Homework 1
second-cycle degree in Physics (27 March 2020)

- Show, using the formalism of the *second quantization*, that, for **both** *bosons* and *fermions* the *total-number* operator and the *Hamiltonian* operator commute: **[N,H]=0** .

- **Degenerate electron gas in 1D :**

Repeat the exercise on the 3D degenerate electron gas in **1D** (replace 3D integrals with 1D integrals !); this could be a *model* for electrons in *carbon nanotubes* (graphite sheets rolled up into cylinders); these long and thin carbon molecules have *extraordinary* material characteristics (they are believed to be the *strongest material* in the world). Depending on the specific way the cylinder is rolled up the nanotubes are either *metallic*, *semiconducting* or *insulating*; a *metallic* nanotube is a nearly *ideal* 1D *metal* wire.

In particular:

(a) **evaluate** the $\mathbf{E_0}$ (*kinetic-energy*) contribution;

(b) **show** that the $\mathbf{E_1}$ (*first-order potential-energy*) **diverges** !

Hint :

$$\int \frac{e^{iqx}}{x} dx = -2[\gamma + \ln(q)]$$

so $\mathbf{4\pi/q^2}$ (in 3D) must be replaced by $\mathbf{-2[\gamma + \ln(q)]}$ in **1D**, ($\gamma = 0.577216\ldots$, Euler constant).

deadline : 7 April 2020

N.B. deliver the **solution** by sending (psil@pd.infn.it) a **file** denoted as **SURNAME-Name_1.pdf** (PDF format only !) , ex. : SMITH-John_1.pdf .