time-based network in which the overall time horizon is divided in discrete periods. The transformation is polynomial with respect to the number of airports and flights. An optimum of the new model corresponds to the optimal solution of the original problem under some slight conditions.

The second mainstream research direction concentrates on simulation and rule based expert systems construction. While "traditional approaches utilizing classical operations research techniques have difficulty with uncertain information and multiple performance criteria and do not adapt well to the needs of real-time operations support" (see [32]), alternatively, many authors focus on the design of so-called rule-based expert systems (see e.g. [33–40]). Based on the knowledge obtained from ground controllers, an expert system uses production rules to produce assignments. Evidently, the number of factors to be taken into account is large. Therefore, the most crucial task is to identify all the rules, order them by importance and list these rules appropriately.

Hamzwawi [37] introduces a rule based system for simulating the assignment of gates to flights and for evaluating the effects of particular rules on gate utilization. Gosling [32] describes an expert system for gate assignment that has been implemented at a major hub of Denver Stapleton airport. Srihari and Muthukrishnan [39] use a similar approach for solving the GAP and also describe how to apply sensitivity analysis.

From a practical point of view, it is even more important to develop simple expert systems that make use of mathematical programming techniques (branch and bound, dynamic programming, local search). Such an integration would help to create a gate scheduling system with the desired flexibility property. For example, Cheng [36,41–43] describes the integration of mathematical programming techniques into a knowledge-based gate assignment system to provide partial parallel assignments with multiple objectives. Both optimization and rule based approaches have been combined with simulation analysis in [33,37].

5. Recent developments

In this section we outline two new promising optimization models for gate scheduling. The choice of these two models is not occasional. The main drawback of all previous models is that they do not take into account a real multiple criteria nature of the problem. The models we propose are multiobjective, i.e. the proper solving of them provides a trade-off between several objectives which are usually in conflict. Finding such a compromise between several goals may positively influence passenger satisfaction with flight and save extra money for airport and airline companies. The first model we describe addresses two traditional goals of flight gate assignment. The first goal is to assign flights to stand positions located directly at the terminal (and, hence, not to the apron). The second goal is to minimize the total passenger walking or baggage transportation distance. In the past the two goals were not considered together inside one model. Contrary to the first model, the second one uses a fairly large number of apron stands for passenger embarking and disembarking because of scarce terminal space. Moreover, we suppose that one aircraft can be assigned to different gates during its ground time. Such assumption gives more freedom for airport managers. For example, if one flight has a long ground time and the aircraft is assigned to a gate, which has to be used very often, then it can be temporally removed to another gate, which is currently cannot be used, or to the apron in order to make the gate free for other assignment. As well as the first model, the second one aims to optimize several objectives which are oriented on both passenger comfort and convenience for airport services. While the first model is considered to be classical, the second model can be treated as quite new and contemporary. Additionally, note that the first (second) model represents the strategy usually adopted for United States (European) airports.

Model 1. This model has been proposed by Ding et al. [26,27]. Generally speaking, the airport gate assignment problem is modeled as a quadratic assignment problem where the objectives are to minimize the number of ungated flights and the total passenger walking distance (or equivalently, connection time). For the sake of shortness we will sketch the basic ideas of [26].

When an aircraft arrives at the airport, it can be either assigned to the terminal gates or, if no terminal stand position is available, it can be assigned to the apron stand position (the model does not distinguish between distinct off-pier stands). All the terminal gates are usually equipped with passenger bridges, whereas passengers from flights assigned to the apron can be transported to the terminal building by transfer busses. Such bus connection may increase connection time and can hardly be regarded as desirable if our goal is to minimize total passenger walking distance and connection time.

The following parameters are given:

- N: set of flights arriving at and/or departing from the airport,
- M: set of available gates at the airport,
- n: total number of flights, i.e. $n = |N|$,
- m: total number of gates, i.e. $m = |M|$,
- a_i: arrival time of flight i,
- d_i: departure time of flight i, $d_i > a_i \ \forall i$,
- $w_{k,l}$: walking distance for passengers from gate k to gate l,
- $f_{i,j}$: number of passengers transferring from flight i to flight j.

Additionally, two dummy gates are introduced. Gate 0 represents the passenger entrance/exit of the airport. Gate $m+1$ represents the apron where flights arrive when no terminal gates are available. The binary variable $y_{i,k}=1$ denotes that flight i is assigned to gate k, $0<k\leqslant m+1$, and $y_{i,k}=0$ otherwise. Then the objectives can be expressed as follows:

$$\text{Min} \sum_{i=1}^{n} y_{i,m+1}$$

$$\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m+1} \sum_{l=1}^{m+1} f_{i,j} w_{k,l} y_{i,k} y_{j,l} + \sum_{i=1}^{n} \sum_{k=1}^{m+1} f_{0,i} w_{0,k} y_{i,k} + \sum_{i=1}^{n} \sum_{k=1}^{m+1} f_{i,0} w_{k,0} y_{i,k}.$$

The first objective represents the number of flights which are not assigned to any terminal gate (i.e., they are assigned to the apron). The second objective represents the total passenger walking distance. It consists of three terms: the walking distance of transfer passengers, originating departure passengers and disembarking arrival passengers.

The set of restrictions is defined by the following system of constraints. The first constraint

$$\sum_{k=1}^{m+1} y_{i,k} = 1, \quad 1 \leqslant i \leqslant n$$

assures that every flight must be assigned to exactly one gate including the apron.

The second constraint

$$y_{i,k} y_{j,k} (d_j - a_i)(d_i - a_j) \leqslant 0,$$

$$1 \leqslant i, \ \ j \leqslant n, \ \ k \neq m+1$$

prohibits schedule overlapping of two flights if they are assigned to the same gate. The last constraint

$$y_{i,k} \in \{0, 1\}, \quad 1 \leqslant i \leqslant n, \ \ 1 \leqslant k \leqslant m+1$$

defines the variables to be boolean.

The mathematical model can also be supplemented with several observations. Firstly, it is very natural to put $f_{ii}=0$. Secondly, in reality, for two distinct flights i and j, f_{ij} and f_{ji} are exclusive. If $f_{ij}>0$ then $f_{ij}=0$ and vice versa.

The problem has been primarily attacked with greedy methods originally proposed by Xu and Bailey [20] to minimize the first objective. The basic idea is to sort all the flights according to departure times and then assign flights one by one to the earliest available gate. If no terminal gates are available for assignment, then the flight is assigned to dummy gate $m+1$. Then the second objective is addressed applying different meta heuristic approaches such as simulated annealing and tabu search. The main new contribution are so-called interval exchange moves. These particular moves generalize a technique proposed in [20], where three types of neighborhood moves have been investigated: insertion moves and two types of exchange moves. With this new approach, experimental results are obtained which are quite good in comparison with previous approaches.

Model 2. The purpose of the second model is to assign available airport flight gates to three possible aircraft activities (arrival; optional intermediate parking activity, the length of which depends on ground time; departure) and to schedule start and completion times of the activities at the positions. A detailed description can be found in [44].

Compared to previous models there are several new contributions. Firstly, the three activities are modeled separately and, hence, can potentially be assigned to different positions. The aircraft can be moved to another assigned position using tow tractors, a procedure which is called towing. Secondly, in contrast to the standard objective function commonly used (which minimizes passenger walking distance) a complex objective function which is a combination of several partial objectives is introduced.

Three objectives were considered to be most important after intensive discussions with airport managers:

- maximization of total flight-gate preferences,
- minimization of the number of towing activities and
- minimization of the absolute deviation of the new gate assignment from a so-called reference schedule.

It should be noted that the total objective only depends on the gate assignment and does not depend on the start and completion times of aircraft processing activities at the assigned positions. Moreover, the overall objective function takes into account both passenger comfort and convenience for airport services.

The major idea behind the flight gate problem presented in [44] is that it can be modeled as a multi-mode (modes in the model represent flight gates) resource-constrained project scheduling problem with a multiple criteria objective function (for resource-constrained project scheduling see, e.g., the survey [45]). For readers convenience we shortly describe the model, thus making our exposition self-contained.

Each aircraft activity (arrival, departure or parking) i can be described by its start time S_i and by its completion time C_i. It is evident that the start time for an arrival activity and the completion time of a departure activity are fixed and given a priory according to some time-table. All other start and completion times are decision variables of the model.

Let V denote the set of all activities as a unification of the sets of arrival, parking and departure activities, respectively, that is $V = V^{\mathrm{a}} \cup V^{\mathrm{p}} \cup V^{\mathrm{d}}$. Each activity i has a minimum required processing time $p_i^{\min}$. Activity i can be assigned to different flight gates (that is, can be processed in modes) M_i from the associated mode set $\mathscr{M}_i$ which is a subset of the set of all possible modes $\mathscr{M}$. To cope with the situation where the constraints do not allow to assign all aircraft to real gates, a fictitious gate 0 with unlimited capacity is introduced. Every set $\mathscr{M}_i$ contains this dummy gate, and an assignment to the dummy gate will be penalized in the objective function.

If two linked activities are assigned to different flight gates, then they require a towing procedure in order to be moved from one position to another one. Two activities are linked if they are subsequently served by the same aircraft (e.g. arrival–parking or parking–departure). Towing takes some fixed processing time $d_{iM_ijM_j}^{\mathrm{tow}}$. Let $\varepsilon^{\mathrm{tow}}$ represent the set of linked activities. It follows that the completion and start times of two linked activities i and j should satisfy the equality $C_i + d_{iM_ijM_j}^{\mathrm{tow}} = S_j$ to provide continuous processing.

Gates are disjunctive resources that can only process one aircraft at a time (the only exception is, of course, dummy gate 0). Between the processing of two activities i and j at the same gate a fixed setup time $d_{iM_ijM_j}^{\mathrm{setup}} \in \mathbb{N}_0$ must pass. The setup time can reflect the time required to push back the first aircraft from the gate and for moving the second aircraft to the gate as well as the duration required for setting up gate equipment. So, the basic disjunctive constraints that forbid simultaneous assignment of two aircraft to the same gate have to be added to the model. Additionally, these constraints must cover so-called "shadowing" restrictions (i, M_i, j, M_j) between gates M_i and M_j that can be interpreted as follows: if mode $M_i \in \mathscr{M}_i$ is assigned to activity i, then activity j must not be processed simultaneously in mode $M_j \in \mathscr{M}_j$. The set of all shadowing restrictions is denoted with $\varepsilon^{\mathrm{shadow}}$.

The model is summarized as follows:

Find a schedule (S, C, M) which assures the following constraints:

Minimal processing time

$$\forall i \in V \quad S_i + p_i^{\min} \leqslant C_i$$

Continuous processing

$$\forall (i, j) \in \varepsilon^{\mathrm{tow}} \quad C_i + d_{iM_ijM_j}^{\mathrm{tow}} = S_j$$

Disjunctive activities and setup times

For any activities $i, j \in V$ *such that either* $M_i = M_j \neq \emptyset$ *or* $\exists (i, M_i, j, M_j) \in \varepsilon^{\mathrm{shadow}}$ *one of the following conditions must be fulfilled*

$$C_i + d_{iM_ijM_j}^{\mathrm{setup}} \leqslant S_j - \textit{activity } i \textit{ must precede } j$$

or

$$C_j + d_{jM_jiM_i}^{\mathrm{setup}} \leqslant S_i - \textit{activity } j \textit{ must precede } i$$

Start and completion time

$$\forall i \in V^{\mathrm{a}} \quad S_i = t_i^{\mathrm{a}}$$

$$\forall i \in V^{\mathrm{d}} \quad C_i = t_i^{\mathrm{d}}$$

$$\forall i \in V \quad S_i,\ C_i \in \mathbb{N}_0$$

Mode selection

$$\forall i \in V \quad M_i \in \mathscr{M}_i.$$

The objective function is a linear combination of several goals: (1) the maximization of the total assignment preference score, (2) the minimization of the number of required towing operations, and (3) the minimization of the deviation from a given reference gate schedule. Using goal weights α_i, which are non-negative real numbers, the objective function $z(M)$ is constructed as follows:

$$z(M) := \min\ \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3,$$

where

$$z_1 := -\sum_{i \in V} w_i u_{iM_i}$$

$$z_2 := -|(i, j) \in \varepsilon^{\mathrm{tow}} : M_i \neq M_j|$$

$$z_3 := -\sum_{i \in V : M_i \neq M_i'} w_i.$$

Typically a $u_{i\mu} \in [0, 1]$ is a preference value associated with every activity-mode combination, $w_i \in [0, 1]$ is a priority weight associated with every activity and M_i' denotes the reference gate of activity i, respectively. It is obvious that the choice of appropriate preference weights and priorities as well as the ordering of the partial goals by importance using parameters $\alpha_1, \alpha_2, \alpha_3$ may have a substantial impact on the optimal gate schedule.

The basic optimization algorithm is a truncated branch and bound procedure (see [46,47]). The algorithm proceeds by assigning modes to the activities and by resolving resource conflicts that might appear. In comparison with a standard branch and bound procedure, it has several distinctive features. First of all, it uses two different types of branching: (1) branching over flight gates (modes) by assigning the best mode to some unscheduled activity according to some rule and accepting or forbidding this assignment afterwards, (2) branching over disjunctive constraints by resolving resource conflicts and defining which activity from the set of already scheduled ones (an activity is considered to be scheduled if it has a mode assignment) must be the predecessor. The second feature of the proposed method is that it uses constraint propagation techniques (see e.g. [46–50]). This means that at each node of the binary search tree induced by the branching scheme constraint propagation techniques are applied in order to reduce the search space until a fixed point has been computed.

For dealing with large instances arising in practice (which have a huge number of aircraft activities and airport gates), the branch and bound procedure was upgraded by combining it with additional problem decomposition (variable partitioning) techniques. Additionally, large neighborhood search techniques have been implemented. Computational experiments with large real-life data as well as with manually constructed small examples demonstrate the effectiveness of the proposed technique especially in comparison with the results of a modern rule based decision support system.

6. Summary and future work

This work describes mathematical models and reviews different research approaches to a general flight gate assignment problem. The models presented in this paper deal with scheduling of such scarce airport resources as terminals over time to different aircraft activities. Predominant approaches are based on the quadratic assignment problem or on integer programming with objective functions that minimize total passenger walking or baggage transport distance. In contrast to this, our emphasis is on a multi-mode version of the resource-constrained project scheduling problem with multiple objectives (with particular emphasis on comfort for passengers as well as convenience for airport services). Solution techniques based on specialized branch and bound procedures and some improvements based on the large neighborhood search principle are mentioned.

There are several open research directions in flight gate scheduling:

1. One problem consists in developing solution techniques for practical gate scheduling with multiple criteria and including all technical and temporal requirements. From the decision makers' (i.e. flight gate managers) perspective we can distinguish three major classes of problems with corresponding solution methods, that is, a priori, interactive and a posteriori methods (see e.g. [51]):

   - The method enabling the decision maker to set up his preferences (as for example the value of weights of the criteria for the minimization of a linear combination of criteria) before problem solving is commonly referred to as a priori.
   - If the decision maker's intervention during the solution process is allowed, then interactive methods have to be considered. Each iteration of such a method provides a solution which probably is not Pareto optimal or which has certain other undesirable properties. Then the aim of the decision maker is to reorient the process (directly or indirectly) by imposing new preference values.
   - Finally, if the decision maker probably intervenes eventually after the problem has been solved the method is called a posteriori.

All these methods aim to provide the set of Pareto optima (for the definition of the Pareto set and its properties see e.g. [52,53]) among which the decision maker has to chose according to his preferences. In case the airport authority can specify preferences in advance we suggest to use a priori methods to tackle the problem.

The technique most frequently used in practice for dealing with multiple objectives (see, for instance, the approach of [44] outlined in the preceding section) is criteria aggregation by adding new parameters (weights or goals) to the problem. These parameters can be interpreted as values of decision makers' preferences, and the partial criteria can be ordered by importance due to preference values. Unfortunately, such an aggregation has several disadvantages: (1) the interpretation of numeric values of an aggregated objective function is difficult due to the different meaning of the partial objectives, (2) the meaning of the weights is not clear for the decision maker, (3) tuning the aggregation function by playing with a set of large weights may be too costly.

Summing up the search for methods which are better suited for flight gate scheduling under multiple objectives is an important area of future research. In particular, we are optimistic that some multiple criteria metaheuristics of [54], like Pareto Simulated Annealing and Genetic Local Search, can be efficiently applied to the above models.

2. Probably one of the major problems not investigated sufficiently consists in constructing so-called robust or stable schedules that (being probably non-optimal in the

original instance but as close as possible to the optimal one; optimizing for instance the worst possible scenario) taking into account possible uncertainty or perturbations of input data (e.g. aircraft earliness or tardiness, flight gates breakdown or failures etc). In turn, this robustness problem can be divided into two subproblems: one where the level of uncertainty is defined deterministically and one where it is given stochastically with some probability measure. An annotated bibliography which covers a variety of different robustness concepts can be found in [55].

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