CREDIT MARKET COMPETITION, CORPORATE INVESTMENT AND INTERMEDIATION VARIETY*

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Abstract

How does competition among financiers affect the nature of borrowers' investments in the economy and how does this, in turn, affect which financial intermediaries arise as lenders in equilibrium? This paper develops a general equilibrium model to address these questions. There are four main results. First, efficient project choices arise in equilibrium for only intermediate levels of competition. Entrepreneurs invest excessively in (riskier) specialized projects at low levels of credit market competition, and invest excessively in (safer) standardized projects at high levels of credit market competition. Second, the emergence of relationship lending eliminates the investment inefficiency for low levels of competition, but not the inefficiency for high levels of competition. Third, this residual investment inefficiency encourages the emergence of highly levered specialized intermediaries that resemble private equity firms and that eliminate the investment inefficiency at high levels of competition. Fourth, these private equity firms arise only in competitive credit markets and fund only specialized projects with high expected returns. The model thus explains the co-existence of arm's-length finance, relationship lending and private equity firms in general equilibrium.

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1 Introduction

There is a large literature which argues that credit market competition affects the supply of credit (e.g. Pagano (1993)) and that this has potentially important consequences for economic growth and welfare. Of course, given that credit market competition affects bank credit supply, it is likely that it also affects the project choices borrowers make in response to credit market conditions. For example, the Stiglitz and Weiss (1981) analysis suggests that higher loan interest rates can cause borrowers to choose riskier projects. Moreover, it is not hard to imagine that credit supply changes—induced by competition—can also affect the types of intermediaries that arise in equilibrium. For example, if credit supply from existing lenders shrinks for some borrowers, it may create incentives for new types of lenders to emerge in order to meet the unmet credit demand of these borrowers.[1] This intuition leads to the two main questions we address in this paper. First, how does credit market competition impact the *types* of projects entrepreneurs invest in? Second, how does this impact influence the *nature* and *variety* of financial intermediation that emerge in equilibrium?

We address these questions by developing a theoretical model in which competition among potential lenders affects the nature of corporate investment, and we analyze how this effect leads endogenously to the emergence of relationship banking[2] and other forms of specialized lending (e.g., private equity). We conduct this analysis in the context of a general equilibrium model with search frictions akin to those in the labor market. The basic insight of the paper is that different degrees of credit market competition lead to different types of investment inefficiencies on the part of borrowers, and that this generates incentives for the emergence of different types of financial intermediation to cope with these inefficiencies. The analysis sheds light on some existing evidence on relationship lending and private equity, and it produces new predictions.

The importance of the question we study is underscored by the fact that, in every country, banks play a dominant role in the allocation of credit, with significant consequences for corporate investment. Moreover, in large credit markets, like the US and the UK, the provision of financing by non-banks, such as private equity firms, is becoming increasingly important as credit markets become more competitive. It is widely recognized that credit market competition impinges on *how* lenders allocate credit and *how much* of it they allocate (see, for example, Cetorelli (2001), and Ratnovski (2013)).

[1] Agarwal, Skiba, and Tobacman (2009) document that borrowers who experience a decline in the availability of liquidity on their credit cards turn to (more expensive) payday loans. This provides one explanation for the prevalence of payday lending.

[2] See, for example, Boot and Thakor (2000) and Petersen and Rajan (1995) for research on relationship banking.

Despite the abundance of previous research on credit market competition, the questions we focus on remain unexplored, to the best of our knowledge. In particular, our analysis shows that one cannot just take the set of financial intermediaries in the economy as given and then look at the kinds of investment inefficiencies that exist among borrowers. Rather, it is a two-way street in general equilibrium—the investment inefficiencies that initially exist determine the variety of intermediaries that endogenously arise in response, and this then determines the residual investment inefficiency that remains.

In our model, each entrepreneur can choose between a "standardized" (low-risk) project and a "specialized" (or "differentiated," higher-risk) project. Specialized projects may be one of two types and the type is the private information of the entrepreneur; whereas, standardized projects have only one type. Therefore, specialized projects are "information-sensitive" and standardized projects are not. Each creditor can become either a bank or a highly-levered non-bank. We will argue that these non-banks strongly resemble private equity firms and, therefore, we will refer to them with the label "PE." A bank can choose between relationship lending and arm's-length finance (or "transaction lending," as in Boot and Thakor (2000)). In our analysis, the distinction between a transaction lender and a relationship lender is that a relationship lender may acquire proprietary information about a borrower in exchange for paying a private cost.[3] We also assume that the value comparison between the standardized and specialized projects is state-contingent. In some states of nature the standardized project has higher NPV and in others the specialized project has higher NPV. We refer to project choice as *efficient* if entrepreneurs always undertake the higher NPV project. The creditor commits to the type of financing before the entrepreneur chooses his project, and the entrepreneur's repayment obligation to the creditor is determined by bargaining over the division of the net surplus.[4] In particular, in the model, banks finance entrepreneurs using debt contracts and PEs finance entrepreneurs using equity. Thus, bargaining determines the face value of the debt that the entrepreneur owes the bank or the size of the equity stake the entrepreneur grants the PE. The relative bargaining positions of creditors and entrepreneurs are determined by competition in the credit market. We model this competition in a search-and-matching framework as the ratio of the supply of creditors to entrepreneurs' demand for funding (this is sometimes called

[3]This assumption finds justification with the micro-foundations in Boot and Thakor (1994), Boot (2000), Rajan (1992), and Sharpe (1990) who show that the repeated transaction between a relationship lender and a borrower leads to the generation of proprietary information about the borrower, giving the relationship lender an informational advantage over competing lenders, something that does not happen with arm's-length finance.

[4]Our main results are not sensitive to the assumption that creditors choose the type of financing they offer before entrepreneurs choose the type of project to undertake.

the "market tightness"). Thus, if there are few creditors and many entrepreneurs, the market is not competitive, whereas if there are many creditors and few entrepreneurs, the market is competitive.

We show the following four main results. First, for only intermediate levels of credit competition efficient project choices occur in equilibrium. For low levels of credit competition, entrepreneurs choose the specialized project excessively, and for high levels of credit competition, entrepreneurs choose the standardized project excessively. Second, relationship lenders help to eliminate the over-specialization problem but not the over-standardization problem. Third, highly levered non-bank intermediaries ("PEs") attenuate the residual over-standardization problem. And, fourth, PEs emerge at only high levels of competition. When PEs emerge, they induce an equilibrium separation in the credit market—banks fund standardized projects and PEs fund specialized projects. PEs never completely displace banks, however, due to a general equilibrium feedback effect.

The key to understanding these results is to note first that the reason why credit market competition affects the investment decision of entrepreneurs is that it affects their bargaining positions. As credit competition increases, it becomes easier for entrepreneurs to find creditors, putting them in stronger bargaining positions with creditors when they negotiate the division of surplus. However, increasing credit market competition affects entrepreneurs with standardized projects differently from entrepreneurs with specialized projects. Because specialized projects are information-sensitive, whereas standardized projects are not, adverse selection is an impediment to locating funding for entrepreneurs with specialized projects even in competitive credit markets. As a result, the bargaining positions of entrepreneurs are elevated more by increased credit market competition when they have standardized projects than when they have specialized projects. This means that increases in credit competition make standardized projects more attractive relative to specialized projects from the entrepreneur's perspective. Further, since credit market competition is a key determinant of the division of surplus and of entrepreneurs' project choices, it affects both the creditor's decision about whether to be in banking or in PE, and the bank's decision about whether to make arm's-length or relationship loans.

We now explain our first main result: if banks offer only arm's-length finance, the general equilibrium involves the entrepreneur making the efficient project choice only for intermediate levels of credit market competition. When competition is sufficiently low, the dominant effect in the entrepreneur's project choice is risk-shifting—borrowing via debt means that the entrepreneur has an incentive to shift risk, preferring a negative-NPV specialized project even when an alternative higher-NPV standardized project is available. As credit-market competition increases, the standardized project, which is not information-sensitive, makes it easier for the entrepreneur to access low-cost funding, therefore becoming relatively more attractive. So, an increase in credit-market competition dampens the entrepreneur's incentive to shift risk, and eventually induces efficient project choice. However, when competition is sufficiently high, this effect induces the firm to choose the standardized project even when the specialized project has a higher value.

Now turn to our second main result, that these inefficiencies create an economic force for the emergence of a relationship lender. In contrast to an arm's-length lender, a relationship lender observes the quality of the entrepreneur's specialized project, so it simply refuses to fund specialized projects with negative NPV. This eliminates the entrepreneur's risk-shifting benefit, preventing overnivestment in the specialized project for low levels of credit-market competition. However, we find that in equilibrium the relationship bank's superior information fails to mitigate the over-standardization problem.

Our third main result is that this *residual* inefficiency encourages the emergence of highly-levered intermediaries. The key feature of these intermediaries is that they raise outside debt to finance the purchase of *equity stakes* in entrepreneurs. Sufficiently high leverage makes them wish to fund only projects with sufficiently high upside, so high leverage emerges endogenously as a *commitment device* for these intermediaries to credibly commit to deny funding to entrepreneurs with standardized projects and to fund only specialized projects. In other words, intermediary leverage serves to discipline entrepreneurs, preventing them from standardizing excessively. Whereas relationship lending fails to mitigate the over-standardization problem because lenders cannot credibly precommit to deny funding to standardized projects, these equity-purchasing intermediaries' own high leverage makes this precommitment possible.

Finally, these highly-levered intermediaries emerge as lenders only when credit market competition is sufficiently high, because under such competitive conditions the rents from traditional banking are low enough that it pays off for the creditor to specialize and become a PE. Further, it is for this reason that the proportion of finance provided by PEs increases as credit competition increases. However, the rate of increase decreases and PEs remain scarce even in the perfect competition limit. This is due to the presence of a general equilibrium feedback loop, which works as follows. As the credit market becomes more competitive, creditors specialize to become highly levered intermediaries, and more intermediation capital shifts to alternative forms of finance. This reduces competition in the traditional banking sector, making banking again more attractive, and ensuring that it does not vanish as competition increases.

In equilibrium, these highly levered intermediaries endogenously have the following properties: they are highly levered; they are scarce; they have high returns; and they invest in specialized projects. These attributes match those of real-world private equity firms (see, for example, Harris, Jenkinson, and Kaplan (2014), Kaplan and Strömberg (2009) and Lerner, Sorensen, and Strömberg (2011)). Hence, we refer to these highly-levered intermediaries as PEs, although we are open to alternative interpretations of them.[5] The label one puts on these intermediaries is less important to our analysis than the economic function these intermediaries serve in our model. However, our interpretation of high PE leverage is also consistent with the compensation of general partners in a PE firm having an equity-like claim and limited partners (who provide most of the capital) having a concave claim (resembling debt); see Axelson, Strömberg, and Weisbach (2009).

Our paper is related to the literature on how credit market competition affects lending. Pagano (1993) develops a model in which lower interbank competition leads to lower equilibrium lending and lower economic growth, a conclusion verified by Guzman (2000) in a general equilibrium model of capital accumulation. However, arrayed against this conclusion are numerous other papers with different results. Shaffer (1997) shows that a more competitive banking system may end up funding a lower-quality borrower pool. Cao and Shi (2000) argue that higher competition would increase loan rates and reduce loan supply. Dell'Ariccia (2000) develops a credit screening model to show that higher competition among banks may dilute their incentives to screen borrowers. Similarly, Manove, Padilla, and Pagano (1998) show that, compared to a competitive banking system, banks in a monopolistic system will screen borrowers more and accept less collateral. There are also numerous theoretical papers that have examined the effect of interbank competition on risk-taking. Some papers have formalized the intuition that, by diminishing the charter values of banks, increased interbank competition generates incentives for banks to take higher risk (e.g. Hellmann, Murdock, and Stiglitz (2000) and Repullo (2004)). However, some others have argued the opposite—by lowering the interest rates that banks charge their borrowers, increased interbank competition can induce *borrowers* to take less risk, thereby diminishing the default risk banks face (e.g. Boyd and De Nicoló (2005)). Martinez-Miera and Repullo (2010) extend this logic to show an inverse U-shaped relationship between bank competition and stability.

Our approach to the question of how competition affects the credit market differs from the previous literature in two significant respects. First, we are concerned with

[5]When we model PE leverage, we have in mind a leveraged buyout. This occurs when an investor, typically a financial sponsor, acquires a controlling interest in a company's equity and it finances a significant percentage of the purchase price through leverage.

how competition impacts the *types* of projects entrepreneurs invest in. Second, we also focus on how this impact leads to the *endogenous* emergence of alternatives to arm's-length bank credit, such as relationship banking and private equity in a general equilibrium framework.

Our paper is also related to theories that examine how interbank competition affects banking industry risk. There is a strand of this literature which argues that monopolistic banking systems are the safest (e.g. Hellmann, Murdock, and Stiglitz (2000), Matutes and Vives (2000), and Repullo (2004)). By contrast, in our analysis such banking systems are excessively risky since specialized projects are riskier than standardized projects. Our model predicts that the most competitive banking systems will involve the least risk, which sheds light on the country-level empirical evidence presented by Schaeck, Cihak, and Wolfe (2009) that more competitive banking systems are less prone to systemic crises. However, the caveat suggested by our theory is that this attainment of safety comes at the cost of over-investment in standardized projects. Our result is also consistent with the empirical finding that firms invest less in R&D-intensive projects when credit competition is high (e.g. Hombert and Matray (2013)).

Another interpretation of our main result is that bank loan portfolios will become most liquid when interbank competition is the highest. This is because a standardized loan is more easily transferable and hence more liquid than a specialized loan. This seems to accord well with casual observation, but we are not aware of existing empirical evidence on this implication.

In addition to the papers discussed above, our work is related to the vast literature on relationship banking (e.g. Berlin and Mester (1992), Boot and Thakor (1994, 2000), Inderst and Mueller (2004), Petersen and Rajan (1995), Rajan (1992), and Sharpe (1990); see Boot (2000) for a review). It is also related to how search frictions in the credit market (e.g. Diamond (1990)) affect credit outcomes, as in Wasmer and Weil (2004).

One of our marginal contributions to this literature is to show that the simplest type of credit intermediation—arm's length finance—creates inefficiencies in project choice, and this creates an incentive for other forms of intermediation to arise naturally to reduce these inefficiencies. This allows us to rationalize relationship banks and PE firms. Our result that the leverage of PE firms helps to attenuate the over investment in standardize projects contrasts with other theories of leverage as a disciplining device (e.g., Hart and Moore (1995)), since it is *intermediary leverage*, not firm leverage, that leads to efficient project choice.

Our paper is also related to the literature on private equity. Campello and Rafael (2010), Chan (1983), Ueda (2004), and Winton and Yerramilli (2008) all provide theories of venture capital and its competitive role in credit allocation vis-à-vis banks. In a security design framework, Axelson, Strömberg, and Weisbach (2009) demonstrate the importance of leverage for private equity firms. Empirical evidence that private equity investments have outperformed public markets appears in Harris, Jenkinson, and Kaplan (2014). Metrick and Yasuda (2010) document that the buyout business in private equity is more scalable than the venture capital business. Metrick and Yasuda (2011) provide a review. Our marginal contribution to this literature is an endogenous rationale for private equity as a function of credit-market competition in a setting in which agents can also form banks.

The rest of the paper is organized in four remaining sections. Section 2 introduces the "toy model." Section 3 develops the actual model. Section 4 contains the analysis. Section 5 develops the extension to analyze private equity. Section 6 concludes. All formal proofs are in the Appendix.

2 Toy Model

In this section we introduce a simplified version of the model that brings out in a straightforward manner the intuition of how the entrepreneur's project choice depends on credit market competition. We derive the first main result that efficient project choice occurs for only intermediate levels of credit market competition within the framework of the toy model. However, to derive the results about intermediation variety, namely when relationship lenders and other specialized lenders emerge, we will require the additional richness of the full model that is developed in the next section.

At the core of the model is the project choice of an entrepreneur who needs outside capital to fund his investment. The entrepreneur chooses between two projects, called *standardized* and *specialized* (or "differentiated"). Both projects cost I to implement. The standardized project has positive NPV with a deterministic cash flow. This project generates $V_{\mathrm{s}} > I$ for sure. The specialized project, in contrast, is both information-sensitive and risky.[6] To capture information sensitivity, we allow the specialized project to be one of two types, which we refer to as "high" and "low." Both the high-type and the low-type specialized projects have binary risky cash flows. They pay off V_{d} when they succeed and zero otherwise. The difference between the types is the success probability. The high-type project yields V_{d} with probability p_h, whereas the low-type project yields V_{d} with probability $p_\ell < p_h$. We assume that the high-type specialized project has a higher NPV than the standardized project and the low-type specialized project has a

[6] It is not crucial to our analysis that the standardized project is safe. What is important is that it is not informationally sensitive, and that it is less risky than the specialized project.

lower NPV, namely

$$p_\ell V_{\mathrm{d}} < V_{\mathrm{s}} < p_h V_{\mathrm{d}}.$$

This assumption implies that *efficient investment requires adapting to circumstances*—the specialized project is better than the standardized project in some circumstances and worse in others. Thus, an entrepreneur with a high-type specialized project should undertake it, while an entrepreneur with a low-type specialized project should undertake a standardized project. All agents are risk neutral and the riskless rate is zero.

The sequence of events is as follows. The entrepreneur first chooses whether to invest in a standardized or a specialized project and then borrows from the creditor via a debt contract with face value F. While creditors can observe whether the entrepreneur has chosen a standardized or a specialized project, they cannot observe whether the specialized project is the low or the high type. Credit competition affects project choice via its effect on the terms of debt, i.e. on this face value F. For simplicity, we assume here that an entrepreneur and a creditor divide the net surplus of their relationship fifty-fifty, where the net surplus is the total value created by the funded project minus the values of the outside options of the two parties. Thus, the outside options will be essential in determining the terms of debt F. We make the following assumptions about the players' outside options, which we micro-found in the full model.

Toy Model Assumption 1. *The creditor's outside option is the value I of its capital.*

The motivation for this assumption is that if the creditor does not lend, the amount I can be "stored" at a zero riskless rate.

Toy Model Assumption 2. *The specialized project has negative NPV on average (when averaged across the low and high types).*

Toy Model Assumption 3. *If the entrepreneur chooses the specialized project, his outside option is zero, written $\pi_e^d = 0$, regardless of the degree of credit market competition.*

The motivation for this assumption is that the specialized project is information-sensitive, and, since by Toy Model Assumption 2 it has negative NPV on average, adverse selection in the credit market (the inability of financiers to tell whether the low type or the high type specialized project was chosen) will make it impossible for the entrepreneur to find funding for his project elsewhere.[7] Thus, if he chooses the specialized project, he is captive to his creditor.

[7]While this is an assumption in the toy model, we micro-found the adverse selection intuition above in the full model (see Subsection 4.1.1 and Appendix A.8).

Toy Model Assumption 4. *If an entrepreneur chooses the standardized project, his outside option, labelled π_e^{s}, increases in value with the competitiveness of the credit market. Further, $\pi_e^{\mathrm{s}} < V_{\mathrm{s}} - I$.*

The motivation for this assumption is as follows. Since the standardized project is information-insensitive, adverse selection will not impede the funding of the entrepreneur's project by the creditor. When the credit market is competitive, it will be easy to find another creditor to fund the project and the entrepreneur's outside option will be high. When the credit market is not competitive, it will be difficult to find another creditor, and the entrepreneur's outside option will be low. Note that the positive monotone relationship we assume between π_e^{s} and credit market competition allows us to use π_e^{s} as a direct proxy for credit market competition.[8] The last part of the assumption, $\pi_e^{\mathrm{s}} < V_{\mathrm{s}} - I$ is a more technical condition. It implies that there is always surplus created by funding the project, so there is always lending in equilibrium.

Toy Model Assumption 5. *The creditor cannot credibly precommit to finance only a pre-determined project.*

This assumption says that the entrepreneur and the creditor cannot agree to a contract that specifies that the entrepreneur will choose a pre-specified project. In particular, if such a contract were written it could be renegotiated—if the entrepreneur asked the creditor to finance a different project from that contracted upon, the creditor would do so as long as it was still more profitable for him than not funding the entrepreneur. In other words, this assumption says that both the entrepreneur and the creditor are free to pursue subgame perfect policies even if they are not ex ante efficient.

We now derive the first main result: for only intermediate values of credit competition will the entrepreneur choose the efficient project. Suppose that an efficient equilibrium exists. We will show how too much or too little credit competition leads the entrepreneur to deviate from his strategy in this conjectured equilibrium. Recall that π_e^{s} proxies for competition. The first step is to calculate the face values of debt F_{s} and F_{d} that the entrepreneur must promise to repay in order to fund the standardized and specialized projects respectively. Recall that the entrepreneur and the creditor divide the net surplus fifty-fifty. The net surplus from standardization is $V_{\mathrm{s}} - I - \pi_e^{\mathrm{s}}$, so F_{s} returns half the net surplus to the creditor on top of its outside option I:

$$F_{\mathrm{s}} = I + \frac{V_{\mathrm{s}} - I - \pi_e^{\mathrm{s}}}{2}.$$

[8] In the full model, we model credit market competition within a search framework and explicitly demonstrate this connection with π_e^{s}.

The conjectured equilibrium is separating, so only high-type entrepreneurs choose the specialized project. Thus, the net surplus given the specialized project is $p_h V_\mathrm{d} - I - \pi_e^\mathrm{d} = p_h V_\mathrm{d} - I$ since $\pi_e^\mathrm{d} = 0$ by assumption. Now the creditor must receive its investment I plus half the net surplus, so

$$p_h F_\mathrm{d} = I + \frac{p_h V_\mathrm{d} - I}{2}.$$

For the efficient outcome to be incentive compatible, the entrepreneur with the high-type project must choose to specialize. He chooses the specialized project as long as

$$p_h (V_\mathrm{d} - F_\mathrm{d}) \geq V_\mathrm{s} - F_\mathrm{s},$$

which can be rewritten as

$$p_h V_\mathrm{d} \geq V_\mathrm{s} + \pi_e^\mathrm{s}$$

or, defining $\Delta V := p_h V_\mathrm{d} - V_\mathrm{s}$,

$$\pi_e^\mathrm{s} \leq \Delta V$$

which says that the entrepreneur with a high-type project specializes only if credit competition is such that the value of his outside option with a standardized project does not exceed the difference between the value of the high-type specialized project and the value of the standardized project. Since the value of the entrepreneur's outside option with the standardized project (π_e^s) is increasing in credit market competition, the above inequality will be violated for sufficiently high competition, causing the entrepreneur to choose the standardized project inefficiently.

Next, when is it incentive compatible for the entrepreneur with the low-type project to standardize? He chooses the standardized project as long as

$$V_\mathrm{s} - F_\mathrm{s} \geq p_\ell (V_\mathrm{d} - F_\mathrm{d})$$

which can be rewritten as

$$V_\mathrm{s} - I + \pi_e^\mathrm{s} \geq \frac{p_\ell}{p_h}(p_h V_\mathrm{d} - I).$$

Here the right-hand side is the payoff of an entrepreneur with a low-type project from borrowing at the terms of the entrepreneur with a high-type project (remember that in the conjectured equilibrium only high types specialize). When competition π_e^s is high, the inequality is satisfied because the entrepreneur's strong bargaining position from standardization results in the efficient choice of the standardized project. When competition is low, however, the inequality is violated and the entrepreneur inefficiently chooses the specialized project.

The analysis above implies immediately that entrepreneurs with both types of projects invest efficiently for only intermediate levels of credit competition, i.e. only if

$$\frac{p_\ell}{p_h}(p_h V_{\mathrm{d}} - I) - (V_{\mathrm{s}} - I) \leq \pi_e^{\mathrm{s}} \leq \Delta V. \tag{1}$$

Both these inefficiencies—over-standardization at high levels of credit market competition and over-specialization at low levels of credit market competition—stem from the entrepreneur attempting to obtain better funding terms. This is the core intuition of our first main result and the starting point for an analysis of the question of whether alternative forms of finance mitigate these inefficiencies, which we undertake in the full model developed in the next section.

The toy model relies on a number of simplifying assumptions that are quite strong: (1) the creditor and entrepreneur split the surplus fifty-fifty; (2) the specialized project has negative NPV on average; (3) the outside option of the entrepreneur with the specialized project is zero; and (4) the outside option of the entrepreneur with the standardized project proxies for credit market competition. The full model is substantially richer in the sense that we either microfound or relax each of these assumptions using a dynamic search-and-matching framework. Apart from the main result above, the full model introduces many additional results, which were discussed in the Introduction. Some of these results rely on a general equilibrium feedback loop that cannot be captured in the toy model.

3 The Full Model

In this section we develop the full version of the model with only banks. It is an infinite-horizon model with search-and-matching. We first describe the agents and projects, then the search market, and then the timing of moves within matches.

3.1 Agents and Projects

Time is discrete and the horizon is infinite. $t \in \{..., -1, 0, 1, ...\}$ denotes the date. There are two kinds of players, creditors (referred to as "c") and entrepreneurs (referred to as "e"). All players are risk-neutral and discount the future at net rate r equal to the return on the money market account. A creditor c provides start-up capital I to a penniless entrepreneur e to fund a project δ. After receiving credit, the entrepreneur e makes an irreversible choice between two projects, called *standardized*, $\delta = \mathrm{s}$, and *specialized*, $\delta = \mathrm{d}$. A standardized project is riskless and information-insensitive. It pays off V_{s} for sure. In contrast, a specialized project is risky and information-sensitive—its cash flow is random and the probability distribution is e's private information. Specifically, a specialized project is one of two types, $\tau \in \{h, \ell\}$, and e will privately observe τ before choosing $\delta \in \{\mathrm{d}, \mathrm{s}\}$. The h-type specialized project pays off V_{d} with probability p_h and zero otherwise, whereas the ℓ-type specialized project pays off V_{d} with probability $p_\ell \in (0, p_h)$ and zero otherwise. The probability that the project is type h is $\alpha \in (0, 1)$. All random variables are pairwise independent. If a project is funded at date t, its cash-flow occurs at the end of the period.

Before granting a loan, a creditor chooses the type of credit η to provide. It either offers *relationship lending*, $\eta = \mathrm{R}$, or *arms-length finance*, $\eta = \mathrm{A}$. The difference between a relationship lender and an arm's-length lender is that the relationship lender can observe the specialized entrepreneur's type, whereas an arm's-length lender cannot.[9] The creditor can always offer arm's-length finance at no cost, but to perform relationship lending it must pay a cost k. Finally, the cost k is entrepreneur-specific, in other words, it allows the lender to learn the type of only one entrepreneur's project.[10]

The entrepreneur applies for credit first and then makes his irreversible project choice in the first round of seeking credit. If the entrepreneur is unable to obtain credit, he searches again; the formal structure of this search-and-matching processes is described later in this section. By the time this next search occurs, the entrepreneur's project choice has already been made (since this choice is irreversible), and it is common knowledge whether the entrepreneur has chosen the standardized or the specialized project.

We now impose restrictions on parameters to capture what we think are two key features of project choice and relationship banking. First, we note that efficient investment requires adapting to circumstances—what may be a good investment in one set of circumstances may be a poor choice in a different set of circumstances. To capture this in the model, we impose conditions so that if an entrepreneur has an h-type specialized project the efficient project is the specialized project, whereas if an entrepreneur has an ℓ-type specialized project the efficient project is the standardized project.[11] Second,

[9] The idea that a relationship lender can obtain proprietary information about the borrower during the course of the relationship is well-established in the literature; see Boot (2000), for example.

[10] The motivation for the cost k is that relationship lending requires the creditor to make investments that are specific to the borrower (see Boot (2000)). For example, the creditor may invest in learning about the borrower's products, customers and competitors.

[11] One can think of this concretely through examples. In a state of the world in which the price of oil is very high, the demand for electric cars may be so high that it pays for an entrepreneur to specialize in order to

we observe that one aspect of relationship lending is that it can increase the NPV of entrepreneurs' projects, i.e. banks can enhance the economic surplus related to their borrowers' projects through relationship lending. The notion that relationship lending adds value to the borrower is a consistent theme in the literature (see, for example, Boot (2000)). However, many different approaches have been used to model the way in which this value enhancement occurs. For example, in Boot, Greenbaum, and Thakor (1993), it is the contractual flexibility of relationship banking that adds value. In Boot and Thakor (1994), the value enhancement comes from a repeated borrowing relationship that results in a reduction in the collateral the borrower must post after establishing a good credit record. In Petersen and Rajan (1995), relationship lending permits the design of intertemporal taxes and subsidies in loan contracts that generate higher surplus than is possible with single-shot contracting.[12] The approach we use is closest to that in Degryse and Van Cayseele (2000) who propose that the offering of multiple services by a bank to the the *same* borrower—such as letters of credit, deposits, check clearing and cash management—in addition to the loan expands the information about the borrower available to the bank and hence permits the bank to add more value to the relationship. Specifically, we capture this in reduced form by assuming that the proprietary information generated by the relationship permits the bank to help the entrepreneur implement a more efficient project choice, and that the efficiency gain attributable to this exceeds the cost of relationship lending. We now formalize these restrictions in the assumptions below.

Assumption 1. *An h-type specialized project has the highest present value, but the standardized project has a higher present value than the ℓ-type specialized project, which has negative NPV, or*

$$p_h V_{\mathrm{d}} - I > V_{\mathrm{s}} - I > 0 > p_\ell V_{\mathrm{d}} - I. \tag{2}$$

Assumption 2. *A standardized project a has higher present value than the average specialized project, or*

$$V_{\mathrm{s}} > \alpha p_h V_{\mathrm{d}} + (1-\alpha) p_\ell V_{\mathrm{d}}. \tag{3}$$

Finally, we assume that the efficiency gains from the efficient project choice exceed

differentiate from incumbents in the industry by investing in battery-powered all-electric cars. In some other state of the world, new oil fields may depress the price of oil so much that the demand for gasoline-powered cars is high, and it pays for the entrepreneur to invest in the standard automobile.

[12] This is not an exhaustive list of the ways in which relationship lending can add value. See Boot (2000) for more.

the cost of relationship lending, i.e.

$$\alpha p_h V_{\mathrm{d}} + (1-\alpha)V_{\mathrm{s}} - k > V_{\mathrm{s}}, \tag{4}$$

which we state more succinctly in the next assumption with the notation $\Delta V := p_h V_{\mathrm{d}} - V_{\mathrm{s}}$.

Assumption 3.

$$k < \alpha \Delta V. \tag{5}$$

The assumption above says that the benefit of flexibility outweighs the cost k of the relationship.

3.2 Search and Matching

Creditors and entrepreneurs find each other by searching in a decentralized market.

At time $t \in \{..., -1, 0, 1, ...\}$ a set E_t of searching entrepreneurs matches with a set C_t of creditors with intensity $m(|E_t|, |C_t|)$. Define $\theta_t := |C_t|/|E_t|$, the credit market competition. Assume the probability that a creditor finds an entrepreneur at time t,

$$q(\theta_t) := \frac{m(|E_t|, |C_t|)}{|C_t|}, \tag{6}$$

and the probability an entrepreneur finds a creditor at time t,

$$Q(\theta_t) := \frac{m(|E_t|, |C_t|)}{|E_t|}, \tag{7}$$

depend only on θ_t (for which, for example, m being homogeneous of degree one suffices).

Assume m is such that q and Q are differentiable with $q' < 0$, $Q' > 0$, $q(0) = 1$, $Q(0) = 0$, and with $q(\theta) \to 0$ and $Q(\theta) \to 1$ as $\theta \to \infty$. As credit competition increases, the likelihood that a creditor finds an entrepreneur decreases and the likelihood that an entrepreneur finds a creditor increases. To make the model stationary, assume that each player that leaves the market is replaced by a player of the same type.

3.3 Stage Game Extensive Form

When an entrepreneur e is born, he learns his type. When $e \in E_t$ matches with a creditor $c \in C_t$ for the first time, they play the extensive form game defined by the timing below:

1. c chooses between relationship lending and arm's-length finance, $\eta \in \{\mathrm{R}, \mathrm{A}\}$. R costs k and enables the creditor to observe the type of the specialized project.
2. e chooses between a specialized project and a standardized project, $\delta \in \{\mathrm{d}, \mathrm{s}\}$. The choice is irreversible.
3. If c has played $\eta = \mathrm{R}$, it observes the type $\tau \in \{h, \ell\}$ of the specialized project.
4. The face value of debt F is determined as follows
   - With probability β, c makes e a take-it-or-leave-it offer F_δ^c; e accepts or rejects.
   - With probability $1 - \beta$, e makes c a take-it-or-leave-it offer F_δ^e; c accepts or rejects.
5. The project pays off $V_\delta \in \{V_\mathrm{s}, V_\mathrm{d}\}$. e and c divide the surplus according to the agreed upon contract.

If the relationship between c and e breaks down, they search again in the market. Since e's choice δ is irreversible, it remains to specify the game played between an entrepreneur committed to a project δ and his new potential creditor c'.

1. c' observes δ.
2. c' chooses between relationship lending and arm's-length finance, $\eta \in \{\mathrm{R}, \mathrm{A}\}$. R costs k and enables the creditor to observe the type of the specialized project.
3. The face value of debt F is determined as follows
   - With probability β, c makes e a take-it-or-leave-it offer $F_\delta^{c'}$; e accepts or rejects.
   - With probability $1 - \beta$, e makes c' a take-it-or-leave-it offer F_δ^e; c' accepts or rejects.

If c and e are searching but are not matched at date t they search again. This sequence of moves is summarized in the timeline in Figure 1.

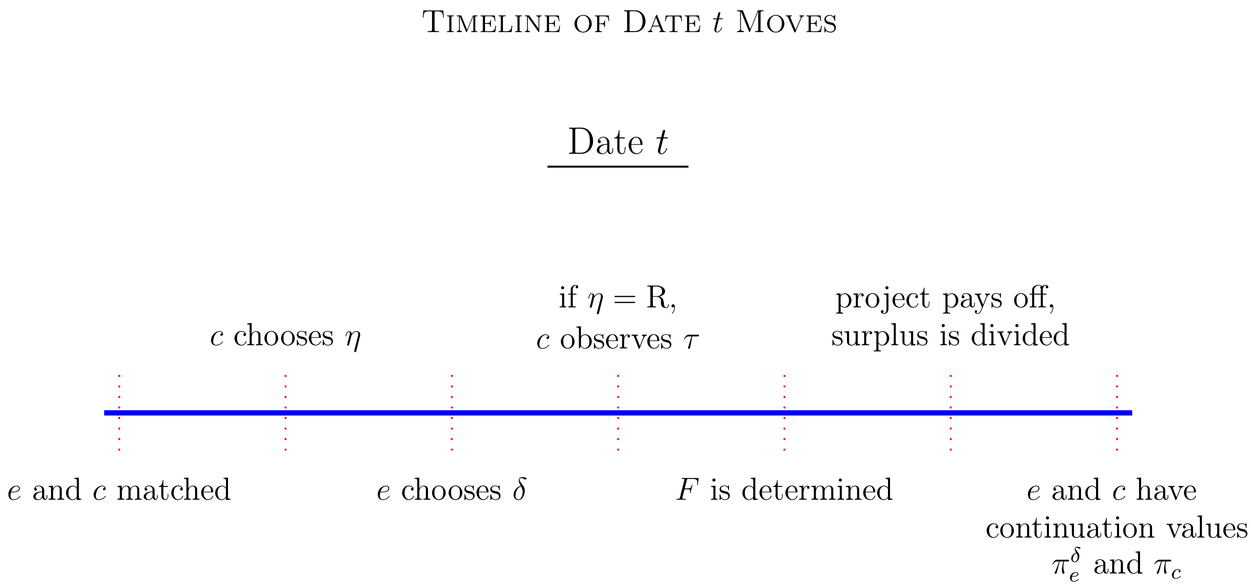

Figure 1: A timeline representation of sequence of moves described in Subsection 3.3.

3.4 Solution Concept

The solution concept we use is Perfect Bayesian Equilibrium.

We focus on pure stationary equilibria; as a result, the infinite horizon serves mainly as a way to endogenize the players' outside options.

4 Results

This section provides the analysis of the model and derivates the main results about project choice and relationship banking. We establish the results first in partial equilibrium, taking the players' continuation values as given. We then endogenize the continuation values and prove the results in terms of the ratio θ of creditors to entrepreneurs, which we call "credit market competition," with higher values of θ signifying a more competitive credit market. The ratio is sometimes referred to as "market tightness," with lower values indicating greater tightness.

4.1 Beliefs and Continuation Values for Already-specialized Entrepreneurs

In this subsection we specify the out-of-equilibrium beliefs of creditors when they encounter already-specialized entrepreneurs who have failed to obtain credit in previous rounds. This equilibrium refinement serves as a reduced-form way to capture the informational advantage of relationship lenders. Further, it induces a restriction on continuation values that we discuss later in the section.

4.1.1 Out-of-Equilibrium Beliefs

Since this is a dynamic game of asymmetric information, off-the-equilibrium path beliefs about the type τ of an entrepreneur's project play a role in the analysis. Throughout, we focus on equilibria supported by the following beliefs: if a creditor matches with an entrepreneur who has already specialized his project (because he failed to obtain funding from the creditor whom he was initially matched with), then the creditor believes the project is type ℓ. We summarize this with the following assumption.

ASSUMPTION 4. *If a creditor c' encounters an entrepreneur with an already specialized project, it believes the project is type ℓ, written as $\mu(\tau = \ell \mid \delta = \mathrm{d}) = 1$.*

We emphasize that this is not an assumption on primitives, but just a statement about which Perfect Bayesian Equilibria we focus on. With this restriction on the set of equilibria, we aim to capture the idea that creditors believe that if an entrepreneur has failed to find funding in the past, it is because previous creditors had private information about the quality of his project. In Appendix A.8 we use a game-theoretic refinement to show that this economic intuition is robust: if we impose a stronger equilibrium concept than Perfect Bayesian Equilibrium (which is akin to Sequential Equilibrium but still well-defined for games with infinite action spaces), then any efficient equilibrium *must* be supported by the beliefs in Assumption 4.

4.1.2 Continuation Values

When c and e are matched, their decisions about whether to engage in relationship lending and whether to undertake a specialized project depend on the proportion of the total surplus from the match that they anticipate earning. For each player, a higher outside option leads to a greater share of the net surplus.

At the time at which the face value of debt is determined, these outside options are equal to the players' continuation values from searching again in the market. Denote the continuation value of agent i by π_i. Thus, the continuation value of the creditor is π_c and the continuation value of the entrepreneur with project δ is π_e^δ. We emphasize that because his project choice is irreversible, the continuation value of the entrepreneur depends on his project choice. Note that, in general, these values could depend on time, but we suppress this possibility since we focus on stationary equilibria. Further, the continuation value of the entrepreneur with the specialized project could also depend on the type τ of his project. However, since the entrepreneur with the specialized project will have his credit completely rationed in the future, this will also not be the case, as the following lemma implies.

Lemma 1. *The continuation value of the already specialized entrepreneur is zero, i.e.* $\pi_e^{\mathrm{d}} = 0$.

4.2 First-best Outcome

The efficient outcome of this model involves the entrepreneur choosing the efficient project, namely $\delta = \mathrm{d}$ when $\tau = h$ and $\delta = \mathrm{s}$ when $\tau = \ell$ and the creditor playing $\eta = \mathrm{A}$, avoiding the cost k of relationship lending. Our question is when is this outcome $(\eta = \mathrm{A}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ implementable in the second-best case in which entrepreneurs cannot be bound to project choices and creditors do not suffer the cost of becoming relationship lenders? Specifically, for which values of credit competition θ can this outcome emerge in equilibrium? We now proceed to find conditions for it to be a stationary Perfect Bayesian Equilibrium of the model, given the out-of-equilibrium beliefs $\mu(\ell \,|\, \mathrm{d}) = 1$.

4.3 When Does the Second-best Equilibrium Implement the First-best Project Choice without Relationship Lending?

For the first-best outcome to be an equilibrium in the second-best case, e must self-select the efficient project without the discipline of his creditor—c cannot observe the type of the specialized project since $\eta = \mathrm{A}$. The main results of this subsection come from finding conditions for e's incentive constraints to be satisfied. We proceed in this subsection as follows. First, we characterize the face value of debt, depending on who is proposing the contract. Then we check the incentive compatibility constraints of the entrepreneurs and creditors to see when the first best can be implemented. Then we derive the value functions in a stationary equilibrium and show how they depend on credit market competition θ. Finally, we characterize the levels of credit market competition under which the first best is attainable in the second-best case.

Before finding these conditions, we emphasize the equilibrium beliefs for clarity. Since beliefs must be consistent in equilibrium, creditors that observe entrepreneurs choose d must believe they have h-type projects.

4.3.1 Face Values

The face value F of debt depends on which player is proposing the contract in round 4 of the stage game (Subsection 3.3).

The proposer always offers the face value that makes his opponent indifferent between accepting and rejecting. Thus there are four cases: (1) when e proposes and he has a standardized project, (2) when c proposes and e has a standardized project, (3) when e proposes and he has a specialized project, and (4) when c proposes and e has a specialized project.

We now compute each of these face values under the equilibrium beliefs. Note that when e chooses s, there is no asymmetric information, so the face value just serves as a means to divide the surplus—it will not enter substantively in the analysis. When the project is specialized, the face value of the debt contract will matter. Here the subscripts on the face values denote the project choice and the superscripts denote the proposer. When e proposes the face value with a standardized project is

$$F_{\mathrm{s}}^{e} = \pi_c \tag{8}$$

and when c proposes and e has chosen a standardized project, the face value is given by

$$V_{\mathrm{s}} - F_{\mathrm{s}}^{c} = \pi_e^{\mathrm{s}}. \tag{9}$$

When e proposes with a specialized project c gets repaid with probability p_h so the face value is given by

$$p_h F_{\mathrm{d}}^{e} = \pi_c \tag{10}$$

and when c proposes and e has chosen a specialized project, the face value is given by

$$p_h(V_{\mathrm{d}} - F_{\mathrm{d}}^{c}) = \pi_e^{\mathrm{d}}. \tag{11}$$

4.3.2 Incentive Constraints of Creditors and Entreprenuers

If c believes that everyone is playing according to the profile $(\eta = \mathrm{A}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$, it never has incentive to deviate to $\eta = \mathrm{R}$ since it comes with a cost k and no informational benefit. Thus, to determine when this outcome is an equilibrium, we can focus entirely on e's incentive constraints. We find conditions first for $\delta_\ell = \mathrm{s}$ and then for $\delta_h = \mathrm{d}$.

In order for $\delta_\ell = \mathrm{s}$, the entrepreneur must prefer to standardize when his project is type ℓ. Recall that in round 4 of the stage game the creditor proposes with probability β. Thus with probability β e receives his outside option and with probability $1 - \beta$ e

pushes c to his outside option. Hence, for e to choose to standardize with the ℓ-type specialized project, it must be that

$$\beta\pi_e^{\mathrm{s}} + (1-\beta)(V_{\mathrm{s}} - F_{\mathrm{s}}^e) \geq \beta\pi_e^{\mathrm{d}} + (1-\beta)p_l(V_{\mathrm{d}} - F_{\mathrm{d}}^e) \tag{12}$$

which, upon simplification, yields:

$$\beta\pi_e^{\mathrm{s}} + (1-\beta)\big(V_{\mathrm{s}} - \pi_c\big) \geq (1-\beta)\frac{p_\ell}{p_h}\left(p_h V_{\mathrm{d}} - \pi_c\right). \tag{IC$_\ell$}$$

For e to choose to specialize with the h-type specialized project it must be that

$$\beta\pi_e^{\mathrm{d}} + (1-\beta)p_h\left(V_{\mathrm{d}} - F_{\mathrm{d}}^e\right) \geq \beta\pi_e^{\mathrm{s}} + (1-\beta)\left(V_{\mathrm{s}} - F_{\mathrm{s}}^e\right) \tag{13}$$

which, upon simplification, yields:

$$(1-\beta)p_h V_{\mathrm{d}} \geq \beta\pi_e^{\mathrm{s}} + (1-\beta)V_{\mathrm{s}}. \tag{IC$_h$}$$

Combining the incentive constraints (IC$_\ell$) and (IC$_h$) implies that the first best can be attained in equilibrium if and only if

$$\left(1 - \frac{p_\ell}{p_h}\right)\pi_c - V_{\mathrm{s}} + p_\ell V_{\mathrm{d}} \;\leq\; \frac{1-\beta}{\beta}\,\pi_e^{\mathrm{s}} \;\leq\; \Delta V, \tag{14}$$

recalling that $\Delta V \equiv p_h V_{\mathrm{d}} - V_{\mathrm{s}}$. These inequalities are the analogs of the inequalities (1) in the toy model described in Section 2 above. Next, we find the values of π_c and π_e^{s} in terms of credit competition θ.

4.3.3 Value Functions

To show that real investment is choked off when credit competition is at the extremes (too low or too high), we compute the continuation values given that players believe $(\eta = \mathrm{A}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ is the stationary action profile of the stage game.

We look for stationary equilibria. The creditor has a single continuation value π_c, whereas the entrepreneur's continuation value depends on his project choice, $\delta \in \{\mathrm{s}, \mathrm{d}\}$. Lemma 1 fixes $\pi_e^{\mathrm{d}} = 0$. It remains to compute π_e^{s} and π_c. Note that π_c and π_e^{s} will be interdependent: a higher π_c lowers π_e^{s} because e anticipates having to give a larger share of the net surplus to c and vice versa.

We look for an equilibrium with no delay, so that on the $(\eta = \mathrm{A}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ equilibrium path, creditors always fund entrepreneurs the first time that they are matched. In order for this to be an equilibrium, it must be incentive compatible for the creditor to agree to fund a standardized entrepreneur rather than search again to wait for a specialized entrepreneur. As long as players are sufficiently impatient, this will always be the case. Specifically, to ensure that standardized projects are funded, we make the following assumption on parameters.

Assumption 5.

$$r > \frac{\alpha\beta\Delta V}{V_{\mathrm{s}} - I}.$$

Note that we view the time between dates as relatively long since it is the time taken to develop a lending relationship and implement a project. Thus we do not consider the assumption that r is large to be overly restrictive.

We now proceed to compute the value functions, maintaining the assumption that standardized entrepreneurs will find funding when they are matched. After we have computed the equilibrium value functions, we will see that Assumption 5 suffices for this indeed to hold in equilibrium.

Entrepreneur's Value Function With Standardized Project: The players' value functions are their expected utilities today from continuing the game. Consider first the standardized entrepreneur. He will be matched tomorrow with probability $Q = Q(\theta)$ and will remain unmatched with probability $1 - Q$. If he is unmatched, he searches again. Since the equilibrium is stationary, he obtains π_e^{s} in this case. If he is matched, the creditor is the proposer with probability $1 - \beta$. In this case again the entrepreneur obtains π_e^{s}. With probability β, however, the entrepreneur is the proposer and in this case he obtains $(1-\beta)(V_{\mathrm{s}} - \pi_c)$ (see Section 4.3.2 above). This description summarizes all the possibilities and allows us to write the expression for the value function of the entrepreneur with the standardized project, π_e^{s}, as:

$$\pi_e^{\mathrm{s}} = \frac{Q\big(\beta\pi_e^{\mathrm{s}} + (1-\beta)(V_{\mathrm{s}} - \pi_c)\big) + (1-Q)\pi_e^{\mathrm{s}}}{1+r},$$

or

$$\pi_e^{\mathrm{s}} = \frac{(1-\beta)Q}{r + (1-\beta)Q}\big(V_{\mathrm{s}} - \pi_c\big). \tag{15}$$

Creditor's Value Function: Next we derive the expression for the creditor's value function π_c. This value function is similar to π_e^{s} above, but it has two additional terms. The first additional term arises because the creditor is matched with an entrepreneur who has a project that can be one of two types—an h-type specialized project, in which case he plays $\delta = \mathrm{d}$, or, an ℓ-type specialized project, in which case he plays $\delta = \mathrm{s}$. The second additional term arises because the creditor earns interest on the capital I that it has not invested.

We now describe all of the terms that determine the creditor's value function. The creditor is matched with probability $q = q(\theta)$. With probability $1 - q$ it is unmatched and searches again to receive π_c, by stationarity. If it is matched, with probability α it is matched with an entrepreneur who has an h-type specialized project and who prefers to invest in this specialized project. In this case, e proposes with probability $1 - \beta$, leaving c with π_c. With probability β, c proposes and its utility is $p_h V_{\mathrm{d}}$, since $\pi_e^{\mathrm{d}} = 0$. Finally, consider the case in which c is matched with an entrepreneur who has an ℓ-type specialized project. Entrepreneur e then chooses to invest in the standardized project. Again, with probability $1 - \beta$, e proposes and c gets π_c. With probability β, c proposes and gets $V_{\mathrm{s}} - \pi_e^{\mathrm{s}}$. The final term is the interest rI that the creditor earns from holding his capital in the money-market account while searching. This description summarizes all the possibilities and allows us to write the creditor's continuation value π_c, as follows:

$$\pi_c = \frac{q\Big[\alpha\Big(\beta p_h V_{\mathrm{d}} + (1-\beta)\pi_c\Big) + (1-\alpha)\Big(\beta\big(V_{\mathrm{s}} - \pi_e^{\mathrm{s}}\big) + (1-\beta)\pi_c\Big)\Big] + (1-q)\pi_c + rI}{1+r}, \tag{16}$$

or

$$\pi_c = \frac{q\Big(\alpha\beta p_h V_{\mathrm{d}} + (1-\alpha)\beta(V_{\mathrm{s}} - \pi_e^{\mathrm{s}})\Big) + rI}{r + \beta q}. \tag{17}$$

We can now solve for the equilibrium value functions, which are the solution of the system of equations (15) and (17).

Lemma 2. *In a stationary equilibrium with action profile* ($\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s}$)*, the value functions are given by*

$$\pi_e^{\mathrm{d}} = 0, \tag{18}$$

$$\pi_e^{s} = \frac{(1-\beta)Q}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)}\Big(r\big(V_{\mathrm{s}} - I\big) - \alpha\beta q \Delta V\Big), \tag{19}$$

$$\pi_c = I + \frac{\beta q}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)}\Big(r\big(\alpha\Delta V + V_{\mathrm{s}} - I\big) + \alpha(1-\beta)Q\big(p_h V_{\mathrm{d}} - I\big)\Big). \tag{20}$$

One of the key shortcuts we took in the toy model of Section 2 was to assume that π_e^{s} proxied for competition. The next lemma says that π_e^{s} is increasing in competition θ, so the shortcut is now micro-founded.

Lemma 3. *In a stationary equilibrium with action profile* ($\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s}$), π_e^{s} *is increasing in* θ.

The intuition for this result is that when θ increases, it decreases the time an entrepreneur expects to wait before he is matched with another creditor. This means that increasing θ improves the entrepreneur's opportunity to find outside funding and thus strengthens his bargaining position against his current creditor.

4.3.4 Credit Market Competition For Which the Efficient (First-best) Outcome Emerges in Equilibrium

The inequalities (14) show that the efficient outcome can be supported in equilibrium for only intermediate values of π_e^{s}. Then, Lemma 3 shows that π_e^{s} indeed proxies for competition θ. This section shows that the intuition established in the toy model of Section 2 is robust: when competition θ is sufficiently high or sufficiently low, the efficient outcome is not an equilibrium. The next proposition summarizes this result given two conditions. These conditions are technical conditions that we use in the proof, which is in Appendix A.3.

Proposition 1. *If*

$$\left(1-\frac{p_\ell}{p_h}\right)\left(I+\frac{\beta}{\beta+r}\right)\left(\alpha\Delta V+V_{\mathrm{s}}-I\right)>V_{\mathrm{s}}-p_\ell V_{\mathrm{d}} \tag{21}$$

and

$$\frac{\beta}{1-\beta+r}\left(V_{\mathrm{s}}-I\right)>\Delta V, \tag{22}$$

then there is a stationary equilibrium with action profile ($\eta=\mathrm{a},\delta_h=\mathrm{d},\delta_\ell=\mathrm{s}$) *only if credit competition θ is neither too high nor too low.*

In the toy model, we assumed that the entrepreneur's outside option was zero if he undertook the specialized project, $\pi_e^{\mathrm{d}}=0$, and that the outside option π_e^{s} could proxy for competition θ (the Toy Model Assumptions 3 and 4). In the full model, we established these assumptions as results in Lemmas 1 and 3, respectively. Given these results, the intuition for why efficiency is lost in the face of extreme credit competition is the same as that presented in the analysis of the toy model.

4.4 Can the Second-best Equilibrium Implement the First-best Project Choice with Relationship Lending?

The result in the preceding subsection—entrepreneurs make inefficient project choices with arm's-length finance when credit market competition is at the extremes—suggests that relationship lending may emerge to attenuate the inefficiencies in project choice.

Thus, we now ask whether the creditor's option to develop a relationship with the entrepreneur can restore efficient project choice even when credit market competition is at one of the extremes. A glimpse of the answer to this question can be had by noting that since relationship lending entails an expense k for a creditor, efficiency cannot be fully restored. However, a welfare gain is possible with relationship lending since Assumptions 1, 2, and 3, say that

$$\big(\alpha p_h + (1-\alpha)p_\ell\big)V_{\mathrm{d}} < V_{\mathrm{s}} < \alpha p_h V_{\mathrm{d}} + (1-\alpha)V_{\mathrm{s}} - k, \tag{23}$$

which implies that the surplus gain from efficient project choice outweighs the cost of relationship lending. Thus, the second-best outcome is $(\eta = \mathrm{R}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$.

The question is: when does this second-best constitute a stationary equilibrium? The analysis to address this question proceeds along the same lines as the previous subsection. First, we examine off-equilibrium-path beliefs that the creditor has when it chooses arm's-length finance and the entrepreneur chooses a specialized project. Then we examine the incentive constraints of the entrepreneurs and creditors. This is followed by a derivation of the value functions, and finally the examination of the levels of credit market competition for which relationship lending reduces the project-choice inefficiency that occurs when creditors offer arm's-length finance.

4.4.1 Off-Equilibrium-Path Behavior

To find the conditions for this action profile to be part of an equilibrium, we must first specify the players' beliefs about behavior off the equilibrium path. When c plays $\eta = \mathrm{R}$, there is no asymmetric information between c and e on the equilibrium path, so we do not need to specify out-of-equilibrium beliefs. In contrast, there is asymmetric information off the equilibrium path when c deviates from relationship lending and plays $\eta = \mathrm{A}$. Thus, when we study the equilibrium with relationship lending, we must specify beliefs about a "double deviation"—i.e. when the creditor deviates and offers arm's-length finance, will the entrepreneur also deviate? More specifically, if c plays $\eta = \mathrm{A}$ and e plays $\delta = \mathrm{d}$, what does c believe about the type of e's project?

We will focus on equilibria in which e plays s (regardless of the type of his specialized project) following c playing A. By Assumption 2, this is the highest surplus deviation; therefore, it yields the most stringent sufficient conditions for $(\eta = \mathrm{R}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ to be an equilibrium. Further, these are the unique reasonable beliefs about off-path behavior as long as the entrepreneur with the h-type specialized project always prefers to play s rather than to pool with the entrepreneur with the ℓ-type project.[13] A sufficient condition for uniqueness is that the average NPV of specialized projects is low, which we make precise with the next assumption.

Assumption 6.

$$\Delta V < \left(\frac{1}{\alpha p_h + (1-\alpha) p_\ell} - 1 \right) I.$$

Further, we maintain our focus on equilibria in which a creditor that encounters an entrepreneur with a specialized project adopts the out-of-equilibrium belief that the project is type ℓ, $\mu(\tau = \ell \,|\, \delta = \mathrm{d}) = 1$.

4.4.2 Face Values

As in Subsection 4.3.1 above, the face value of debt depends on who is the proposer in round 4 of the stage game and on the project choice. Since we are looking for an equilibrium in which $\eta = \mathrm{R}$, the creditor also observes the type of the specialized project. Note that the entrepreneur with the ℓ-type specialized project will not obtain funding for it—it is negative NPV and the creditor observes the type τ. Thus, following $\eta = \mathrm{R}$, e always plays $\delta = \mathrm{s}$ if he has an ℓ-type specialized project. Thus, again there are four face values to compute: (1) when e proposes and he has a standardized project, (2) when c proposes and e has a standardized project, (3) when e proposes and he has an h-type specialized project, and (4) when c proposes and e has an h-type specialized project. The expressions for the face values are identical to those in Subsection 4.3.1. The key difference is that the continuation values π_e^{s} and π_c are different. Also, even though the notation is unchanged, it is useful to keep in mind that when the project is specialized, the relationship creditor observes whether $\tau = \ell$ or $\tau = h$ whereas this was not observed by the creditor in an arm's-length transaction. To summarize, the face values are

$$F_{\mathrm{s}}^{e} = \pi_c, \tag{24}$$

$$F_{\mathrm{s}}^{c} = V_{\mathrm{s}} - \pi_e^{\mathrm{s}}, \tag{25}$$

$$F_{\mathrm{d}}^{e} = \pi_c / p_h, \tag{26}$$

$$F_{\mathrm{d}}^{c} = V_{\mathrm{d}}. \tag{27}$$

4.4.3 Entrepreneurs' Incentive Constraints

As mentioned in the previous section, if c has played $\eta = \mathrm{R}$, then whenever e has an ℓ-type specialized project, he plays $\delta = \mathrm{s}$. Thus, to check whether there is a stationary

[13] If self-selected separation were possible we could implement first-best anyway.

equilibrium with action profile ($\eta = \mathrm{R}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s}$), we need to check the incentive constraint for only the entrepreneur with the h-type project. As in the first-best, he plays $\delta = \mathrm{d}$ whenever

$$(1-\beta)p_h V_{\mathrm{d}} \geq \beta \pi_e^{\mathrm{s}} + (1-\beta)V_{\mathrm{s}}$$

or

$$\pi_e^{\mathrm{s}} \leq \frac{1-\beta}{\beta}\,\Delta V. \tag{28}$$

This is the incentive constraint (IC$_h$) already written down in Subsection 4.3.2, but recall that π_e^{s} depends on the equilibrium.

4.4.4 Creditors' Incentive Constraint

Unlike the case in which the creditor offers arm's-length finance, a creditor's incentive constraint now has bite. If it deviates to $\eta = \mathrm{A}$, it saves the cost k of relationship lending, but forgoes the increased rents it gains when e has an h-type specialized project. In fact, if it chooses $\eta = \mathrm{A}$, it anticipates that the entrepreneur will standardize (see Assumption 6 and the discussion in Subsection 4.4.1). Therefore, the creditor's expected payoff is $\beta F_{\mathrm{s}}^c + (1-\beta)\pi_c$. If it chooses $\eta = \mathrm{R}$, the expression for its payoff is more complicated for two reasons: (1) the creditor must take into account the possibility that it is matched with an entrepreneur who has an ℓ-type specialized project as well as the possibility that it is matched with an entrepreneur who has an h-type specialized project; and (2) it must pay the cost k of relationship lending. If the creditor is matched with an entrepreneur with an ℓ-type specialized project, e standardizes and c's payoff is again $\beta F_{\mathrm{s}}^c + (1-\beta)\pi_c$, whereas if c is matched with an entrepreneur with an h-type specialized project, e specializes and c's payoff is $\beta p_h F_{\mathrm{d}}^c + (1-\beta)\pi_c$. Thus, c plays $\eta = \mathrm{R}$ whenever

$$\alpha\Big(\beta p_h F_{\mathrm{d}}^c + (1-\beta)\pi_c\Big) + (1-\alpha)\Big(\beta F_{\mathrm{s}}^c + (1-\beta)\pi_c\Big) - k \geq \beta F_{\mathrm{s}}^c + (1-\beta)\pi_c.$$

This simplifies to

$$p_h F_{\mathrm{d}}^c - \frac{k}{\alpha\beta} \geq F_{\mathrm{s}}^c,$$

or

$$\pi_e^{\mathrm{s}} \geq \frac{k}{\alpha\beta} - \Delta V. \tag{IC$_c$}$$

Note that the incentive constraints (IC_h) and (IC_c) combine to say that the second-best action profile ($\eta = \text{R}, \delta_h = \text{d}, \delta_\ell = \text{s}$) is attainable if and only if

$$\frac{k}{\alpha\beta} - \Delta V \;\leq\; \pi_e^{\text{s}} \;\leq\; \frac{1-\beta}{\beta}\,\Delta V. \tag{29}$$

Thus, taking π_e^{s} again as a proxy for competition, we see that, even with relationship lending, efficient project choice is possible only for intermediate levels of credit competition.

It may be illustrative to compare these bounds, which describe when efficient project choice is implementable with relationship lending, with the bounds we derived on π_e^{s} in Subsection 4.3.2, which describe when efficient project choice is implementable with arm's-length finance (inequalities (14)). Observe first that when the cost of relationship lending k is small, the lower bound on π_e^{s} in equation (29) above is negative. Therefore, when k is small, relationship lending always solves the over-differentiation problem that arises for low levels of credit market competition. Relationship lending, however, does not affect the upper bound on π_e^{s}.[14] This foreshadows the results that Subsection 4.4.6 establishes in general equilibrium, i.e. that relationship lending can solve the over-differentiation problem, but not the over-standardization problem.

4.4.5 Value Functions

To express the range of competition for which ($\eta = \text{R}, \delta_h = \text{d}, \delta_\ell = \text{s}$) is the stationary action profile in equilibrium, we compute the players' continuation values as value functions. This is similar to the calculations in Subsection 4.3.3. In fact, the expression for π_e^{s} is identical:

$$\pi_e^{\text{s}} = \frac{Q\big(\beta\pi_e^{\text{s}} + (1-\beta)(V_{\text{s}} - \pi_c)\big) + (1-Q)\pi_e^{\text{s}}}{1+r},$$

or

$$\pi_e^{\text{s}} = \frac{(1-\beta)Q}{r + (1-\beta)Q}\,\big(V_{\text{s}} - \pi_c\big). \tag{30}$$

The only difference between π_c in the efficient equilibrium (that with the stationary action profile ($\eta = \text{A}, \delta_h = \text{d}, \delta_\ell = \text{s}$) above) and π_c in the equilibrium under consideration (with the stationary action profile ($\eta = \text{R}, \delta_h = \text{d}, \delta_\ell = \text{s}$)) is that when c is matched with an entrepreneur, it first pays k to invest in a relationship with e. Thus, its value function has an extra $-k$ term with probability q relative to equation (16) in

[14] Keep in mind that π_e^{s} is endogenous, so this does not imply that the upper bounds on credit market competition θ coincide in these two cases.

Section 4.3.3:

$$\pi_c = \frac{q\Big[\alpha\Big(\beta p_h V_{\mathrm{d}} + (1-\beta)\pi_c\Big) + (1-\alpha)\Big(\beta\big(V_{\mathrm{s}} - \pi_e^{\mathrm{s}}\big) + (1-\beta)\pi_c\Big) - k\Big] + (1-q)\pi_c + rI}{1+r},$$

or

$$\pi_c = \frac{q\Big(\alpha\beta p_h V_{\mathrm{d}} + (1-\alpha)\beta(V_{\mathrm{s}} - \pi_e^{\mathrm{s}}) - k\Big) + rI}{r + \beta q}. \tag{31}$$

We can now solve for the equilibrium value functions, which are the solutions of the system of equations (30) and (31). The next lemma summarizes them.

Lemma 4. *In a stationary equilibrium with action profile* $(\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$, *the value functions are given by*

$$\pi_e^{\mathrm{d}} = 0, \tag{32}$$

$$\pi_e^s = \frac{(1-\beta)Q}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)}\left(r\big(V_{\mathrm{s}} - I\big) + q\big(k - \alpha\beta\Delta V\big)\right), \tag{33}$$

$$\pi_c = I + \frac{\beta q\Big(r\big(\alpha\Delta V + V_{\mathrm{s}} - I\big) + \alpha(1-\beta)Q\big(p_h V_{\mathrm{d}} - I\big) - \big(r + (1-\beta)Q\big)k\Big)}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)}. \tag{34}$$

4.4.6 Under What Credit Market Competition Conditions Can Relationship Lending Restore Efficient Project Choice?

This subsection presents our two main results about relationship lending: first, that relationship lending can solve the over-differentiation problem that arises for low levels of credit market competition; and second, that relationship lending cannot solve the over-standardization problem that arises for high levels of credit market competition.

Proposition 2. *Relationship lending restores efficiency for low levels of credit competition if and only if* $k \leq \alpha\beta\Delta V$. *That is, for low* θ, *there is a stationary equilibrium with action profile* $(\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ *if and only if* $k \leq \alpha\beta\Delta V$.

This proposition results from two forces. First, a creditor will never fund an ℓ-type specialized project when it observes the project's type because it has negative NPV by Assumption 1. Thus, the entrepreneur with the ℓ-type specialized project knows that if he plays $\delta = \mathrm{d}$, he will receive payoff zero, which makes him prefer to standardize. However, the creditor must pay the entire cost k to perform relationship lending. Even though the total surplus gain is positive by Assumption 3, the creditor's share of this surplus may not suffice to induce the creditor to bear the cost *privately*. Due to this hold-up problem, the creditor may choose *not* to become a relationship lender, so there may still be an inefficient project choice by the borrower for low levels of competition. Note that this intuition is already present from the creditor's incentive constraint (IC_c) taking π_e^{s} as a proxy for competition. The proposition above combined with Lemma 4 confirms the intuition in general equilibrium.

The next main result relies more heavily on the general equilibrium framework provided by the search model. It says that the entrepreneur's incentive to over-standardize is stronger in the equilibrium with action profile $(\eta = \mathrm{R}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$.

PROPOSITION 3. *Relationship lending never restores efficiency for high levels of credit competition. That is, if there is no stationary equilibrium with action profile* $(\eta = \mathrm{r}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ *for* $\theta > \bar{\theta}$*, then there is no stationary equilibrium with action profile* $(\eta = \mathrm{a}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ *for* $\theta > \bar{\theta}$.

Put simply, if, for any level of competition θ, the entrepreneur with the h-type specialized project plays $\delta = \mathrm{s}$ in the equilibrium with action profile $(\eta = \mathrm{A}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$, then the entrepreneur with the h-type specialized project plays $\delta = \mathrm{s}$ in the equilibrium with action profile $(\eta = \mathrm{R}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$. That is, the incentive to standardize is even stronger when the creditor chooses relationship lending than when the creditor chooses arm's-length finance. The reason is that the creditor anticipates being in a weaker bargaining position with relationship lending. Because c bears the cost k, its outside option is relatively low, so the entrepreneur e takes advantage of c's low outside option to negotiate better loan terms and capture more of the surplus. This means that standardization is even more attractive for e when c is playing $\eta = \mathrm{R}$ than when c is playing $\eta = \mathrm{A}$. This, in turn, means that relationship lending can only exacerbate the over-standardization problem.

5 The Role for Non-bank Funding

The result that even relationship lending does not entirely eliminate investment inefficiencies creates a natural economic rationale for the emergence of forms of intermediation other than banks to improve efficiency. In this section we extend the model to consider a class of investors which are levered, and (in equilibrium) fund only h-type specialized projects, taking equity positions in these projects. We call these investors "private equity firms" or "PEs."

5.1 Preliminaries

We use the label "PEs" because the intermediaries we model in this section have a number of similarities with real-world private equity firms. Private equity firms are typically informed investors which use high leverage to acquire equity stakes in firms (see, for example, Kaplan and Strömberg (2009)). Our PEs share these properties: they learn the types of the projects they invest in, they raise additional capital via debt, and they take equity stakes in entrepreneurs' projects. Further, in equilibrium, our model captures a number of stylized facts about PE behavior and returns.

What we have in mind is a PE firm that engages in a leveraged buyout (LBO), rather than a venture capitalist. If the PE firm has only one project (LBO), then it is exactly the levered intermediary we model, since all of the debt used in the LBO is essentially part of the PE firm's capital structure, and this capital structure involves very high leverage. If the PE firm has multiple projects, then one should view our analysis as modeling the decision-making incentives of the general partner (GP). As Axelson, Strömberg, and Weisbach (2009) point out, the compensation structures of the GP and the limited partners (LPs) look like those in the Figure 2 below.

Illustration of Private Equity Compensation Scheme

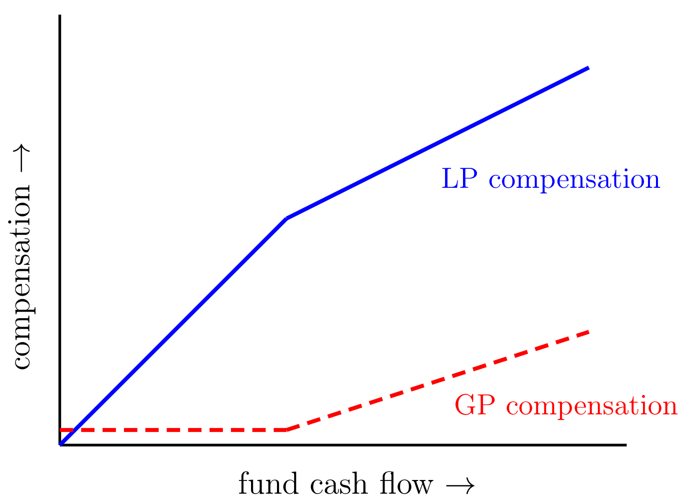

Figure 2: An illustration of how limited partners (LPs) and general partners (GPs) are compensated as a function of the cash flow of a private equity fund.

As is evident from the figure, the GP's claim resembles an equity claim and the LP's claim is concave in the PE firm's cash flow because, beyond a threshold, the GP gets 20 percent of the capital gains, whereas below the threshold all the cash flows go to the LPs (except for a management fee). Given the relatively small amount of equity capital provided by the GP, one can view both the contribution of the LPs as well as the debt used by the PE firm as "leverage" from the standpoint of the incentives of the GP.

The analysis shows that the emergence of PEs can improve efficiency, but cannot eliminate investment inefficiencies entirely. We find that PEs enter the market only for high levels of credit market competition. They fund only h-type specialized projects, and, as a result, they restore efficiency when they are matched with entrepreneurs with h-type specialized projects. However, they make entrepreneurs with ℓ-type specialized projects wait inefficiently to be matched with banks in order to obtain funding for their standardized projects. Further, banks continue to exist in equilibrium, and some entrepreneurs with h-type specialized projects still over-standardize because they are matched with banks.

The key mechanism that arises in equilibrium to improve efficiency is that PE leverage acts as a *commitment device* to fund only h-type specialized projects. The reason is that, given the PE's debt, the standardized project becomes effectively negative NPV for the PE.

In our model, one difference between banks and PEs is that PEs take equity positions and banks take debt positions in the projects they finance. This conforms to what we see in practice. Another difference in our model is that PE firms are levered, whereas banks are not. Of course, in reality banks are also levered, but we think that this modeling choice captures a fundamental difference between the effect of leverage on banks' behavior and its effect on PEs' behavior. PE leverage induces PEs to choose investments with high upside even if they are not the highest NPV investments. Because a PE makes few investments,[15] an equity stake in a project constitutes a large proportion of its assets.[16] Therefore, the PE is likely to default when a project does not return high cash flows. Because the PE is highly levered and is protected by limited liability, its payoff depends only on the upside of the project—it is not exposed to the downside. Observe that this argument depends on the causal link from project failure to PE default, which, in turn, depends on the equity stake in the project being a significant proportion of the PE's investment portfolio. This is obviously true when a PE firm has a single LBO as its project, as pointed out earlier. But it is also true, by extension of

[15]Metrick and Yasuda (2010) shows that the median private equity firm makes a total of twelve investments over the entire life of the fund, which is typically ten years.

[16]In fact, the contemporary theory of banking relies on the intermediary making a large number of loans, each of which constitutes a very small fraction of its portfolio.

the same logic, when the PE firm has just a few (LBO) projects.

In contrast, leverage does not have the same effect on a bank's behavior. Even if we incorporated bank leverage into the model explicitly, banks would still choose the project with the highest NPV.[17] Because a bank makes many investments, a loan to an entrepreneur constitutes only a small proportion of its assets. Therefore, the bank is unlikely to default because the entrepreneur's project does not return high cash flows. Despite its leverage and limited liability protection, the bank is still exposed to the downside risk of the entrepreneur's project. The reason is that when the entrepreneur defaults, the bank itself probably does not default and, hence, its limited liability protection does not bite. We can thus conclude that the role of leverage is fundamentally different across PEs and banks. The key role of intermediary leverage in inducing PEs to commit not to fund standardized projects is absent in the case of banks. Thus, our simplifying assumption that banks are unlevered is rather innocuous. Note, finally, that the difference between bank leverage and PE leverage is amplified by the difference between their funding contracts. Because PEs fund via equity, they are exposed to the upside of the projects they invest in. In contrast, because banks fund via debt, they are exposed only to the downside of the projects they invest in.

Despite parsimony in the assumptions about the nature of PEs, we can capture several salient stylized facts about them. In particular, in the model, PEs fund only specialized projects and they have high returns on investment but are still scarce, even in the perfect-competition limit.[18] In our model, as in reality, PEs and banks fund very different types of projects: PEs fund (risky) specialized projects and banks fund (safe) standardized projects. In our model, this heterogeneity arises despite the absence of ex ante heterogeneity among either entrepreneurs or creditors—both banks and PEs emerge in equilibrium even though all creditors are ex ante identical. A key insight we offer is that the type of creditor an entrepreneur finds may be an important determinant of the type of investment the entrepreneur makes.

5.2 Model with Private Equity

In this section we outline how we model PEs. The entrepreneurs and their projects are identical to those in the baseline model. Creditors, on the other hand, make a one-time choice when they enter the market that was not in the baseline model. They can either remain traditional banks (and later give relationship or arm's-length loans to

[17]To be precise, this holds as long as the size of the loan is small relative to the size of the bank's other assets, and the project's success is not correlated with the value of the bank's other assets.

[18]Harris, Jenkinson, and Kaplan (2014) document that the returns delivered by the PE funds have consistently exceeded those of public equity markets.

entrepreneurs) or they can specialize in private equity. If they remain banks, the model is identical to the model considered previously. If they specialize in private equity, they must borrow l from a competitive market *at the time of investment.* l will represent the operational cost of private equity; it helps us capture the important role of PE leverage.[19] We call the face value of PE debt F_{PE}. A PE searches in the same way as a bank, but when matched with an entrepreneur e, the stage game a PE plays is different. It is summarized by the following timing:

1. e chooses between a specialized and a standardized project, $\delta \in \{\mathrm{d}, \mathrm{s}\}$.
2. PE either borrows l in exchange for the promise to repay F_{PE} and invests, or chooses not to invest and therefore keeps searching in the market.
   - If PE invests l it observes the type τ of the entrepreneur's specialized project.
3. e and PE either negotiate the PE's equity stake γ, which is determined as follows:
   - with probability β, PE makes e a take-it-or-leave-it offer $\gamma_\delta^{\mathrm{PE}}$; e accepts or rejects
   - with probability $1-\beta$, e makes PE a take-it-or-leave-it offer γ_δ^e

   or the PE diverts I.
4. The project succeeds or fails, with e and PE dividing the surplus according to the agreed equity stake. The PE repays his debt or defaults.

Note that because the project type is observable, we have $\gamma_\delta \in \{\gamma_{\mathrm{s}}, \gamma_\ell, \gamma_h\}$. Moreover, when negotiating the PE's equity stake, the PE's outside option is I, since it has the option to divert but, since its capital l is sunk, it does not have the option to keep searching. We justify the assumption that a PE can divert its capital with the following assumption on observability: if a PE's creditor observes that the project that the PE has funded does not pay off, the creditor does not know whether it failed randomly or whether it returned nothing because the PE withheld funds, i.e. diverted capital. This assumption serves to keep the PE's outside option sufficiently high at the point of bargaining over the equity stake. Further, we want to emphasize that F_{PE} is the face value of the debt that the PE owes to an external market—it is different from the face

[19] The assumption that the PE must raise additional capital l generates a reason for the PE to raise money from external capital markets and, therefore, to become levered themselves. We interpret the cost l as required payments to PE employees or a costly investments in a monitoring technology. Note that we could have alternatively assumed that the PE did not have all of the capital I required to fund entrepreneurs' projects and, thus, that it needed to raise the remainder from the markets. We chose to include the operational cost l to keep banks and PEs as similar as possible except for leverage. In particular, in our model they are ex ante identical.

value of *entrepreneurs'* debt in the case of bank lending; PEs fund entrepreneurs with the equity stakes γ_{d}. Finally, the PE levers up at the time of investment, after it is already matched with an entrepreneur. Thus, it repays within the same period and, as a result, the face value F_{PE} does not have to compensate the market for the discount rate r.

Everything else, including the off-equilibrium beliefs and games played by off-equilibrium matches, coincides with the baseline model.

To generate the results below, we focus on the high-competition region in which θ is large enough to induce entrepreneurs to over-standardize. Further, we make the following assumptions.

Assumption 7.

$$p_h V_{\text{d}} - I > l.$$

This assumption simply ensures that h-type specialized projects have positive NPV, even net of the PE's monitoring cost l.

Assumption 8.

$$\frac{l}{p_h} > V_{\text{s}} - I.$$

This assumption serves to ensure that PEs require enough leverage—l is high enough—that their debt makes it undesirable for them to fund standardized projects.

Assumption 9.

$$r > \beta - \frac{\Delta V}{p_h V_{\text{d}} - I}.$$

This assumption serves to ensure that an entrepreneur always accepts funding from his first match, whether it is a bank or a PE.

5.3 Private Equity Analysis and Results

We now characterize an equilibrium of the extended model in which creditors may become either banks, denoted $c = b$, or private equity firms, denoted $c = \text{PE}$. Our main results of this section are the equilibrium characterization and comparative statics related to the proportion φ of PEs in the market. We begin by characterizing the equity stakes of the PE firm and the entrepreneurs, followed by an analysis of the entrepreneur's and PE's incentive constraints, then an analysis of the value functions of entrepreneurs, banks and VCs, and, finally, a characterization of the equilibrium and the proportion of PE firms in equilibrium.

5.3.1 Equity Stakes

The PE's equity stake γ depends on which player is proposing the contract in round 3 of the stage game (Subsection 3.3).

The proposer always offers a stake that makes his opponent indifferent between accepting and rejecting. There are four cases: (1) when e proposes and he has a standardized project, (2) when c proposes and e has a standardized project, (3) when e proposes and he has a specialized project, and (4) when c proposes and e has a specialized project. When e has a specialized project, it can be either the h-type or the ℓ-type.

Note that whenever e and the PE negotiate the equity stake, the PE has paid l and has learned e's type, so there is no asymmetric information. Here the subscripts on the equity stakes denote the project choice, where the subscript τ indicates a specialized project of type τ. The superscripts denote the proposer. When e proposes, the equity stake with a standardized project is given by

$$\gamma_{\mathrm{s}}^{e} V_{\mathrm{s}} = I$$

and when c proposes and e has chosen a standardized project, the equity stake is given by

$$\left(1 - \gamma_{\mathrm{s}}^{\mathrm{PE}}\right) V_{\mathrm{s}} = \pi_{e}^{\mathrm{s}}.$$

When e proposes with a specialized project, the equity stake is given by

$$\gamma_{\tau}^{e}\, p_{\tau} V_{\mathrm{d}} = I$$

and when c proposes and e has chosen a specialized project, the equity stake is given by

$$\left(1 - \gamma_{\tau}^{\mathrm{PE}}\right) p_{\tau} V_{\mathrm{d}} = \pi_{e}^{\mathrm{d}},$$

where $\tau \in \{h, \ell\}$. Recall that the PE observes e's type before negotiating the equity stake.

5.3.2 Entrepreneurs' Incentive Constraints

We look for equilibria in which PEs fund only h-type specialized projects. In this section, we find e's incentive constraint given that he believes that he will find funding from a PE only if he has an h-type specialized project. We then show in Section 5.3.3 that these beliefs are consistent in equilibrium.

First consider an entrepreneur who is matched with a PE and has an ℓ-type specialized project. He knows that: (1) he will not obtain funding from the PE, and (2) if he searches and is matched with a bank, he is better off if his project is standardized than if it is specialized. Recall that we are focusing on the high competition region, in which entrepreneurs over-standardize as shown in Subsections 4.2 and 4.4. Thus, this entrepreneur with an ℓ-type specialized project will always standardize.

Now consider an entrepreneur who is matched with a PE and has an h-type specialized project. He either specializes and gets funding from a PE or standardizes, does not obtain funding from a PE and continues searching for a bank. Thus, he specializes whenever

$$\beta\pi_e^{\mathrm{d}} + (1-\beta)(1-\gamma_h^e)p_h V_{\mathrm{d}} \geq \pi_e^{\mathrm{s}}$$

which simplifies to

$$(1-\beta)(p_h V_{\mathrm{d}} - I) \geq \pi_e^{\mathrm{s}}. \tag{$\mathrm{IC}_h^{\mathrm{PE}}$}$$

Finally, consider an entrepreneur who is matched with a bank. Since we are focusing on the high competition region, the entrepreneur will standardize as in Subsections 4.2 and 4.4. Recall that an entrepreneur who has specialized but is still searching is believed to have an ℓ-type specialized project by Assumption 4. This implies that an entrepreneur who is matched with a bank does not have the option of differentiating and obtaining funding from a PE in the future because the PE will observe that he failed to obtain funding in the past and will assume that it was because his project was the ℓ-type.

5.3.3 PEs' Incentive Constraints

In order to show that a PE funds only h-type specialized projects, we must check that: (1) it prefers to divert capital rather than fund an ℓ-type specialized project, and (2) it prefers to wait than to fund a standardized project. The first incentive constraint, that it prefers to divert capital when confronted with an ℓ-type specialized project, is given by

$$\beta\, p_\ell \left(\gamma_\ell^{\mathrm{PE}} V_{\mathrm{d}} - F_{\mathrm{PE}}\right) + (1-\beta)I \leq I$$

or, simplifying,

$$p_\ell V_{\mathrm{d}} - I \leq F_{\mathrm{PE}},$$

which is always satisfied since $p_\ell V_{\mathrm{d}} - I < 0$ and $F_{\mathrm{PE}} \geq 0$.

The second incentive constraint, that the PE prefers to search again than to be matched with a standardized entrepreneur, is given by

$$\pi_{\mathrm{PE}} \geq \beta\big(\gamma_{\mathrm{s}}^{\mathrm{PE}} V_{\mathrm{s}} - F_{\mathrm{PE}}\big) + (1-\beta)I$$

which simplifies to:

$$\pi_{\mathrm{PE}} \geq \beta(V_{\mathrm{s}} - \pi_e^{\mathrm{s}} - F_{\mathrm{PE}} - I) + I. \tag{$\mathrm{IC}_{\mathrm{s}}^{\mathrm{PE}}$}$$

5.3.4 Value Functions

In this section we compute the value functions that will pin down players' continuation values in the incentive constraints demonstrated in the section above.

***Entrepreneur's Value Function*:** First, note that if e specializes, his outside option vanishes due to Assumption 4. Recall that entrepreneurs believe that PEs fund only h-type specialized projects. Thus, if e standardizes, he faces the new risk of being matched with a PE and being forced to search again. As in the baseline model, his value function is his expected utility before he searches again. If he is unmatched, which occurs with probability $1-Q$, he searches again and therefore stationarity implies that he obtains π_e^{s}. If he is matched, a new term appears that is absent in the baseline model. With probability φ, he is matched with a PE. In this case, he does not obtain funding and searches again, obtaining π_e^{s}. With probability $1-\varphi$, he is matched with a bank. In this case, he obtains his outside option π_e^{s} when the bank proposes the contract, which occurs with probability β. With complementary probability $1-\beta$, e proposes the contract and gets the surplus $V_{\mathrm{s}} - \pi_b$. Thus we can write the equation for e's value function as

$$\pi_e^{\mathrm{s}} = \frac{Q\Big(\varphi\pi_e^{\mathrm{s}} + (1-\varphi)\big((1-\beta)(V_{\mathrm{s}} - \pi_b) + \beta\pi_e^{\mathrm{s}}\big)\Big) + (1-Q)\pi_e^{\mathrm{s}}}{1+r}$$

which gives

$$\pi_e^{\mathrm{s}} = \frac{(1-\beta)(1-\varphi)Q}{r + (1-\beta)(1-\varphi)Q}\big(V_{\mathrm{s}} - \pi_b\big). \tag{35}$$

***Bank's Value Function*:** If a creditor becomes a bank, $c = b$, then it funds only standardized projects in equilibrium, as derived in Subsections 4.2 and 4.4. Thus, its value function must take into account only the probability of finding a match, who proposes the contract, and the interest earned overnight. Specifically, the terms are as follows. The banker b earns interest rI on his principal over the next period. If it does not find a match, which occurs with probability $1-q$, it obtains π_b. If it does find a match, which occurs with probability q, the entrepreneur proposes the contract and pushes the creditor to its outside option π_b with probability $1-\beta$. Finally, if the banker finds a match and proposes the contract, which occurs with joint probability βq, it obtains the surplus $V_{\mathrm{s}} - \pi_e^{\mathrm{s}}$. The equation for the creditor's value function when it becomes a bank is thus

$$\pi_b = \frac{q\Big(\beta(V_{\mathrm{s}} - \pi_e^{\mathrm{s}}) + (1-\beta)\pi_b\Big) + (1-q)\pi_b + rI}{1+r},$$

which gives

$$\pi_b = I + \frac{\beta q\big(V_{\mathrm{s}} - \pi_e^{\mathrm{s}} - I\big)}{r+\beta q}. \tag{36}$$

We can now solve the system of equations (35) and (36) for π_e^{s} and π_b to obtain the value functions:

$$\pi_e^{\mathrm{s}} = \frac{(1-\beta)(1-\varphi)Q}{r+q\beta+(1-\beta)(1-\varphi)Q}(V_{\mathrm{s}} - I) \tag{37}$$

and

$$\pi_b = I + \frac{\beta q}{r+q\beta+(1-\beta)(1-\varphi)Q}(V_{\mathrm{s}} - I). \tag{38}$$

***PE's Value Function*:** We need to calculate the value function of the private equity lender, $c = \mathrm{PE}$. As in the case in which $c = b$, the PE earns interest rI overnight. As usual, if the PE does not find an entrepreneur next period, it obtains π_{PE} by stationarity. This occurs with probability $1-q$. If it does find a match, which occurs with probability q, there are several cases to consider. If it is matched with an entrepreneur with an ℓ-type specialized project, which occurs with conditional probability $1-\alpha$, the PE does not fund it and searches again, obtaining payoff π_{PE}.[20] If it is matched with an entrepreneur with an h-type specialized project, there are two cases: with probability $1-\beta$ the entrepreneur proposes the equity stake, and with probability β the PE proposes the equity stake. If e is the proposer, he pushes the PE to its outside option I. Remember that his operational cost l is now sunk, so he does not have the option of searching again. Hence, its outside option if bargaining breaks down is to divert capital and obtain I. If, on the other hand, the PE makes the offer, it obtains the surplus but repays its own debt, F_{PE}. Thus, it obtains $p_h(V_{\mathrm{d}} - F_{\mathrm{PE}})$ because the entrepreneur's outside option is zero, $\pi_e^{\mathrm{d}} = 0$. We can now write the equation for the

[20]It may be worth noting that PEs never divert capital along the equilibrium path; however, the threat that PEs may divert is important in that it forces the entrepreneur with the ℓ-type specialized project to standardize. The reason is that the entrepreneur with the ℓ-type specialized project knows that if he specializes, then the PE will later observe that the project is ℓ-type and choose to divert capital instead of investing I to fund it. Thus, the entrepreneur with the ℓ-type specialized project will never choose the specialized project when he is matched with a PE. (If he chooses to standardize he will find funding from a bank in a subsequent period.) As a result, when entrepreneurs are matched with PEs, they self-select to separate in equilibrium—entrepreneurs matched with PEs choose to specialize if and only if they have h-type specialized projects.

PE's value function:

$$\pi_{\mathrm{PE}} = \frac{q\left[\alpha\Big(\beta p_h(V_{\mathrm{d}} - F_{\mathrm{PE}}) + (1-\beta)I\Big) + (1-\alpha)\pi_{\mathrm{PE}}\right] + (1-q)\pi_{\mathrm{PE}} + rI}{1+r}.$$

Recall that the PE borrows from a competitive capital market. The market anticipates that the PE will fund only h-type entrepreneurs and repay if and only if these projects succeed (there is no diversion on the equilibrium path because ℓ-type entrepreneurs standardize). Thus, the equilibrium face value of the PE's debt is

$$F_{\mathrm{PE}} = \frac{l}{p_h}.$$

Hence, the PE's value function is

$$\pi_{\mathrm{PE}} = I + \frac{q\alpha\beta\Big(p_h V_{\mathrm{d}} - l - I\Big)}{r + \alpha q}, \tag{39}$$

having just simplified the equation above.

5.3.5 Equilibrium Characterization

The next two propositions (Proposition 4 and Proposition 5) characterize the equilibrium. The key features of the equilibrium are the following: (1) PEs enter the market for only sufficiently high levels of credit competition θ; (2) PEs and banks coexist; (3) PEs fund only h-type specialized projects; (4) PE returns are high; and (5) banks fund only standardized projects.

Proposition 4. *For sufficiently high θ, there is an equilibrium characterized as follows*

- *All banks play $\eta = \mathrm{a}$.*
- *PEs invest l if and only if they observe entrepreneurs play $\delta = \mathrm{d}$.*
- *An entrepreneur with an h-type specialized projects plays $\delta = \mathrm{d}$ if and only if matched with a PE.*
- *All entrepreneurs play $\delta = \mathrm{s}$ when matched with a bank.*
- *An entrepreneur with an h-type specialized project obtains funding from the first creditor he is matched with; an entrepreneur with an ℓ-type specialized project obtains funding from the first bank he is matched with.*

We have already established the main elements of the proposition in the preceding subsections. The proof in the Appendix verifies the incentive constraints ($\mathrm{IC}_h^{\mathrm{PE}}$) and ($\mathrm{IC}_{\mathrm{s}}^{\mathrm{PE}}$), given the equilibrium value functions.

It is worth noting that what allows the PE to improve efficiency is that it forces entrepreneurs not to over-standardize even though competition in the credit market is high. Thus, when an entrepreneur is matched with a PE, he anticipates that he will *not* obtain funding from the PE if he plays $\delta = \mathrm{s}$.

5.3.6 Equilibrium Proportion of PE

In sufficiently competitive markets, banks and PEs coexist. Coexistence requires creditors to be indifferent between devoting their capital to banking and devoting it to PE. They maintain indifference in equilibrium due to the presence of a *feedback loop*, which works as follows. Start by considering an equilibrium with only banks in the high competition region in which entrepreneurs are over-standardizing. As the credit market becomes more competitive, the rents from traditional banking decrease. When these rents are sufficiently low, the creditors benefit from specializing in PE. But then since more intermediation services are devoted to PE, the competition in the traditional banking sector decreases, making banking again more attractive. That is, the payoff from traditional banking is increasing in the proportion of creditors which specialize in PE. This feedback loop is the key force in the model that allows for the coexistence of disparate types of credit.

The feedback mechanism appears directly from equating the value functions of the two types of creditors,

$$\pi_b(\theta, \varphi(\theta)) = \pi_{\mathrm{PE}}(\theta)$$

or, simplifying:

$$I + \frac{\beta q}{r + \beta q + (1-\beta)(1-\varphi)Q}\big(V_{\mathrm{s}} - I\big) = I + \frac{\alpha\beta q}{r + \alpha q}\big(p_h V_{\mathrm{d}} - l - I\big).$$

Rearranging gives

$$\varphi = 1 - \frac{(r+\alpha q)(V_{\mathrm{s}} - I) - \alpha(r+\beta q)(p_h V_{\mathrm{d}} - l - I)}{\alpha(1-\beta)Q(p_h V_{\mathrm{d}} - l - I)}. \tag{40}$$

The Proportion of Intermediaries Specialized in PE as a Function of Credit Market Competition

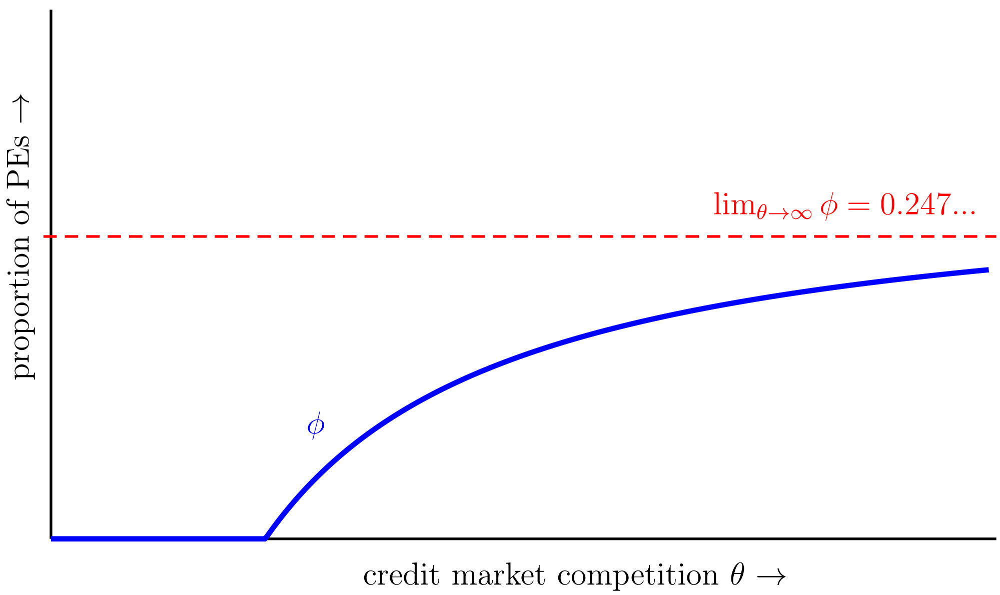

Figure 3: The graph depicts the formula for ϕ in equation (40) for paramaterization $V_{\rm d} = 10$, $p_h = .8$, $V_{\rm s} = 7$, $I = 5$, $\alpha = .5$, $\beta = .5$, $r = .2$, $l = 1.6$, $Q(\theta) = \theta/(1+\theta)$, $q(\theta) = 1/(1+\theta)$.

Proposition 5. *As long as*

$$(r+\alpha)(V_{\rm s} - I) > \alpha(r+\beta)(V_{\rm d} - I - l)$$

and

$$V_{\rm s} - I > \beta(V_{\rm d} - I - l),$$

the proportion φ of PEs in the market is:

1. *positive only for sufficiently high credit competition θ;*
2. *always less than one; and*
3. *increasing in credit market competition θ.*

The proposition above says that PEs emerge only in highly competitive credit markets, and they continue to become more important as credit markets become more competitive. However, PEs never completely oust banks from the market. The reason is, again, the feedback loop between competition in the banking market and the presence of PE: the more capital that is devoted to PE, the more attractive banking becomes. This general equilibrium effect is crucial for the *coexistence* of PE and banking in the model.

6 Conclusion

This paper develops a general equilibrium model of competition in the credit market to investigate the question of how credit market competition affects corporate investment, and how the different investment inefficiencies at different levels of credit market competition generate incentives for the emergence of a variety of intermediaries. In the model, entrepreneurs have two projects, a (safe) "standardized" project and a (risky) "specialized" (or "differentiated") project. Which project is efficient to undertake depends on the state of nature, so efficient investment requires adapting to circumstances.

The paper has four main results. The first is that project choice is inefficient when creditors offer arm's-length finance and when banking competition is at either extreme (too high or too low). Specifically, when competition in the banking market is very low, entrepreneurs over-specialize, inefficiently forgoing the standardized project. When competition is very high, entrepreneurs over-standardize, inefficiently foregoing the specialized project. The key force behind this result is that entrepreneurs with standardized projects can find funding more easily in the future, thus they can obtain better terms of debt from their creditors. This pushes entrepreneurs toward the standardized project, and the effect is strongest in the most competitive credit markets.

The second main result is that this inefficiency with arm's-length finance encourages the emergence of relationship banking. We show that relationship lending mitigates the over-specialization inefficiency that emerges for low levels of credit competition but not the over-standardization inefficiency that emerges for high levels of competition. The reason is that relationship banking allows banks to make more informed lending decisions, mitigating inefficiencies that arise from the asymmetric information that is associated with a specialized project. However, relationships with borrowers do not affect banks' ability to fund standardized projects; relationship banking cannot force entrepreneurs not to standardize.

Third, the residual inefficiency that relationship lending is unable to eliminate leads to the emergence of private equity (PE). These firms eliminate the over-standardization investment inefficiency.

Fourth, PE firms emerge only when the credit market is sufficiently competitive. A natural separation arises in general equilibrium as PE firms lend only to entrepreneurs who invest in h-type specialized projects and banks lend only to those who invest in standardized projects.

To the best of our knowledge, this is the first paper that develops a theory in which arm's-length or relationship banks coexist with PE firms, and this coexistence has ramifications for the nature of entrepreneurial investment. The paper thus exposes a natural economic link between credit market competition, investment inefficiencies and intermediation variety, with general equilibrium effects playing a crucial role in the co-existence of various types of intermediaries. Our result that PE firms arise only when the credit market is very competitive accords well with the observation that PE firms are ubiquitous in the highly competitive credit market found in the US. Moreover, our theory also sheds light on why PE firms tend to be highly levered. High leverage is not just a tax-driven effect for PE firms. Rather, it is essential to the economic role they play.

A Appendix: Proofs

A.1 Proof of Lemma 1

Given the Assumption 4 on out-of-equilibrium beliefs, the creditor believes that it knows the quality of the project so it will not pay k to gain information via relationship lending. Further, given the Assumption 1 that the ℓ-type specialized project has negative NPV, there is no face value that will deliver a positive expected payoff to the creditor. Since the creditor's outside option is positive, in fact $\pi_c \geq I$, no lending can take place.

A.2 Proof of Lemma 3

First recall the expression for π_e^{s} from Lemma 2:

$$\pi_e^{\mathrm{s}} = \frac{(1-\beta)Q}{r(r+\beta q)+(1-\beta)Q(r+\alpha\beta q)}\bigg(r\big(V_{\mathrm{s}}-I\big)-\alpha\beta q\Delta V\bigg).$$

Now observe that the first term in the product,

$$f(\theta) := \frac{(1-\beta)Q}{r(r+\beta q)+(1-\beta)Q(r+\alpha\beta q)},$$

is increasing in θ. Compute the derivative with the quotient rule and group terms:

$$\begin{aligned} f'(\theta) &= \frac{\partial}{\partial\theta}\left(\frac{(1-\beta)Q}{r(r+\beta q)+(1-\beta)Q(r+\alpha\beta q)}\right) \\ &= \frac{Q'r(r+\beta q)-Qq'\beta\big(r+\alpha(1-\beta)Q\big)}{\Big(r(r+\beta q)+(1-\beta)(r+\alpha\beta q)Q\Big)^2}. \end{aligned}$$

To see that this expression is positive, recall the assumptions on the matching function from Subsection 3.2. Namely $q' < 0$ and $Q' > 0$. Thus $-q'$ is positive and so are all other terms, so $f' > 0$.

Now

$$\pi_e^{\mathrm{s}} = f(\theta)\Big(r\big(V_{\mathrm{s}}-I\big)-\alpha\beta q\Delta V\Big)$$

so

$$\frac{\partial\pi_e^{\mathrm{s}}}{\partial\theta} = f'(\theta)\Big(r\big(V_{\mathrm{s}}-I\big)-\alpha\beta q\Delta V\Big)-\alpha\beta q'\Delta V f(\theta).$$

Assumption 5 and the result above that $f' > 0$ imply that the first term is positive. f is positive because all its terms are positive, $\Delta V > 0$ and $-q' > 0$ as above. Thus, π_e^{s} is increasing in competition θ.

A.3 Proof of Proposition 1

First note that π_e^{s} and π_c as written in Lemma 2 are continuous in θ since q and Q are continuous in θ and the denominators are always positive. Thus, we must just show that an entrepreneur who has an ℓ-type specialized project chooses $\delta = \mathrm{d}$ when $\theta \to 0$ and that an entrepreneur who has an h-type specialized project chooses $\delta = \mathrm{s}$ when $\theta \to \infty$. That is to say that inequality (IC_ℓ) is violated for low θ and inequality (IC_h) is violated for high θ.

Before checking e's incentive constraints, note the limits of the value functions from Lemma 2:

$$\begin{aligned}
\lim_{\theta\to 0} \pi_c &= I + \frac{\beta}{r+\beta}\Big(\alpha\Delta V + V_{\mathrm{s}} - I\Big),\\
\lim_{\theta\to 0} \pi_e^{\mathrm{s}} &= 0,\\
\lim_{\theta\to \infty} \pi_e^{\mathrm{s}} &= \frac{1-\beta}{r+1-\beta}\Big(V_{\mathrm{s}} - I\Big).
\end{aligned}$$

Consider first the incentive constraint of the entrepreneur with an ℓ-type specialized project. The constraint reads

$$p_\ell V_{\mathrm{d}} - V_{\mathrm{s}} + \left(1 - \frac{p_\ell}{p_h}\right)\pi_c \leq \frac{\beta}{1-\beta}\,\pi_e^{\mathrm{s}}$$

or, as $\theta \to 0$,

$$p_\ell V_{\mathrm{d}} - V_{\mathrm{s}} + \left(1 - \frac{p_\ell}{p_h}\right)\left(I + \frac{\beta}{\beta+r}\Big(\alpha\Delta V + V_{\mathrm{s}} - I\Big)\right) \leq 0.$$

This is violated by the first condition in the statement of the proposition. Therefore there is no efficient equilibrium when θ is small.

Now consider the incentive constraint of the entrepreneur with an h-type specialized project. The constraint reads

$$\frac{\beta}{1-\beta}\,\pi_e^{\mathrm{s}} \;\leq\; \Delta V$$

or, for $\theta \to \infty$,

$$\frac{\beta}{r+1-\beta}\Big(V_{\mathrm{s}} - I\Big) \;\leq\; \Delta V.$$

This is violated by the second condition in the statement of the proposition. Therefore there is no efficient equilibrium when θ is large.

A.4 Proof of Proposition 2

From Lemma 4 observe that $\pi_e^{\mathrm{s}} \to 0$ as $\theta \to 0$. Recall from the creditor's incentive constraint (IC_c) that it plays $\eta = \mathrm{R}$ if and only if

$$\pi_e^{\mathrm{s}} \geq \frac{k}{\alpha\beta} - \Delta V.$$

Thus, this holds as $\theta \to 0$ if and only if

$$0 \geq \frac{k}{\alpha\beta} - \Delta V.$$

That is to say that relationship lending restores efficiency for low θ—it eliminates the over-specialization inefficiency—whenever $k \leq \alpha\beta\Delta V$.

A.5 Proof of Proposition 3

This proof involves comparing the incentive constraint (IC_h) of the entrepreneur with the high-type specialized project across the equilibria described in Lemma 2 and Lemma 4. This is the incentive constraint that says the entrepreneur who has an h-type specialized project chooses to play $\delta = \mathrm{d}$. It reads always

$$\pi_e^{\mathrm{s}} \leq \frac{1-\beta}{\beta}\Delta V,$$

but π_e^{s} depends on the equilibrium. Write $\pi_e^{\mathrm{s}}|_{\eta=\mathrm{A}}$ for the entrepreneur's value function as written in Lemma 2 and $\pi_e^{\mathrm{s}}|_{\eta=\mathrm{R}}$ for the entrepreneur's value function as written in Lemma 4.

Now, immediately from the expressions written in the lemmas,

$$\pi_e^{\mathrm{s}}|_{\eta=\mathrm{R}} - \pi_e^{\mathrm{s}}|_{\eta=\mathrm{A}} = \frac{(1-\beta)qQk}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)} > 0.$$

So immediately, $\pi_e^{\mathrm{s}}|_{\eta=\mathrm{R}} > \pi_e^{\mathrm{s}}|_{\eta=\mathrm{A}}$ which means that if

$$\pi_e^{\mathrm{s}}|_{\eta=\mathrm{A}} > \frac{1-\beta}{\beta}\Delta V$$

then

$$\pi_e^{\mathrm{s}}|_{\eta=\mathrm{R}} > \frac{1-\beta}{\beta}\Delta V.$$

That is to say if e over-standardizes given the $(\eta = \mathrm{A}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ equilibrium then he would also over-standardize given the $(\eta = \mathrm{R}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s})$ equilibrium.

A.6 Proof of Proposition 4

The bulk of the argument lies in subsections preceding the statement of the proposition (Subsections 5.3.1, 5.3.2, 5.3.3, and 5.3.4). It remains to check the ICs stated in Subsections 5.3.2 and 5.3.3.

First, check inequality ($\mathrm{IC}_h^{\mathrm{PE}}$), which says that the entrepreneur with an h-type project prefers to stay matched with a PE than to standardize and wait to be matched with a bank. Plugging in for π_e^{s} from equation (37), the inequality reads

$$(1-\beta)(p_h V_{\mathrm{d}} - I) \geq \frac{(1-\beta)(1-\varphi)Q}{r + q\beta + (1-\beta)(1-\varphi)Q}(V_{\mathrm{s}} - I).$$

Note that the right-hand side of the inequality above is decreasing in φ, thus sufficient for the inequality above to be satisfied is to set $\varphi = 0$:

$$p_h V_{\mathrm{d}} - I \geq \frac{Q}{r + q\beta + (1-\beta)Q}(V_{\mathrm{s}} - I). \tag{41}$$

Since the right-hand side is maximized when $q = 0$ and $Q = 1$ it suffices to show that

$$p_h V_{\mathrm{d}} - I \geq \frac{1}{r + 1 - \beta}(V_{\mathrm{s}} - I)$$

or, equivalently,

$$r \geq \beta - \frac{\Delta V}{p_h V_{\mathrm{d}} - I}$$

which is guaranteed by Assumption 5.

It remains to check that inequality ($\mathrm{IC}_{\mathrm{s}}^{\mathrm{PE}}$) is satisfied, namely that a PE prefers to search again than to fund a standardized entrepreneur. The constraint reads that

$$\pi_{\mathrm{PE}} > \beta(V_{\mathrm{s}} - \pi_e^{\mathrm{s}} - F_{\mathrm{PE}} - I) + I.$$

π_{PE} is always at least I, so the IC holds if

$$V_{\mathrm{s}} - \pi_e^{\mathrm{s}} - F_{\mathrm{PE}} - I < 0. \tag{42}$$

Since in equilibrium $F_{\mathrm{PE}} = l/p_h$, Assumption 8 implies that

$$F_{\mathrm{PE}} > V_{\mathrm{s}} - I$$

which suffices to guarantee that inequality (42) holds.

A.7 Proof of Proposition 5

Start with the expression for φ:

$$\varphi = 1 - \frac{(r+\alpha q)(V_{\mathrm{s}} - I) - \alpha(r+\beta q)(p_h V_{\mathrm{d}} - I - l)}{\alpha(1-\beta)Q(p_h V_{\mathrm{d}} - I - l)}$$

First observe that φ is increasing by direct specialization. For simplicity use the short-hands $\xi_{\mathrm{d}} := p_h V_{\mathrm{d}} - I$ and $\xi_{\mathrm{s}} := V_{\mathrm{s}} - I$, to write

$$\varphi' = -\frac{\alpha\Big(\xi_{\mathrm{S}} - \beta(\xi_{\mathrm{D}} - l)\Big)Qq' - \Big(r\big(\xi_{\mathrm{S}} - \alpha(\xi_{\mathrm{D}} - l)\big) + \alpha\big(\xi_{\mathrm{S}} - \beta(\xi_{\mathrm{D}} - l)\big)q\Big)Q'}{\alpha(1-\beta)(\xi_{\mathrm{D}} - l)Q^2}.$$

Since $q' < 0$, $Q' > 0$ and $\xi_{\mathrm{S}} - \beta(\xi_{\mathrm{D}} - l) > 0$, by hypothesis, $\varphi' > 0$ as long as $\xi_{\mathrm{S}} - \alpha(\xi_{\mathrm{D}} - l) > 0$. This condition is implied by Assumption 2. Thus, we can conclude that $\varphi' > 0$.

Note that the numerator in the fraction in the expression for φ above is positive by hypothesis. Since $Q \to 0$ as $\theta \to 0$ the expression approaches minus infinity for low credit market competition, $\varphi \to -\infty$ as $\theta \to 0^+$. Since to be well-defined we must have $\varphi \in [0, 1]$ and φ is continuous and increasing, we see that $\varphi > 0$ for only sufficiently high θ.

Finally, to demonstrate that $\varphi < 1$ even in the limit as $\theta \to \infty$, we just compute the limit. Recall that $q \to 0$ and $Q \to 1$ as $\theta \to \infty$. Thus, as $\theta \to \infty$,

$$\varphi \to 1 - \frac{r(V_{\mathrm{s}} - I) - \alpha r(p_h V_{\mathrm{d}} - I - l)}{\alpha(1-\beta)(p_h V_{\mathrm{d}} - I - l)}$$

which is less than one since

$$r(V_{\mathrm{s}} - I) > \alpha r(p_h V_{\mathrm{d}} - I - l)$$

by the Assumption 2.

A.8 Equilibrium Refinement

In Subsection 4.1.1 we argued that the out-of-equilibrium beliefs we restricted attention to in Assumption 4 were the most economically reasonable. We now present a game-theoretic argument to justify this restriction on the out-of-equilibrium beliefs.

We consider a refinement akin to Sequential Equilibrium. Sequential Equilibrium is a well-defined solution concept for only games with finite action spaces (see Myerson and Reny (2014)), thus we impose the same refinement as Sequential Equilibrium only for the actions drawn from finite action spaces.

Definition 1. *A Refined Perfect Bayesian Equilibrium is a Perfect Bayesian Equilibrium which is the limit of a sequence of strategy profiles and associated sensible beliefs in which the strategies are totally mixed whenever the action space is finite.*

Proposition 6. *In any efficient Refined Perfect Bayesian Equilibrium, the out-of-equilibrium beliefs specified in Assumption 4 are unique.*

Proof. Consider a sequence of assessments converging to the (efficient) stationary action profile ($\eta = \mathrm{A}, \delta_h = \mathrm{d}, \delta_\ell = \mathrm{s}$) that are totally mixed over the actions η and δ. In particular, suppose that the creditor plays R with probability ε^η, the entrepreneur with the h-type specialized project plays s with probability ε^h, and the entrepreneur with the ℓ-type specialized project plays d with probability ε^ℓ where $\varepsilon^\eta, \varepsilon^h, \varepsilon^\ell \to 0^+$.

We proceed to show that for $\varepsilon^\eta, \varepsilon^h, \varepsilon^\ell$ small and positive, only entrepreneurs with ℓ-type specialized projects will fail to find funding from their initial creditors. This implies that if a creditor with an already-specialized project is still searching it must be because he is has an ℓ-type project and failed to find funding from his initial creditor. This will imply the desired result that the only reasonable out-of-equilibrium belief is that an entrepreneur's already-specialized project is the ℓ-type.

Since we are supposing that the equilibrium is efficient, if e plays s or e plays d and c plays A, c lends to e as in the equilibrium in Proposition 1. If, in contrast, c plays R and e plays d, c observes that e's project is ℓ-type and therefore negative NPV. This occurs with probability $\varepsilon^\eta\varepsilon^\ell$. In this case c denies e credit and both parties search again. Since this is the only scenario in which c denies e credit, it must be believed that if an entrepreneur has already specialized his project then it is type ℓ. □

References

Agarwal, Sumit, Paige M. Skiba, and Jeremy Tobacman, 2009, Payday Loans and Credit Cards: New Liquidity and Credit Scoring Puzzles?, NBER Working Papers 14659 National Bureau of Economic Research, Inc.

Axelson, Ulf, Per Strömberg, and Michael Weisbach, 2009, Why are buyouts levered? the financial structure of private equity firms, *Journal of Finance* 64, 1549–1582.

Berlin, Mitchell, and Loretta J. Mester, 1992, Debt Covenants and Renegotiation, *Journal of Financial Intermediation* 2, 95–133.

Boot, Arnoud W. A., 2000, Relationship Banking: What Do We Know?, *Journal of Financial Intermediation* 9, 7–25.

———, Stuart I. Greenbaum, and Anjan V. Thakor, 1993, Reputation and Discretion in Financial Contracting, *American Economic Review* 83, 1165–83.

Boot, Arnoud W. A., and Anjan V. Thakor, 1994, Moral Hazard and Secured Lending in an Infinitely Repeated Credit Market Game, *International Economic Review* 35, 899–920.

———, 2000, Can relationship banking survive competition?, *Journal of Finance* 55, 679–713.

Boyd, John, and Gianni De Nicoló, 2005, The theory of bank risk-taking and competition revisited, *Journal of Finance* 60, 1329–1343.

Campello, Murillo, and De Matta Rafael, 2010, How are venture capitalists rewarded? the economics of venture capital partnerships, Discussion paper, Working Paper.

Cao, Melanie, and Shouyong Shi, 2000, Screening, Bidding, and the Loan Market Tightness, Center for Financial Institutions Working Papers 00-09 Wharton School Center for Financial Institutions, University of Pennsylvania.

Cetorelli, Nicola, 2001, Competition among Banks: Good or Bad?, *Economic Perspectives* pp. 38–48.

Chan, Yuk-Shee, 1983, On the Positive Role of Financial Intermediation in Allocation of Venture Capital in a Market with Imperfect Information, *Journal of Finance* 38, 1543–68.

Degryse, Hans, and Patrick Van Cayseele, 2000, Relationship Lending within a Bank-Based System: Evidence from European Small Business Data, *Journal of Financial Intermediation* 9, 90–109.

Dell'Ariccia, Giovanni, 2000, Learning by Lending, Competition, and Screening Incentives in the Banking Industry, Center for Financial Institutions Working Papers 10 Wharton School Center for Financial Institutions, University of Pennsylvania.

Diamond, Peter, 1990, Pairwise Credit in Search Equilibrium, *The Quarterly Journal of Economics* 105, 285–319.

Guzman, Mark G., 2000, Bank structure, capital accumulation and growth: a simple macroeconomic model, *Economic Theory* 16, 421–455.

Harris, Rober S., Tim Jenkinson, and Steven N. Kaplan, 2014, Private equity performance: What do we know?, *The Journal of Finance*.

Hart, Oliver, and John Moore, 1995, Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management, *American Economic Review* 85, 567–85.

Hellmann, Thomas F., Kevin C. Murdock, and Joseph E. Stiglitz, 2000, Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?, *American Economic Review* 90, 147–165.

Hombert, Johan, and Adrien Matray, 2013, The real effects of hurting lending relationships: Evidence from banking deregulation and innovation, Working paper.

Inderst, Roman, and Holger M. Mueller, 2004, The effect of capital market characteristics on the value of start-up firms, *Journal of Financial Economics* 72, 319–356.

Jayaratne, Jith, and Philip Strahan, 1996, The financial growth nexus: Evidence from bank branch deregulation, *Quarterly Journal of Economics* 111, 639–670.

———, 1998, Entry restrictions, industry evolution, and dynamic efficiency: Evidence from commercial banking, *Journal of Law and Economics* 41, 239–275.

Kaplan, Steven N., and Per Strömberg, 2009, Leveraged Buyouts and Private Equity, *Journal of Economic Perspectives* 23, 121–46.

Lerner, Josh, Morten Sorensen, and Per Strömberg, 2011, Private equity and long-run investment: The case of innovation, *The Journal of Finance* 66, 445–477.

Manove, Michael, A. Jorge Padilla, and Marco Pagano, 1998, Collateral vs. Project Screening: A Model of Lazy Banks, Discussion Paper, 10 Centre for Studies in Economics and Finance (CSEF), University of Naples, Italy.

Martinez-Miera, David, and Rafael Repullo, 2010, Does Competition Reduce the Risk of Bank Failure?, *Review of Financial Studies* 23, 3638–3664.

Matutes, Carmen, and Xavier Vives, 2000, Imperfect competition, risk taking, and regulation in banking, *European Economic Review* 44, 1–34.

Metrick, Andrew, and Ayako Yasuda, 2010, The Economics of Private Equity Funds, *Review of Financial Studies* 23, 2303–2341.

——— , 2011, Venture capital and other private equity: a survey, *European Financial Management* 17, 619–654.

Myerson, Roger, and Phil Reny, 2014, Sequential equilibrium in multistage games with infinite sets of actions and types, Discussion paper, .

Pagano, Marco, 1993, Financial markets and growth: An overview, *European Economic Review* 37, 613–622.

Petersen, Mitchell A, and Raghuram G Rajan, 1995, The effect of credit market competition on lending relationships, *The Quarterly Journal of Economics* 110, 407–43.

Rajan, Raghuram G, 1992, Insiders and outsiders: The choice between informed and arm's-length debt, *Journal of Finance* 47, 1367–400.

Rajan, Raghuram G., 2012, Presidential Address: The Corporation in Finance, *Journal of Finance* 67, 1173–1217.

Ratnovski, Lev, 2013, Competition Policy for Modern Banks, IMF Working Papers 13/126 International Monetary Fund.

Repullo, Rafael, 2004, Capital requirements, market power, and risk-taking in banking, *Journal of Financial Intermediation* 13, 156–182.

Schaeck, Klaus, Martin Cihak, and Simon Wolfe, 2009, Are more competitive banking systems more stable?, *Journal of Money, Credit and Banking* 41, 711–734.

Shaffer, Sherril, 1997, The winner's curse in banking, Working Papers 97-25 Federal Reserve Bank of Philadelphia.

Sharpe, Steven A, 1990, Asymmetric Information, Bank Lending, and Implicit Contracts: A Stylized Model of Customer Relationships, *Journal of Finance* 45, 1069–87.

Stiglitz, Joseph E, and Andrew Weiss, 1981, Credit Rationing in Markets with Imperfect Information, *American Economic Review* 71, 393–410.

Ueda, Masako, 2004, Banks versus venture capital: Project evaluation, screening, and expropriation, *Journal of Finance* 59, 601–621.

Wasmer, Etienne, and Philippe Weil, 2004, The Macroeconomics of Labor and Credit Market Imperfections, *American Economic Review* 94, 944–963.

Winton, Andrew, and Vijay Yerramilli, 2008, Entrepreneurial finance: Banks versus venture capital, *Journal of Financial Economics* 88, 51–79.