Formulae for Digital Signal Processing

Hogeschool van Arnhem en Nijmegen

ir drs E.J Boks

Convolutie in het tijddomein

An LTI system that is characterized by the impulse response h [n], will convert a signal x [n] to a result y [n] according to:

$$y[n]=\sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

Fourier Transform

Ω is the sampled frequency which runs from 0 to 2π. Ω=π corresponds with $f=\frac{f_{sample}}{2}$

Description	***Signal in time domain***	***Signal in frequency domain***
Transformation	x[n]	$\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$
Linearity	$a*x_1[n] + b*x_2[n]$	$a*X_1(\Omega) + b*X_2(\Omega)$
Time shift	x[n-a]*u[n-a]	$X(\Omega)e^{-ja\Omega}$
Modulation	$x_1[n]x_2[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\lambda)X_2(\Omega-\lambda)d$
Convolution	$x_1[n]*x_2[n]$	$X_1(\Omega)X_2(\Omega)$
Time difference	x[n] –x[n-1]	$X(\Omega)(1-e^{-j\Omega})$

Description	***Signal in time domain x[n]***	***Signal in frequency domain X(Ω)***
Impulse	δ[n]	1
Delayed impulse	$\delta[n-n_0]$	$e^{-j\Omega n_0}$
Step	u[n]	$\frac{1}{1-e^{-j\Omega}}$
Rectangular pulse in the time domain	$u[n+n_0] - u[n-n_0+1]$	$\frac{\sin((n_0+\frac{1}{2})\Omega)}{\sin(\frac{1}{2}\Omega)}$
Rectangular pulse in the frequency domain	$\frac{\sin(n\Omega_0)}{n\pi}=\frac{\Omega_0}{\pi}sinc(n\Omega_0)$	$u[\Omega+\Omega_0] - u[\Omega-\Omega_0]$
Power	$a^n u[n]$	$\frac{1}{1-ae^{-j\Omega}}$

Z Transform

Description	***Signal in time domain***	***Signal in Z domain***
Transformation	x[n]	$\sum_{n=0}^{\infty} x[n] z^{-n}$
Linearity	$a*x_1[n] + b*x_2[n]$	$a*X_1(z) + b*X_2(z)$
Time shift	x[n-a]*u[n-a]	$X(z)z^{-a}$
Time integration	$\sum_{k=0}^{N} x[k]$	$X(z)\frac{z}{z-1}$
Time difference	x[n] –x[n-1]	$X(z)(1-z^{-1})$
Convolution	$x_1[n]*x_2[n]$	$X_1(z)X_2(z)$
Final value theorem	$\lim_{n\to\infty} x[n]$	$\lim_{z\to 1} (\frac{z-1}{z})X(z)$

Description	***Signal in time domain***	***Signal in Z domain***
Impulse	δ[n]	1
Step	u[n]	$\frac{z}{z-1}$ of $\frac{1}{1-z^{-1}}$
Slope	r[n]	$\frac{z}{(z-1)^2}$
Power	$a^n u[n]$	$\frac{z}{z-a}$

Transformatie van s- naar z-domein

A transfer function H (s), which exists after performing the Laplace transformation on a linear differential equation, can be converted to the z-domain by means of the bilinear z-transform. This BZT transformation is carried out according to:

$$s \equiv \frac{2}{T_s}\frac{z-1}{z+1}$$

Where T_S represents the sample time.

Leonhard Euler's rule:

$$e^{jx} = \cos x + j\ \sin x$$

$$\cos(x) = \frac{e^{jx}+e^{-jx}}{2} \quad \text{En} \quad \sin(x) = \frac{e^{jx}-e^{-jx}}{2j}$$

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