MASTER'S DEGREE IN PHYSICS
Academic Year 2020-2021

STRONGLY CORRELATED SYSTEMS

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STUFF TO REMEMBER

In this file I will collect some elements from the class by prof. dell'Anna.

Path Integrals

Schroedinger Equation is:

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = H(\vec{\mathbf{p}},\vec{\mathbf{x}})|\psi\rangle \tag{1}$$

with solution

$$|\psi(t')\rangle = e^{\frac{i}{\hbar}}H(\vec{\mathbf{x}},\vec{\mathbf{p}})(t'-t)|\psi(t)\rangle \tag{2}$$

Coordinate representation:

$$\begin{aligned}\langle x'|\psi(t')\rangle &= \langle x'|U(t',t)|\psi(t)\rangle && (3)\\ &= \int \mathrm{d}\vec{\mathbf{x}}\ \langle x'|U(t',t)|x\rangle\langle x|\psi(t)\rangle && (4)\\ &= \int \mathrm{d}\vec{\mathbf{x}}\, U(x',t',x,t)\psi(x,t) && (5)\end{aligned}$$

The generic matrix element of U is:

$$U(\vec{\mathbf{x}}_f,t_f,\vec{\mathbf{x}}_i,t_i) = \langle\vec{\mathbf{x}}_f|U(t_f,t_i)|\vec{\mathbf{x}}_i\rangle = \langle\vec{\mathbf{x}}_f|e^{\frac{-i}{\hbar}H(\vec{\mathbf{p}},\vec{\mathbf{x}})(t'-t)}|\vec{\mathbf{x}}_i\rangle \tag{6}$$

Splitting the time interval into M small pieces and sending $m\to\infty$ one is able (after some long calculations) to write U as:

$$U(\vec{\mathbf{x}}_f,t_f,\vec{\mathbf{x}}_i,t_i) = \int \mathcal{D}[\vec{\mathbf{x}}(t)]\mathcal{D}[p(t)]e^{\frac{i}{\hbar}\int_{t_i}^{t_f}\mathrm{d}t\left[p(t)\dot{\vec{\mathbf{x}}}(t)-H(\vec{\mathbf{x}},\vec{\mathbf{p}})\right]} \tag{7}$$

or in the equivalent form

$$U(\vec{\mathbf{x}}_f,t_f,\vec{\mathbf{x}}_i,t_i) = \int \mathcal{D}[\vec{\mathbf{x}}(t)]e^{\frac{i}{\hbar}\int_{t_i}^{t_f}\mathrm{d}t\left[\frac{m}{2}\left(\frac{\mathrm{d}\vec{\mathbf{x}}}{\mathrm{d}t}\right)^2-V(\vec{\mathbf{x}}(t))\right]} \tag{8}$$

the analogous for **statistical mechanics** is derived from the previous using a **Wick's rotation** $t=-i\tau$:

$$U(\vec{\mathbf{x}}_f,t_f,\vec{\mathbf{x}}_i,t_i) = \int \mathcal{D}[\vec{\mathbf{x}}(\tau)]e^{-\frac{1}{\hbar}\int_{\tau_i}^{\tau_f}\mathrm{d}\tau\left[\frac{m}{2}\left(\frac{\mathrm{d}\vec{\mathbf{x}}}{\mathrm{d}\tau}\right)^2-V(\vec{\mathbf{x}}(\tau))\right]} \tag{9}$$

Partition function in statistical mechanics is defined as $\mathrm{Tr}\big(e^{-\beta H}\big)$, which is

$$\begin{aligned}Z = \mathrm{Tr}\big(e^{-\beta H}\big) &= \int \mathrm{d}\vec{\mathbf{x}}\ \langle x|e^{-\beta H}|x\rangle && (10)\\ &= \int \mathrm{d}\vec{\mathbf{x}} \int_{\vec{\mathbf{x}}(0)=\vec{\mathbf{x}}(\beta\hbar)} \mathcal{D}[\vec{\mathbf{x}}(\tau)]e^{-\frac{1}{\hbar}\int_0^{\beta\hbar}\mathrm{d}\tau\left[\frac{m}{2}\left(\frac{\mathrm{d}\vec{\mathbf{x}}}{\mathrm{d}\tau}\right)^2-V(\vec{\mathbf{x}}(\tau))\right]} && (11)\end{aligned}$$