WAREHOUSE BANKING*

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Abstract

We develop a new theory of banking that provides a view of liquidity creation that differs significantly from the existing paradigm. The theory links modern-day banks to their historical origin as warehouses. When the warehouse accepts deposits, it issues "warehouse receipts," claims on deposits it safeguards, as "proof" of deposits, but this suffices neither to create liquidity nor to make the warehouse a bank. Our main result is that the warehouse becomes a bank and creates ex ante aggregate liquidity only when it *makes loans* in "fake" receipts—i.e. receipts that are observationally identical to authentic receipts, but are not backed by deposits. In other words, what creates liquidity and economic growth is not banks' deposit-taking per se—banks issuing receipts in exchange for the deposit of goods—but rather the combination of this with account-keeping (i.e. warehousing) and fake-receipt-based lending. On the policy front our theory suggests that the currently-contemplated suggestions to have narrow banking or impose liquidity requirements will inhibit aggregate liquidity creation, whereas increasing bank capital will enhance bank liquidity creation. This stands in contrast with the policy prescriptions of models of bank liquidity creation based on deposit-taking and lending but without warehousing and fake-receipt issuance. We also show that a tighter central bank monetary policy does not always reduce liquidity creation—we provide conditions under which it leads to higher liquidity creation by banks.

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The banks in their lending business are not only not limited by their own capital; they are not, at least immediately, limited by any capital whatever; by concentrating in their hands almost all payments, they themselves create the money required...

Wicksell (1907)

1 Introduction

Banking is an old business. The invention of banking precedes the invention of coinage by several thousand years.[1] Banks evolved from ancient warehouses, where cattle, grain, and precious metals were deposited for storage. For example, in ancient Egypt, grain harvests were "deposited" (or stored) in centralized warehouses and depositors could write orders for the withdrawal of certain quantities of grain as means of payment. These orders constituted some of the earliest paper money. Eventually the warehouses for the safe storage of commodities began making loans, thereby evolving into modern banks (see Williams (1986) and Lawson (1855), for example). By extending credit, these institutions transformed from simple warehouses of liquidity to *creators* of liquidity. To this day, the same institutions that provide safekeeping services also engage in the bulk of lending in the economy and are responsible for significant liquidity creation (e.g., Berger and Bouwman (2009)). Modern commercial and retail banks keep deposit accounts, provide payment services and act as custodians as well as make corporate and consumer loans.

Why do modern banks offer deposit-taking, account-keeping, payment, and custodial—namely, warehousing—services within the same institution that provides lending services? *How* do banks that combine warehousing and lending services create liquidity? What does a theory of banking that addresses these questions have to say about regulatory initiatives like bank capital and liquidity requirements and proposals like narrow banking?

In this paper, we address these questions by developing a model of banking based on the warehousing function of the bank. In our model, the institutions that provide the warehousing services endogenously perform the lending in the economy. The model relies on two key assumptions. First, warehouses have an efficient storage technology and, second, no firm's output is pledgeable,[2] so debt written on a firm's future cash flow is not readily enforceable. Warehouses use their superior storage technology to

[1]Banking seems to have originated in ancient Mesopotamia and some of the earliest recorded laws pertaining banks (banking regulation) were part of the Code of Hammurabi.

[2]See Holmström and Tirole (2011) for a list of "...several reasons why this [non-pledgeability] is by and large reality" (p. 3).

circumvent the non-pledgeability problem. The warehouse's ability to do this stems from the fact that it is costly for firms to store their output privately, so they want to deposit their output in a warehouse that can store it at a lower cost.[3] However, once a firm deposits with the warehouse, the deposit can then be seized by the warehouse. Thus, when the costs to private storage are high enough, it becomes incentive compatible for firms to repay their debts in order to access warehouses' storage services. This mechanism explains why the same institutions should provide both the warehousing and lending services in the economy.[4]

Our model of warehouse banking leads to a new perspective on banks' liquidity creation. In our model, as in ancient Egypt, the receipts that warehouses issue for deposits circulate as a medium of exchange—they constitute private money. This is the first step in the bank's creation of funding liquidity, which we define as the bank-linked elevation of the initial liquidity that is allocated to productive investments.[5] The second step in the liquidity creation process—and the key reason that banks increase aggregate funding liquidity—is that they make loans in warehouse receipts rather than in deposited goods. When banks make loans, they issue new receipts—"fake receipts"—that are not backed by any deposits. Despite the lack of deposit-backing, these receipts provide firms with working capital, allowing them to make more productive investments. Note that banks create ex ante liquidity only when they make loans in fake receipts. If the warehouse only takes deposits and issues receipts for them, it creates no liquidity because it is simply exchanging one liquid asset for another—exchanging a liquid receipt for a liquid deposit of real goods or central bank money.

Our warehousing view of banking provides some new insights into financial regulatory policy. One proposal is narrow banking. We interpret a narrow bank as an institution that can invest its deposits in only "safe" assets, namely in cash or marketable liquid securities such as sovereign bonds (see Pennacchi (2012)). The proposal forces the separation of the warehousing and lending functions of banks. Our analysis suggests that banks create liquidity only when they perform this dual function, implying that narrow banks create no liquidity. Less extreme proposals, such as the liquidity (reserve) ratio in Basel III, demand that banks invest at least a specified fraction of their assets in cash and marketable liquid securities. These too stifle banks' liquidity

[3]The assumption is somewhat similar to the assumption in Allen and Gale (1998) that the storage technology available to banks is strictly more productive than the storage technology available to consumers. As we discuss later, this cost advantage for the warehouse may also stem from being in a privileged position of power, like the ancient warehouses from which banks evolved; these were linked to places of worship and/or the king's treasury.

[4]In support of this mechanism, Skrastins (2015) uses a differences-in-differences research design to demonstrate that agricultural lenders in Brazil extend more credit when they merge with grain silos, i.e. banks lend more when they are also warehouses.

[5]We use the term "funding liquidity" in the sense in which it is used by Brunnermeier and Pedersen (2009).

creation in our set-up, since they prevent banks from issuing fake receipts to expand the supply of liquidity. In an extension, we contrast the effect of liquidity requirements with the effect of capital (equity) requirements, which can actually enhance banks' ability to create liquidity by reducing moral hazard problems between banks and depositors, making warehousing relatively more efficient. We also extend the model to include a central bank and argue that a tighter monetary policy does not always lead to lower liquidity creation. We establish conditions under which such a policy can actually *encourage* lending by warehouse-banks.

Our model stands in contrast to the contemporary literature on why banks exist. The only assumption we make on banks is that they have a storage technology. They have *no* superior ability to screen or to monitor loans in an environment of asymmetric information, as in Diamond (1984) and Ramakrishnan and Thakor (1984). Because we assume that all agents are risk neutral, banks also do not provide better risk sharing for risk averse depositors as in Bryant (1980) and Diamond and Dybvig (1983). Technological and financial developments have diminished informational frictions and provided alternatives to banks for risk-sharing (see the discussion in Coval and Thakor (2005)). This should have led to a decline in financial intermediaries' share of output (and corporate profits) in developed economies, but their financial sectors have continued to grow. This suggests that other forces also determine the demand for banking services; we suggest that warehousing-type financial services may be one important determinant, one linked with the very origins of banking. In fact, the largest deposit bank in the world today, Bank of New York–Mellon, is usually classified as a custodian bank, i.e. an institution responsible for the safeguarding, or warehousing, of financial assets.

Our paper is also related to the literature on liquidity creation by banks which relies on the role of intermediaries as providers of liquidity insurance. Important contributions include Allen and Gale (1998), Allen, Carletti, and Gale (2014), Bryant (1980), Diamond and Dybvig (1983), and Postlewaite and Vives (1987). In our model, banks create *liquid securities*, since they issue receipts that constitute private money, and they also create *aggregate liquidity* since more capital is directed into productive investment with the bank than without. In other words, banks enhance funding liquidity. This contrasts with the model in Diamond and Dybvig (1983) and other models of that genre in at least five important respects, discussed below.

First, in Diamond and Dybvig (1983), the bank creates liquid securities, namely demand-deposit contracts, to raise funds to invest in illiquid projects. However, the bank does not create aggregate *funding* liquidity, since no more is invested in the illiquid projects than would be absent the bank. In our model, the bank creates aggregate funding liquidity precisely because it can create liquid securities—warehouse receipts—with which to make loans. Since the receipts circulate as a medium of exchange, they provide working capital to firms, expanding the total supply of liquidity available for investment. Second, in existing models of bank liquidity creation, aggregate investment is typically bounded by the total initial endowment of liquidity in the economy. Banks are limited to taking deposits and lending them out. Not so in our model. Indeed, in aggregate firms invest in *more* capital and labor than the initial aggregate endowment of liquidity in the economy, something that is impossible with the non-bank direct investment outcome. That is, in our model, *lending also creates deposits*. Third, models like Bryant (1980) and Diamond and Dybvig (1983) focus on the bank's interactions with depositors as the source of liquidity creation, with uncertainty about time preferences creating a preference for liquidity by (risk-averse) depositors. In other words, liquidity creation is viewed as being synonymous with consumption insurance—the bank provides risk averse depositors with higher interim consumption than possible with direct investment in the project that the bank invests its deposits in. Thus, even though the depositor has just as liquid an asset by keeping his money in his pocket as he does by depositing it in a bank, he obtains higher interim consumption and hence better risk sharing with a deposit.[6] In contrast, our model focuses on the asset side of the bank's balance sheet as a result of our different definition of liquidity creation. Fourth, in the existing models of bank liquidity creation, coordination failures among depositors can lead to bank runs, an issue that does not arise in our model. Finally, while in the existing models bank capital either plays no role in liquidity creation or is viewed as impeding bank liquidity creation, in our model we show that it enhances bank liquidity creation.

This focus on the bank's assets, along with our assumption that all agents are risk neutral, means that bank liquidity creation is decoupled from risk preferences. Risk sharing for depositors is thus neither necessary nor sufficient for ex ante liquidity creation by the bank in our framework, although deposits are an important reason why the bank exists in the first place. Liquidity creation is manifested through the lending of fake receipts. In this sense, our approach in which bank lending creates deposits is reminiscent of Hahn (1920):[7]

> We thus maintain—contrary to the entire literature on banking and credit—that the primary business of banks is not the liability business, especially the deposit business, but in general and in each and every case an asset transaction of a bank must have previously taken place, in order to allow the possibility of a liability business and to cause it. The liability business of banks is nothing but a reflex of prior credit extension.... (Hahn, 1920, p. 29)

[6]Jacklin (1987) shows that if the owner of the project that the bank invests in were to issue traded equity that paid a dividend at the interim date, the same risk sharing can be achieved as provided by bank deposits.

[7]See also Werner (2014) and the "goldsmith anecdote" in Greenbaum, Thakor, and Boot (2015)

Keynes makes a related point:

> It is not unnatural to think of deposits of a bank as being created by the public through the deposits of cash representing either savings or amounts which are not for the time being required to meet expenditures. But the bulk of the deposits arise out of the action of the banks themselves, for by granting loans, allowing money to be drawn on an overdraft or purchasing securities, a bank creates a credit in its books which is the equivalent of a deposit. (Keynes in his contribution to the Macmillan Committee, 1931, p. 34)

We view our paper as offering a view of bank liquidity creation that *complements* the consumption insurance view in the existing literature, a view that has provided deep insights into a variety of phenomena like bank runs and deposit insurance (Bryant (1980) and Diamond and Dybvig (1983)), financial crises (e.g. Allen and Gale (1998)), and the role of financial intermediaries *vis-à-vis* markets (e.g. Allen and Gale (2004)). Juxtaposing our analysis with the existing literature, liquidity creation is seen to have two important dimensions: consumption insurance for depositors/savers and elevated funding liquidity for entrepreneurs/borrowers. These mutually-reinforcing views of bank liquidity creation are consistent with the idea that banks create liquidity by both taking in deposits and selling loan commitments (e.g. Kashyap, Rajan, and Stein (2002)).

In addition to providing a different perspective on the role of banks and what they do to create liquidity, our paper also provides a microfoundation for the assumption that cash flow diversion destroys value, which is common in the finance literate (e.g. Bolton and Scharfstein (1990) and Hart and Moore (1998)). In these models, if a borrower diverts his cash flow rather than repays his creditor, his cash flow typically decreases by an exogenous, borrower-specific constant. In our model, such a number arises endogenously as the ratio of depreciation given private storage to the returns on the warehouses' technology. Note that this suggests that the costs of diversion should not be borrower-specific in general, in the sense that they depend not only on the borrower's technology, but on the creditor's technology as well.

The rest of the paper is organized as follows. Section 2 provides an example in a simplified set-up in which all the key forces of the model are at work. Section 3 develops the formal model. Section 4 contains the solution of the model. Section 5 solves two benchmark models: (i) one in which warehouses cannot issue fake receipts and (ii) the first-best allocation. Section 6 contains the main results. It presents analysis of liquidity creation and fractional reserves. Section 7 considers the welfare implications of four policies: liquidity requirements, narrow banking, capital requirements, and monetary policy. Finally, Section 8 concludes. The appendix contains all proofs and a glossary of notation.

2 Motivating Example

In this subsection, we write a numerical example that illustrates the main mechanism at work in a simplified setup. We write the example with just three players, one farmer, one laborer, and one warehouse. We examine a sequence of increasingly rich cases to demonstrate the efficiency gains from warehousing and from issuing fake receipts. Specifically, we consider: (1) the case without a warehouse, (2) the case in which a warehouse provides only safe-keeping services but does not lend, (3) the case in which a warehouse provides both safe-keeping and lending services, and (4) the first-best case, in which the allocation is efficient.

The analysis of the example shows that, even without lending, warehousing alone increases efficiency by providing more efficient storage. This efficiency gain is merely technological, however; introducing a better storage technology increases terminal output. But when a warehouse can issue fake receipts, it does more to improve efficiency. By issuing fake receipts, the warehouse creates liquidity that the farmer invests productively. This efficiency gain is allocational and is much more important than the simple technological efficiency gain. When the warehouse issues fake receipts, it allows *other players* in the economy to invest in more efficient technologies. Finally, the analysis of the first-best allocation suggests that there is an efficiency loss in the second best even when the warehouse can issue fake receipts, it still creates less liquidity than in the first-best.

The setup of the example is as follows. There are three dates, Date 0, Date 1, and Date 2. The farmer has an endowment e of twelve units of grain at Date 0 and no one else has any grain. At Date 0 the farmer can borrow B from the warehouse at gross rate one. We assume that warehouse deposit rates and wages w are also all set equal to one.[8] The farmer produces over the period from Date 0 to Date 1 and he stores his output over the period from Date 1 to Date 2. If the farmer stores his grain privately, it depreciates at δ percent, and we choose to set $\delta = 20\%$; if he stores it in a warehouse, it does not depreciate. This can be viewed as the warehouse having better defense against theft, for reasons we explain later. The farmer's production technology transforms a unit of labor and a unit of grain at Date 0 into four units of grain at Date 1 with constant returns. In other words, the farmer has a Leontief production function in the first period that takes grain investment i, which we will refer to as "capital investment," and labor ℓ and produces output $y = 4 \min \{i, \ell\}$ at Date 1. We assume that this output is not pledgeable; however, a warehouse can seize the deposits it holds. Suppose there is no discounting, so workers are willing to store grain in warehouses at the deposit rate

[8] These prices—i.e. rates and wages—result from competition in the full model, leaving rents to the farmer. We take them as given in this example for simplicity.

of one. Everyone consumes only at Date 2.

The parameter values are summarized in Figure 1 and the timing is illustrated in Figure 2.

Figure 1: Summary of notation and values in example in Section 2 .

Quantity	Notation	Value in Example
Farmer's endowment	e	12
Farmer's technology	y	$4 \min\{i, \ell\}$
Depreciation rate	δ	20%
Wage	w	1

Figure 2: A Simplified Timeline Representation of the Sequence of Moves

Date 0

the farmer has endowment e and borrows B from the warehouse
he invests i in grain, and $w\ell$ in labor

the laborer exerts labor ℓ and deposits his wages $w\ell$ in the warehouse

Date 1

the farmer produces y
he either repays and deposits or diverts and stores privately

Date 2

the farmer, laborer, and warehouse consume

Definition of liquidity creation: We refer to the farmer's expenditure on capital and labor $i + w\ell$ as the "total investment." We measure liquidity creation by the ratio Λ of the farmer's total investment to his initial endowment,

$$\Lambda = \frac{i + w\ell}{e}.$$

No warehousing. Consider first the case in which there is no warehousing. Thus, the farmer must pay the laborer in grain. To maximize his Date 2 consumption, the farmer maximizes his Date 1 output and then stores his output from Date 1 to Date 2. To maximize his Date 1 output, he invests in equal amounts of capital i and labor ℓ (as a result of the Leontief technology). Since his endowment is twelve and wages are one, he sets $i = \ell = 6$ and produces $y = 4 \times 6 = 24$ units of grain. He then stores his grain privately from Date 1 to Date 2 and this grain depreciates by twenty percent; the farmer's final payoff is $(1 - 20\%) \times 24 = 19.2$ units. $\Lambda_{\text{nw}} = 1$, so there is no liquidity creation.

Warehousing but no fake receipts. Now consider the case where there is a warehouse, but that it performs only the function of safekeeping. When a depositor (the farmer or the laborer) deposits grain in the warehouse, the warehouse issues receipts and holds the grain until it is withdrawn. In this case, the farmer again maximizes his Date 1 output in order to maximize his Date 2 consumption. Again, he will invest equal amounts of capital and labor. He cannot borrow from the warehouse, so he again just divides his endowment fifty-fifty between capital investment and labor, setting $i = \ell = 6$ and producing $y = 4 \times 6 = 24$ units of grain. He now stores his grain in the warehouse from Date 1 to Date 2. Since it is warehoused, the grain does not depreciate; the farmer's final payoff is 24 units. Warehousing has added 4.8 units to the farmer's consumption by increasing efficiency in storage. But the warehouse has not created any liquidity for the farmer since the initial investment in the technology $i + w\ell = e = 12$ is the same as in the case in which there is no warehouse. There is no liquidity creation, $\Lambda_{\text{nr}} = 1$.

Warehousing with fake receipts. Now consider the case in which there is a warehouse that provides not only safe-keeping but also lends. Since the farmer's technology is highly productive, he wishes to borrow to scale it up. But the farmer already holds all twelve units of grain in the economy, so how can he scale up his production even further? The key is that the farmer can borrow from the warehouse in warehouse receipts. Observe that the receipts the warehouse uses to make loans are *not* backed by grain; they are "fake receipts." However, if the laborer accepts payment from the farmer in these fake receipts, they are still valuable to the farmer—they provide him with "working capital" to pay the laborer.

The farmer again sets his capital investment equal to his labor investment, $i = \ell$. Given that he can borrow B in receipts from the warehouse, however, he can now invest a total up to $i + w\ell = e + B$. Thus, recalling that wages w are one, his optimal investment is

$$i = \frac{e + B}{2} = 6 + \frac{B}{2} \tag{1}$$

and the corresponding Date 1 output is

$$y = 4i = 24 + 2B. \tag{2}$$

Given that this technology is highly productive and has constant returns to scale, the farmer wishes to expand production as much as possible. The amount he can borrow from the warehouse, however, is limited by the amount that he can credibly promise to repay. Since we have assumed that the farmer's output is not pledgeable, his creditor (i.e., the warehouse) cannot enforce the repayment of his debt. However, if the farmer deposits in the warehouse, it is possible for the warehouse to seize the deposit. Thus, after the farmer produces, he faces a tradeoff between not depositing and depositing. If he does not deposit, he stores privately, so his grain depreciates, but he avoids repayment. If he does deposit, he avoids depreciation, but the warehouse can seize his deposit and force repayment. The warehouse lends to the farmer only if repayment is incentive compatible. For the repayment to be incentive compatible, the farmer must prefer to deposit in the warehouse and repay his debt rather than to store the grain privately and default on his debt. This is the case if the following inequality holds

$$y - B \geq (1 - 20\%)y. \tag{3}$$

The maximum the farmer can borrow B is thus given by

$$\big(24 + 2B\big) - B = \big(1 - 20\%\big)\big(24 + 2B\big), \tag{4}$$

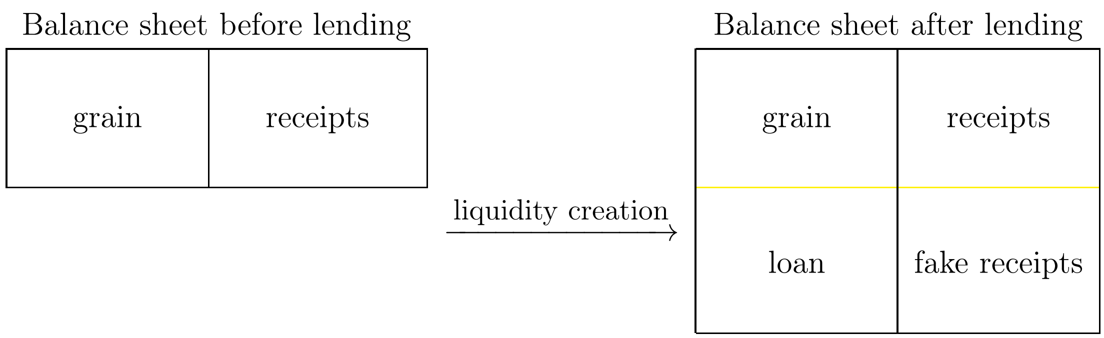
or $B = 8$. This corresponds to $i = \ell = 10$. The liquidity creation is given by

$$\Lambda = \frac{i + w\ell}{e} = \frac{10 + 10}{12} = \frac{5}{3}. \tag{5}$$

The farmer is able to scale up his production only when the warehouse makes loans by writing fake receipts. In other words, liquidity is created on the asset side, not the deposit side, of the warehouse's balance sheet. When a warehouse makes a loan, it creates both an illiquid asset, the loan, and a liquid liability, the fake receipt. This is liquidity transformation, one of the fundamental economic roles of banks. Our model suggests, however, that liquidity transformation occurs entirely when banks make loans—it is not necessary that banks fund themselves with liquid deposits and then, separately, make illiquid loans.

If the farmer could pay the labor on credit, he would not need to borrow from the warehouse and he could expand production even further. However, an impediment to this is that the laborer cannot enforce repayment from the farmer (because the output is not pledgeable and the laborer has no way to seize it). Therefore, the farmer's promise

Figure 3: THE WAREHOUSE'S BALANCE SHEET EXPANDS WHEN IT MAKES A LOAN, CREATING LIQUIDITY.



to the laborer is not credible.

First-best. We now consider the first best allocation of resources in order to emphasize that the incentive compatibility constraint limits liquidity creation. Allowing warehouses to make loans in fake receipts moves the economy closer to the first best level of liquidity creation, but does not achieve it. In the first-best allocation the farmer invests his entire endowment in capital $i = e = 12$ and laborers exert equal labor $\ell = 12$. In this allocation output $y = 4i = 48$ and liquidity creation is given by

$$\Lambda_{\text{fb}} = \frac{i + w\ell}{e} = \frac{12 + 12}{12} = 2. \tag{6}$$

Figure 4: SUMMARY OF LIQUIDITY CREATION THE EXAMPLE IN SECTION 2.

Case	Date 1 Output y	Date 2 Output	Liquidity Λ
No warehouses	24	19.2	1
Warehouses without lending	24	24	1
Warehouses with lending	40	40	5/3
First-best	48	48	2

3 Model

There are three dates, Date 0, Date 1, and Date 2 and three groups of players, farmers, warehouses, and laborers. There is a unit continuum of each type of player. There is one real good, called grain, which serves as the numeraire. There are also receipts issued by warehouses, which entail the right to withdraw grain from a warehouse.

All players are risk neutral and consume only at Date 2. Denote farmers' consumption by c^f, laborers' consumption by c^l, and the warehouses' consumption by c^b (the index b stands for "bank"). Farmers begin life with an endowment e of grain. No other player has a grain endowment. Laborers have labor at Date 0. They can provide labor ℓ at the constant marginal cost of one. So their utility is $c^l - \ell$. Farmers have access to the following technology. At Date 0, a farmer invests i units of grain and ℓ units of labor. At Date 1, this investment yields

$$y = A \min\{\alpha i, \ell\}, \tag{7}$$

i.e. the production function is Leontief. The output y is *not pledgeable*. At Date 1 farmers have no special production technology: they can either store grain privately or store it in a warehouse.

If grain is not invested in the technology, it is either stored privately or stored in a warehouse. If the grain is stored privately (by either farmers or laborers), it depreciates at rate $\delta \in [0,1)$. If player j stores s_t^j units of grain privately from Date t to Date $t+1$, he has $(1-\delta)s_t^j$ units of grain at Date $t+1$. If grain is stored in a warehouse, it does not depreciate. Further, if grain is stored in the warehouse, the warehouse can seize it.

The assumption that the warehouse can store grain more efficiently than the individual farmer has a natural interpretation in the context of the original warehouses from which banks evolved. These ancient warehouses tended to be (protected) temples or the treasures of sovereigns, so they had more power than ordinary individuals and a natural advantage in safeguarding valuables and enforcing contracts.[9] Our assumption on δ can thus be viewed either as a technological advantage arising from specialization acquired through (previous) investment and experience or as a consequence of the power associated with the warehouse.[10]

When players store grain in warehouses, warehouses issues receipts as "proof" of these deposits. The bearers of receipts can trade them among themselves. Warehouses can also issue receipts that are not proof of deposits. These receipts, which we refer to as "fake receipts," still entail the right to withdraw grain from a warehouse, and thus they are warehouses' liabilities that are not backed by the grain they hold. Receipts

[9] We thank Charles Goodhart for this interpretation. Thus, whereas our focus is on private money (fake receipts), the alternative, sovereign-power-linked interpretation of the storage advantage of the warehouse over individuals means that our model may complement the chartalist view of money creation by the state (e.g. Knapp (1924) and Minsky (2008)).

[10] This power was important for several reasons that complement our approach and provide alternative interpretations of the deep parameters in the model. First, it enabled grain, gold, or other valuable commodities to be stored safely, without fear of robbery. Second, power enabled the creditor to impose greater penalties on defaulting borrowers. Third, power also generated a greater likelihood of continuation of the warehouse, and hence of engaging in a repeated game with depositors. This created reputational incentives for the warehouse not to abscond with deposits.

backed by grain are indistinguishable from fake receipts.

The markets for labor, warehouse deposits, and loans are competitive.

3.1 Financial Contracts

There are three types of contracts in the economy: labor contracts, deposit contracts, and lending contracts. We restrict attention to bilateral contracts, although warehouse receipts are tradeable and loans are also tradeable in an interbank market.

Labor contracts are between farmers and laborers. Farmers pay laborers $w\ell$ in exchange for laborers' investing ℓ in their technology, which then produces $y = y(i, \ell)$ units of grain at Date 1.

Deposit contracts are between warehouses and the other players, i.e., laborers, farmers, and (potentially) other warehouses. Warehouses accept grain deposits with gross rate R_t^D over one period, i.e. if player $j \in \{l, f\}$ makes a deposit of d_t^j units of grain at Date t he has the right to withdraw $R_t^D d_t^j$ units of grain at Date $t+1$. When a warehouse accepts a deposit of one unit of grain, it issues a receipt in exchange as "proof" of the deposit.

Lending contracts are between warehouses and farmers. Warehouses lend L to farmers at Date 0 in exchange for farmers' promise to repay $R^L L$ at Date 1, where R^L is the lending rate. Warehouses can lend in grain or in receipts. A loan made in receipts is tantamount to a warehouse offering a farmer a deposit at Date 0 in exchange for the farmer's promise to repay grain at Date 1. When a warehouse makes a loan in receipts, we say that it is "issuing fake receipts." We refer to a warehouse's total deposits at Date t as D_t. These deposits include both those deposits backed by grain and those granted as fake receipts.

Lending contracts are subject to a form of limited commitment on the farmers' side. Because farmers' Date 1 output is not pledgeable, they are free to divert their output. However, if the farmers do divert, they must store their grain privately. The reason is that if they deposit their output in a warehouse, it may be seized by the warehouse.

We now formalize how we capture farmers' inability to divert output if they deposit in warehouses. We define the variable T as the total transfer from farmers to warehouses at Date 1; T includes both the repayment of farmers' debt to warehouses and farmers' new deposits d_1^f in warehouses. If farmers have borrowed B at Date 0, then they have to repay $R^L B$ to warehouses at Date 1. When they make a transfer T to a warehouse at Date 1, the "first" $R^L B$ units of grain they transfer to the warehouse are used to repay the debt. Only after full repayment of $R^L B$ do warehouses store grain for farmers as deposits. Thus, the farmers' deposits at Date 1 are given by

$$d_1^f = T - \min\left\{T, R^L B\right\} = \max\left\{T - R^L B, 0\right\}. \tag{DC}$$

This says that if farmers have not repaid their debt at Date 1, their Date 1 deposits are constrained to be zero; we call this the *deposit constraint.*

The argument above glosses over one important subtlety. The farmer could borrow from one warehouse at Date 0 and divert his output at Date 1, only to deposit his grain in a different warehouse. Would this allow the farmer to avoid both repaying his debt and allowing his grain to depreciate? The answer is no. The reason is as follows. If the farmer deposits his grain in a warehouse that is not his original creditor, that warehouse will buy the farmer's debt from his original creditor on the interbank market, allowing him seize the grain the farmer owes. Thus, no matter which warehouse the farmer deposits with, he will end up repaying his debt. We model the interbank market and discuss this reasoning in more formally in Appendix A.2.

Since there is no uncertainty, without loss of generality we can restrict attention to lending contracts where default at Date 1 never happens in equilibrium. The farmer will never default on his debt as long as repayment is incentive compatible. In other words, he must prefer to repay his debt and deposit in the warehouse rather than to default on his debt and store the grain privately. If g_1^f denotes the farmer's total Date 1 grain holding and $R^L B$ denotes the face value of his debt, then he repays his debt if the following inequality is satisfied[11]

$$R_1^D \left(g_1^f - R^L B \right) \geq (1-\delta) g_1^f. \tag{IC}$$

The timeline of moves for each player and their contractual relationships are illustrated in a timeline in Figure 5.

[11] In equilibrium, the farmer's total Date 1 grain holding g_1^f comprises his Date 1 output y, his Date 0 deposits gross of interest, $R_0^D d_0^f$, and his depreciated savings $(1-\delta)s_0^f$, or $g_1^f = y\big(i, \ell^f\big) + R_0^D d_0^f + (1-\delta)s_0^f$.

Figure 5: A Timeline Representation of Sequence of Moves

Date 0	Date 1	Date 2
Warehouses accept deposits D_0 lend L to farmers store s_0^b	**Warehouses** receive T from farmers accept deposits D_1 repay $R_0^D D_0$ to depositors store s_1^b	**Warehouses** repay $R_1^D D_1$ to depositors consume $c^b = s_1^b - R_1^D D_1$
Farmers borrow B from warehouses invest i and ℓ in technology y pay laborers $w\ell$ deposit d_0^f in warehouses store s_0^f	**Farmers** receive cash flow $y(i, \ell)$ transfer T to warehouses receive $R_0^D d_0^f$ from warehouses have total grain holding g_1^f deposit d_1^f in warehouses store s_1^f	**Farmers** receive $R_1^D d_1^f$ from warehouses consume $c^f = R_1^D d_1^f + (1-\delta)s_1^f$
Laborers exert labor ℓ accept wage $w\ell$ deposit d_0^l in warehouses store s_0^l	**Laborers** receive $R_0^D d_0^l$ from warehouses deposit d_1^l in warehouses store s_1^l	**Laborers** receive $R_1^D d_1^l$ from warehouses consume $c^l = R_1^D d_1^l + (1-\delta)s_1^l$

3.2 Summary of Key Assumptions

In this subsection, we restate and briefly discuss the major assumptions underlying the results.

Assumption 1. *Output is not pledgeable.*

This assumption prevents farmers' from paying laborers directly in equity or debt. If farmers' output were pledgeable, they could pay laborers after their projects paid off. Without this assumption, there would be no frictions and we could achieve the first-best allocation. Thus, this assumption creates a role for banks as providers of liquidity.

Assumption 2. *Warehouses can seize the deposits they hold.*

This assumption implies that a warehouse can enforce repayment from a farmer *as long as that farmer chooses to deposit in it.* Whenever farmers deposit in a warehouse, the warehouse has a seizure technology that allows it to demand repayment from farmers. This is one key ingredient to understand the connection between warehousing/account-keeping and lending. As discussed above and in Appendix A.2, the interbank market for loans prevents farmers from avoiding repayment by depositing in warehouses that are not their creditors.

ASSUMPTION 3. *Grain depreciates relatively more slowly if stored in a warehouse.*

Specifically, we use the normalization that grain does not depreciate inside a warehouse and depreciates at rate $\delta \in (0, 1]$ outside a warehouse. This assumption gives farmers a reason to deposit in warehouses rather than store grain themselves—farmers deposit in warehouses to take advantage of more efficient storage. This, in turn, ensures that farmers will repay their debt, since warehouses can seize grain owed to them if it is deposited with them, by Assumption 2. Thus, Assumption 2 and Assumption 3 together imply that warehouses have a superior technology to enforce repayment.

In some other theories of banking, the agents with the power to enforce contracts (or "monitor borrowers") naturally perform banking activities (e.g., Holmstrom and Tirole (1997)). In our model, warehouse-banks' enforcement power is an endogenous consequence of their warehousing technology. Note that warehousing efficiency and enforcement power are natural complements, as a superior warehousing technology may result from the power to store assets safely, without fear of robbery, as explained earlier. Further, Assumption 2 implies that if a borrowers defaults, his access to warehousing is limited in the future; this is analogous to warehouses' having the power to impose penalties on defaulting borrowers in a dynamic setting.

3.3 Individual Maximization Problems

All players take prices as given and maximize their Date 2 consumption subject to their budget constraints. Farmers' maximization problems are also subject to their incentive compatibility constraint (IC).

We now write down each player's maximization problem.

The warehouses' maximization problem is

$$\text{maximize} \quad c^b = s_1^b - R_1^D D_1 \tag{8}$$

over s_1^b, s_0^b, D_0, D_1, and L subject to

$$s_1^b = R^L L + s_0^b - R_0^D D_0 + D_1, \tag{BC$_1^b$}$$

$$s_0^b + L = D_0, \tag{BC$_0^b$}$$

and the non-negativity constraints $D_t \geq 0, s_t^b \geq 0, L \geq 0$. To understand this maximization program, note that equation (8) says that the warehouse maximizes its profit (consumption) c^b, which consists of the difference between what is stored in the warehouse at Date 1, s_1^b, and what is paid to depositors, $R_1 D_1$. Equation (BC$_1^b$) is the warehouse's budget constraint at Date 1, which says that what is stored in the warehouse at Date 1, s_1^b, is given by the sum of the interest on the loan to the farmer, $R^L L$, the warehouse's savings at Date 0, the deposits at Date 1 D_1 minus the interest the warehouse must pay on its time 0 deposits, $R_0^D D_0$. Similarly, Equation (BC_0^b) is the warehouse's budget constraint at Date 0, which says that the sum of the warehouse's savings at Date 0, s_0^b, and its loans L must equal the sum of the Date 0 deposits, D_0.

The farmers' maximization problem is

$$\text{maximize} \quad c^f = R_1^D d_1^f + (1-\delta) s_1^f \tag{9}$$

over $s_1^f, s_0^f, d_0^f, T, i, \ell^f$, and B subject to

$$d_1^f = \max\left\{T - R^L B, 0\right\}, \tag{DC}$$

$$\left(R_1^D - 1 + \delta\right)\left(y\big(i, \ell^f\big) + R_0^D d_0^f + (1-\delta) s_0^f\right) \geq R_1^D R^L B, \tag{IC}$$

$$T + s_1^f = y\big(i, \ell^f\big) + R_0^D d_0^f + (1-\delta) s_0^f, \tag{BC_1^f}$$

$$d_0^f + s_0^f + i + w\ell^f = e + B, \tag{BC_0^f}$$

and the non-negativity constraints $s_t^f \geq 0, d_t^f \geq 0, B \geq 0, i \geq 0, \ell^f \geq 0, T \geq 0$. The farmer's maximization program can be understood as follows. In equation (9) the farmer maximizes his Date 2 consumption c^f, which consists of his Date 1 deposits gross of interest, $R_1^D d_1^f$, and his depreciated private savings, $(1-\delta)s_1^f$. Equations (DC) and (IC) are, respectively, the deposit constraint and the incentive compatibility constraint (for an explanation see Subsection 3.1). The incentive compatibility constraint follows directly from equation (IC) in Subsection 3.1, since the farmer's Date 1 grain holding g_1^f comprises his Date 1 output y, his Date 0 deposits gross of interest, $R_0^D d_0^f$, and his depreciated savings $(1-\delta)s_0^f$, or $g_1^f = y\big(i, \ell^f\big) + R_0^D d_0^f + (1-\delta)s_0^f$. Equation ($\mathrm{BC}_1^f$) is the farmer's budget constraint that says that the sum of his Date 1 savings, s_1^f, and his overall transfer to the warehouse, T, must equal the sum of his output y, his Date 0 deposits gross of interest, $R_0^D d_0^f$, and his depreciated savings, $(1-\delta)s_0^f$. Equation (BC_0^f) is the farmer's budget constraint at Date 0 which says that the sum of his Date 0 deposits, d_0^f, his Date 0 savings, s_0^f, his investment in grain i and his investment in labor, $w\ell^f$, must equal the sum of his initial endowment, e, and the amount he borrows, B.

The laborers' maximization problem is

$$\text{maximize} \quad c^l = R_1^D d_1^l + (1-\delta) s_1^l - \ell^l \tag{10}$$

over $s^l_1, s^l_0, d^l_1, d^l_0$, and ℓ^l subject to

$$d^l_1 + s^l_1 = R^D_0 d^l_0 + (1-\delta)s^l_0, \tag{BC$^l_1$}$$

$$d^l_0 + s^l_0 = w\ell^l, \tag{BC$^l_0$}$$

and the non-negativity constraints $s^l_t \geq 0, d^l_t \geq 0, \ell^l \geq 0$. The laborer's maximization program can be understood as follows. In equation (10) the laborer maximizes his Date 2 consumption c^l, which consists of his Date 1 deposits gross of interest, $R^D_1 d^l_1$, and his depreciated private savings, $(1-\delta)s^l_1$. Equation (BC$^l_1$) is the laborer's budget constraint that says that the sum of his Date 1 savings, s^l_1, and his Date 1 deposits, d^l_1, must equal the sum of his Date 0 deposits gross of interest, $R^D_0 d^l_0$, and his depreciated savings, $(1-\delta)s^l_0$. Equation (BC$^l_0$) is the laborer's budget constraint at Date 0 which says that the sum of his Date 0 deposits, d^l_0, and his Date 0 savings, s^l_0, must equal his labor income $w\ell^l$.

3.4 Equilibrium

The equilibrium is a profile of prices $\langle R^D_t, R^L, w\rangle$ for $t \in \{1, 2\}$ and a profile of allocations $\langle s^j_t, d^f_t, d^l_t, D_t, L, B, \ell^l, \ell^f\rangle$ for $t \in \{1, 2\}$ and $j \in \{b, f, l\}$ that solves the warehouses' problem, the farmers' problem, and the laborers' problem defined in Section 3.3 and satisfies the market clearing conditions for the labor market, the lending market, the grain market and deposit market at each date:

$$\ell^f = \ell^l \tag{MC$^\ell$}$$

$$B = L \tag{MC^L}$$

$$i + s^f_0 + s^l_0 + s^b_0 = e \tag{MC$^g_0$}$$

$$s^f_1 + s^l_1 + s^b_1 = (1-\delta)s^f_0 + (1-\delta)s^l_0 + s^b_0 + y \tag{MC$^g_1$}$$

$$D_0 = d^f_0 + d^l_0 \tag{MC$^D_0$}$$

$$D_1 = d^f_1 + d^l_1. \tag{MC$^D_1$}$$

3.5 Parameter Restrictions

In this section we make two restrictions on parameters. The first ensures that farmers' production technology generates sufficiently high output that the investment is positive NPV in equilibrium and the second ensures that the incentive problem that results from the non-pledgeablity of farmers' output is sufficiently severe to generate a binding borrowing constraint in equilibrium. Note that since the model is linear, if the farmers' IC does not bind, they will scale their production infinitely.

Parameter Restriction 1. *The farmers' technology is sufficiently productive,*

$$A > 1 + \frac{1}{\alpha}. \tag{11}$$

Parameter Restriction 2. *Depreciation from private storage is not too fast,*

$$\delta A < 1. \tag{12}$$

4 Model Solution

In this subsection, we solve the model to characterize the equilibrium. We proceed as follows. First, we pin down the equilibrium deposit rates, lending rates and wages. Then we show that the model collapses to the farmer's problem which we then solve to characterize the equilibrium.

4.1 Preliminary Results

Here we state three results that completely characterize all the prices in the model, namely the two deposit rates R_0^D and R_1^D, the lending rate R^L, and the wage w. We then show that given the equilibrium prices, farmers and laborers will never store grain privately. The results all follow from the definition of competitive equilibrium with risk-neutral agents.

The first two results say that the risk-free rate in the economy is one. This is natural, since the warehouses have a scalable storage technology with return one.

Lemma 1. *Deposit rates are one at each date, $R_0^D = R_1^D = 1$.*

Now we turn to the lending rate. Since the farmers' incentive compatibility constraint ensures that loans are riskless and warehouses are competitive, warehouses also lend to farmers at rate one.

Lemma 2. *Lending rates are one, $R^L = 1$.*

Finally, since laborers have a constant marginal cost of labor, the equilibrium wage must be equal to this cost; this says that $w = 1$, as summarized in Lemma 3 below. [12]

Lemma 3. *Wages are one, $w = 1$.*

[12]Note that we have omitted the effect of discounting in the preceding argument—laborers work at Date 0 and consume at Date 2; discounting is safely forgotten, though, since the laborers have access to a riskless storage technology with return one via the warehouses, as established above.

These results establish that the risk-free rate offered by warehouses exceeds the rate of return from private storage, or $R_0^D = R_1^D = 1 > 1 - \delta$. Thus, farmers and laborers do not wish to make use of their private storage technologies. The only time a player may choose to store grain outside a warehouse is if a farmer diverts his output; however, the farmer's incentive compatibility constraint ensures he will not do this. Corollary 1 below summarizes this reasoning.

Corollary 1. *Farmers and laborers do not store grain,* $s_0^l = s_0^f = s_1^l = s_1^f = 0$.

4.2 Equilibrium Characterization

In this section we characterize the equilibrium of the model. We proceed as follows. First, we show that given the equilibrium prices established in Subsection 4.1 above, laborers and warehouses are indifferent among all allocations. We then establish that a solution to the farmers' maximization problem given the equilibrium prices is a solution to the model.

The prices R_0^D, R_1^D, R^L, and w are determined exactly so that the markets clear given that agents are risk-neutral. In other words, they are the unique prices that prevent the demands of warehouses or laborers from being infinite. This is the case only if warehouses are indifferent between demanding and supplying deposits and loans at rates R_0^D, R_1^D, and R^L and laborers are indifferent between supplying or not supplying labor at wage w. This implies that all prices are one in equilibrium, as summarized in Lemma 4 below.

Lemma 4. *Given the equilibrium prices,* $R_0^D = R_1^D = R^L = w = 1$, *warehouse are indifferent among all deposit and loan amounts and laborers are indifferent among all labor amounts.*

Lemma 4 implies that, given the equilibrium prices, they will absorb any excess demand left by the farmers. In other words, given the equilibrium prices established in Subsection 4.1 above, for any solution to the farmer's individual maximization problem, laborers' and warehouses' demands are such that markets clear.

We have thus established that the equilibrium allocation is given by the solution to the farmers' problem given the equilibrium prices, where the laborers' and warehouses' demands are determined by the market clearing conditions. Thus, to find the equilibrium, we maximize the farmers' Date 2 consumption subject to his budget and incentive constraints given the equilibrium prices.

Lemma 5. *The equilibrium allocation solves the problem to*

$$\text{maximize} \quad d_1^f \tag{13}$$

subject to

$$\delta\Big(y\big(i,\ell^f\big)+d_0^f\Big)\geq B, \tag{IC}$$

$$d_1^f+B=y\big(i,\ell^f\big)+d_0^f, \tag{BC$_1^f$}$$

$$d_0^f+i+\ell^f=e+B, \tag{BC$_0^f$}$$

and $i\geq 0,\ell^f\geq 0,B\geq 0,d_0^f\geq 0,$ *and* $d_1^f\geq 0.$

Proposition 1. *The equilibrium allocation is as follows:*

$$B=\frac{\delta A\alpha e}{1+\alpha\big(1-\delta A\big)}, \tag{14}$$

$$\ell=\frac{\alpha e}{1+\alpha\big(1-\delta A\big)}, \tag{15}$$

$$i=\frac{e}{1+\alpha\big(1-\delta A\big)}. \tag{16}$$

5 Benchmarks

In this section we consider two benchmarks, one in which warehouses cannot issue fake receipts but must back all deposits with grain and another in which contracts are perfectly enforceable, i.e. the first best.

5.1 Benchmark: No Fake Receipts

Consider a benchmark model in which warehouses cannot issue any receipts that are not backed by grain. Since farmers have the entire endowment at Date 0, the warehouse cannot lend. Thus, farmers simply divide their endowment between their capital investment i and their labor investment ℓ; their budget constraint reads

$$i+w\ell=e. \tag{17}$$

The Leontief production function implies that they will always make capital investments equal to the fraction α of their labor investments, or

$$\alpha i=\ell. \tag{18}$$

We summarize the solution to this benchmark model in Proposition 2 below.

Proposition 2. *In the benchmark model in which warehouses cannot issue fake re-*

ceipts, the equilibrium is as follows:

$$\ell_{\text{nr}} = \frac{\alpha e}{1+\alpha}, \tag{19}$$

$$i_{\text{nr}} = \frac{e}{1+\alpha}. \tag{20}$$

Note that allocation in the benchmark without fake receipts coincides with what the allocation would be if there were no warehouses, since warehouses are not storing any grain at Date 0. Warehouses nonetheless lead to efficiency gains, because they provide efficient storage of grain from Date 1 to Date 2; however, they have no effect at Date 0.

5.2 Benchmark: First-best

We now consider the first-best allocation. Here we consider the allocation that would maximize total output subject only to market clearing conditions. Since the utility, cost, and production functions are all linear, in the first-best allocation, all resources are allocated to the most productive players at each date. At Date 0 the farmers are the most productive and at Date 1 the warehouses are the most productive. Thus, all grain is held by farmers at Date 0 and by warehouses at Date 1. Laborers exert labor in proportion $1/\alpha$ of the total grain invested to maximize production.

PROPOSITION 3. *In the first-best benchmark, the allocation is as follows:*

$$\ell_{\text{fb}} = \alpha e, \tag{21}$$

$$i_{\text{fb}} = e. \tag{22}$$

In Appendix A.3 we discuss the connection between our model and the classical relending model in fractional reserve banking. In this model, warehouses make loans, some fraction of these loans are redeposited by laborers and then loaned again by the warehouse, some of these loans are then redeposited by laborers, and so on. With no constraints, this effective reuse of deposits yields the first-best allocation above.

6 Analysis of the Second-best

In this section we present the analysis of the second-best equilibrium. We show two main results. First, warehouses create liquidity only when they make loans by issuing fake receipts and, second, warehouses still hold grain in equilibrium, i.e. the incentive constraint leads to "endogenous fractional reserves" that prevent the economy from reaching the first-best benchmark.

6.1 Liquidity Creation

In this section we turn to the liquidity a warehouse creates by lending in fake receipts. We begin with the definition of a *liquidity multiplier*, which describes the total investment (grain investment plus labor investment) that farmers can undertake at Date 0 relative to the total endowment e.

DEFINITION 1. *The liquidity multiplier Λ is the ratio of the equilibrium investment in production $i + w\ell$ to the total grain endowment in the economy e,*

$$\Lambda := \frac{i + w\ell}{e}. \tag{23}$$

Note that we will refer to the *total liquidity created* by warehouses as the total investment $i + w\ell$ less the initial liquidity e, which is given by $i + w\ell - e = (\Lambda - 1)e$.

The next result compares the liquidity created in equilibrium, given that warehouses can issue fake receipts, with the liquidity created in the benchmark in which warehouses must back all receipts by grain.

PROPOSITION 4. *Banks create liquidity only when they can issue fake receipts. In equilibrium, the liquidity multiplier is*

$$\Lambda = \frac{1 + \alpha}{1 + \alpha\big(1 - \delta A\big)} > 1, \tag{24}$$

whereas, in the benchmark model with no receipts, the liquidity multiplier is one, denoted $\Lambda_{\text{nr}} = 1$.

This result implies that it is the warehouse's ability to *make loans in fake receipts, not its ability to take deposits, that creates liquidity in the economy.* Warehouses lubricate the economy because they lend in fake receipts rather than in grain. They can do this because of their dual function: they keep accounts (i.e. warehouse grain) and also make loans. This is the crux of farmers' incentive constraint: because warehouses provide valuable warehousing services, farmers go to these warehouse-banks and deposit their grain, which is then also the reason why they repay their debts.

To cement the argument that liquidity creation results only from warehouses' lending in fake receipts, we now relate the quantity of fake receipts that the warehouse issues to the liquidity multiplier. The number of fake receipts the warehouse issues at Date 0 is given by the total number of receipts it issues D_0 less the total quantity of grain it stores s_0^b; in equilibrium, this is given by

$$D_0 - s_0^b = \frac{\alpha\delta A e}{1 + \alpha\big(1 - \delta A\big)}. \tag{25}$$

The expression in Proposition 4 reveals that the number of fake receipts is proportional to the amount of liquidity created in the economy. This reiterates our main point: warehouses create liquidity only by lending in fake receipts. We summarize this in Corollary 2 below.

Corollary 2. *The total liquidity created at Date 0 equals the number of fake receipts the warehouse issues*

$$(\Lambda - 1)\, e = D_0 - s_0^b. \tag{26}$$

We now analyze the effect of the private storage technology—i.e. the depreciation rate δ—on warehouses' liquidity creation. We find that the amount of liquidity Λ that warehouses create is increasing in warehouses' storage advantage, as measured by δ. The reason is that the more desirable it is for farmers to deposit in a warehouse at Date 1 rather than store privately, the looser is their incentive constraint. As a result, warehouses are more wiling to lend to them at Date 0—they know they can lend more and it will still be incentive compatible for farmers to repay at Date 1. We can see this immediately by differentiating the liquidity multiplier with respect to δ:

$$\frac{\partial \Lambda}{\partial \delta} = \frac{(1-\alpha)A}{\big(1+\alpha(1-\delta A)\big)^2} > 0. \tag{27}$$

We summarize this result in Corollary 3 below.

Corollary 3. *The more efficiently warehouses can store grain relative to farmers (the higher is δ), the more liquidity warehouses create by issuing fake receipts.*

We note that Corollary 3 may seem counterintuitive: a *decrease* in the efficiency in private storage leads to an *increase* in overall efficiency. The reason is that it allows banks to create more liquidity by weakening farmer's incentive to divert capital. We return to this result when we discuss central bank policy in Subsection 7.3 below.

This result also suggests an empirical implication. To the extent that warehouses have "power" in enforcing contracts, we should expect warehousing services to be more important in countries with weaker property rights.

6.2 Fractional Reserves

We now proceed to analyze warehouse balance sheets. Absent reserve requirements, do they still store grain? We find that the answer is yes, even though the farmers' technology is constant-returns-to-scale, and farmers would therefore prefer to invest all grain in the economy in their technology. The reason that warehouses store grain in equilibrium is that the farmers' incentive constraint puts an endogenous limit on the amount that farmers can borrow in dollars and, therefore, on the amount of grain that they can invest productively.

PROPOSITION 5. *Warehouses hold a positive fraction of grain at $t = 0$, in equilibrium,*

$$s_0^b = e - i = \frac{\alpha(1-\delta A)e}{1+\alpha(1-\delta A)} > 0, \tag{28}$$

i.e. the incentive constraint leads to endogenous fractional reserves.

Note that in our model, the storage of grain by warehouses at Date 0 is inefficient. Grain could be put to better use by farmers (in conjunction with labor paid for in fake receipts). Thus, a policy maker in our model actually wishes to reduce warehouse holdings or bank capital reserves, to have the economy operate more efficiently.

7 Welfare and Policy

In this section we consider the implications of four policies, all of which have been advocated by policy-makers after the financial crisis of 2007–2009. These are: (1) narrow banking, (ii) liquidity requirements for banks, (iii) capital requirements for banks, and (iv) tightening monetary policy.

7.1 Liquidity Requirements and Narrow Banks

Basel III, the Basel Committee on Banking Supervision's third accord, extends international financial regulation to include so-called liquidity requirements. Specifically, Basel III mandates that banks must hold a sufficient quantity of liquidity to ensure a "liquidity ratio" called the Liquidity Coverage Ratio (LCR) is satisfied. The ratio effectively forces banks to invest a portion of their assets in cash and cash-proximate marketable securities. The rationale is that banks should be able to liquidate a portion of their balance sheets expeditiously to withstand withdrawals in a crisis.

In our model, the LCR imposes a limit on the ratio of loans that a bank (warehouse) can make relative to deposits (grain) it stores. This is exactly a limit on the quantity of fake receipts a bank can issue or a limit on liquidity creation.

We now make this more formal. Consider a liquidity regulation that, like the LCR, mandates that a bank hold a proportion θ of its assets in liquid assets, or

$$\frac{\text{liquid assets}}{\text{total assets}} \geq \theta. \tag{29}$$

In our model, the warehouses' liquid assets are the grain they store and their total assets are the grain they store plus the loans they make. Thus, within the model, the liquidity regulation described above prescribes that, at Date 0,

$$\frac{s_0^b}{B + s_0^b} \geq \theta. \tag{30}$$

We see immediately by rewriting this inequality that this regulation imposes a cap on bank lending,

$$B \leq \frac{1-\theta}{\theta} s_0^b. \tag{31}$$

The next proposition states the circumstance in which liquidity regulation constrains liquidity creation to a level below the equilibrium level.

Proposition 6. *Whenever the required liquidity ratio θ is such that*

$$\theta > 1 - \delta A, \tag{32}$$

liquidity regulation inhibits liquidity creation—and thus farmers' investment—below the equilibrium level.

Advocates of so-called *narrow banking* have argued that banks should hold *only* liquid securities as assets, with some arguing for banks to invest only in treasuries, effectively requiring one-hundred percent reserves.[13] In our model, this corresponds to $\theta = 1$ in the analysis of the LCR, which reduces to the benchmark in which warehouses cannot make loans. We state this as a proposition for emphasis.

Proposition 7. *The requirement of narrow banking is equivalent to the benchmark in which warehouse cannot issue fake receipts (Section 5). In this case there is no liquidity creation,* $\Lambda_{\text{nr}} = 1$.

7.2 Bank Capital and Liquidity Creation

In this subsection we analyze the implications of changes in bank capital for warehouse liquidity creation.[14] In particular, we show that increasing warehouse equity increases liquidity creation. To do this we extend the model in the following way. Between Date 1 and Date 2 we introduce the possibility that grain may spoil even if stored in a warehouse and that the warehouse may reduce the probability of spoilage by exerting (unobservable) costly effort at the end of Date 1. Specifically, with probability $1 - \Delta$ warehoused grain does not depreciate, and with probability Δ warehoused grain depreciates entirely. Warehouses exert effort to decrease the probability Δ, where the

[13]See the review by Pennacchi (2012).

[14]Because we do not have bank failures and crises in our model, our analysis understates the value and role of bank capital. Calomiris and Nissim (2014) document that the market is attaching a higher value to bank capital after the 2007–09 crisis.

cost of effort is $\gamma(1-\Delta)^2/2$. We also assume that the warehouse has an exogenous endowment E of grain at Date 1. We impose a parameter restrictions that equity is neither too small nor too large, so we obtain an interior solution.

Parameter Restriction 3.

$$\gamma(1-\delta) < E \leq \gamma. \tag{33}$$

If the grain in the warehouse spoils, the warehouse has no grain to repay its depositors and it defaults. Finally, note that we leave the model unchanged at Date 0 for simplicity.

We proceed to show that increasing bank capital, i.e. increasing warehouses' Date 1 equity E, increases the liquidity multiplier Λ, thereby elevating liquidity creation. This result contrasts with Proposition 6 above, demonstrating that liquidity requirements and capital requirements are by no means substitutes; the two regulatory policies have opposite effects on liquidity creation in our model.

We solve the model backwards, first finding the equilibrium level of effort that warehouses exert to prevent spoilage. Note that the warehouses choose their effort to maximize their Date 2 payoffs, given their grain holdings s_1^b, their deposits D_1, and the deposit rate R_1^D:

$$c^b = (1-\Delta)\big(s_1^b - R_1^D D_1\big) - \frac{\gamma}{2}(1-\Delta)^2. \tag{34}$$

We solve for the maximum via the first-order condition. The equilibrium effort level is stated in Lemma 6 below.

Lemma 6. *In equilibrium, warehouses determine their effort such that*

$$1-\Delta = \frac{s_1^b - R_1^D D_1}{\gamma}. \tag{35}$$

Next, we state a lemma describing the prices in the economy. The prices are all one, as in the baseline model (Subsection 4.1). We state the result in Lemma 7 below, but we first mention one subtlety here. At Date 1, competitive banks offer the deposit rate that makes them indifferent between accepting more deposits and not accepting more deposits. Note that in the event of spoilage, warehouses always default, so the deposit rate is irrelevant. In the event of no spoilage, their return on deposits is one, as above. Thus, competition leads them to set $R_1^D = 1$. Depositors prefer to deposit in the warehouse as long as their *expected* payoffs from depositing exceed their payoffs from storing privately. Taken together, Parameter Restriction 3 and Lemma 6 above ensure that this is always the case and farmers and laborers still prefer to deposit in a warehouse than to store privately in equilibrium.

LEMMA 7. $R_0^D = R_1^D = R^L = w = 1$.

We can now analyze the effect of warehouse capital on the farmers' incentive constraint. Farmers will deposit in a warehouse as long as the benefits provided by warehouses' superior storage technology outweigh the costs of repaying their debt, or, given $R_1^D = R_L = 1$,

$$(1-\delta)y \leq (1-\Delta)(y-B). \tag{36}$$

Observe that the right-hand side of the incentive constraint above takes into account that farmers receive nothing with probability Δ because their grain spoils. We can substitute in for Δ from Lemma 6 above and find that the amount farmers can borrow is

$$B \leq \frac{s_1^b - R_1^D D_1 - \gamma(1-\delta)}{s_1^b - R_1^D} y. \tag{37}$$

Solving for s_1^b from the warehouses' Date 1 budget constraint (remembering to take into account their additional endowment E) and substituting in for the prices from Lemma 7 above, we see that

$$s_1^b = E + R^L L + D_0 - L - R_0^D D_0 + D_1 \tag{38}$$

$$= E + D_1. \tag{39}$$

Hence, we find that the amount that farmers can borrow is bounded by a function of warehouses equity, specifically

$$B \leq \left(1 - \frac{\gamma(1-\delta)}{E}\right) y. \tag{40}$$

The bound on the right-hand side above is a function of warehouse equity E. In other words, the warehouse's willingness to lend at Date 0 is increasing in its capital. In equilibrium, this implies that the more capital the warehouse has, the more fake receipts it prints and the more liquidity it creates at Date 0. This is summarized in Proposition 8 below.

PROPOSITION 8. *Increasing warehouse equity increases liquidity creation,*

$$\frac{\partial \Lambda}{\partial E} > 0. \tag{41}$$

This result stands in sharp contrast to the existing models literature on bank liquidity creation. In models like Bryant (1980) and Diamond and Dybvig (1983), there is no discernable role for bank capital, and models like Diamond and Rajan (2001) argue that higher bank capital will diminish liquidity creation by banks. In our model, higher bank capital expands the bank's lending capacity and hence enhances ex ante liquidity creation. Nonetheless, we have no central bank and no frictions that generate a rationale for regulatory capital requirements.

7.3 Monetary Policy

In this subsection we analyze the implications of changes in monetary policy on warehouse liquidity creation. We define the central bank rate R^{CB} as the (gross) rate at which warehouses can deposit with the central bank.[15] This is analogous to the storage technology of the warehouse yielding return R^{CB}. In this interpretation of the model, grain is central bank money and fake receipts are private money.

We first state the necessary analogs of the parameter restrictions in Subsection 3.5. Note that they coincide with Parameter Restriction 1 and Parameter Restriction 2 when $R^{\mathrm{CB}} = 1$, as in the baseline model.

Parameter Restriction 1′. *The farmers' technology is sufficiently productive,*

$$A > \frac{1}{R^{\mathrm{CB}}} + \frac{R^{\mathrm{CB}}}{\alpha}. \tag{42}$$

Parameter Restriction 2′. *Depreciation from private storage is not too fast,*

$$A\left(R^{\mathrm{CB}} - 1 + \delta\right) < 1. \tag{43}$$

The preliminary results of Subsection 4.1 lead to the natural modifications of the prices. In particular, due to competition in the deposit market, the deposit rates equal the central bank rate. Further, because laborers earn interest on their deposits, they accept lower wages. We now summarize these results in Lemma 8.

Lemma 8. *When warehouses earn the central bank rate R^{CB} on deposits, in equilibrium, the deposit rates, lending rate, and wage are as follows:*

$$R_0^D = R_1^D = R^L = R^{\mathrm{CB}} \tag{44}$$

and

$$w = \left(R^{\mathrm{CB}}\right)^{-2}. \tag{45}$$

The crucial takeaway from the result is that the warehouse pays a higher deposit rate when the central bank rate is higher. This means that the farmer's incentive constraint takes into account a higher return from depositing in a warehouse, but the

[15] We are considering a rather limited aspect of central bank monetary policy here, thereby ignoring things like the role of the central bank in setting the interest rate on interbank lending, as in Freixas, Martin, and Skeie (2011), for example.

same depreciation rate from private storage. Formally, with the central bank rate R^{CB}, the farmers' incentive constraint at Date 1 reads

$$(1-\delta)y \leq R^{\text{CB}}\big(y - R^{\text{CB}}B\big) \tag{46}$$

or

$$B \leq \frac{1}{R^{\text{CB}}}\left(1 - \frac{1-\delta}{R^{\text{CB}}}\right)y. \tag{47}$$

Observe that whenever farmers are not too highly levered—$B < y\left(2R^{\text{CB}}\right)^{-2}$—increasing R^{CB} loosens the incentive constraint. The reason is that it makes warehouse storage relatively more attractive at Date 1, inducing farmers to repay their debt rather than to divert capital. (The reason that increasing R^{CB} does not loosen the constraint when B is high, it that it also increases the lending rate between Date 0 and Date 1.)

Proposition 9. *A tightening of monetary policy (an increase in R^{CB}) increases liquidity creation Λ as long as $\alpha + 2R^{\text{CB}}(1-\delta) > \left(R^{\text{CB}}\right)^2$ (otherwise it decreases liquidity creation).*

This contrasts with the established idea that low interest rates (easy money policy) stimulate bank lending and help the economy recover from a recession.[16] In our model, *high interest rates allow banks to lend more* because they loosen the farmers' incentive compatibility constraint by giving warehouses a greater advantage in storage. This result complements Corollary 3, that says that liquidity creation is increasing in the depreciation rate δ. Both results say that the better warehouses are at storing grain relative to farmers, the more warehouses can lend.

8 Conclusion

Summary of paper. In this paper we have developed a new theory of banking that is tied to the origins of banks as commodity warehouses. The *raison d'être* for banks does not require asymmetric information, screening, monitoring or risk aversion. Rather, we show that the institutions with the best storage (warehousing) technology have an advantage in enforcing contracts, and are therefore not only natural deposit-takers but are also natural lenders—i.e. they are natural banks. With this theory we show how banks create liquidity even when they do *not* provide superior risk sharing. While the existing literature views bank liquidity creation as being synonymous with improved risk sharing for risk-averse depositors, we focus on ex ante *funding* liquidity creation, which is the bank-attributed increase in the initially available liquidity that can be channeled

[16]See, for example, Keeton (1993). Mishkin (2010) provides a broad assessment of monetary policy, bank lending, and the role of the central bank.

into aggregate investment in productive activities. The key to the bank's ability to do this is the issuance of "fake" warehouse receipts by the bank. This creates a striking contrast with the existing literature, which views the process of liquidity creation as banks accepting deposits that are then loaned out, i.e., deposits create loans. In our theory, loans also create deposits. We thus decouple the notion of creating liquidity from risk preferences and show that risk aversion is neither a sufficient nor necessary condition for ex ante liquidity creation. In this way, our analysis of bank liquidity creation *complements* the focus of the existing literature on the bank creating liquidity through better risk sharing.

Our theory has regulatory implications. It shows that proposals like narrow banking and liquidity requirements on banks will diminish bank liquidity creation and be inimical to economic growth. By contrast, higher levels of bank capital will enhance bank liquidity creation. Moreover, we establish conditions under which a tighter monetary policy induces more liquidity creation.

Empirical implications. Our paper generates numerous predictions that could be tested. First, across countries, banks that provide warehousing services should play a more important role in countries with weaker property rights. Second, more aggregate funding liquidity will be created in the economy when banks have higher capital. Third, liquidity requirements on banks will reduce aggregate liquidity creation.

Future research. We view this paper as a first step in the direction of formalizing the funding liquidity creation role of banks in the context of safeguarding. Much remains to be done. A few extensions come immediately to mind. One is to investigate the effects of frictions in the interbank loan market. Another is to examine the implications of alternative production technologies for farmers, e.g. Cobb–Douglas. Finally, we have not considered the potential bank fragility associated with the creation of liquidity in our model of warehouse banking, an issue that will create a natural role for a central bank, and one that should be addressed in future research.[17] But our paper emphasizes that the creation of funding liquidity is not one that inherently generates fragility—other ingredients need to be added to the recipe for that to happen.

[17]See Goodhart (2010) for a thorough discussion of the role of central banks and how it is changing.

A Appendix

A.1 Proofs

A.1.1 *Proof of Lemma 1*

We show the result by contradiction. If $R_t^D \neq 1$ in equilibrium, deposit markets cannot clear.

First suppose (in anticipation of a contradiction) that $R_t^D < 1$ in equilibrium (for either $t \in \{0, 1\}$). Now set $s_t^b = D_t$ in the warehouse's problem in Subsection 3.3. The warehouse's objective function (equation (8)) goes to infinity as $D_t \to \infty$ without violating the constraints. The deposit markets therefore cannot clear if $R_t < 1$, a contradiction. We conclude that $R_t^D \geq 1$.

Now suppose (in anticipation of a contradiction) that $R_t^D > 1$ in equilibrium (for either $t \in \{0, 1\}$). Now set $s_t^b = D_t$ in the warehouses problem. The warehouse's objective function goes to infinity as $D_t \to -\infty$ without violating the budget constraints. Thus, if $R_t^D > 1$, it must be that $D_t = 0$. However, since the depreciation rate $\delta > 0$, the demand from laborers and farmers to store grain is strictly positive for $R_t^D > 1 - \delta$. Thus, again, deposit markets cannot clear, a contradiction. We conclude that $R_t^D \leq 1$.

The two contradictions above taken together imply that $R_t^D = 1$ for $t \in \{0, 1\}$. □

A.1.2 *Proof of Lemma 2*

We show the result by contradiction. If $R^L \neq 1$ in equilibrium, loan markets cannot clear.

First suppose (in anticipation of a contradiction) that $R^L > 1$ in equilibrium. Now set $L = D_t$ in the warehouse's problem in Subsection 3.3. Given that $R_0^D = 1$ from Lemma 1 above, the warehouse's objective function (equation 8) goes to infinity as $L \to \infty$ without violating the constraints. The deposit markets therefore cannot clear if $R^L > 0$, a contradiction. We conclude that $R^L \leq 1$.

Now suppose (in anticipation of a contradiction) that $R^L < 1$ in equilibrium. Now set $L = D_0$ in the warehouse's problem. Given that $R_0^D = 1$ from Lemma 1 above, the warehouse's objective function goes to infinity as $L \to -\infty$ without violating the budget constraints. Thus, if $R^L < 1$, it must be that $D_0 = 0$. However, since the depreciation rate $\delta > 0$, the demand from laborers and farmers to store grain is always strictly positive for $R_t^D > 1 - \delta$. Thus, again, deposit markets cannot clear, a contradiction. We conclude that $R^L \geq 1$.

The two contradictions above taken together imply that $R^L = 1$. □

A.1.3 Proof of Corollary 1

Given Lemma 1 above, the result is immediate from inspection of the farmer's problem and the laborer's problem in Subsection 3.3 given that $R_0^D = R_1^D = 1 > 1 - \delta$, the return from private storage. $\square$

A.1.4 Proof of Lemma 3

We show the result by contradiction. If $w \neq 1$ in equilibrium, labor markets cannot clear.

First suppose (in anticipation of a contradiction) that $w > 1$ in equilibrium. From Corollary 1, $d_0^l = w\ell^l$ and $d_1^l = R_0^D d_0^l$ in the laborer's problem in Subsection 3.3. The constraints collapse, and the laborer's objective function (equation (10)) is $R_1^D R_0^D w\ell^l - \ell^l = (w-1)\ell^l$, having substituted $R_0^D = R_1^D = 1$ from Lemma 1 above. Since $w > 1$ by supposition, the objective function approaches infinity as $\ell^l \to \infty$ without violating the constraints. The labor market therefore cannot clear if $w > 1$, a contradiction. We conclude that $w \leq 1$.

Now suppose (in anticipation of a contradiction) that $w < 1$ in equilibrium. As above, the laborer's objective function is $(w-1)\ell^l$. Since $w < 1$ by supposition, the laborer sets $\ell^l = 0$. The farmer, however, always has a strictly positive demand for labor if $w < 1$—he produces nothing without labor and his productivity $A > 1 + 1/\alpha$ by Parameter Restriction 1. The labor market therefore cannot clear if $w < 1$, a contradiction. We conclude that $w \geq 1$.

The two contradictions above taken together imply that $w = 1$. $\square$

A.1.5 Proof of Lemma 4

The result follows immediately from the proofs of Lemma 1, Lemma 2, and Lemma 3, which pin down the prices in the model by demonstrating that if prices do not make these players indifferent, markets cannot clear, contradicting equilibrium. $\square$

A.1.6 Proof of Lemma 5

The result follows from Lemma 4 and substituting in prices and demands from the preliminary results in Subsection 4.1. In short, since, given the equilibrium prices, laborers and warehouses are indifferent among allocations, they will take on the excess demand left by the farmers to clear the market. $\square$

A.1.7 *Proof of Proposition 1*

We begin by rewriting the farmer's problem in Lemma 5 as

$$\text{maximize} \;\; d_1^f \tag{48}$$

subject to

$$\delta\Big(A \max\left\{\alpha i, \ell^f\right\} + d_0^f\Big) \geq B, \tag{IC}$$

$$d_1^f + B = A \max\left\{\alpha i, \ell^f\right\} + d_0^f, \tag{BC_1^f}$$

$$d_0^f + i + \ell^f = e + B, \tag{BC_0^f}$$

and $i \geq 0, \ell^f \geq 0, B \geq 0, d_0^f \geq 0$ and $d_1^f \geq 0$.

Now observe that at the optimum, $\max\left\{\alpha i, \ell^f\right\} = \ell^f$ and $\ell^f = \alpha i$. Further, eliminate the d_1^f of the objective from the budget constraint. Now we can write the problem as

$$\text{maximize} \;\; A\ell^f + d_0^f - B \tag{49}$$

subject to

$$\delta\Big(A\ell^f + d_0^f\Big) \geq B, \tag{IC}$$

$$d_0^f + i + \ell^f = e + B, \tag{BC_0^f}$$

$$\ell^f = \alpha i \tag{50}$$

and $i \geq 0, \ell^f \geq 0, B \geq 0$, and $d_0^f \geq 0$.

We see that the budget constraint and $\ell^f = \alpha i$ imply that

$$B = d_0^f + \frac{1+\alpha}{\alpha}\ell^f - e \tag{51}$$

and, thus, the objective is

$$A\ell^f - \frac{1+\alpha}{\alpha}\ell^f + e = \frac{\alpha(A-1)-1}{\alpha}\ell^f + e. \tag{52}$$

This is increasing in ℓ^f by Parameter Restriction 1, so ℓ^f is maximal at the optimum. Thus, the incentive constraint binds, or

$$\delta\big(A\ell^f + d_0^f\big) = B = d_0^f + \frac{1+\alpha}{\alpha}\ell^f - e. \tag{53}$$

or

$$e - (1-\delta)d_0^f = \left(1 - \delta A + \frac{1}{a}\right)\ell^f. \tag{54}$$

Since, by Parameter Restriction 2, $\delta A < 1$, setting $d_0^f = 0$ maximizes ℓ^f. Hence,

$$\ell^f = \frac{\alpha e}{1 + \alpha\big(1 - \delta A\big)}. \tag{55}$$

Combining this with the budget constraint and the equation $i = \ell^f/\alpha$ gives the expressions in the proposition. □

A.1.8 Proof of Proposition 2

The equilibrium allocation again solves the farmer's problem, but in this case the warehouse cannot issue more receipts than has deposits so $s_0^b \geq 0$. Further, since the warehouse has no endowment, its budget constraint reads $L + s_0^b = D_0$. Thus, $L = 0$. Market clearing implies $B = 0$. The farmer's problem is thus to

$$\text{maximize} \;\; d_1^f \tag{56}$$

subject to

$$d_1^f = A \max\Big\{\alpha i, \ell^f\Big\}, \tag{57}$$

$$i + \ell^f = e. \tag{58}$$

The solution to the problem is

$$\ell_{\text{nr}} = \frac{\alpha e}{1+\alpha}, \tag{59}$$

$$i_{\text{nr}} = \frac{e}{1+\alpha}, \tag{60}$$

as expressed in the proposition. □

A.1.9 Proof of Proposition 3

As discussed in the text preceding the statement of the proposition, in the first-best all grain is invested in its first-best use at Date 0. This corresponds to $i_{\text{fb}} = e$, since the farmer's technology is the most productive. The production function requires $\ell_{\text{fb}} = \alpha i = \alpha e$ units of labor to be productive, and any more is unproductive. In summary, $i_{\text{fb}} = e$ and $\ell_{\text{fb}} = \alpha e$, as stated in the proposition. □

A.1.10 Proof of Proposition 4

The result follows immediately from comparison of the equilibrium expression for $i + w\ell$ given in Proposition 1 with the expression for $i_{\text{nr}} + w\ell_{\text{nr}}$ given in Proposition 2. Note that $w = 1$ in the benchmark with no receipts as well as in the full model. The proofs of the results for the prices (in particular for the wage w) in Subsection 4.1 are unchanged for the benchmark. $\square$

A.1.11 Proof of Corollary 2

The result follows from direct calculation given the equilibrium expressions for Λ, D_0 and s_0^b. $\square$

A.1.12 Proof of Corollary 3

The result is immediate from differentiation, as expressed in equation (27). $\square$

A.1.13 Proof of Proposition 5

The expression given in the proposition is positive as long as $1 - \delta A > 0$. This holds by Parameter Restriction 2. The result follows immediately. $\square$

A.1.14 Proof of Proposition 6

The liquidity ration inhibits liquidity creation whenever warehouses' equilibrium Date 0 grain holdings s_0^b are insufficient to satisfy their liquidity requirements. In other words, given equation (31), if

$$B < \frac{1-\theta}{\theta} s_0^b, \tag{61}$$

then liquidity requirements inhibit liquidity creation. Given the equilibrium values of s_0^b and B, this can be rewritten as

$$\frac{\delta A \alpha e}{1 + \alpha\big(1 - \delta A\big)} < \frac{1-\theta}{\theta} \frac{\alpha\big(1 - \delta A\big)e}{1 + \alpha\big(1 - \delta A\big)}. \tag{62}$$

This holds only if

$$\theta < 1 - \delta A. \tag{63}$$

Whenever this inequality is violated, liquidity requirements inhibit liquidity requirements. It is the negation of the above equality stated in the proposition. $\square$

A.1.15 Proof of Lemma 6

The result is immediate from maximizing the objective function in equation (35). $\square$

A.1.16 Proof of Lemma 7

The proof is identical to those of the lemmata in Subsection 4.1. $\square$

A.1.17 *Proof of Proposition 8*

In order to show that increasing warehouse equity E increases liquidity creation Λ, we first characterize the equilibrium in the extended model of Subsection 7.2. This amounts to solving an analogous version of the farmer's problem stated in Lemma 5. Specifically, the equilibrium allocation maximizes d_1^f subject to the modified incentive constraint in equation (40) and the budget constraints. This reduces to solving the system of equations

$$B = \left(1 - \frac{\gamma(1-\delta)}{E}\right) y, \tag{64}$$

$$\ell^f = \alpha i, \tag{65}$$

$$e + B = i + \ell^f \tag{66}$$

where $y = A\ell$. The solution of the system gives

$$B = \frac{(1-\gamma(1-\delta)/E)\, A\alpha e}{1+\alpha-\alpha A\,(1-\gamma(1-\delta)/E)} \tag{67}$$

$$\ell^f = \frac{\alpha e}{1+\alpha-\alpha A\,(1-\gamma(1-\delta)/E)}, \tag{68}$$

$$i = \frac{e}{1+\alpha-\alpha A\,(1-\gamma(1-\delta)/E)} \tag{69}$$

Note that this allocation coincides with the allocation in Proposition 1 when warehouses exert sufficient effort to prevent spoilage, that is when $\Delta = 1$ or $E = \gamma$.

Given the above equilibrium allocation, the liquidity multiplier is given by

$$\Lambda = \frac{i + w\ell}{e} = \frac{1+\alpha}{1+\alpha-\alpha A\,(1-\gamma(1-\delta)/E)}, \tag{70}$$

which is increasing in E, as desired. □

A.1.18 *Proof of Lemma 8*

The proofs that $R_0^D = R_1^D = R^L = R^{\text{CB}}$ are all identical to the proofs of the analogous results in Subsection 4.1 with the warehouses' return on storage (which is one in the baseline model) replaced with the central bank rate R^{CB}. The result is simply that warehouses lend and borrow at their cost of storage, which is a result of warehouses being competitive.

The result that $w = \left(R^{\text{CB}}\right)^{-2}$ is also nearly the same as the proof of the analogous result (Lemma 3) in Subsection 4.1. The modification is that the laborer's objective function (equation (10)) reduces to $c^l = \left(R^{\text{CB}}\right)^2 w\ell - \ell$, since the laborer invests its income in the warehouse for two periods at gross rate R^{CB}. In order for the laborer not to supply infinite (positive or negative) labor ℓ, it must be that $w = \left(R^{\mathrm{CB}}\right)^{-2}$.

A.1.19 *Proof of Proposition 9*

Solving for the equilibrium again reduces to solving the farmer's problem with binding incentive and budget constraints. With the prices given in Lemma 8 these equations are

$$R^{\mathrm{CB}}\left(y - R^{\mathrm{CB}}\right) = (1-\delta)y \tag{71}$$

and

$$i + \left(R^{\mathrm{CB}}\right)^{-2}\ell^f = e + B \tag{72}$$

where $y = A\max\{\alpha i, \ell\}$ and, in equilibrium, $i = \alpha\ell^f$. From the budget constraint we find that

$$\ell^f = \frac{\alpha\left(R^{\mathrm{CB}}\right)^2(e+B)}{\alpha + (R^{\mathrm{CB}})^2} \tag{73}$$

and, combining the above with the incentive constraint,

$$B = \frac{\alpha A\left(R^{\mathrm{CB}} - 1 + \delta\right)e}{\alpha + (R^{\mathrm{CB}})^2 - \alpha A\left(R^{\mathrm{CB}} - 1 + \delta\right)}. \tag{74}$$

This gives the following equilibrium allocation:

$$\ell = \frac{\alpha\left(R^{\mathrm{CB}}\right)^2 e}{\alpha + (R^{\mathrm{CB}})^2 - \alpha A\left(R^{\mathrm{CB}} - 1 + \delta\right)}, \tag{75}$$

$$i = \frac{\left(R^{\mathrm{CB}}\right)^2 e}{\alpha + (R^{\mathrm{CB}})^2 - \alpha A\left(R^{\mathrm{CB}} - 1 + \delta\right)}. \tag{76}$$

We use the allocation to write down the liquidity multiplier Λ as

$$\Lambda = \frac{i + w\ell}{e} \tag{77}$$

$$= \frac{\alpha + \left(R^{\mathrm{CB}}\right)^2}{\alpha + (R^{\mathrm{CB}})^2 - \alpha A\left(R^{\mathrm{CB}} - 1 + \delta\right)}. \tag{78}$$

We now compute the derivative of Λ with respect to R^{CB} to show when increasing R^{CB} increases Λ:

$$\frac{\partial\Lambda}{\partial R^{\mathrm{CB}}} = \frac{2R^{\mathrm{CB}}\left[\alpha + \left(R^{\mathrm{CB}}\right)^2 - \alpha A\left(R^{\mathrm{CB}} - 1 + \delta\right)\right] - \left(\left(R^{\mathrm{CB}}\right)^2 + \alpha\right)\left(2R^{\mathrm{CB}} - \alpha A\right)}{\left[\alpha + (R^{\mathrm{CB}})^2 - \alpha A\left(R^{\mathrm{CB}} - 1 + \delta\right)\right]}$$

$$= \frac{\alpha + 2(1-\delta)R^{\mathrm{CB}} - \left(R^{\mathrm{CB}}\right)^2}{\left[\alpha + (R^{\mathrm{CB}})^2 - \alpha A\left(R^{\mathrm{CB}} - 1 + \delta\right)\right]^2}.$$

This is positive exactly when $\alpha + 2R^{\text{CB}}(1-\delta) > \left(R^{\text{CB}}\right)^2$ as stated in the proposition.

A.2 The Interbank Market and the Incentive Constraint

The observation that a warehouse's superior storage technology also serves as a superior enforcement technology is at the center of our theory of warehouse banking. The key to our argument is that farmers want to deposit their grain in warehouses to prevent depreciation and that a warehouse holding a deposit can seize it. In Subsection 3.1, we argued that this mechanism is robust to the possibility that a farmer may switch warehouses, i.e. borrow from one warehouse at Date 0 and deposit in a different warehouse at Date 1. The reason is that an interbank debt market ensures that whatever warehouse he deposits in, the warehouse will always buy his debt and seize his grain. We explore this in more detail and show that for *any* reasonable interbank market price and any positive cost of switching warehouses, a farmer will always strictly prefer to deposit with the warehouse he borrowed from (if the switching cost is zero, he still weakly prefers to deposit in the warehouse he borrowed from). We include this as a separate argument outside the baseline model because the analysis is game theoretic, while our solution concept in the baseline model is competitive equilibrium.

Consider a farmer with grain g and outstanding debt with face value $F < g$ to a warehouse, called Warehouse 1. Assume that deposit rates are one (this is a result of competition in the full model, stated in Lemma 7). The farmer can deposit his grain in Warehouse 1 or in a different warehouse, Warehouse 2. We assume that if the farmer deposits in Warehouse 2 he bears a switching cost ε. After the farmer has deposited in a warehouse, Warehouse 1 may sell the farmer's debt to Warehouse 2 at an exogenous price p. If a warehouse has both the farmer's debt and his deposit, the warehouse may seize an amount F of the farmer's deposit; otherwise, the warehouse with the farmer's debt does not collect. Finally, the warehouse that has accepted the deposit repays the farmer (net of any seized grain). We focus on $\varepsilon > 0$ and $p \in (0, F)$. Formally, the timing is as follows.

1. The farmer deposits g in Warehouse 1 or Warehouse 2
2. Warehouse 1 sells the farmer's debt to Warehouse 2 or does not
3. If Warehouse 1 or Warehouse 2 has both the debt and the deposit, it seizes an amount F of the deposit
4. The warehouse holding the deposit repays the farmer's deposit (net of seized grain)

Note that in the game there are only two choices: first the farmer chooses Warehouse 1 or Warehouse 2 and second Warehouse 1 chooses to sell or not to sell. We have assumed seizure and deposit repayment as automatic.

We now proceed to solve the game by backward induction. We first consider the case in which the farmer deposits in Warehouse 1. In this case, Warehouse 1 gets p if it sells the farmer's debt and F if it does not sell the farmer's debt. Since $F > p$, Warehouse 1 does not sell the farmer's debt. The farmer's payoff from depositing in Warehouse 1 is thus $g - F$.

Now consider the case in which the farmer deposits in Warehouse 2, bearing the switching cost ε. In this case, Warehouse 1 gets p if it sells the farmer's debt and zero if it does not sell the farmer's debt. Since $p > 0$, Warehouse 1 sells the farmer's debt to Warehouse 2. Warehouse 2 now holds both the farmer's deposit and his debt and therefore seizes an amount F. The farmer's payoff from depositing in Warehouse 2 is thus $g - F - \varepsilon$.

Now turn do the farmer's choice of where to deposit. If he deposits in Warehouse 1 he receives $g - F$ and if he deposits in Warehouse 2 he receives $g - F - \varepsilon$. Since $\varepsilon > 0$, the farmer prefers to deposit in Warehouse 1. We state this result in a proposition for emphasis.

PROPOSITION 10. *For any positive switching cost and any positive interbank price less than the face value of debt, the farmer deposits in the warehouse he borrowed from in the subgame perfect equilibrium.*

This result says that a farmer cannot circumvent a warehouse's ability to enforce contracts by depositing in a warehouse different from the one he borrowed from. Note that this result does not depend on the fair pricing of debt in the interbank market, it holds for any price less than the face value of debt. Since the farmer anticipates that no matter the interbank price of his debt, the warehouse he deposits with will ultimately hold his debt and then seize his deposit, the farmer prefers to deposit in the warehouse he borrowed from. A warehouse anticipate this when it makes loans, so the deposit constraint is as described in equation (DC) in Subsection 3.1.

A.3 Connection with the Relending Model

Here we explain how the first-best outcome above has an interpretation in the *relending* model. In the relending model, banks take deposits, they use these deposits to make loans, which are later deposited again, and then lent out again, and so on. That the bank has a deposit before before it makes loans is the "deposits-first" view of bank balance sheets. It contrasts with the "loans-first" perspective that we have stressed thus far. In this loans-first view, banks create deposits (viz. fake receipts) when they make loans. We show below that relending model yields the same first-best outcome as stated in Proposition 3, implying that there is not necessarily a conflict between the deposits-first and loans-first perspectives.

In relending model, there is a *money multiplier* which describes how much money the banking system creates from an initial deposit. In textbook treatments the money multiplier is the reciprocal of the reserve requirement, but we demonstrate below that in our model it depends on the production technology.

Here we consider a sequential view of the equilibrium, analogous to Walrasian tâtonnement, in which each player acts optimally in sequence of rounds. In Round n, farmers divide their grain e_n between capital investment i_n and labor investment $w\ell_n$. Since the farmers' production function is $A \min\{\alpha i, \ell\}$, they set $\alpha i_n = \ell_n$ in each round. Further, since wages are one, $i_n + \ell_n = e_n$ and $w\ell_n = \ell_n$. Thus,

$$i_n = \frac{e_n}{1+\alpha} \quad \text{and} \quad \ell_n = \frac{\alpha e_n}{1+\alpha}. \tag{79}$$

Laborers then receive wages $w\ell_n = \alpha e_n/(1+\alpha)$ which they deposit directly in warehouses, $d^l_n = \alpha e_n/(1+\alpha)$. Finally, in the first-best, warehouses lend out *all* their grain, $L_n = d^l_n = \alpha e_n/(1+\alpha)$. Round $n+1$ begins with farmers having the borrowed grain $e_{n+1} = L_n$. One round of this process is represented pictorially in Figure 6.

Now, note from the computations above that

$$e_{n+1} = \frac{\alpha e_n}{1+\alpha} \tag{80}$$

From here, we compute e_n recursively in terms of e_0 as

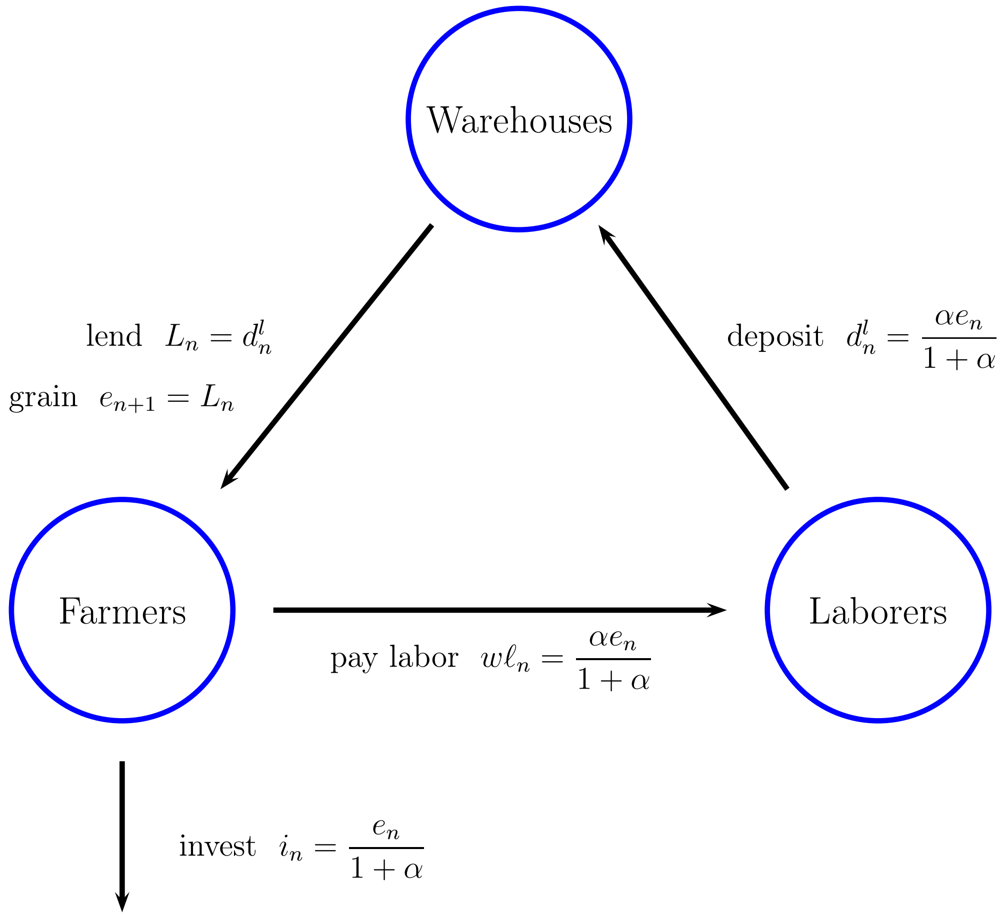
$$e_n = \left(\frac{\alpha}{1+\alpha}\right)^n e_0. \tag{81}$$

We now compute the first-best allocations of capital and labor investment given the relending model. To do so, we sum up the per-round investments i_n and ℓ_n over an infinite number of rounds, given the initial endowment $e_0 = e$, to recover

$$\begin{aligned}
i_{\text{fb}} &= \sum_{n=0}^{\infty} i_n && (82)\\
&= \sum_{n=0}^{\infty} \frac{e_n}{1+\alpha} && (83)\\
&= \sum_{n=0}^{\infty} \frac{1}{1+\alpha}\left(\frac{\alpha}{1+\alpha}\right)^n e && (84)\\
&= e, && (85)
\end{aligned}$$

having used equation (81) and the formula for the sum of a geometric series. Likewise,

Figure 6: A Representation of the Relending Interpretation of the First-best



we find that

$$\ell_{\text{fb}} = \sum_{n=0}^{\infty} \ell_n \tag{86}$$

$$= \sum_{n=0}^{\infty} \frac{\alpha e_n}{1+\alpha} \tag{87}$$

$$= \sum_{n=0}^{\infty} \frac{\alpha}{1+\alpha} \left(\frac{\alpha}{1+\alpha} \right)^n e \tag{88}$$

$$= \alpha e. \tag{89}$$

The expressions for i_{fb} and ℓ_{fb} under the relending model coincide with those in Proposition 3. Further, they suggest a natural money multiplier, stemming from the production function. Of each unit of grain in the economy, a proportion $1/(1+\alpha)$ is invested in capital and cannot be reused. The remainder is invested in labor, or working capital, and therefore deposited back in the warehouse for reuse. These deposits are lend out, and a proportion $1/(1+\alpha)$ is again invested in capital and the remainder is invested in labor, which is deposited and reused. Thus, for every unit of grain, the amount that can be invested is

$$\Lambda_{\text{fb}} = \sum_{n=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^n = 1 + \alpha. \tag{90}$$

The multiplier Λ_{fb} corresponds to a money multiplier in classical banking models. This is the liquidity multiplier of Section 6. Note that the more important working capital (labor ℓ) is to production relative to physical capital (grain i), i.e. the higher is α, the higher is the multiplier. Private money expands the money supply only to create working capital. It cannot, of course, create physical capital out of thin air.

A.4 Table of Notations

	Indices
f	farmer index
l	laborer index
b	warehouse (bank) index
$t \in \{0, 1, 2\}$	time index
	Prices
R_t^D	deposit rate at Date t
R^L	lending rate at Date 0
w	wages at Date 0
	Demand and Supply
i	grain farmers invest at Date 0
ℓ^f	labor farmers demand at Date 0
ℓ^l	labor laborers supply at Date 0
s_t^j	grain stored by player j at Date t
d_t^f	grain deposited in warehouses by farmers at Date t
d_t^l	grain deposited in warehouses by laborers at Date t
B	loans demanded by farmers at Date 0
L	loans supplied by warehouses at Date 0
D_t	overall deposits in warehouse at Date t
	Production and Consumption
y	farmers' output at Date 1
c^j	consumption of player j at Date 2
	Parameters
δ	depreciation rate with private storage
A	productivity
α	ratio of labor to grain in farmers' production
e	farmers' (Date 0) endowment
E	warehouses' (Date 1) endowment (extension in Subsection 7.2)
γ	cost parameter (extension in Subsection 7.2)
	Other Variables
g_1^f	farmer's grain holding at Date 1
Λ	liquidity multiplier
θ	liquidity ratio (extension in Subsection 7.1)
Δ	probability of spoilage (extension in Subsection 7.2)
R^{CB}	central bank rate (extension in Subsection 7.3)

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