DEADLOCK ON THE BOARD[*]

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Abstract

We develop a dynamic model of board decision making. We show that a board may be unable to make a decision even if all directors agree on what the best decision is. Thus, the board may be unable to fire a CEO all directors agree is bad—a deadlocked board leads to an entrenched CEO. We explore how to compose the board and appoint directors to mitigate the costs of deadlock. We find that board diversity and long director tenure can exacerbate deadlock. Moreover, we rationalize why CEOs and incumbent directors have the power to appoint new directors: it helps to avoid deadlock. Our model also gives a new perspective on staggered boards and short-termism.

1. CEO tenure

2. Capital structure example

3. Good deadlock

   Deadlock can be good even if the incumbent policy is bad

4. Is deadlock just disagreement

   - Draw out empirical implications that distinguish our model from simple disagreement (and from Hermalin–Weisback)
   - Use Ilona's slide 2

5. Condorcet winner?

6. John Morgan on advisors with different biases

7. Highlight that the power to appoint result is a theoretical contribution

8. Few such problems are as relevant as those faced by boards of directors, the highest decision making authority in corporations, where directors sit side by side for many years. Our goal is to understand how this inaction arises in the boardroom, whether it should be prevented, and if so, how.

9. Introduce "tyranny of a fully biased board"

1 Introduction

The board of directors is the highest decision-making authority in a corporation. But sometimes boards struggle to make decisions. In surveys, 67% of directors report the inability to decide about some issues in the boardroom. Moreover, 30% say they have encountered a boardroom dispute threatening the very survival of the corporation (**?**, p. 2).[1] Such a "division among the directors" that "may render the board unable to take effective management action"—such deadlock on the board—can even lead directors to "vote wholly in disregard of the interests of the corporation" (**?**, pp. 113, 120).[2] Deadlock on the board can be so costly to US corporations that most states have adopted deadlock statutes, which often give courts the power to dissolve a deadlocked corporation, a power they rarely have otherwise, except in the event of default or fraud. A substantial legal literature studies how corporations can resolve deadlock ex post.[3] In this paper, we ask how deadlock can be avoided ex ante. Can the right mix of directors ensure a board makes efficient decisions? And, if so, how should director elections be structured to help achieve the right board composition? Should director elections be staggered, or should all directors be chosen at once? Should director tenure be limited? And should shareholders have all the power to choose directors, or should executives and incumbent directors have some power as well?

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[1]Further, from 2004–2006, 166 directors experienced disputes so severe that they publicly resigned from their boards at US public corporations, accepting potential damage to their careers (**?**; see also **?**).

[2]In summer 2017, deadlock on the board made it hard for Uber to appoint a CEO. According to Shervin Pishevar, an early shareholder in the company, there was one director, Benchmark Capital, who was "holding the company hostage and not allowing it to move forward in its critical executive search.... It has threatened to block any funding deals until Mr. Kalanick [the former CEO] relinquished the board seats" ("Uber Shareholders Fight for Control Amid Leadership Vacuum" *Wall Street Journal*, August 16, 2017). Moreover, deadlock on the board led one frontrunner for the job, Meg Whitman, to withdraw her name from consideration, saying "it was becoming clear that the board was still too fractured to make progress on the issues that were important to me" ("Inside Uber's Wild Ride in a Search of a New C.E.O." *New York Times*, August 29, 2017). Whitman's description of Uber's board mirrors the dictionary definition of deadlock: "a situation, typically one involving opposing parties, in which no progress can be made" (*New Oxford American Dictionary*).

[3]See, e.g., **?**, **?**, **?**, **?** (**?**, **?**), **?**, and **?**.

To address these questions, we develop a dynamic model of board decision making, in which we build on a notion of deadlock in the political economy literature (discussed below). We find that some directors refuse to replace a current policy with a new one just because they fear that other directors will refuse to replace the new policy in the future. There is complete deadlock: the board cannot move forward with any policy, even if all directors prefer it to the status quo. Shareholders suffer, since a deadlocked board cannot remove low-quality policies or executives—a deadlocked board leads to an entrenched CEO. Our model gives a new perspective on boardroom diversity and director tenure, two hotly debated policy issues: increased diversity and longer tenures can both exacerbate deadlock (their benefits notwithstanding).[4]

We study how the anticipation of deadlock can affect board composition via director elections. Should shareholders have all the power over the future of the board? We find that the answer is no. Indeed, shareholders themselves are better off if incumbent directors have some power over new directors elections: with more power, directors anticipate fewer disagreements in the future, and hence are less likely to become deadlocked today. This result rationalizes why it is typically the CEO and incumbent directors who nominate new directors in practice.

Model preview. In the model, a board made up of multiple directors decides on a corporate policy at each date. The model is based on three key assumptions, reflecting how real-world boards operate. (i) Directors have different preferences over policies. We refer to these different preferences as "biases," as they could reflect private benefits or misspecified beliefs. However, they could also reflect reasonable diversity of opinion (as we formalize in Subsection 8.1). For example, in the context of CEO turnover decisions, an activist's representative on the board could be biased toward an outside candidate with a history of asset divestitures, and an executive director could be biased toward an internal candidate with experience at the firm.

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[4]See **?** for a survey of the literature on boardroom diversity. And see, e.g., "Big Investors Question Corporate Board Tenures" (*Wall Street Journal*, March 23, 2016) and **?** for discussions of director tenure.

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(ii) The set of feasible policies changes over time. For example, different candidates are available to replace the incumbent CEO at each date. (iii) The incumbent stays in place whenever the board does not come to a decision. For example, if the board cannot decide on a replacement, the current CEO keeps the job.

Results preview. First, we ask when the board will replace an existing policy with a new one. We find that directors may knowingly retain a Pareto-dominated policy: there is deadlock on the board.[5] In the context of CEO turnover, this implies that a CEO can be so severely entrenched that he is not fired even if all directors prefer a replacement. To see why, consider a firm with a bad incumbent CEO, whom the board is considering replacing with an alternative. Suppose all directors agree that the alternative is better than the incumbent, but some directors are especially biased toward him. For example, activist representatives could be biased toward an alternative with a history of divestiture, as touched on above. Then, if the alternative becomes the new CEO, the biased directors will try to keep him in place, voting down alternatives in the future, no matter how much other directors prefer them—the new CEO will become entrenched. To prevent this, other (sufficiently patient) directors block the alternative today—the incumbent CEO becomes entrenched. Directors keep the bad incumbent CEO in place to preserve their option to get their way in the future, since a low quality CEO is relatively easy to replace. But by preventing the new CEO from becoming entrenched later on, they entrench the incumbent CEO today—the fear of entrenchment begets entrenchment.

This mechanism resonates with practice. For example, when Uber's recent executive search resulted in deadlock on the board, in which the company stayed without a CEO for several months, one director was pushing for a weak CEO who would be easy to replace in the future. According to *Bloomberg*:

> The company hopes to lock in a CEO by early September. The big question is whether the board can get on the same page. Getting a

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[5]This first result has close precedents in the political economy literature; see the discussion of the related literature and Section 4 below.

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majority of the eight-person group to support a single candidate is looking to be difficult.... Some...have argued...that Kalanick [a current director and former CEO] would *prefer a weak CEO just to increase his chance of making a comeback* ("Behind Uber's Messy CEO Search Is a Divided Boardroom," *Bloomberg Technology*, July 28, 2017, emphasis added).

Such deadlock arises only in a dynamic model: it relies on the linkage between decisions today and decisions in the future. The only reason a director votes to keep a Pareto dominated incumbent in place is to have a weak incumbent in place in the future, one that will be easy to replace with her preferred policy. In other words, a director votes strategically to manipulate the endogenous status quo for decisions in the future. Thus, there is no deadlock in the static benchmark, in which there is no future. Likewise, there is no deadlock in the (dynamic) benchmark with an exogenous status quo, in which there is no linkage between today and the future.

Second, we ask how director tenure affects deadlock. In the current debate (e.g., ?),[6] arguments against long director tenure focus on concerns about independence and the lack of fresh ideas. Our analysis suggests a distinct yet complementary argument for shorter tenures: in anticipation of a long tenure, directors behave strategically, blocking good candidates, and creating deadlock. This provides a counterpoint to the broadly negative view of corporate short-termism.

Third, we ask how board composition affects deadlock. We find that board diversity has a downside: it can exacerbate deadlock. For example, the deadlock caused by an activist's bias toward divestiture is not resolved by adding some executive directors biased toward investment. These directors will block divestiture-oriented CEOs, even if they agree that they are optimal today, just to preserve a strong bargaining position for the future. More generally, heterogeneous director biases do not cancel out—they do not yield a board that implements policies in shareholders'

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[6]While many countries, such as the UK, France, Spain, Hong Kong, and Singapore, have adopted some form of term limits for independent directors, the US and Canada have not as yet. However, many institutional investors, such as BlackRock and State Street, deem director tenures in the US too long and are voting against reappointments, leading commentators to suggest that director tenure is "the next boardroom battle" (?, p. 5; ?).

interest. Rather, they can yield a board that does not implement any policies at all. This is in line with empirical findings about diversity among directors: diversity in their skills, experience, or the blockholders they represent is associated with more persistent corporate policies (**?**), lower likelihood of strategic change (**?**), and lower firm value (**?**, **?**, and **?**); cf. Subsection 8. Our results thus offer a counterpoint to the blanket view that a "board should reflect a diversity of thought, backgrounds, skills, experiences and expertise" (**?**, p. 11).

Although diversity exacerbates deadlock in our model, it is not necessarily a bad thing. The short-term deadlock created by opposing biases can benefit shareholders—by blocking policies that some directors are biased toward, a diverse board can prevent low-quality policies from becoming entrenched forever, as can happen if all directors are biased in the same way. Diversity prevents this tyranny of a fully biased board. So there is such a thing as good deadlock.

Continuing the analysis of board composition, we ask what happens to deadlock if directors with no bias whatsoever join the board. Of course, such unbiased directors are unlikely to exist in reality. Even independent directors, with no explicit ties to a firm, have their own opinions and conflicts of interest (cf. Subsection 8.1). However, we find that even if unbiased directors represent a theoretical ideal and act purely in the interest of shareholders, they can still make these shareholders strictly worse off by joining the board. The reason is that adding an unbiased director to the board can induce other directors to engage in strategic blocking, deadlocking the board.

These results also speak to an important policy question: should director elections be staggered, with only a subset of directors being replaced at a time? Our framework is suited to address this question—it is a question about the connection between dynamic board decisions and board composition, and hence difficult to address with the models in the literature. We show that staggered elections can be a double-edged sword. They can mitigate deadlock on a diverse board, but can exacerbate it on a fully biased board. The reason is that with staggered elections directors anticipate sharing the board with different directors in the future, whose biases they do not

know when they vote today. Thus, directors on a diverse board anticipate that they may be joined by new directors biased in the same way and, symmetrically, those on a fully biased board anticipate that they may be joined by new directors biased in the opposite way.

Fourth, we ask who should have the power to choose new directors. This power does not rest entirely with shareholders in practice. As corporate governance advocate Adrian Cadbury puts it,

> The classical theory of the board is that shareholders elect the directors.... In practice, however, the shareholders of most public companies have little say in the appointment of directors, other than to nod through the nominations presented by the current board..... The legitimacy of the board as the appointee of the shareholders is something of a fiction (**?**, p. 66).

Why do current directors have so much power over the future of the board? Cadbury seems to think it is a bad idea, but could such an institution be optimal? We find that indeed it could be. The reason is that by ceding power to incumbent directors, shareholders can commit not to block their preferred policies in the future. This helps to prevent deadlock today. However, they should not give the current directors full power over board appointments; otherwise, current directors' bias can take over the whole board. Indeed, we find that it is typically optimal for shareholders and incumbent directors to share the power. This result rationalizes the real-world institution by which incumbent directors nominate new directors and shareholders vote on them. It also applies if there is an executive director on the board and hence explains why the CEO often exerts influence over the appointment of new board members in practice (e.g., **?**, **?**, and **?**).

Finally, given CEOs' influence over director appointments, we also explore how a CEO will choose directors if he wants to avoid being fired in the future. To our surprise, we find that he will not always appoint directors who are biased towards him. He may prefer directors who are unbiased, or even biased against him. The

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reason is that such directors may exacerbate deadlock on the board. Since deadlock makes it hard to fire the CEO, such strategic director appointments can help the CEO entrench himself. Colloquially, deadlock on the board can be better for the CEO than buddies on the board.

Related literature. A relatively small number of theory papers studies decision making by multiple directors on a corporate board.[7] We contribute to this literature by including dynamic interactions, which are almost entirely absent from this literature, even though they alter behavior dramatically. Indeed, none of our results obtain with a one-shot decision since deadlock does not arise (Proposition 1). The only other paper that includes board dynamics with multiple directors is **?**. As we discuss further in Section 4, in that paper, directors are not strategic, but group dynamics still generate inefficiencies.

We also add to the broader theory literature on boards.[8] Our finding that board diversity can exacerbate deadlock complements existing work on the downsides of director independence, since independent directors are likely to have different views than insiders on the board.[9] And our finding that a CEO may prefer to appoint unbiased directors, even when they may fire him in the future, contrasts with **?**, another paper in which a CEO appoints directors with the power to fire him.

Our result on deadlock causing entrenchment is related to results in the political economy literature on dynamic collective choice models with endogenous status quo (see Section 4).[10] Our other results do not have parallels in this literature (to our knowledge). Hence, our results on board/committee composition, director appointments/elections, and especially on the power of current directors to appoint new ones, which might be applicable to some political situations, may contribute to this literature. In contrast, our other results, for example, on shareholders and CEOs appointing directors, are more specific to corporate boards.

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[7]See **?**, **?**, **?**, **?**, and **?**; see also **?** for a model with multiple directors, but not strategic decision making.

[8]See **?** for a survey.

[9]See **?**, **?**, **?**, and **?**.

[10]See also **?**, **?**, **?**, and **?**.

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Our explanation of CEO entrenchment, which is based only on directors' strategic behavior, contrasts with those in the finance literature, which are based largely on a CEO's actively entrenching himself (e.g., "invest[ing] in businesses related to their own background and experience")[11] or directors' direct utility costs of voting against a CEO.[12]

Layout. Section 2 presents the model. Section 3 analyzes two benchmarks: without dynamic interaction and with exogenous status quo. Section 4 describes the baseline mechanism of deadlock on the board and entrenched policies. Section 5 studies optimal board composition. Section 6 examines who should have the power to appoint directors. Section 7 studies how a CEO should appoint directors. Section 8 discusses robustness and presents several extensions. Section 9 presents empirical implications and how to test them. Section 10 discusses our results, their limitations, and their implications for future research, and concludes.

2   Model

There is a board comprising two directors, $i \in \{1, 2\}$, who decide on a policy at each of two dates, $t \in \{1, 2\}$. (See Section 8 for infinite-horizon and N-director extensions.) At date t, the board can replace the current "incumbent" policy x_{t-1} with an alternative policy y_t. Decisions are made by strict majority voting: if both directors vote for the alternative y_t, then y_t becomes the incumbent policy, $x_t = y_t$; otherwise, the incumbent policy stays in place, $x_t = x_{t-1}$.[13] The policy in place

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[11]**?**, p. 125. See also, e.g., **?**.

[12]See, e.g., **?**, **?**, **?**, and **?**.

[13]These binary decisions between x_{t-1} and y_t can represent many real-world decision problems, which are often formally binary—e.g., whether or not to fire an executive or whether or not to divest a division—or meaningfully approximated by binary decisions—e.g., the question of how much cash to pay out can be approximated by the question of whether or not to cut a dividend. Just as importantly, with binary decisions there is only one way to disagree, and as a result different decision-making protocols (e.g., unanimity and majority) lead to the same outcomes. This is not the case with many alternatives, since there are many ways to disagree. (See **?** pp. 1161–1162 for a discussion of this problem.)

creates value $v(x_t)$ at date t, so shareholders get $v(x_1) + \delta v(x_2)$, where δ is the rate of time preference. (We allow for $\delta > 1$, since date 2 may represent more calendar time than date 1.) Directors care about firm value, but they can be biased. Each director i maximizes the sum $v(x_1) + b_i(x_1) + \delta\big(v(x_2) + b_i(x_2)\big)$, where b_i is her bias.[14] We discuss different interpretations of directors' biases in Section 8.1.

Policies differ in two dimensions: in how much value they create for shareholders and in how much they appeal to biased directors. We capture shareholder value with the "quality" $q \in \{h, \ell\}$. If the date-t policy x_t is of high quality h, then $v(x_t) = v_h$; if x_t is of low(er) quality ℓ, then $v(x_t) = v_\ell < v_h$. We capture the appeal to biased directors by adding a "bias" type $\tau \in \{\alpha, \beta\}$ to each policy and allowing directors to be either α- or β-biased, where a τ-biased director gets $b_i(x_t) = b_\tau$ if the policy x_t is type τ and $b_i(x_t) = 0$ otherwise. We also allow for unbiased directors, for whom $b_i(x_t) = 0$ for all policies x_t.

We assume that the qualities and bias types are i.i.d. at date 1 and date 2 (although we relax this in Appendix A.7.1). p_q and p_τ denote the probabilities that an alternative y_t is of quality $q \in \{h, \ell\}$ and of bias type $\tau \in \{\alpha, \beta\}$, respectively. $\bar{v} := p_h v_h + p_\ell v_\ell$ denotes the average quality of y_t and $v_0 := v(x_0)$ denotes the quality of the initial incumbent policy x_0. Thus, although each alternative policy y_t must be one of four types $h\alpha$, $h\beta$, $\ell\alpha$, and $\ell\beta$, the initial policy x_0 need not be.

Assumptions. We make two assumptions on parameters. The first says that directors prefer *any* alternative y_t to x_0.

Assumption 1 *The initial policy x_0 is "very bad," in that it is worse for shareholders than low-quality alternatives, $v_0 < v_\ell$, and no director is biased toward it, $b_i(x_0) = 0$ for $i \in \{1, 2\}$.*

We make this assumption because we want to highlight that a policy x_0 can be entrenched even though it is "worse" than *any* alternative y_t (see Section 4).

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[14]$v(x_t)$ need not represent the common value of all shareholders, but could rather represent the average value of shareholders with heterogenous biases, e.g., half of the shareholders could value x_t above $v(x_t)$ and half below. Thus, directors' biases could also reflect the heterogenous biases/preferences of individual shareholders.

The second assumption says that directors' biases can be large, capturing the diversity of interests or opinions on real-world boards.[15]

Assumption 2 *Biased directors are sufficiently biased: for $\tau \in \{\alpha, \beta\}$,*

$$b_\tau > \max \left\{ \frac{v_h - v_0 + \delta p_\ell (v_h - v_\ell)}{\delta p_\tau p_\ell} \, , \, v_h - v_\ell \right\}. \tag{1}$$

We make this assumption to generate a tradeoff between directors' incentives to maximize the common value v_q vs. the private value b_τ that reflects their individual biases. This is necessary for deadlock, which arises because directors block policies that may be good for the firm today to preserve their option of getting b_τ in the future. (To see where the specific terms on the RHS of equation (1) come from, see the proofs of Lemma 1, Lemma 2, and Proposition 3.)

Note that the assumption requires b_τ to be of the same order of magnitude as v_q. However, it does not imply that b_τ is on the same scale as the value of the entire firm—rather, it is on the same scale as the extent to which directors care about firm value. In particular, v_q in a director's payoff should be interpreted as her value-linked compensation, her equity stake in the firm, or her intrinsic motivation. Thus, it is natural to think that directors' private benefits (such as those of insiders, large blockholders, or employee/creditor representatives), captured by b_τ, could be just as large as their interest in firm value. As we discuss in Subsection 8.1, directors' heterogenous beliefs can also be captured by b_τ. In this case, b_τ is automatically of a similar size to v_q.

Solution concept. We solve for subgame perfect equilibria—sequentially rational strategies for each director $i \in \{1, 2\}$ to vote for/against y_t for $t \in \{1, 2\}$ given consistent beliefs—such that directors use the following tie-breaking rules:

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[15]For example, a recent survey of global directors emphasizes the importance of having different opinions on boards as follows: "In the boardroom, disagreements are often unavoidable—especially when the board is composed of independent-minded, skilled, and outspoken directors. This is not a bad thing. There should be a debate in the boardroom" (**?**, p. 2).

(i) When indifferent, directors do not vote against strictly Pareto-dominant poli-cies.[16]

(ii) If both directors are indifferent, the incumbent stays in place.

Unless stated otherwise, our results below pertain to the unique subgame perfect equilibrium that satisfies these tie-breaking rules.

Note that this definition implies directors cannot commit to do something individ-ually suboptimal at date 2. As we discuss in Section 10, this is a source of inefficiency in our model, since it implies directors cannot bargain to reach the efficient outcome at date 1.

Board composition. If a director is unbiased we indicate her type with ν. If a director is biased toward τ-policies, we refer to her as τ-biased and indicate her type with τ (so τ can represent a director type as well as a policy type). We use primes to denote the opposite director or policy: if $\tau = \alpha$, then $\tau' = \beta$, and vice versa. Hence, a ν-ν board is an "unbiased" board in which both directors are unbiased; a τ-τ board is a "fully biased" board in which directors have the same bias; a τ-ν board is a "partially biased" board in which one director is τ-biased and the other is unbiased; and a τ-τ' board is a "diverse" board in which directors have opposing biases.

3   Benchmarks

Before we turn to the main analysis, we present two benchmarks to emphasize that our results are driven by (i) the dynamic interaction between directors across dates

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[16]In particular, if (under the equilibrium beliefs) both directors weakly prefer the alternative y_t to the incumbent x_{t-1} and one director strictly prefers y_t to x_{t-1}, then (i) if one director is indifferent between y_t and x_{t-1}, she votes in the interest of the director with a strict preference and (ii) if the director with the strict preference is indifferent between voting for and against (because she is not pivotal), she votes for her preferred policy. (This assumption just serves to rule out the equilibrium in which both directors vote against even though one of them prefers the alternative. This is a Nash equilibrium since if one director votes against, the incumbent stays in place no matter how the other votes and, hence, it is a weak best response for her to vote against too.)

and (ii) the linkage between the dates through the endogenous status quo, i.e. the fact that if a policy is implemented today, it is the default choice tomorrow.

In the first benchmark, we suppose that the game ends after date 1: either x_0 or the alternative y_1 is implemented. In this case, there is no deadlock no matter how large the biases b_τ are. This underscores that deadlock in our model is not an immediate consequence of directors' disagreement.

Proposition 1 (ONE-SHOT BENCHMARK.) *Suppose the board votes only once. The incumbent policy x_0 is always replaced, regardless of board composition.*

In the second benchmark, we consider two dates, but suppose that the status quo is exogenous at date 2. At date 1, directors vote for/against implementing y_1 over x_0 (as in the baseline model), and at date 2, they vote for/against implementing y_2 over an exogenous status quo policy (rather than over the policy chosen at date 1, as in the baseline model). It follows immediately from the one-shot benchmark that x_0 is never chosen, since directors have no incentive to vote strategically to manipulate the future status quo.

Proposition 2 (EXOGENOUS STATUS QUO.) *Suppose that the status quo at date 2 is exogenous. Then x_0 is always replaced, regardless of board composition.*

4   Entrenchment

Given that directors always replace x_0 in the benchmarks above, do they also replace it in our dynamic model in which the status quo is endogenous? Not if the board is diverse, since directors with opposing biases vote strategically. In particular, with a diverse board, the α-biased director votes against all β-alternatives and the β-biased director votes against all α-alternatives. This leaves x_0 in place at date 1, even though both directors would be strictly better off with any other policy.

Proposition 3 (ENTRENCHMENT.) *Given a diverse (τ-τ') board, the incumbent policy x_0 is entrenched: no alternative y_1 is ever appointed at date 1.*

Intuitively, the τ-biased director knows that if a τ'-alternative is chosen, the τ'-biased director will vote against replacing it with any τ-alternative at date 2. In contrast, if the bad policy x_0 stays in place, the τ'-biased director will vote for any τ-alternative at date 2. Because the τ-biased director's bias toward τ-policies is large (by Assumption 1), she blocks any τ'-policy at date 1. There is complete deadlock: the very bad policy x_0 stays in place because each director votes against the other's preferred policies to preserve the option to get her way in the future.

This extends the standard real options intuition that it can be optimal to delay irreversible decisions (see, e.g., **?**). Unlike in the standard case, there are no explicit costs of reversing the decision: it is irreversible only because it is made by a group—directors choose not to exercise the option to replace the incumbent today only to ensure that the other director has incentive to exercise her option to replace it in the future.

This result does not rely on our assumption that the board comprises only two directors or even that it comprises an even number of directors. Deadlock is not about a fifty-fifty split of the board, but rather about strategic stasis. Indeed, as we spell out formally below (see the proof of Proposition 3 and Subsection 8.2), with $N > 2$ directors and either N possible alternatives or uncertainty about future directors' biases, the same intuition leads to the same result: directors block Pareto-dominating policies today just to preserve the option to implement their preferred policies in the future. Moreover, this idea is not special to the finite-horizon setup. In Subsection 8.3, we extend the model to the infinite horizon and show that the same intuition leads to the same result there too.

In fact, this result is a manifestation of a general phenomenon: if the status quo is endogenous in a dynamic collective decision problem, deadlock arises from strategic blocking.[17] It is also closely related to the mechanism in **?**, in which directors pass

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[17]See Proposition 4 in **?**. One difference between our environment and theirs is that in our model today's decision is linked to future decisions via the set of available policies, not just the status quo—if a policy is not implemented today, it may not be available to implement tomorrow. This exacerbates the inefficiencies of deadlock, since strategic blocking reduces access to high-quality policies. This extra effect matters for optimal board composition, as we spell out in Appendix

up an investment they all believe is good anticipating that it will not be managed the way they want after new information arrives. Like our mechanism, this relies on the pivotal director changing over time. In our model, who is pivotal tomorrow depends on tomorrow's status quo, which changes with today's decision. In theirs, who is pivotal tomorrow depends on the intensity of directors' preferences tomorrow, which changes with the arrival of new information today. Broadly, these mechanisms both suggest that directors can prefer inaction to losing control (by giving up being pivotal).

Perhaps the most important function of real-world boards is appointing CEOs. If the incumbent policy x_0 represents the incumbent CEO, and the alternatives y_t represent potential replacement CEOs, our model generates CEO entrenchment, which seems to be a major source of corporate inefficiency (**?**). In our model, unlike in others, entrenchment arises without any opportunistic behavior by the CEO or director disutility of voting against him. Rather, it arises only due to dynamic decision making by multiple strategic directors.

In our model, deadlock is the result of directors' voting strategically to increase their chances of implementing their preferred policies later in their tenure on the board. The time preference parameter δ in our model can be viewed as a measure of directors' remaining tenure: if a director has a short tenure, she does not care about future policies, so δ is low; in contrast, if she has a long tenure, she cares a lot about them, so δ is high. This interpretation yields the next corollary.

Corollary 1 (TENURE.) *Suppose (instead of Assumption 2) that $b_\tau > (v_h - v_\ell)/p_\tau$ for each τ. Given a diverse board, increasing director tenure leads to entrenchment in the sense that x_0 is always replaced at date 1 for δ sufficiently small but never replaced for δ sufficiently large.*

Deeming director tenures too long, a number of institutional investors are now voting against reappointments, leading commentators to suggest that director tenure is

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A.7.1.

"the next boardroom battle" (**?**, p. 5; **?**). The argument for shorter tenures has centered around the idea that after a long tenure, a director may become too close to management and may also lack fresh ideas about the business. Our analysis offers a new, complementary perspective on the downside of long tenures: in anticipation of a long tenure, directors behave strategically, creating deadlock.

More generally, our analysis uncovers a cost of long-termism: it can incentivize strategic voting, exacerbating deadlock. This provides a counterpoint to the broadly negative view of corporate short-termism; see, e.g., the former Vice President Joe Biden's opinion that short-termism "saps the economy" (**?**).

5 Board Composition

Our results so far show that board diversity can lead to deadlock: directors with opposing biases will vote against each others' preferred policies. We now examine how board composition more generally can mitigate/aggravate deadlock. How does having an unbiased (ν-) director on the board affect deadlock? Is it always good for shareholders? Below, we point out that although the presence of unbiased directors has an immediate benefit—it prevents biased directors from implementing low-quality policies of their preferred type, it can also have a cost—it leads biased directors to respond strategically, blocking high-quality policies of other types. The shareholder-optimal board composition trades off this cost and benefit.

We start with the benefit of an unbiased director on the board. She can counteract a biased director on the board, who could make some low-quality policies hard to replace. To do so, the unbiased director votes strategically and prevents these policies from being implemented in the first place.

Lemma 1 (BENEFIT OF DIRECTOR HETEROGENEITY.) *Define*

$$\Delta_\tau := \delta(1 - p_\tau)p_h\left(v_h - v_\ell\right) - \left(v_\ell - v_0\right). \tag{2}$$

Consider a τ-ν-partially biased board. The unbiased director votes against the low-quality τ-alternative if and only if $\Delta_\tau > 0$ and votes in favor of all other alternatives.

Realizing that the τ-biased director will resist replacing a low-quality τ-alternative if it is chosen today, the unbiased director blocks this alternative to preserve the option to implement a high-quality τ'-alternative in the future. To benefit shareholders in the long term, she is effectively deadlocking the board in the short term: there is such a thing as good deadlock. In other words, sometimes the long-term benefit of deadlock—the option to implement a high-quality policy in the future—is higher than the short-term cost of keeping x_0 in place. (This benefit and cost correspond to the two terms in Δ_τ.)

The unbiased director's strategic blocking can make her appear passive, or even biased, in the short-term. She votes against some low-quality alternatives even though the incumbent policy is even worse ($v_0 < v_\ell$). However, she is actually actively doing what is best for shareholders. She is preventing them from getting stuck with low-quality policies in the long-term.

But an unbiased director on the board can also have a cost: she can create bad deadlock. Yes, she always acts in the interest of shareholders. However, anticipating as much, a biased director on the board responds strategically. Just as in the case of a diverse board, she strategically blocks high-quality policies of her less-preferred type today, anticipating that the unbiased director will make them hard to replace in the future.

Lemma 2 (COST OF DIRECTOR HETEROGENEITY.) *Consider a τ-ν-partially biased board. The τ-biased director votes against the high-quality τ'-alternative and votes in favor of all other alternatives.*

Perhaps a biased director can resolve deadlock? Yes, in fact. If both directors are biased in the same way, neither does any strategic blocking, since they both know that the other will agree to their preferred policies in the future. Whether this is

in the interest of shareholders depends on the relative costs and benefits of good deadlock vs. bad deadlock.

Proposition 4 (SHAREHOLDER OPTIMAL BOARD COMPOSITION.) *Shareholders are better off with a fully τ-biased board than a τ-ν-partially biased board if and only if*

$$p_\ell p_\tau \Delta_\tau < p_{\tau'} p_h \Big(v_h - v_0 + \delta p_{\tau'} p_\ell (v_h - v_\ell) \Big), \tag{3}$$

and are always better off with a fully biased or partially biased board than a diverse board.

Shareholders are better off with a fully biased board than with a partially biased board whenever the benefit of the partially biased board—the unbiased director strategically blocking low-quality τ-alternatives, is outweighed by its cost—the τ-biased director strategically blocking high-quality τ'-alternatives. (This benefit and cost correspond to the left-hand side and right-hand side of equation (3).)

What do these results on shareholder-optimal board composition have to say about director appointments? To address this question, we suppose there is a board with a τ-biased director in place and an empty seat to be filled at date 0, and we ask how shareholders will choose to fill it. When will they appoint an unbiased director who will act in their interest? When will they appoint a τ'-biased director who will counteract the τ-biased director? And when will they appoint a τ-biased director to avoid deadlock entirely? The characterization in Proposition 4 gives the answer.

Corollary 2 (SHAREHOLDERS' DIRECTOR APPOINTMENTS.) *Suppose there is a τ-biased director in place and an empty board seat. Shareholders appoint a τ-biased director if condition (3) holds. Otherwise, they appoint an unbiased director. (They never appoint a τ'-biased director.)*

There are two caveats that we should stress here. First, like a partially biased board, a diverse τ-τ' board also has the benefit of good deadlock: it prevents some

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low-quality policies from becoming entrenched in the long-term. Although this benefit is never large enough to make shareholders prefer a diverse board to a fully biased board in our baseline specification, it can be so if we relax the assumption that the distribution of alternatives is identical across periods (see Appendix A.7.1 for a formal analysis).

Second, in our setting, a board with two unbiased directors is ideal for shareholders. However, as we elaborate on in Subsection 8.1, we believe that unbiased directors could be impossible to find in reality. Hence, our point here is that even if such an unbiased director could be appointed to the board, it might not be good for shareholders. Moreover, even if shareholders could replace all directors at once with unbiased directors, there are (unmodeled) practical concerns that could make this unattractive. For example, shareholders may not want to replace a biased director who has indispensable expertise. Hence, they must account for her bias when they appoint a new director to join her on the board. Given the costs of deadlock, the best response may be to exacerbate her bias, rather than to attenuate it.

Application to staggered boards. One important policy question that our framework can speak to is whether boards should be staggered, i.e., whether shareholders should be prevented "from replacing a majority of the board of directors without the passage of at least two annual elections" (**?**, p. 410). The literature emphasizes that this can prevent efficient takeovers and proxy fights by forcing bidders and activists to win two far-apart elections (**?**). But, absent a model like ours, in which multiple strategic directors interact over time, a formal theoretical analysis has been missing.

We use our framework to take a small step toward filling this gap, by capturing a staggered board in a stylized way. We assume that there are two directors at date 1 and one of them is replaced before date 2 with another director, whose bias is unknown before she joins the board. This captures the key feature that only half of the board is replaced at a time. We find that uncertainty about the future of the board can mitigate deadlock on a diverse board, because each director is less likely

to block policies if she thinks she may be joined by another director who shares her bias in the future. On the other hand, it can create deadlock on a fully biased board, because each director is more likely to block policies if she worries she will be joined by a director with an opposing bias in the future.

Proposition 5 *Consider a board with a τ-biased incumbent director and a new director joining at date 1. If the new director is τ-biased with probability q and τ'-biased with probability $1 - q$, then the incumbent blocks τ'-policies at date 1 whenever*

$$b_\tau > \frac{v_h - v_0 + \delta p_\ell (1 + q p_\tau)(v_h - v_\ell)}{\delta(1 - q)p_\tau}. \tag{4}$$

We should add the caveat that we are not holding director tenure constant in this example. It is shorter on staggered boards by construction. Since we still find that a staggered board can create deadlock, even though short tenure helps mitigate deadlock (Corollary 1), on net we see this result as supporting the skepticism toward staggered boards.

6 Who Should Appoint Directors?

When we asked which directors shareholders would appoint in Corollary 2, we assumed that they had the power to make these appointments. But, as the Adrian Cadbury quote that we cite in the Introduction makes clear, this assumption does not always reflect reality. Indeed, it may be "something of a fiction." Hence, we now ask what happens if incumbent directors have the power over new director appointments, and whether it can be optimal for shareholders to give them this power. We find that it can be, because it helps prevent deadlock.

To analyze what happens if incumbent directors have the power to appoint new directors, we twist the model slightly. Suppose that both directors vote on a policy at date 1. One of the directors, who we assume is τ-biased, stays in place for two periods—we refer to her as the incumbent director. The other director, who can

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have any bias or be unbiased, steps down after the date-1 vote and is replaced by a new director. After that, the date-2 alternative y_2 is realized, and the new board, composed of the incumbent director and the new director, votes on it at date 2. How the new director is chosen depends on the power of the incumbent director relative to shareholders. We capture this power by assuming that the incumbent director chooses the replacement director with probability π and shareholders choose her with probability $1 - \pi$.

Ceding power to the incumbent director can help prevent deadlock. When the incumbent director controls the board at date 2, she does not block policies at date 1, since she does not need to improve her future bargaining position, as in a fully biased board. In other words, by ceding power to the incumbent director, shareholders are able to overcome a commitment problem: they effectively commit not to block her preferred policies at date 2, which can improve date 1 outcomes. However, this can come at the cost of worse date 2 outcomes, again as in a fully biased board. The next result summarizes how much power shareholders optimally give to the incumbent director to manage the tradeoff between avoiding deadlock at date 1 and not getting their preferred policy at date 2.

Proposition 6 *Define*

$$\bar{\pi} := 1 - \frac{v_h - v_0 + \delta p_l p_{\tau'} (v_h - v_l)}{\delta p_l p_\tau [b_\tau - (v_h - v_l)]} \in (0, 1). \tag{5}$$

The amount of power shareholders optimally give the incumbent director is

$$\pi^* = \begin{cases} \bar{\pi} & \text{if } v_0 + \delta \bar{v} < v_h + \delta(1 - \bar{\pi})v_h + \delta\bar{\pi}\big(v_h (1 - p_\tau p_\ell) + v_\ell p_\tau p_\ell\big) \\ \pi \in [0, \bar{\pi}) & \text{otherwise.} \end{cases} \tag{6}$$

This result implies that shareholders never give the incumbent director full power over new appointments ($\pi^* = 1$ is never optimal). Rather, they give her just enough

power over the future to prevent deadlock today, and still retain some power to prevent her from implementing low-quality policies that she favors. Thus, by giving her partial power, shareholders get the full benefit of a fully biased board without bearing all of the cost.

Overall, this result suggests that the institution that Cadbury and others find so perverse can be optimal in some circumstances, and thereby rationalizes why directors have (some but not all) power over director appointments. This result also applies if there is an executive director on the board. Hence, it explains why the CEO often exerts influence over the appointment of new board members in practice (e.g., **?**, **?**).

7 CEO Appoints Directors

Given CEOs' influence over director appointments, we also explore how a CEO should choose directors. This adds the wrinkle that the CEO is hiring someone who could fire him in the future. Thus, we suppose that his sole objective is to keep his position. Will he appoint directors who are biased toward him whenever he can? We show below that the answer is no. In fact, a CEO can be better off with a diverse board than a board that is fully biased toward him. The reason is that a diverse board is likely to be deadlocked, and thus struggle to fire the CEO.

We return to the interpretation of the incumbent policy x_0 as the incumbent CEO and of the alternatives y_t as potential replacement CEOs, and ask how this incumbent CEO will choose directors at date 0. We assume that his only objective is to keep his job as CEO, which we capture by assuming he maximizes a weighted average of the probabilities he is employed at each date: $U = \mathbb{P}\big[\text{employed at date } 1\big] w_1 + \mathbb{P}\big[\text{employed at date } 2\big] w_2$ for some weights or "wages" w_1 and w_2.

We start with the case of a "very bad" CEO, who does not appeal to any directors. It follows from Proposition 3 that a "very bad" CEO prefers a more diverse board to entrench himself.

Corollary 3 ("Very bad" CEO's board ranking.) *Given $v_0 < v_\ell$ and $b_i(x_0) = 0$, the incumbent CEO prefers a more diverse board in the sense that his ranking over boards is as follows:*

$$\text{diverse} \; \succ \; \text{partially biased} \; \succ \; \text{unbiased} \; \sim \; \text{fully biased}. \tag{7}$$

Now, we relax our assumption that the incumbent CEO x_0 is "very bad." To capture the idea that a CEO could choose directors who are biased toward him, we assume that the CEO is of type τ. And to capture the idea that this could lead bad quality CEOs to become entrenched, we assume he is low quality. We find that, like the very bad CEO above, a low-quality CEO has a preference for diversity, because he wants to exploit deadlock on the board to avoid being fired. In fact, deadlock on the board can be more valuable for him than favoritism from the board.

Proposition 7 (Low-quality CEO's board ranking.) *Suppose $v_0 = v_\ell$ and $b_i(x_0) = b_\tau$. If p_τ is sufficiently large,[18] the incumbent $\ell\tau$-CEO can be better off with a τ'-director than without one, in the sense that his ranking over boards is as follows:*

$$
\begin{aligned}
\tau'\text{-}\nu\text{-partially biased} \; &\succ \; \text{diverse} \; \sim \text{fully } \tau\text{-biased} \; \sim \\
\tau\text{-}\nu\text{-partially biased} \; &\succ \; \text{unbiased} \; \succ \; \text{fully } \tau'\text{-biased}.
\end{aligned} \tag{8}
$$

8 Robustness and Extensions

8.1 Interpretation of Biases

Heterogenous biases. Heterogeneous director biases are the key driver of our results. These biases capture realistic heterogeneity among directors. For example, the interests of inside directors can be different from those of outside directors. In start-ups, founding entrepreneurs often sit on boards beside capital providers like

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[18]Specifically, we require $(p_\tau - p_{\tau'})p_h w_1 > \Big(p_{\tau'} - p_\tau p_h(p_{\tau'}p_h + p_\tau p_\ell)\Big)w_2$, which is always satisfied if p_τ is close to one. In the proof we also give the low-quality τ-CEO's rankings for other parameters.

VCs, which have different objectives for the corporation. Indeed, early this year at Applied Cleantech, a technology start-up, deadlock on the board was so severe that the investors on the board sued the founder for control. In mature firms, equity blockholders typically sit on the board. These could be heirs to family firms, with an interest in preserving their legacies, or activist investors, with interests in preserving their reputations for fast value-enhancement. Other kinds of director heterogeneity are common. For example, in Germany it is common for directors to represent stakeholders such as bank creditors or employees/unions.

In fact, we view the "unbiased" directors in our model as a theoretical ideal, unlikely even to exist in reality. Even independent directors, those without a material relationship to the corporation, have their own opinions and conflicts of interest, e.g., due to connections with the CEO[19] or their own career concerns.[20]

Director heterogeneity can also reflect heterogeneity among shareholders themselves, who have different preferences, e.g., due to different beliefs and portfolio positions. In close corporations, diverse shareholders sit directly on the board. But even in public corporations, diverse shareholders appoint directors to represent their diverse interests.

Preferences vs. beliefs. We have described directors' biases as reflecting differences in their preferences (i.e. tastes) over policies. But they can also reflect differences in beliefs. To see why, consider the following setup. At the end of each date the policy x_t either "succeeds," generating value V or "fails,"An unbiased director believes the policy succeeds with probability $\pi_\nu(x_t)$, so that her value of the policy coincides with shareholder value, i.e. $\pi_\nu(x_t)V = v(x_t)$, so $\pi_\nu(x_t) = v(x_t)/V$. A τ-biased director believes the success probability of a τ-policy is $\pi_\tau(x_t) > \pi_\nu(x_t)$,

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[19]Independent directors can be connected to the CEO because, for example, the CEO appointed them (**?**), they have overlapping social networks (**?**), or they serve on interlocking boards with the CEO (**?**).

[20]**?** show that time to re-election affects directors' decisions, implying they care about their own careers, not just maximizing shareholder value.

so that her value of the policy is $v(x_t) + b_\tau$, i.e. $\pi_\tau(x_t)V = v(x_t) + b_\tau$ or

$$\pi_\tau(x_t) = \frac{v(x_t) + b_\tau}{V} = \pi_\nu(x_t) + \frac{b_\tau}{V}. \tag{9}$$

8.2 $N > 2$ Directors and Uncertain Biases

We next show that our results are not specific to boards with just two directors (we do this in another way in the proof of Proposition 3, where we consider a board with $N > 2$ different director types). We argue that deadlock can arise even if the majority of the board is biased the same way as long as directors are not sure about other directors' biases about future decisions. For example, directors may be uncertain about what personal connections other directors have with future candidate CEOs, what views they have on corporate policy decisions that come up in the future, or what other directors will be on the board in future.

Specifically, suppose there are $N > 2$ directors, but still just two types of policies, and decisions are made by majority voting. All directors are either τ-biased or τ'-biased. Each knows how she is biased, but is uncertain about the biases of her fellow directors in the future. Define q_τ as the probability that the majority of directors are τ-biased at date 2, i.e.,

$$q_\tau := \mathbb{P}\left[\text{at least } \frac{N+1}{2} \text{ directors are } \tau\text{-biased at date 2}\right]. \tag{10}$$

The next result shows that the very bad incumbent policy x_0 can still be entrenched in this setup.

Proposition 8 *With N directors and uncertain biases, a τ-biased director votes against any τ'-alternative at date 1 if her bias is sufficiently large,*

$$b_\tau > \frac{v_h - v_0 + \delta p_\ell \left(1 - q_\tau p_\tau\right) \left(v_h - v_\ell\right)}{\delta p_\tau \left(1 - q_\tau\right)}. \tag{11}$$

Deadlock arises for the same reason as in the baseline model: directors block

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alternatives that they worry might be hard to replace in the future. In the baseline model with known biases, they block alternatives because they know other directors are biased toward them. With uncertainty about biases, directors block alternatives because they worry other directors might be biased toward them in the future.

Observe, however, that if $q_\tau \to 1$, the condition in the proposition is never satisfied. In words, if τ-directors know they will always have the majority, and hence will always have control over decisions, they do not strategically block good alternatives. Thus, a board where τ-biased directors have the majority and will retain their majority in the future, behaves like a fully biased board: the group decision problem has no teeth, so the board acts like a single biased agent.

8.3   Infinite Horizon

We now ask whether our results are specific to the two-date setup. We show that a version of deadlock can arise even if there is no final date in which all directors know their preferred policy can get through. Even with an infinite horizon, directors do strategic blocking, keeping Pareto-dominated policies in place, as long as it improves their chances to get their way in the future.

Consider an infinite-horizon setup. Everything is as in the baseline model, except we now require $\delta \in (0, 1)$ to ensure that directors' value functions are well defined.

We define x_0 as entrenched due to deadlock if all ℓ-quality policies are blocked in favor of retaining x_0. This definition is stronger than in the baseline model, since it applies to all dates (not just date 1), but weaker in that it applies only to ℓ-quality policies (directors do not block h-quality policies). The next result gives conditions under which such entrenchment due to deadlock occurs.

Proposition 9 (INFINITE-HORIZON ENTRENCHMENT.) *Suppose $\delta \in (0, 1)$. Given a diverse board, there is an equilibrium in which all ℓ-quality policies are blocked (there is entrenchment), as long as directors' biases are neither too high nor too low,*

i.e.,

$$\frac{b_\tau}{1-\delta} \in \left[\max\left\{ \frac{v_h - v_\ell}{1 - \delta + \delta p_\tau p_h}, \frac{(1 - \delta p_\ell)(v_\ell - v_0) - \delta p_\tau p_h (v_h - v_0)}{\delta p_\tau p_h (1 - \delta + \delta p_{\tau'} p_h)} \right\}, \frac{v_h - v_0}{\delta p_\tau p_h} \right],$$
(12)

where the interval above is non-empty.

Intuitively, by retaining the very bad incumbent x_0 and blocking a low-quality τ'-alternative, the τ-director increases his chances of appointing an $h\tau$-alternative in the future. This intuition is analogous to that in the baseline model: directors block Pareto-dominating alternatives, holding out to get higher payoffs in the future. Observe that this would not occur if high-quality policies were also blocked and hence x_0 were to stay in place forever: in that case, directors would never get high payoffs in the future, and hence would be better off implementing any alternative today. This is why Proposition 9 requires not only that directors' biases are large enough, as in the baseline model, but also that they are not too large—it ensures that directors do not block high-quality alternatives.

9 Empirical Implications

Turning to our model's empirical content, we discuss empirical proxies for our model's key quantities and empirical predictions corresponding to its main results.

Proxies. Boards meet in the privacy of the boardroom without disclosing their minutes. Hence, deadlock is often not revealed publicly except in extreme cases, such as those that wind up in court or result in director resignations.[21] One exception is China, where firms must publicly disclose if their independent directors vote in dissent (e.g., **?**).[22]

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[21]Translation company Transperfect and startup Applied Cleantech are recent examples of deadlock cases that have gone all the way to court. **?** and **?** analyze director resignations resulting from board disputes, which US companies must disclose by a 2004 SEC law.

[22]Of course, companies typically want to keep such disagreements private, so boards that disclose their directors' voting in dissent should make up only a fraction of deadlocked boards.

In addition, our model suggests a way to test for deadlock indirectly: deadlock is manifested in boards' retaining incumbent policies, even when superior alternatives are available (Proposition 3). Applied to boards' key decisions, CEO turnover and corporate strategy, deadlock can be measured/proxied for by the following:

(i) longer CEO tenure;

(ii) conditional on CEO termination, longer periods to appoint a new CEO (as with Uber's deadlocked board);

(iii) slow changes in strategy in response to a changing environment, even at the expense of the firm's competitiveness (as is common in corporations, **?**, **?**);

(iv) more persistent corporate policies.

A number of our predictions require proxies not only for deadlock, but also for directors' "biases" b_τ representing their preferences/private benefits or beliefs (Subsection 8.1). Proxies for directors' preferences include the stakeholders they represent—directors could represent employee unions, outside creditors, corporate executives, and a variety of equity blockholders, such as VC investors, activists, and founding families; these diverse stakeholders are likely all to have different preferences over/private benefits from different company policies. Proxies for directors' beliefs include diversity in directors' experience, expertise, backgrounds, or skills, all of which are likely to lead to different views on the best policy for a company.

Predictions. Our main results correspond to testable predictions on the determinants of deadlock.

Prediction 1 *All else equal, deadlock is more likely on more diverse boards (cf. Proposition 3).*

This is consistent with the following empirical findings.

(i) **?** (**?**) construct a board diversity index and show that a high index is associated with persistent corporate policies.

(ii) **?** (**?**) show that diversity in directors' occupational or professional backgrounds is associated with less strategic change, such as fewer divestitures and reorganizations.

(iii) **?** (**?**) and **?** (**?**) find that diversity in directors' skills, expertise, and incentives is associated with lower firm value.

(iv) **?** (**?**) finds that blockholder diversity has a negative influence on company value and operations, since proposed policy changes receive little support (if blockholders have their representatives on the board, diversity in blockholders' preferences will translate into diversity in directors' preferences).

Prediction 2 *All else equal, deadlock is more likely when directors' remaining tenures are longer (cf. Corollary 1).*

In contrast to much of the literature, which focuses on directors' past tenure, this prediction underscores the costs and benefits of directors' future tenure. Strategic voting and deadlock in our model result from directors' incentive to preserve a strong bargaining position in anticipation of future negotiations. Hence, our model predicts that deadlock is less likely to arise if many directors are likely to leave a board soon, e.g., because they are nearing retirement or they are reaching the legal maximum tenure in jurisdictions where such a maximum exists, such as the UK, France, Spain, Hong Kong, and Singapore (e.g., **?**).

In our model, directors strategically block policies preferred by other directors to improve their future bargaining positions. Hence, given data on individual director voting (as is available for Chinese firms), we have the following testable predictions:

Prediction 3 *All else equal, a director is more likely to vote against a policy if*

(i) there are other directors on the board who especially favor this alternative;

(ii) these other directors have long expected remaining tenure;

(iii) the director himself has longer expected remaining tenure.

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This prediction reflects the real-options intuition at the core of our model: to preserve the option to get their way in the future, directors want to ensure that other directors are dissatisfied with the status quo. This prediction contrasts with other theory models of boards, making it useful to distinguish our model from them.

Practically, it suggests that a director is relatively likely to vote against a CEO candidate nominated by an influential blockholder on the board, since the blockholder is likely to nominate someone she is biased toward. For example, hedge fund activist campaigns are increasingly likely to include the demand to replace the incumbent CEO. Our model suggests that directors on the board are relatively likely to vote against the activist's candidate if the activist has (or will get) board representation. This happened during Paul Hilal's 2017 activist campaign at railroad company CSX.[23] Something similar happened at Uber during its CEO search last summer, when some directors were opposed to hiring Meg Whitman because they viewed her as "potentially compromised by her strong affiliation with Benchmark," a VC blockholder with a seat on the board.[24]

10   Discussion and Conclusion

We argue that deadlock on the board can cause pervasive entrenchment, and hence explain why corporations are often too slow to turn over their top management and to adapt their strategies to a changing competitive environment. Our results hinge on the dynamic interaction between multiple directors' strategic decisions, something new to the literature on corporate boards. Indeed, deadlock in our model is entirely a consequence of dynamic consistency: the board is deadlocked because it fears it

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[23]Hilal demanded that CSX replace the incumbent CEO with veteran railroad executive Hunter Harrison and, in addition, give Hilal and Harrison six seats on the board. Although Harrison was widely considered to be the perfect candidate to lead CSX, directors were for a long time reluctant to agree to the activist's demands: they probably worried that, given support from the new directors, the new CEO would be hard to replace in the future. See, e.g., "The $10 Billion Battle for CSX Stock Will Be Decided Shortly" (Fortune, February 15, 2017).

[24]"Inside Uber's Wild Ride in a Search of a New C.E.O." *New York Times*, August 29, 2017.

will become deadlocked in the future. Our dynamic model gives a new take on board composition, director appointments, and director tenure.

When a board implements a policy, it is akin to exercising a real option. But the group-decision problem gives the standard real-options intuition a twist. When a board needs to agree on a policy to implement, it is easier for any director to implement her preferred policy if other directors' preferred policies are not available. Hence, directors gain flexibility by keeping a low-quality incumbent in place—it makes it easier to exercise the option to replace it.

This mechanism relies on our assumption that today's decision is linked to future decisions via the endogenous status quo. This reflects the institutional processes under which real-world boards operate. Directors sit beside each other on boards for years at a time, and make decisions by voting. They do not write contracts to determine future policies or trade votes for money. In other words, boards operate under conditions in which the Coase theorem does not apply.

Deadlock arises naturally in this dynamic environment. We explore when and how it should be prevented within the current institutional structure of the corporation. Notably, we find that transferring control rights to incumbent directors by giving them power over director appointments can help restore the Coase theorem, rationalizing the real-world institution by which the current board nominates new directors. But other kinds of horse trading could also help directors make partial transfers, and move toward efficient bargaining. One thing they might do is to bundle small policies together, so that some directors get their way on some policies and other directors do on other policies. Such arrangements might not have much bite for the most important policies, like CEO appointments, on which directors are unlikely to compromise for concessions on other policies, and over which real-world boards seem to become deadlocked. However, digging deeper into how they can help resolve deadlock in other cases could help us to understand more about the broader issues boards face, for example how to set the agendas for board meetings and how frequent board meetings should be.

A   Proofs

A.1   Proof of Proposition 1

Given Assumption 1 that x_0 is "very bad," voting for the alternative is a strict best response if the other director votes for. Hence, replacing the incumbent is always an equilibrium.

In all other candidate equilibria, one director must vote against. But keeping the incumbent in place is Pareto dominated. Hence, the tie-breaking rules rule them out (see the solution concept in Section 2).[25]

A.2   Proof of Proposition 2

In the game at date 2, directors vote on whether to keep an exogenous status quo in place or implement a new alternative. Hence, the game at date 2 does not depend on what policy is implemented at date 1. Given this, the game at date 1 decouples from that at date 2. Hence, it can be solved exactly as the one-shot benchmark, implying that x_0 is always replaced.

A.3   Proof of Proposition 3

To prove the proposition, we solve the model backwards. The key observation is that if the "very bad" incumbent policy x_0 is in place at date 2, the date-2 game is equivalent to the one-shot benchmark in Proposition 1, and hence no alternative is blocked at that date. This means that if directors' biases are large, directors have incentive to keep x_0 in place at date 1 to preserve the option to implement their preferred alternatives at date 2. Thus, at date 1, the τ-biased director blocks all τ'-alternatives and, symmetrically, the τ'-biased director blocks all τ-alternatives.

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[25]Without the tie-breaking rules, both directors voting against would be an equilibrium, since if one director votes against, the incumbent always stays in place, making voting against a weak best response to voting against.

To prove this formally, we next characterize the τ-biased director's payoffs and show that given Assumption 2, she blocks all τ'-alternatives at date 1. The argument for the τ'-biased director is identical.

Date 2. To ask whether a τ-biased director will block a τ'-policy at date 1, we consider the subgames following (i) blocking, so x_0 stays in place ($x_1 = x_0$), and (ii) not blocking, so y_1 of type τ' is put in place (x_1 is of type τ').

(i) First, suppose $x_1 = x_0$. Then, we are back to the one-shot benchmark, so any y_2 is implemented. Since $\bar{v}$ is the expected value of y_2 and p_τ is the probability that y_2 is of type τ, the τ-director's payoff is

$$v_0 + \delta\big(\bar{v} + p_\tau b_\tau\big). \tag{13}$$

(ii) Second, suppose that x_1 is of type τ'. By the same argument above and given that $b_{\tau'} > v_h - v_\ell$ by Assumption 2, x_1 can only be replaced if y_2 is of type $h\tau'$, and therefore the τ-director's payoff in this case is

$$\begin{cases} v_\ell + \delta\Big(p_{\tau'}p_h v_h + (1 - p_{\tau'}p_h)v_\ell\Big) & \text{if } x_1 \text{ is type } \ell\tau', \\ v_h + \delta v_h & \text{if } x_1 \text{ is type } h\tau'. \end{cases} \tag{14}$$

Date 1. Now, consider the τ-biased director's decision between blocking alternative y_1 of type τ' and voting for it, assuming that the other director votes for y_1. If the τ-director blocks y_1, her payoff is given by (13). If the τ-director votes for y_1, her payoff is given by one of the expressions in (14), depending on the quality of y_1. Thus, she prefers to block the $h\tau'$-policy if and only if

$$v_0 + \delta\big(\bar{v} + p_\tau b_\tau\big) > v_h + \delta v_h, \tag{15}$$

which is equivalent to

$$b_\tau > \frac{v_h - v_0 + \delta p_\ell\,(v_h - v_\ell)}{\delta p_\tau}, \tag{16}$$

which is implied by Assumption 2. Since her expected payoff from the $\ell\tau'$-policy is even smaller than from the $h\tau'$-policy, she blocks the $\ell\tau'$-policy as well. Thus, she blocks any τ'-policy.

A.3.1 The case of N directors of N types

Here, we extend Proposition 3 to the case where the board consists of N directors, and there are N policies of types $\tau_1, ..., \tau_N$, where each type of policy τ_n arrives with probability p_{τ_n}. Suppose that the decision rule is such that the alternative is implemented if at least T directors vote for it, and consider any $T > 1$ (i.e., a director cannot unilaterally accept a policy). We adapt Assumption 2 to this setting as follows:

Assumption 3 *Biased directors are sufficiently biased: for any $n \in \{1, ..., N\}$,*

$$b_{\tau_n} > \max \left\{ \frac{v_h - v_0 + \delta p_\ell (v_h - v_\ell)}{\delta p_{\tau_n} p_\ell} , \, v_h - v_\ell \right\}. \tag{17}$$

We also have to modify the second tie-breaking rule in our baseline solution concept slightly (cf. Section 2) as follows: at date 2, if a director is indifferent between the alternative y_2 and the incumbent x_1 and neither policies is Pareto dominant, then she votes for the incumbent.

Consider a diverse board, with one director of each bias type. The next result extends Proposition 3 to this setting.

Proposition 10 *Given a diverse (τ_1-τ_2-...-τ_N) board, the incumbent policy x_0 is entrenched: no alternative y_1 is ever appointed at date 1.*

This result shows that complete deadlock arises with multiple directors as well. The intuition is also unchanged. Each director knows that she will be more likely to be able to implement her preferred policy at date 2 if the incumbent x_0 stays in place, so she votes against all alternatives not of her preferred type. Moreover, this argument is not sensitive to the decision rule.

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Proof. The proof follows that of Proposition 3 closely. However, there are a couple of extra subtleties in the N-type setup. The reason is that if a low quality τ_n-policy is in place at date 2, only the τ_n-biased director does not want to replace it with a high-quality policy of another type. Thus, it is replaced unless the decision rule is unanimity $(T = N)$. Hence, we consider this case separately after proving the result for $T < N$.

Date 2. There are three cases.

(i) If x_0 is in place at date 2, it is always replaced with the alternative y_2 as per the one-shot benchmark.

(ii) If a high-quality policy is in place at date 2, it is never replaced.

To see why, suppose that the incumbent policy x_1 is of type τ_1 and the alternative y_2 is of type τ_2, so the τ_1-director always prefers x_1 and the τ_2-director always prefers y_2. The remaining $N - 2$ directors are indifferent, since neither policy is Pareto-dominant the tie-breaking rule implies they vote for the incumbent: x_1 stays in place.

(ii) If a low-quality policy is in place at date 2, it is replaced with high-quality alternatives but with not low-quality alternatives.

The reason a low-quality policy is replaced with a high quality alternative is that $N - 1$ directors strictly prefer the alternative, and hence vote for it.

The reason that a low-quality policy is not replaced by a low-quality alternative is exactly the same as the reason that a high-quality policy is not replaced with a high quality alternative, as explained in (ii) above.

Date 1. There are two cases:

(i) If the date-1 alternative y_1 is high quality, say $h\tau_1$, then any director who is not τ_1-biaed, say the τ_2-biased director, prefers to block it if

$$v_0 + \delta \left(\bar{v} + p_{\tau_2} b_{\tau_2} \right) > v_h + \delta v_h. \tag{18}$$

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This condition is equivalent to (15) and, analogously to the proof of Proposition 3, it is satisfied by Assumption 3.

(ii) If the date-1 alternative y_1 is low quality, say $\ell\tau_1$, then any director who is not τ_1-biased, say the τ_2-biased director, prefers to block it if

$$v_0 + \delta\left(\bar{v} + p_{\tau_2}b_{\tau_2}\right) > v_\ell + \delta\left[p_{\tau_2}p_h b_{\tau_2} + \bar{v}\right] \tag{19}$$

This is satisfied if and only if $b_{\tau_2} > \frac{v_\ell - v_0}{\delta p_{\tau_2}p_\ell}$, which is satisfied by Assumption 3. Thus, each director finds it optimal to vote against any alternative y_1, that is not her own preferred type. Hence, no alternative is ever chosen at date 1.

Unanimity. Under the unanimity rule, the analysis is similar, except ℓ-quality policies are not replaced at date 2, except with h-quality policies of the same type. This changes directors' incentive to block ℓ quality policies at date 1. Hence, equation (19) is replaced by the following:

$$v_0 + \delta\left(\bar{v} + p_{\tau_2}b_{\tau_2}\right) > v_\ell + \delta\left[v_h p_h p_{\tau_1} + v_\ell\left(1 - p_h p_{\tau_1}\right)\right]. \tag{20}$$

Since the right-hand side of (20) is smaller than the right-hand side of (19), (20) follows from (19). ∎

A.4   Proof of Corollary 1

The result follows from two observations.

(i) For $\delta = 0$, directors care only about today's policy, and the date-1 game is equivalent to the ne-shot benchmark in Proposition 1. Hence, directors replace x_0 with any alternative at date 1.

(ii) Starting from Assumption 2 from the text and taking the limit as directors

become patient, we have that

$$b_\tau > \frac{v_h - v_\ell}{p_\tau} + \frac{v_h - v_0}{\delta p_\tau p_\ell} \rightarrow \frac{v_h - v_\ell}{p_\tau} \tag{21}$$

as δ becomes large. This says that Assumption 2 holds for large δ as long as $b_\tau > (v_h - v_\ell)/p_\tau$, which is satisfied by hypothesis. Thus, since there is entrenchment under Assumption 2 by Proposition 3, there is also entrenchment under the assumption that $b_\tau > (v_h - v_\ell)/p_\tau$ for large δ.

The argument above completes the proof, but the effect of δ may merit another comment. It may seem like there should be a threshold δ, above which x_0 is always retained and below which x_0 is never retained. But this is not the whole story. Yes, there is one threshold above which x_0 is always retained (as in the proof) but there is a different threshold below which x_0 is never retained. There is also a region between these thresholds in which x_0 is replaced with a positive probability but not always: whether x_0 is replaced depends on the quality and type of the alternative y_1.

A.5   Proof of Lemma 1

Suppose the date-1 alternative y_1 is $\ell\tau$. Consider the unbiased director on the τ-ν board. If the unbiased director blocks the $\ell\tau$-policy and x_0 remains in place, both directors will vote for any alternative at date 2 by Proposition 1, so her payoff is

$$v_0 + \delta\bar{v}. \tag{22}$$

If she votes for the $\ell\tau$-policy, it becomes the new status quo. In this case, the τ-director will block the $h\tau'$-alternative if it arrives at date 2, since $v_h - v_\ell < b_\tau$ by Assumption 2. Hence, the $\ell\tau$-policy can only be replaced by an $h\tau$-policy at date 2, and so the unbiased director's payoff from voting for it is

$$v_\ell + \delta\Big(p_\tau p_h v_h + (1 - p_\tau p_h) v_\ell\Big). \tag{23}$$

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Comparing these payoffs, the unbiased director blocks the $\ell\tau$-alternative if and only if

$$v_\ell - v_0 < \delta(1 - p_\tau)p_h\left(v_h - v_\ell\right) \tag{24}$$

or $\Delta_\tau > 0$, which is the condition in the proposition.

The unbiased director votes for all other policies: she votes for any high-quality policy because it delivers the highest value, and she does not block the $\ell\tau'$-policy because the other director will always agree to replace it by a high-quality alternative in the future.

A.6   Proof of Lemma 2

Consider the τ-biased director on the τ-ν board. If an $\ell\tau'$-policy comes, she knows that at date 2, the other director will agree to replace it by both an $\ell\tau$-policy (given the tie-breaking rule in the solution concept in Section 2) and any h-policy. Hence, she votes for the $\ell\tau'$-policy because it increases her payoff at date 1 and she can replace it with her more preferred policy at date 2, if one is available. She also votes for any τ-policy. However, if an $h\tau'$-policy arrives, she knows that the unbiased director will not agree to replace it with an $\ell\tau$-policy at date 2 and will only agree to replace it with an $h\tau$-policy (given the tie-breaking rule). Thus, the τ-biased director blocks the $h\tau'$-alternative at date 1 whenever

$$v_0 + \delta\left(\bar{v} + p_\tau b_\tau\right) > v_h + \delta v_h + \delta p_\tau p_h b_\tau, \tag{25}$$

or

$$b_\tau > \frac{v_h - v_0 + \delta p_\ell(v_h - v_\ell)}{\delta p_\tau p_\ell}, \tag{26}$$

which holds by Assumption 2.

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A.7 Proof of Proposition 4

τ-τ **board vs.** τ-ν **board.** On a fully τ-biased board, directors always agree at date 2. Hence, there is no strategic blocking at date 1. Since $v_0 < v_\ell$, directors will always replace the inferior manager at date 1. At date 2, the board keeps any τ-policy in place: given that $v_h - v_\ell < b_\tau$ by Assumption 2, directors will replace an $h\tau'$-policy with an $\ell\tau$-policy, but will not replace an $\ell\tau$ policy with an $h\tau'$-policy.

It follows that shareholders' expected payoff from a τ-τ board is

$$V_{\tau\text{-}\tau} = p_\tau p_h \Big(v_h + \delta v_h\Big) + p_{\tau'} p_h \Big(v_h + \delta p_\tau p_\ell v_\ell + \delta \left(1 - p_\tau p_\ell\right) v_h\Big) \tag{27}$$
$$+ p_\tau p_\ell \Big(v_\ell + \delta p_\tau p_h v_h + \delta(1 - p_\tau p_h) v_\ell\Big) + p_{\tau'} p_\ell \Big(v_\ell + \delta \bar{v}\Big).$$

For example, the second term follows from the fact that if y_1 is of type $\tau' h$, it is implemented at date 1, and is only replaced by a low-quality policy at date 2 if y_2 is of type $\tau\ell$, which occurs with probability $p_\tau p_\ell$. The other terms are calculated analogously.

On a τ-ν board, the analysis follows from Lemma 2 and Lemma 1. Recall that the ν-director's strategy depends on whether $\Delta_\tau \lessgtr 0$. Hence, we consider these two cases separately.

Case 1: $\Delta_\tau < 0$. The ν-director votes for $\tau\ell$-policy at date 1, and the τ-director blocks the $\tau'h$-policy at date 1. Shareholders' expected payoff $V_{\tau\text{-}\nu}^{\Delta_\tau < 0}$ is therefore

$$V_{\tau\text{-}\nu}^{\Delta_\tau < 0} = p_\tau p_h \Big(v_h + \delta v_h\Big) + p_{\tau'} p_h \Big(v_0 + \delta \bar{v}\Big) + \tag{28}$$
$$+ p_\tau p_\ell \Big(v_\ell + \delta p_\tau p_h v_h + \delta \left(1 - p_\tau p_h\right) v_\ell\Big) + p_{\tau'} p_\ell \Big(v_\ell + \delta \bar{v}\Big).$$

Hence

$$V_{\tau\text{-}\tau} - V_{\tau\text{-}\nu}^{\Delta_\tau < 0} = p_{\tau'} p_h \Big(v_h + \delta p_\tau p_\ell v_\ell + \delta \left(1 - p_\tau p_\ell\right) v_h - v_0 - \delta \bar{v}\Big)$$
$$= p_{\tau'} p_h \Big(v_h - v_0 + \delta p_{\tau'} p_\ell (v_h - v_\ell)\Big), \tag{29}$$

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which is positive because $v_0 < v_\ell < v_h$.

Case 2: $\Delta_\tau > 0$. The ν-director blocks the $\tau\ell$-policy, and the τ-director blocks the $\tau'h$-policy. Shareholders' expected payoff $V_{\tau\text{-}\nu}^{\Delta_\tau>0}$ is therefore

$$V_{\tau\text{-}\nu}^{\Delta_\tau>0} = p_\tau p_h \left(v_h + \delta v_h\right) + p_{\tau'} p_h \left(v_0 + \delta\bar{v}\right) + p_\tau p_\ell \left(v_0 + \delta\bar{v}\right) + p_{\tau'} p_\ell \left(v_\ell + \delta\bar{v}\right). \quad (30)$$

Hence,

$$\begin{aligned}
V_{\tau\text{-}\tau} - V_{\tau\text{-}\nu}^{\Delta>0} &= p_{\tau'} p_h \left(v_h + \delta p_\tau p_\ell v_\ell + \delta \left(1 - p_\tau p_\ell\right) v_h - v_0 - \delta\bar{v}\right) + \\
&\quad + p_\tau p_\ell \left(v_\ell + \delta p_\tau p_h v_h + \delta \left(1 - p_\tau p_h\right) v_\ell - v_0 - \delta\bar{v}\right) \\
&= V_{\tau\text{-}\tau} - V_{\tau\text{-}\nu}^{\Delta<0} - p_\tau p_\ell \Delta_\tau = p_{\tau'} p_h \left(v_h - v_0 + \delta p_{\tau'} p_\ell (v_h - v_\ell)\right) - p_\tau p_\ell \Delta_\tau.
\end{aligned} \quad (31)$$

This is positive exactly when condition (3) in the statement of the proposition is satisfied.

τ-ν board vs. τ-τ' board. Here, we show that shareholders always prefer a τ-ν board to a τ-τ' board. By Proposition 3, shareholders' expected payoff from a τ-τ' board is

$$V_{\tau\text{-}\tau'} = v_0 + \delta\bar{v}. \quad (32)$$

Shareholders' expected payoff from a τ-ν board is given by (28) if $\Delta_\tau \leq 0$ and by (30) if $\Delta_\tau \geq 0$. Again, consider the two cases for $\Delta_\tau \lesseqgtr 0$.

Case 1: $\Delta_\tau < 0$. Using (32), (28), and simplifying, $V_{\tau\text{-}\tau'} > V_{\tau\text{-}\nu}^{\Delta_\tau<0}$ is equivalent to

$$p_\tau p_h (v_h - v_0) + p_\ell (v_\ell - v_0) + \delta p_\tau^2 p_\ell p_h (v_h - v_\ell) > 0. \quad (33)$$

This is always satisfied since $v_h > v_\ell > v_0$.

Case 2: $\Delta_\tau > 0$. Using (32), (30), and simplifying, $V_{\tau\text{-}\tau'} > V_{\tau\text{-}\nu}^{\Delta_\tau>0}$ is equivalent to

$$p_\tau p_h \left(v_h - v_0 + \delta(v_h - \bar{v})\right) + p_{\tau'} p_\ell (v_\ell - v_0) > 0. \quad (34)$$

This is always satisfied since $v_h > v_\ell > v_0$.

τ-τ board vs. τ-τ' board. Here, we show that a τ-biased board is always

preferred to a diverse board. Using (27) and (32), and simplifying, $V_{\tau\text{-}\tau} > V_{\tau\text{-}\tau'}$ is equivalent to

$$\bar{v} - v_0 + \delta \left(p_\tau^2 + p_{\tau'}^2 \right) p_\ell p_h \left(v_h - v_\ell \right) > 0, \tag{35}$$

which is always satisfied since $v_h > v_\ell > v_0$.

A.7.1 Non-stationary Qualities

Here, we relax the assumption that the probability of a high-quality policy at date 2 is the same as at date 1, i.e., that the qualities of alternatives are identically distributed across periods. In this setup, a diverse board can be preferred to a biased board: a diverse board has the benefit of preventing some low-quality policies from being implemented and becoming entrenched (as a partially biased board does (Lemma 1)). In the baseline model, this benefit of a diverse board is present as well, but it is always outweighed by another cost of a diverse board: when one of the directors blocks a high-quality policy of the other's preferred type at date 1, the set of policies the board can choose from at date 2 becomes worse. In particular, if a high-quality policy is not blocked at date 1 (as happens with a fully biased board), the set of policies at date 2 is this high-quality policy and the date-2 alternative; in contrast, with strategic blocking, the set at date 2 is the very bad incumbent x_0 and the date-2 alternative.

To show that the positive effect of a diverse board can dominate the negative effect when qualities are not identically distributed across periods, consider the following setup. As above, p_h denotes the probability that the alternative is of type h at date 1, but now let $\hat{p}_h$ denote the probability that the alternative is of type h at date 2, which can generally be different from p_h. Analogously, as above, $\bar{v} = p_h v_h + p_\ell v_\ell$ denotes the average value of date-1 alternatives, and let $\hat{v} := \hat{p}_h v_h + \hat{p}_\ell v_\ell$ denote the average value of the date-2 alternative (where $\hat{p}_\ell := 1 - \hat{p}_h$).

We now compare the value $V_{\tau\text{-}\tau}$ of a τ-τ board with the value $V_{\tau\text{-}\tau'}$ of a τ-τ' board: $V_{\tau,\tau} \geq V_{\tau,\tau'}$ if and only if

$$p_\tau p_h \left(v_h + \delta v_h \right) + p_{\tau'} p_h \left(v_h + \delta p_\tau \hat{p}_\ell v_\ell + \delta \left(1 - p_\tau \hat{p}_\ell \right) v_h \right) +$$
$$+ p_\tau p_\ell \left(v_\ell + \delta p_\tau \hat{p}_h v_h + \delta \left(1 - p_\tau \hat{p}_h \right) v_\ell \right) + p_{\tau'} p_\ell \left(v_\ell + \delta \hat{v} \right) - \left(v_0 + \delta \hat{v} \right), \geq 0 \qquad (36)$$

or, after some manipulation, if

$$\frac{\bar{v} - v_0}{\delta} + p_\tau \left[p_\tau p_\ell \hat{p}_h \left(v_h - v_\ell \right) + \bar{v} - \hat{v} \right] + p_{\tau'}^2 p_h \hat{p}_\ell (v_h - v_\ell) < 0. \qquad (37)$$

To see that this may be satisfied, set $v_\ell = 0$ and $\hat{p}_h = 1$, so that $\hat{p}_\ell = 0$, $\hat{v} = v_h$, and $\bar{v} = p_h v_h$. The condition becomes

$$\frac{\bar{v} - v_0}{\delta} + p_\tau \left[p_\tau p_\ell v_h + p_h v_h - v_h \right] < 0, \qquad (38)$$

or

$$\frac{\bar{v} - v_0}{\delta} < p_\tau p_\ell p_{\tau'} v_h, \qquad (39)$$

which is satisfied if $p_\ell > 0$ and δ is large enough. Intuitively, in this case, the negative effect of a diverse board on the date-2 value does not exist (because even if a high-quality policy is blocked at date 2, the date-2 alternative is always high-quality as well), while the positive effect on the date-2 value exists.

A.8   Proof of Corollary 2

The result follows immediately from Proposition 4.

A.9   Proof of Proposition 5

A τ-director blocks an $h\tau'$-policy if

$$v_0 + \delta \left(\bar{v} + p_\tau b_\tau \right) > v_h + \delta \left[q \left(p_\tau \bar{v} + (1 - p_\tau) v_h + p_\tau b_\tau \right) + (1 - q) v_h \right], \qquad (40)$$

or, rearranging,

$$b_\tau > \frac{v_h - v_0 + \delta p_\ell(1 - qp_\tau)(v_h - v_\ell)}{\delta(1 - q)p_\tau}, \tag{41}$$

which is the condition in the proposition.

To see that staggered elections can mitigate deadlock on a diverse board, observe that the threshold b_τ above which deadlock arises is higher by comparing this condition with the condition for deadlock on a diverse board in equation (16):

$$\frac{v_h - v_0 + \delta p_\ell(1 - qp_\tau)(v_h - v_\ell)}{\delta(1 - q)p_\tau} > \frac{v_h - v_0 + \delta p_\ell(v_h - v_\ell)}{\delta p_\tau}. \tag{42}$$

On the other hand, to see that staggered elections can exacerbate deadlock on a fully biased board, simply recall that deadlock does not arise on a fully biased board.

A.10   Proof of Proposition 6

We prove the proposition in three steps. First, we solve for the optimal director appointment decision by whoever gets the power to do it. Second, we solve for the voting equilibrium given this appointment decision and show that the continuing τ-biased director does not block any policies at date 1 if her power to appoint is above a certain cutoff, $\pi \geq \bar{\pi}$, but blocks $h\tau'$-policies at date 1 if she has little power to appoint, $\pi < \bar{\pi}$. Third, we derive and compare shareholders' payoffs for $\pi < \bar{\pi}$ and $\pi \geq \bar{\pi}$, and solve for the optimal power to appoint π^* given this comparison.

Appointments. Consider the appointment decision after one of the directors steps down. Since the new board will be making a one-shot decision, then, similarly to Proposition 1, at date 2, all directors will vote sincerely for their preferred policy. Hence, whoever makes the appointment decision chooses the director who represents their interest: shareholders appoint an unbiased director, while the incumbent director (who, we assumed, is τ-biased) appoints a director aligned with her, i.e., a τ-biased director.

Voting strategies.

Date 2. At date 2, the board is fully biased (τ-τ) with probability π and partially biased (τ-ν) with probability $1 - \pi$.

First, consider a fully biased board. There are five possibilities for the incumbent policy x_1 (depending on the date-1 outcome). If a $\tau\ell$-policy is in place, it is replaced only with a τh-policy and kept in place otherwise. If a τh-policy is in place, it is never replaced. If a $\tau'\ell$-policy is in place, it is replaced by every alternative except $\tau'\ell$. If a $\tau'h$-policy is in place, it is replaced by a $\tau\ell$- and τh-policy and is not replaced otherwise. If x_0 is in place, it is always replaced.

Second, consider the partially biased board and the five possibilities for the incumbent policy x_1. If a $\tau\ell$-policy is in place, it is replaced only with a τh-policy because the τ-biased director blocks all other alternatives. If a τh-policy is in place, it is never replaced. If a $\tau'\ell$-policy is in place, it is replaced by every alternative except $\tau'\ell$ by the tie-breaking rule. If a $\tau'h$-policy is in place, it is replaced only by a τh-policy because the unbiased director votes against low-quality alternatives. If x_0 is in place, it is always replaced.

Comparing the voting strategies of the two boards, we see that they make the same decisions unless a $\tau'h$-policy is in place.

Date 1. Since one director retires at the end of date 1, she only maximizes her date-1 payoff. Hence, she does not vote strategically, but rather votes for any alternative regardless of her type, as in the one-shot benchmark of Proposition 1.

Consider the continuing τ-director's voting decision given the date-2 voting outcomes considered above. There are four alternatives for y_1. If y_1 is of type τ ($\tau\ell$ or τh), she votes for it given her bias. If y_1 is of type $\tau'\ell$, she votes for it as well, since with this policy in place, she will still be able to implement any τ-policy at date 2, regardless of whether the other director will be τ-biased or unbiased. However, if y_1 is of type $\tau'h$, voting for/against comes with a tradeoff. If the director votes for, and the policy becomes the incumbent, her payoff is

$$v_h + \delta(1 - \pi)\big(v_h + b_\tau p_\tau p_h\big) + \delta\pi\Big(p_\tau p_\ell v_\ell + (1 - p_\tau p_\ell)\, v_h + b_\tau p_\tau\Big). \qquad (43)$$

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If she votes against, her payoff is

$$v_0 + \delta\big(\bar{v} + b_\tau p_\tau\big). \tag{44}$$

Comparing the two payoffs, the continuing τ-director votes for the $\tau'h$-alternative if and only if

$$\delta p_\tau p_\ell \left[b_\tau - (v_h - v_\ell) \right] \pi \geq \delta p_\ell \left[p_\tau b_\tau - (v_h - v_\ell) \right] - (v_h - v_0). \tag{45}$$

Because, by Assumption 2, $b_\tau - (v_h - v_\ell) > 0$, this is equivalent to $\pi > \bar{\pi}$, given by the expression in the proposition. In addition, Assumption 2 implies that $\bar{\pi} \in (0, 1)$.

Shareholders' optimal power. Now we calculate shareholders' expected payoffs in case $\pi \geq \bar{\pi}$ and $\pi < \bar{\pi}$, denoted by $V(\pi | \pi \geq \bar{\pi})$ and $V(\pi | \pi < \bar{\pi})$, respectively. Recall from the derivation above that the date-2 voting strategies of the fully biased and partially biased board that the two boards make different decisions only if a $\tau'h$-policy is in place at date 1 and a $\tau\ell$-alternative arrives at date 2: the partially biased board retains the high-quality policy, while the fully biased τ-τ board replaces it with a low-quality τ-alternative. Hence, if $\pi < \bar{\pi}$, so the τ-biased director blocks the $\tau'h$-policy at date 1, shareholder value $V(\pi | \pi < \bar{\pi})$ does not depend on π at all. On the other hand, if $\pi \geq \bar{\pi}$, so the τ-biased director votes for the $\tau'h$-policy at date 1, shareholder value $V(\pi | \pi \geq \bar{\pi})$ is maximized if π is the minimized, i.e. $\pi = \bar{\pi}$, because this minimizes the probability of a low-quality policy being chosen at date 2.

Now we have to compare shareholders' payoffs for $\pi = \bar{\pi}$ and $\pi < \bar{\pi}$. These payoffs are only different if the $\tau'h$-policy arrives at date 1. In this case, when $\pi = \bar{\pi}$, the $\tau'h$-policy is voted for at date 1 and is replaced by a low-quality policy at date 2 if and only if a $\tau\ell$-alternative arrives and the board is fully biased (i.e., with probability $p_\ell p_\tau \bar{\pi}$). On the other hand, if $\pi < \bar{\pi}$, x_0 is retained at date 1 and is replaced by any

alternative that arrives at date 2. Therefore,

$$V\left(\pi|\pi=\bar{\pi}\right)-V\left(\pi|\pi<\bar{\pi}\right)=p_{\tau'}p_h\left[v_h+\delta\left(v_\ell p_\ell p_\tau\bar{\pi}+v_h\left(1-\bar{\pi}p_\ell p_\tau\right)\right)\right]-p_{\tau'}p_h\left[v_0+\delta\bar{v}\right]$$

Hence, the optimal $\pi^*=\bar{\pi}$ if and only if

$$v_0+\delta\bar{v}<v_h+\delta\left(v_\ell\bar{\pi}p_\ell p_\tau+v_h\left(1-\bar{\pi}p_\ell p_\tau\right)\right)$$

and $\pi^*\in[0,\bar{\pi})$ otherwise.

It is also worth pointing out that if $v_0+\delta\bar{v}<v_h+\delta\left(v_\ell p_\ell p_\tau+v_h\left(1-p_\ell p_\tau\right)\right)$, then $\pi=1$ is better for shareholders than $\pi=0$: full control of the existing board over director appointments can be better for shareholders than full shareholder control.

A.11 Proof of Corollary 3

First observe that, since $v_0<v_\ell$ (the incumbent CEO is "very bad"), he is always fired at date 2. Hence, he just wants to minimize the probability that he is fired at date 1, which varies with board composition as follows.

(i) With a τ-τ or ν-ν board, he is always fired at date 1, since there is no strategic blocking at date 1.

(ii) With a τ-τ' board, he is never fired at date 1—he is entrenched by Proposition 3.

(iii) With a τ-ν board (for any τ), he is retained when y_1 is of type $h\tau'$ (by Lemma 2) and, for some parameters, when y_1 is of type $\ell\tau$ (by Lemma 1) and is fired otherwise.

This yields the ranking stated in the corollary.

A.12 Proof of Proposition 7

CEO's payoff given board composition. Consider each of the six possible boards.

(i) τ-τ *board.* Since both directors prefer the $\ell\tau$-CEO to the $h\tau'$-CEO by Assumption 2, the CEO is fired the first time there is an $h\tau$-alternative. Hence,

$$U_{\tau\text{-}\tau} = (1 - p_\tau p_h)w_1 + (1 - p_\tau p_h)^2 w_2. \tag{46}$$

(ii) τ-ν *board.* This board's decision rule coincides with that of the τ-τ board: the τ-biased director blocks all τ'-alternatives, and if an $h\tau$-alternative arrives, it brings the highest possible utility to both directors, so they both vote for it. Hence, $U_{\tau\text{-}\nu} = U_{\tau\text{-}\tau}$ given by (46).

(iii) τ-τ' *board.* This board's decision rule also coincides with that of the τ-τ board, since there is no strategic blocking at date 1. In particular, if an $h\tau$-policy arrives at date 1, the τ'-biased director does not gain anything from blocking it: whether she blocks it or not, the τ-biased director will block *any* τ'-alternative at date 2 (since she prefers the $\ell\tau$-CEO to an $h\tau'$-CEO by Assumption 2). As a result, the τ'-director knows she can never hire a τ'-CEO, and hence wants to hire a high-quality τ-CEO as soon as possible. Thus, the CEO is fired the first time there is a $h\tau$-alternative, as with the τ-τ and τ-ν boards. Hence, $U_{\tau\text{-}\tau'} = U_{\tau\text{-}\tau}$ given by (46).

(iv) ν-ν *board.* The CEO is fired the first time there is a high-quality alternative. Hence,

$$U_{\nu\text{-}\nu} = (1 - p_h)w_1 + (1 - p_h)^2 w_2 = p_\ell w_1 + p_\ell^2 w_2. \tag{47}$$

(v) τ'-τ' *board.* The CEO is fired the first time there is a τ'- or a high-quality alternative. Hence,

$$U_{\tau'\text{-}\tau'} = p_\tau p_\ell w_1 + (p_\tau p_\ell)^2 w_2. \tag{48}$$

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(vi) τ'-ν *board.* With this board, it is automatic that the CEO is replaced by an $h\tau'$ alternative, but not by an $\ell\tau$-alternative. What happens with the other two types, $h\tau$ and $\ell\tau'$ is not automatic, because they induce strategic blocking. It turns out that both are blocked, leaving the CEO in place, as we now explain.

- Suppose y_1 is of type $h\tau$. The τ'-biased director blocks it by an argument analogous to that of Lemma 2: specifically the τ'-biased director finds it optimal to block $h\tau$-alternatives as long as

$$v_h + \delta\left(v_h + p_{\tau'}p_h b_{\tau'}\right) < v_\ell + \delta\left(\bar{v} + p_{\tau'}b_{\tau'}\right), \tag{49}$$

or

$$b_{\tau'} > \frac{(v_h - v_\ell)(1 + \delta p_\ell)}{\delta p_{\tau'}}, \tag{50}$$

which is implied by Assumption 1 since $v_0 < v_\ell$.

- Suppose y_1 is of type $\ell\tau'$. The unbiased director blocks it by argument analogous to that of Lemma 1: if $\ell\tau'$ is appointed today, the τ'-biased director will block an $h\tau$-alternative in the future, whereas if $\ell\tau$ stays in place, the τ'-biased director will vote for any high-quality alternative at date 2.

In summary, given a τ'-ν board, the CEO is replaced at date 1 only if an $h\tau'$-alternative arrives. If he is not replaced at date 1, he only stays in place at date 2 if an $\ell\tau$-alternative arrives. Hence,

$$U_{\tau'\text{-}\nu} = (1 - p_{\tau'}p_h)w_1 + (1 - p_{\tau'}p_h)p_\tau p_\ell w_2. \tag{51}$$

CEO's ranking. From the computations above, we can observe immediately that

$$U_{\tau\text{-}\tau} = U_{\tau\text{-}\nu} = U_{\tau\text{-}\tau'} > U_{\nu\text{-}\nu} > U_{\tau'\text{-}\tau'}. \tag{52}$$

It remains to compare $U_{\tau'\text{-}\nu}$ with the above.

- $U_{\tau'-\nu} > U_{\tau-\tau}$: Using (51) and (46) and simplifying, $U_{\tau'-\nu} > U_{\tau-\tau}$ is equivalent to

$$(p_\tau - p_{\tau'})p_h w_1 > \left(p_{\tau'} - p_\tau p_h(p_{\tau'}p_h + p_\tau p_\ell)\right)w_2. \tag{53}$$

When p_τ goes to one, the left-hand side of (53) is positive, while the right-hand side is negative, and hence (53) is satisfied for p_τ sufficiently large.

- $U_{\tau'-\nu} > U_{\nu-\nu}$ if

$$(p_\ell p_\tau + p_h p_{\tau'})w_1 + (p_\ell p_\tau + p_h p_{\tau'})(1 - p_\ell p_\tau)w_2 > p_\ell w_1 + p_\ell^2 w_2. \tag{54}$$

- $U_{\tau'-\nu} > U_{\tau-\tau'}$ if

$$(p_\ell p_\tau + p_h p_{\tau'})w_1 + (p_\ell p_\tau + p_h p_{\tau'})(1 - p_\ell p_\tau)w_2 > (1 - p_\tau p_h)w_1 + (1 - p_\tau p_h)^2 w_2. \tag{55}$$

To summarize, τ-$\tau \sim \tau$-$\nu \sim \tau$-$\tau' \succ \nu$-$\nu \succ \tau'$-τ', and the ranking of τ'-ν depends on the inequalities (53), (54), and (55) above, as stated in the proposition.

A.13 Proof of Proposition 8

At date 2, all directors vote sincerely (it is a weakly dominant strategy). Thus, if the high-quality alternative is in place, it is retained unless the date-2 alternative is type-τ and the majority of directors are τ-biased. Hence, given a high-quality τ'-incumbent, a τ-biased director's expected date-2 payoff is

$$\tau\text{-biased director's date-2 payoff}\Big|_{x_1 \text{is } h\tau'} = q_{\tau'} v_h + q_\tau \left(p_\tau(\bar{v} + b_\tau) + p_{\tau'} v_h\right). \tag{56}$$

Whereas her payoff, given the incumbent is x_0, is as in the two-director model, since the alternative is always implemented at date 2:

$$\tau\text{-biased director's date-2 payoff}\Big|_{x_1 = x_0} = \bar{v} + p_\tau b_\tau. \tag{57}$$

Adding the date-1 payoffs to the expressions above, we get the following condition for when a τ-biased director prefers to retain the incumbent x_0 than to implement a high-quality τ'-alternative:

$$v_0 + \delta\Big(\bar{v} + p_\tau b_\tau\Big) > v_h + \delta\Big(q_{\tau'} v_h + q_\tau\Big(p_\tau\big(\bar{v} + b_\tau\big) + p_{\tau'} v_h\Big)\Big), \tag{58}$$

which is satisfied given the condition in the proposition.

A.14   Proof of Proposition 9

I generalized the proof to $v_0 \neq 0$ and $p_\tau \neq 0.5$. Could you recheck those formulas?

To prove that the equilibrium described in the proposition exists, we compute directors' value functions at each date (as a function of the incumbent policy at the beginning of that date) in this equilibrium. Then, we use these value functions to show that the strategies described in the proposition indeed form an equilibrium.

Directors' strategies. Note that in equilibrium described in the proposition, the status quo policy at each period is either x_0 or a high-quality policy. However, we also need to define voting outcomes off-equilibrium, when a low-quality policy is in place. Consider the following strategies for any $\tau \in \{\alpha, \beta\}$ and $\tau' \neq \tau$:

(i) If x_0 is in place, the τ-biased director votes for τ- and $h\tau'$-alternatives, but against $\ell\tau'$-alternatives.

(ii) If an $h\tau$-policy is in place, the τ-biased director votes against all alternatives, and the τ'-biased director votes against all τ-alternatives.

(iii) If an $\ell\tau$-policy were in place (off equilibrium), the τ-biased director would only vote for an $h\tau$-alternative and against all other alternatives, and the τ'-director would vote for any alternative except $\ell\tau$.

Continuation values. Define u_τ^x as the τ-director's continuation value at any date t given that x is chosen at date t (but before the date-t flow payoffs from

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x are realized). We state these value functions in the following auxiliary lemma. For clarity, we compute them for general parameters even though we formulate the proposition only for $v_0 = 0$ and $p_\tau = p_{\tau'} = 1/2$.

Lemma 3 (VALUE FUNCTIONS.) *The value functions are as follows:*

$$
\begin{aligned}
u_\tau^{h\tau} &= \tfrac{v_h + b_\tau}{1-\delta}, \\
u_\tau^{h\tau'} &= \tfrac{v_h}{1-\delta}, \\
u_\tau^{\ell\tau} &= \tfrac{1}{1-\delta(1-p_\tau p_h)} \left(v_\ell + b_\tau + \delta p_\tau p_h \tfrac{v_h + b_\tau}{1-\delta} \right), \\
u_\tau^{\ell\tau'} &= \tfrac{1}{1-\delta(1-p_{\tau'} p_h)} \left(v_\ell + \delta p_{\tau'} p_h \tfrac{v_h}{1-\delta} \right), \\
u_\tau^{x_0} &= \tfrac{1}{1-\delta p_\ell} \left(v_0 + \delta p_\tau p_h \tfrac{v_h + b_\tau}{1-\delta} + \delta p_{\tau'} p_h \tfrac{v_h}{1-\delta} \right).
\end{aligned}
\tag{59}
$$

Proof. These expressions follow by direct computation given the supposed strategies above. Indeed:

- $u_\tau^{h\tau}$ and $u_\tau^{h\tau'}$. If an h-policy is in place, it stays in place forever since at least one director votes against it. Hence, we can write $u_\tau^{h\tau}$ and $u_\tau^{h\tau'}$ recursively as

$$
u_\tau^{h\tau} = v_h + b_\tau + \delta u_\tau^{h\tau}
\tag{60}
$$

and

$$
u_\tau^{h\tau'} = v_h + \delta u_\tau^{h\tau'}.
\tag{61}
$$

Solving for $u_\tau^{h\tau}$ and $u_\tau^{h\tau'}$ gives the expressions in the lemma.

- $u_\tau^{\ell\tau}$ and $u_\tau^{\ell\tau'}$. If an ℓ-policy is in place (off equilibrium), it stays in place until it is replaced with an h-policy of the same bias-type. Hence, we can write the value functions $u_\tau^{\ell\tau}$ and $u_\tau^{\ell\tau'}$ recursively as

$$
u_\tau^{\ell\tau} = v_\ell + b_\tau + \delta \left(p_\tau p_h u_\tau^{h\tau} + (1 - p_\tau p_h) u_\tau^{\ell\tau} \right)
\tag{62}
$$

and

$$
u_\tau^{\ell\tau'} = v_\ell + \delta \left(p_{\tau'} p_h u_\tau^{h\tau'} + (1 - p_{\tau'} p_h) u_\tau^{\ell\tau'} \right).
\tag{63}
$$

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Substituting for $u_\tau^{h\tau}$ and $u_\tau^{h\tau'}$ from above and solving for $u_\tau^{\ell\tau}$ and $u_\tau^{\ell\tau'}$ gives the expressions in the lemma.

- $u_\tau^{x_0}$. If x_0 is in place, it stays in place until it is replaced with an h-policy of either type. Hence, we can write the value function $u_\tau^{x_0}$ recursively as

$$u_\tau^{x_0} = v_0 + \delta\left(p_\ell u_\tau^{x_0} + p_\tau p_h u_\tau^{h\tau} + p_{\tau'} p_h u_\tau^{h\tau'}\right). \tag{64}$$

Substituting for $u_\tau^{h\tau}$ and $u_\tau^{h\tau'}$ from above and solving for $u_\tau^{x_0}$ gives the expression in the lemma.

∎

We next derive conditions under which the above strategies form an equilibrium.

Equilibrium. For the strategies above to form a subgame perfect equilibrium, we need the following inequalities to be satisfied for each $\tau \in \{\alpha, \beta\}$ and $\tau' \neq \tau$, respectively:

(i) $u_\tau^y \geq u_\tau^{x_0} \geq u_\tau^{\ell\tau'}$ for any $y \in \{h\tau', \ell\tau, h\tau\}$;

(ii) $u_\tau^{h\tau} \geq u_\tau^y$ for any y, and $u_\tau^{h\tau'} \geq u_\tau^{q\tau'}$ for any $q \in \{h, \ell\}$;

(iii) $u_\tau^{h\tau} \geq u_\tau^{\ell\tau} \geq u_\tau^{q\tau'}$ for any $q \in \{h, \ell\}$, and $u_\tau^{\ell\tau'} \leq u_\tau^y$ for any y.

These sets of inequalities are satisfied if and only if

$$u_\tau^{h\tau} \geq u_\tau^{\ell\tau} \geq u_\tau^{h\tau'} \geq u_\tau^{x_0} \geq u_\tau^{\ell\tau'}, \tag{65}$$

for any $\tau \in \{\alpha, \beta\}$ and $\tau' \neq \tau$. In particular, the last inequality reflects incentives for strategic blocking, which leads to deadlock.[26]

Now we show that (65) are satisfied given the above expressions for the value functions and the conditions in the statement of the proposition.

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[26]This preference ordering implies that the equilibrium is not the result of directors' indifference. Directors are not driven by the fact that if one director votes against, the other director is never pivotal. However, this ranking is only a sufficient condition, and other equilibria could exist that do not satisfy it.

- $u_\tau^{h\tau} \geq u_\tau^{\ell\tau}$ immediately follows from the comparison of the two expressions.

- $u_\tau^{\ell\tau} \geq u_\tau^{h\tau'}$ reduces to

$$b_\tau \geq \frac{(v_h - v_\ell)(1 - \delta)}{1 - \delta + \delta p_\tau p_h}.$$

- $u_\tau^{h\tau'} \geq u_\tau^{x_0}$ reduces to

$$b_\tau \leq \frac{(v_h - v_0)(1 - \delta)}{\delta p_\tau p_h}.$$

- $u_\tau^{x_0} \geq u_\tau^{\ell\tau'}$ reduces to

$$b_\tau \geq (1 - \delta) \frac{(1 - \delta p_\ell)(v_\ell - v_0) - \delta p_\tau p_h (v_h - v_0)}{\delta p_\tau p_h (1 - \delta + \delta p_{\tau'} p_h)}.$$

Together, the inequalities above yield the condition in the proposition.

Finally, note that the interval in the proposition is non-empty, since for any $v_h > v_\ell > v_0$ we have that

$$\frac{v_h - v_\ell}{1 - \delta + \delta p_\tau p_h} < \frac{v_h - v_\ell}{\delta p_\tau p_h} < \frac{v_h - v_0}{\delta p_\tau p_h}, \tag{66}$$

and

$$\frac{(1 - \delta p_\ell)(v_\ell - v_0) - \delta p_\tau p_h (v_h - v_0)}{\delta p_\tau p_h (1 - \delta + \delta p_{\tau'} p_h)} < \frac{(1 - \delta p_\ell - \delta p_\tau p_h)(v_h - v_0)}{\delta p_\tau p_h (1 - \delta + \delta p_{\tau'} p_h)} = \frac{v_h - v_0}{\delta p_\tau p_h}. \tag{67}$$