Homework 4
second-cycle degree in Physics (5 June 2020)

4.13. Consider the diagrams in Fig. 12.3 for an arbitrary potential $V(\mathbf{q})$, and show that only $\Pi^{\star}_{(1)b}$ contributes to E_2 in Eq. (12.21). Use Eq. (12.21) to evaluate E_2^b, and show that it agrees with that in Prob. 1.4.

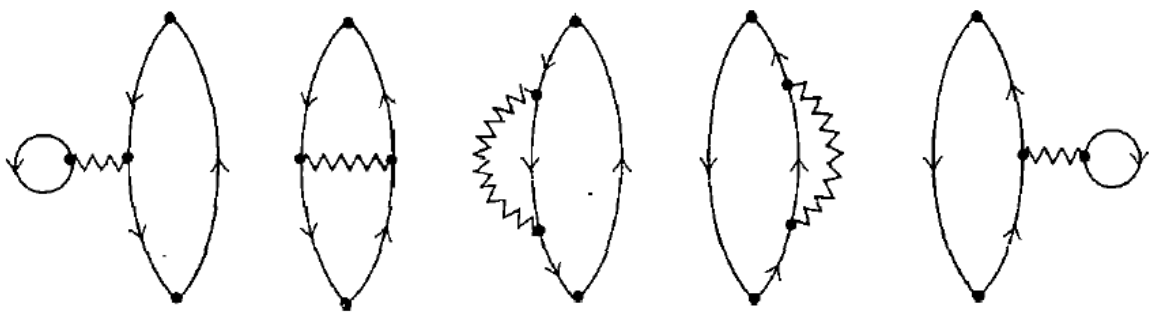

Fig. 12.3 All first-order contributions to proper polarization.

and e) vanish in the present example [$V(0) = 0$], and we are left with the middle three. Correspondingly, the correlation energy has the expansion

$$E_{\text{corr}} = E_2^r + E_2^b + E_2^c + E_2^d + \cdots \tag{12.19}$$

where the various second-order contributions are given by

$$E_2^r = \tfrac{1}{2} iV\hbar(2\pi)^{-4} \int_0^1 d\lambda\, \lambda^{-1} \int d^4q\, [\lambda U_0(q)\, \Pi^0(q)]^2 \tag{12.20}$$

$$E_2^{b,c,d} = \tfrac{1}{2} iV\hbar(2\pi)^{-4} \int_0^1 d\lambda\, \lambda^{-1} \int d^4q\, [\lambda U_0(q)\, \Pi^{\star}_{(1)b,c,d}(q)] \tag{12.21}$$

1.4.‡ Show that the second-order contribution to the ground-state energy of an electron gas is given by $E^{(2)} = (Ne^2/2a_0)(\epsilon_2^r + \epsilon_2^b)$, where

$$\epsilon_2^r = -\frac{3}{8\pi^5} \int \frac{d^3q}{q^4} \int_{|\mathbf{k}+\mathbf{q}|>1} d^3k \int_{|\mathbf{p}+\mathbf{q}|>1} d^3p\, \frac{\theta(1-k)\,\theta(1-p)}{\mathbf{q}^2 + \mathbf{q}\cdot(\mathbf{k}+\mathbf{p})}$$

$$\epsilon_2^b = \frac{3}{16\pi^5} \int \frac{d^3q}{q^2} \int_{|\mathbf{k}+\mathbf{q}|>1} d^3k \int_{|\mathbf{p}+\mathbf{q}|>1} d^3p\, \frac{\theta(1-k)\,\theta(1-p)}{(\mathbf{q}+\mathbf{k}+\mathbf{p})^2\,[\mathbf{q}^2 + \mathbf{q}\cdot(\mathbf{k}+\mathbf{p})]}$$

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7.1. Define the two-particle temperature Green's function by

$$\mathcal{G}_{\alpha\beta;\gamma\delta}(\mathbf{x}_1\tau_1, \mathbf{x}_2\tau_2; \mathbf{x}_1'\tau_1', \mathbf{x}_2'\tau_2') = \mathrm{Tr}\{\hat{\rho}_G T_\tau[\hat{\psi}_{K\alpha}(\mathbf{x}_1\tau_1)\,\hat{\psi}_{K\beta}(\mathbf{x}_2\tau_2)\,\hat{\psi}^\dagger_{K\delta}(\mathbf{x}_2'\tau_2')\,\hat{\psi}^\dagger_{K\gamma}(\mathbf{x}_1'\tau_1')]\}$$

Prove that the ensemble average of the two-body interaction energy is

$$\langle\hat{V}\rangle = \tfrac{1}{2}\int d^3x \int d^3x'\, V(\mathbf{x},\mathbf{x}')_{\mu'\lambda',\mu\lambda}\,\mathcal{G}_{\lambda\lambda';\mu\mu'}(\mathbf{x}'\tau, \mathbf{x}\tau; \mathbf{x}'\tau^+, \mathbf{x}\tau^+)$$

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deadline : 15 June 2020

N.B. deliver the **solution** by sending (psil@pd.infn.it) a **file** denoted as **SURNAME-Name_4.pdf** (PDF format only !) , ex. : SMITH-John_4.pdf .