Efficient Aggregation in Heterogeneous Agent Models with Bounded Rationality*

David Evans
University of Oregon
devans@uoregon.edu

Giorgi Nikolaishvili
Wake Forest University
nikolag@wfu.edu

November 6, 2025

Abstract

A key challenge in heterogeneous agent models with bounded rationality is the intensive computational burden of repeatedly aggregating policy functions within equilibrium solvers. This cost scales with belief heterogeneity, creating a severe bottleneck. We propose a fast aggregation method that replaces repeated summations with a compact representation of aggregate demand as a function of prices, delivering speedups of several orders of magnitude over conventional approaches while preserving accuracy. Demonstrated in a model with several dimensions of belief heterogeneity, our method directly overcomes a central obstacle to simulating boundedly rational heterogeneous agent economies and extends the scope of feasible applications.

JEL Classifications: C63; C68

Keywords: Heterogeneous agents; belief heterogeneity; bounded rationality; equilibrium approximation; policy function aggregation

*This work benefited from access to the University of Oregon high performance computing cluster, Talapas, and the Wake Forest University High Performance Computing Facility. All errors are our own.

1 Introduction

Recent work has called to move beyond rational expectations in heterogeneous-agent (HA) macroeconomics – a natural way forward is to allow for boundedly rational, heterogeneous beliefs (Moll, 2025). However, this added realism introduces a potentially prohibitive computational cost. Simulating boundedly rational HA economies requires solving for a temporary equilibrium (TE) in each period. Each TE involves integrating over the cross-sectional distribution of idiosyncratic states – including agents' individual beliefs – multiple times per period throughout the simulation. As the dimensionality of beliefs expands, this aggregation step becomes increasingly burdensome, scaling rapidly with the number of belief dimensions. Consequently, computational tractability places a tight constraint on how richly we can model heterogeneous and boundedly rational belief formation in HA settings. In this paper, we introduce a method that resolves this computational bottleneck by streamlining the aggregation step required to compute TE in boundedly rational HA models.

In a TE, market-clearing conditions jointly determine agents' decisions and prices, which in turn govern aggregate dynamics. Solving for the TE typically involves approximating individual policy functions over both idiosyncratic and aggregate states, and then using a root-finding algorithm to recover equilibrium prices from market-clearing conditions, which depend on market-specific aggregates of individual policy rules (Bakota, 2022). Even with projection methods (Judd, 1992, 1996) and discretization schemes (Young, 2010), repeatedly computing and aggregating policy rules at each step of the root-solver across periods imposes a substantial computational burden (Den Haan et al., 2010). Each additional idiosyncratic dimension introduced by incorporating another belief parameter into the model expands the coordinate grid of the idiosyncratic state space – this causes the number of policy mappings required for aggregation to grow exponentially at each iteration of the root-solver.

We address this computational bottleneck in simulating boundedly rational HA economies by introducing a fast new method of approximating TE that removes a key redundancy in conventional projection methods. Our approach modifies the basis-function representation of individual policy rules so that the aggregation over idiosyncratic states can be performed only once per period, rather than at every iteration of the root solver. Specifically, by reorganizing the projection of individual demand functions, we isolate the price-dependent components from the idiosyncratic-state components, allowing the latter to be pre-aggregated into constant coefficients that summarize the distributional information. The resulting formulation expresses aggregate demand purely as a function of the price vector, producing a compact, low-dimensional representation of market demand. This pre-aggregation eliminates the repeated summations across agents embedded in standard TE solvers, leading to speedups of several orders of magnitude while maintaining the same level of accuracy as traditional methods.

We begin by defining the TE problem for a general class of HA models allowing for heterogeneous beliefs. Next, we present conventional methods of solving for TE, and propose our method as a modification to the standard projection approach. Finally, we apply all methods to a stylized HA model with boundedly rational expectations to compare their performance. We conclude with a brief discussion of the implications of our method for the feasibility of incorporating bounded rationality in HA macroeconomics.

2 The TE Problem

Suppose that we aim to find the TE of an HA economy featuring heterogeneous beliefs. We model this as a set of demand functions $\widetilde{x}(z, P, Y)$, which depend on individual states z (including agent-specific beliefs) with distribution Ω, prices P, and a finite-dimensional vector of observable aggregates Y. The aggregate state, $Z = (\Omega, Y)$, follows a law of motion $Z' = G(Z, \nu')$, where Z' and ν' denote the realization of the aggregate state and shocks in the next period, respectively. Our objective is to find the set of market-clearing prices P^* given Z, such that

$$\widetilde{X}(P^*, Z) = \overline{X}(P^*, Z)\,, \tag{1}$$

where $\widetilde{X}$ and $\overline{X}$ denote aggregate demand and supply, respectively, and the former is determined by

$$\widetilde{X}(P, Z) \equiv \int \tilde{x}(z, P, Y)\, d\Omega(z)\,. \tag{2}$$

In a simulation, the numerical bottleneck in solving Eq. (1) is the repeated evaluation of the aggregation operator in Eq. (2) inside a root-finding routine over prices. Consider approximating the cross-sectional distribution Ω with weighted nodes $\{(z_i, \omega_i)\}_{i=1}^{N}$. The aggregate demand for good $j = 1, \ldots, J$ at a candidate price vector P and aggregate state Z becomes

$$\widehat{X}_j(P, Z) \;\equiv\; \sum_{i=1}^{N} \omega_i\, \widehat{x}_j(z_i, P, Y), \tag{3}$$

where $\widehat{x}$ is an approximation of the individual demand function $\widetilde{x}$. Then, the TE problem can be expressed as the functional

$$F(P; Z) \;\equiv\; \overline{X}(P, Z) - \widehat{X}(P, Z) = 0\,. \tag{4}$$

A generic solver iterates $P_{m+1} = f(P_m, F(P_m; Z))$, where each evaluation of F requires N calls to the individual policy map $z \mapsto \widehat{x}(z; P, Y)$ to compute $\widehat{X}$. Given M solver steps, the number of individual policy evaluations scales as $M \times N \times J$ and the wall-clock cost in each period of a simulation is $O\big(MNJ\,T_{\text{eval}}(d_z)\big)$, where $T_{\text{eval}}(d_z)$ is the time to compute a single $\widehat{x}$ given the dimension d_z of the idiosyncratic state. Even for moderate M, the aggregation inside Eq. (2) quickly dominates runtime in a multi-period simulation. This issue worsens with an increasing number of markets J,[1] as well as an increase in the dimensionality of the idiosyncratic state.[2]

3 TE Approximation Methodology

We present three distinct approaches to approximating TE in HA models, and later apply all of them in our example setting. The first method involves explicitly solving the policy rules for each individual agent in the simulation and aggregating their respective decisions in each iteration of the root solver at each period. The second method involves interpolating agent-level policy rules via basis function approximation at the start of a simulation and using these interpolated functions in the market-clearing root solver. Although pre-computing policy rules could improve performance, we demonstrate that this approach is inefficient as individual policy functions must still be aggregated at each step of the root solver.

We propose the third method as a modification to the second method that achieves a significant reduction in computational cost. We show that the structure of the interpolated policy functions allows us to aggregate over the idiosyncratic state space separately from prices, which yields an approximation of aggregate demand conditional on the aggregate state using the basis functions over prices. This in turn allows us to pre-compute the corresponding component of the aggregate demand for each good as a constant at the beginning of each

[1]Although this paper focuses on the context of belief heterogeneity, HA models with multiple markets – such as those featuring endogenous labor supply (Krusell and Smith, 1997; Chang and Kim, 2006, 2007; Krusell et al., 2010) and multiple assets (Krusell and Smith, 1997) – may also present the same challenge if the number of markets is large.

[2]An increase in d_z necessarily increases executive time by increasing $T_{\text{eval}}(d_z)$. However, if using a histogram approach to approximate Ω (Young, 2010), the size of the grid N explodes with the dimension of the idiosyncratic state z. Let $d_z = d_{\text{base}} + d_{\text{beliefs}}$, where d_{beliefs} and d_{base} are the dimensionality of individual beliefs and all other remaining idiosyncratic states, respectively. Under a tensor-product rule with q_ℓ nodes along dimension ℓ, the total node count is $N = \prod_{\ell=1}^{d_z} q_\ell$, so if $q_\ell \equiv q$ then $N = q^{\,d_z}$. Because d_{beliefs} enters d_z linearly while N grows exponentially in d_{beliefs}, richer belief heterogeneity tightens the aggregation bottleneck inside the root solver by orders of magnitude (the curse of dimensionality).

period of a simulation. In doing so, we eliminate the computational cost of aggregating the policy functions in each iteration of the root solver. This approach significantly outperforms the first method in terms of execution time, without suffering from a loss in accuracy.

Method #1 (Naive Global Approximation): Let Ω be approximated with N points $\{\overline{z}_i\}_{i\in\mathbb{N}_N}$ with corresponding weights $\{\omega_i\}_{i\in\mathbb{N}_N}$. The approximation for the demand schedule $\widetilde{X}(P,Z)\in\mathbb{R}^J$ for a given P and Z may be expressed as the following weighted sum:

$$\widehat{X}^j(P,Z)\equiv\sum_i \omega_i\,\tilde{x}^j(\overline{z}_i,P,Y)\,, \tag{5}$$

where $\tilde{x}$ is derived individually for each $(\overline{z}_i,P,Y)$ tuple,[3] and $j=1,\ldots,J$ is an index for the set of goods in the economy. Repeating the above approximation for all J goods yields $\widetilde{X}(P,Z)$, which can then be used to solve the equilibrium vector of prices P^* using the market clearing condition in Eq. (1). See Algorithm 1 in Appendix A for a step-by-step implementation of Method 1.

Method #2 (Interpolated Global Approximation): This method adds a layer of approximation to Method 1. Suppose that $\tilde{x}$ is approximated by $\hat{x}$ via projection, such that the demand for the j-th good is represented by

$$\hat{x}^j(z,P,Y)=\sum_k\sum_l\sum_m c^j_{klm}\,\Phi^z_k(z)\,\Phi^P_l(P)\,\Phi^Y_m(Y)\,, \tag{6}$$

where Φ^P_l, Φ^z_k and Φ^Y_m are the basis functions that each depend on prices, idiosyncratic states, and the aggregate observables, respectively. The coefficients for this approximation,

[3] Given a fixed set of beliefs, the individual decision problem $\widetilde{x}$ can typically be solved exactly (rather than being approximated) by finding the root of the agent's first-order conditions – that is, by numerically solving the system of equations implied by the intratemporal optimality condition, the Euler equation, and the budget constraint at the candidate price vector P. In practice, this amounts to computing each agent's optimal decisions by applying a root-finding routine conditional on their idiosyncratic states z and perceived law of motion for aggregate variables. This approach is standard in heterogeneous-agent models with endogenous labor supply, where labor choice is recovered as the unique solution to the nonlinear system characterizing within-period optimality.

c^j_{klm}, need to be computed only once prior to simulation. The approximation for the demand schedule $\widetilde{X}(P,Z)$ may be expressed as the following weighted sum:

$$\widehat{X}^j(P,Z) \equiv \sum_i \omega_i\, \hat{x}^j(\overline{z}_i, P; Y)\,, \tag{7}$$

where $\overline{z}_i$ and ω_i are defined as before. Once again, Eq. (7) may be used to solve for P^* in Eq. (1). Algorithm 2 in Appendix A provides a detailed procedural statement of Method 2.

This approach requires the policy function for the j-th good to be approximated only once, after which it is inputted into Eq. (7) to compute the sum. However, given a large N, the sum in Eq. (7) may still take a substantial amount of time to compute. Moreover, while precomputing individual policy functions through projection may seem to improve efficiency, this approach can in fact become more computationally demanding than directly solving each agent's problem as in Method 1. The reason is that evaluating the interpolated policy function $\hat{x}$ in Eq. (7) requires summing over all combinations of basis functions across the idiosyncratic states, prices, and aggregate variables for every agent in the simulated population. When the number of basis terms or the dimensionality of heterogeneity is large, the resulting nested summations impose a heavy computational burden – often involving millions of evaluations per iteration of the root solver. In contrast, the "naive" approach in Method 1, which solves each agent's decision problem exactly as a small-scale root-finding task, can be relatively fast if those individual problems are low-dimensional. Thus, the interpolation scheme in Method 2 may paradoxically yield higher execution times despite its analytical convenience, since the computational gains from precomputing coefficients are offset by the cost of repeatedly evaluating the full basis expansion for each agent during aggregation.

Method #3 (Fast Approximation): Finally, we present our proposed method. Notice that substituting Eq. (6) into Eq. (7) yields

$$\widehat{X}^j(P,Z) = \sum_i \omega_i \sum_l \sum_k \sum_m c^j_{klm}\, \Phi^z_k(\overline{z}_i)\, \Phi^P_l(P)\, \Phi^Y_m(Y)\,, \tag{8}$$

which can be rearranged by distributing ω_i, switching the order of summation, and factoring Φ^P_l in the following manner:

$$\widehat{X}^j(P,Z) = \sum_l \Phi^P_l(P) \sum_k \sum_m c^j_{klm} \left(\sum_i \omega_i \Phi^z_k(\bar{z}_i) \right) \Phi^Y_m(Y)\,. \tag{9}$$

Notice that by integrating over the grid of idiosyncratic states, we obtain the following composite mapping for each k-th basis function over idiosyncratic states:

$$\overline{\Phi}^z_k(Z) \equiv \sum_i \omega_i \Phi^z_k(\bar{z}_i)\,, \tag{10}$$

so that $\overline{\Phi}^z_k(Z)$ can be computed independent of the price vector P. Furthermore, we may define

$$C^j_l(Z) \equiv \sum_k \sum_m c^j_{klm}\, \overline{\Phi}^z_k(Z)\, \Phi^Y_m(Y)\,, \tag{11}$$

which is a mapping of the aggregate state that is also independent of P. Collapsing the price-independent components of each term of the sum over price basis functions in Eq. (9) into $C^j_l(Z)$ allows us to express the aggregate demand in Eq. (9) as

$$\widehat{X}^j(P,Z) = \sum_l C^j_l(Z)\, \Phi^P_l(P)\,. \tag{12}$$

With this formulation of $\widehat{X}$, since C^j_l is independent of P, it suffices to compute C^j_l only once before initializing the root solver to find P^*. A formal algorithmic description is provided in Algorithm 3 in Appendix A.

This approach strictly dominates Method 2 by relying on the same initial projections but fewer mappings in the aggregation stage, so that execution times are lower but the accuracy is identical. More importantly, this method outpaces Method 1 without much loss in accuracy, as discussed in Section 3.

The execution speed of a TE solver becomes crucial in practice when the dynamic recursive equilibrium of an HA economy is simulated over multiple periods of time. In other words, when the TE solver is used repeatedly, the speedup offered by our method is particularly attractive. The case becomes even stronger in the context of simulating stationary recursive equilibria, in which the idiosyncratic distribution Ω is time-invariant – it becomes sufficient to compute C_l^j only once at the initialization stage of the entire simulation, beyond which it can be reused in all periods.

4 The Model

We use the boundedly rational HA model developed by Evans et al. (2023) with the notation adapted to the general setting presented in Section 2 to tangibly illustrate the challenge of solving the general class of TE shown in Eq. (1). Refer to Table 2 in Appendix A for a side-by-side comparison of the objects in the model to those defined in Section 2. We use this particular model to compare the execution time and accuracy of our approach with that of Methods 1 and 2.

Let time be discrete. The economy is populated by a continuum of agents such that a given agent is endowed with a unit of labor per period and derives utility from consumption c and leisure l according to the instantaneous utility function $u(c, l)$. Each agent has a unique effective unit of labor for each unit of nominal labor supplied and receives a corresponding wage that can be separated into the following two components: (1) a common aggregate component w, and (2) an idiosyncratic efficiency component ε distributed across the population. We assume $\{\varepsilon\}$ to be a Markov process with time-invariant transition function Π distributed i.i.d. across agents. In each period, an agent can trade one-period claims to capital for net return r, limited by the exogenous borrowing constraint $\underline{a}$. The goods and factor markets are assumed to be competitive.

In period t, an agent holds claims a, experiences idiosyncratic efficiency ε, and faces factor prices $P = (r, w)$. Additionally, an agent has a vector of beliefs $\psi \in \mathbb{R}^n$, which comprises coefficients of a forecasting model used to form expectations of next period's shadow price λ', where the current shadow price is defined as

$$\lambda(a, \varepsilon, \psi) \equiv (1 + r)\, u_c(c(a, \varepsilon, \psi), l(a, \varepsilon, \psi))\,. \tag{13}$$

The set of individual states $z = (a, \varepsilon, \psi)$ has a distribution Ω.

The representative firm rents capital k at the real rental rate $r + \delta$, hires effective labor n at the real wage w, and produces output under perfect competition using CRTS technology $\theta f(k, n)$, where δ is the capital depreciation rate. θ is a stationary process that affects total factor productivity with dynamics given by $\theta' = \nu' \theta^\rho$, where $|\rho| < 1$ and ν' is i.i.d. with a log-normal distribution. There are no capital installation costs. Profit-maximizing behavior by the firm implies that factors earn their respective marginal products:

$$w = \theta\, f_n(k, n) \quad \text{and} \quad r + \delta = \theta\, f_k(k, n)\,. \tag{14}$$

The aggregate state is $Z = (\Omega, Y)$, where Y is a vector of observable aggregates, including θ, that all agents observe in the current period and could use to condition their forecasts of λ'. The transition dynamics of the aggregate state is governed by $Z' = G(Z, \nu')$, where ν' is the total factor productivity shock of the next period.

The individual consumption, labor, and claims demands of agents are functions of idiosyncratic states, prices, and observable aggregates. We may express the individual demand function as $\tilde{x}(z, P, Y) = (\tilde{c}(z, P, Y), \tilde{a}(z, P, Y), \tilde{l}(z, P, Y))$, which satisfies the following conditions:

$$u_c(\tilde{c}(z, P, Y), \tilde{l}(z, P, Y)) \geq \beta \, \tilde{\lambda}^e(\tilde{a}(z, P, Y), \varepsilon, \psi, Y) \tag{15}$$

$$\text{and } \tilde{a}(z, P, Y) \geq \underline{a}, \text{ with c.s.}$$

$$u_l(\tilde{c}(z, P, Y), \tilde{l}(z, P, Y)) = u_c(\tilde{c}(z, P, Y), \tilde{l}(z, P, Y))w \tag{16}$$

$$\tilde{a}(z, P, Y) = (1 + r)a + w \cdot \varepsilon \cdot (1 - \tilde{l}(z, P, Y)) - \tilde{c}(z, P, Y)\,, \tag{17}$$

where $\tilde{\lambda}^e$ represents an agent's forecast of λ' based on Y. Specifically, each agent forms a forecast using the following perceived law of motion (PLM):

$$\log \tilde{\lambda}^e = \log \overline{\lambda}^e + \langle \psi, Y \rangle \ , \tag{18}$$

where $\langle \cdot, \cdot \rangle$ is the standard inner product on $\mathbb{R}^n$, and $\overline{\lambda}^e$ is a time-t forecast of λ' in a corresponding stationary recursive equilibrium without any aggregate risk. We may express

$$\overline{\lambda}^e(\bar{a}(a, \varepsilon), \varepsilon) = \int \overline{\lambda}(\bar{a}(a, \varepsilon), \varepsilon')\Pi(\varepsilon, d\varepsilon')\,, \tag{19}$$

where ε' is an individual efficiency shock in period $t + 1$, $\bar{a}$ is the stationary equilibrium level of claims in the current period, and $\overline{\lambda}(a, \varepsilon) = (1 + \overline{r})u_c(\overline{c}(a, \varepsilon), \overline{l}(a, \varepsilon))$ such that $\overline{c}$, $\overline{l}$, and $\overline{r}$ represent the stationary equilibrium levels of consumption, labor, and the capital rate of return, respectively.[4][5]

[4]In essence, we assume that agents' forecasts of the following period's shadow price consists of two components: (1) a rational forecast of next period's shadow price in a corresponding stationary equilibrium without aggregate risk; and (2) a boundedly rational forecast of the expected deviation of next period's shadow price from its stationary level attributed to variation in aggregate risk, as predicted by variation in the observed aggregate variables. For an in-depth discussion of this PLM and how agents update their beliefs, refer to Evans and McGough (2021) and Evans et al. (2023).

[5]Giusto (2014) offers an alternative approach to incorporating agent-level bounded rationality in an HA

This brings us to the principal challenge addressed in this paper. Given agent-specific states $z = (a, \varepsilon, \psi)$ and observable aggregates Y, conditions (15)–(17) determine agents' decision rules as a function of prices $P = (r, w)$. However, the realized value of prices and other endogenous aggregates is determined by market clearing, i.e., TE. Mechanically, this determination requires tracking the evolving distribution of agent-specific states. TE imposes $r = \theta f_k(k, n) - \delta$ and $w = \theta f_n(k, n)$, where (k, n) is determined by the following market clearing conditions:

$$k = \int a \cdot \Omega(dz) \quad \text{and} \quad n = \int (1 - \tilde{l}(z, P, Y))\, \Omega(dz)\,. \tag{20}$$

Notice that n depends on the agent-level policy rules $\tilde{l}(z, P, Y)$, which in turn depend implicitly on current factor prices P. All must be determined jointly in the TE as a solution to a system of nonlinear equations. Solving the system necessitates aggregating over the individual decisions of all agents for each candidate solution (n, r, w), which may pose a significant computational hurdle in a corresponding simulation with a large number of agents.

5 Performance

We simulate a single period of the example economy, conditioning on a TFP realization $\theta = 1$ and on an ergodic cross-sectional distribution of individual states $\widehat{\Omega}$.[6] Following Evans et al. (2023), we define the vector of observables as $Y \equiv (1,\ \log(k/\overline{k}),\ \log\theta)$, where $\overline{k}$ denotes the steady-state capital stock. Under Eq. (18), this specification implies that each agent forecasts the log-deviation of their shadow price from its stationary value via a linear projection on a constant, the log deviation of aggregate capital from its steady state, and

model.

[6] In principle, runtime and accuracy can depend on θ and Ω; we found this not to be the case in our application.

the log of aggregate productivity. Each agent's belief vector contains the same number of parameters as the number of regression coefficients in their forecast model. Accordingly, the belief vector is $\psi \in \mathbb{R}^3$, and the idiosyncratic state is $z = (a, \varepsilon, \psi) \in \mathbb{R}^5$.

In our application of Method 1, for a given wage level w and distribution $\widehat{\Omega}$ of individual states (a, ε, ψ), we analytically solve the level of capital k and the return on capital r. Using the above objects, we then solve the quantity of labor supplied individually by each agent, $1 - l_i$, according to conditions (15)–(17). Averaging these individual labor supply decisions across the set of all agents, as in Eq. (5), yields the aggregate labor supply n_t, as shown in Eq. (20).

We compare the performance of the described TE solution methods across three dimensions: (1) the execution time of the aggregation procedure – in other words, the time it takes to find the sum of individual policy rules given all necessary inputs; (2) the execution time of the multidimensional mapping characterizing the TE, which needs to be computed at each step of the nonlinear solver; (3) the execution time of the nonlinear solver that approximates the set of equilibrium prices. We generate samples of these execution times and present the corresponding summary statistics in Table 1. We find that our proposed method (Method 3) offers significant speed-ups across all stages of the TE solution procedure.

In addition to comparing execution speed, we compare the accuracy with which our method estimates the aggregate labor supply schedule. In Figure 1, we plot the percentage deviation of the labor supply schedule approximated using Methods 2 and 3 from that obtained using Method 1 over a large interval surrounding the steady state wage level.[7] We find that our approach offers accuracy similar to Method 1, since the two labor supply schedules appear to be practically identical.

[7] Note that Methods 2 and 3 must yield the same approximation.

	Method #1	Method #2	Method #3
Aggregation	3.30e-2	4.20e+1	4.00e-6
	(2.40e-2)	(3.94e+1)	(3.20e-6)
Temporary Eq. Mapping	6.11e-2	4.33e+1	3.99e-6
	(5.01e-2)	(4.32e+1)	(3.44e-6)
Nonlinear Solver	6.82e-1	4.86e+2	3.36e-4
	(6.57e-1)	(4.39e+2)	(3.22e-4)
Sample Size			1000

Table 1: Temporary equilibrium solution methods execution times. *Note:* Mean execution time in seconds (minimum execution time in parentheses).

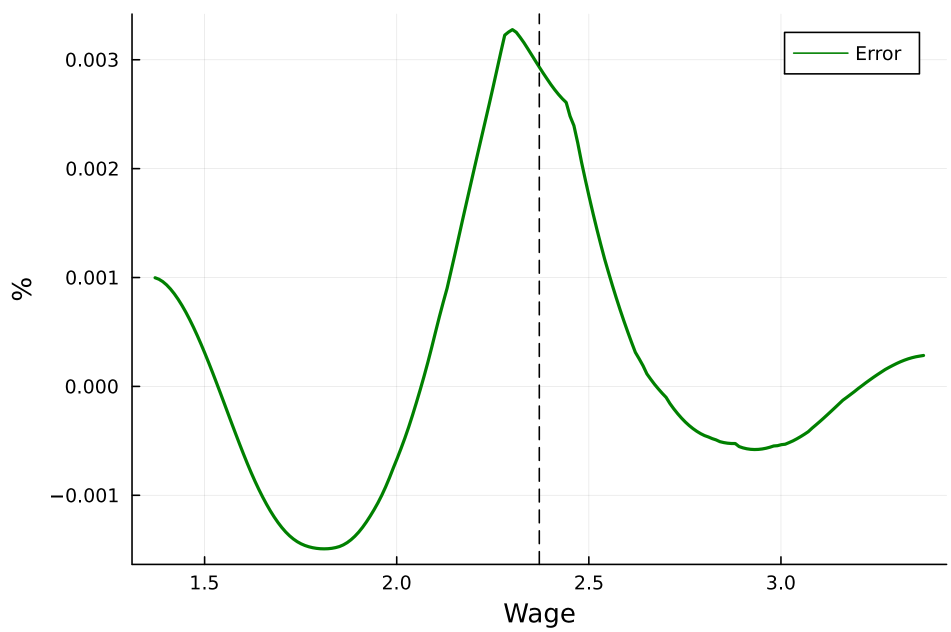

Figure 1: Percentage deviation of aggregate labor supply as a function of wage approximated using Method 3 from that obtained using Method 1. *Note:* The steady state wage level is represented by the dashed vertical line.

6 Conclusion

We address a challenge in simulating heterogeneous agent temporary equilibria with boundedly rational beliefs: repeated aggregation of individual policy rules nested inside the nonlinear price solver. After laying out the standard formulation of the TE problem and its computational scaling, we show that it is possible to avoid aggregating over the idiosyncratic state space at each instance of the solver in a given period by pre-computing a portion of the interpolated policy rules. In our application, this reformulation delivers large speed gains for aggregation, the TE mapping, and the overall solve, while preserving accuracy relative to a more computationally-intensive conventional benchmark. Because our proposed method replaces the within-solver aggregation with a compact representation of aggregate demand, it may also be applied to speed up computation in HA models with multiple markets, such as those with endogenous labor supply (Chang and Kim, 2006, 2007; Krusell et al., 2010) and multiple assets (Krusell and Smith, 1997).

Our proposed TE solver is a practical building block for the program to move beyond rational expectations in heterogeneous agent macroeconomics. Moll (2025) calls for alternative approaches in HA modeling that (i) are computationally tractable, (ii) discipline beliefs with empirical evidence, and (iii) render beliefs sufficiently endogenous to be credible under policy counterfactuals; as part of this agenda, he highlights temporary equilibrium frameworks and adaptive learning as promising directions. By collapsing the repeated cross-sectional summations inside the price solver into a low-dimensional price-basis representation, our method directly delivers criterion (i) and makes the TE mapping essentially insensitive to the dimensionality of belief heterogeneity after a one-time pre-aggregation in each period. This reduction in marginal cost allows researchers to incorporate forecasting rules or learning dynamics without facing the aggregation bottleneck, and the same pre-aggregated coefficients can be reused across periods when the cross-sectional distribution is stationary. Taken together, these features lower the fixed and variable costs of building boundedly rational HA economies and expand the feasible frontier – bringing within reach the kinds of non-linear, aggregate-risk questions proposed by Moll (2025).

References

Bakota, I. (2022). Market clearing and krusell-smith algorithm in an economy with multiple assets. *Computational Economics*, 62:1007–1045. https://doi.org/10.1007/s10614-022-10290-2.

Chang, Y. and Kim, S.-B. (2006). From individual to aggregate labor supply: A quantitative analysis based on a heterogeneous agent macroeconomy. *International Economic Review*, 47(1):1–27. https://doi.org/10.1111/j.1468-2354.2006.00370.x.

Chang, Y. and Kim, S.-B. (2007). Heterogeneity and aggregation: Implications for labor-market fluctuations. *American Economic Review*, 97(5):1939–1956. https://doi.org/10.1257/aer.97.5.1939.

Den Haan, W. J., Judd, K. L., and Juillard, M. (2010). Computational suite of models with heterogeneous agents: Incomplete markets and aggregate uncertainty. *Journal of Economic Dynamics and Control*, 34(1):1–3. https://doi.org/10.1016/j.jedc.2009.07.001.

Evans, D., Li, J., and McGough, B. (2023). Local rationality. *Journal of Economic Behavior & Organization*, 205:216–236. https://doi.org/10.1016/j.jebo.2022.11.005.

Evans, G. W. and McGough, B. (2021). Agent-level adaptive learning. In *Oxford Research Encyclopedia of Economics and Finance*. Oxford University Press. https://doi.org/10.1093/acrefore/9780190625979.013.620.

Giusto, A. (2014). Adaptive learning and distributional dynamics in an incomplete markets model. *Journal of Economic Dynamics and Control*, 40:317–333. https://doi.org/10.1016/j.jedc.2014.01.007.

Judd, K. L. (1992). Projection methods for solving aggregate growth models. *Journal of Economic Theory*, 58(2):410–452. https://doi.org/10.1016/0022-0531(92)90061-L.

Judd, K. L. (1996). Chapter 12 approximation, perturbation, and projection methods in economic analysis. In *Handbook of Computational Economics*, volume 1, pages 509–585. Elsevier. https://doi.org/10.1016/S1574-0021(96)01014-3.

Krusell, P., Mukoyama, T., and Şahin, A. (2010). Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations. *The Review of Economic Studies*, 77(4):1477–1507. https://doi.org/10.1111/j.1467-937X.2010.00700.x.

Krusell, P. and Smith, A. A. (1997). Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns. *Macroeconomic Dynamics*, 1(2):387–422. https://doi.org/10.1017/S1365100597003052.

Moll, B. (2025). The trouble with rational expectations in heterogeneous agent models: A challenge for macroeconomics. *arXiv*. https://doi.org/10.48550/arXiv.2508.20571.

Young, E. R. (2010). Solving the incomplete markets model with aggregate uncertainty using the krusell–smith algorithm and non-stochastic simulations. *Journal of Economic Dynamics and Control*, 34(1):36–41. https://doi.org/10.1016/j.jedc.2008.11.010.

Appendix A

Algorithm 1: Temporary Equilibrium via Agent Simulation (Method 1)

Inputs: Observable aggregates Y; simulated agents $\mathcal{I} = \{1, \ldots, N\}$ with realized idiosyncratic states $\{z_i\}_{i \in \mathcal{I}}$; individual policy map $\tilde{x}(z, P, Y)$; aggregate supply mapping $\overline{X}(P, Z)$; solver settings $(P_0, \mathrm{tol}, \mathrm{maxit})$.

Outputs: Equilibrium prices P^*; approximated aggregate demand $\widehat{X}(P^*, Z)$.

Define agent–based aggregation $\mathrm{Agg}_1(P \mid Z)$:;

$\widehat{X}(P, Z) \leftarrow \dfrac{1}{N} \sum_{i=1}^{N} \tilde{x}(z_i, P, Y)$ (componentwise in $j = 1, \ldots, J$).;

Residual: $F(P; Z) \leftarrow \overline{X}(P, Z) - \mathrm{Agg}_1(P \mid Z)$.;

Solve TE: Apply a root solver to $F(P; Z) = 0$ from P_0 until $\|F\| < \mathrm{tol}$ (at most `maxit` steps).;

Result: P^* and $\widehat{X}(P^*, Z)$.

Algorithm 2: Temporary Equilibrium with Interpolated Policies (Method 2)

Inputs: Observable aggregates Y; simulated agents $\mathcal{I} = \{1, \ldots, N\}$ with realized states $\{z_i\}$; basis families $\{\Phi_k^z\}_{k=1}^K$, $\{\Phi_\ell^P\}_{\ell=1}^L$, $\{\Phi_m^Y\}_{m=1}^{M_Y}$; precomputed coefficients $c_{k\ell m}^j$ for $\hat{x}_j(z, P, Y) = \sum_{k,\ell,m} c_{k\ell m}^j \, \Phi_k^z(z) \, \Phi_\ell^P(P) \, \Phi_m^Y(Y)$; aggregate supply mapping $\overline{X}(P, Z)$; solver settings $(P_0, \mathrm{tol}, \mathrm{maxit})$.

Outputs: P^*; $\widehat{X}(P^*, Z)$.

Cache (per period):;

Precompute $\{\Phi_m^Y(Y)\}_{m=1}^{M_Y}$; optionally cache $\{\Phi_k^z(z_i)\}_{i,k}$.;

Define interpolated aggregation $\mathrm{Agg}_2(P \mid Z)$:;

$\widehat{X}(P, Z) \leftarrow \dfrac{1}{N} \sum_{i=1}^N \hat{x}(z_i, P, Y)$, with components $j = 1, \ldots, J$ given by

$$\hat{x}_j(z_i, P, Y) = \sum_{k=1}^{K} \sum_{\ell=1}^{L} \sum_{m=1}^{M_Y} c_{k\ell m}^j \, \Phi_k^z(z_i) \, \Phi_\ell^P(P) \, \Phi_m^Y(Y).$$

return $\widehat{X}(P, Z)$.;

Residual: $F(P; Z) \leftarrow \overline{X}(P, Z) - \mathrm{Agg}_2(P \mid Z)$.;

Solve TE: Apply a root solver to $F(P; Z) = 0$ from P_0 until $\|F\| < \mathrm{tol}$ (at most `maxit` steps).;

Result: P^* and $\widehat{X}(P^*, Z)$.

Algorithm 3: Temporary Equilibrium with Fast Aggregation (Method 3)

Inputs: Observable aggregates Y; simulated agents $\mathcal{I} = \{1, \dots, N\}$ with realized states $\{z_i\}$; basis families $\{\Phi_k^z\}_{k=1}^K$, $\{\Phi_\ell^P\}_{\ell=1}^L$, $\{\Phi_m^Y\}_{m=1}^{M_Y}$; projection coefficients $c_{k\ell m}^j$ (same $\hat{x}_j$ as Method 2); aggregate supply mapping $\overline{X}(P, Z)$; solver settings $(P_0, \mathrm{tol}, \mathrm{maxit})$.

Outputs: P^*; $\widehat{X}(P^*, Z)$.

One-time pre-aggregation (per period):;

Compute and store $\{C_\ell^j(Z)\}_{j=1,\dots,J;\ \ell=1,\dots,L}$ via

$$C_\ell^j(Z) \;=\; \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{m=1}^{M_Y} c_{k\ell m}^j \, \Phi_k^z(z_i) \, \Phi_m^Y(Y).$$

Define fast aggregation $\mathrm{Agg}_3(P \mid Z)$:;

For $j = 1, \dots, J$ set

$$\widehat{X}_j(P, Z) \;\leftarrow\; \sum_{\ell=1}^{L} C_\ell^j(Z) \, \Phi_\ell^P(P).$$

Return $\widehat{X}(P, Z) \equiv (\widehat{X}_1, \dots, \widehat{X}_J)$.;

Residual: $F(P; Z) \;\leftarrow\; \overline{X}(P, Z) \;-\; \mathrm{Agg}_3(P \mid Z)$.;

Solve TE: Apply a root solver to $F(P; Z) = 0$ from P_0 until $\|F\| < \mathrm{tol}$ (at most `maxit` steps).;

Result: P^* and $\widehat{X}(P^*, Z)$.

Object	General Notation	Example Environment	Notes
Individual state vector	z	(a, ε, ψ)	Current asset/claims a, idiosyncratic efficiency ε, belief coefficients $\psi \in \mathbb{R}^n$ used to forecast next-period shadow price.
Distribution of individual states	Ω	Distribution over (a, ε, ψ)	Used to aggregate to capital k and effective labor n.
Price vector	P	(r, w)	Net return on one-period claims r and real wage per efficiency unit w.
Observable aggregate state variables	Y	$(1,\ \log(k/\bar{k}),\ \log\theta))$	Includes a constant, the log deviation of aggregate capital from its steady state, and the log of aggregate productivity
Aggregate state	$Z = (\Omega, Y)$	$Z = (\Omega, Y)$	Collects the cross-sectional distribution Ω over (a, ε, ψ) and the observables Y (incl. θ).
Law of motion for aggregate state	$Z' = G(Z, \nu')$	$\theta' = \nu' \theta^{\rho}$	Z' updates via this aggregate shock and the induced evolution of Ω.
Individual decision (demand) vector	$\tilde{x}(z, P, Y)$	$(\tilde{c},\ \tilde{a},\ \tilde{l})$	Consumption, next-period claims, and leisure; determined by Euler (with borrowing constraint), intratemporal labor-leisure condition, and budget.
Aggregate demand	$\widetilde{X}(P, Z) = \int \tilde{x}(z, P, Y)\, d\Omega(z)$	$k = \int a\, \Omega(dz),\ \ n = \int \big(1 - \tilde{l}(z, P, Y)\big)\, \Omega(dz)$	Specializes to the aggregates needed for factor markets; see (13).
Aggregate supply	$\overline{X}(P, Z)$	$w = \theta f_n(k, n),\ \ r + \delta = \theta f_k(k, n)$	Firm-side requirements implied by FOCs under perfect competition (i.e., factor demands at prices P); see (7).
Goods/markets index	$j = 1, \ldots, J$	$J = 2$ (capital, effective labor)	Two factor markets solved for (r, w).

Table 2: Mapping from general objects in Section 2 to their counterparts in the example environment.