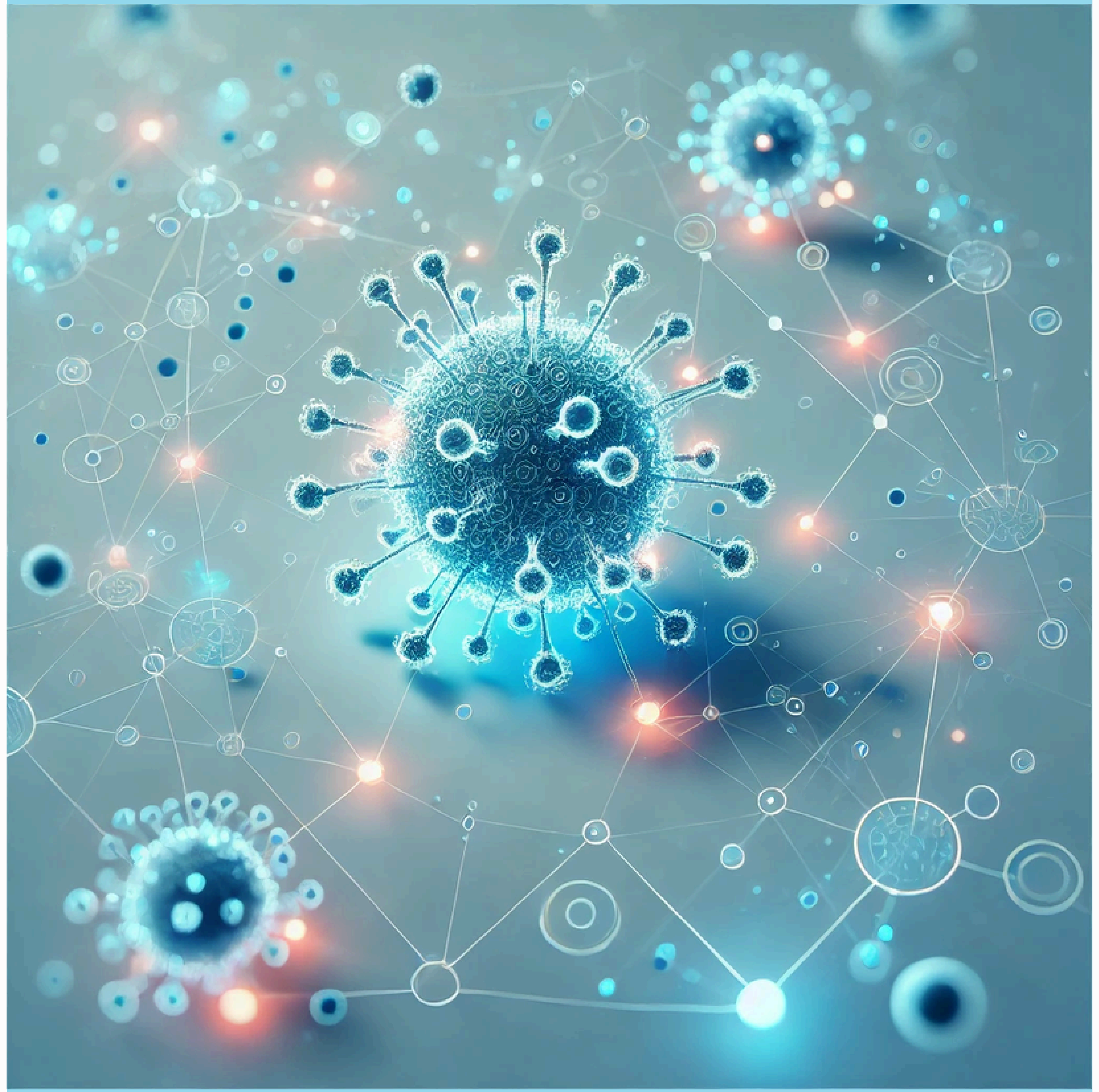

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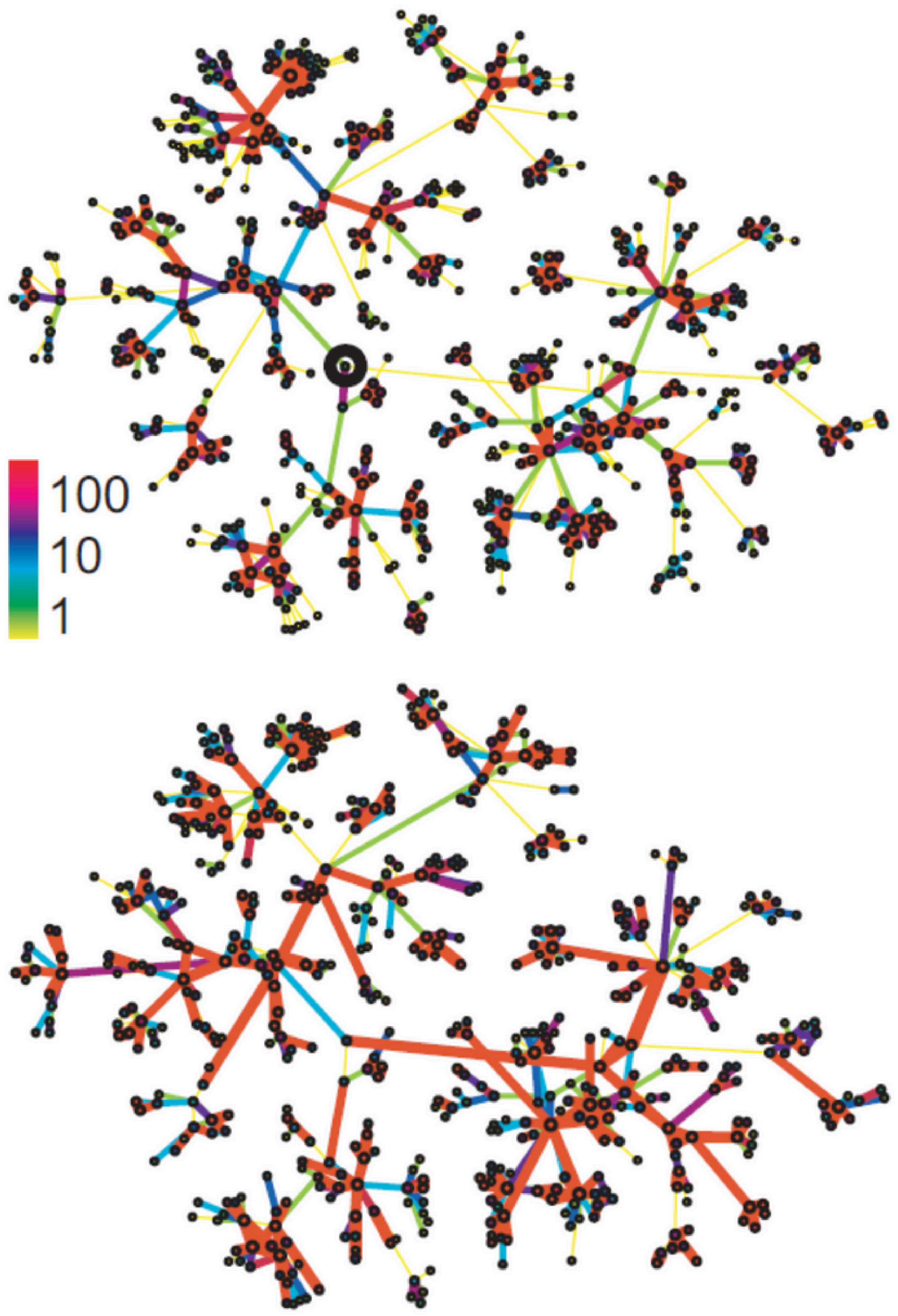
Spreading Processes on Networks

Network Science of Socio-Economic Systems

Giordano De Marzo



Recap

Communities in Networks

We introduced the concept of communities, community detection and modularity.

Community Detection Algorithms

We introduced some of the most known community detection algorithms, pointing out their limitations and strengths.

Homophily and Communities Formation

Homophily plays a central role in the formation of communities.

The Strength of Weak Ties

Weak ties connect communities in social networks acting as bridges

Outline

1. Processes on Complex Networks
2. Epidemic Spreading
3. Epidemic Spreading on Networks
4. Complex Contagion

Processes on Complex Networks

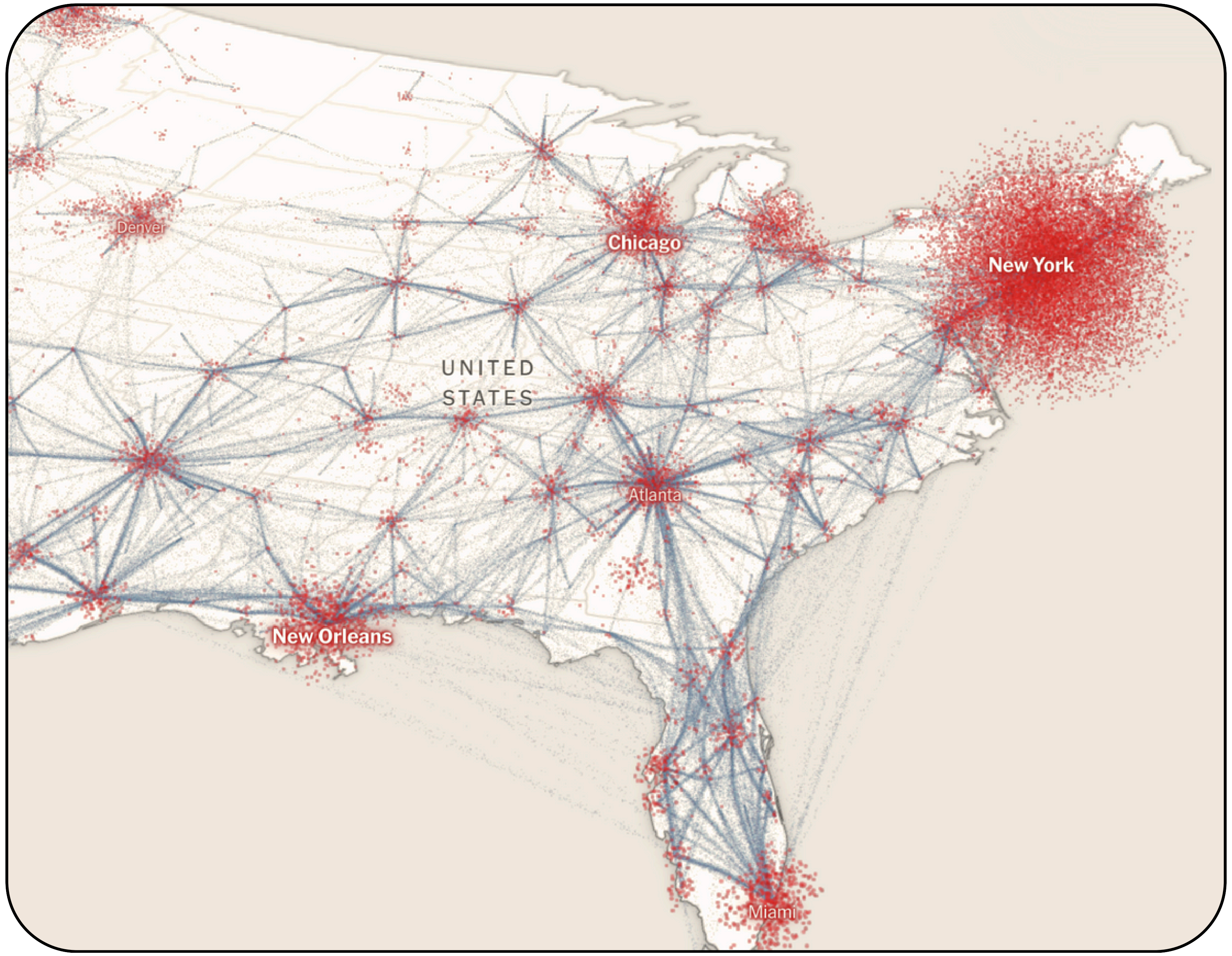
Processes on Networks

Many different processes take place on a network

- traffic on road networks
- content diffusing on online platforms
- opinion and behaviors spreading on social networks

For many of these processes the network structure plays a very important role, strongly influencing their outcomes

Spreading Processes

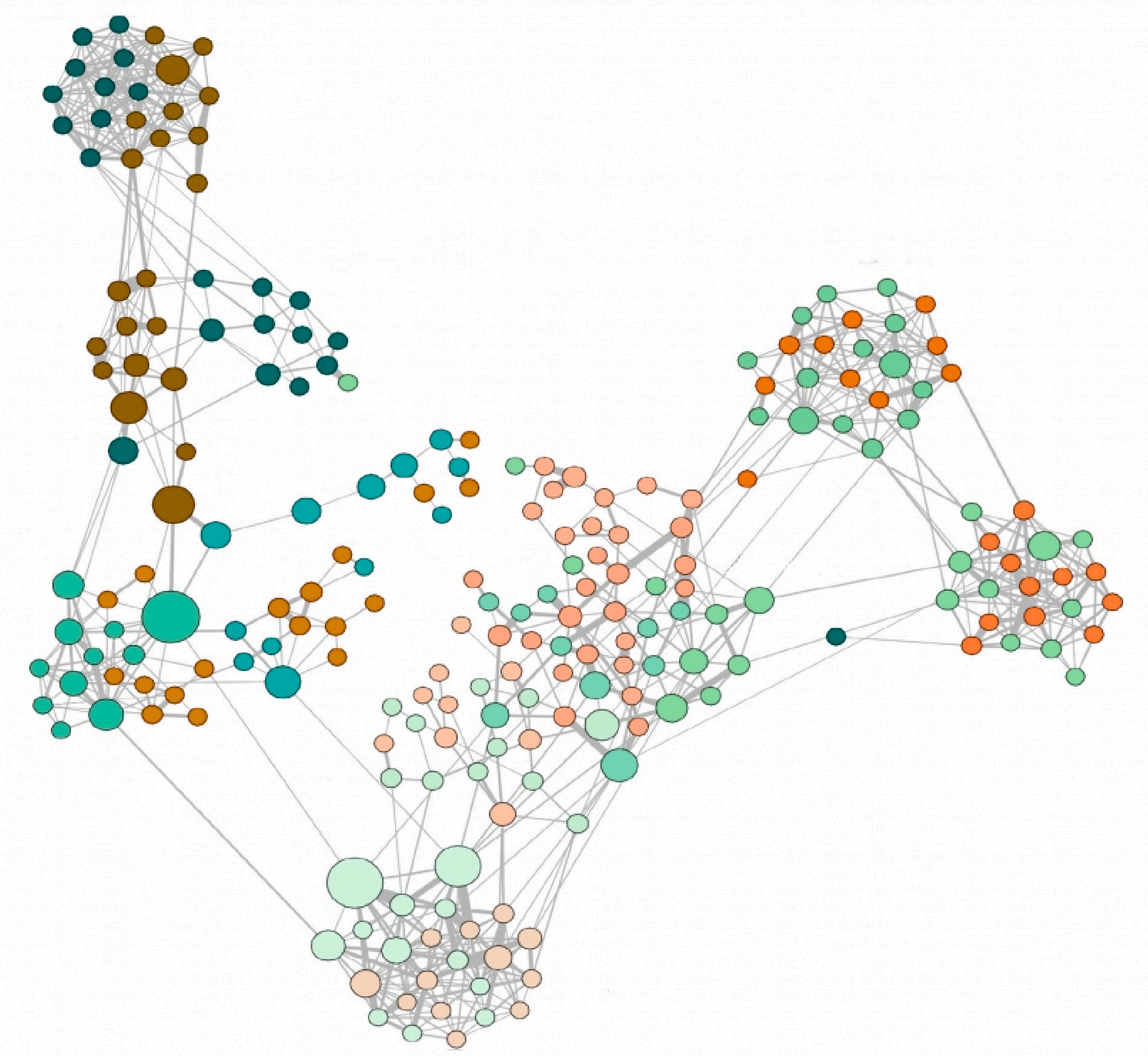


Many processes involve the spreading or diffusion of something

- how a virus can diffuse at the local or global level
- how information is propagated on online social networks
- how behaviors are adopted by populations

Roughly speaking, each node on the network can be “infected” and infect its neighbors, thus spreading the virus or behavior

Face to Face Interactions



At the local level, epidemic spreading take places on face to face networks

- students in a school
- employees on their workplace

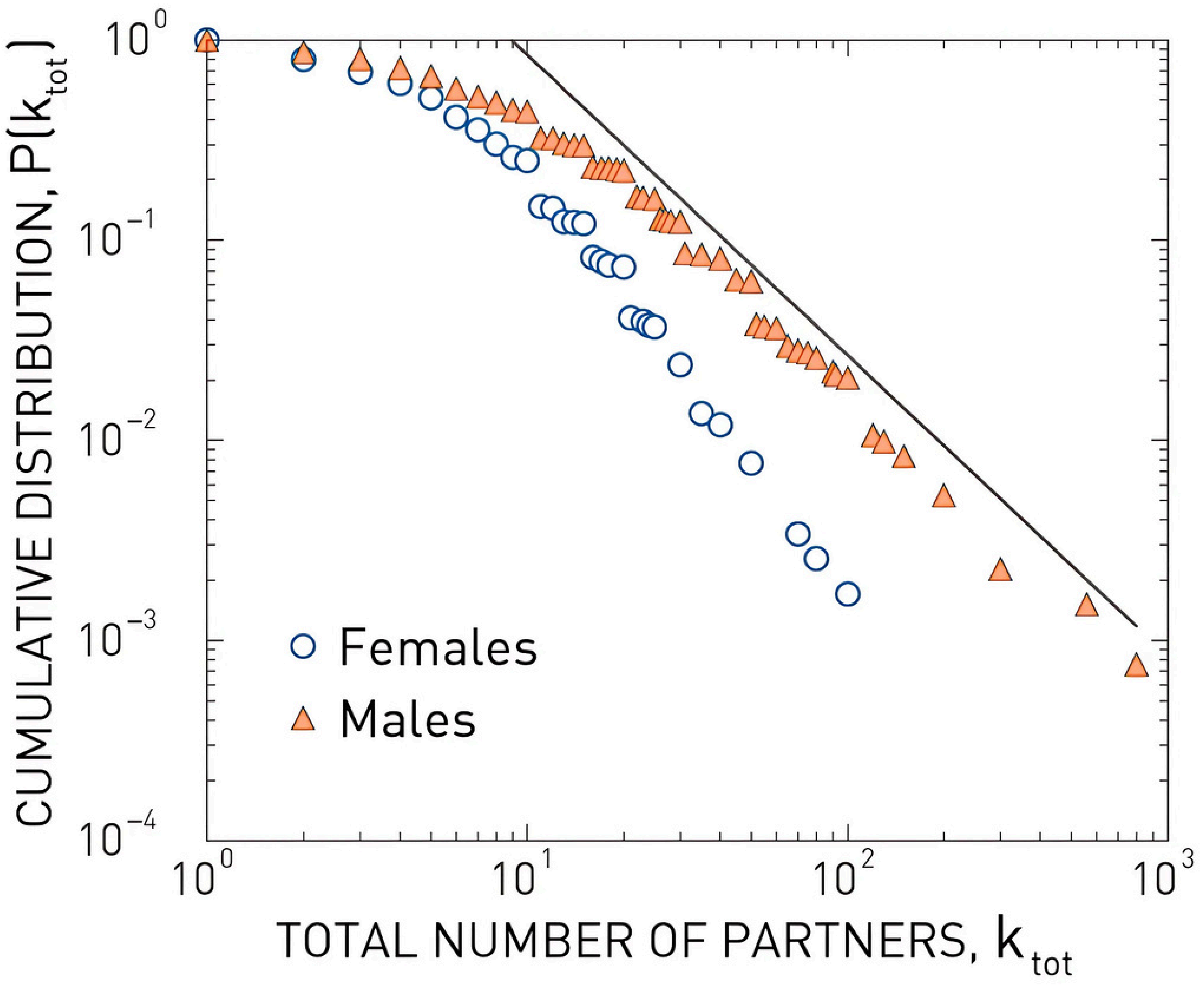
These networks are typically reconstructed by tracking the interaction of individuals with cameras or sensors

- they generally present a block structure
- often there are nodes with many connections

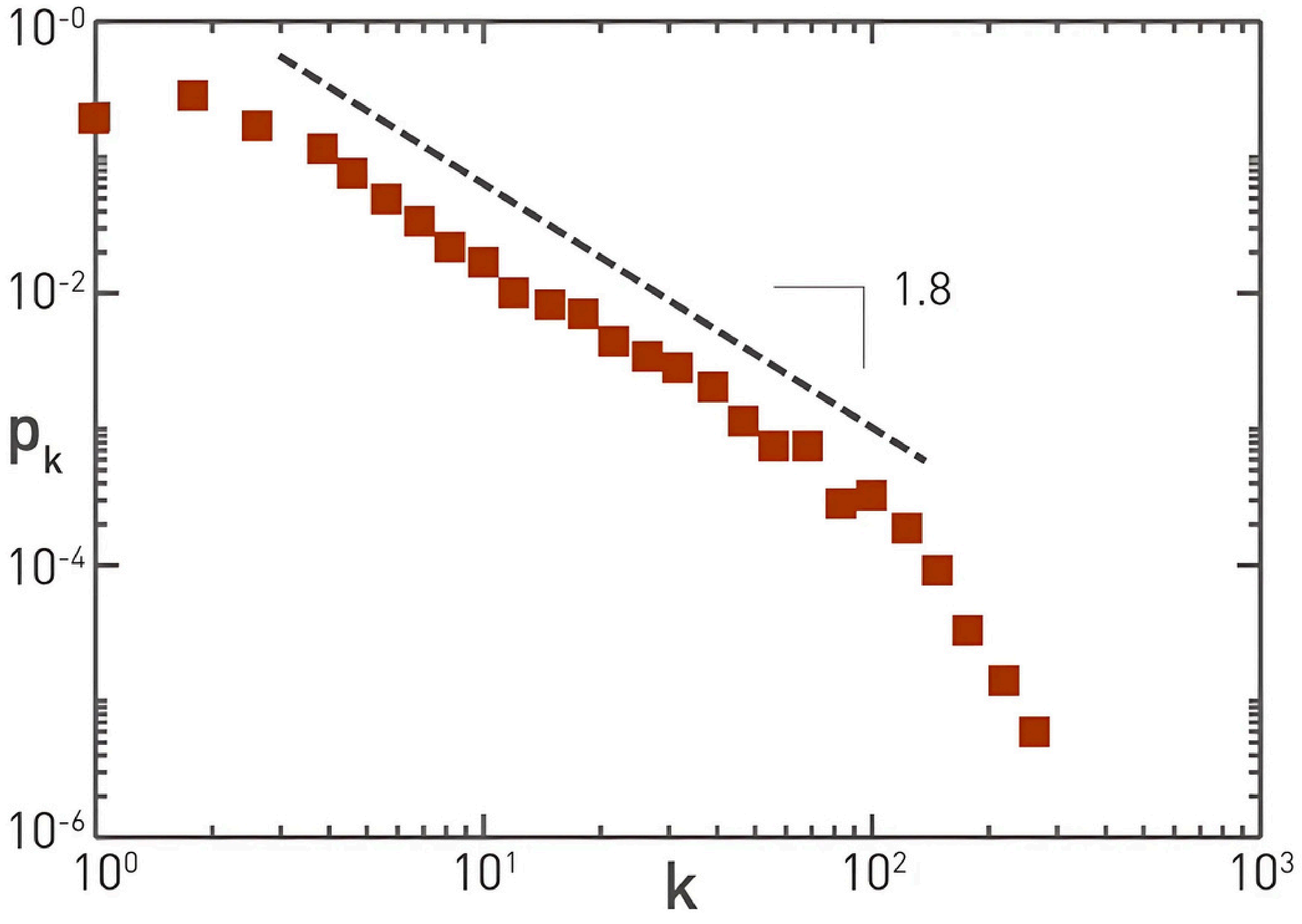
The Web of Sex

Face to face networks work well for air-transmitted illnesses
- many infections are transmitted by blood or other fluids
- an example are sexually transmitted infections

In this case we have to analyze dating or sex networks
- also in this case we observe a scale free structures
- there are hubs with the potential of infecting many sexual partners



Air Transportation Network



At the global level, viruses and bacteria may travel following the air transportation network routes

- this networks is characterized by the presence of hubs
- the structure is scale free
- the diameter is very small

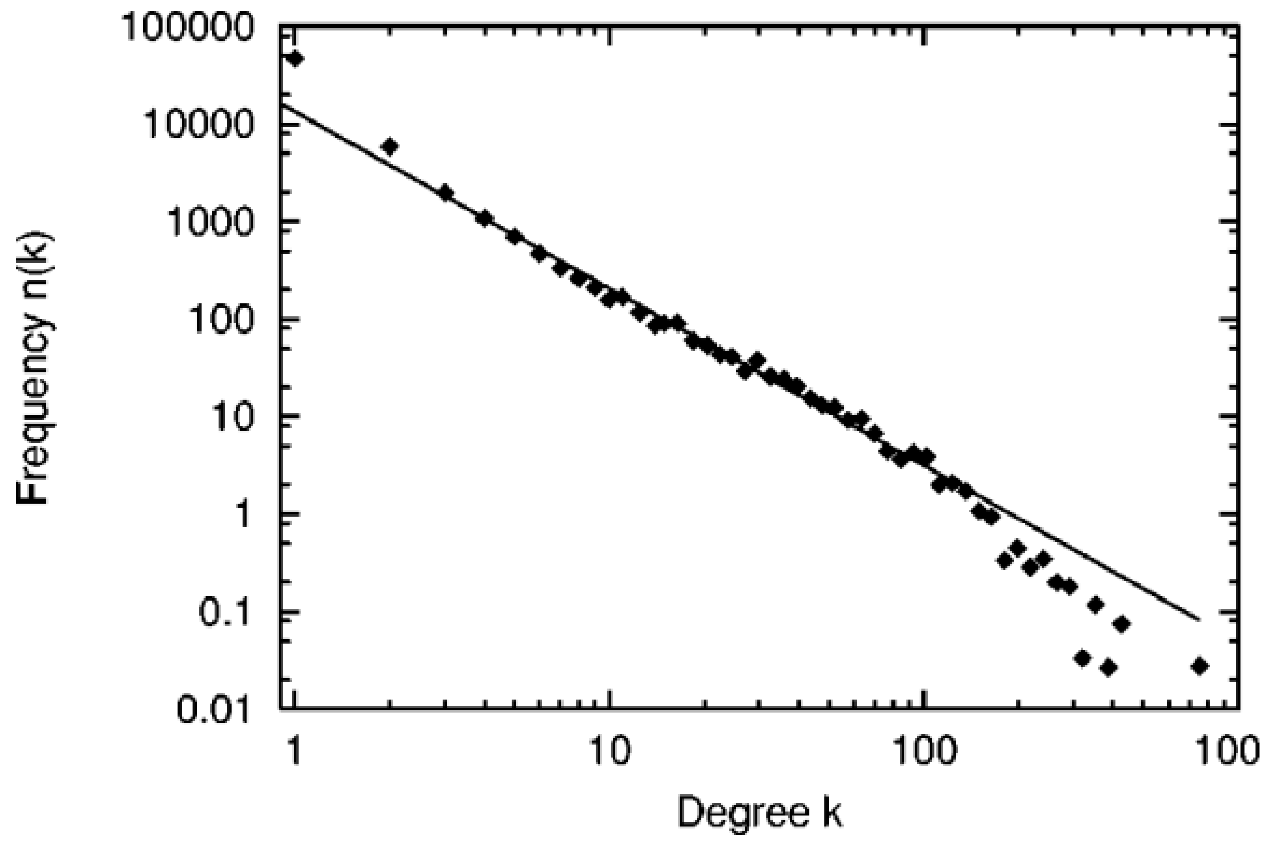
We thus expect a virus can very easily diffuse at the global level using the hubs and the presence of bridges

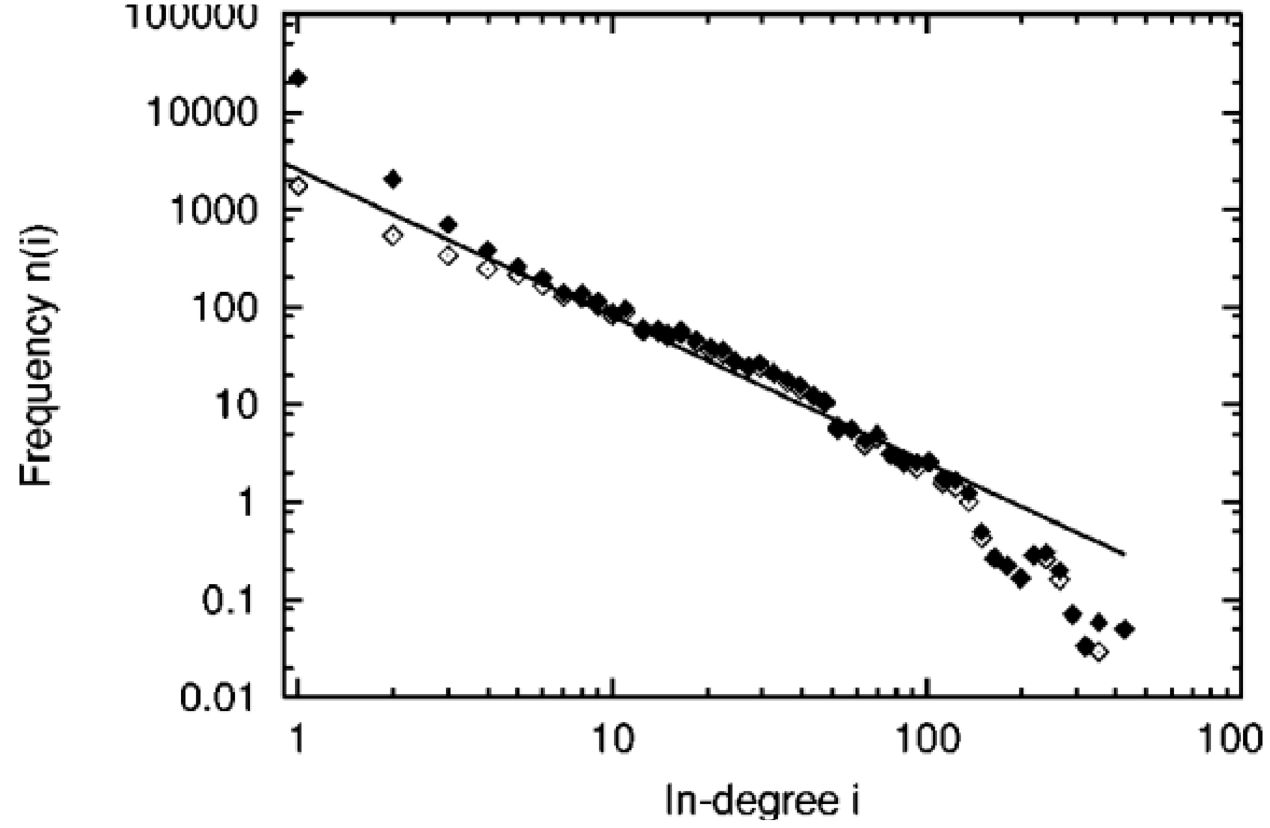
Email Networks

Viruses can affect also computers and digital viruses spread very similarly to biological ones

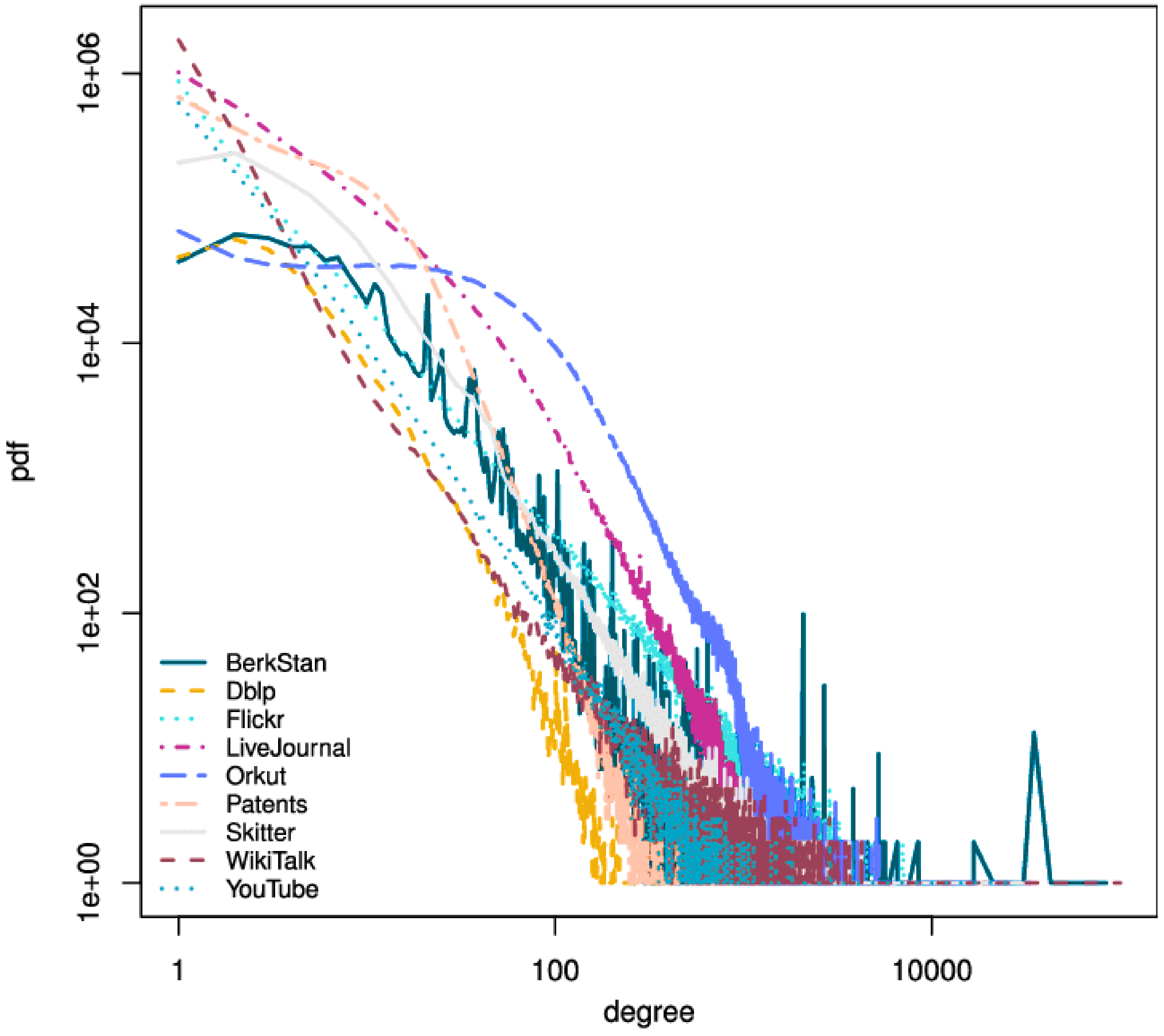
- in this case the process takes place on a network of computers
- for instance links between computers could represent sharing of email

Also in this case the networks are typically scale free





Online Social **Networks**



Online social platform are another example of networks “hosting” spreading processes

- trends
- memes
- shorts and reels

But also

- social movements
- protests
- revolutions

Most online social networks are characterized by a scale free structure and a strong clustering

Epidemic Spreading

Modeling Epidemic Spreading

A very relevant process taking place on networks is epidemic spreading

- epidemic spreading models describe how an illness spread in a group of individuals
- they have been crucial in mitigating the effects of Covid and in guiding policies

Given a set on initial infected people:

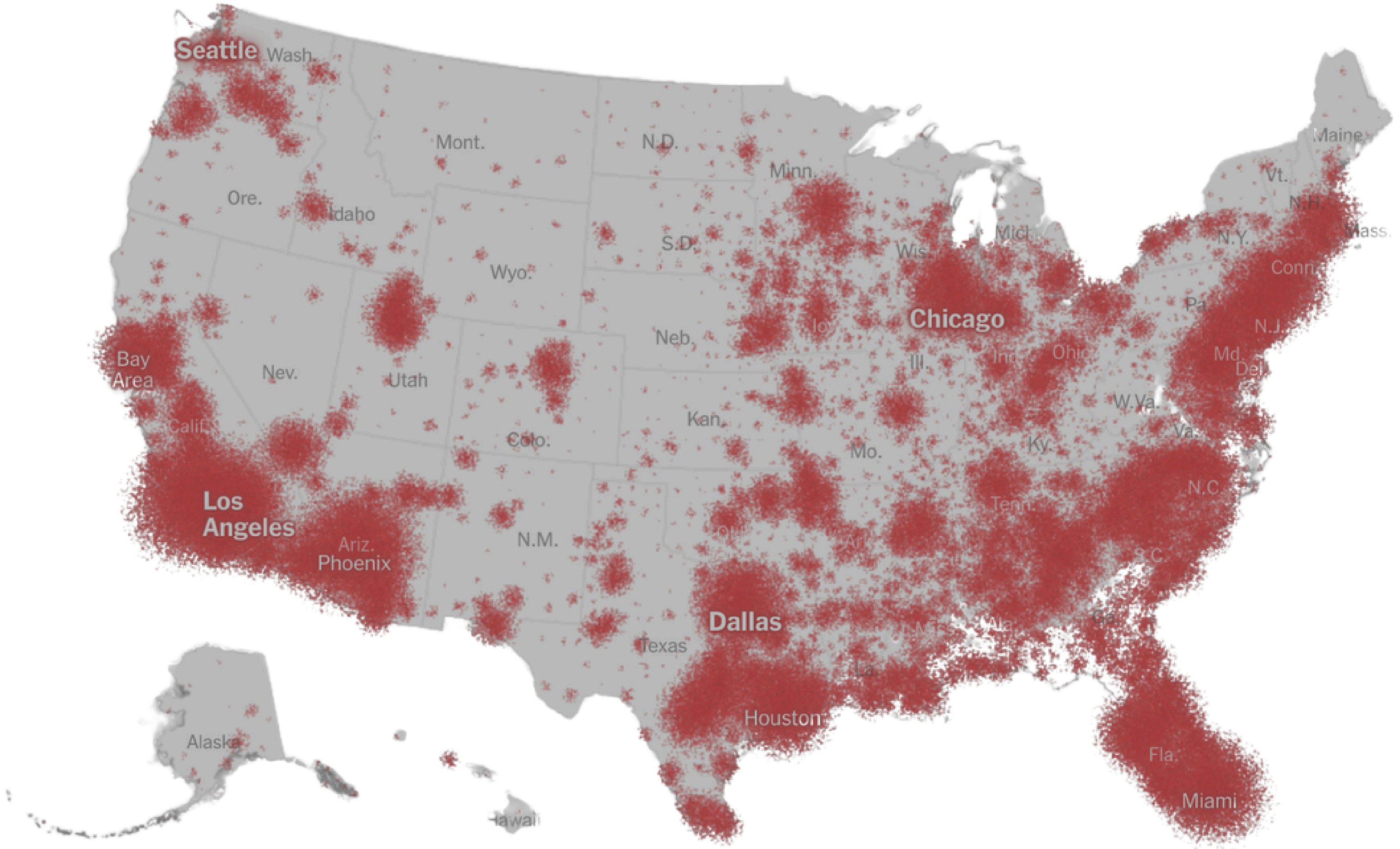
Modeling Epidemic Spreading

A very relevant process taking place on networks is epidemic spreading

- epidemic spreading models describe how an illness spread in a group of individuals
- they have been crucial in mitigating the effects of Covid and in guiding policies

Given a set on initial infected people:

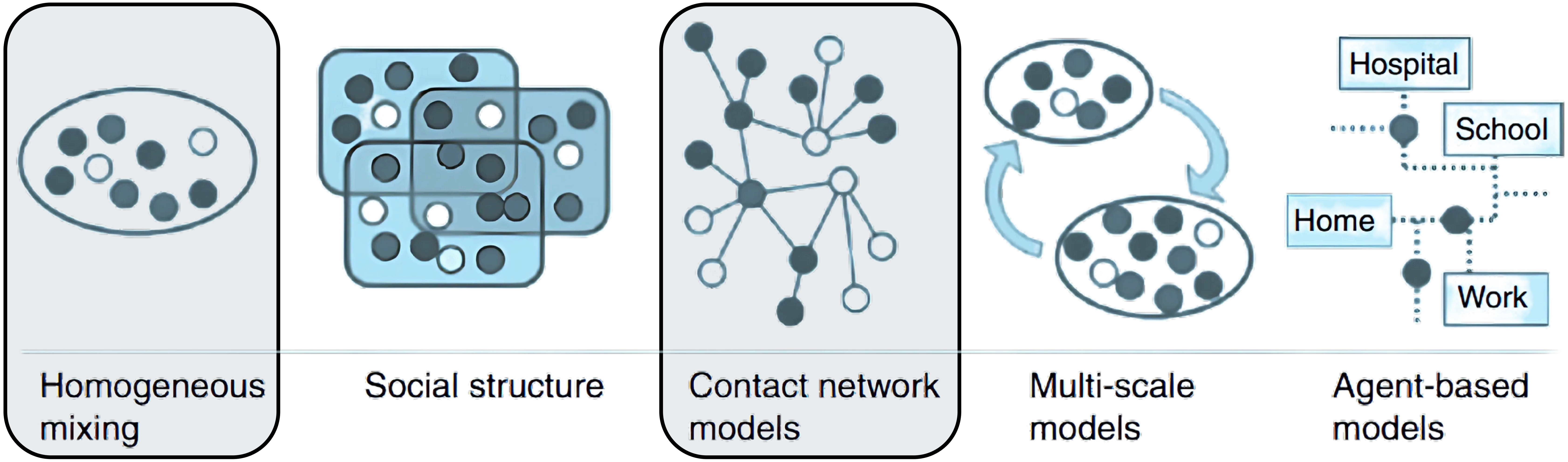
- will the epidemic die out or spread?
- how many people will be affected?



Different Levels of Modeling

Epidemic modeling can be performed at different scales. In this lecture we focus on two possible choices

- homogeneous mixing
- contact network models



Epidemic Models

Most epidemics model consider individual in 3 possible conditions

- **Susceptible S:** not infected, could be infected
- **Infected I:** has disease and is contagious
- **Recovered R:** not contagious and immune

The epidemics is governed by the transition probabilities between the different states

There are 3 main epidemic models

- SI model
- SIS model
- SIR model

Susceptible
S

Infected
I

Recovered
R

SI Model

In the SI model individuals can only be in 2 possible different states

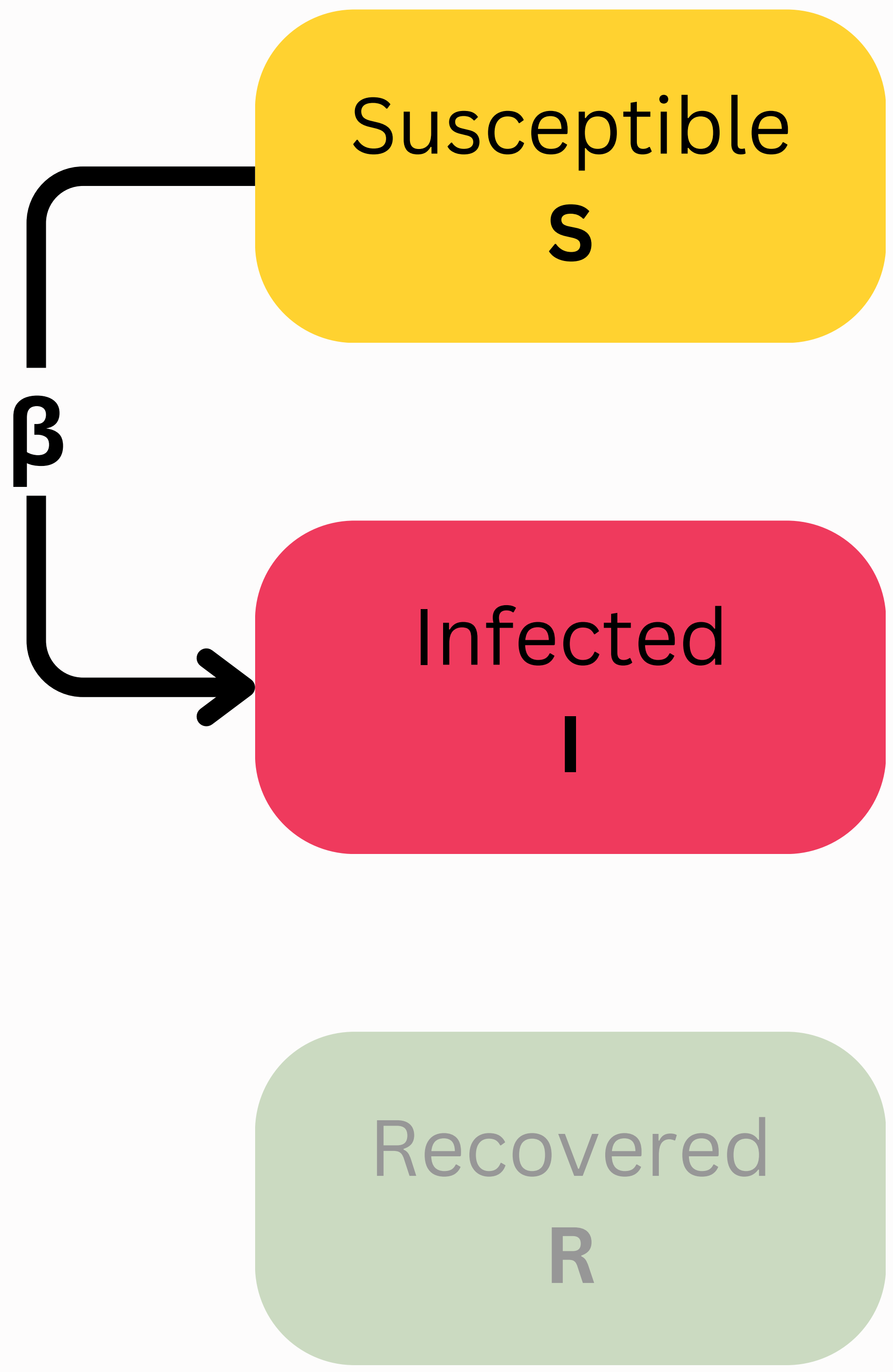
- **Susceptible S:** not infected, could be infected
- **Infected I:** has disease and is contagious

There are (probabilistic) transitions between states:

- From S to I: infection from another infected individual with prob. β
- From I to S: recovery is impossible

Examples:

- HIV/AIDS
- Human Papillomavirus (HPV)



SIS Model

In the SIS model individuals can only be in 2 possible different states

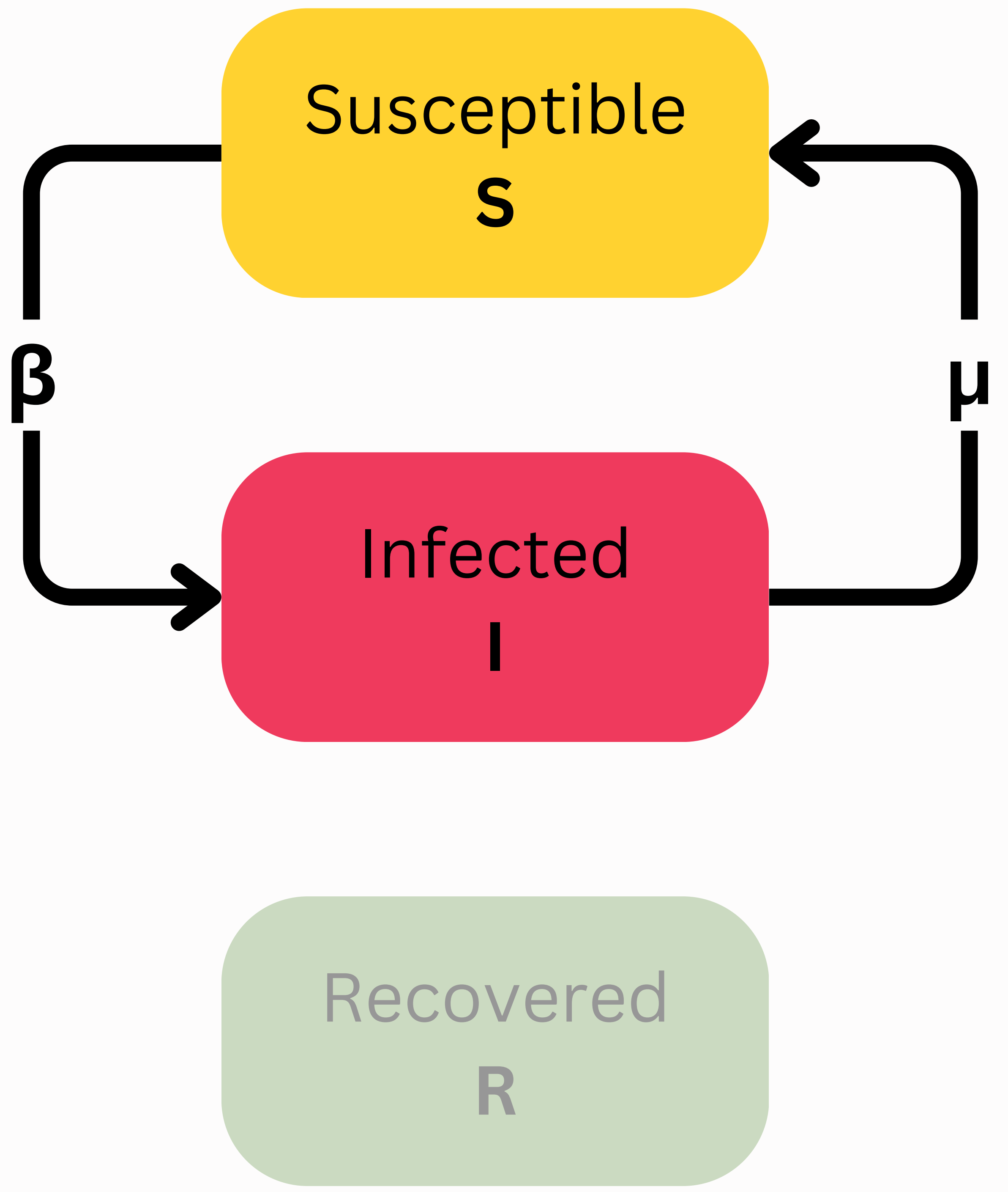
- **Susceptible S:** not infected, could be infected
- **Infected I:** has disease and is contagious

Also in this case there are (probabilistic) transitions between states:

- From S to I: infection from another infected individua with prob. β
- From I to S: recovery from disease with prob. μ

Examples:

- seasonal influenza
- Covid-19



SIR Model

In the SIR model individuals can be in 3 possible different states

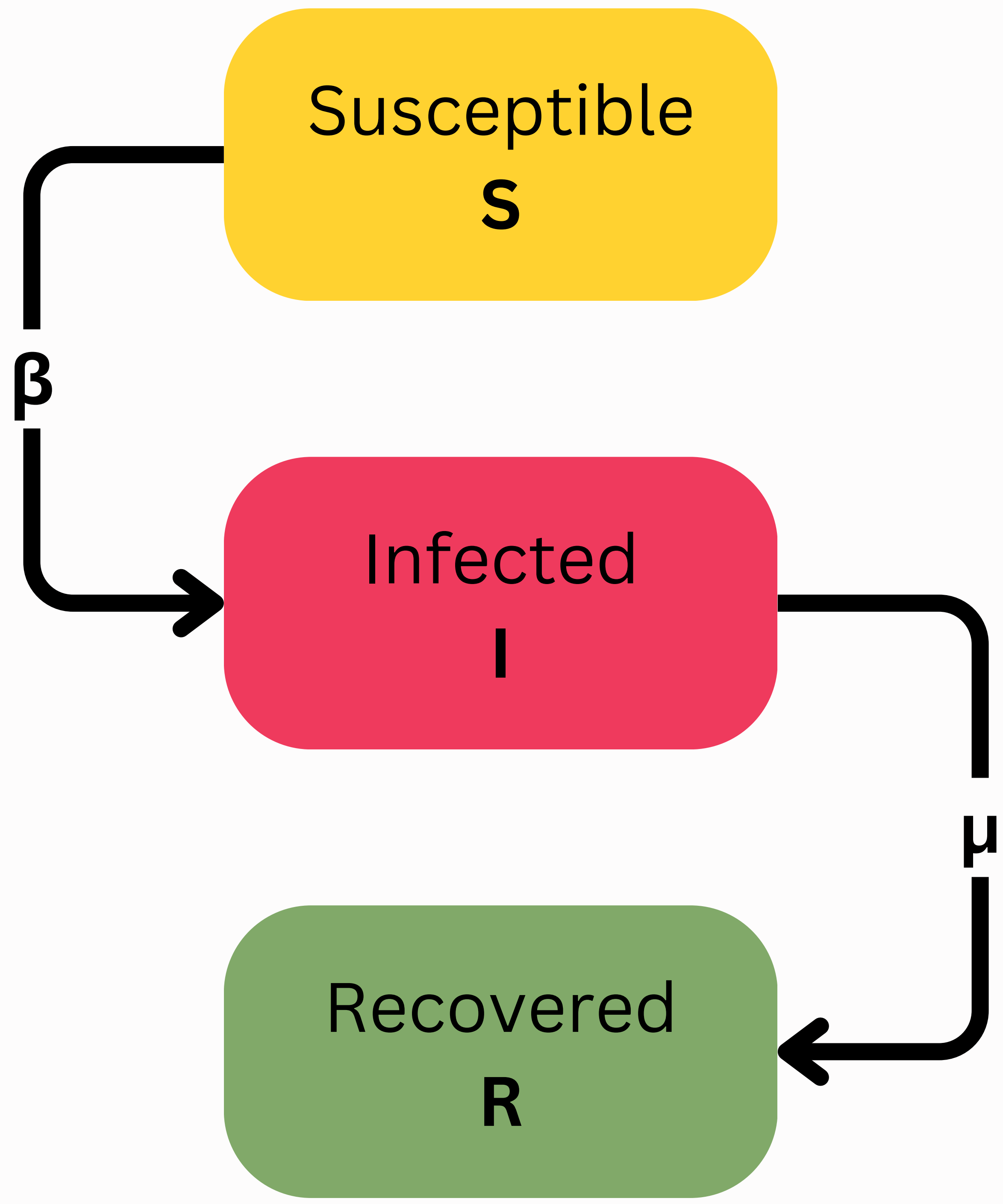
- **Susceptible S:** not infected, could be infected
- **Infected I:** has disease and is contagious
- **Recovered R:** not contagious and immune

The allowed transitions are:

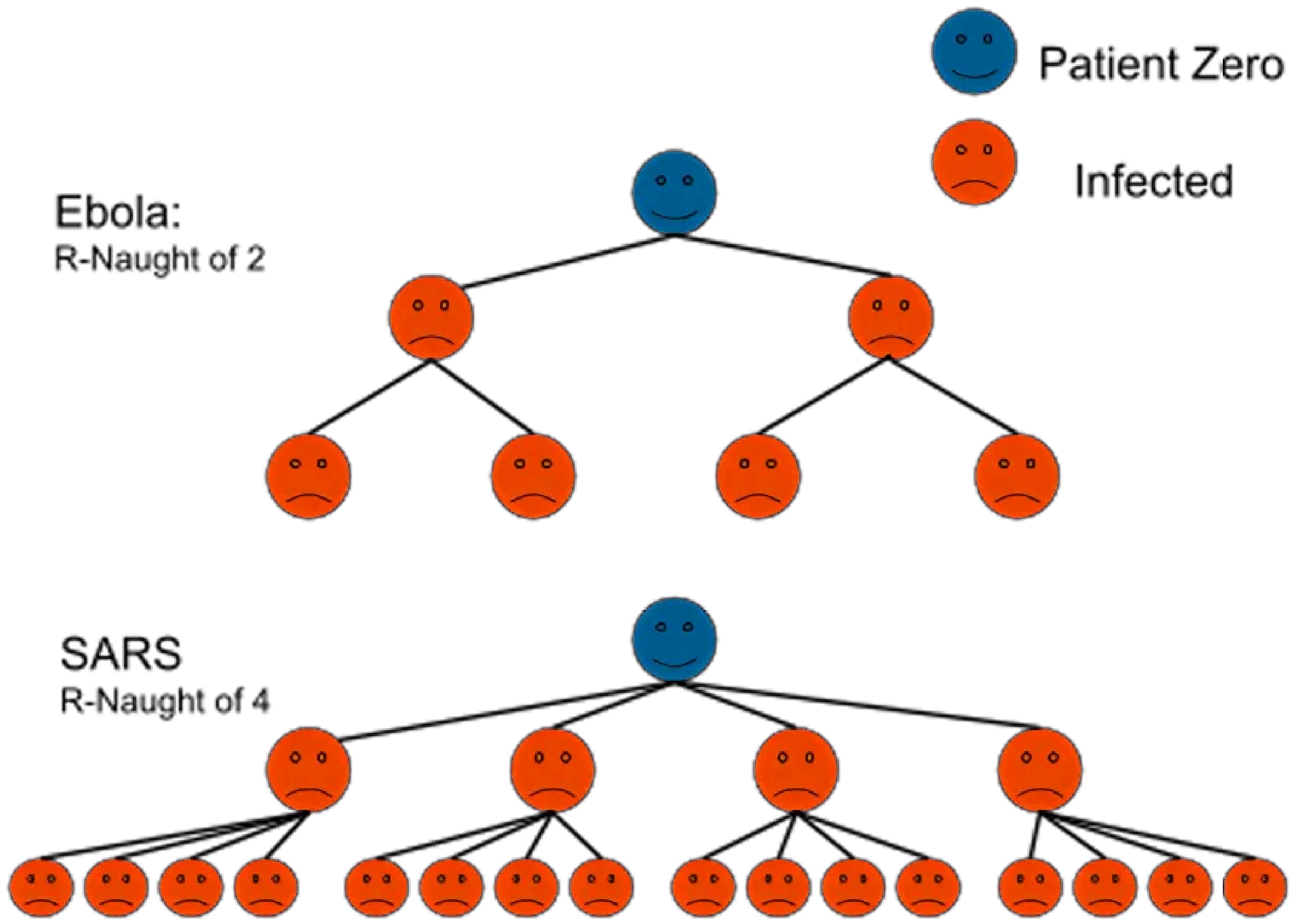
- From S to I: infection from another infected individual with prob. β
- From I to R: recovery from disease, death, or permanent isolation with prob. μ

Examples:

- measles
- varicella



Spreading Rate and Basic Reproduction Number



The spreading capabilities of an illness are described by the parameters β and μ

- **β** gives the probability for an infected individual to infected another person
- **μ** give the probability of recovery

We can then define

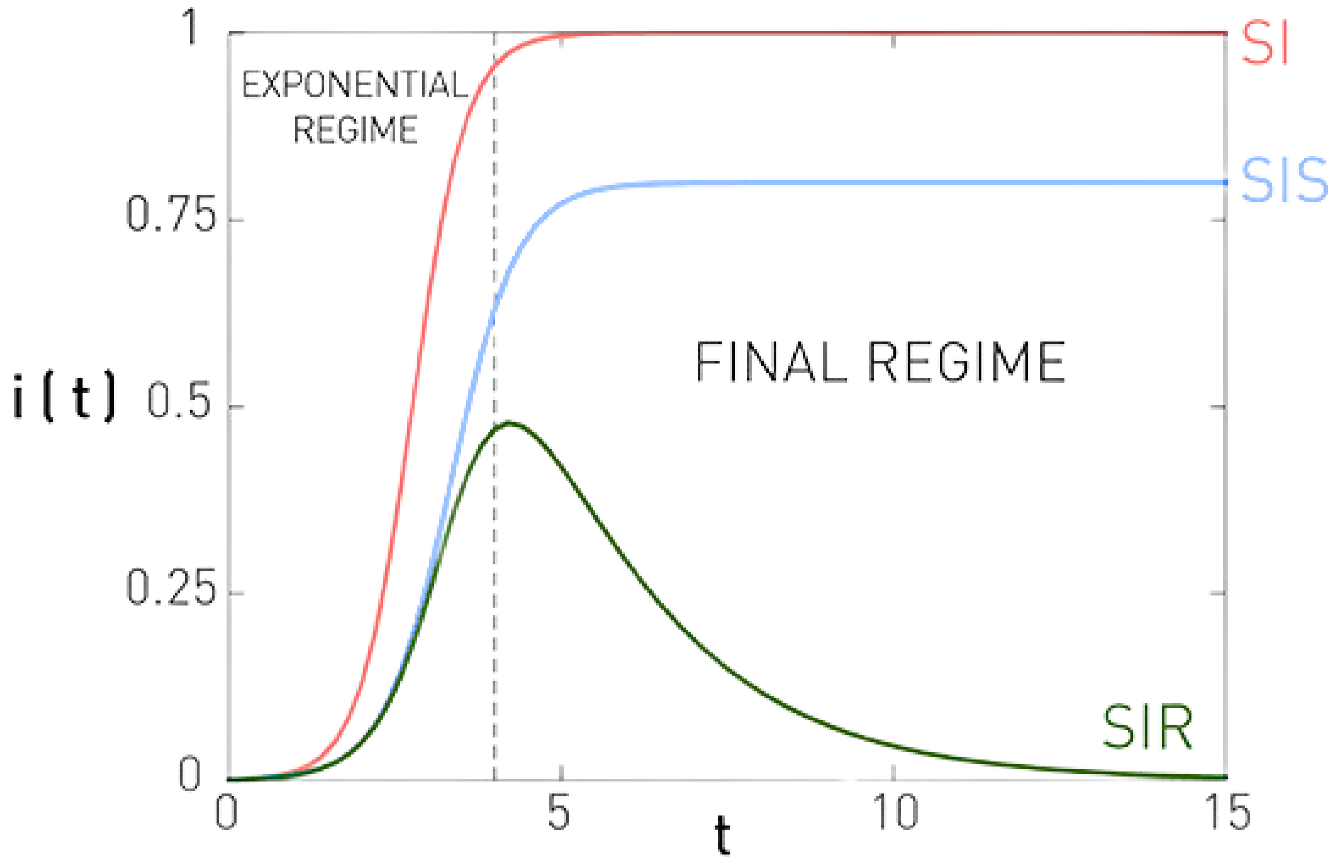
- **Spreading rate**
  $\lambda=\beta/\mu$
- **Basic reproduction number**
  $R_0=\langle k\rangle\lambda=\langle k\rangle\beta/\mu$

λ is an intrinsic feature of the illness and the population, R_0 depends on the networks

Epidemics Evolution

In the case of homogeneous mixing ⟨k⟩=N) the outcome of the epidemics depends on R_0

- **SI** all population is always infected
- **SIS** the epidemics dies out for $R_0<1$, while becomes endemics for $R_0>1$
- **SIR** the epidemics dies out for $R_0<1$, while it temporarily infects a relevant fraction of the population for $R_0>1$



	SI	SIS	SIR
Final Regime: Saturation at t→=∞	$i(\infty)=1$	$i(\infty)=1-\frac{\mu}{\beta\langle k\rangle}$	$i(\infty)=0$
Epidemic Threshold: Disease does not always spread	No threshold	$R_0=1$	$R_0=1$

Epidemic Spreading on Networks

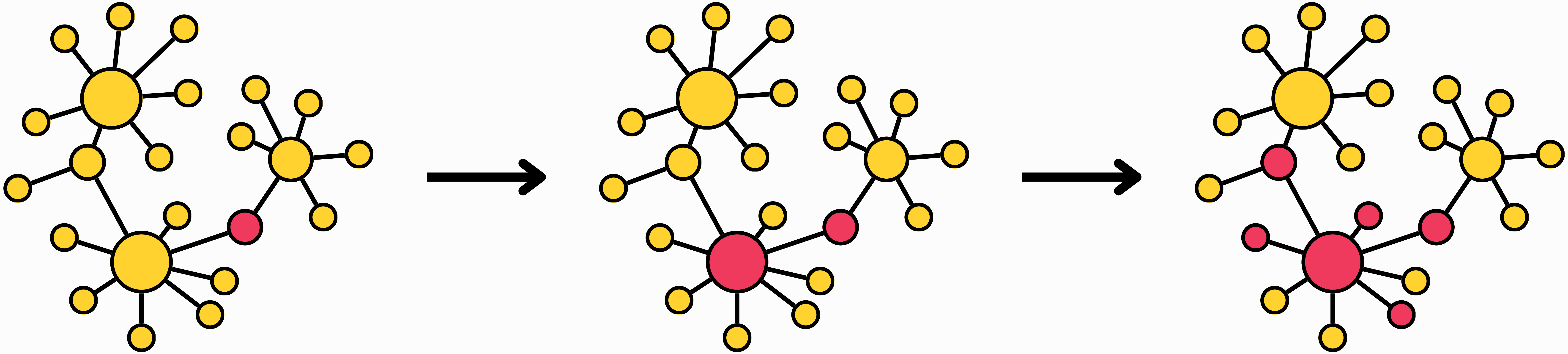
Epidemics on Networks

Homogeneous mixing works well for describing situations like

- students in a class
- people in a waiting room

If instead we want to describe how an epidemics would spread e.g. in a university, we need to reconstruct the network of interaction among individuals

- an infected subject can only infected those people they are linked to

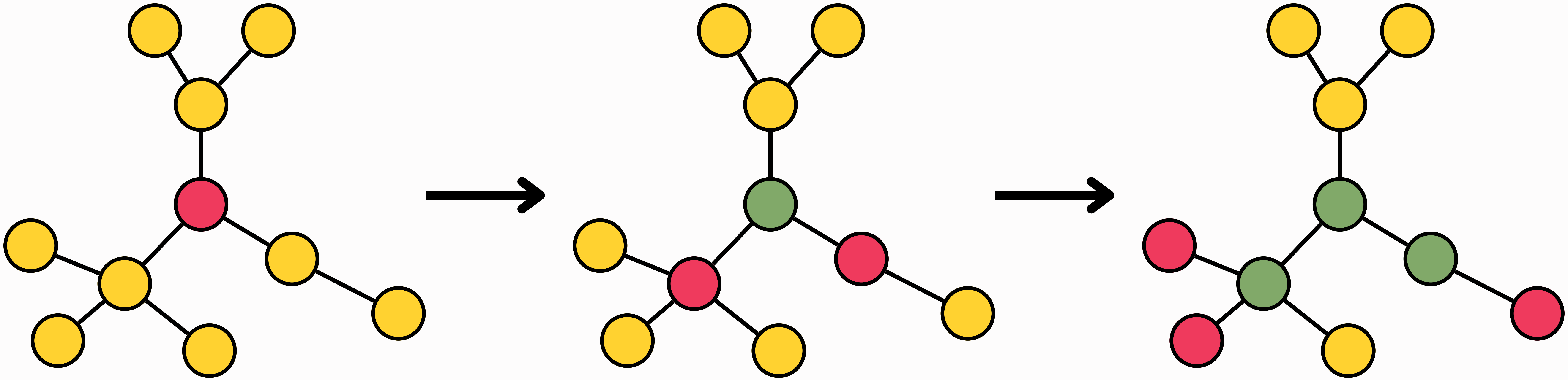


Epidemic Threshold

Like in the homogeneous mixing, we want to understand the critical spreading reate λ_c above which an epidemics can propagate in the network

- we have to study the second neighbors, like for the Molloy-Reed criterion
- remember that the number of second neighbors is $Z_2 = \langle k^2 \rangle - \langle k \rangle$

Epidemics Propagates



Epidemic Threshold

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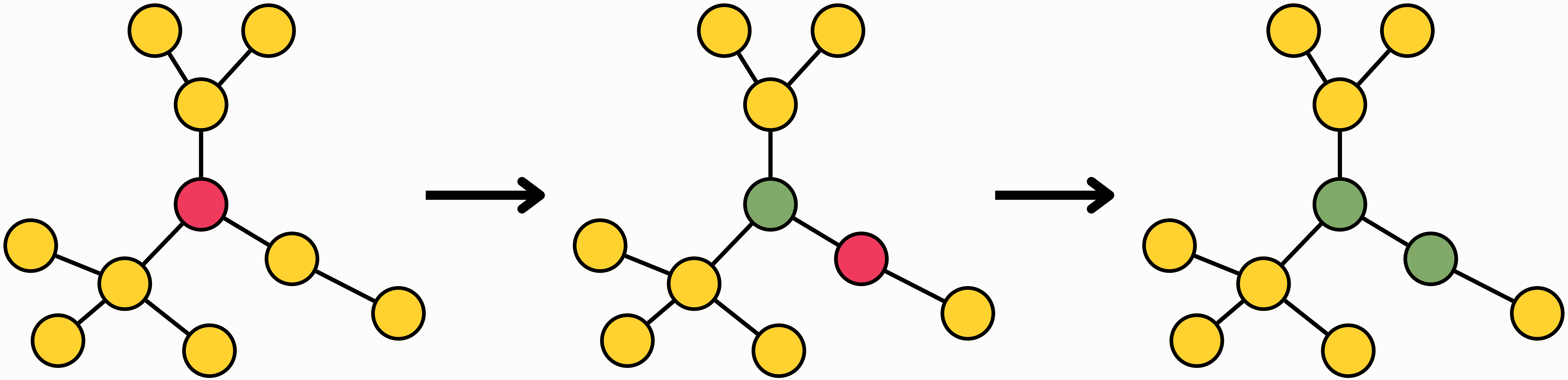
Epidemics Dies Out



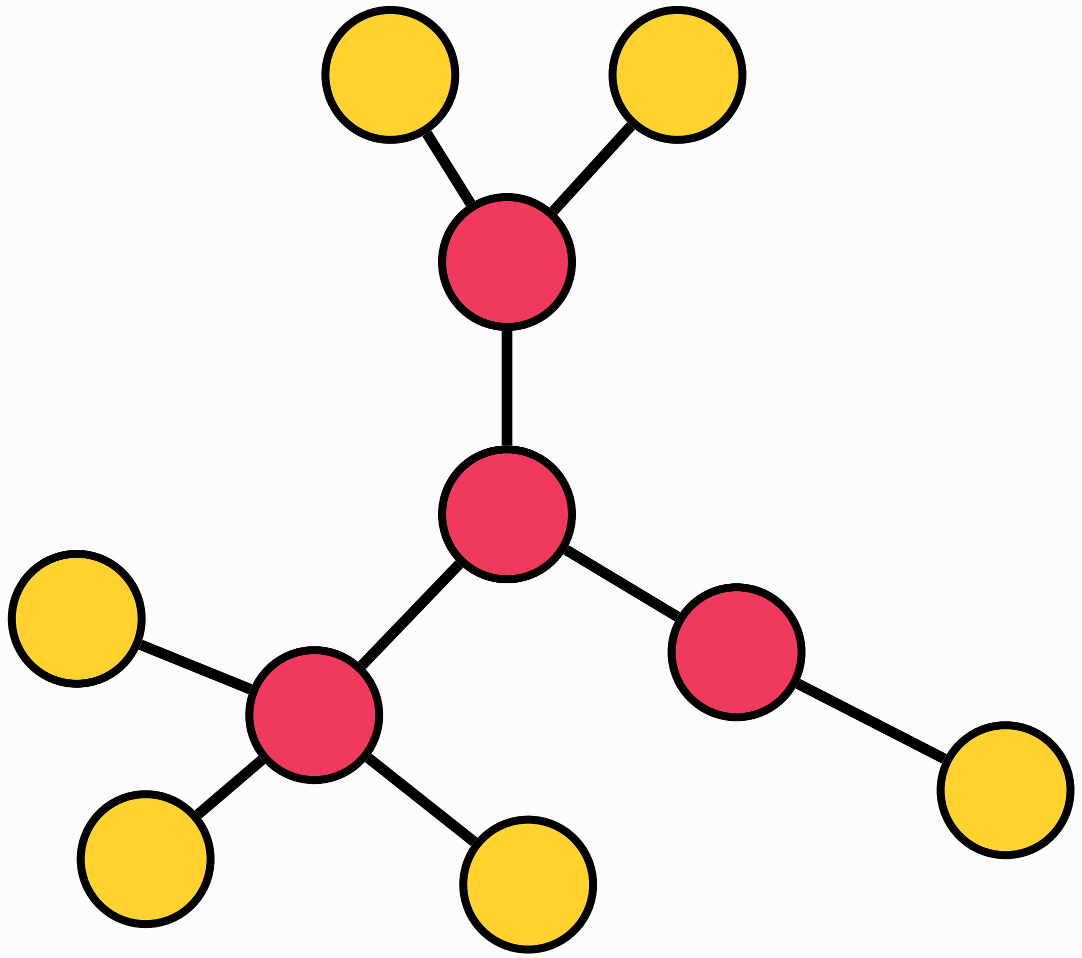
Epidemic Threshold for SI Model

We consider the situation in figure

- the epidemics has spread to the first neighbors
- will the second neighbors keep spreading it?

There are $Z_2 = \langle k^2 \rangle - \langle k \rangle$ second neighbors

- each of them can be infected with probability β
- the average number of infected second neighbors is then $N_i = \beta(\langle k^2 \rangle - \langle k \rangle)$
- if this number is larger than zero the infection grows over time



The condition for the SI model is then

$$\langle k^2 \rangle - \langle k \rangle > 0 \rightarrow \lambda_c = 0$$

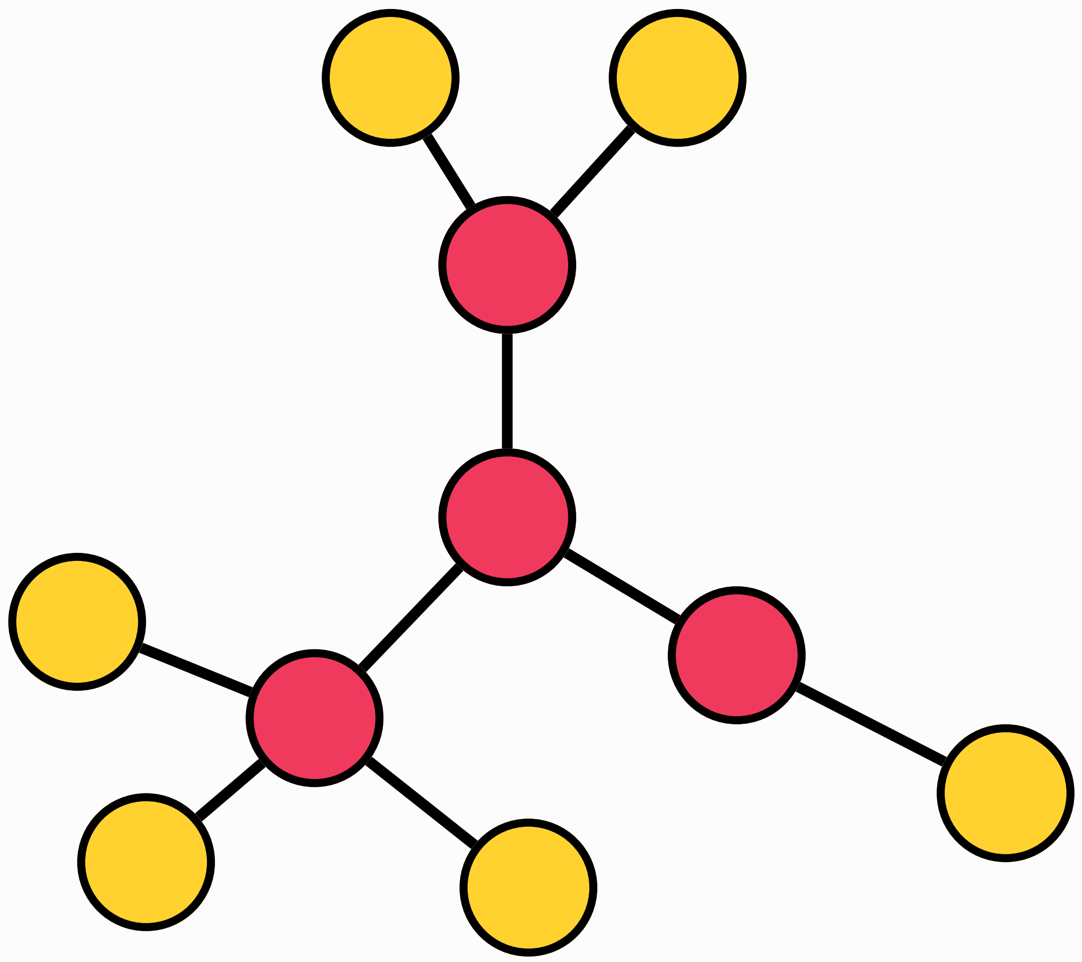
Epidemic Threshold for SIR Model

We consider the same situation with Z_2 susceptible second neighbors

- each of them can be infected with probability β
- however the first neighbors can heal with probability μ
- the variation in infected people is then $N_i=\beta(\langle k^2\rangle-\langle k\rangle)-\mu\langle k\rangle$
- if this number is larger than zero the infection grows over time

The condition for the SIR model is then

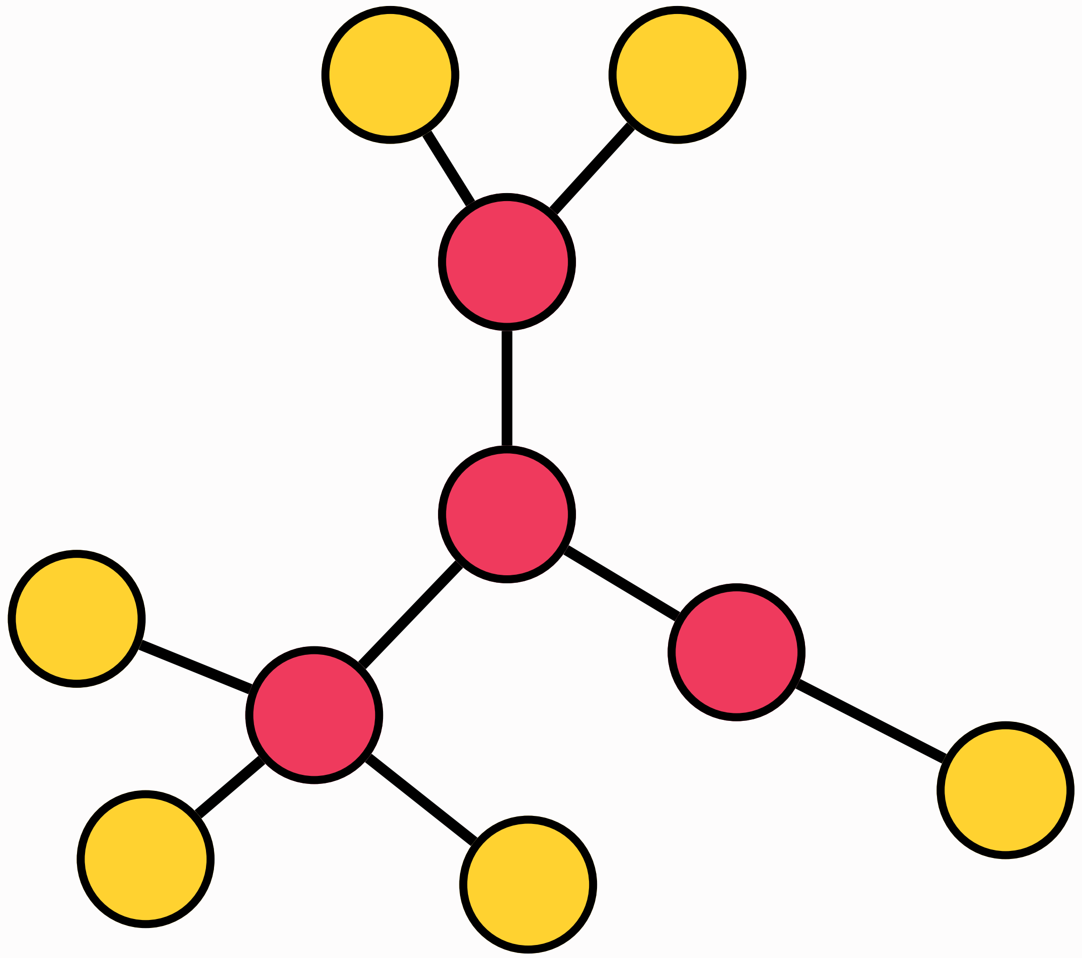
$$\beta(\langle k^2\rangle - \langle k\rangle) - \mu\langle k\rangle > 0 \rightarrow \lambda_c = \frac{\langle k\rangle}{\langle k^2\rangle - \langle k\rangle}$$



Epidemic Threshold for SIR Model

We consider the same situation with Z_2 susceptible second neighbors

- each of them can be infected with probability β
- the first neighbors can heal with probability μ
- however they can also get reinfected with probability β
- the variation in infected people is then $N_i=\beta(\langle k^2\rangle-\langle k\rangle)-\mu\langle k\rangle+\beta\langle k\rangle$
- if this number is larger than zero the infection grows over time

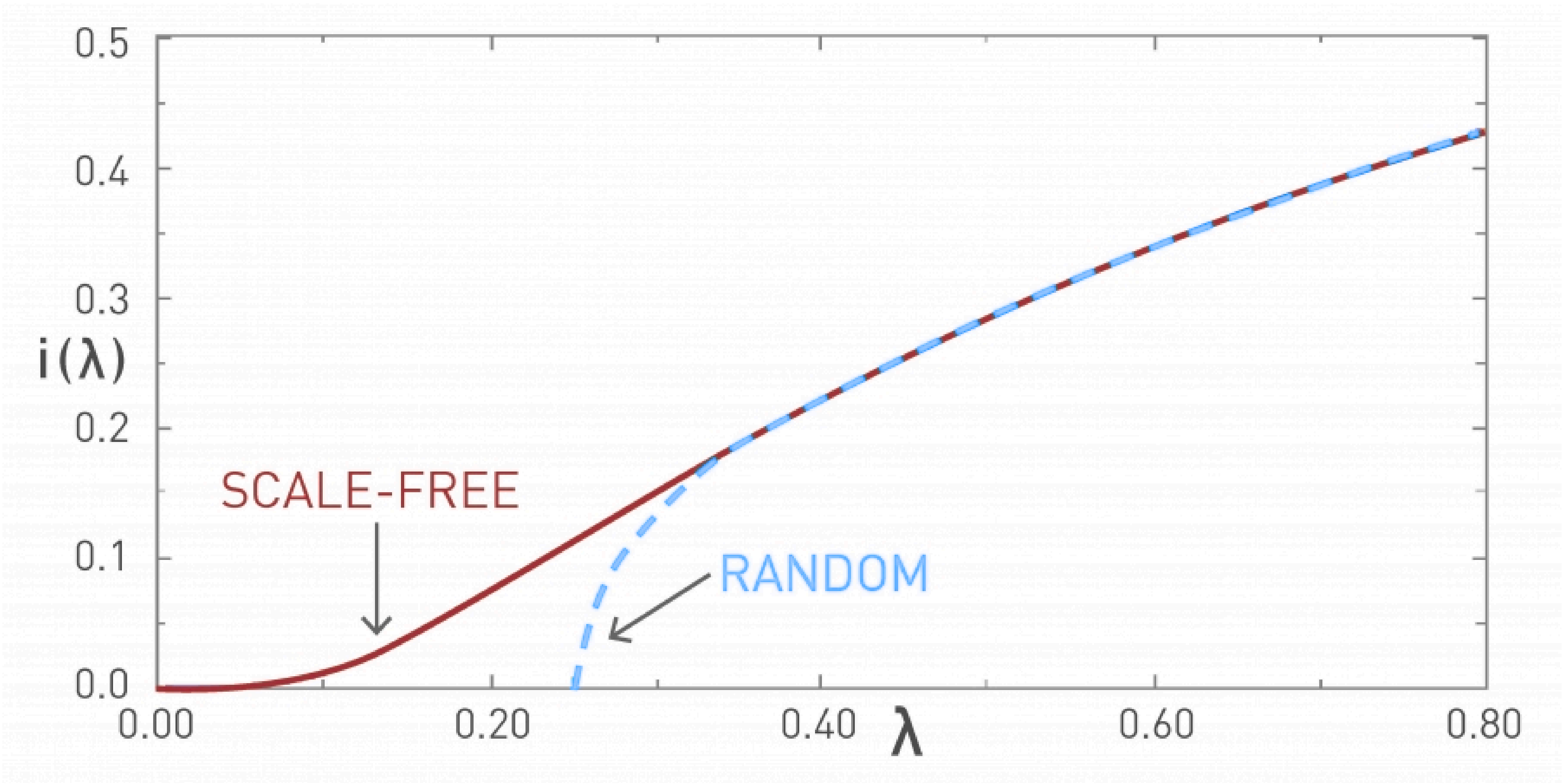


The condition for the SIS model is then

$$\beta(\langle k^2\rangle - \langle k\rangle) - \mu\langle k\rangle + \beta\langle k\rangle > 0 \to \lambda_c = \frac{\langle k\rangle}{\langle k^2\rangle}$$

Epidemic Threshold Vanishing

In scale free networks the expectation of k^2 diverges when the scaling exponent is smaller than 3. In this case the epidemic threshold vanishes, meaning than basically any virus can spread infecting a non null fraction of the population



Immunization Strategies

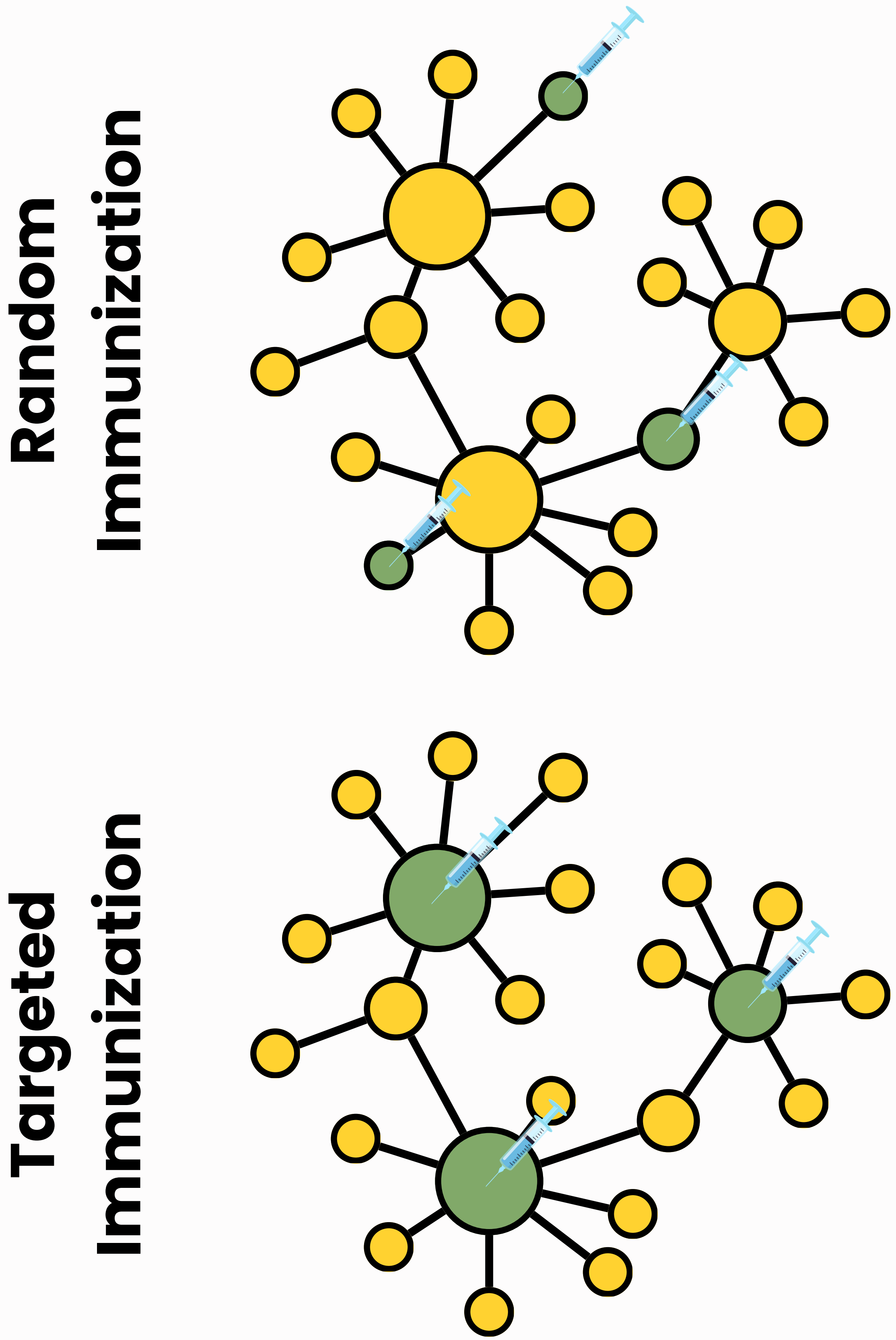
Vaccines are one of the main weapons against virus epidemics

- they alter the network over which the epidemics spread
- this can reduce the basic reproductive number

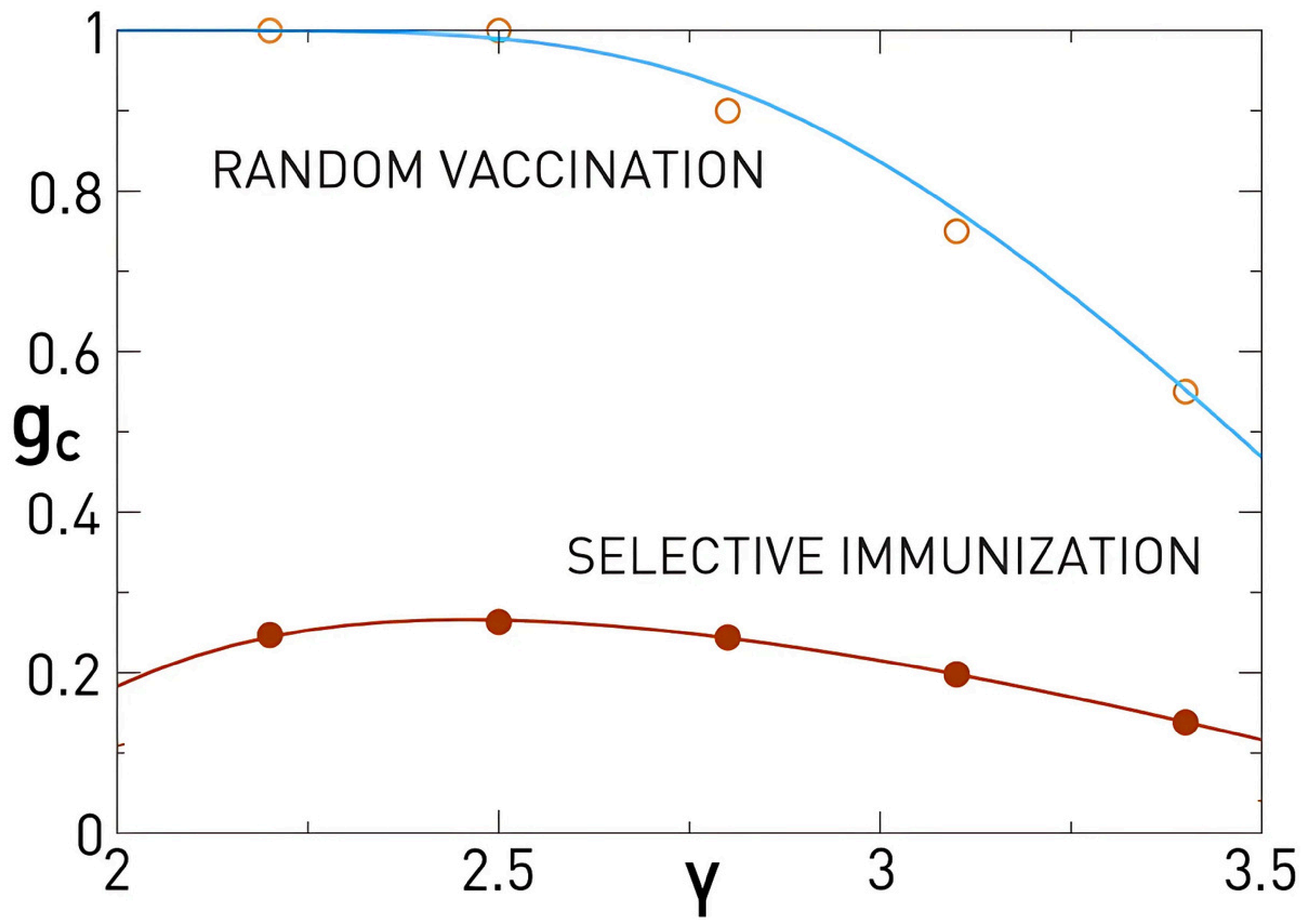
There are different possible approaches to immunization

- random immunization
- targeted immunization

The latter is more effective but also harder to implement



Random vs Targeted Immunization



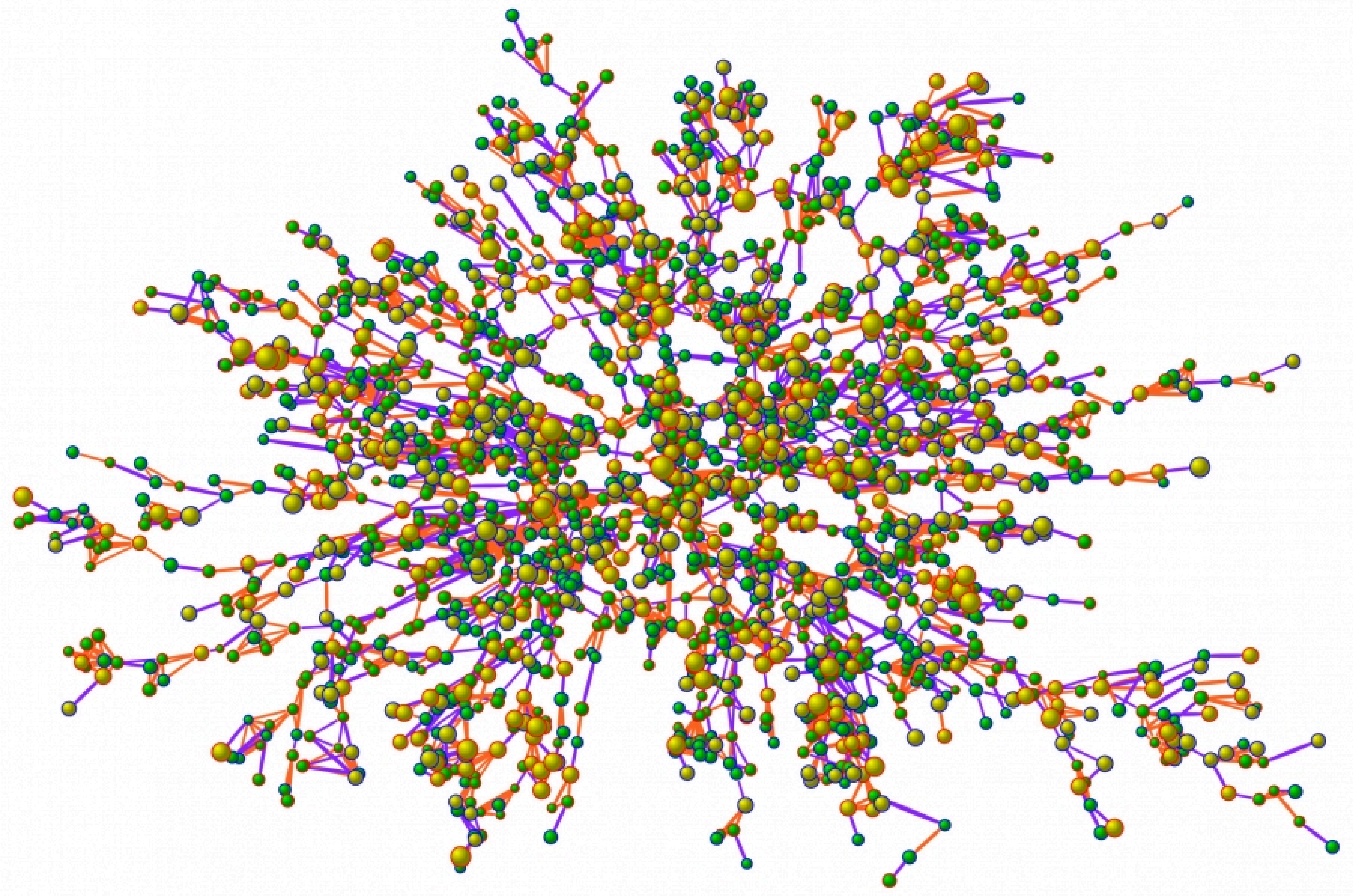
Targeted immunization is particularly relevant in scale-free networks

- random immunization produces no effects if $\gamma<3$ and little effects otherwise
- selective immunization instead, by targeting high degree nodes, has a much stronger impact

The same properties that make scale-free networks robust to random failures, make also them susceptible to epidemic spreading

Complex Contagion

Behavior Spreading



Viruses are not the only things that can spread

- there are many spreading processes on social networks
  - ideas
  - behaviors
  - fashions and trends

We often say that ideas can spread like a virus, is this true?

- there are many similarities between the two phenomena
- however there are also important differences

Simple Contagion

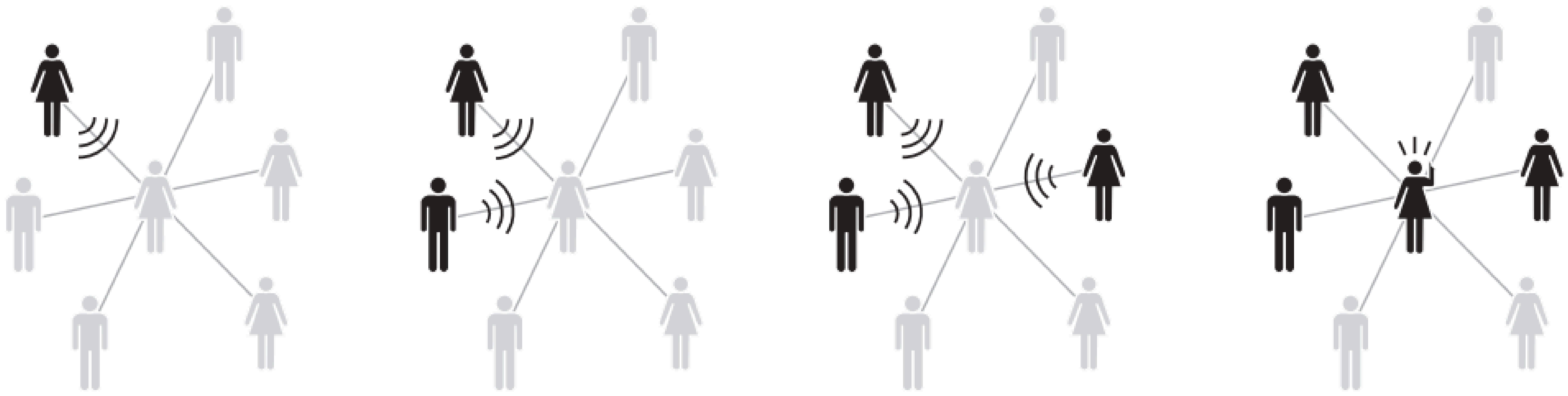
Epidemics are **Simple Contagion** processes

- a single individual is **enough** for infecting other people
- the probability of being infected grows **linearly** with the number of infected people you are exposed to
- there is no role played by how many connections you have, a single infected contact can lead to an infection

Complex Contagion

Behaviors and ideas spreading is instead described by **Complex Contagion**

- a single individual is **not enough** for infecting other people
- there is a **threshold** of infected contacts above which the probability of getting infected becomes large than zero
- the more connections an individual has, the more infected people are needed for it to be infected



The Weakness of Long Ties

We saw the long ties play an important role in Granovetter's theory

- the situation is very different in complex contagion processes
- weak ties have low overlap, so they can hardly propagate the contagion

Complex contagion works better on networks with very high clustering

- introducing bridges in the network may inhibit the propagation
- this is opposite with respect to simple contagion

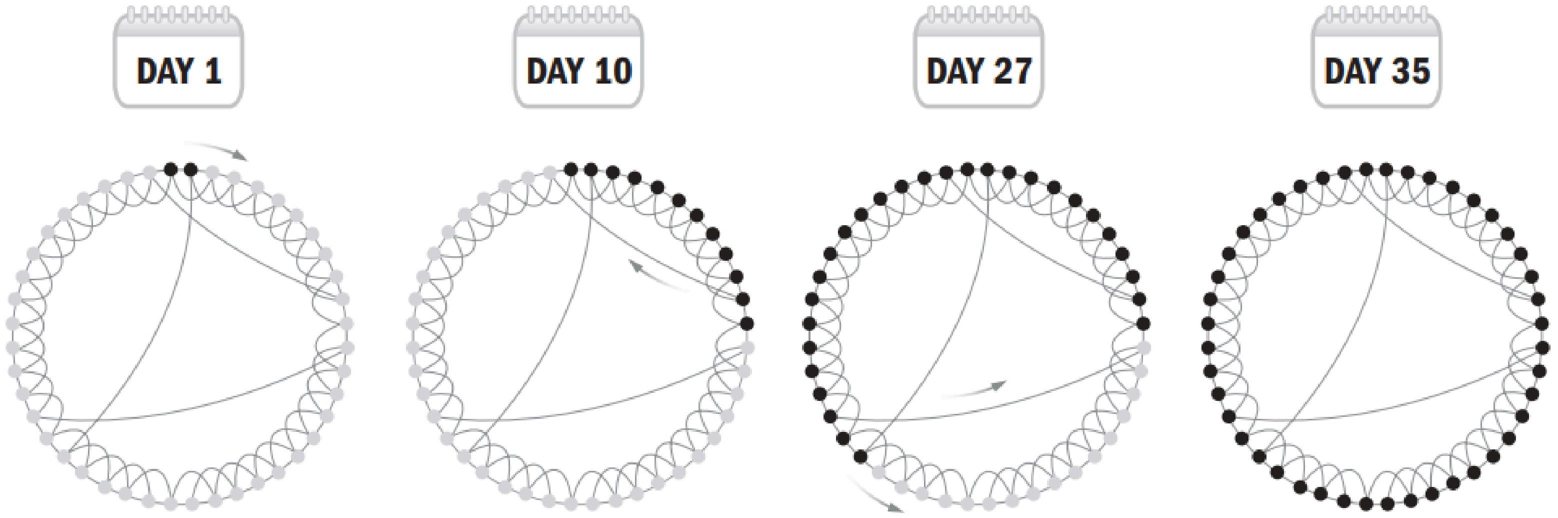


Figure 3.4 Diffusion with Weak Ties

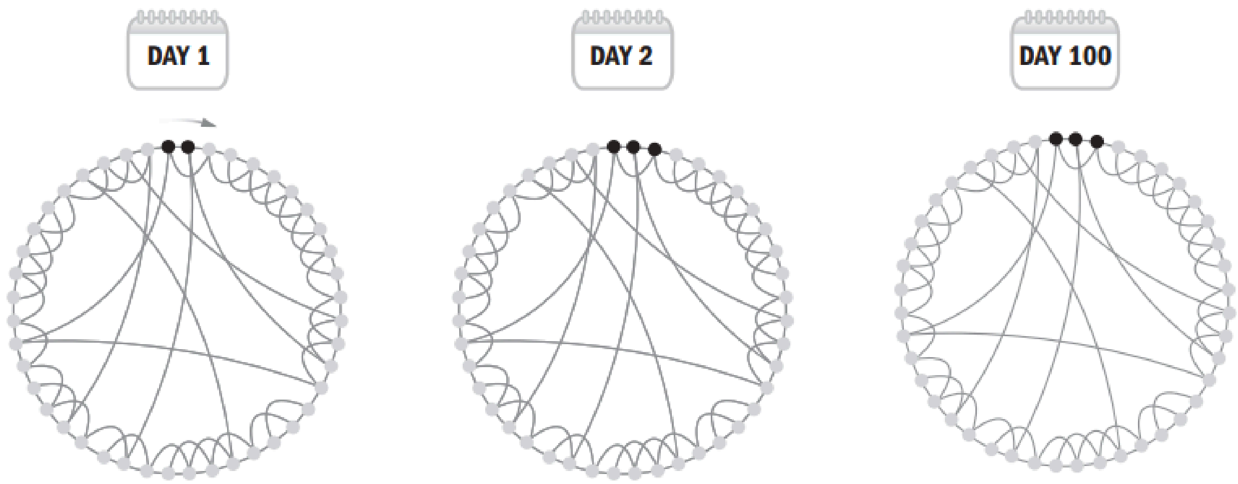
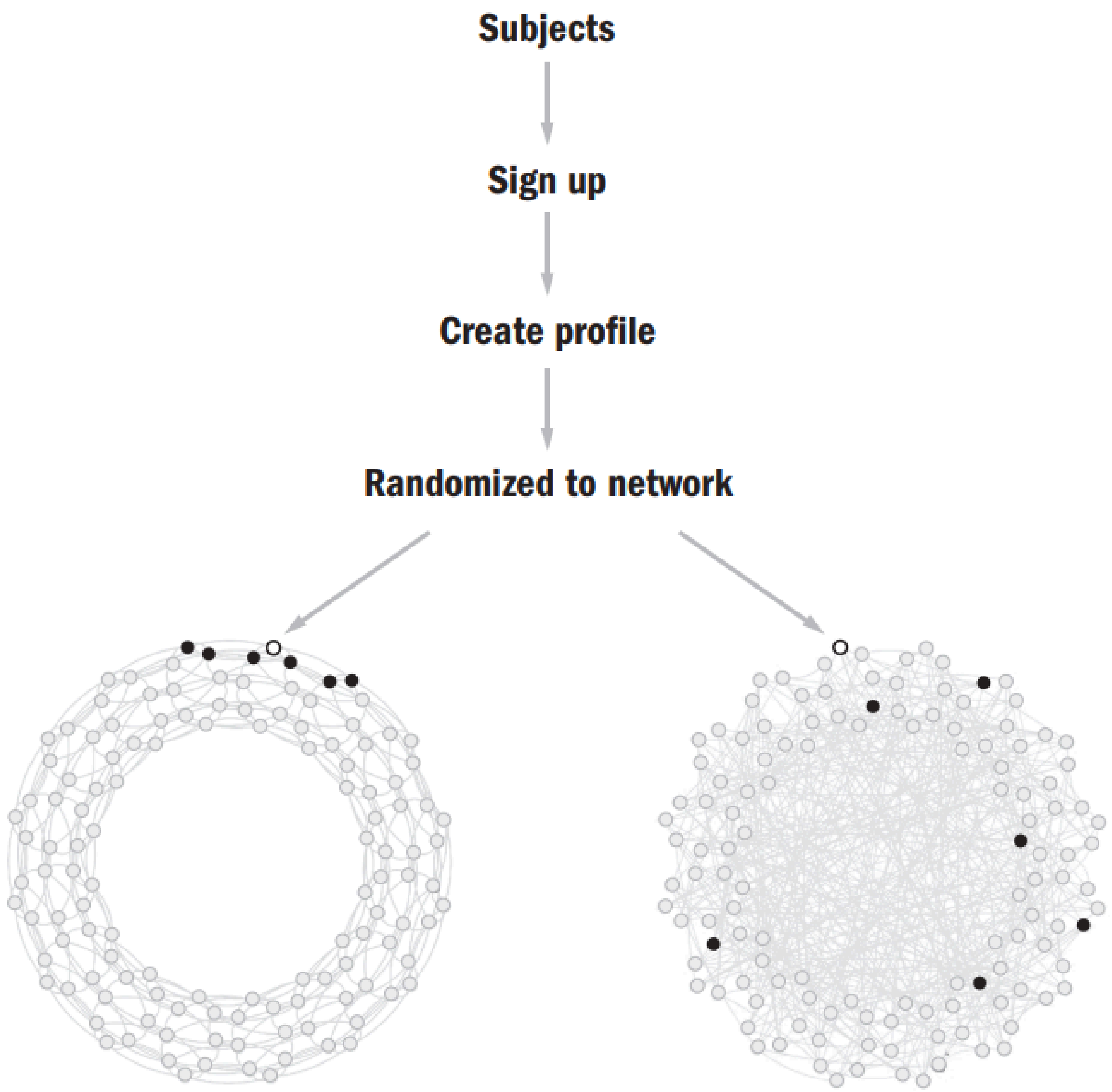


Figure 3.5 Diffusion with More Weak Ties

"*How behavior spreads: the science of complex contagions*: by Damon Centola, Princeton University Press, 2018." (2019): 231-232.



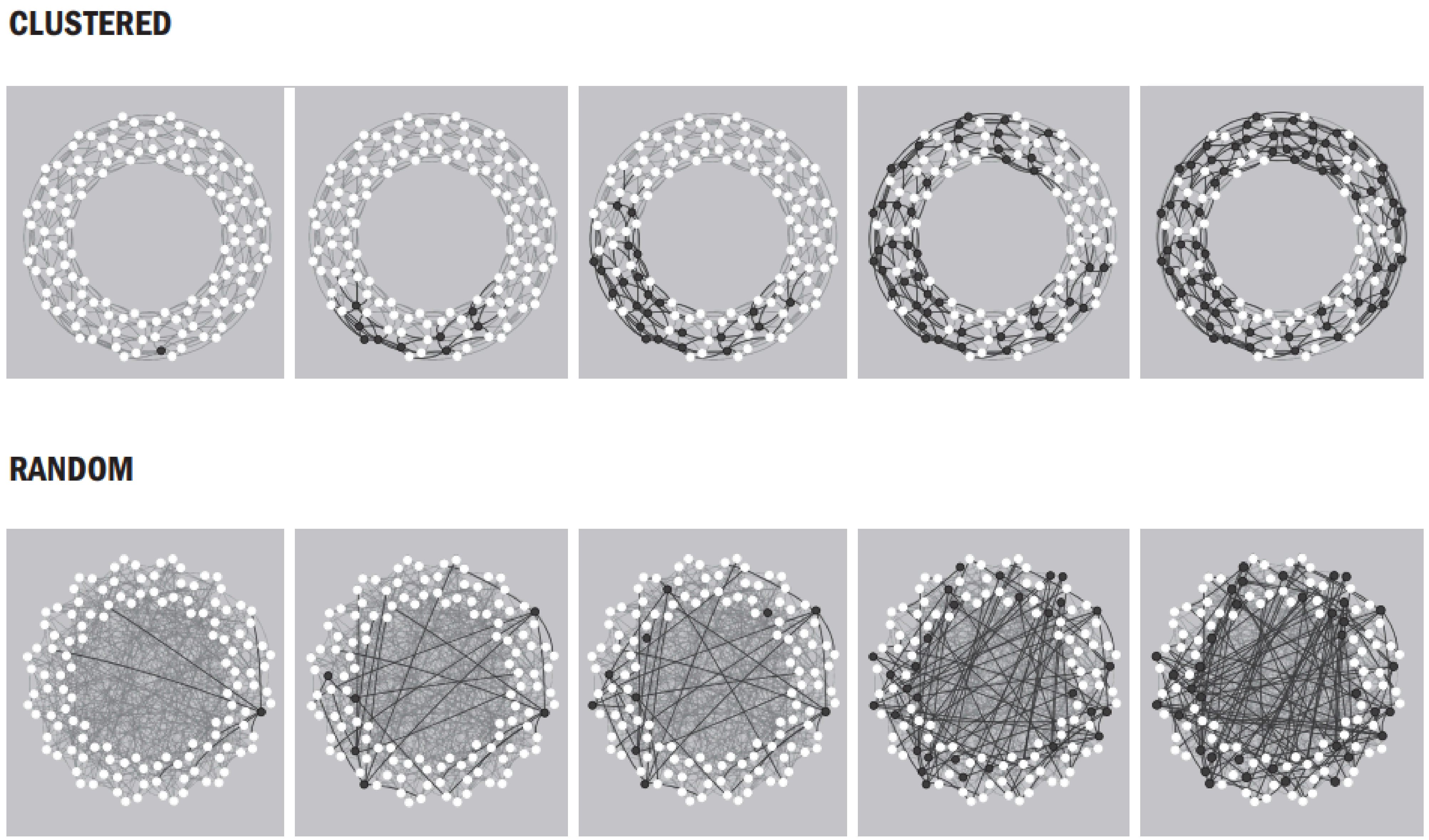
A Social **Experiment** on the **Internet**

This theory has been tested with an online experiment

- participants are dividend into groups with different topologies
  - clustered network
  - random network
- a user is the initial spreader of a behavior (subscribing to a forum)
- users get the information of what their neighbors are doing

The spreading of the behavior on the two different topologies is then compared

Spreading on Different Topologies



Different behaviors depending on the network are observed

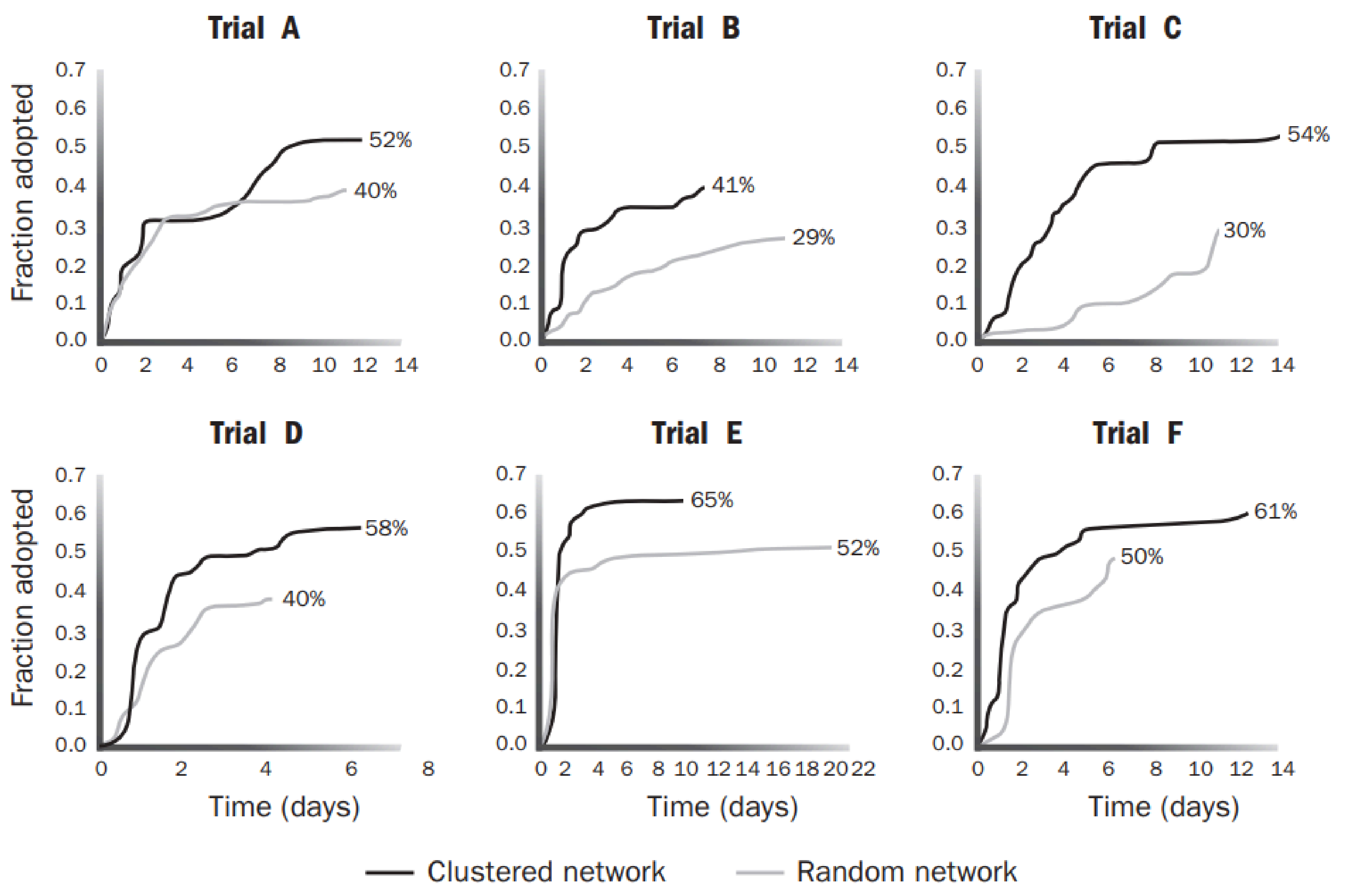
- in the clustered network
  - it takes longer to reach far regions in the networks
  - many nodes adopt the behavior
- in the random network
  - bridges diffuse the behavior far
  - however only few nodes adopt it

Behavior Adoption Dynamics

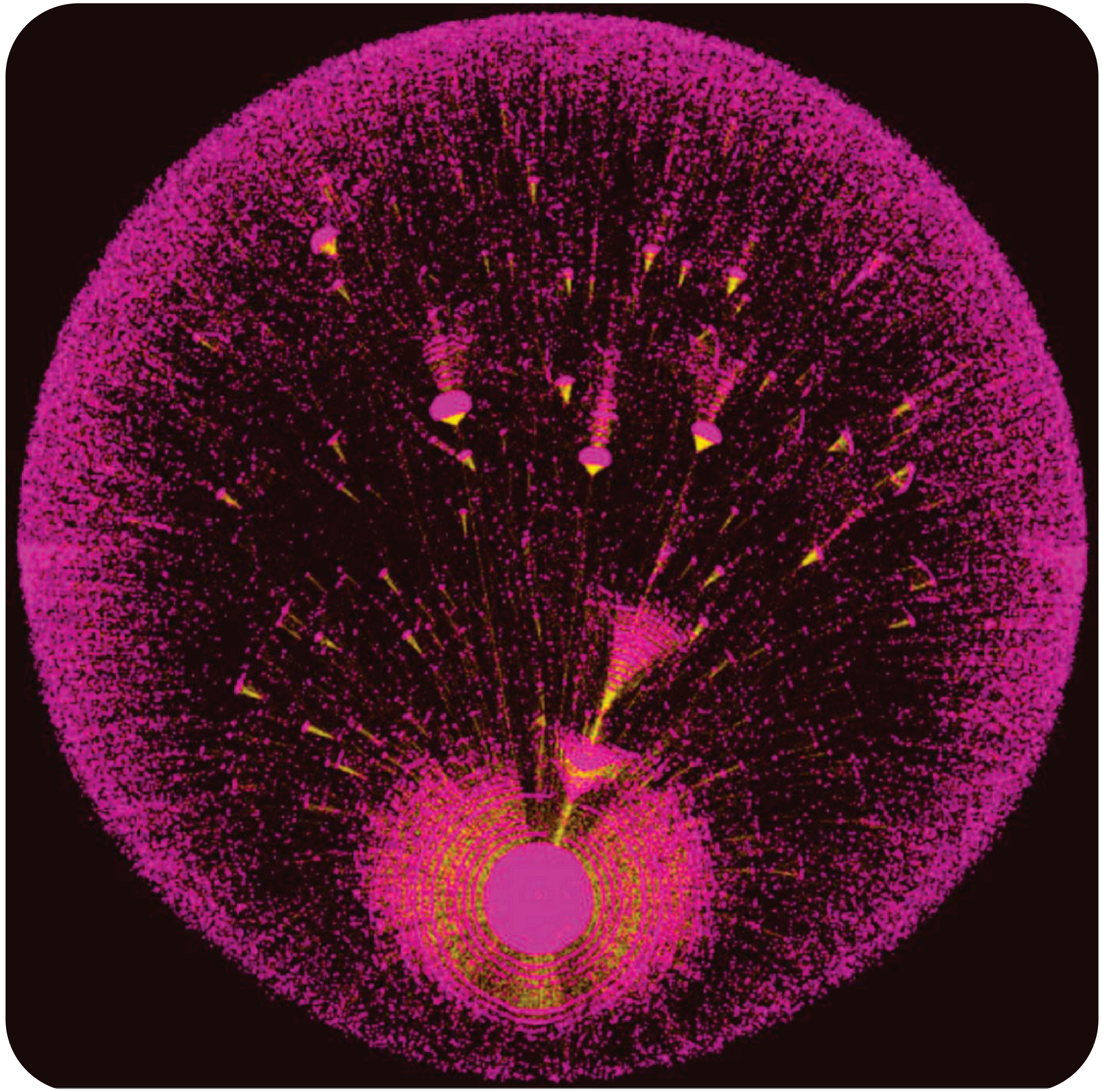
Participants were divided into 6 groups, each containing a random and a clustered configuration

- the behavior spreads faster in the clustered network
- more people adopt the behavior in clustered networks

These results confirm the fact that behaviors spread following a Complex Contagion process



Spreading Viral Content



Feng, Ling, et al. "Competing for attention in social media under information overload conditions." PloS one 10.7 (2015): e0126090.

Also spreading of viral content on social networks can be described by Complex Contagion processes

- users are in an “information overload” state
- they receive much more content than they can process

This means that in order for a post to be visibile, it must be shared by a large number of our connections

- also in this case a single “infected” individual is not enough

Fractional SIR Model

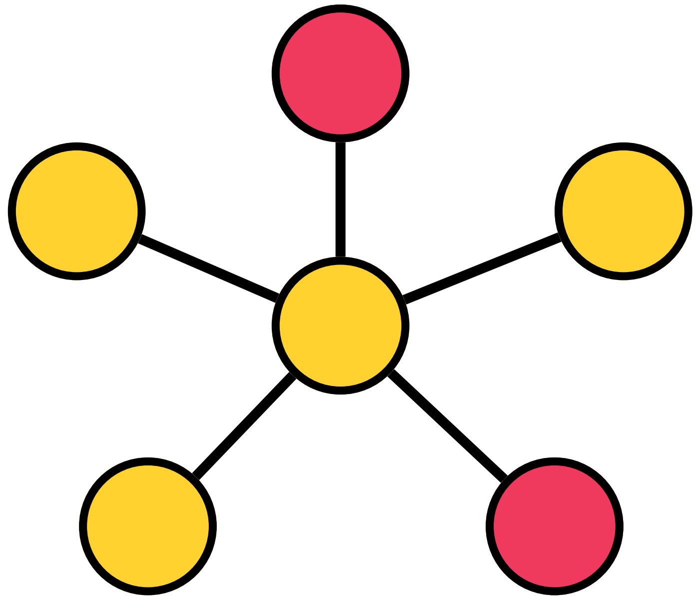
We want a model that capture the following features

- on online platform the situation is different
- the more friends we have, the harder it is for any of them to “infect” us with a meme

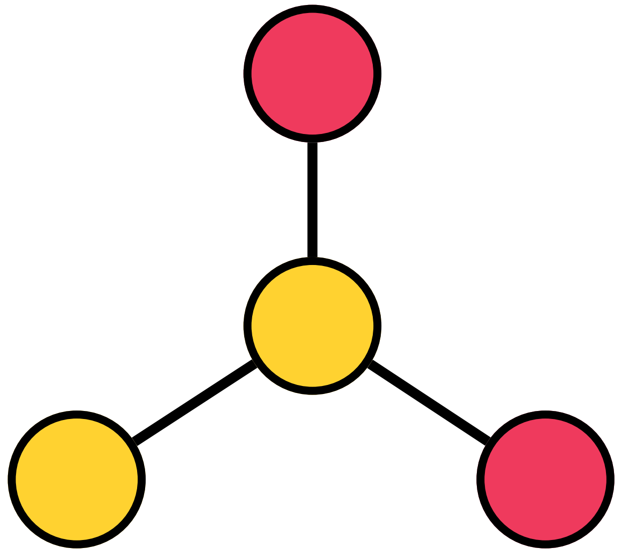
These properties are described by the Fractional SIR (FSIR) model

- individuals recover with probability μ
- instead of the infection rate β we use β/k_u
- when k_u is large, an individual is infected only if many of its contacts are infected

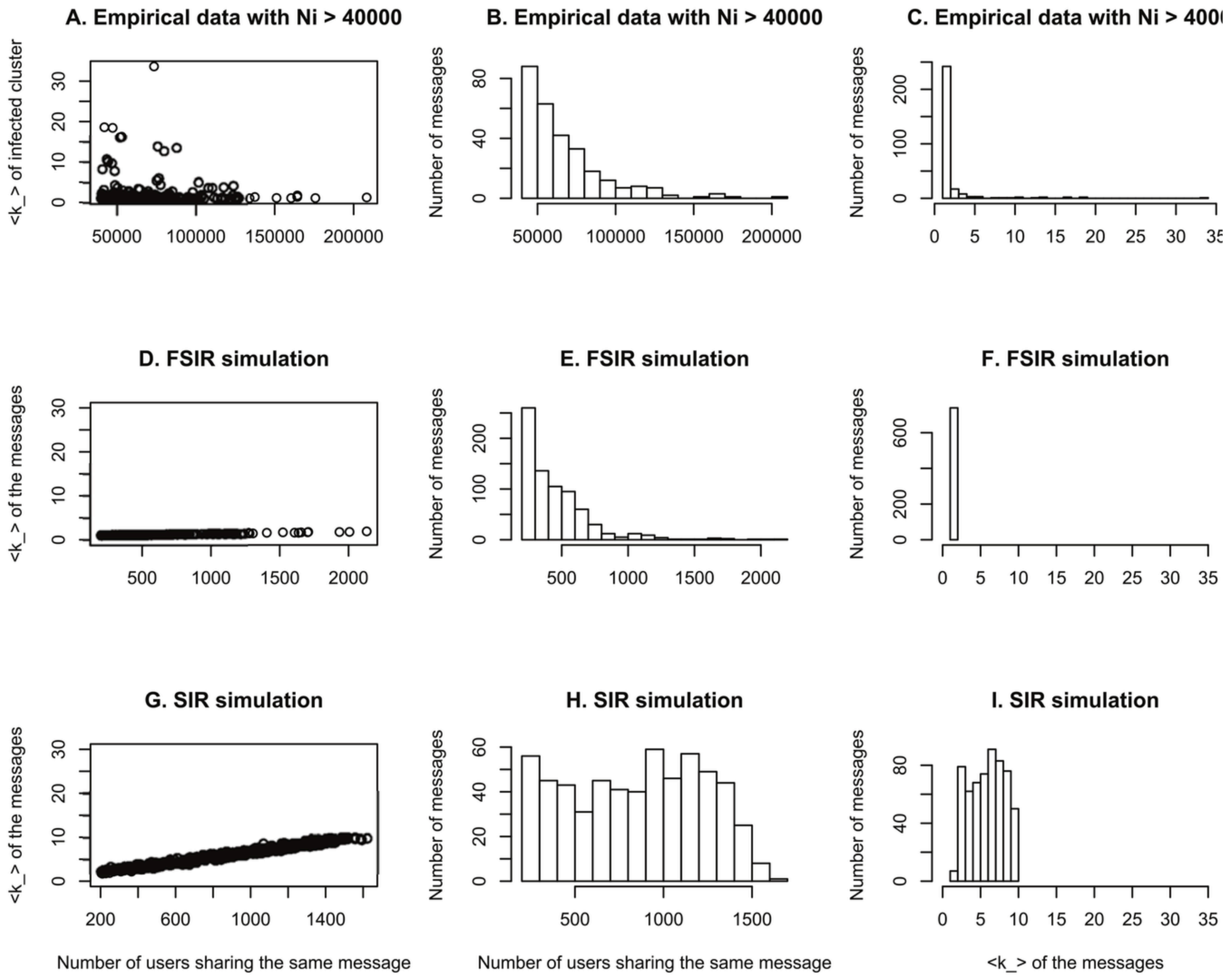
Prob. Infection = $2/5\ \beta$



Prob. Infection = $2/3\ \beta$



Testing the Model



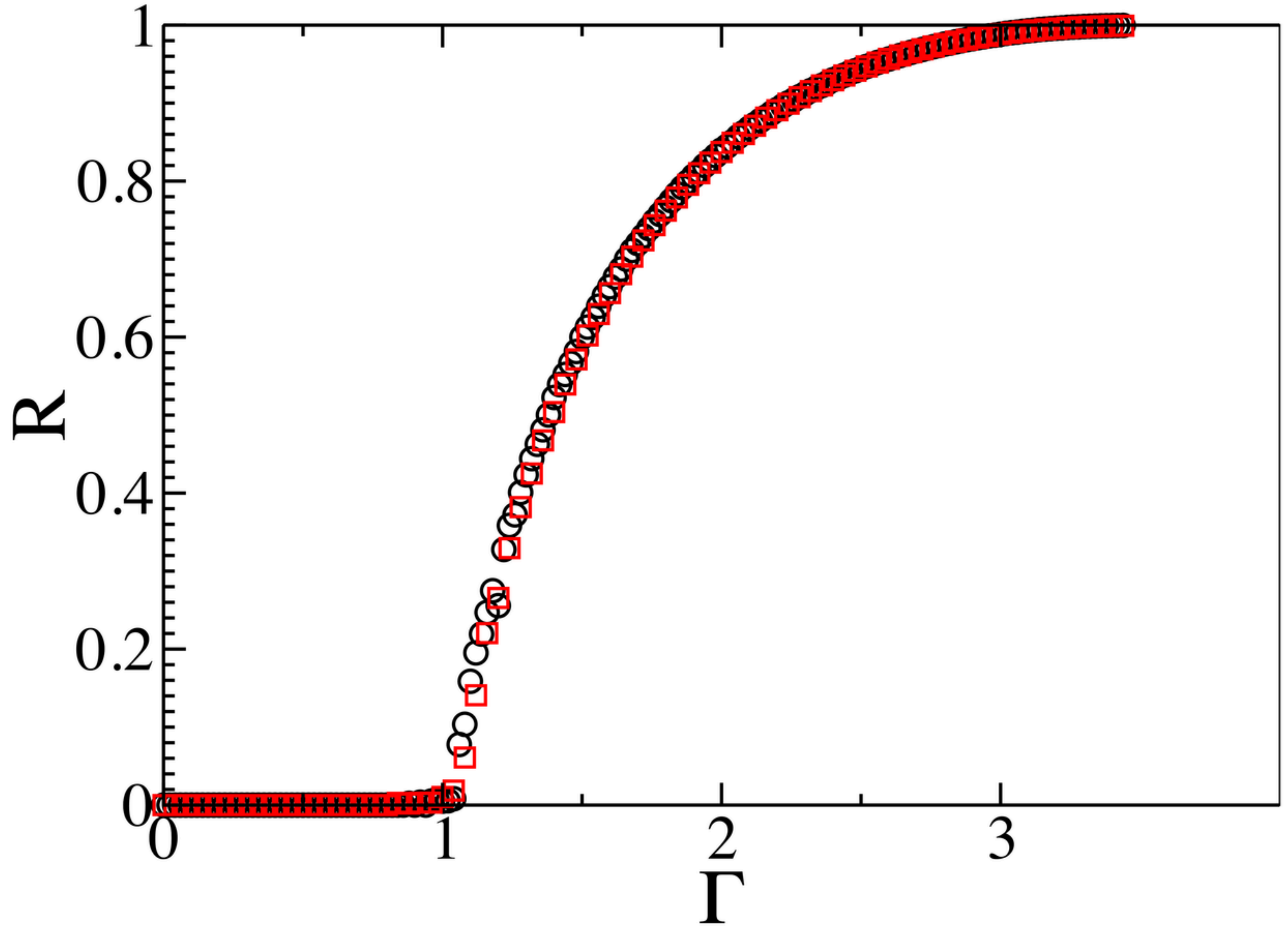
The authors compared the results of epidemic cascades in real data (Weibo social network) with those obtained using the SIR model and the FSIR model.

- The FSIR model better describes the data
- The SIR model creates cascades that are larger on average, but with much less viral content

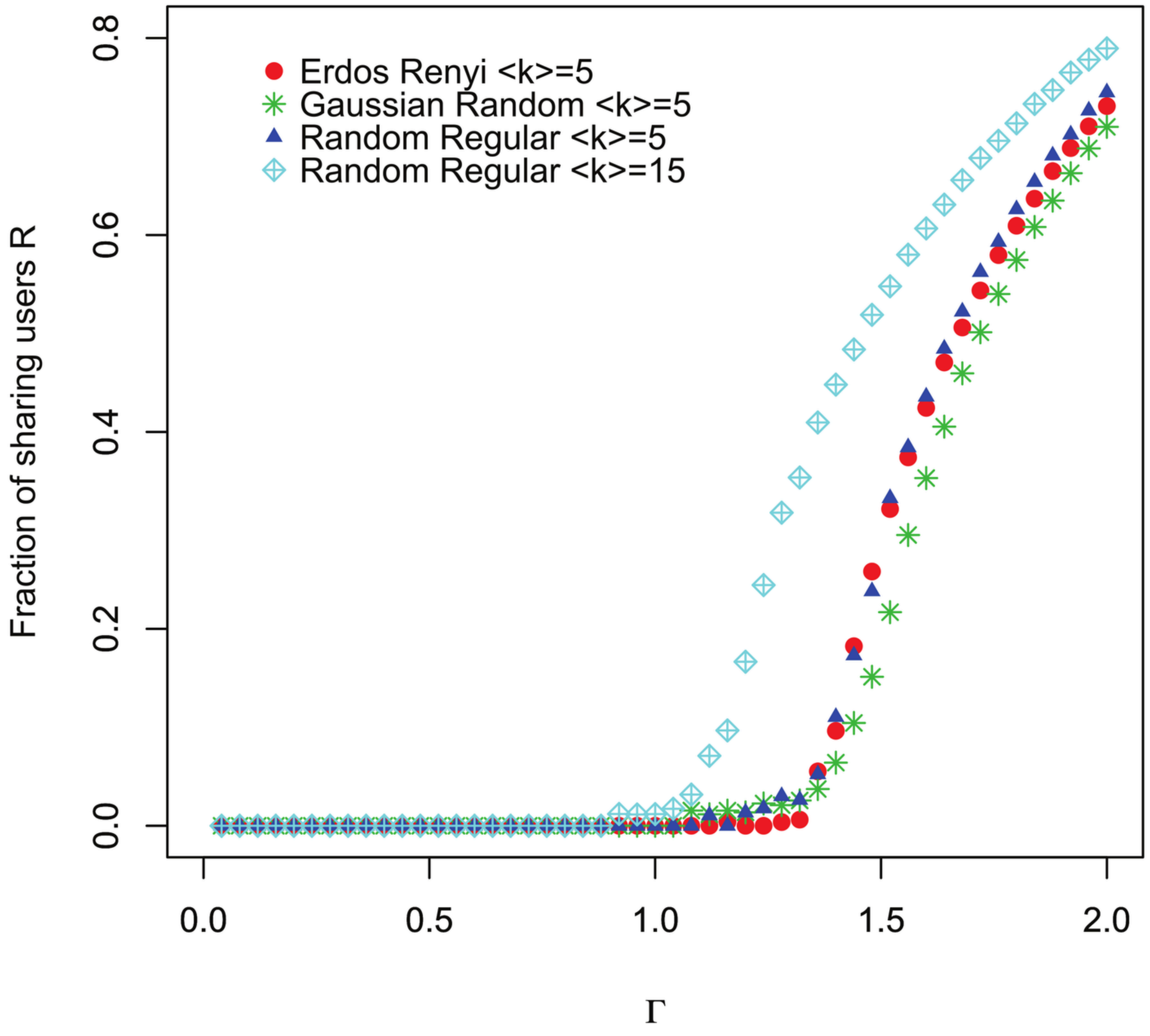
Phase Transition in the FSIR Model

The FSIR model shows a continuous phase transition in the variable $\Gamma=\gamma\tau$

- for small values of Γ there are no viral messages
- the ratio R of infected individuals is null
- for $\Gamma=1$ there is a phase transition and viral content appears
- when Γ is large some messages spread in the whole network



The Role of Topology



It is possible to compute the expression of the critical point analytically

$$\Gamma_c = \frac{\langle k \rangle}{\langle k \rangle - 1}$$

The critical point only depends on the average degree, not on the network topology. For large values of the degree, it tends to one as we already saw.

Conclusions

Processes on Complex Networks

Many processes take place on a network, in particular spreading processes that include epidemics and diffusion of viral content online

Epidemic Spreading

Epidemic spreading can be modeled at different levels. The most common approaches are the SI, the SIS and the SIR model

Epidemic Spreading on Networks

The network topology plays an important role in determining the size of the epidemics. In particular, on scale free networks the epidemic threshold is null

Complex Contagion

Behaviors spread differently from viruses. They follow complex contagion processes, where a single exposition is not enough for getting infected