WAREHOUSE BANKING*

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Abstract

This paper develops a theory of banking that is rooted in the evolution of banks from warehouses of commodities and precious goods, which occurred even before the invention of coinage or fiat money. The theory helps to explain why modern banks offer warehousing (custodial and deposit-taking) services within the same institutions that provides lending services and how banks create *funding liquidity* by creating private money. In our model, the warehouse endogenously becomes a bank because its superior storage technology—the *raison d'etre* for its existence in the first place—allows it to enforce loan repayment most effectively. The warehouse makes loans by issuing "fake" warehouse receipts—those not backed by actual deposits—rather than by lending out deposited goods. The model provides a rationale for banks that take deposits, make loans, and have circulating liabilities, even in an environment without risk or asymmetric information. Our analysis provides new perspectives on narrow banking, liquidity ratios and reserves requirements, capital regulation, and monetary policy.

> The banks in their lending business are not only not limited by their own capital; they are not, at least immediately, limited by any capital whatever; by concentrating in their hands almost all payments, they themselves create the money required....

<div align="right">Wicksell (1907)</div>

1 Introduction

Banking is an old business. The invention of banking precedes the invention of coinage by several thousand years.[1] Banks evolved from ancient warehouses, where cattle, grain, and precious metals were deposited for storage. For example, in ancient Egypt, grain harvests were "deposited" (or stored) in centralized warehouses and depositors could write orders for the withdrawal of grain as means of payment. These orders constituted some of the earliest paper money. Eventually the warehouses for the safe storage of commodities began making loans, thereby evolving into banks (see Williams (1986) and Lawson (1855), for example). The more recent antecedents of modern-day banks were Venetian goldsmiths.[2] By extending credit, these institutions transformed from simple warehouses of liquidity to creators of liquidity. To this day, the same institutions that provide safekeeping services also engage in the bulk of lending in the economy and are thereby responsible for significant liquidity creation (see, e.g., Berger and Bouwman (2009)). Modern commercial and retail banks keep deposit accounts, provide payment services and act as custodians as well as make corporate and consumer loans.

Historically, why did banks start out as warehouses? And why, still today, do banks offer deposit-taking, account-keeping, payment, and custodial services—namely, warehousing services—within the same institution that provides lending services? How do banks that combine warehousing and lending services create liquidity? And how does banks' creation of private money contribute to this liquidity creation? Finally, what does a theory of banking that addresses these questions have to say about regulatory initiatives like bank capital and liquidity requirements and proposals like narrow banking?

In this paper, we address these questions by developing a theory of banking based on the warehousing function of the bank. In our model, the institutions that provide

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[1]Banking seems to have originated in ancient Mesopotamia and some of the earliest recorded laws pertaining banks (banking regulation) were part of the Code of Hammurabi.

[2]These goldsmiths owned safes that gave them an advantage in safe-keeping. This interpretation is emphasized in He, Huang, and Wright (2005, 2008) as well in many historical accounts of banking, including, for example, the Encyclopedia Britannica, which states that "The direct ancestors of modern banks were the goldsmiths. At first the goldsmiths accepted deposits merely for safe keeping; but early in the 17th century their deposit receipts were circulating in place of money." (1954, vol. 3, p. 41)

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the warehousing services endogenously perform the lending in the economy. The model relies on two key assumptions. First, warehouses have an efficient storage technology.[3] For example, warehouses may prevent spoilage like grain silos in ancient Egypt or may protect against theft like goldsmiths' safes in Medieval Europe. Second, no firm's output is pledgeable,[4] so a debt contract written on a firm's future cash flow is not readily enforceable. This impedes a firm's access to credit. But warehouses use their superior storage technology to circumvent this problem. The reason is as follows. A firm wants to deposit its output in a warehouse to take advantage of the warehouse's efficient storage technolgy. However, once a firm deposits with the warehouse, the deposit can then be seized by the warehouse. Hence, as long as the benefits of warehouse storage (relative to private storage) are high enough, it is incentive compatible for a firm that borrows from a warehouse to repay its debt in order to access the warehouse's storage services. This mechanism explains why the same institutions should provide both the warehousing and lending services in the economy. Empirical evidence supporting this result appears in Skrastins (2015). Using a differences-in-differences research design, Skrastins (2015) documents that agricultural lenders in Brazil extend more credit when they merge with grain silos, i.e. banks lend more when they are also warehouses.
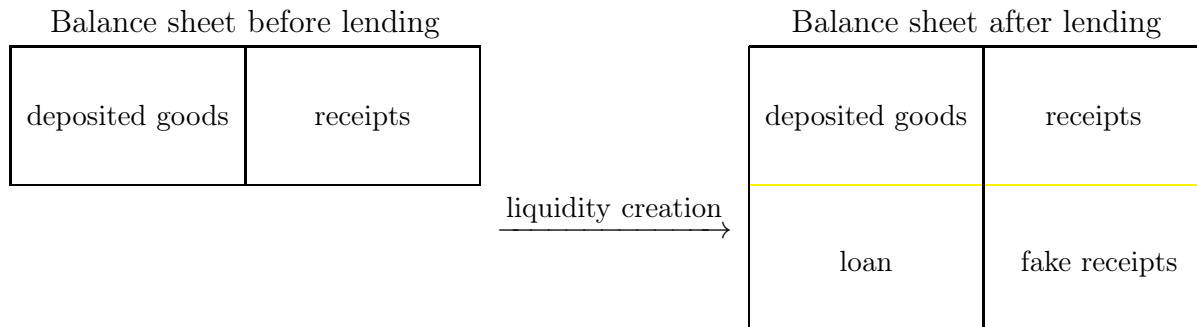
Our model of warehouse banking leads to a new perspective on banks' liquidity creation. In our model, as in ancient Egypt, the receipts that warehouses issue for deposits circulate as a medium of exchange—they constitute private money. This is the first step in the bank's creation of *funding liquidity*, which we define as the initial liquidity that is used to fund productive investments.[5] The second step in the liquidity creation process—and the key reason that banks increase aggregate funding liquidity— is that they make loans in warehouse receipts rather than in deposited goods. When warehouses make loans, they issue new receipts that are not backed by any new deposits. Due to their the lack of deposit-backing, we sometimes refer to these new receipts as *"fake receipts,"* although we emphasize that they are good value IOUs. These fake receipts provide firms with working capital, allowing them to make more productive investments. Thus, when a warehouse-bank makes a loan, it is not reallocating assets on the left-hand side of its balance sheet. Rather, because it is lending by issuing new receipts, it is creating a new liability. Making a loan thus expands both sides of the balance sheet. This is depicted in Figure 1.

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[3]Allen and Gale (1998) also assume that the storage technology available to banks is strictly more productive than the storage technology available to consumers.

[4]See Holmström and Tirole (2011) for a list of "...several reasons why this [non-pledgeability] is by and large reality" (p. 3).

[5]Our use of the term "funding liquidity" resembles that of Brunnermeier and Pedersen (2009) or Holmström and Tirole (1998).

Figure 1: THE WAREHOUSE'S BALANCE SHEET EXPANDS WHEN IT MAKES A LOAN, CREATING LIQUIDITY.

Balance sheet before lending

deposited goods	receipts

liquidity creation $\longrightarrow$

Balance sheet after lending

deposited goods	receipts
loan	fake receipts

The reason that fake warehouse receipts are useful to firms is that they cannot pay their suppliers or laborers on credit, due to the non-pledgeability of their output. However, they can circumvent this problem by borrowing from a warehouse. Warehouse deposits do not suffer from the non-pledgeability problem, so their receipts are readily accepted as payments. Thus, when a warehouse extends credit by issuing new receipts, it creates liquidity for borrowers. In other words, liquidity is created on the asset side, not the deposit side, of the warehouse's balance sheet. When a warehouse makes a loan, it creates both an illiquid asset, the loan, and a liquid liability, the receipt. This is liquidity transformation, one of the fundamental economic roles of banks. Our model suggests, however, that liquidity transformation occurs only when banks make loans. The usual process in which banks first receive deposits of cash, issue liquid (demandable) claims against them to depositors, and then make illiquid loans is turned on its head. The lending that creates liquidity occurs *before* cash is deposited in the bank.

That warehouse-banks make loans by issuing new liabilities is a realistic feature of our model, as applied not only to ancient warehouses, but also as applied to modern banks. Today, when a bank makes a loan, it does not transfer physical currency to the borrower, but it rather creates a deposit in the borrower's name. In other words, a bank loan is an exchange of IOUs: the borrower gives the bank a promise to repay later (the loan) and in exchange the bank gives the borrower a promise to repay later (the deposit). Our model explains how these exchanges of IOUs can create funding liquidity by providing working capital for firms, thereby casting light on why banks lend in private money as opposed to in currency. Our approach in which bank lending creates deposits is reminiscent of Hahn (1920):[6]

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[6]See also Werner (2014) and the "goldsmith anecdote" in Greenbaum, Thakor, and Boot (2015)

We thus maintain—contrary to the entire literature on banking and credit—that the primary business of banks is not the liability business, especially the deposit business, but in general and in each and every case an asset transaction of a bank must have previously taken place, in order to allow the possibility of a liability business and to cause it. The liability business of banks is nothing but a reflex of prior credit extension.... (Hahn, 1920, p. 29)

Keynes makes a related point:

It is not unnatural to think of deposits of a bank as being created by the public through the deposits of cash representing either savings or amounts which are not for the time being required to meet expenditures. But the bulk of the deposits arise out of the action of the banks themselves, for by granting loans, allowing money to be drawn on an overdraft or purchasing securities, a bank creates a credit in its books which is the equivalent of a deposit. (Keynes in his contribution to the Macmillan Committee, 1931, p. 34)

Quinn and Roberds (2014) show empirically that this ability of banks to originate loans in private money has important real consequences. They exploit a 17th century policy change that allowed the Bank of Amsterdam to create unbacked private money. They show that this helped the Bank finance its loans and, further, resulted in the Bank florin becoming the dominant international currency throughout Europe.

Our warehousing view of banking provides some new insights into financial regulatory policy. One proposal is narrow banking. We interpret a narrow bank as an institution that can invest its deposits in only "safe" assets, namely in cash or marketable liquid securities such as sovereign bonds (see Pennacchi (2012)). The proposal forces the separation of the warehousing and lending functions of banks. Our analysis suggests that banks create liquidity only when they perform this dual function, implying that narrow banks create no liquidity. Less extreme proposals, such as the liquidity (reserve) ratio in Basel III, demand that banks invest at least a specified fraction of their assets in cash and marketable liquid securities. These too stifle banks' liquidity creation in our set-up, since they prevent banks from issuing new receipts to expand the supply of liquidity. However, advocates of liquidity requirements typically argue that forcing banks to hold more liquid securities will make the financial system less fragile. The argument is that banks can use liquid securities to withstand shocks to their funding such as bank runs. In an extension, we show that increasing warehouses' liquidity ratios makes them *more* susceptible to runs, note less. The reason is that although warehouses that hold more liquidity can withstand bigger runs, their depositors are more prone to withdraw.

We also include an extension to examine the effect of bank capital (equity). The effect of increasing bank capital contrasts with the effect of increasing bank liquidity.

Increasing bank capital actually *enhances* banks' ability to create funding liquidity by reducing non-pledgeability problems between banks and depositors, making warehousing relatively more efficient.

We also extend the model to include a central bank and argue that a higher policy rate does not always lead to lower liquidity creation. We establish conditions under which such a policy can actually *encourage* lending by warehouse-banks. In other words, "tighter" monetary policy can loosen credit in some circumstances.

Among other issues, our paper addresses questions related to the *raison d'etre* of banks, the identity of bankers and the role of circulating bank liabilities. A paper that also addresses these questions is Gu, Mattesini, Monnet, and Wright (2013). They show that players that have more ability to commit to repay depositors endogenously emerge as banks, in the sense that they make delegated investments and their liabilities circulate to facilitate payments among other players. In our model, we go one step further by asking not only who should make delegated investments, but also who should make loans. In our model, a bank is an intermediary between a depositor and a borrower, not only between a depositor and an investment technology. Unlike in Gu, Mattesini, Monnet, and Wright (2013), our focus is mainly on the ability of banks to enforce contracts with borrowers, rather than on their ability to commit to repay depositors (except in the extension in Subsection 7.3, which is all about banks' limited commitment in connection with bank capital).

Gu, Mattesini, Monnet, and Wright (2013) take a mechanism design approach to explaining why banks exist. In other words, they show that banks are necessary to implement the best incentive-feasible allocation. We apply this approach to our environment in Subsection 7.5. There, we show that if warehouses act as banks and they can trade loans in an interbank market, then warehouse-banking implements the best incentive-feasible allocation. Thereby, we provide a rationale for banks and markets together.

Another paper that emphasizes the circulation of bank liabilities is Kiyotaki and Moore (2001). In that paper, someone with verifiable collateral (specifically, a "Scottish laird...[who's] castle is publicly visible" (p. 22)) becomes a banker. This is because his collateral guarantees his liabilities, allowing them to circulate freely. Thus, Kiyotaki and Moore emphasize banks' advantage in taking deposits and creating private money, but, unlike us, do not analyze banks' advantage in enforcing loans.

In our model, warehouses endogenously function as financial intermediaries. In contrast to most of the contemporary literature explaining why banks exist, there is no asymmetric information or risk in our model—the warehouses' function as intermediaries results entirely from their superior storage technology.[7] Specifically, they have *no*

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[7]Thus, the contract between the bank and its depositors does not confront an incentive problem that

superior ability to screen or to monitor loans in an environment of asymmetric information, as in Diamond (1984) and Ramakrishnan and Thakor (1984). Further, because we assume that all agents are risk neutral, banks also do not provide better risk sharing for risk-averse depositors as in Bryant (1980) and Diamond and Dybvig (1983). Technological and financial developments have diminished informational frictions and provided alternatives to banks for risk-sharing (see the discussion in Coval and Thakor (2005)). This should have led to a decline in financial intermediaries' share of output (and corporate profits) in developed economies, but their financial sectors have continued to grow. This suggests that other forces also determine the demand for banking services; we suggest that warehousing-type financial services may be one important determinant, one linked with the very origins of banking. In fact, the largest deposit bank in the world today, Bank of New York–Mellon, is usually classified as a custodian bank, i.e. an institution responsible for the safeguarding, or warehousing, of financial assets. Further, in less developed markets lending services are linked with storage services for commodities.[8]

Our paper is also related to the literature in which bank liquidity creation is linked to the provision of consumption insurance. Important contributions include Allen and Gale (1998), Allen, Carletti, and Gale (2014), Bryant (1980), Diamond and Dybvig (1983), and Postlewaite and Vives (1987). We view our paper as offering a view of bank liquidity creation that *complements* the consumption insurance view in the existing literature, a view that has provided deep insights into a variety of phenomena like bank runs and deposit insurance (Bryant (1980) and Diamond and Dybvig (1983)), financial crises (e.g. Allen and Gale (1998)), and the role of financial intermediaries *vis-à-vis* markets (e.g. Allen and Gale (2004)). Juxtaposing our analysis with the existing literature, liquidity creation is seen to have two important dimensions: consumption insurance for depositors/savers and elevated funding liquidity for entrepreneurs/borrowers. These mutually-reinforcing views of bank liquidity creation are consistent with the idea that banks create liquidity by both taking in deposits and selling loan commitments (e.g. Kashyap, Rajan, and Stein (2002)).[9]

Two recent papers that emphasize that banks create money (or deposits) when they lend are Bigio and Bianchi (2015) and Jakab and Kumhof (2015). As an incremental contribution to Bigio and Bianchi (2015), we micro-found why warehouse receipts (bank deposits) are used as a means of payment rather than real goods. As an incremental contribution to Jakab and Kumhof (2015), we explain why banks' lending by creating

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needs to be solved by contract design as, for example, in Calomiris and Kahn (1991).

[8]For example, see Skrastins (2015).

[9]Our paper is also related to papers in which debt serves as inside money. For example, Kahn and Roberds (2007) develop a model that shows the advantage of circulating liabilities (transferable debt) over simple chains of credit. Townsend and Wallace (1987) develop a model of pure intertemporal exchange with informationally separated markets to explain the role of circulating liabilities in exchange.

deposits increases aggregate output without assuming that money is a direct input in the production function.

The rest of the paper is organized as follows. Section 2 provides an example in a simplified set-up in which all the key forces of the model are at work. Section 3 develops the formal model. Section 5 contains the solution of the model. Section 4 solves two benchmark models: (i) one in which warehouses cannot issue fake receipts and (ii) the first-best allocation. Section 6 contains the main results. It presents analysis of liquidity creation and fractional reserves. Section 7 considers the welfare implications of four policies: liquidity requirements, narrow banking, capital requirements, and monetary policy. In that section we also analyze our environment from the point of view of mechanism design. Section 8 is the Conclusion. The appendix contains a formal analysis of the interbank market, an alternative analysis of the role of warehouse-bank equity, and a study of the relending model and money multiplier in connection with our environment. It also contains all proofs and a glossary of notation.

2   Motivating Example

In this subsection, we write a numerical example that illustrates the main mechanism at work in a simplified setup. We write the example with just three players, one farmer, one laborer, and one warehouse. We examine a sequence of increasingly rich cases to demonstrate the efficiency gains from warehousing and from issuing fake receipts. Specifically, we consider: (i) the case without a warehouse, (ii) the case in which a warehouse provides only safe-keeping services but does not lend, (iii) the case in which a warehouse provides both safe-keeping and lending services, and (iv) the first-best case, in which the allocation is efficient.

The analysis of the example shows that, even without lending, warehousing alone increases efficiency by providing more efficient storage. This efficiency gain is merely technological, however; introducing a better storage technology increases terminal output. But when a warehouse can issue fake receipts, it does more to improve efficiency—it creates liquidity that the farmer invests productively. This efficiency gain is allocational and is much more important than the simple technological efficiency gain because it involes *other players* in the economy investing in more efficient technologies. Finally, the analysis of the first-best allocation suggests that there is an efficiency loss in the second best in that even when the warehouse can issue fake receipts, it still creates less liquidity than in the first-best.

The setup of the example is as follows. There are three dates, Date 0, Date 1, and Date 2. The farmer has an endowment e of twelve units of grain at Date 0 and no one else has any grain. At Date 0 the farmer can borrow B from the warehouse at

gross rate one. We assume that warehouse deposit rates and wages w are also all set equal to one.[10] The farmer produces over the period from Date 0 to Date 1 and he stores his output over the period from Date 1 to Date 2. If the farmer stores his grain privately, it depreciates at δ percent, and we choose to set $\delta = 20\%$; if he stores it in a warehouse, it does not depreciate. Suppose there is no discounting, so workers are willing to store grain in warehouses at the deposit rate of one. The farmer's production technology transforms a unit of labor and a unit of grain at Date 0 into four units of grain at Date 1 with constant returns. In other words, the farmer has a Leontief production function in the first period that takes grain investment i, which we will refer to as "capital investment," and labor ℓ and produces output $y = 4 \min \{i, \ell\}$ at Date 1. We assume that this output is not pledgeable; however, a warehouse can seize the deposits it holds. Everyone consumes only at Date 2.

The parameter values are summarized in Figure 2 and the timing is illustrated in Figure 3.

Figure 2: SUMMARY OF NOTATION AND VALUES IN EXAMPLE IN SECTION 2.

Quantity	Notation	Value in Example
Farmer's endowment	e	12
Farmer's technology	y	$4 \min \{i, \ell\}$
Depreciation rate	δ	20%
Wage	w	1

[10]These prices—i.e. rates and wages—result from competition in the full model, leaving rents to the farmer. We take them as given in this example for simplicity.

Date 0

··· the farmer has endowment e and borrows B from the warehouse
 he invests i in grain, and $w\ell$ in labor

··· the laborer exerts labor ℓ and deposits his wages $w\ell$ in the warehouse

Date 1

··· the farmer produces y
 he either repays and deposits or diverts and stores privately

Date 2

··· the farmer, laborer, and warehouse consume
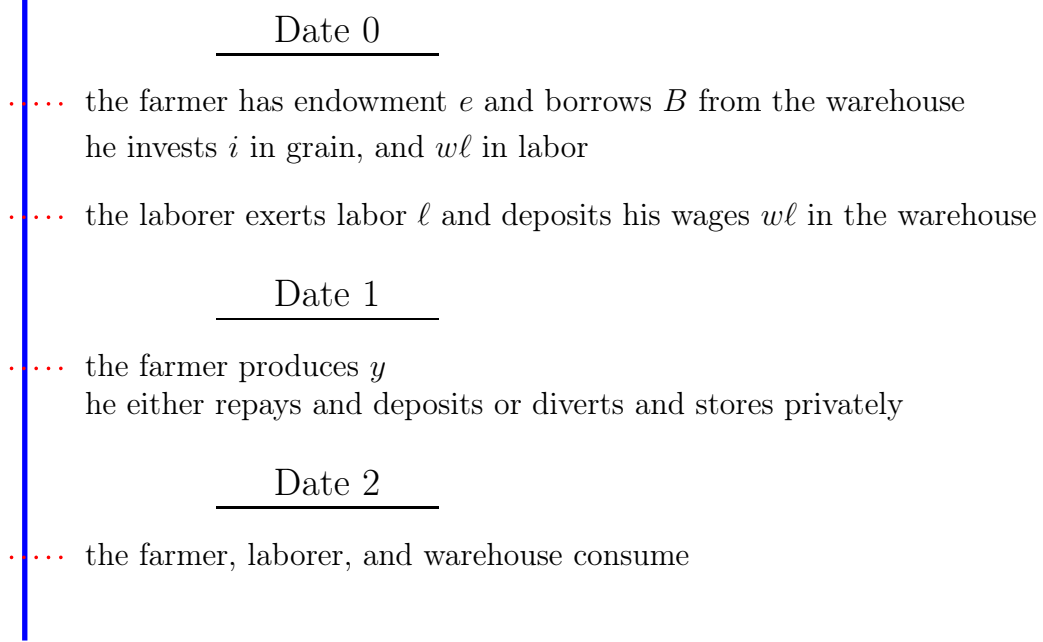
Definition of liquidity: We refer to the farmer's expenditure on capital and labor $i + w\ell$ as the "total investment." We measure liquidity by the ratio Λ of the farmer's total investment to his initial endowment, which we refer to as the *liquidity multiplier*,

$$\Lambda = \frac{i + w\ell}{e}.$$

To see this definition clearly, observe that if the farmer invests only his endowment in production, the liquidity multiplier is one. If the liquidity multiplier is greater than one, that indicates that he has obtained credit (or *outside liquidity* in the sense of Holmström and Tirole (2011)) to scale up his investment. Further, since the farmer has the entire initial grain endowment (no one else has any grain in the model), this measure of outside liquidity is also a measure of total funding liquidity in the model.

No warehousing. Consider first the case in which there is no warehousing. Thus, the farmer must pay the laborer in grain. To maximize his Date 2 consumption, the farmer maximizes his Date 1 output and then stores his output from Date 1 to Date 2. Given that he cannot borrow, his budget constraint reads $i + w\ell = 12$. To maximize his Date 1 output, he invests in equal amounts of capital i and labor ℓ (as a result of the Leontief technology). Since his endowment is twelve and wages are one, he sets $i = \ell = 6$ and produces $y = 4 \times 6 = 24$ units of grain. He then stores his grain privately

from Date 1 to Date 2 and this grain depreciates by twenty percent; the farmer's final payoff is $(1 - 20\%) \times 24 = 19.2$ units. $\Lambda_{\mathrm{nw}} = 1$, so there is no liquidity creation.

Warehousing but no fake receipts. Now consider the case where there is a warehouse, but that it performs only the function of safekeeping. When a depositor (the farmer or the laborer) deposits grain in the warehouse, the warehouse issues receipts and holds the grain until it is withdrawn. In this case, the farmer again maximizes his Date 1 output in order to maximize his Date 2 consumption. Again, he will invest equal amounts of capital and labor. He cannot borrow from the warehouse, so he again just divides his endowment fifty-fifty between capital investment and labor, setting $i = \ell = 6$ and producing $y = 4 \times 6 = 24$ units of grain. He now stores his grain in the warehouse from Date 1 to Date 2. Since it is warehoused, the grain does not depreciate; the farmer's final payoff is 24 units. Warehousing has added 4.8 units to the farmer's consumption by increasing efficiency in storage. But the warehouse has not created any liquidity for the farmer since the initial investment in the technology $i + w\ell = e = 12$ is the same as in the case in which there is no warehouse. There is no liquidity creation, $\Lambda_{\mathrm{nr}} = 1$.

Warehousing with fake receipts. Now consider the case in which there is a warehouse that not only can provide safe-keeping but can also issue fake receipts to make loans. Since the farmer's technology is highly productive, he wishes to borrow to scale it up. But the farmer already holds all twelve units of grain in the economy, so how can he scale up his production even further? The key is that the farmer can borrow from the warehouse in warehouse receipts. Observe that the receipts the warehouse uses to make loans are *not* backed by grain; they are "fake receipts." However, if the laborer accepts payment from the farmer in these fake receipts, they are still valuable to the farmer—they provide him with "working capital" to pay the laborer.

The farmer again sets his capital investment equal to his labor investment, $i = \ell$. Given that he can borrow B in receipts from the warehouse, however, he can now invest a total up to $i + w\ell = e + B$. Thus, recalling that wages w are one, his optimal investment is

$$i = \frac{e + B}{2} = 6 + \frac{B}{2} \tag{1}$$

and the corresponding Date 1 output is

$$y = 4i = 24 + 2B. \tag{2}$$

Given that this technology is highly productive and has constant returns to scale, the farmer wishes to expand production as much as possible. The amount he can borrow from the warehouse, however, is limited by the amount that he can credibly promise to repay. Since we have assumed that the farmer's output is not pledgeable, his creditor

(i.e., the warehouse) cannot enforce the repayment of his debt. However, if the farmer deposits in the warehouse, it is possible for the warehouse to seize the deposit. Thus, after the farmer produces, he faces a tradeoff between not depositing and depositing. If he does not deposit, he stores privately, so his grain depreciates, but he avoids repayment. If he does deposit, he avoids depreciation, but the warehouse can seize his deposit and force repayment. The warehouse lends to the farmer only if repayment is incentive compatible. For the repayment to be incentive compatible, the farmer must prefer to deposit in the warehouse and repay his debt rather than to store the grain privately and default on his debt. This is the case if the following inequality holds

$$y - B \geq (1 - 20\%)y. \tag{3}$$

The maximum the farmer can borrow B is thus given by

$$\left(24 + 2B\right) - B = \left(1 - 20\%\right)\left(24 + 2B\right), \tag{4}$$

or $B = 8$. This corresponds to $i = \ell = 10$.

Observe that intermediation has emerged endogenously. The warehouse lends to the farmer in fake receipts. The farmer then transfers these fake receipts to the laborer. Thus, the laborer has a claim on the warehouse and the warehouse has a claim on the farmer, who has made a productive investment. This is a canonical *banking* arrangement because (i) the warehouse is both debtor to the laborer and creditor to the farmer and (ii) the warehouse's liabilities circulate as a means of payment. In this environment, the warehouse endogenously becomes the bank. This result relies only on the warehouse having a superior storage technology.

The warehouse's superior storage technology allows it to enforce loans because the farmer relies on the warehouse for storage at Date 1. It may seem that this argument depends critically on there being only one warehouse—the idea being that if there were multiple warehouses, the farmer could borrow from one warehouse at Date 0 and then deposit his output in another warehouse at Date 1. This way, the farmer could avoid both repayment and depreciation. However, as long as there is an interbank (or "inter-warehouse") market in which warehouses can trade the farmer's debt, this is not the case. We explain why this is in Subsection 3.2 and in Appendix A.2.

With warehousing and fake-receipts, the liquidity creation is given by

$$\Lambda = \frac{i + w\ell}{e} = \frac{10 + 10}{12} = \frac{5}{3}. \tag{5}$$

The farmer is able to scale up his production only when the warehouse makes loans by writing fake receipts. Hence, liquidity is created on the asset side, not the deposit

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side, of the warehouse's balance sheet, as discussed in the introduction.

If the farmer could pay the labor on credit, he would not need to borrow from the warehouse and he could expand production even further. However, an impediment to this is that the laborer cannot enforce repayment from the farmer (because the output is not pledgeable and the laborer has no way to seize it). Therefore, the farmer's promise to the laborer is not credible.

First-best. We now consider the first best allocation of resources in order to emphasize that the incentive compatibility constraint limits liquidity creation. Allowing warehouses to make loans in fake receipts moves the economy closer to the first-best level of liquidity creation, but does not achieve it. In the first-best allocation the farmer invests his entire endowment in capital $i = e = 12$ and laborers exert equal labor $\ell = 12$. In this allocation output $y = 4i = 48$ and liquidity creation is given by

$$\Lambda_{\text{fb}} = \frac{i + w\ell}{e} = \frac{12 + 12}{12} = 2. \tag{6}$$

Figure 4: SUMMARY OF LIQUIDITY CREATION THE EXAMPLE IN SECTION 2.

Case	Date 1 Output y	Date 2 Output	Liquidity Λ
No warehouses	24	19.2	1
Warehouses without lending	24	24	1
Warehouses with lending	40	40	5/3
First-best	48	48	2

3 Model

In this section, we describe the base model. We also specify the maximization programs of the different agents.

3.1 Time Line, Production Technology and Warehouses

There are three dates, Date 0, Date 1, and Date 2 and three groups of players, farmers, warehouses, and laborers. There is a unit continuum of each type of player. There is one real good, called grain, which serves as the numeraire. There are also receipts issued by warehouses, which entail the right to withdraw grain from a warehouse.

All players are risk neutral and consume only at Date 2. Denote farmers' consumption by c^f, laborers' consumption by c^l, and warehouses' consumption by c^b (the index b stands for "bank"). Farmers begin life with an endowment e of grain. No other player

has a grain endowment. Laborers have labor at Date 0. They can provide labor ℓ at the constant marginal cost of one. So their utility is $c^l - \ell$. Farmers have access to the following technology. At Date 0, a farmer invests i units of grain and ℓ units of labor. At Date 1, this investment yields

$$y = A \min \{\alpha i, \ell\}, \tag{7}$$

i.e. the production function is Leontief. The output y is *not pledgeable*. At Date 1 farmers have no special production technology: they can either store grain privately or store it in a warehouse.

If grain is not invested in the technology, it is either stored privately or stored in a warehouse. If the grain is stored privately (by either farmers or laborers), it depreciates at rate $\delta \in [0, 1)$. If player j stores s_t^j units of grain privately from Date t to Date $t+1$, he has $(1-\delta)s_t^j$ units of grain at Date $t+1$. If grain is stored in a warehouse, it does not depreciate. Further, if grain is stored in the warehouse, the warehouse can seize it.

The assumption that the warehouse can store grain more efficiently than the individual farmer has a natural interpretation in the context of the original warehouses from which banks evolved. These ancient warehouses tended to be (protected) temples or the treasures of sovereigns, so they had more power than ordinary individuals and a natural advantage in safeguarding valuables.[11] In more recent times, warehouses tended to be the safes of goldsmiths, so they had a physical advantage in safeguarding valuables. Note that physical safes play an important role in banking today, even in developed economies. For example, custodians like Clearstream hold physical certificates for all publicly listed companies in Germany. Our assumption on δ can thus be viewed either as a technological advantage arising from specialization acquired through (previous) investment and experience or as a consequence of the power associated with the warehouse.[12]

When players store grain in warehouses, warehouses issues receipts as "proof" of these deposits. The bearers of receipts can trade them among themselves. Warehouses can also issue receipts that are not proof of deposits. These receipts, which we refer to as "fake receipts," still entail the right to withdraw grain from a warehouse, and thus

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[11]We thank Charles Goodhart for this interpretation. Thus, whereas our focus is on private money (fake receipts), the alternative, sovereign-power-linked interpretation of the storage advantage of the warehouse over individuals means that our model may complement the chartalist view of money creation by the state (e.g. Knapp (1924) and Minsky (2008)).

[12]This power was important for several reasons that complement our approach and provide alternative interpretations of the deep parameters in the model. First, it enabled grain, gold, or other valuable commodities to be stored safely, without fear of robbery. Second, power enabled the creditor to impose greater penalties on defaulting borrowers. Third, power also generated a greater likelihood of continuation of the warehouse, and hence of engaging in a repeated game with depositors. This created reputational incentives for the warehouse not to abscond with deposits.

they are warehouses' liabilities that are not backed by the grain they hold. Receipts backed by grain are indistinguishable from fake receipts.

The markets for labor, warehouse deposits, and loans are competitive.

3.2   Financial Contracts

There are three types of contracts in the economy: labor contracts, deposit contracts, and lending contracts. We restrict attention to bilateral contracts, although warehouse receipts are tradeable and loans are also tradeable in an interbank market.

Labor contracts are between farmers and laborers. Farmers pay laborers $w\ell$ in exchange for laborers' investing ℓ in their technology, which then produces $y = y(i, \ell)$ units of grain at Date 1.

Deposit contracts are between warehouses and the other players, i.e., laborers, farmers, and (potentially) other warehouses. Warehouses accept grain deposits with gross rate R_t^D over one period, i.e. if player j makes a deposit of d_t^j units of grain at Date t he has the right to withdraw $R_t^D d_t^j$ units of grain at Date $t + 1$. When a warehouse accepts a deposit of one unit of grain, it issues a receipt in exchange as "proof" of the deposit.

Lending contracts are between warehouses and farmers. Warehouses lend L to farmers at Date 0 in exchange for farmers' promise to repay $R^L L$ at Date 1, where R^L is the lending rate. Warehouses can lend in grain or in receipts. A loan made in receipts is tantamount to a warehouse offering a farmer a deposit at Date 0 in exchange for the farmer's promise to repay grain at Date 1. When a warehouse makes a loan in receipts, we say that it is "issuing fake receipts." We refer to a warehouse's total deposits at Date t as D_t. These deposits include both those deposits backed by grain and those granted as fake receipts.

Lending contracts are subject to a form of limited commitment on the farmers' side. Because farmers' Date 1 output is not pledgeable, they are free to divert their output. However, if the farmers do divert, they must store their grain privately. The reason is that if they deposit their output in a warehouse, it may be seized by the warehouse.

We now formalize how we capture a farmer's inability to divert output if he deposits in a warehouse. We define the variable T as the total transfer from a farmer to a warehouse at Date 1; T includes both the repayment of the farmer's debt to the warehouse and the farmer's new deposit d_1^f in the warehouse. If the farmer has borrowed B at Date 0, then he has to repay $R^L B$ to the warehouse at Date 1. When he makes a transfer T to the warehouse at Date 1, the "first" $R^L B$ units of grain he transfers to the warehouse are used to repay the debt. Only after full repayment of $R^L B$ does the warehouse store grain for the farmer as deposit. Thus, the farmer's deposit at Date 1

is given by

$$d_1^f = T - \min\left\{T, R^L B\right\} = \max\left\{T - R^L B, 0\right\}. \tag{DC}$$

This says that if the farmer has not repaid his debt at Date 1, his Date 1 deposit is constrained to be zero; we call this the *deposit constraint*.

The argument above glosses over one important subtlety. The farmer could borrow from one warehouse at Date 0 and divert his output at Date 1, only to deposit his grain in a different warehouse. Would this allow the farmer to avoid both repaying his debt and allowing his grain to depreciate? The answer is no. The reason is as follows. If the farmer deposits his grain in a warehouse that is not his original creditor, that warehouse will buy the farmer's debt from his original creditor on the interbank market, allowing it to seize the grain the farmer owes. Thus, no matter which warehouse the farmer deposits with, he will end up repaying his debt. We model the interbank market and discuss this reasoning more formally in Appendix A.2.

Since there is no uncertainty, without loss of generality we can restrict attention to lending contracts where default at Date 1 never happens in equilibrium. The farmer will never default on his debt as long as repayment is incentive compatible. In other words, he must prefer to repay his debt and deposit in the warehouse rather than to default on his debt and store the grain privately. If g_1^f denotes the farmer's total Date 1 grain holding[13] and $R^L B$ denotes the face value of his debt, then he repays his debt if the following inequality is satisfied

$$R_1^D\left(g_1^f - R^L B\right) \geq (1-\delta)g_1^f. \tag{IC}$$

The timeline of moves for each player and their contractual relationships are illustrated in a timeline in Figure 5.

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[13]In equilibrium, the farmer's total Date 1 grain holding g_1^f comprises his Date 1 output y, his Date 0 deposits gross of interest, $R_0^D d_0^f$, and his depreciated savings $(1-\delta)s_0^f$, or $g_1^f = y(i, \ell^f) + R_0^D d_0^f + (1-\delta)s_0^f$.

Date 0	Date 1	Date 2
Warehouses	**Warehouses**	**Warehouses**
accept deposits D_0	receive T from farmers	repay $R_1^D D_1$ to depositors
lend L to farmers	accept deposits D_1	consume $c^b = s_1^b - R_1^D D_1$
store s_0^b	repay $R_0^D D_0$ to depositors	
	store s_1^b	
Farmers	**Farmers**	**Farmers**
borrow B from warehouses	receive cash flow $y(i, \ell)$	receive $R_1^D d_1^f$ from warehouses
invest i and ℓ in technology y	transfer T to warehouses	consume $c^f = R_1^D d_1^f + (1 - \delta) s_1^f$
pay laborers $w\ell$	receive $R_0^D d_0^f$ from warehouses	
deposit d_0^f in warehouses	have total grain holding g_1^f	
store s_0^f	deposit d_1^f in warehouses	
	store s_1^f	
Laborers		**Laborers**
exert labor ℓ	**Laborers**	receive $R_1^D d_1^l$ from warehouses
accept wage $w\ell$	receive $R_0^D d_0^l$ from warehouses	consume $c^l = R_1^D d_1^l + (1 - \delta) s_1^l$
deposit d_0^l in warehouses	deposit d_1^l in warehouses	
store s_0^l	store s_1^l	

3.3   Summary of Key Assumptions

In this subsection, we restate and briefly discuss the main assumptions of the model.

ASSUMPTION 1. *Output is not pledgeable.*

This assumption prevents farmers' from paying laborers directly in equity or debt. If farmers' output were pledgeable, they could pay laborers after their projects paid off. Without this assumption, there would be no frictions and we could achieve the first-best allocation. Thus, this assumption creates a role for banks as providers of liquidity.

ASSUMPTION 2. *Warehouses can seize the deposits they hold.*

This assumption implies that a warehouse can enforce repayment from a farmer *as long as that farmer chooses to deposit in it*. Whenever farmers deposit in a warehouse, the warehouse has a seizure technology that allows it to demand repayment from farmers. This is one key ingredient to understand the connection between warehousing/account-keeping and lending. As discussed above and in Appendix A.2, the interbank market for loans prevents farmers from avoiding repayment by depositing in warehouses that are not their creditors.

ASSUMPTION 3. *Grain depreciates relatively more slowly if stored in a warehouse.*

Specifically, we use the normalization that grain does not depreciate inside a warehouse and depreciates at rate $\delta \in (0, 1]$ outside a warehouse.

Assumption 3 gives farmers a reason to deposit in warehouses rather than store grain themselves—farmers deposit in warehouses to take advantage of more efficient storage. This, in turn, ensures that farmers will repay their debt, since warehouses can seize grain owed to them if it is deposited with them, by Assumption 2. Thus, Assumption 2 and Assumption 3 together imply that warehouses have a superior technology to enforce repayment.

3.4 Individual Maximization Problems

All players take prices as given and maximize their Date 2 consumption subject to their budget constraints. Farmers' maximization problems are also subject to their incentive compatibility constraint (IC).

We now write down each player's maximization problem.

The warehouse's maximization problem is

$$\text{maximize} \quad c^b = s_1^b - R_1^D D_1 \tag{8}$$

over s_1^b, s_0^b, D_0, D_1, and L subject to

$$s_1^b = R^L L + s_0^b - R_0^D D_0 + D_1, \tag{BC$_1^b$}$$
$$s_0^b + L = D_0, \tag{BC$_0^b$}$$

and the non-negativity constraints $D_t \geq 0, s_t^b \geq 0, L \geq 0$. To understand this maximization program, note that equation (8) says that the warehouse maximizes its profit (consumption) c^b, which consists of the difference between what is stored in the warehouse at Date 1, s_1^b, and what is paid to depositors, $R_1 D_1$. Equation (BC$_1^b$) is the warehouse's budget constraint at Date 1, which says that what is stored in the warehouse at Date 1, s_1^b, is given by the sum of the interest on the loan to the farmer, $R^L L$, the warehouse's savings at Date 0, the deposits at Date 1 D_1 minus the interest the warehouse must pay on its time 0 deposits, $R_0^D D_0$. Similarly, Equation (BC$_0^b$) is the warehouse's budget constraint at Date 0, which says that the sum of the warehouse's savings at Date 0, s_0^b, and its loans L must equal the sum of the Date 0 deposits, D_0.

The farmer's maximization problem is

$$\text{maximize} \quad c^f = R_1^D d_1^f + (1 - \delta)s_1^f \tag{9}$$

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over $s_1^f, s_0^f, d_0^f, T, i, \ell^f$, and B subject to

$$d_1^f = \max\left\{T - R^L B, 0\right\}, \tag{DC}$$

$$\left(R_1^D - 1 + \delta\right)\left(y(i, \ell^f) + R_0^D d_0^f + (1 - \delta)s_0^f\right) \geq R_1^D R^L B, \tag{IC}$$

$$T + s_1^f = y(i, \ell^f) + R_0^D d_0^f + (1 - \delta)s_0^f, \tag{BC_1^f}$$

$$d_0^f + s_0^f + i + w\ell^f = e + B, \tag{BC_0^f}$$

and the non-negativity constraints $s_t^f \geq 0, d_t^f \geq 0, B \geq 0, i \geq 0, \ell^f \geq 0, T \geq 0$. The farmer's maximization program can be understood as follows. In equation (9) the farmer maximizes his Date 2 consumption c^f, which consists of his Date 1 deposits gross of interest, $R_1^D d_1^f$, and his depreciated private savings, $(1 - \delta)s_1^f$. Equations (DC) and (IC) are, respectively, the deposit constraint and the incentive compatibility constraint (for an explanation see Subsection 3.2). The incentive compatibility constraint follows directly from equation (IC) in Subsection 3.2, since the farmer's Date 1 grain holding g_1^f comprises his Date 1 output y, his Date 0 deposits gross of interest, $R_0^D d_0^f$, and his depreciated savings $(1 - \delta)s_0^f$, or $g_1^f = y(i, \ell^f) + R_0^D d_0^f + (1 - \delta)s_0^f$. Equation ($\text{BC}_1^f$) is the farmer's budget constraint that says that the sum of his Date 1 savings, s_1^f, and his overall transfer to the warehouse, T, must equal the sum of his output y, his Date 0 deposits gross of interest, $R_0^D d_0^f$, and his depreciated savings, $(1 - \delta)s_0^f$. Equation (BC_0^f) is the farmer's budget constraint at Date 0 which says that the sum of his Date 0 deposits, d_0^f, his Date 0 savings, s_0^f, his investment in grain i and his investment in labor, $w\ell^f$, must equal the sum of his initial endowment, e, and the amount he borrows, B.

The laborer's maximization problem is

$$\text{maximize} \quad c^l = R_1^D d_1^l + (1 - \delta)s_1^l - \ell^l \tag{10}$$

over $s_1^l, s_0^l, d_1^l, d_0^l$, and ℓ^l subject to

$$d_1^l + s_1^l = R_0^D d_0^l + (1 - \delta)s_0^l, \tag{BC_1^l}$$

$$d_0^l + s_0^l = w\ell^l, \tag{BC_0^l}$$

and the non-negativity constraints $s_t^l \geq 0, d_t^l \geq 0, \ell^l \geq 0$. The laborer's maximization program can be understood as follows. In equation (10) the laborer maximizes his Date 2 consumption c^l, which consists of his Date 1 deposits gross of interest, $R_1^D d_1^l$, and his depreciated private savings, $(1 - \delta)s_1^l$. Equation (BC_1^l) is the laborer's budget constraint that says that the sum of his Date 1 savings, s_1^l, and his Date 1 deposits, d_1^l, must equal the sum of his Date 0 deposits gross of interest, $R_0^D d_0^l$, and his depreciated

savings, $(1 - \delta)s_0^l$. Equation (BC_0^l) is the laborer's budget constraint at Date 0 which says that the sum of his Date 0 deposits, d_0^l, and his Date 0 savings, s_0^l, must equal his labor income $w\ell^l$.

3.5 Equilibrium Definition (Second Best)

The equilibrium is a profile of prices $\langle R_t^D, R^L, w \rangle$ for $t \in \{1, 2\}$ and a profile of allocations $\langle s_t^j, d_t^f, d_t^l, D_t, L, B, \ell^l, \ell^f \rangle$ for $t \in \{1, 2\}$ and $j \in \{b, f, l\}$ that solves the warehouses' problem, the farmers' problem, and the laborers' problem defined in Section 3.4 and satisfies the market clearing conditions for the labor market, the lending market, the grain market and deposit market at each date:

$$\ell^f = \ell^l \tag{MC$^\ell$}$$

$$B = L \tag{MC^L}$$

$$i + s_0^f + s_0^l + s_0^b = e \tag{MC$_0^g$}$$

$$s_1^f + s_1^l + s_1^b = (1 - \delta)s_0^f + (1 - \delta)s_0^l + s_0^b + y \tag{MC$_1^g$}$$

$$D_0 = d_0^f + d_0^l \tag{MC$_0^D$}$$

$$D_1 = d_1^f + d_1^l. \tag{MC$_1^D$}$$

3.6 Parameter Restrictions

In this section we make two restrictions on parameters. The first ensures that farmers' production technology generates sufficiently high output that the investment is positive NPV in equilibrium and the second ensures that the incentive problem that results from the non-pledgeablity of farmers' output is sufficiently severe to generate a binding borrowing constraint in equilibrium. Note that since the model is linear, if a farmer's IC does not bind, he will scale his production infinitely.

PARAMETER RESTRICTION 1. *The farmers' technology is sufficiently productive,*

$$A > 1 + \frac{1}{\alpha}. \tag{11}$$

PARAMETER RESTRICTION 2. *Depreciation from private storage is not too high,*

$$\delta A < 1. \tag{12}$$

4 Benchmarks

In this section we consider two benchmarks, before proceeding to solve for the second best. First, we solve for the first-best allocation. Second, we solve for the outcome given that the warehouse cannot issue fake receipts.

4.1 Benchmark: First Best

We now consider the first-best allocation. Here we consider the allocation that would maximize total output subject only to market clearing conditions. Since the utility, cost, and production functions are all linear, in the first-best allocation, all resources are allocated to the most productive players at each date. At Date 0 the farmers are the most productive and at Date 1 the warehouses are the most productive. Thus, all grain is held by farmers at Date 0 and by warehouses at Date 1. Laborers exert labor in proportion $1/\alpha$ of the total grain invested to maximize production.

PROPOSITION 1. (FIRST-BEST ALLOCATION) *The first-best allocation is as follows:*

$$\ell_{\text{fb}} = \alpha e, \tag{13}$$

$$i_{\text{fb}} = e. \tag{14}$$

In Appendix A.3 we discuss the connection between our model and the classical relending model in fractional reserve banking. In this model, warehouses make loans, some fraction of these loans are redeposited by laborers and then loaned again by the warehouse, some of these loans are then redeposited by laborers, and so on. With no constraints, this effective reuse of deposits yields the first-best allocation above.

4.2 Benchmark: No Fake Receipts

Consider a benchmark model in which warehouses cannot issue any receipts that are not backed by grain. This corresponds to adding an additional constraint in the warehouses' problem in Subsection 3.4. Since farmers have the entire endowment at Date 0, the warehouse cannot lend. Thus, farmers simply divide their endowment between their capital investment i and their labor investment ℓ; their budget constraint reads

$$i + w\ell = e. \tag{15}$$

The Leontief production function implies that they will always make capital investments equal to the fraction α of their labor investments, or

$$\alpha i = \ell. \tag{16}$$

We summarize the solution to this benchmark model in Proposition 2 below.

PROPOSITION 2. (BENCHMARK CASE WITH NO FAKE RECEIPTS) *When warehouses cannot issue fake receipts, the equilibrium is as follows:*

$$\ell_{\mathrm{nr}} = \frac{\alpha e}{1 + \alpha}, \tag{17}$$

$$i_{\mathrm{nr}} = \frac{e}{1 + \alpha}. \tag{18}$$

Note that even though warehouses do not improve allocational efficiency by extending credit to farmers, they nonetheless lead to efficiency gains, because they provide efficient storage of grain from Date 1 to Date 2.

5 Second-Best Solution (With Fake Recipts)

In this subsection, we solve the model to characterize the second best. We proceed as follows. First, we pin down the equilibrium deposit rates, lending rates and wages. Then we show that the model collapses to the farmer's problem which we then solve to characterize the equilibrium.

5.1 Preliminary Results

Here we state three results that completely characterize all the prices in the model, namely the two deposit rates R_0^D and R_1^D, the lending rate R^L, and the wage w. We then show that given the equilibrium prices, farmers and laborers will never store grain privately. The results all follow from the definition of competitive equilibrium with risk-neutral agents.

The first two results say that the risk-free rate in the economy is one. This is natural, since the warehouses have a scalable storage technology with return one.

LEMMA 1. (DEPOSIT RATES AT $t = 0$ AND $t = 1$) $R_0^D = R_1^D = 1$.

Now we turn to the lending rate. Since the farmers' incentive compatibility constraint ensures that loans are riskless and warehouses are competitive, warehouses also lend to farmers at rate one.

LEMMA 2. (LENDING RATES) $R^L = 1$.

Finally, since laborers have a constant marginal cost of labor, the equilibrium wage must be equal to this cost; this says that $w = 1$, as summarized in Lemma 3 below. [14]

LEMMA 3. (WAGES) $w = 1$.

These results establish that the risk-free rate offered by warehouses exceeds the rate of return from private storage, or $R_0^D = R_1^D = 1 > 1 - \delta$. Thus, farmers and laborers do not wish to make use of their private storage technologies. The only time a player may choose to store grain outside a warehouse is if a farmer diverts his output; however, the farmer's incentive compatibility constraint ensures he will not do this. Corollary 1 below summarizes this reasoning.

COROLLARY 1. (GRAIN STORAGE) *Farmers and laborers do not store grain, i.e.,* $s_0^l = s_0^f = s_1^l = s_1^f = 0$.

5.2 Equilibrium Characterization (Second Best)

In this section we characterize the equilibrium (second-best outcome) of the model. We proceed as follows. First, we show that given the equilibrium prices established in Subsection 5.1 above, laborers and warehouses are indifferent among all allocations. We then establish that a solution to the farmers' maximization problem given the equilibrium prices is a solution to the model.

The prices R_0^D, R_1^D, R^L, and w are determined exactly so that the markets clear given that agents are risk-neutral. In other words, they are the unique prices that prevent the demands of warehouses or laborers from being infinite. This is the case only if warehouses are indifferent between demanding and supplying deposits and loans at rates R_0^D, R_1^D, and R^L and laborers are indifferent between supplying or not supplying labor at wage w. This implies that all prices are one in equilibrium, as summarized in Lemma 4 below.

LEMMA 4. (WAREHOUSE AND LABOR PREFERENCES) *Given the equilibrium prices, $R_0^D = R_1^D = R^L = w = 1$, warehouses are indifferent among all deposit and loan amounts and laborers are indifferent among all labor amounts.*

Lemma 4 implies that, given the equilibrium prices, warehouses will absorb any excess demand left by the farmers. In other words, given the equilibrium prices established in Subsection 5.1 above, for any solution to the farmer's individual maximization problem, laborers' and warehouses' demands are such that markets clear.

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[14]Note that we have omitted the effect of discounting in the preceding argument—laborers work at Date 0 and consume at Date 2; discounting is safely forgotten, though, since the laborers have access to a riskless storage technology with return one via the warehouses, as established above.

We have thus established that the equilibrium allocation is given by the solution to the farmer's problem given the equilibrium prices. Thus, to find the equilibrium, we maximize the farmer's Date 2 consumption subject to his budget and incentive constraints given the equilibrium prices. In other words, we have reduced the problem of solving for the equilibrium—solving the warehouse's problem, farmer's problem, and laborer's problem in Subsection 3.4 and the market clearing conditions in Subsection 3.5—to solving a single constrained maximization problem. We state this problem in Lemma 5 below.

LEMMA 5. (SECOND-BEST PROGRAM) *The equilibrium allocation solves the problem to*

$$\text{maximize} \quad d_1^f \tag{19}$$

subject to

$$\delta\Big(y\big(i, \ell^f\big) + d_0^f\Big) \geq B, \tag{IC}$$

$$d_1^f + B = y\big(i, \ell^f\big) + d_0^f, \tag{BC_1^f}$$

$$d_0^f + i + \ell^f = e + B, \tag{BC_0^f}$$

and $i \geq 0, \ell^f \geq 0, B \geq 0, d_0^f \geq 0,$ *and* $d_1^f \geq 0.$

Solving the program above allows up to characterize the equilibrium allocations.

PROPOSITION 3. (SECOND-BEST EQUILIBRIUM VALUES OF DEBT, LABOR AND IN-VESTMENT) *The second-best equilibrium allocation is as follows:*

$$B = \frac{\delta A \alpha e}{1 + \alpha\big(1 - \delta A\big)}, \tag{20}$$

$$\ell = \frac{\alpha e}{1 + \alpha\big(1 - \delta A\big)}, \tag{21}$$

$$i = \frac{e}{1 + \alpha\big(1 - \delta A\big)}. \tag{22}$$

The equilibrium above is the solution of a system of linear equations, from the binding budget constraint and the farmers' binding incentive constraints. Observe that lending, labor, and investment are increasing in depreciation. That is to say that the worse is the farmer's private storage technology, the better is the equilibrium outcome. The reason is that it loosens his incentive constraint, making depositing more attractive at Date 1. This intuition will reappear in the next section, in which we analyze the equilibrium in detail.

6 Analysis of the Second Best

In this section we present the analysis of the second-best equilibrium. We show two main results. First, warehouses create liquidity only when they make loans by issuing fake receipts and, second, warehouses still hold grain in equilibrium, i.e. the incentive constraint leads to "endogenous fractional reserves" that prevent the economy from reaching the first-best benchmark.

6.1 Liquidity Creation

In this section we turn to the liquidity a warehouse creates by lending in fake receipts. We begin with the definition of a *liquidity multiplier*, which describes the total investment (grain investment plus labor investment) that farmers can undertake at Date 0 relative to the total endowment e.

DEFINITION 1. *The liquidity multiplier Λ is the ratio of the equilibrium investment in production $i + w\ell$ to the total grain endowment in the economy e,*

$$\Lambda := \frac{i + w\ell}{e}. \tag{23}$$

The liquidity multiplier Λ reflects farmers' total investment at Date 0. To focus on the role of warehouses in creating *additional* liquidity, we will refer to the *total liquidity created by warehouses* as the total investment $i + w\ell$ minus the initial liquidity e, which is given by $i + w\ell - e = (\Lambda - 1)e$.

The next result compares the liquidity created in equilibrium, given that warehouses can issue fake receipts, with the liquidity created in the benchmark in which warehouses must back all receipts by grain.

PROPOSITION 4. (FAKE RECEIPTS AND LIQUIDITY CREATION) *Banks create liquidity only when they can issue fake receipts. In equilibrium, the liquidity multiplier is*

$$\Lambda = \frac{1 + \alpha}{1 + \alpha(1 - \delta A)} > 1, \tag{24}$$

whereas, in the benchmark model with no receipts, the liquidity multiplier is one, denoted $\Lambda_{\mathrm{nr}} = 1$.

This result implies that it is the warehouse's ability to *make loans in fake receipts, not its ability to take deposits, that creates liquidity in the economy.* Warehouses lubricate the economy because they lend in fake receipts rather than in grain. They can do this because of their dual function: they keep accounts (i.e. warehouse grain) and also make loans. This is the crux of a farmers' incentive constraints: because warehouses provide

valuable warehousing services, farmers go to these warehouse-banks and deposit their grain, which is then also the reason why they repay their debts.

To cement the argument that liquidity creation results only from warehouses' lending in fake receipts, we now relate the quantity of fake receipts that the warehouse issues to the liquidity multiplier. The number of fake receipts the warehouse issues at Date 0 is given by the total number of receipts it issues D_0 less the total quantity of grain it stores s_0^b;[15] in equilibrium, this is given by

$$D_0 - s_0^b = \frac{\alpha \delta A e}{1 + \alpha(1 - \delta A)}.$$ (25)

Comparing the expression above with the formula for Λ in Proposition 4 reveals that the number of fake receipts is equal to the liquidity created by warehouses. This reiterates our main point of this section: warehouses create liquidity only by lending in fake receipts. We summarize this in Corollary 2 below.

COROLLARY 2. (TOTAL LIQUIDITY CREATION) *The total liquidity created by warehouses equals the number of fake receipts the warehouse issues*

$$(\Lambda - 1) e = D_0 - s_0^b.$$ (26)

We now analyze the effect of the private storage technology—i.e. the depreciation rate δ—on warehouses' liquidity creation. We find that the amount of liquidity Λ that warehouses create is increasing in warehouses' storage advantage, as measured by δ. The reason is that the more desirable it is for farmers to deposit in a warehouse at Date 1 rather than store privately, the looser is their incentive constraint. As a result, warehouses are more wiling to lend to them at Date 0—they know they can lend more and it will still be incentive compatible for farmers to repay at Date 1. We can see this immediately by differentiating the liquidity multiplier with respect to δ:

$$\frac{\partial \Lambda}{\partial \delta} = \frac{\alpha A}{\left(1 + \alpha(1 - \delta A)\right)^2} > 0.$$ (27)

We summarize this result in Corollary 3 below.

COROLLARY 3. (WAREHOUSE EFFICIENCY AND LIQUIDITY CREATION) *The more efficiently warehouses can store grain relative to farmers (the higher is δ), the more liquidity warehouses create by issuing fake receipts.*

We note that Corollary 3 may seem counterintuitive: a *decrease* in the efficiency in private storage leads to an *increase* in overall efficiency. The reason is that it allows

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[15]The grain market clearing condition implies that $s_0^b = e - i$ and warehouses' Date-0 budget constraint says $D_0 = s_0^b + L$, where $L = B$ by loan market clearing and B is given in Proposition 3.

banks to create more liquidity by weakening farmers' incentive to divert capital. We return to this result when we discuss central bank policy in Subsection 7.4 below.

This result also suggests an empirical implication. To the extent that warehouses have "power" in enforcing contracts, we should expect warehousing services to be more important in countries with weaker property rights.

6.2 Fractional Reserves

We now proceed to analyze warehouses' balance sheets. Absent reserve requirements, do they still store grain? We find that the answer is yes, even though farmers' technology is constant-returns-to-scale, and farmers would therefore prefer to invest all grain in the economy in their technology. The reason that warehouses store grain in equilibrium is that farmers' incentive constraints put an endogenous limit on the amount that each farmer can borrow and, therefore, on the amount of grain that each can invest productively.

PROPOSITION 5. (DEPOSIT RESERVES HELD BY WAREHOUSES) *Warehouses hold a positive fraction of grain at $t = 0$, in equilibrium,*

$$s_0^b = e - i = \frac{\alpha(1 - \delta A)e}{1 + \alpha(1 - \delta A)} > 0, \tag{28}$$

i.e. the incentive constraint leads to endogenous fractional reserves.

Note that in our model, the storage of grain by warehouses at Date 0 is inefficient. Grain could be put to better use by farmers (in conjunction with labor paid for in fake receipts). Thus, a policy maker in our model actually wishes to reduce warehouse holdings or bank capital reserves, to have the economy operate more efficiently.

7 Welfare and Policy

In this section we consider the implications of four policies, all of which have been advocated by policy-makers after the financial crisis of 2007–2009. These are: (1) narrow banking, (ii) liquidity requirements for banks, (iii) capital requirements for banks, and (iv) tightening monetary policy. We also include a section in which we apply the mechanism design approach to our environment.

7.1 Liquidity Requirements and Liquidity Creation

Basel III, the Basel Committee on Banking Supervision's third accord, extends international financial regulation to include so-called liquidity requirements. Specifically,

Basel III mandates that banks must hold a sufficient quantity of liquidity to ensure a "liquidity ratio" called the Liquidity Coverage Ratio (LCR) is satisfied. The ratio effectively forces banks to invest a portion of their assets in cash and cash-proximate marketable securities. The rationale is that banks should be able to liquidate a portion of their balance sheets expeditiously to withstand withdrawals in a crisis.

In our model, the LCR imposes a limit on the ratio of loans that a bank (warehouse) can make relative to deposits (grain) it stores. This is exactly a limit on the quantity of fake receipts a bank can issue or a limit on liquidity creation.

We now make this more formal. Consider a liquidity regulation that, like the LCR, mandates that a bank hold a proportion θ of its assets in liquid assets, or

$$\frac{\text{liquid assets}}{\text{total assets}} \geq \theta. \tag{29}$$

In our model, the warehouses' liquid assets are the grain they store and their total assets are the grain they store plus the loans they make. Thus, within the model, the liquidity regulation described above prescribes that, at Date 0,

$$\frac{s_0^b}{B + s_0^b} \geq \theta. \tag{30}$$

We see immediately by rewriting this inequality that this regulation imposes a cap on bank lending,

$$B \leq \frac{1 - \theta}{\theta} s_0^b. \tag{31}$$

The next proposition states the circumstance in which liquidity regulation constrains liquidity creation to a level below the equilibrium level.

PROPOSITION 6. (EFFECT OF LIQUIDITY REQUIREMENTS) *Whenever the required liquidity ratio θ is such that*

$$\theta > 1 - \delta A, \tag{32}$$

liquidity regulation inhibits liquidity creation—and thus farmers' investment—below the equilibrium level.

Advocates of so-called *narrow banking* have argued that banks should hold *only* liquid securities as assets, with some arguing for banks to invest only in treasuries, effectively requiring one-hundred percent reserves.[16] In our model, this corresponds to $\theta = 1$ in the analysis of the LCR, which reduces to the benchmark in which warehouses cannot make loans. We state this as a proposition for emphasis.

PROPOSITION 7. (NARROW BANKING) *The requirement of narrow banking is equivalent to the benchmark in which warehouses cannot issue fake receipts (Section 4). In*

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[16]See the review by Pennacchi (2012).

this case there is no liquidity creation, $\Lambda_{\mathrm{nr}} = 1$.

7.2 Liquidity Requirements and Financial Fragility

In the previous section we showed that higher liquidity requirements decrease bank liquidity creation by inhibiting lending. However, the oft-stated purpose of liquidity requirements is to enhance financial stability.[17] The argument goes that a bank with more liquid reserves will be able to withstand more withdrawals or a larger "run" from its creditors, creating stability. In this section we deal with this by extending the model to include a bank run game among depositors. We show that more liquid reserves make a financial system more *fragile* in our setting. The reason is that while liquid reserves do indeed allow a bank to withstand a bigger run, they also make depositors more prone to running the bank.

We consider a warehouse with total deposits θ at Date 0 and add an additional stage immediately after Date 0, called Date 0^+. At this stage, each depositor may demand to withdraw grain or leave it in the warehouse. We use the notation λ to refer to the total amount of grain demanded by all depositors at Date 0^+. If $\lambda \leq \theta$, then the warehouse has sufficient reserves to pay all withdrawing depositors. It remains solvent. Withdrawing depositors take out their grain and store it privately, letting it depreciate at rate δ. Non-withdrawing depositors do not take out their grain, but can claim it at Date 1. Thus, they avoid depreciation. If $\lambda > \theta$, then the warehouse does not have sufficient reserves to pay all withdrawing depositors. The warehouse is insolvent. It distributes reserves among withdrawing depositors according to a pro rata rule, i.e. for each unit of grain demanded, a withdrawing depositor receives θ/λ units of grain. Since the warehouse is insolvent and closes, depositors who have not withdrawn cannot cash in their receipts at Date 1. They receive zero. The payoffs from withdrawing or not withdrawing a unit of grain as a function of λ are summarized in Figure 6.

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[17]The Basel Committee on Banking Supervision states that "The LCR is one of the Basel Committee's key reforms to strengthen global capital and liquidity regulations with the goal of promoting a more resilient banking sector. The LCR promotes the short-term resilience of a bank's liquidity risk profile" (see http://www.bis.org/publ/bcbs238.htm).

	$\lambda \leq \theta$	$\lambda > \theta$
Withdraw	$1 - \delta$	$\dfrac{(1-\delta)\theta}{\lambda}$
Don't withdraw	1	0

Figure 6: Payoff matrix of the bank run game at Date 0^+.

At Date 0^+ depositors now play a coordination game with multiple Nash equilibria. In particular, there are two pure strategy Nash equilibria, one in which all depositors withdraw (a bank run) and another in which no depositors withdraw.

We now discuss the effect of reserves on financial fragility. To do this, we define a notion of fragility that captures the propensity of depositors to run. Specifically, we refer to the system as fragile if depositors are likely to withdraw regardless of their beliefs about other depositors' withdrawals. In other words, a financial system is fragile if runs are likely to arise as a result of strategic uncertainty.[18] We consider the beliefs of a single depositor about the withdrawal decisions of other depositors. For simplicity, we focus on beliefs for which all depositors act the same way. Call μ a depositor's belief that others do not withdraw, so μ is the belief that $\lambda = 0$. Thus, $1 - \mu$ is the belief that everyone else withdrawals, so $\lambda = D_0$ (recall that D_0 is the total number of receipts the warehouse issues at Date 0).

Now define μ^* as the belief that makes the depositor indifferent between withdrawing and not withdrawing. We refer to μ^* as *financial fragility*. To see why μ^* captures financial fragility, consider the extreme cases when $\mu^* = 0$ and $\mu^* = 1$. If $\mu^* = 0$, a depositor strictly prefers to leave his grain in the warehouse even if he is almost certain that every other depositor will withdraw. Thus, all depositors will leave their grain in the warehouse and there is no run. Thus, if $\mu^* = 0$ the financial system is stable. In contrast, if $\mu^* = 1$, a depositor strictly prefers to withdraw his grain from the warehouse even if he is almost certain that every other depositor will not withdraw. Thus, all depositors will withdraw their grain from the warehouse and there is a run. Thus, if $\mu^* = 1$ the financial system is fragile. Thus, μ^* measures financial fragility.

We now analyze how financial fragility μ^* varies with reserves. Given the payoff

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[18]Our view of financial fragility as the sensitivity to strategic uncertainty is in the spirit of the literature on global games (see Goldstein and Pauzner (2005) for a study of bank runs and Morris and Shin (2003) for a survey). However, our model does not lend itself to a formal analysis within the global games framework because at least one key requirement of the global games approach is violated: each action must be a strictly dominant strategy for some parameters. In our model not withdrawing is always preferable to withdrawing if others are not withdrawing, since $\delta \in [0, 1/A]$ by Parameter Restriction 2.

matrix of the bank-run game, we can express financial fragility μ^* as a function of liquid reserves θ. The indifference condition gives

$$\mu^*(1 - \delta) + (1 - \mu^*)\frac{(1 - \delta)\theta}{D_0} = \mu^*, \tag{33}$$

so

$$\mu^* = \frac{1 - \delta}{\frac{\delta D_0}{\theta} + 1 - \delta}. \tag{34}$$

This equation implies that μ^* is increasing in θ, which is the main result of this section.

PROPOSITION 8. (LIQUIDITY RESERVES AND FINANCIAL FRAGILITY) *An increase in reserves θ causes an increase in financial fragility μ^*.*

The intuition is that an increase in reserves increases the attractiveness (to a depositor) of withdrawing early. To see this consider the extreme case in which the warehouse holds no reserves or $\theta = 0$. In this case, a depositor never wishes to withdraw early because he never receives any grain, regardless of whether there is a run. This finding contrasts with the classic bank run model of Diamond and Dybvig (1983). The reason is that in their model, liquidating the long-term investment at the interim date has a positive value (recovery of the initial investment). Knowing this depositors may run to get a fraction of that liquidating payoff. That is, the illiquid project has partial interim liquidity. In contrast, in our model, the illiquid project cannot generate any liquidity before the terminal date, because the warehouse does not have the possibility of prematurely calling in loans from farmers to pay depositors.

7.3 Bank Capital and Liquidity Creation

In this subsection we analyze the implications of changes in bank capital for warehouse liquidity creation.[19] In particular, we show that increasing warehouse equity increases liquidity creation. To do this we extend the model in the following way. We endow warehouses with equity e^w at Date 1 and add a pledgeablity problem for the warehouse when it accepts deposits. Specifically, after a warehouse accepts deposits, it has the following choice: it can either divert grain and store it privately or not divert grain and store it in the warehouse. If it diverts the grain, the depositors will not be able to claim it, but it will depreciate at rate δ. If the warehouse does not divert, depositors will be able to claim it, but it will not depreciate. In this section, we show that warehouse equity has an important function: it gives the warehouse the incentive *not*

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[19]Because we do not have bank failures and crises in our baseline model, our analysis likely understates the value and role of bank capital. Calomiris and Nissim (2014) document that the market is attaching a higher value to bank capital after the 2007–09 crisis.

to divert deposits. For low levels of warehouse equity, an increase in warehouse equity can increase lending and liquidity creation. However, beyond a threshold, an increase in warehouse equity has no effect on the real economy.

The results of this section follow from the analysis of the warehouse's incentive constraint: depositors store in a warehouse at Date 1 only if the warehouse prefers not to divert deposits. Its payoff, if it diverts, is given by the depreciated value of its equity plus its deposits, or $(1 - \delta)(e^w + D_1)$. Its payoff, if it does not divert, is the value of its equity plus its deposits less its repayment to its depositors, or $e^w + D_1 - R_1^D D_1$. Since $R_1^D = 1$ by Lemma 1, the warehouses incentive constraint at Date 1[20] is

$$(1 - \delta)(e^w + D_1) \leq e^w. \tag{35}$$

Thus, in this extension in which warehouses also face a pledgeability problem, the second-best equilibrium outcome summarized in Proposition 3 is obtained only if

$$\frac{e^w}{D_1} \geq \frac{1 - \delta}{\delta}. \tag{36}$$

This is a constraint on the capital ratio. It says that the second-best is attained only if warehouse's capital ratio is sufficiently high. In the next proposition we substitute the equilibrium value of D_1 to write the minimum amount of equity a warehouse must have in order for it not to impede liquidity creation.

PROPOSITION 9. (ROLE OF WAREHOUSE EQUITY) *The second-best equilibrium in Proposition 3 is attained only if warehouse equity is sufficiently high, or*

$$e^w \geq \hat{e}^w := \frac{(1 + A)e}{2 - \delta A}. \tag{37}$$

Warehouse equity above this threshold has no effect on liquidity creation. The proposition gives a threshold above which the second-best is attained. The reason why increasing bank equity above this threshold has no effect on liquidity creation or investment in the economy is that the borrowers' (farmers') incentive constraints bind, and if they were to take larger loans they would not repay their debt. This may shed light on why banks were reluctant to lend even after they refurbished their capital following the financial crisis of 2008.

If warehouse equity is low, then the second-best is not attained. In this case, the warehouse's incentive constrain binds (and the farmer's incentive constraint does not) and an increase in warehouse equity loosens the warehouse's incentive constraint. This allows it to accept more deposits. Since accepting more deposits allows it to obtain a

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[20]If the incentive constraint is satisfied at Date 1, when deposits are high because farmers deposit their output in warehouses, it is also satisfied at Date 0, when deposits are relatively low.

larger repayment from borrowers, this also allows the warehouse to make more loans. In other words, increasing warehouse equity at Date 1 allows the warehouse to provide more liquidity to farmers at Date 0. This is summarized in the following proposition.

PROPOSITION 10. (LIQUIDITY CREATION WHEN WAREHOUSE CAPITAL IS BELOW $\hat{e}^w$) *When warehouse equity is below the threshold in Proposition 9, liquidity creation is strictly increasing in warehouse equity e^w whenever warehouses lend; specifically,*

$$\Lambda = \frac{1+\alpha}{\alpha A - 1} \max\left\{ \frac{\delta}{1-\delta} \frac{e^w}{e} - 1\,,\, 0 \right\}. \tag{38}$$

The expression in the proposition above says that when warehouse equity is very low, the incentive problem is so severe that warehouses do not lend at all. As equity increases, warehouse start lending and the amount they lend increases linearly until the farmer's incentive constraint binds. Above this threshold, an increase in equity has no further effect as described above.

Our results in the section stand in contrast to the existing literature on bank liquidity creation. In models like Bryant (1980) and Diamond and Dybvig (1983), there is no discernable role for bank capital, and models like Diamond and Rajan (2001) argue that higher bank capital will diminish liquidity creation by banks. In our model, higher bank capital expands the bank's lending capacity and hence enhances ex ante liquidity creation. Nonetheless, we have no central bank and no frictions that generate a rationale for regulatory capital requirements.

7.4 Monetary Policy

In this subsection we analyze the implications of changes in monetary policy on warehouse liquidity creation. We define the central bank rate R^{CB} as the (gross) rate at which warehouses can deposit with the central bank.[21] This is analogous to the storage technology of the warehouse yielding return R^{CB}. In this interpretation of the model, grain is central bank money and fake receipts are private money.

We first state the necessary analogs of the parameter restrictions in Subsection 3.6. Note that they coincide with Parameter Restriction 1 and Parameter Restriction 2 when $R^{\text{CB}} = 1$, as in the baseline model.

PARAMETER RESTRICTION 1′. *The farmers' technology is sufficiently productive,*

$$A > \frac{1}{R^{\text{CB}}} + \frac{R^{\text{CB}}}{\alpha}. \tag{39}$$

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[21]We are considering a rather limited aspect of central bank monetary policy here, thereby ignoring things like the role of the central bank in setting the interest rate on interbank lending, as in Freixas, Martin, and Skeie (2011), for example.

PARAMETER RESTRICTION 2′. *Depreciation from private storage is not too fast,*

$$A\left(R^{\mathrm{CB}} - 1 + \delta\right) < 1. \tag{40}$$

The preliminary results of Subsection 5.1 lead to the natural modifications of the prices. In particular, due to competition in the deposit market, the deposit rates equal the central bank rate. Further, because laborers earn interest on their deposits, they accept lower wages. We now summarize these results in Lemma 6.

LEMMA 6. (INTEREST RATES AND WAGES WITH A CENTRAL BANK) *When warehouses earn the central bank rate R^{CB} on deposits, in equilibrium, the deposit rates, lending rate, and wage are as follows:*

$$R_0^D = R_1^D = R^L = R^{\mathrm{CB}} \tag{41}$$

and

$$w = \left(R^{\mathrm{CB}}\right)^{-2}. \tag{42}$$

The crucial takeaway from the result is that the warehouse pays a higher deposit rate when the central bank rate is higher. This means that the farmer's incentive constraint takes into account a higher return from depositing in a warehouse, but the same depreciation rate from private storage. Formally, with the central bank rate R^{CB}, the farmers' incentive constraint at Date 1 reads

$$(1 - \delta)y \le R^{\mathrm{CB}}\left(y - R^{\mathrm{CB}}B\right) \tag{43}$$

or

$$B \le \frac{1}{R^{\mathrm{CB}}}\left(1 - \frac{1 - \delta}{R^{\mathrm{CB}}}\right)y. \tag{44}$$

Observe that whenever farmers are not too highly levered—$B < y\left(2R^{\mathrm{CB}}\right)^{-2}$—increasing R^{CB} loosens the incentive constraint. The reason is that it makes warehouse storage relatively more attractive at Date 1, inducing farmers to repay their debt rather than diverting capital.[22]

PROPOSITION 11. (MONETARY POLICY AND LIQUIDITY CREATION) *A tightening of monetary policy (an increase in R^{CB}) increases liquidity creation Λ as long as $\alpha + 2R^{\mathrm{CB}}(1 - \delta) > \left(R^{\mathrm{CB}}\right)^2$ (otherwise it decreases liquidity creation).*

This contrasts with the established idea that low interest rates (easy money policy)

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[22]The reason that increasing R^{CB} does not loosen the constraint when B is high, it that it also increases the lending rate between Date 0 and Date 1.

stimulate bank lending and help the economy recover from a recession.[23] In our model, *high interest rates allow banks to lend more* because they loosen the farmers' incentive compatibility constraints by giving warehouses a greater relative advantage in storage. This result complements Corollary 3 which says that liquidity creation is increasing in the depreciation rate δ. Both results say that the better warehouses are at storing grain relative to farmers, the more warehouses can lend.

7.5   The Mechanism Design Approach to the Second-best

In this section, we briefly discuss the mechanism design approach to our model and results. This approach suggests considering all incentive feasible allocations given only the preferences and technologies of the players, rather than focusing on a particular equilibrium concept. In other words, rather than considering markets a primitive of the model, we consider all the allocations that can be implemented with any general mechanism for players' interactions. The main message of this section is that our model implements the best incentive-feasible outcome. In other words, the setting in the model in which markets are Walrasian and warehouses have the ability to seize grain deposited in them is an optimal mechanism. This provides a new rationale for why warehouses do the lending as well as why there are interbank markets.

We state our main finding as a proposition for emphasis and provide a verbal proof in the text.

PROPOSITION 12. (MECHANISM DESIGN AND THE SECOND-BEST EQUILIBIRUM) *If the worst feasible punishment for any player is autarky, then the second-best equilibrium summarized in Proposition 3 is optimal in the sense that it maximizes output and utilitarian welfare among all incentive-feasible allocations.*

We divide the proof of the proposition into three steps. In Step 1, we explain that any mechanism that implements the most severe feasible punishments can implement the (constrained) optimal outcome. In Step 2, we argue that the most severe punishments in our environment are the exclusion from warehousing. In Step 3, we show that our environment with Walrasian markets in which warehouses can seize their deposits implements these punishments.

Step 1. A mechanism can implement an outcome if the outcome is incentive compatible given the mechanism. Increasing the severity of punishments corresponds to loosening incentive constraints, which expands the set of implementable outcomes. Hence, increasing the severity of punishments expands the set of implementable outcomes.

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[23]See, for example, Keeton (1993). Mishkin (2010) provides a broad assessment of monetary policy, bank lending, and the role of the central bank.

Step 2. In our environment, punishments must be administered at Date 1 (at Date 2 agents consume, so we are effectively already in autarky and at Date 0 it is too early to punish them for anything). At Date 1, there are only two technologies, private storage and warehouse storage. Thus, the only benefit the environment provides beyond autarky is access to warehousing. In other words, the worst possible punishment is exclusion from warehousing.

Step 3. The only limit to commitment in our environment comes from the non-pledgeability of farmers' output—the farmer is the only player who might not fulfill his promise. However, given the interbank market, anything the farmer deposits in the warehouse ultimately can be seized (see Appendix A.2). Thus, the only way that a farmer can avoid repayment at Date 1 is by storing privately. This is equivalent to saying that if a farmer breaks his promise, he cannot store in a warehouse—he receives the autarky payoff. Thus, our model imposes the most severe feasible punishments on defecting players. As a result (from Step 1), our model implements the optimal incentive-feasible outcome.

8   Conclusion

Summary of paper. In this paper we have developed a new theory of banking that is tied to the origins of banks as commodity warehouses. The *raison d'être* for banks does not require asymmetric information, screening, monitoring or risk aversion. Rather, we show that the institutions with the best storage (warehousing) technology have an advantage in enforcing contracts, and are therefore not only natural deposit-takers but are also natural lenders—i.e. they are natural banks. With this theory we show how banks create liquidity even when they do *not* provide superior risk sharing. While the existing literature views bank liquidity creation as being synonymous with improved risk sharing for risk-averse depositors, we focus on ex ante *funding* liquidity creation, which is the bank-attributed increase in the initially available liquidity that can be channeled into aggregate investment in productive activities. The key to the bank's ability to do this is the issuance of "fake" warehouse receipts by the bank. This creates a striking contrast with the existing literature, which views the process of liquidity creation as banks accepting deposits that are then loaned out, i.e., deposits create loans. In our theory, loans also create deposits. We thus decouple the notion of creating liquidity from risk preferences and show that risk aversion is neither a sufficient nor necessary condition for ex ante liquidity creation. In this way, our analysis of bank liquidity creation *complements* the focus of the existing literature on the bank creating liquidity through better risk sharing.

Our theory has regulatory implications. It shows that proposals like narrow banking

and liquidity requirements on banks will diminish bank liquidity creation and be inimical to economic growth. By contrast, higher levels of bank capital will enhance bank liquidity creation. Moreover, we establish conditions under which a tighter monetary policy induces more liquidity creation.

Empirical implications. Our paper generates numerous predictions that could be tested. First, across countries, banks that provide warehousing services should play a more important role in countries with weaker property rights. Second, more aggregate funding liquidity will be created in the economy when banks have higher capital. Third, liquidity requirements on banks will reduce aggregate liquidity creation.

Future research. We view this paper as a first step in the direction of formalizing the funding liquidity creation role of banks in the context of safeguarding. Much remains to be done. A few extensions come immediately to mind. One is to investigate the effects of frictions in the interbank loan market. Another is to examine the implications of alternative production technologies for farmers, e.g. Cobb–Douglas. Finally, we have not considered the potential bank fragility associated with the creation of liquidity in our model of warehouse banking, an issue that will create a natural role for a central bank, and one that should be addressed in future research.[24] But our paper emphasizes that the creation of funding liquidity is not one that inherently generates fragility— other ingredients need to be added to the recipe for that to happen.

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[24]See Goodhart (2010) for a thorough discussion of the role of central banks and how it is changing.

A Appendix

A.1 Proofs

A.1.1 Proof of Proposition 1

As discussed in the text preceding the statement of the proposition, in the first-best all grain is invested in its first-best use at Date 0. This corresponds to $i_{\text{fb}} = e$, since the farmer's technology is the most productive. The production function requires $\ell_{\text{fb}} = \alpha i = \alpha e$ units of labor to be productive, and any more is unproductive. In summary, $i_{\text{fb}} = e$ and $\ell_{\text{fb}} = \alpha e$, as stated in the proposition. □

A.1.2 Proof of Proposition 2

In this proof we make use of the results in Subsection 5.1 that state the prices and of Lemma 5 that simplifies the problem of solving for the equilibrium. Note that although these results come after this proposition in the next, they do not depend on it. Thus, we may employ them in this proof.

The equilibrium allocation again solves the farmer's problem, but in this case the warehouse cannot issue more receipts than has deposits so $s_0^b \geq 0$. Further, since the warehouse has no endowment, its budget constraint reads $L + s_0^b = D_0$. Thus, $L = 0$. Market clearing implies $B = 0$. The farmer's problem is thus to

$$\text{maximize} \quad d_1^f \tag{45}$$

subject to

$$d_1^f = A \max\left\{\alpha i, \ell^f\right\}, \tag{46}$$

$$i + \ell^f = e. \tag{47}$$

The solution to the problem is

$$\ell_{\text{nr}} = \frac{\alpha e}{1 + \alpha}, \tag{48}$$

$$i_{\text{nr}} = \frac{e}{1 + \alpha}, \tag{49}$$

as expressed in the proposition. □

A.1.3 Proof of Lemma 1

We show the result by contradiction. If $R_t^D \neq 1$ in equilibrium, deposit markets cannot clear.

First suppose (in anticipation of a contradiction) that $R_t^D < 1$ in equilibrium (for either $t \in \{0, 1\}$). Now set $s_t^b = D_t$ in the warehouse's problem in Subsection 3.4. The warehouse's objective function (equation (8)) goes to infinity as $D_t \to \infty$ without violating the constraints. The deposit markets therefore cannot clear if $R_t < 1$, a contradiction. We conclude that $R_t^D \geq 1$.

Now suppose (in anticipation of a contradiction) that $R_t^D > 1$ in equilibrium (for either $t \in \{0, 1\}$). Now set $s_t^b = D_t$ in the warehouses problem. The warehouse's objective function goes to infinity as $D_t \to -\infty$ without violating the budget constraints. Thus, if $R_t^D > 1$, it must be that $D_t = 0$. However, since the depreciation rate $\delta > 0$, the demand from laborers and farmers to store grain is strictly positive for $R_t^D > 1 - \delta$. Thus, again, deposit markets cannot clear, a contradiction. We conclude that $R_t^D \leq 1$.

The two contradictions above taken together imply that $R_t^D = 1$ for $t \in \{0, 1\}$. $\square$

A.1.4 Proof of Lemma 2

We show the result by contradiction. If $R^L \neq 1$ in equilibrium, loan markets cannot clear.

First suppose (in anticipation of a contradiction) that $R^L > 1$ in equilibrium. Now set $L = D_t$ in the warehouse's problem in Subsection 3.4. Given that $R_0^D = 1$ from Lemma 1 above, the warehouse's objective function (equation 8) goes to infinity as $L \to \infty$ without violating the constraints. The deposit markets therefore cannot clear if $R^L > 0$, a contradiction. We conclude that $R^L \leq 1$.

Now suppose (in anticipation of a contradiction) that $R^L < 1$ in equilibrium. Now set $L = D_0$ in the warehouse's problem. Given that $R_0^D = 1$ from Lemma 1 above, the warehouse's objective function goes to infinity as $L \to -\infty$ without violating the budget constraints. Thus, if $R^L < 1$, it must be that $D_0 = 0$. However, since the depreciation rate $\delta > 0$, the demand from laborers and farmers to store grain is always strictly positive for $R_t^D > 1 - \delta$. Thus, again, deposit markets cannot clear, a contradiction. We conclude that $R^L \geq 1$.

The two contradictions above taken together imply that $R^L = 1$. $\square$

A.1.5 Proof of Corollary 1

Given Lemma 1 above, the result is immediate from inspection of the farmer's problem and the laborer's problem in Subsection 3.4 given that $R_0^D = R_1^D = 1 > 1 - \delta$, the return from private storage. $\square$

A.1.6 Proof of Lemma 3

We show the result by contradiction. If $w \neq 1$ in equilibrium, labor markets cannot clear.

First suppose (in anticipation of a contradiction) that $w > 1$ in equilibrium. From Corollary 1, $d_0^l = w\ell^l$ and $d_1^l = R_0^D d_0^l$ in the laborer's problem in Subsection 3.4. The constraints collapse, and the laborer's objective function (equation (10)) is $R_1^D R_0^D w\ell^l - \ell^l = (w - 1)\ell^l$, having substituted $R_0^D = R_1^D = 1$ from Lemma 1 above. Since $w > 1$ by supposition, the objective function approaches infinity as $\ell^l \to \infty$ without violating the constraints. The labor market therefore cannot clear if $w > 1$, a contradiction. We conclude that $w \leq 1$.

Now suppose (in anticipation of a contradiction) that $w < 1$ in equilibrium. As above, the laborer's objective function is $(w - 1)\ell^l$. Since $w < 1$ by supposition, the laborer sets $\ell^l = 0$. The farmer, however, always has a strictly positive demand for labor if $w < 1$—he produces nothing without labor and his productivity $A > 1 + 1/\alpha$ by Parameter Restriction 1. The labor market therefore cannot clear if $w < 1$, a contradiction. We conclude that $w \geq 1$.

The two contradictions above taken together imply that $w = 1$. □

A.1.7 Proof of Lemma 4

The result follows immediately from the proofs of Lemma 1, Lemma 2, and Lemma 3, which pin down the prices in the model by demonstrating that if prices do not make these players indifferent, markets cannot clear, contradicting equilibrium. □

A.1.8 Proof of Lemma 5

The result follows from Lemma 4 and substituting in prices and demands from the preliminary results in Subsection 5.1. In short, since, given the equilibrium prices, laborers and warehouses are indifferent among allocations, they will take on the excess demand left by the farmers to clear the market. □

A.1.9 Proof of Proposition 3

We begin by rewriting the farmer's problem in Lemma 5 as

$$\text{maximize} \quad d_1^f \tag{50}$$

subject to

$$\delta\left(A\max\left\{\alpha i, \ell^f\right\} + d_0^f\right) \geq B, \tag{IC}$$

$$d_1^f + B = A\max\left\{\alpha i, \ell^f\right\} + d_0^f, \tag{BC$_1^f$}$$

$$d_0^f + i + \ell^f = e + B, \tag{BC$_0^f$}$$

and $i \geq 0, \ell^f \geq 0, B \geq 0, d_0^f \geq 0$ and $d_1^f \geq 0$.

Now observe that at the optimum, $\max\left\{\alpha i, \ell^f\right\} = \ell^f$ and $\ell^f = \alpha i$. Further, eliminate the d_1^f of the objective from the budget constraint. Now we can write the problem as

$$\text{maximize} \quad A\ell^f + d_0^f - B \tag{51}$$

subject to

$$\delta\left(A\ell^f + d_0^f\right) \geq B, \tag{IC}$$

$$d_0^f + i + \ell^f = e + B, \tag{BC$_0^f$}$$

$$\ell^f = \alpha i \tag{52}$$

and $i \geq 0, \ell^f \geq 0, B \geq 0$, and $d_0^f \geq 0$.

We see that the budget constraint and $\ell^f = \alpha i$ imply that

$$B = d_0^f + \frac{1+\alpha}{\alpha}\ell^f - e \tag{53}$$

and, thus, the objective is

$$A\ell^f - \frac{1+\alpha}{\alpha}\ell^f + e = \frac{\alpha(A-1) - 1}{\alpha}\ell^f + e. \tag{54}$$

This is increasing in ℓ^f by Parameter Restriction 1, so ℓ^f is maximal at the optimum. Thus, the incentive constraint binds, or

$$\delta\left(A\ell^f + d_0^f\right) = B = d_0^f + \frac{1+\alpha}{\alpha}\ell^f - e. \tag{55}$$

or

$$e - (1-\delta)d_0^f = \left(1 - \delta A + \frac{1}{a}\right)\ell^f. \tag{56}$$

Since, by Parameter Restriction 2, $\delta A < 1$, setting $d_0^f = 0$ maximizes ℓ^f. Hence,

$$\ell^f = \frac{\alpha e}{1 + \alpha(1 - \delta A)}. \tag{57}$$

Combining this with the budget constraint and the equation $i = \ell^f/\alpha$ gives the expres-

sions in the proposition. □

A.1.10 Proof of Proposition 4

The result follows immediately from comparison of the equilibrium expression for $i + w\ell$ given in Proposition 3 with the expression for $i_{\mathrm{nr}} + w\ell_{\mathrm{nr}}$ given in Proposition 2. Note that $w = 1$ in the benchmark with no receipts as well as in the full model. The proofs of the results for the prices (in particular for the wage w) in Subsection 5.1 are unchanged for the benchmark. □

A.1.11 Proof of Corollary 2

The result follows from direct calculation given the equilibrium expressions for Λ, D_0 and s_0^b. □

A.1.12 Proof of Corollary 3

The result is immediate from differentiation, as expressed in equation (27). □

A.1.13 Proof of Proposition 5

The expression given in the proposition is positive as long as $1 - \delta A > 0$. This holds by Parameter Restriction 2. The result follows immediately. □

A.1.14 Proof of Proposition 6

The liquidity ration inhibits liquidity creation whenever warehouses' equilibrium Date 0 grain holdings s_0^b are insufficient to satisfy their liquidity requirements. In other words, given equation (31), if

$$B < \frac{1 - \theta}{\theta} s_0^b, \tag{58}$$

then liquidity requirements inhibit liquidity creation. Given the equilibrium values of s_0^b and B, this can be rewritten as

$$\frac{\delta A \alpha e}{1 + \alpha(1 - \delta A)} < \frac{1 - \theta}{\theta} \frac{\alpha(1 - \delta A)e}{1 + \alpha(1 - \delta A)}. \tag{59}$$

This holds only if

$$\theta < 1 - \delta A. \tag{60}$$

Whenever this inequality is violated, liquidity requirements inhibit liquidity requirements. It is the negation of the above equality stated in the proposition. □

A.1.15 Proof of Proposition 10

This result follows from solving for the equilibrium with the warehouse's incentive constraint binding. We proceed assuming that lending L is positive. If it is negative, the formulae do not apply and $L = 0$.

Begin with the warehouses' binding incentive constraint, which gives a formula for D_1, the total grain deposited at Date 1,

$$D_1 = \frac{\delta}{1 - \delta} e^w. \tag{61}$$

The Date 1 deposit market clearing condition implies that the total amount of deposits equals the total amount of grain at Date 1. This is the sum of the farmer's output y and the grain stored in the warehouse at Date 0 s_0^b,

$$D_1 = y + s_0^b \tag{62}$$
$$= A\alpha i + e - i. \tag{63}$$

Combining this with the warehouses' incentive constraint gives

$$i = \frac{1}{\alpha A - 1} \left(\frac{\delta}{1 - \delta} e^w - e \right). \tag{64}$$

(Note that $\alpha A - 1 > 0$ by Parameter Restriction 1.) Now, since the farmers' technology is Leontief, $\ell = \alpha i$. This allows us to write down the expression for the liquidity multiplier Λ:

$$\Lambda = \frac{i + w\ell}{e} \tag{65}$$
$$= \frac{1 + \alpha}{\alpha A - 1} \left(\frac{\delta}{1 - \delta} \frac{e^w}{e} - 1 \right). \tag{66}$$

This expression applies when it is positive (and e^w is below the threshold in Proposition 9). Otherwise, liquidity creation is zero.

A.1.16 Proof of Lemma 6

The proofs that $R_0^D = R_1^D = R^L = R^{CB}$ are all identical to the proofs of the analogous results in Subsection 5.1 with the warehouses' return on storage (which is one in the baseline model) replaced with the central bank rate R^{CB}. The result is simply that warehouses lend and borrow at their cost of storage, which is a result of warehouses being competitive.

The result that $w = \left(R^{CB} \right)^{-2}$ is also nearly the same as the proof of the analogous result (Lemma 3) in Subsection 5.1. The modification is that the laborer's objective

function (equation (10)) reduces to $c^l = \left(R^{\mathrm{CB}}\right)^2 w\ell - \ell$, since the laborer invests its income in the warehouse for two periods at gross rate R^{CB}. In order for the laborer not to supply infinite (positive or negative) labor ℓ, it must be that $w = \left(R^{\mathrm{CB}}\right)^{-2}$.

A.1.17 Proof of Proposition 11

Solving for the equilibrium again reduces to solving the farmer's problem with binding incentive and budget constraints. With the prices given in Lemma 6 these equations are

$$R^{\mathrm{CB}} \left(y - R^{\mathrm{CB}}\right) = (1 - \delta)y \tag{67}$$

and

$$i + \left(R^{\mathrm{CB}}\right)^{-2} \ell^f = e + B \tag{68}$$

where $y = A \max \{\alpha i, \ell\}$ and, in equilibrium, $i = \alpha \ell^f$. From the budget constraint we find that

$$\ell^f = \frac{\alpha \left(R^{\mathrm{CB}}\right)^2 (e + B)}{\alpha + \left(R^{\mathrm{CB}}\right)^2} \tag{69}$$

and, combining the above with the incentive constraint,

$$B = \frac{\alpha A \left(R^{\mathrm{CB}} - 1 + \delta\right) e}{\alpha + \left(R^{\mathrm{CB}}\right)^2 - \alpha A \left(R^{\mathrm{CB}} - 1 + \delta\right)}. \tag{70}$$

This gives the following equilibrium allocation:

$$\ell = \frac{\alpha \left(R^{\mathrm{CB}}\right)^2 e}{\alpha + \left(R^{\mathrm{CB}}\right)^2 - \alpha A \left(R^{\mathrm{CB}} - 1 + \delta\right)}, \tag{71}$$

$$i = \frac{\left(R^{\mathrm{CB}}\right)^2 e}{\alpha + \left(R^{\mathrm{CB}}\right)^2 - \alpha A \left(R^{\mathrm{CB}} - 1 + \delta\right)}. \tag{72}$$

We use the allocation to write down the liquidity multiplier Λ as

$$\Lambda = \frac{i + w\ell}{e} \tag{73}$$

$$= \frac{\alpha + \left(R^{\mathrm{CB}}\right)^2}{\alpha + \left(R^{\mathrm{CB}}\right)^2 - \alpha A \left(R^{\mathrm{CB}} - 1 + \delta\right)}. \tag{74}$$

We now compute the derivative of Λ with respect to R^{CB} to show when increasing R^{CB}

increases Λ:

$$\frac{\partial \Lambda}{\partial R^{\mathrm{CB}}} = \frac{2R^{\mathrm{CB}}\left[\alpha + \left(R^{\mathrm{CB}}\right)^2 - \alpha A\left(R^{\mathrm{CB}} - 1 + \delta\right)\right] - \left(\left(R^{\mathrm{CB}}\right)^2 + \alpha\right)\left(2R^{\mathrm{CB}} - \alpha A\right)}{\left[\alpha + (R^{\mathrm{CB}})^2 - \alpha A\left(R^{\mathrm{CB}} - 1 + \delta\right)\right]}$$

$$= \frac{\alpha + 2(1 - \delta)R^{\mathrm{CB}} - \left(R^{\mathrm{CB}}\right)^2}{\left[\alpha + (R^{\mathrm{CB}})^2 - \alpha A\left(R^{\mathrm{CB}} - 1 + \delta\right)\right]^2}.$$

This is positive exactly when $\alpha + 2R^{\mathrm{CB}}(1 - \delta) > \left(R^{\mathrm{CB}}\right)^2$ as stated in the proposition.

A.2 The Interbank Market and the Incentive Constraint

In Subsection 3.2 we argue that, as long as there is an interbank market, the mechanism by which warehouses' superior storage technology circumvents the non pledgeability problem is robust to the possibility that a borrower may deposit in a different warehouse than he borrowed from. Here we show that for *any* reasonable interbank market price and any positive cost of switching warehouses, a farmer will always strictly prefer to deposit with the warehouse he borrowed from (if the switching cost is zero, he still weakly prefers to deposit in the warehouse he borrowed from). We include this as a separate argument outside the baseline model because the analysis is game theoretic, while our solution concept in the baseline model is competitive equilibrium.

Consider a farmer with grain g and outstanding debt with face value $F < g$ to a warehouse, called Warehouse 1. Assume that deposit rates are one (this is a result of competition in the full model, stated in Lemma **??**). The farmer can deposit his grain in Warehouse 1 or in a different warehouse, Warehouse 2. We assume that if the farmer deposits in Warehouse 2 he bears a switching cost ε. After the farmer has deposited in a warehouse, Warehouse 1 may sell the farmer's debt to Warehouse 2 at an exogenous price p. If a warehouse has both the farmer's debt and his deposit, the warehouse may seize an amount F of the farmer's deposit; otherwise, the warehouse that holds the farmer's debt cannot collect on it. Finally, the warehouse that has accepted the deposit repays the farmer (net of any seized grain). We focus on $\varepsilon > 0$ and $p \in (0, F)$. Formally, the timing is as follows.

1. The farmer deposits g in Warehouse 1 or Warehouse 2

2. Warehouse 1 sells the farmer's debt to Warehouse 2 or does not

3. If Warehouse 1 or Warehouse 2 has both the debt and the deposit, it seizes an amount F of the deposit

4. The warehouse holding the deposit repays the farmer's deposit (net of seized grain)

Note that in the game there are only two choices: first the farmer chooses Warehouse 1 or Warehouse 2 and second Warehouse 1 chooses to sell or not to sell. We have assumed seizure and deposit repayment as automatic.

We now proceed to solve the game by backward induction. We first consider the case in which the farmer deposits in Warehouse 1. In this case, Warehouse 1 gets p if it sells the farmer's debt and F if it does not sell the farmer's debt. Since $F > p$, Warehouse 1 does not sell the farmer's debt. The farmer's payoff from depositing in Warehouse 1 is thus $g - F$.

Now consider the case in which the farmer deposits in Warehouse 2, bearing the switching cost ε. In this case, Warehouse 1 gets p if it sells the farmer's debt and zero if it does not sell the farmer's debt. Since $p > 0$, Warehouse 1 sells the farmer's debt to Warehouse 2. Warehouse 2 now holds both the farmer's deposit and his debt and therefore seizes an amount F. The farmer's payoff from depositing in Warehouse 2 is thus $g - F - \varepsilon$.

Now turn do the farmer's choice of where to deposit. If he deposits in Warehouse 1 he receives $g - F$ and if he deposits in Warehouse 2 he receives $g - F - \varepsilon$. Since $\varepsilon > 0$, the farmer prefers to deposit in Warehouse 1. We state this result in a proposition for emphasis.

PROPOSITION 13. *For any positive switching cost and any positive interbank price less than the face value of debt, the farmer deposits in the warehouse he borrowed from in the subgame perfect equilibrium.*

This result says that a farmer cannot circumvent a warehouse's ability to enforce contracts by depositing in a warehouse different from the one he borrowed from. Note that this result does not depend on the fair pricing of debt in the interbank market, it holds for any price less than the face value of debt. Since the farmer anticipates that no matter the interbank price of his debt, the warehouse he deposits with will ultimately hold his debt and then seize his deposit, the farmer prefers to deposit in the warehouse he borrowed from. A warehouse anticipates this when it makes loans, so the deposit constraint is as described in equation (DC) in Subsection 3.2.

A.3   Connection with the Relending Model

Here we explain how the first-best outcome above has an interpretation in the *relending* model. In the relending model, banks take deposits, they use these deposits to make loans, which are later deposited again, and then lent out again, and so on. That the bank has a deposit before before it makes loans is the "deposits-first" view of bank balance sheets. It contrasts with the "loans-first" perspective that we have stressed thus far. In this loans-first view, banks create deposits (viz. fake receipts) when they

make loans. We show below that relending model yields the same first-best outcome as stated in Proposition 1, implying that there is not necessarily a conflict between the deposits-first and loans-first perspectives.

In relending model, there is a *money multiplier* which describes how much money the banking system creates from an initial deposit. In textbook treatments the money multiplier is the reciprocal of the reserve requirement, but we demonstrate below that in our model it depends on the production technology.

Here we consider a sequential view of the equilibrium, analogous to Walrasian tâtonnement, in which each player acts optimally in sequence of rounds. In Round n, farmers divide their grain e_n between capital investment i_n and labor investment $w\ell_n$. Since the farmers' production function is $A \min\{\alpha i, \ell\}$, they set $\alpha i_n = \ell_n$ in each round. Further, since wages are one, $i_n + \ell_n = e_n$ and $w\ell_n = \ell_n$. Thus,

$$i_n = \frac{e_n}{1+\alpha} \quad \text{and} \quad \ell_n = \frac{\alpha e_n}{1+\alpha}. \tag{75}$$

Laborers then receive wages $w\ell_n = \alpha e_n/(1+\alpha)$ which they deposit directly in warehouses, $d_n^l = \alpha e_n/(1+\alpha)$. Finally, in the first-best, warehouses lend out *all* their grain, $L_n = d_n^l = \alpha e_n/(1+\alpha)$. Round $n+1$ begins with farmers having the borrowed grain $e_{n+1} = L_n$. One round of this process is represented pictorially in Figure 7.

Now, note from the computations above that

$$e_{n+1} = \frac{\alpha e_n}{1+\alpha} \tag{76}$$

From here, we compute e_n recursively in terms of e_0 as

$$e_n = \left(\frac{\alpha}{1+\alpha}\right)^n e_0. \tag{77}$$

We now compute the first-best allocations of capital and labor investment given the relending model. To do so, we sum up the per-round investments i_n and ℓ_n over an infinite number of rounds, given the initial endowment $e_0 = e$, to recover
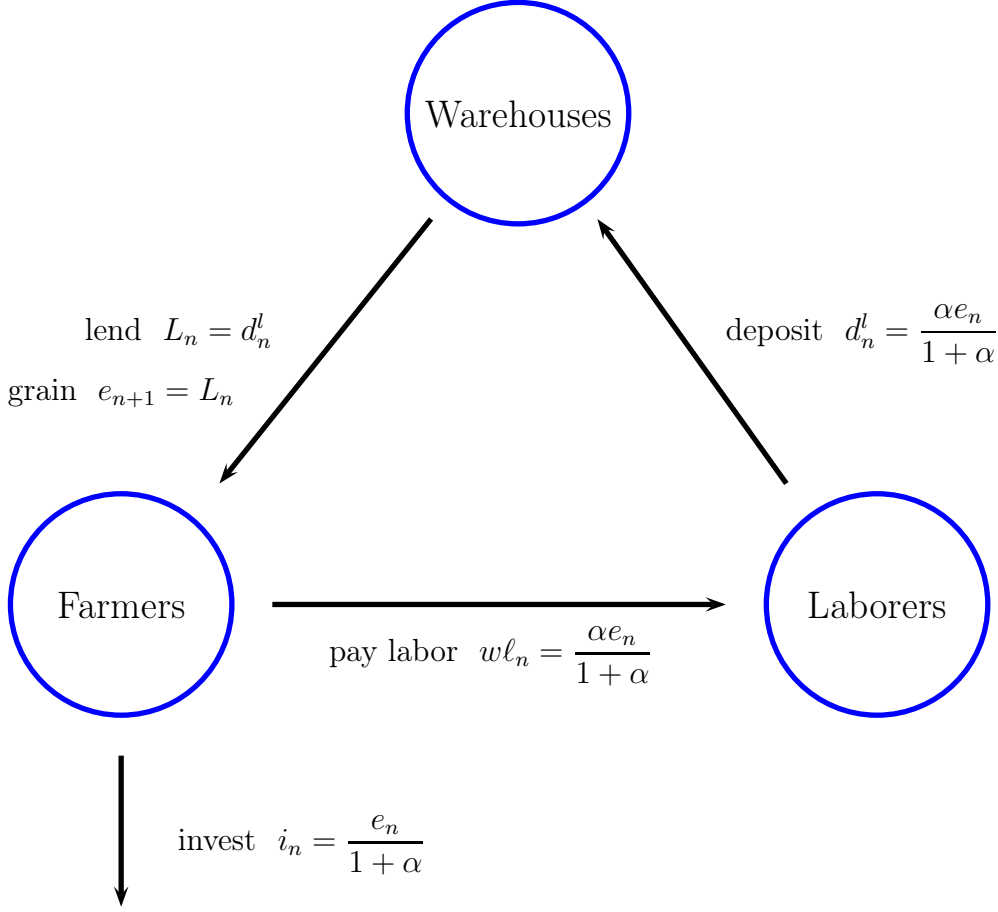
$$i_{\text{fb}} = \sum_{n=0}^{\infty} i_n \tag{78}$$

$$= \sum_{n=0}^{\infty} \frac{e_n}{1+\alpha} \tag{79}$$

$$= \sum_{n=0}^{\infty} \frac{1}{1+\alpha}\left(\frac{\alpha}{1+\alpha}\right)^n e \tag{80}$$

$$= e, \tag{81}$$

having used equation (77) and the formula for the sum of a geometric series. Likewise, we find that

$$\ell_{\text{fb}} = \sum_{n=0}^{\infty} \ell_n \tag{82}$$

$$= \sum_{n=0}^{\infty} \frac{\alpha e_n}{1+\alpha} \tag{83}$$

$$= \sum_{n=0}^{\infty} \frac{\alpha}{1+\alpha} \left(\frac{\alpha}{1+\alpha}\right)^n e \tag{84}$$

$$= \alpha e. \tag{85}$$

The expressions for i_{fb} and ℓ_{fb} under the relending model coincide with those in Proposition 1. Further, they suggest a natural money multiplier, stemming from the production function. Of each unit of grain in the economy, a proportion $1/(1+\alpha)$ is invested in

capital and cannot be reused. The remainder is invested in labor, or working capital, and therefore deposited back in the warehouse for reuse. These deposits are lend out, and a proportion $1/(1 + \alpha)$ is again invested in capital and the remainder is invested in labor, which is deposited and reused. Thus, for every unit of grain, the amount that can be invested is

$$\Lambda_{\mathrm{fb}} = \sum_{n=0}^{\infty} \left(\frac{\alpha}{1 + \alpha} \right)^n = 1 + \alpha. \tag{86}$$

The multiplier Λ_{fb} corresponds to a money multiplier in classical banking models. This is the liquidity multiplier of Section 6. Note that the more important working capital (labor ℓ) is to production relative to physical capital (grain i), i.e. the higher is α, the higher is the multiplier. Private money expands the money supply only to create working capital. It cannot, of course, create physical capital out of thin air.

A.4 Table of Notations

Indices	
f	farmer index
l	laborer index
b	warehouse (bank) index
$t \in \{0,1,2\}$	time index

Prices	
R_t^D	deposit rate at Date t
R^L	lending rate at Date 0
w	wages at Date 0

Demand and Supply	
i	grain farmers invest at Date 0
ℓ^f	labor farmers demand at Date 0
ℓ^l	labor laborers supply at Date 0
s_t^j	grain stored by player j at Date t
d_t^f	grain deposited in warehouses by farmers at Date t
d_t^l	grain deposited in warehouses by laborers at Date t
B	loans demanded by farmers at Date 0
L	loans supplied by warehouses at Date 0
D_t	overall deposits in warehouse at Date t

Production and Consumption	
y	farmers' output at Date 1
c^j	consumption of player j at Date 2

Parameters	
δ	depreciation rate with private storage
A	productivity
α	ratio of labor to grain in farmers' production
e	farmers' (Date 0) endowment
E	warehouses' (Date 1) endowment (extension in Subsection 7.3)
γ	cost parameter (extension in Subsection 7.3)

Other Variables	
g_1^f	farmer's grain holding at Date 1
Λ	liquidity multiplier
θ	liquidity ratio (extension in Subsection 7.1)
Δ	probability of spoilage (extension in Subsection 7.3)
R^{CB}	central bank rate (extension in Subsection 7.4)

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