

# INVESTMENT MANDATES AND THE DOWNSIDE OF PRECISE CREDIT RATINGS \*

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## Abstract

In a problem of delegated portfolio choice, competitive risk-averse agents offer a risk-averse investor contracts depending on portfolio weights and final wealth as well as on a public signal, for example an asset's credit rating. The optimal contract is affine in wealth and implements both efficient investment and optimal risk-sharing for each realization of the public signal, but agents' competition drives them to write the public signal into their contracts and prevent risk-sharing over it, a result reminiscent of Hirshleifer (1971). We comment on applications to asset managers' investment mandates and advocate regulation of credit rating agencies to prohibit their publishing information in forms conducive to inclusion in rigid contracts.

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# 1 Introduction

Delegated asset managers hold upwards of seventy percent of US publicly traded equity, assuming responsibility for private wealth management based on expertise they have and their clients lack. Unfortunately, finance professionals' incentives can never be perfectly aligned with the interests of their capital providers; the problem epitomizes the theoretical trade-off between information and incentives for the economics of delegation and contracting and our model builds on a rich theoretical literature. The problem, in the partial equilibrium portfolio choice setting with asymmetric information, originates with Bhattacharya and Pfleiderer (1985), who consider the problem of an investor who must simultaneously screen talent and induce truth-telling. Their agents have CARA preferences and they focus on approximately risk-neutral investors. Stoughton (1993) modifies the setting to include a moral hazard problem by demanding that managers take a costly action in order to become informed and demonstrates the importance of nonlinear contracts. Palomino and Prat (2003) study the problem when the agent chooses the portfolio's riskiness unobservably and demonstrate that the optimal non-linear contract need not be complicated: his optimal contract is a bonus contract that pays a fixed fee above a threshold.

The economic spirit of our model resembles Palomino and Prat's, since we study agents' incentives to shift risk in an optimal contracting setting, but our structure is more truly a simplification of Bhattacharya and Pfleiderer's as we consider a problem of portfolio choice with hidden information but dispense with agents' heterogeneity. We add a public contracting variable that correlates with the agents' information and focus on risk-averse investors—thereby bringing risk-sharing to the foreground—and, further, solve for the optimal direct mechanism as a function of the players' risk aversions and agents' reservation payoff.

While motivated by the suspicion that contracting on public infor-

mation could mitigate incentive problems—inspired in particular by funds’ investment mandates based on credit ratings—our results are reminiscent of papers relating risk-sharing to truthful revelation of private information on the one hand and speculation in the presence of public information on the other. In a 1984 paper about information revelation and joint production given a social planner’s sharing rule, Wilson demonstrates that private knowledge may not lead to inefficient risk-sharing. A similar result in our model obviates the usefulness of the public signal; in fact, decreasing the informativeness of public information leads to Pareto improvements. Hirshleifer applied his famous 1971 argument that traders may be uniformly better off if they agree not to obtain privately valuable information to a market setting very different from our model of strategic agency, but his economics are robust: public information destroys risk-sharing and, since it fails to mitigate the agency problem, it does only harm.

## Model and Results

In the model, competitive agents compete in contracts before learning their private information or observing the correlated public signal. They offer contracts to the investor that can depend on the portfolio allocation between a risky and a riskless asset, the final wealth, and the public signal. The investor, knowing that the agents learn the true state, employs one to invest his wealth on his behalf. All players have quadratic utility, but the investor’s risk aversion differs from the agents’.

We firstly demonstrate that our extensive form game is equivalent to a family of principal-agent problems—one for each realization of the public signal—in which the investor offers the contract take-it-or-leave-it to a single agent; then we apply the revelation principle before transforming the agency problems into social planners’ problems for appropriate welfare weights. Since the efficient risk-sharing rule does not depend on the true state (the agent’s type in the formalism) and

the optimal investment does not depend on the welfare weight, the efficient sharing rule composed with the optimal investment implements the agent's truth-telling and thus efficiency. The contracts do depend on the public signal, which proves a valuable tool for the agents to compete for the investor's business.

We then rank the public signals by informativeness according to the coarseness of the sigma algebras they generate and demonstrate that ex ante—namely, in expectation across the family of principal-agent problems—coarser public information Pareto dominates finer public information. Our proof uses the law of iterated expectations to show that one random variable second-order stochastically dominates another and then the result that the expectation of a concave function of a dominated random variable is less than the expectation of a dominating one.

## **Credit Ratings and Investment Mandates: An Application and Policy Prescription**

Our 2012 paper “Overrating Agencies” examines credit rating agencies’ and securities issuers’ strategic behaviour when investors can purchase only rated securities, an assumption motivated firstly by regulatory restrictions under which many US asset managers operate. Learning that many such portfolio restrictions are endogenous, written by funds themselves in their investment mandates to clients, incited our work on the current paper. Credit ratings are a prime example of public information that investment funds contract upon, and our paper explains why even expert asset managers write contracts on signals that are to them uninformative: it gives them a competitive edge in boom times. Our results suggest that such mandates, ostensibly imposed to protect investors, only impair risk-sharing and thus welfare.

Both the global financial crisis that climaxed in 2008-2009 and the ensuing Eurozone sovereign debt crisis (the climax of which EU

politicians continue to fight to deter/postpone) have brought scrutiny to the major credit rating agencies. Much academic attention has focused on the agencies' incentives and information-provision (notably, Mathis et al. (2009), Bolton et al. (2009), Skreta and Veldkamp (2008), and Doherty et al. (2012) among many others), but few papers have addressed the question of the effect of credit ratings on financial institutions and markets. Kurlat and Veldkamp (2011) do explore the problem in a two-asset rational expectations equilibrium and also rediscover some of Hirshleifer's reasoning: announcing credit ratings makes investors worse off, since more information about the payoff of the risky asset makes their securities too alike, thus preventing diversification—viz. public information impedes risk-sharing. Their paper uses a cardinal welfare measure to suggest that government enforcement of information disclosure may hurt investors. Given our model examines only a narrow channel of the effect of credit ratings, our regulatory advice is less bold: broaden ratings categories and focus on qualitative reporting, i.e. coarsen the contractible public information partition. Our suggestion jives with regulators' assertions that institutions should quit responding robotically to ratings, as rigid contingent contracts fine-tuned to CRA announcements force them to. For example, the Financial Stability Board told the G20 Finance Ministers that “Institutional investors must not mechanistically rely on CRA ratings...[by limiting] the proportion of a portfolio that is CRA ratings-reliant.”

## 2 Model

The model constitutes an extensive game of incomplete information in which agents first compete in contracts in the hope of being employed by a single investor and then invest his capital on his behalf in assets with exogenous returns. The solution concept is perfect bayesian equilibrium.

## The Economy

The economy comprises a large number of agents, viewed as asset managers, with von Neuman–Morganstern utility  $u_A$  and outside option  $\bar{u}$  as well as a single investor with von Neuman–Morgenstern utility  $u_I$  and one unit of initial wealth. There are two securities, a risk-free bond with gross return  $R_f$  and a risky asset with random gross return  $\tilde{R}$ ; initially no one knows the distribution of  $\tilde{R}$ . Finally, a public signal  $\tilde{\rho}$  is informative about the distribution of returns. Call  $\tilde{\rho}$  the credit rating of the risky security.

Two key assumptions give the model structure. Firstly, all players have quadratic utility.

$$u_n(W) = -\frac{1}{2}(\alpha_n - W)^2 \quad (1)$$

for  $n \in \{A, I\}$ . The investor differs from the agents in his risk aversion (note that the coefficient of absolute risk aversion is  $(\alpha_i - W)^{-1}$  so  $\alpha_i$  represents risk tolerance).

Secondly, the mean return  $\bar{R}$  of the risky asset is known. Since, with quadratic utility, players' expected utility depends only on the mean and variance of the distribution, summarize the unknown payoff-relevant component of the distribution with the random variance  $\tilde{\sigma}^2$ ,

$$\sigma^2 := \text{Var} \left[ \tilde{R} \mid \tilde{\sigma} = \sigma \right]. \quad (2)$$

With this notation the assumption that all players know the mean return of the risky asset reads

$$\mathbb{E} \left[ \tilde{R} \mid \tilde{\sigma} = \sigma \right] = \bar{R} \quad (3)$$

for each  $\sigma^2$ . Note that this assumption implies that the credit rating is informative only about the asset's risk and not about its expected return,

$$\mathbb{E} \left[ \tilde{R} \mid \tilde{\rho} = \rho \right] = \mathbb{E}[\tilde{R}] \quad (4)$$

but, in general,

$$\mathbb{E} [\tilde{\sigma}^2 \mid \tilde{\rho} = \rho] \neq \mathbb{E} [\tilde{\sigma}^2] . \quad (5)$$

With these preferences, players' marginal utility is decreasing when their wealth is large. To prevent its unrealistic implications, we aim to restrict the set of possible realizations of final wealth so that

$$\text{supp } \tilde{w} \subset [0, \alpha_I + \alpha_A), \quad (6)$$

which will ensure the equilibrium contract satisfies our feasibility conditions (cf. equation 10). To this end, make the technical assumption that return on the risky asset is not too fat-tailed according to

$$(\bar{R} - R_f)(R - \bar{R}) \leq \sigma^2 \quad (7)$$

for all pairs  $(\sigma, R)$ .<sup>1</sup>

## **Actions and Contracts**

The investor's only action is employing an agent  $a$  who then forms a verifiable portfolio with weight  $x$  in the risky asset and  $1 - x$  in the bond with the investor's capital. The investor wishes to delegate investment to an agent because he is better informed but anticipates a misalignment of investment incentives since the investor's risk tolerance differs from the agents'.

Contracts attempt to align incentives to mitigate the downside of delegated asset management, allocating decision rights to the players with the most information. Critically, credit ratings are verifiable but agents' true information about the distribution of returns is not. Thus the contracting variables are credit rating  $\rho$ , the portfolio weight  $x$ ,

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<sup>1</sup>Conditions 6 and 7 are sufficient conditions for increasing marginal utility given our equilibrium.

and the final wealth, denoted

$$\tilde{w} \equiv w(x, \tilde{R}) := R_f + x(\tilde{R} - R_f). \quad (8)$$

Assume that agents' contracts do not depend on other agents' contracts. Thus write that each agent  $a$  offers a contract

$$\Phi_a : (w, x, \rho) \mapsto \Phi_a(w, x, \rho), \quad (9)$$

but to economize on notation we often omit the contract's arguments and write  $\Phi(\tilde{w})$  for  $\Phi((w(x, \tilde{R}), x, \rho))$ . A feasible contract is a Lebesgue measurable function such that

$$w - \alpha_I < \Phi(w) < \alpha_A, \quad (10)$$

which ensures marginal utility is positive.

There is full commitment.

The dependence of contracts on portfolio weights and credit ratings are the investment mandates in the model.

## Timing

Aiming to understand why asset managers themselves use investment mandates in addition to or instead of performance incentives—contract on  $x$  and  $\rho$  and not just  $w$ —we have agents offer the contracts in our model. While the problem is ultimately equivalent to one in which the investor offers the contract to a single agent take-it-or-leave-it, we think that our set-up is important both to get the information structure right in the single-agent model and to understand applications and larger economics better.

After agents announce their contracts, the investor observes the credit rating and employs an agent who, knowing the true distribution of returns, goes on to form a portfolio with the investor's wealth. Finally, the assets pay off and players divide final wealth according to



the initial contract. Formally, the timing is as follows:

1. Agents simultaneously offer contracts  $\Phi_a$ .
2. The variance of the risky security realizes,  $\tilde{\sigma}^2 = \sigma^2$  and ratings are released,  $\tilde{\rho} = \rho$ .
3. The principal observes the profile of contracts  $\{\Phi_a\}_a$  and credit rating  $\rho$  and hires an agent  $a^*$ .
4. Agent  $a^*$  invests  $x^*$  in the risky asset.
5. The return of the risky asset realize,  $\tilde{R} = R$ , and final wealth

$$w = R_f + x^*(R - R_f) \quad (11)$$

is distributed such that agent  $a^*$  is awarded  $\Phi_{a^*}(w)$  and the investor keeps  $w - \Phi_{a^*}(w)$ .

Note that key to our timing is that players learn ratings after agents offer contracts but before investors have parted with their cash. Since employing lawyers to formalize the documents is both slow and costly for delegated asset managers, agents' fixing  $\Phi$  before knowing  $\rho$  is consistent with our application. As the next section's results demonstrate, assuming ratings realize before, but do not change after, the investor's delegation decision is equivalent (in welfare and allocation terms) to the richer model in which credit ratings are also updated after the investor has committed to an agent. (The model, however, does not speak to the fully dynamic situation in which asset managers invest the investor's capital in different assets over time.)

### 3 Results

The main result that coarser credit ratings lead to Pareto improvements follows from first transforming the extensive form to look like a classical principal-agent problem and then rewriting it as a social planner's problem, where the challenge is to implement truth-telling

and optimal risk-sharing simultaneously; the relationship of our result to Wilson's 1984 theorem on optimal sharing rules for joint production with dispersed information becomes apparent in this final formulation.

### Competition Is Rating-by-Rating

The first lemma states that competition in contracts is Bertrand-like in the sense that the employed agent will receive his reservation utility conditional on any realization of the credit rating  $\tilde{\rho}$ ; further the agents act so as to maximize the investor's expected utility conditional on every  $\rho$  subject to their participation constraints.

**Lemma 3.1.** *If  $\Phi$  is the contract of the agent employed given rating  $\hat{\rho}$  and there is another contract  $\hat{\Phi}$  such that*

$$\mathbb{E} \left[ u_I(\tilde{w} - \hat{\Phi}(\tilde{w})) \mid \tilde{\rho} = \hat{\rho} \right] > \mathbb{E} \left[ u_I(\tilde{w} - \Phi(\tilde{w})) \mid \tilde{\rho} = \hat{\rho} \right], \quad (12)$$

*then*

$$\mathbb{E} \left[ u_A(\hat{\Phi}(\tilde{w})) \mid \tilde{\rho} = \hat{\rho} \right] < \bar{u}. \quad (13)$$

*Proof.* Suppose, in anticipation of a contradiction, an equilibrium in which the employed agent offers contract  $\Phi$  given credit rating  $\hat{\rho}$  and there is another contract  $\hat{\Phi}$  such that

$$\mathbb{E} \left[ u_I(\tilde{w} - \hat{\Phi}(\tilde{w})) \mid \tilde{\rho} = \hat{\rho} \right] > \mathbb{E} \left[ u_I(\tilde{w} - \Phi(\tilde{w})) \mid \tilde{\rho} = \hat{\rho} \right] \quad (14)$$

and

$$\mathbb{E} \left[ u_A(\hat{\Phi}(\tilde{w})) \mid \tilde{\rho} = \hat{\rho} \right] \geq \bar{u}. \quad (15)$$

Suppose that agent  $\hat{A}$  offers the contract  $\hat{\Phi}_\varepsilon$  constructed from  $\hat{\Phi}$  given  $\hat{\rho}$

$$\hat{\Phi}_\varepsilon(w, x, \hat{\rho}) := \alpha_A - \sqrt{(\alpha_A - \hat{\Phi}(w, x, \hat{\rho}))^2 - 2\varepsilon} \quad (16)$$

and that his action is according to the supposed equilibrium if  $\rho \neq \hat{\rho}$ .

Note that

$$u_A(\hat{\Phi}_\varepsilon) = u_A(\hat{\Phi}) + \varepsilon \quad (17)$$

immediately by construction and the quadric form of the agents' utility. The contract does not change agents' incentives and the same portfolio weight  $x$  is chosen under either contract. Since  $x$  is unchanged and  $u_I(w - \hat{\Phi}_\varepsilon(w))$  is continuous in  $\varepsilon$ , for  $\varepsilon > 0$  sufficiently small

$$\mathbb{E} \left[ u_I(\tilde{w} - \hat{\Phi}_\varepsilon(\tilde{w})) \mid \tilde{\rho} = \hat{\rho} \right] > \mathbb{E} \left[ u_I(\tilde{w} - \Phi(\tilde{w})) \mid \tilde{\rho} = \rho \right]. \quad (18)$$

Thus the investor will employ agent  $\hat{A}$  who will receive utility greater than his utility at the supposed equilibrium given rating  $\hat{\rho}$  where he was unemployed and obtaining  $\bar{u}$  (and the same utility given all other ratings). Thus  $\hat{\Phi}_\varepsilon$  is a profitable deviation for agent  $\hat{A}$  and  $\Phi$  cannot be the contract of an agent employed at equilibrium given  $\hat{\rho}$ .  $\square$

### Principal-Agent Formulation and Revelation Principle

Lemma 3.1 asserts that agents compete rating-by-rating, maximizing investor welfare subject to their participation constraints, that is to say that for every realization  $\rho$  of the credit ratings the contract of the employed agent and the corresponding portfolio weight solve the principal-agent problem:

$$\left\{ \begin{array}{l} \text{Maximize } \mathbb{E} \left[ u_I(w(x, \tilde{R}) - \Phi(w(x, \tilde{R}), x, \rho)) \mid \tilde{\rho} = \rho \right] \\ \text{subject to } \mathbb{E} \left[ u_A(\Phi(w(x, \tilde{R}), x, \rho)) \mid \tilde{\rho} = \rho \right] \geq \bar{u} \text{ and} \\ x \in \arg \max \left\{ \mathbb{E} \left[ u_A(\Phi(w(\xi, \tilde{R}), \xi, \rho)) \mid \tilde{\sigma} = \sigma \right] ; \xi \in \mathbb{R} \right\} \end{array} \right. \quad (\text{P-A})$$

over all feasible contracts  $\Phi$ . Applying the revelation principle allows us to restrict attention to direct mechanisms

$$\varphi(w; \hat{\sigma}, \rho) := \Phi(w, x(\hat{\sigma}), \rho) \quad (19)$$

where  $x$  is an incentive compatible portfolio weight given  $\Phi$ .

Replace the incentive compatibility of the portfolio allocation  $x$  with the truth-telling condition  $\hat{\sigma} = \text{Id}$ :

$$\left\{ \begin{array}{l} \text{Maximize } \mathbb{E} \left[ u_I \left( W(\tilde{\sigma}, \tilde{R}) - \varphi(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho) \right) \mid \tilde{\rho} = \rho \right] \\ \text{subject to } \mathbb{E} \left[ u_A \left( \varphi(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho) \right) \mid \tilde{\rho} = \rho \right] \geq \bar{u} \text{ and} \\ \sigma \in \arg \max \left\{ \mathbb{E} \left[ u_A \left( \varphi(W(\hat{\sigma}, \tilde{R}), \hat{\sigma}, \rho) \mid \tilde{\sigma} = \sigma \right) \right] ; \hat{\sigma} \in \mathbb{R} \right\} \end{array} \right. \quad (\text{P-A(D)})$$

over all feasible contracts  $\varphi$  where  $W$  denotes the wealth as a function of the report  $\hat{\sigma}$  rather than of the portfolio weight  $x$  directly,

$$W(\hat{\sigma}, R) := w(x(\hat{\sigma}), R). \quad (20)$$

Note while the contract and wealth do not depend directly on the true variance  $\sigma^2$ , we already plugged  $\hat{\sigma}(\sigma) = \sigma$  from the truth-telling condition into the statement of the problem.

### Equilibrium Contract as the Solution of a Social Planner's Problem

Use the method of Lagrange multipliers to eliminate the participation constraint and say that the problem is to maximize

$$\mathbb{E} \left[ u_I \left( W(\tilde{\sigma}, \tilde{R}) - \varphi(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho) \right) + \mu \left( u_A \left( \varphi(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho) \right) - \bar{u} \right) \mid \tilde{\rho} = \rho \right] \quad (21)$$

subject to

$$\sigma \in \arg \max \left\{ \mathbb{E} \left[ u_A(\varphi(W(\hat{\sigma}, \tilde{R}), \hat{\sigma}, \rho) \mid \tilde{\sigma} = \sigma) \right] ; \hat{\sigma} \in \mathbb{R} \right\} \quad (22)$$

over feasible  $\varphi$  and  $\mu \in \mathbb{R}$ . Defining the social welfare given credit rating  $\rho$  (with weight one on the investor and  $\mu$  on the agent) as

$$\begin{aligned} \mathcal{S}_{\mu, \rho}(x)[\varphi] := & \mathbb{E} \left[ u_I \left( W(\tilde{\sigma}, \tilde{R}) - \varphi(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho) \right) \mid \tilde{\rho} = \rho \right] + \\ & + \mu \mathbb{E} \left[ u_A \left( \varphi(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho) \right) \mid \tilde{\rho} = \rho \right], \end{aligned} \quad (\text{SW})$$

observe that (since lemma 3.1 says that the agent's participation constraint binds) the principal-agent problem is the social planner's problem  $\text{SP}(\mu, \rho)$  to maximize  $\mathcal{S}$  given  $\rho$  subject to truth-telling whenever  $\mu$  is the welfare weight such that the agent breaks even,

$$\mathbb{E} \left[ u_A \left( \varphi_{\mu, \rho}(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho) \right) \mid \tilde{\rho} = \rho \right] = \bar{u}, \quad (23)$$

where  $\varphi_{\mu, \rho}$  is the solution to the problem.

Transforming the game into a social planner's problem combined with the fixed-point problem reveals that the task is to trade off efficient risk sharing with implementing truth-telling.

## The Efficient Sharing Rule Implements Truth-telling

Step back from the game under scrutiny to observe that the optimal risk sharing rule is linear for all  $\mu$  and  $\rho$  by maximizing

$$\mathbb{E} \left[ u_I(w - \phi(w)) + \mu u_A(\phi(w)) \mid \tilde{\sigma} = \sigma \right] \quad (24)$$

unconstrained over all feasible  $\phi$ , which immediately decouples into a family of one-dimensional optimization problems solvable by differentiation:

$$u_I'(w - \phi_\mu(w)) = \mu u_A'(\phi_\mu(w)) \quad (25)$$

or, plugging in quadratic utility,

$$w - \phi_\mu(w) - \alpha_I = \mu(\phi_\mu(w) - \alpha_A) \quad (26)$$

for all  $w$ . Thus the unconstrained efficient sharing rule is

$$\phi_\mu(w) = \alpha_A + \frac{w - \alpha_I - \alpha_A}{1 + \mu}, \quad (27)$$

which is feasible whenever  $\mu > 0$  and assumption 6 holds. Since the standard deviation  $\sigma$  does not enter the expression, the social planner need not know the true variance to implement optimal risk sharing.

Given the optimal sharing rule, the expression for the corresponding optimal investment  $X_\mu$  in the risky security will be useful. The social planner finds it by computing the maximum of

$$\begin{aligned} & \mathbb{E} \left[ u_I \left( R_f + x(\tilde{R} - R_f) - \phi_\mu(R_f + x(\tilde{R} - R_f)) \right) \middle| \tilde{\sigma} = \sigma \right] \\ & + \mu \mathbb{E} \left[ u_A \left( \phi_\mu(R_f + x(\tilde{R} - R_f)) \right) \middle| \tilde{\sigma} = \sigma \right], \end{aligned} \quad (28)$$

over all  $x$ . Mechanical computations collected in Appendix A.1 reveal that the optimal investment is

$$X_\mu(\sigma) \equiv X(\sigma) = \frac{(\bar{R} - R_f)(\alpha_I + \alpha_A - R_f)}{\sigma^2 + (\bar{R} - R_f)^2}. \quad (29)$$

Since the optimal investment does not depend on the welfare weight  $\mu$ , it remains unchanged in the extreme when the social planner cares only about the agent and thus

$$\begin{aligned} & \mathbb{E} \left[ u_A \left( \phi_\mu(R_f + X(\sigma)(\tilde{R} - R_f)) \right) \middle| \tilde{\sigma} = \sigma \right] \\ & \geq \mathbb{E} \left[ u_A \left( \phi_\mu(R_f + x(\tilde{R} - R_f)) \right) \middle| \tilde{\sigma} = \sigma \right] \end{aligned} \quad (30)$$

for all  $x \in \mathbb{R}$ , in particular for all  $x \in \text{Im } X$  so

$$\begin{aligned} & \mathbb{E} \left[ u_A \left( \phi_\mu \left( R_f + X(\sigma)(\tilde{R} - R_f) \right) \right) \middle| \tilde{\sigma} = \sigma \right] \\ & \geq \mathbb{E} \left[ u_A \left( \phi_\mu \left( R_f + X(\hat{\sigma})(\tilde{R} - R_f) \right) \right) \middle| \tilde{\sigma} = \sigma \right] \end{aligned} \quad (31)$$

for all  $\hat{\sigma}$ , which proves the following essential lemma.

**Lemma 3.2.** *The efficient sharing rule composed with the optimal investment  $\phi_\mu \circ X$ ,*

$$\varphi \left( R_f + X(\hat{\sigma})(\tilde{R} - R_f), \hat{\sigma}, \rho \right) = \phi_\mu \left( R_f + X(\hat{\sigma})(\tilde{R} - R_f) \right), \quad (32)$$

*implements the agent's truth-telling for any credit rating  $\rho$ .*

Lemma 3.2 is closely related to Wilson's 1984 result on the "revelation of information for joint production", where he proves that when the efficient sharing rule is affine, truthful revelation is a Nash equilibrium. We import the methodology for connecting risk-sharing with implementation into the principal-agent setting, emphasizing the explicit (direct) implementation and, further, that the optimal sharing rule is the investor's optimal contract by the equivalence of the principal-agent problem and social planner's problem above. Note that Wilson's proof exploits that when the efficient sharing rule is affine its derivative is constant and cancels out of his problem's first-order condition; we instead use that in our case the optimal allocation is independent of the welfare weight.

### The Break-even Welfare Weight and Ex Ante Utility

In order to characterize the employed agent's contract via the social planner's problem, determine the welfare weight  $\mu_\rho$  given the credit rating  $\rho$ ; thanks to truth-telling, the equilibrium allocation depends

on the credit rating only via the participation constraint:

$$\mathbb{E} \left[ u_A \left( \phi_{\mu_\rho} \left( R_f + X(\tilde{\sigma})(\tilde{R} - R_f) \right) \right) \middle| \tilde{\rho} = \rho \right] = \bar{u}, \quad (33)$$

which, via string of calculations employing the law of iterated expectations (cf. Appendix A.2), says

$$(1 + \mu_\rho)^2 = \frac{(\alpha_I + \alpha_A - R_f)^2}{2|\bar{u}|} \mathbb{E} \left[ \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + (\tilde{R} - R_f)^2} \middle| \tilde{\rho} = \rho \right]. \quad (34)$$

A tangential remark: the mapping

$$\tilde{\sigma}^2 \mapsto \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + (\tilde{R} - R_f)^2} \quad (35)$$

under the expectation operator is concave, so that if the distribution of  $\tilde{\sigma}^2$  spreads out (for example in the the second-order stochastic dominance sense) then  $\mu_\rho$  decreases, suggesting that the more distribution risk the agent faces, the less the investor must compensate him despite his risk aversion, as captured by the social planner's lower welfare weight. The reason is that his investment decision comes after the realization of the variance, and thus the riskier decisions come with option value: when  $\tilde{\sigma}^2$  is very low he will invest a lot in the risky asset, while when it is high he will invest relatively more in the riskless bond.

Further, the equilibrium welfare weight provides a handy formula for the investor's equilibrium expected utility given the rating  $\rho$ ,

$$\mathbb{E} \left[ u_I \left( W(\tilde{\sigma}, \tilde{R}) - \varphi(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho) \right) \middle| \tilde{\rho} = \rho \right] = \bar{u} \mu_\rho^2 \quad (36)$$

(see Appendix A.3 for the short calculation) and thus his ex ante expected utility

$$\mathbb{E} \left[ u_I \left( W(\tilde{\sigma}, \tilde{R}) - \varphi(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho) \right) \right] = \bar{u} \mathbb{E} [\mu_\rho^2]. \quad (37)$$



## Main Result: Coarser Credit Ratings Are Pareto-Improving

Since competition means that agents always receive their reservation utilities, the main result that coarsening credit ratings makes everyone better-off follows from directly comparing the ex ante expected utility of the investor across ratings systems, using the formula above combined with the connection between convex functions, second-order stochastic dominance, and the law of iterated expectations.

**Proposition 3.1.** *Coarser credit ratings Pareto-dominate finer ones: for any ratings  $\tilde{\rho}_C$  and  $\tilde{\rho}_F$  such that  $\sigma(\tilde{\rho}_C) \subset \sigma(\tilde{\rho}_F)$ , the ex ante equilibrium utility of all agencies is weakly higher given  $\tilde{\rho}_C$  than  $\tilde{\rho}_F$ .*

*Proof.* Our proof has two main steps, firstly to show that the investor's ex ante expected utility is minus the expectation of a convex function,

$$\bar{u} \mathbb{E} [\mu_{\tilde{\rho}}^2] = -c \mathbb{E} \left[ f \left( \mathbb{E} [Y \mid \tilde{\rho}] \right) \right] \quad (38)$$

for  $c > 0$ ,  $f'' > 0$  and a random variable  $Y$ ; and secondly to show that the expectation conditional on coarse ratings second-order stochastically dominates the expectation conditional on fine ratings,

$$\mathbb{E} [Y \mid \tilde{\rho}_C] \stackrel{\text{SOSD}}{\succ} \mathbb{E} [Y \mid \tilde{\rho}_F], \quad (39)$$

whence utility is greater under coarse ratings because minus a convex function is a concave function, and, à la risk aversion, the expectation of a concave function of a stochastically dominated random variable is greater than the expectation of the function of the dominated variable.

Step 1: Rewrite the investor's ex ante expected utility:

$$\begin{aligned}
\bar{u} \mathbb{E} [\mu_{\tilde{\rho}}^2] &= \bar{u} \mathbb{E} \left[ \left( \sqrt{\frac{(\alpha_P + \alpha_A - R_f)^2}{2|\bar{u}|}} \mathbb{E} \left[ \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \mid \tilde{\rho} \right] - 1 \right)^2 \right] \\
&= \frac{\bar{u}(\alpha_P + \alpha_A - R_f)^2}{\sqrt{2}|\bar{u}|} \mathbb{E} \left[ \left[ \sqrt{\mathbb{E} \left[ \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \mid \tilde{\rho} \right]} - 1 \right]^2 \right] \\
&= -c \mathbb{E} \left[ f \left( \mathbb{E} [Y | \tilde{\rho}] \right) \right]
\end{aligned} \tag{40}$$

where

$$c := \sqrt{|\bar{u}|/2} (\alpha_P + \alpha_A - R_f)^2, \tag{41}$$

$$f(z) := (\sqrt{z} - 1)^2, \tag{42}$$

and

$$Y := \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2}. \tag{43}$$

Note that  $c > 0$  and  $f''(z) = z^{3/2}/2 > 0$ .

Step 2: By definition,

$$\mathbb{E} [Y | \tilde{\rho}_C] \stackrel{\text{SOSD}}{\succ} \mathbb{E} [Y | \tilde{\rho}_F] \tag{44}$$

if there exists a random variable  $\tilde{\varepsilon}$  such that

$$\mathbb{E} [Y | \tilde{\rho}_F] = \mathbb{E} [Y | \tilde{\rho}_C] + \tilde{\varepsilon} \tag{45}$$

and

$$\mathbb{E} [\tilde{\varepsilon} | \mathbb{E} [Y | \tilde{\rho}_C]] = 0. \tag{46}$$

For  $\tilde{\varepsilon} = \mathbb{E} [Y | \tilde{\rho}_F] - \mathbb{E} [Y | \tilde{\rho}_C]$  from the above, the condition is

$$\mathbb{E} \left[ \mathbb{E} [Y | \tilde{\rho}_F] - \mathbb{E} [Y | \tilde{\rho}_C] \mid \mathbb{E} [Y | \tilde{\rho}_C] \right] = 0 \tag{47}$$

or

$$\mathbb{E} \left[ \mathbb{E} [Y \mid \tilde{\rho}_F] \mid \mathbb{E} [Y \mid \tilde{\rho}_C] \right] = \mathbb{E} [Y \mid \tilde{\rho}_C]. \quad (48)$$

Given the assumption  $\sigma(\tilde{\rho}_C) \subset \sigma(\tilde{\rho}_F)$  and since conditioning destroys information— $\sigma(\mathbb{E} [Y \mid \tilde{\rho}_C]) \subset \sigma(\tilde{\rho}_C)$ —apply the law of iterated expectations firstly to add and then to delete conditioning information to calculate that

$$\mathbb{E} \left[ \mathbb{E} [Y \mid \tilde{\rho}_F] \mid \mathbb{E} [Y \mid \tilde{\rho}_C] \right] = \mathbb{E} \left[ \mathbb{E} \left[ \mathbb{E} [Y \mid \tilde{\rho}_F] \mid \rho_C \right] \mid \mathbb{E} [Y \mid \tilde{\rho}_C] \right] \quad (49)$$

$$= \mathbb{E} \left[ \mathbb{E} [Y \mid \tilde{\rho}_C] \mid \mathbb{E} [Y \mid \tilde{\rho}_C] \right] \quad (50)$$

$$= \mathbb{E} [Y \mid \rho_C], \quad (51)$$

as desired.  $\square$

## A Appendices

### A.1 Computation of Optimal Investment

The problem stated in line 28 to find the optimal investment  $X_\mu$  given the optimal sharing rule

$$\phi_\mu(w) = a + bw, \quad (52)$$

where the constants  $a$  and  $b$  are as in equation 27, is to maximize the expectation

$$\begin{aligned} -\frac{1}{2} \mathbb{E} \left[ \left( R_f + x(\tilde{R} - R_f) - a - b \left( R_f + x(\tilde{R} - R_f) \right) - \alpha_I \right)^2 \right. \\ \left. + \mu \left( \left( a + \left( R_f + x(\tilde{R} - R_f) \right) - \alpha_A \right)^2 \right) \mid \tilde{\sigma} = \sigma \right] \end{aligned} \quad (53)$$

over all  $x$ . Thus the first-order condition says that for optimum  $X_\mu$

$$\begin{aligned} \mathbb{E} \left[ (1-b)(\tilde{R} - R_f) \left( R_f + X_\mu(\tilde{R} - R_f) - a - b \left( R_f + X_\mu(\tilde{R} - R_f) \right) - \alpha_I \right) \right. \\ \left. + \mu b(\tilde{R} - R_f) \left( a + b \left( R_f + X_\mu(\tilde{R} - R_f) \right) - \alpha_A \right) \mid \tilde{\sigma} = \sigma \right] = 0 \end{aligned} \quad (54)$$

$$X_\mu = \frac{(\bar{R} - R_f)}{\mathbb{E}[(\tilde{R} - R_f)^2 \mid \tilde{\sigma} = \sigma]} \left( \frac{(1-b)(a + \alpha_I) - \mu b(a - \alpha_A)}{(1-b)^2 + \mu b^2} - R_f \right). \quad (55)$$

Substituting in for  $a$  and  $b$  from the expression in equation 27 gives that

$$(1-b)(a + \alpha_I) - \mu b(a - \alpha_A) = \frac{\mu(\alpha_A + \alpha_I)}{1 + \mu} \quad (56)$$

and

$$(1-b)^2 + b^2\mu = \frac{\mu}{1 + \mu} \quad (57)$$

therefore

$$X_\mu = \frac{(\bar{R} - R_f)(\alpha_I + \alpha_A - R_f)}{\mathbb{E}[(\tilde{R} - R_f)^2 \mid \tilde{\sigma} = \sigma]} \quad (58)$$

$$= \frac{(\bar{R} - R_f)(\alpha_I + \alpha_A - R_f)}{\sigma^2 + (\bar{R} - R_f)^2}. \quad (59)$$

## A.2 Computation of Optimal Investment

Immediately from plugging in the expressions for  $u_A$ ,  $\phi_{\mu_\rho}$ , and  $X$  into equation 33, observe that

$$\begin{aligned} 2|\bar{u}|(1 + \mu_\rho)^2 &= \mathbb{E} \left[ \left( R_f + \frac{(\bar{R} - R_f)(\alpha_I + \alpha_A - R_f)}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} (\tilde{R} - R_f) - \alpha_I - \alpha_A \right)^2 \middle| \tilde{\rho} = \rho \right] \\ &= (\alpha_I + \alpha_A - R_f)^2 \mathbb{E} \left[ \left( \frac{(\bar{R} - R_f)(\tilde{R} - R_f)}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} - 1 \right)^2 \middle| \tilde{\rho} = \rho \right] \\ &= (\alpha_I + \alpha_A - R_f)^2 \left\{ 1 - 2\mathbb{E} \left[ \frac{(\bar{R} - R_f)(\tilde{R} - R_f)}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \middle| \tilde{\rho} = \rho \right] + \right. \\ &\quad \left. + \mathbb{E} \left[ \left( \frac{(\bar{R} - R_f)(\tilde{R} - R_f)}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \right)^2 \middle| \tilde{\rho} = \rho \right] \right\}. \end{aligned} \quad (60)$$

Applying the law of iterated expectations gives

$$\begin{aligned} 1 - \frac{2|\bar{u}|(1 + \mu_\rho)^2}{(\alpha_I + \alpha_A - R_f)^2} &= 2\mathbb{E} \left[ \mathbb{E} \left[ \frac{(\bar{R} - R_f)(\tilde{R} - R_f)}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \middle| \tilde{\sigma} \right] \middle| \tilde{\rho} = \rho \right] - \mathbb{E} \left[ \mathbb{E} \left[ \left( \frac{(\bar{R} - R_f)(\tilde{R} - R_f)}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \right)^2 \middle| \tilde{\sigma} \right] \middle| \tilde{\rho} = \rho \right] \\ &= 2\mathbb{E} \left[ \frac{(\bar{R} - R_f)\mathbb{E}[(\tilde{R} - R_f) \mid \tilde{\sigma}]}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \middle| \tilde{\rho} = \rho \right] + \mathbb{E} \left[ \frac{(\bar{R} - R_f)^2 \mathbb{E}[(\tilde{R} - R_f)^2 \mid \tilde{\sigma}]}{(\tilde{\sigma}^2 + (\bar{R} - R_f)^2)^2} \middle| \tilde{\rho} = \rho \right] \end{aligned} \quad (61)$$

and since

$$\mathbb{E} \left[ (\tilde{R} - R_f)^2 \mid \tilde{\sigma} \right] = \tilde{\sigma}^2 + (\bar{R} - R_f)^2 \quad (62)$$

we have

$$\begin{aligned} & 1 - \frac{2|\bar{\mu}|(1 + \mu_\rho)^2}{(\alpha_I + \alpha_A - R_f)^2} \\ &= (\bar{R} - R_f)^2 \left\{ \mathbb{E} \left[ \frac{2}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \mid \tilde{\rho} = \rho \right] - \mathbb{E} \left[ \frac{1}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \mid \tilde{\rho} = \rho \right] \right\} \\ &= \mathbb{E} \left[ \frac{(\bar{R} - R_f)^2}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \mid \tilde{\rho} = \rho \right]. \end{aligned} \quad (63)$$

Finally, solve for  $\mu_\rho$  in equation 60 and cross multiply to recover equation 34.

### A.3 Computation of Expected Utility Given $\rho$

Plug in to equation 36 and compute, maintaining at first the shorthand

$$\tilde{w} = W(\sigma, R) = R_f + X(\sigma)(R - R_f), \quad (64)$$

that is:

$$\begin{aligned}
& \mathbb{E} \left[ u_I \left( W(\tilde{\sigma}, \tilde{R}) - \varphi(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho) \right) \middle| \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2} \mathbb{E} \left[ \left( \alpha_I - \tilde{w} + \phi_{\mu_\rho}(\tilde{w}) \right)^2 \middle| \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2} \mathbb{E} \left[ \alpha_I - \tilde{w} + \alpha_A + \frac{\tilde{w} - \alpha_I - \alpha_A}{1 + \mu_\rho} \middle| \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2} \mathbb{E} \left[ \alpha_I - \tilde{w} + \alpha_A + \frac{\tilde{w} - \alpha_I - \alpha_A}{1 + \mu_\rho} \middle| \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2} \left( \frac{\mu_\rho}{1 + \mu_\rho} \right)^2 \mathbb{E} \left[ (\alpha_I + \alpha_A - \tilde{w})^2 \middle| \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2} \left( \frac{\mu_\rho}{1 + \mu_\rho} \right)^2 \mathbb{E} \left[ \left( \alpha_I + \alpha_A - R_f - X(\tilde{\sigma})(\tilde{R} - R_f) \right)^2 \middle| \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2} \left( \frac{\mu_\rho}{1 + \mu_\rho} \right)^2 \mathbb{E} \left[ \left( \alpha_I + \alpha_A - R_f - (\alpha_I + \alpha_A - R_f) \frac{(\bar{R} - R_f)(\tilde{R} - R_f)}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \right)^2 \middle| \tilde{\rho} = \rho \right] \\
&= -\frac{(\alpha_I + \alpha_A - R_f)^2}{2} \left( \frac{\mu_\rho}{1 + \mu_\rho} \right)^2 \mathbb{E} \left[ \left( 1 - \frac{(\bar{R} - R_f)(\tilde{R} - R_f)}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \right)^2 \middle| \tilde{\rho} = \rho \right].
\end{aligned} \tag{65}$$

Now, from equation 60 above,

$$\mathbb{E} \left[ \left( 1 - \frac{(\bar{R} - R_f)(\tilde{R} - R_f)}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \right)^2 \middle| \tilde{\rho} = \rho \right] = 2|\bar{u}| \left( \frac{1 + \mu_\rho}{\alpha_I + \alpha_A - R_f} \right)^2, \tag{66}$$

so, finally,

$$\mathbb{E} \left[ u_I \left( W(\tilde{\sigma}, \tilde{R}) - \varphi(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho) \right) \middle| \tilde{\rho} = \rho \right] = \bar{u} \mu_\rho^2. \tag{67}$$

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