

# Efficient Aggregation in Heterogeneous-Agent Models with Bounded Rationality\*

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## Abstract

A key challenge in heterogeneous-agent models with bounded rationality is the intensive computational burden of repeatedly aggregating policy functions when solving for temporary equilibrium within a given period. This cost scales with belief heterogeneity, creating a severe bottleneck. We propose a fast aggregation method that replaces repeated summations with a compact representation of aggregate demand as a function of prices, delivering speedups of several orders of magnitude over conventional approaches while preserving accuracy. Demonstrated in a model with multiple dimensions of belief heterogeneity, our method directly overcomes a central obstacle to simulating boundedly rational heterogeneous-agent economies and extends the scope of feasible applications.

**JEL Classifications:** C63; C68

**Keywords:** Heterogeneous agents; belief heterogeneity; bounded rationality; equilibrium approximation; policy function aggregation

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# 1 Introduction

Recent work has called to move beyond rational expectations in heterogeneous-agent (HA) macroeconomics — a natural way forward is to allow for boundedly rational, heterogeneous beliefs (Moll, 2025). However, this added realism introduces a potentially prohibitive computational cost. Simulating boundedly rational HA economies requires solving for a temporary equilibrium (TE) in each period. Each TE involves integrating over the cross-sectional distribution of idiosyncratic states — including agents’ individual beliefs — multiple times per period throughout the simulation. As the dimensionality of beliefs expands, this aggregation step becomes increasingly burdensome, scaling rapidly with the number of belief dimensions. Consequently, computational tractability places a tight constraint on how richly we can model heterogeneous and boundedly rational belief formation in HA settings. In this paper, we introduce a method that resolves this computational bottleneck by streamlining the aggregation step required to compute TE in boundedly rational HA models.

In a TE, market-clearing conditions jointly determine agents’ decisions and prices, which in turn govern aggregate dynamics. Solving for the TE typically involves approximating individual policy functions over both idiosyncratic and aggregate states, and then using a root-finding algorithm to recover equilibrium prices from market-clearing conditions, which depend on market-specific aggregates of individual policy rules (Bakota, 2022). Even with projection methods (Judd, 1992, 1996) and discretization schemes (Young, 2010), repeatedly computing and aggregating policy rules at each step of the root-solver across periods imposes a substantial computational burden (Den Haan et al., 2010). Each additional idiosyncratic dimension introduced by incorporating another belief parameter into the model expands the coordinate grid of the idiosyncratic state space — this causes the number of policy mappings required for aggregation to grow exponentially at each iteration of the root-solver.

We address this key computational bottleneck in simulating boundedly rational HA

economies by introducing an efficient method for approximating TE. Our approach exploits the structure of high-dimensional basis-function approximations to remove redundant computations when aggregating policy rules. To begin, policy rules for individual demand are approximated using basis functions defined over idiosyncratic states, prices, and observable aggregates; this step can be performed once prior to the simulation. Rather than evaluating these basis functions at every gridpoint, integration is carried out only over the components that depend on idiosyncratic states. This integration, performed once per period, yields a vector of coefficients on the price-dependent basis functions that compactly represent aggregate demand. The resulting pre-aggregation eliminates the repeated summations across agents embedded in standard TE solvers, producing speedups of several orders of magnitude while maintaining the accuracy of conventional methods.

We begin by defining the TE problem for a general class of HA models allowing for heterogeneous beliefs. Next, we present a conventional method of solving for TE, and propose our method as a modification to a projection approach. Finally, we apply all methods to a stylized HA model with boundedly rational expectations to compare their performance. We conclude with a brief discussion of the implications of our method for the feasibility of incorporating bounded rationality in HA macroeconomics.

## 2 The TE Problem

Suppose that we aim to find the TE of an HA economy featuring heterogeneous beliefs. We model this as a set of demand functions  $\tilde{x}(z, P, Y)$ , which depend on individual states  $z$  (including agent-specific beliefs) with distribution  $\Omega$ , prices  $P$ , and a finite-dimensional vector of observable aggregates  $Y$ . The aggregate state,  $Z = (\Omega, Y)$ , follows a law of motion  $Z' = G(Z, \nu')$ , where  $Z'$  and  $\nu'$  denote the realization of the aggregate state and shocks in the next period, respectively. Our objective is to find the set of market-clearing prices  $P^*$

given  $Z$ , such that

$$X^D(P^*, Z) = X^S(P^*, Z), \quad (1)$$

where  $X^D$  and  $X^S$  denote aggregate demand and supply, respectively. The former is determined by

$$X^D(P, Z) \equiv \int \tilde{x}(z, P, Y) d\Omega(z), \quad (2)$$

while the latter is characterized by the first-order necessary conditions of a representative firm.<sup>1</sup>

In a simulation, the numerical bottleneck in solving Eq. (1) is the repeated evaluation of the aggregation operator in Eq. (2) inside a root-finding routine over prices. Consider approximating the cross-sectional distribution  $\Omega$  with weighted nodes  $\{(z_i, \omega_i)\}_{i=1}^N$ .<sup>2</sup> The aggregate demand for good  $j = 1, \dots, J$  at a candidate price vector  $P$  and aggregate state  $Z$  becomes

$$\hat{X}^j(P, Z) \equiv \sum_{i=1}^N \omega_i \hat{x}_j(z_i, P, Y), \quad (3)$$

where  $\hat{X}$  and  $\hat{x}$  are approximations of the aggregate and individual demand functions,  $X^D$  and  $\tilde{x}$ , respectively. Then, the TE problem can be expressed as the functional

$$F(P; Z) \equiv X^S(P, Z) - \hat{X}(P, Z) = 0. \quad (4)$$

A generic solver iterates  $P_{m+1} = f(P_m, F(P_m; Z))$ , where each evaluation of  $F$  requires  $N$

<sup>1</sup>Our methodology applies equally to models with heterogeneous firms as well as households; for simplicity, we present a model with a representative firm.

<sup>2</sup>These can be obtained by approximating the distribution using the histogram method of [Young \(2010\)](#) or by Monte Carlo simulation with a finite number of agents.

calls to the individual policy map  $z \mapsto \hat{x}(z; P, Y)$  to compute  $\hat{X}$ . Given  $M$  solver steps, the number of individual policy evaluations scales as  $M \times N \times J$  and the wall-clock cost in each period of a simulation is  $O(MNJT_{\text{eval}}(d_z))$ , where  $T_{\text{eval}}(d_z)$  is the time to compute a single  $\hat{x}$  given the dimension  $d_z$  of the idiosyncratic state. Even for moderate  $M$ , the aggregation inside Eq. (2) quickly dominates runtime in a multi-period simulation. This issue worsens with an increasing number of markets  $J$ ,<sup>3</sup> as well as an increase in the dimensionality of the idiosyncratic state.<sup>4</sup>

### 3 TE Approximation Methodology

We present two approaches to approximating TE in HA models, and later apply both in our example setting. The first method involves explicitly solving for the decisions for each individual agent in the simulation and aggregating their respective decisions in each iteration of the root solver at each period. Although straightforward to implement, this approach becomes computationally expensive when the number of idiosyncratic state dimensions is large or when the TE solver requires many evaluations of the aggregate mapping.

The second method we consider is a based around approximating individual policy rules using projection methods. This replaces repeated agent-level evaluations with a compact representation of aggregate demand as a function of prices. This method builds on the structure of basis function approximations commonly used in the HA literature,

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<sup>3</sup>Although this paper focuses on the context of belief heterogeneity, HA models with multiple markets – such as those featuring endogenous labor supply (Krusell and Smith, 1997; Chang and Kim, 2006, 2007; Krusell et al., 2010) and multiple assets (Krusell and Smith, 1997) – may also present the same challenge if the number of markets is large.

<sup>4</sup>An increase in  $d_z$  necessarily increases execution time by increasing  $T_{\text{eval}}(d_z)$ . However, if using a histogram approach to approximate  $\Omega$  (Young, 2010), the size of the grid  $N$  explodes with the dimension of the idiosyncratic state  $z$ . Let  $d_z = d_{\text{base}} + d_{\text{beliefs}}$ , where  $d_{\text{beliefs}}$  and  $d_{\text{base}}$  are the dimensionality of individual beliefs and all other remaining idiosyncratic states, respectively. Under a tensor-product rule with  $q_\ell$  nodes along dimension  $\ell$ , the total node count is  $N = \prod_{\ell=1}^{d_z} q_\ell$ , so if  $q_\ell \equiv q$  then  $N = q^{d_z}$ . Because  $d_{\text{beliefs}}$  enters  $d_z$  linearly while  $N$  grows exponentially in  $d_{\text{beliefs}}$ , richer belief heterogeneity tightens the aggregation bottleneck inside the root solver by orders of magnitude (the curse of dimensionality).

but introduces an additional step that eliminates redundant summation over idiosyncratic states within each iteration of the root solver. We show that this reorganization allows us to integrate out the idiosyncratic state space once per period, after which the remaining dependence of aggregate demand on prices is captured entirely by basis functions defined over the price vector. In doing so, we remove the computational bottleneck associated with repeatedly aggregating individual decision rules inside the root-finding routine. This approach dramatically reduces execution time while maintaining accuracy of projection-based approaches.

**Method #1 (Naive Global Approximation):** Let  $\Omega$  be approximated with  $N$  points  $\{\bar{z}_i\}_{i \in \mathbb{N}_N}$  with corresponding weights  $\{\omega_i\}_{i \in \mathbb{N}_N}$ . The approximation for the demand schedule  $X^D(P, Z) \in \mathbb{R}^J$  for a given  $P$  and  $Z$  may be expressed as the following weighted sum:

$$\hat{X}^j(P, Z) \equiv \sum_i \omega_i \tilde{x}^j(\bar{z}_i, P, Y), \quad (5)$$

where  $\tilde{x}$  is derived individually for each  $(\bar{z}_i, P, Y)$  tuple.<sup>5</sup> Repeating the above approximation for all  $J$  goods yields  $\hat{X}(P, Z)$ , which can then be used to solve the equilibrium vector of prices  $P^*$  using the market clearing condition in Eq. (1). See Algorithm 1 in the Appendix for a step-by-step implementation of Method 1.

**Method #2 (Fast Basis-Function Approximation):** We now develop an approach based on approximating the policy rules with basis functions (Judd, 1992, 1996) that does not require aggregating individual policy rules at each iteration of the TE solver. Suppose that the individual policy functions are approximated by a set of basis functions according

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<sup>5</sup>Given a fixed set of beliefs, the individual decision problem  $\tilde{x}$  can typically be solved exactly by finding the root of the agent's first-order conditions. In practice, this entails applying a root-finding routine to recover each agent's decisions, conditional on their idiosyncratic state  $z$ , prices  $P$ , and the observed aggregates  $Y$ .

to

$$\hat{x}^j(z, P, Y) = \sum_k \sum_l \sum_m c_{klm}^j \Phi_k^z(z) \Phi_l^P(P) \Phi_m^Y(Y), \quad (6)$$

where  $\Phi_k^z$ ,  $\Phi_l^P$ , and  $\Phi_m^Y$  denote basis functions defined over idiosyncratic states, prices, and aggregate observables, respectively. The coefficients  $c_{klm}^j$  can be computed once prior to simulation and remain fixed.<sup>6</sup>

Substituting the projection expansion into the aggregation operator yields

$$\hat{X}^j(P, Z) = \sum_i \omega_i \sum_l \sum_k \sum_m c_{klm}^j \Phi_k^z(\bar{z}_i) \Phi_l^P(P) \Phi_m^Y(Y). \quad (7)$$

Rearranging summations and collecting terms that do not depend on prices produces

$$\hat{X}^j(P, Z) = \sum_l \Phi_l^P(P) \sum_k \sum_m c_{klm}^j \left( \sum_i \omega_i \Phi_k^z(\bar{z}_i) \right) \Phi_m^Y(Y). \quad (8)$$

Combining terms allows us to express aggregate demand as

$$\hat{X}^j(P, Z) = \sum_l C_l^j(Z) \Phi_l^P(P), \quad (9)$$

where

$$C_l^j(Z) \equiv \sum_k \sum_m c_{klm}^j \bar{\Phi}_k^z(Z) \Phi_m^Y(Y) \quad (10)$$

are coefficients on the price basis functions representing aggregate demand. These coefficients are constructed from the basis functions for the idiosyncratic states averaged over the current distribution:  $\bar{\Phi}_k^z(Z) \equiv \sum_i \omega_i \Phi_k^z(\bar{z}_i)$ . A key feature of Eq. (8) is that all dependence on the

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<sup>6</sup>In principle, one could evaluate Eq. (5) directly using Eq. (6). In practice, however, Eq. (6) requires evaluating the basis functions across all dimensions and is generally slower than directly using a root-solver to find  $\tilde{x}(z, P, Y)$ .

idiosyncratic distribution and on the aggregate observables can be summarized by  $\{C_l^j(Z)\}$ , which can be computed once at the start of each period of the simulation. Only the evaluation of the price basis functions remains inside the root solver.

By collapsing repeated cross-sectional summations into a low-dimensional mapping over price basis functions, this method removes the main aggregation bottleneck inside the TE solver. A full algorithmic description is provided in Algorithm 2 in the Appendix.

## 4 The Model

We employ the boundedly rational HA model of Evans et al. (2023), adapting the notation to the general setting of Section 2. This application tangibly illustrates the challenges involved in solving the general class of TE problems defined in Eq. (1). Table 2 in the Appendix provides a concordance between the model objects and the general notation of Section 2. We use this model to benchmark the execution time and accuracy of our proposed approach against Method 1.

### 4.1 Environment

Time is discrete and infinite. The economy is populated by a continuum of agents. Each agent is endowed with one unit of time per period and derives utility from consumption,  $c$ , and leisure,  $l$ , via the instantaneous utility function  $u(c, l)$ . Agents supply labor and earn a wage comprised of two components: (1) a common aggregate component,  $w_t$ , and (2) an idiosyncratic efficiency component,  $\varepsilon$ , which is i.i.d. across agents and follows a time-invariant Markov process with transition function  $\Pi$ . Agents trade one-period claims to capital with net return  $r_t$ , subject to an exogenous borrowing constraint  $\underline{a}$ . Markets for goods and factors are competitive.

In period  $t$ , an agent's state is defined by assets  $a$ , labor productivity  $\varepsilon$ , and beliefs  $\psi$ , where  $\psi$  represents the coefficients of the forecasting model used to form expectations of the future shadow price of wealth. Let  $\Omega_t$  denote the joint distribution of these characteristics. Agents observe a vector of common aggregates,  $Y_t \in \mathbb{R}^n$ , and condition their shadow price forecasts,  $\hat{\lambda}_t^e$ , on these observables. Given current prices  $(r_t, w_t)$ , agents choose consumption  $\hat{c}_t(a, \varepsilon, \psi)$ , labor supply  $1 - \hat{l}_t(a, \varepsilon, \psi)$ , and savings  $\hat{a}_t(a, \varepsilon, \psi)$  to satisfy:

$$u_c \left( \hat{c}_t(a, \varepsilon, \psi), \hat{l}_t(a, \varepsilon, \psi) \right) \geq \beta \hat{\lambda}_t^e (\hat{a}_t(a, \varepsilon, \psi), \varepsilon, \psi) \text{ and } \hat{a}_t(a, \varepsilon, \psi) \geq \underline{a}, \text{ with c.s.} \quad (11)$$

$$u_l \left( \hat{c}_t(a, \varepsilon, \psi), \hat{l}_t(a, \varepsilon, \psi) \right) = u_c \left( \hat{c}_t(a, \varepsilon, \psi), \hat{l}_t(a, \varepsilon, \psi) \right) w_t \quad (12)$$

$$\hat{a}_t(a, \varepsilon, \psi) = (1 + r_t) a + w_t \cdot \varepsilon \cdot (1 - \hat{l}_t(a, \varepsilon, \psi)) - \hat{c}_t(a, \varepsilon, \psi). \quad (13)$$

Eqs. (11)–(13) represent agents' behavioral primitives: Eq. (11) governs the inter-temporal consumption/savings decision, Eq. (12) determines the intra-temporal labor/leisure choice, and Eq. (13) is the budget constraint.

Following the *local rationality* framework of [Evans et al. \(2023\)](#), agents lack perfect knowledge of price dynamics under aggregate risk. Instead, they use a boundedly rational forecasting mechanism that extrapolates deviations from a known Stationary Recursive Equilibrium (SRE). In the absence of aggregate risk, agents are assumed to know the optimal SRE policy functions  $\bar{c}(a, \varepsilon)$ ,  $\bar{a}(a, \varepsilon)$ , and  $\bar{l}(a, \varepsilon)$ , as well as the stationary prices  $\bar{r}$  and  $\bar{w}$ . The stationary shadow price of wealth is:

$$\bar{\lambda}(a, \varepsilon) = (1 + \bar{r}) u_c(\bar{c}(a, \varepsilon), \bar{l}(a, \varepsilon)). \quad (14)$$

With aggregate risk, agents form expectations relative to the rational forecasts they would make in the stationary environment:

$$\bar{\lambda}^e(a', \varepsilon) = \int \bar{\lambda}(a', \varepsilon') \Pi(\varepsilon, d\varepsilon'). \quad (15)$$

Operationally, agents use a Perceived Law of Motion (PLM) to forecast deviations from this stationary baseline:

$$\log \hat{\lambda}_t = \log \bar{\lambda} + \langle \psi, Y_{t-1} \rangle, \quad (16)$$

where  $\langle \cdot, \cdot \rangle$  is the standard inner product on  $\mathbb{R}^n$ . Thus, an agent with beliefs  $\psi$  forecasts

$$\hat{\lambda}_t^e(a', \varepsilon, \psi) = \bar{\lambda}^e(a', \varepsilon) \cdot \exp(\langle \psi, Y_t \rangle), \quad (17)$$

where  $\bar{\lambda}^e$  is defined in Eq. (15). The term  $\langle \psi, Y_t \rangle$  represents the forecasted log-deviation of the shadow price from its stationary expectation.

Beliefs evolve via recursive least squares learning. An agent with state  $(a, \varepsilon)$  and beliefs  $\psi$  updates their beliefs by regressing the log-deviation of the realized shadow price,

$$\hat{\lambda}_t(a, \varepsilon, \psi) = (1 + r_t) u_c \left( \hat{c}_t(a, \varepsilon, \psi), \hat{l}_t(a, \varepsilon, \psi) \right) \quad (18)$$

from its stationary counterpart  $\bar{\lambda}(a, \varepsilon)$  on lagged observables  $Y_{t-1}$ . The constant-gain updating rule based on recursive least squares is

$$\hat{\psi}_t(a, \varepsilon, \psi) = \psi + g \cdot R_{t+1}^{-1} Y_{t-1} \left( \log \left( \hat{\lambda}_t(a, \varepsilon, \psi) / \bar{\lambda}(a, \varepsilon) \right) - \langle \psi, Y_{t-1} \rangle \right), \quad (19)$$

where  $R$  is an estimate of the second moment of  $Y$  and evolves according to  $R_{t+1} = R_t + g \cdot (Y_{t-1} \otimes Y_{t-1} - R_t)$ . The term  $R_{t+1}^{-1}$  depends only on aggregates and is common across agents. The distribution  $\Omega_t$  evolves consistent with the policy rules, belief updating, and the exogenous process for  $\varepsilon$ .

A representative firm closes the model. The firm rents capital  $k_t$  at rate  $r_t + \delta$  and hires effective labor  $n_t$  at wage  $w_t$  to produce output using a constant returns to scale (CRTS) technology  $\theta_t f(k_t, n_t)$ . Here,  $\delta$  is the depreciation rate and  $\theta_t$  is TFP, which follows  $\theta_{t+1} = \nu_{t+1} \theta_t^\rho$  with  $\nu_{t+1}$  log-normally distributed. Profit maximization implies  $r_t = \theta_t f_k(k_t, n_t) - \delta$

and  $w_t = \theta_t f_n(k_t, n_t)$ . Market clearing requires:

$$k_t = \int a \cdot \Omega_t(da, d\epsilon, d\psi) \quad \text{and} \quad n_t = \int (1 - \hat{l}_t(a, \epsilon, \psi)) \Omega_t(da, d\epsilon, d\psi). \quad (20)$$

Since labor supply  $n_t$  depends on policy rules  $\hat{l}_t$ , which implicitly depend on prices, the system must be solved jointly.

## 4.2 Temporary Equilibrium

The principal computational challenge in this paper is to find, in every period, the vector of factor prices  $(w_t, r_t)$  such that the implied decision rules  $\hat{l}_t$  and the resultant aggregate capital  $k_t$  and labor supply  $n_t$  satisfy the no-arbitrage conditions of the representative firm. This requires evaluating the off-equilibrium behavior of households.

Let  $Z_t = (\Omega_t, Y_t, Y_{t-1})$  represent the aggregate state at date  $t$ .<sup>7</sup> Let  $P = (w, r)$  represent the candidate vector of prices. Individual demand functions are denoted by  $\tilde{x}(z, P, Y) = (\tilde{c}(z, P, Y), \tilde{a}(z, P, Y), \tilde{l}(z, P, Y))$ , and satisfy the following Euler, intratemporal optimality, and budget conditions:

$$u_c(\tilde{c}(z, P, Y), \tilde{l}(z, P, Y)) \geq \beta \tilde{\lambda}^e(\tilde{a}(z, P, Y), \epsilon, \psi, Y) \quad (21)$$

and  $\tilde{a}(z, P, Y) \geq \underline{a}$ , with c.s.

$$u_l(\tilde{c}(z, P, Y), \tilde{l}(z, P, Y)) = u_c(\tilde{c}(z, P, Y), \tilde{l}(z, P, Y))w \quad (22)$$

$$\tilde{a}(z, P, Y) = (1 + r)a + w \cdot \epsilon \cdot (1 - \tilde{l}(z, P, Y)) - \tilde{c}(z, P, Y), \quad (23)$$

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<sup>7</sup>This model-specific definition of the aggregate state  $Z$  slightly extends the general formulation in Section 2 by incorporating lagged realizations of observable aggregates as an additional component used for learning. These lagged observables are required because locally rational agents use them to update their beliefs  $\psi$  at the end of each period, as described in Eq. (19).

where  $\tilde{\lambda}^e$  represents an agent's forecast of  $\lambda'$  based on observation  $Y$ , satisfying

$$\tilde{\lambda}^e(a', \varepsilon, Y) = \bar{\lambda}^e(a', \varepsilon) \exp(\langle \psi, Y \rangle). \quad (24)$$

We define the effective labor supply of the household as

$$\tilde{n}(z, P, Y) = \varepsilon \cdot (1 - \tilde{l}(z, P, Y)). \quad (25)$$

By construction, the off-equilibrium policy rules equal the equilibrium policy rules at the market clearing prices:  $\hat{c}_t(a, \varepsilon, \psi) = \tilde{c}(z, P_t, Y_t)$ , where  $P_t = (w_t, r_t)$ .

Market clearing prices equate the capital and labor supplied by the household to those demanded by the firm. In period  $t$ , capital is supplied inelastically by the household,  $k_t = \int ad\Omega_t(z)$ . Aggregate labor supply is constructed by integrating over the current distribution of individual states

$$n^S(P, Z_t) = \int \tilde{n}(z, P, Y_t) d\Omega_t(z). \quad (26)$$

Constant returns to scale production combined with the inelastic supply of capital implies that the labor demanded by the firm and the interest rate will be functions of the wage:  $n^D(w, Z_t)$  and  $\tilde{r}(w, Z_t)$ . Finding the market clearing wage amounts to setting labor demand equal to labor supply:

$$n^D(w_t, Z_t) = n^S(w_t, \tilde{r}(w_t, Z_t), Z_t). \quad (27)$$

Eq. (27) maps directly into our framework from Section 2. As documented below, our computational approach speeds up the solution of the TE in any given period by several orders of magnitude.

## 5 Performance

We show that our approach yields substantial reductions in execution time while preserving the accuracy of TE solutions. These computational gains arise from collapsing repeated cross-sectional summations into a low-dimensional mapping over price basis functions. The benefits are especially pronounced in multi-period simulations, where the TE solver is invoked repeatedly.

First, we simulate a single period of the example economy, conditioning on a TFP realization  $\theta = 1$  and on an ergodic cross-sectional distribution of individual states  $\hat{\Omega}$ .<sup>8</sup> Following Evans et al. (2023), we define the vector of observables as  $Y_t \equiv (1, \log(k_t/\bar{k}), \log \theta_t)$ , where  $\bar{k}$  denotes the steady-state capital stock. Under Eq. (14), this specification implies that each agent forecasts the log-deviation of their shadow price from its stationary value via a linear projection on a constant, the log deviation of aggregate capital from its steady state, and the log of aggregate productivity. Each agent's belief vector contains the same number of parameters as the number of regression coefficients in their forecast model. Accordingly, the belief vector is  $\psi \in \mathbb{R}^3$ , and the idiosyncratic state is  $z = (a, \varepsilon, \psi) \in \mathbb{R}^5$ .

In our application of Method 1, for a given wage level  $w$  and distribution  $\hat{\Omega}$  of individual states  $(a, \varepsilon, \psi)$ , we analytically solve the level of capital  $k$  and the return on capital  $r$ . Using the above objects, we then solve the quantity of labor supplied individually by each agent,  $1 - l_i$ , according to conditions (21)–(23). Averaging these individual labor supply decisions across the set of all agents, as in Eq. (5), yields the aggregate labor supply  $n_t$ , as defined in Eq. (26).

We compare the performance of the described TE solution methods across three dimensions: (1) the execution time of the aggregation procedure – in other words, the time it

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<sup>8</sup>In principle, runtime can depend on  $\theta$  and  $\Omega$ ; we found this not to be the case in our application.

takes to find the sum of individual policy rules given all necessary inputs; (2) the execution time of the multidimensional mapping characterizing the TE, which needs to be computed at each step of the nonlinear solver; (3) the execution time of the nonlinear solver that approximates the set of equilibrium prices. We generate samples of these execution times and present the corresponding summary statistics in Table 1. We find that our proposed method (Method 2) offers significant speed-ups across all stages of the TE solution procedure.

	Method #1	Method #2
<b>Aggregation</b>	3.30e-2 (2.40e-2)	4.00e-6 (3.20e-6)
<b>TE Mapping</b>	6.11e-2 (5.01e-2)	3.99e-6 (3.44e-6)
<b>Nonlinear Solver</b>	6.82e-1 (6.57e-1)	3.36e-4 (3.22e-4)
Sample Size	1000	

Table 1: TE solution method execution times. *Note:* Mean execution time in seconds (minimum execution time in parentheses).

In addition to execution speed, we assess the numerical accuracy of our fast aggregation method by comparing its implied equilibrium outcomes to those obtained under the benchmark solution. Figure 1 displays the percentage deviations of the equilibrium return on capital, wage, and aggregate labor supply computed using Method 2 relative to Method 1 over a 1,000-period simulation. These deviations represent approximation errors arising from the use of basis functions to approximate individual policy rules. While we observe a slight bias, the errors are sufficiently small that they do not materially affect the equilibrium solution. Specifically, mean absolute errors remain below 0.01% across all variables, while the maximum error is approximately 0.025%. This demonstrates that our method preserves the accuracy of the underlying economic relationships while achieving orders-of-magnitude reductions in computation time. The efficiency gains documented in Table 1 therefore come

at a minimal cost to precision.

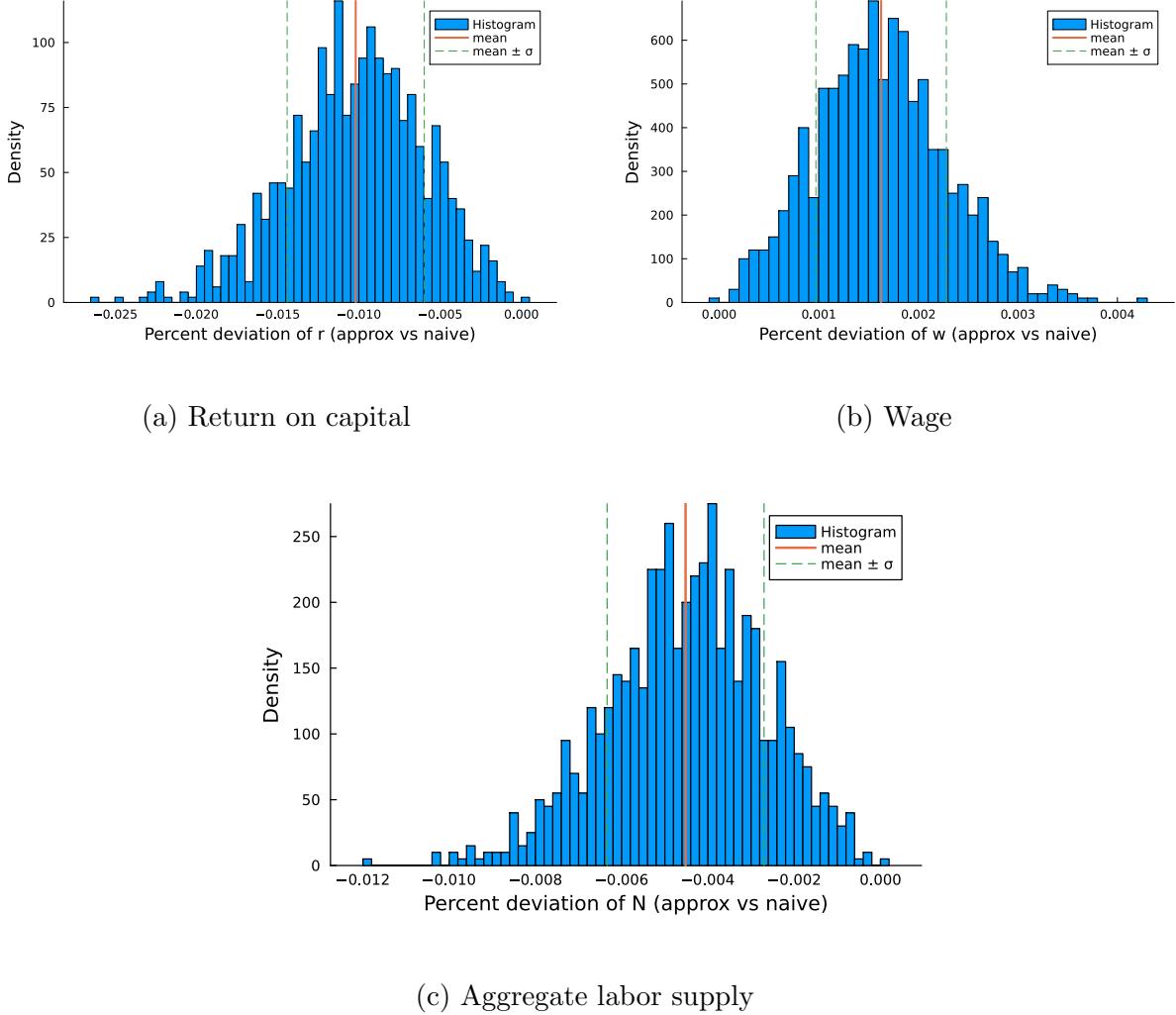


Figure 1: Distributions of the percentage deviation of equilibrium outcomes approximated using Method 2 relative to Method 1 in a 1,000-period simulation.

## 6 Conclusion

We address a challenge in simulating heterogeneous-agent temporary equilibria with boundedly rational beliefs: repeated aggregation of individual policy rules nested inside the nonlinear price solver. After laying out the standard formulation of the TE problem and its computational scaling, we show that it is possible to avoid aggregating over the idiosyncratic

state space at each instance of the solver in a given period by pre-computing a portion of the interpolated policy rules. In our application, this reformulation delivers large speed gains for aggregation, the TE mapping, and the overall solve, while preserving accuracy relative to a more computationally-intensive conventional benchmark. Because our proposed method replaces the within-solver aggregation with a compact representation of aggregate demand, it may also be applied to speed up computation in HA models with multiple markets, such as those with endogenous labor supply ([Chang and Kim, 2006, 2007](#); [Krusell et al., 2010](#)) and multiple assets ([Krusell and Smith, 1997](#)).

Our methodology is a practical building block for the program to move beyond rational expectations in HA macroeconomics. [Moll \(2025\)](#) calls for alternative approaches in HA modeling that (i) are computationally tractable, (ii) discipline beliefs with empirical evidence, and (iii) render beliefs sufficiently endogenous to be credible under policy counterfactuals. As part of this agenda, he highlights TE frameworks and adaptive learning as promising directions. By collapsing the repeated cross-sectional summations inside the price solver into a low-dimensional price-basis representation, our method directly delivers criterion (i); allowing researchers to incorporate richer forecasting rules and learning dynamics without facing an aggregation bottleneck.

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# Appendix

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**Algorithm 1:** Temporary Equilibrium via Agent Simulation (Method 1)

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**Inputs:** Observable aggregates  $Y$ ; simulated agents  $\mathcal{I} = \{1, \dots, N\}$  with realized idiosyncratic states  $\{z_i\}_{i \in \mathcal{I}}$ ; individual policy map  $\tilde{x}(z, P, Y)$ ; aggregate supply mapping  $X^S(P, Z)$ ; solver settings  $(P_0, \text{tol}, \text{maxit})$ .

**Outputs:** Equilibrium prices  $P^*$ ; approximated aggregate demand  $\hat{X}(P^*, Z)$ .

- 1 **Define agent-based aggregation**  $\text{Agg}_1(P | Z) :=$
  - 2      $\hat{X}(P, Z) \leftarrow \frac{1}{N} \sum_{i=1}^N \tilde{x}(z_i, P, Y)$     (componentwise in  $j = 1, \dots, J$ );
  - 3 **Residual:**  $F(P; Z) \leftarrow X^S(P, Z) - \text{Agg}_1(P | Z)$ ;
  - 4 **Solve TE:** Apply a root solver to  $F(P; Z) = 0$  from  $P_0$  until  $\|F\| < \text{tol}$  (at most **maxit** steps);.
- Result:**  $P^*$  and  $\hat{X}(P^*, Z)$ .
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**Algorithm 2:** Temporary Equilibrium with Fast Aggregation (Method 2)

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**Inputs:** Observable aggregates  $Y$ ; simulated agents  $\mathcal{I} = \{1, \dots, N\}$  with realized states

$\{z_i\}$ ; basis families  $\{\Phi_k^z\}_{k=1}^K$ ,  $\{\Phi_\ell^P\}_{\ell=1}^L$ ,  $\{\Phi_m^Y\}_{m=1}^{M_Y}$ ; projection coefficients  $c_{k\ell m}^j$  for  $\hat{x}^j(z, P, Y)$  defined in Eq. (6); aggregate supply mapping  $X^S(P, Z)$ ; solver settings  $(P_0, \text{tol}, \text{maxit})$ .

**Outputs:**  $P^*$ ;  $\hat{X}(P^*, Z)$ .

**1 One-time pre-aggregation (per period):;**

**2** Compute and store  $\{C_\ell^j(Z)\}_{j=1, \dots, J; \ell=1, \dots, L}$  via

$$C_\ell^j(Z) = \frac{1}{N} \sum_{i=1}^N \sum_{k=1}^K \sum_{m=1}^{M_Y} c_{k\ell m}^j \Phi_k^z(z_i) \Phi_m^Y(Y).$$

**3 Define fast aggregation  $\text{Agg}_2(P | Z)$ :**

**4** For  $j = 1, \dots, J$  set

$$\hat{X}_j(P, Z) \leftarrow \sum_{\ell=1}^L C_\ell^j(Z) \Phi_\ell^P(P).$$

Return  $\hat{X}(P, Z) \equiv (\hat{X}_1, \dots, \hat{X}_J)$ .

**5 Residual:**  $F(P; Z) \leftarrow X^S(P, Z) - \text{Agg}_2(P | Z)$ .

**6 Solve TE:** Apply a root solver to  $F(P; Z) = 0$  from  $P_0$  until  $\|F\| < \text{tol}$  (at most  $\text{maxit}$  steps).

**Result:**  $P^*$  and  $\hat{X}(P^*, Z)$ .

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Object	General Notation	Example Environment	Notes
Individual state vector	$z$	$(a, \varepsilon, \psi)$	Current asset/claims $a$ , idiosyncratic efficiency $\varepsilon$ , belief coefficients $\psi \in \mathbb{R}^n$ used to forecast next-period shadow price.
Distribution of individual states	$\Omega$	Distribution over $(a, \varepsilon, \psi)$	Used to aggregate to capital $k$ and effective labor $n$ .
Price vector	$P$	$(r, w)$	Net return on one-period claims $r$ and real wage per efficiency unit $w$ .
Observable aggregate state variables	$Y$	$(1, \log(k/\bar{k}), \log \theta)$	In our application, $Y$ includes a constant, the log deviation of aggregate capital from its steady state, and the log of aggregate productivity
21 Aggregate state	$Z = (\Omega, Y)$	$Z = (\Omega, Y, Y_-)$	Collects the cross-sectional distribution $\Omega$ over $(a, \varepsilon, \psi)$ and both the current and lagged observable aggregates $(Y, Y_-)$ .
Law of motion for aggregate state	$Z' = G(Z, \nu')$	$\theta' = \nu' \theta^\rho$	$Z'$ updates via the aggregate TFP shock $\theta$ and the induced evolution of $\Omega$ .
Individual decision (demand) vector	$\tilde{x}(z, P, Y)$	$(\tilde{c}, \tilde{a}, \tilde{l})$	Consumption, next-period claims, and leisure; determined by Euler (with borrowing constraint), intratemporal labor-leisure condition, and budget.
Aggregate demand	$X^D(P, Z) = \int \tilde{x}(z, P, Y) d\Omega(z)$	$k = \int a \Omega(dz), n = \int (1 - \tilde{l}(z, P, Y)) \Omega(dz)$	Specializes to the aggregates needed for factor markets.
Aggregate supply	$X^S(P, Z)$	$w = \theta f_n(k, n), r + \delta = \theta f_k(k, n)$	Firm-side requirements implied by FOCs under perfect competition (i.e., factor demands at prices $P$ ).
Goods/markets index	$j = 1, \dots, J$	$J = 2$ (capital, effective labor)	Two factor markets solved for $(r, w)$ .

Table 2: Mapping from general objects in Section 2 to their counterparts in the example environment.