# MONEY RUNS*

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**Abstract**

We present a banking model in which bank debt, like banknotes and repos, circulates as "money" in decentralized secondary markets. We find that bank debt is susceptible to runs because secondary-market liquidity is subject to sudden, self-fulfilling dry-ups. When debt fails to circulate it is redeemed on demand in a "money run." Even though demandable debt exposes banks to costly runs, banks still want to issue it. To facilitate creating demandable money, banks pool investments and transform maturity/liquidity—they endogenously do something that resembles real-world banking. This money-creation rationale for banking does not rely on diversification of bank assets or liabilities.

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> In the use of money, every one is a trader.
>
> David Ricardo (1876)

# 1 Introduction

Bank debt was a major form of money in the early nineteenth-century United States. To get beer from the barman, you would exchange private banknotes over the counter (OTC). Banknotes were redeemable on demand and sudden redemptions—bank runs—were common.[1] If you held bank debt, you could get liquidity either by trading OTC or, alternatively, by demanding redemption from the issuing bank. But demanding redemption comes with the risk of a run. Why would you run on a bank rather than trade its debt in the market? In other words, why is bank debt susceptible to costly runs, even though it is tradeable? Moreover, why do banks choose to borrow via demandable debt, even though it exposes them to costly runs?

To give a new perspective on these questions, we focus on how banks create money by issuing liabilities that circulate in OTC markets, like banknotes did in the nineteenth century and much bank debt does today (see below). In the model, bank debt is susceptible to runs because liquidity in the OTC market is subject to sudden, self-fulfilling dry-ups. When debt fails to circulate it is redeemed on demand in a bank run, or "money run." Such runs were common in the nineteenth-century US, when depositors ran on banks after "the bank note that passed freely yesterday was rejected this morning."[2] Still, banks choose to issue

[1] Gorton (2012b) argues that it remains a theoretical challenge to understand how these runs arise and how they affect the design of bank liabilities that circulate as money. He says

> In the U.S. under state free banking laws banks were required to back their notes with state bonds. In the case of a bank failure—an inability to honor requests for cash from noteholders—the state bonds would be sold (by the state government) and the note holders paid off pro rata. Note holders were paid off pro rata, so there was no common pool problem. Yet, there was a run on banks (banknotes and deposits) during the Panic of 1857 (p. 15).

Further, he says

> Generating such [a run] event in a model seems harder when...the form of money [is such that] each "depositor" receives a bond as collateral. There is no common pool of assets on which bank debt holders have a claim. So, strategic considerations about coordinating with other agents do not arise. This is a challenge for theory and raises issues concerning notions of liquidity and collateral, and generally of the design of trading securities—private money (p. 2).

We generate such runs on bank debt in a model in which banks optimally design securities that circulate in secondary markets.

[2] Treasury Secretary Howell Cobb (1858), quoted in Gorton (2012a), p. 36. Cobb goes on to suggest that the failure of banks can cause the failure of their debt to circulate as money. We emphasize that the chain of causation can run in the other direction, in line with Gorton's (2012a) interpretation of bank panics in the

demandable debt, even though it exposes them to costly runs. In our model, the reason is that it increases their debt capacity, since the option to redeem on demand increases the price it trades at—unlike in Jacklin (1987), demandability and tradeability are complements. To create this circulating demandable debt, banks pool investments and transform maturity/liquidity, even absent the benefits of diversification emphasized in the literature. I.e., just to create money, they do something that looks like real-world banking. But, to do it effectively, they cannot hold enough liquidity to meet all redemptions at once. Hence, bank fragility is a necessary evil. Overall, our model reveals a new type of run, a new rationale for demandable debt, and a new raison d'être for banking, all of which are based on how bank liabilities circulate as money in the secondary market.

**Model preview.** A borrower B has an investment opportunity and needs to borrow from a creditor $C_0$ to fund it. The model is based on two key assumptions. First, there is a horizon mismatch, similar to that in Diamond and Dybvig (1983): $C_0$ may be hit by a liquidity shock before B's investment pays off. Second, B's debt is traded in an OTC market, similar to those in Trejos and Wright (1995) and Duffie, Gârleanu, and Pedersen (2005): if $C_0$ is hit by a liquidity shock before B's investment pays off, $C_0$ can match with a counterparty $C_1$ and bargain bilaterally to trade B's debt. Likewise, $C_1$ may be hit by a liquidity shock before B's investment pays off, in which case it can match with a counterparty $C_2$ and bargain bilaterally to trade B's debt, and so on. If B's debt is demandable, then a creditor may redeem it before the investment pays off, forcing B to liquidate inefficiently to pay the redemption value.

**Results preview.** Our first main result is that B's debt capacity is highest if it issues tradeable, demandable debt, which we refer to as a "banknote." In particular, as long as the horizon mismatch is sufficiently severe, B cannot fund its investment with non-tradeable debt (e.g., a bank loan), even if it is demandable, or with non-demandable debt (e.g., a bond), even if it is tradeable. To see why, consider $C_0$'s decision whether or not to lend to B. $C_0$ knows that he may be hit by a liquidity shock before B's investment pays off, in which case $C_0$ liquidates B's debt, either by redeeming on demand or by trading in the OTC market. If B's debt is not tradeable (but is demandable), then $C_0$ must redeem on demand, forcing B into inefficient liquidation and recovering less than his initial investment. If the horizon mismatch is severe, then this loss from early redemption is so likely that $C_0$ is unwilling to lend in the first place. In contrast, if B's debt is tradeable (but is not demandable), then $C_0$ can avoid early redemption by trading with $C_1$ in the OTC market. However, $C_0$'s liquidity shock puts him in a weak bargaining position with $C_1$: $C_0$ has a low outside option because

National Banking Era, when "the fear of a sudden appearance of a discount on checks [i.e. on bank money] led to bank runs" (p. 21).

he has no way to get liquidity if trade fails. As a result, he sells B's debt at a discounted price, recovering less than his initial investment. If the horizon mismatch is severe, then this loss from selling at a discount is so likely that $C_0$ is unwilling to lend in the first place. But if B's debt is demandable as well as tradeable, then debt does not trade at such a high discount in the secondary market. This is because demandability improves $C_0$'s bargaining position with $C_1$. It increases his outside option, since he can redeem on demand when trade fails. As a result, $C_0$ can trade B's debt at a high price following a liquidity shock. Thus, $C_0$ is insured against liquidity shocks, making him willing to fund B's investment. This result contrasts with existing models of demandable debt, in which, roughly, you do not need the option to redeem debt on demand if you can just trade it in the secondary market (e.g., Jacklin (1987)). Here, in contrast, you do: just the option to redeem on demand props up the resale price of debt in the secondary market, even if the option is never exercised, and the debt is not actually redeemed in any state of the world.

Our second main result is that banknotes are susceptible to a new kind of bank run, which results directly from the dry-up of secondary-market liquidity. Specifically, a sudden (but rational) change in beliefs can cause secondary-market trading to stop, leading the creditor to redeem on demand and forcing B to liquidate inefficiently to pay the redemption value. The belief change may be precipitated by a shock to fundamentals, in which case the run amplifies a downturn, or by a "sunspot" unrelated to fundamentals, in which case the run constitutes a panic in itself. Either way, the run occurs even though B has only a single creditor—there is no static coordination problem in which multiple creditors race to withdraw as in Diamond and Dybvig (1983); rather, there is a dynamic coordination problem in the secondary market in which a counterparty does not accept B's debt today because he is worried that his future counterparty will not accept B's debt tomorrow. Due to this self-fulfilling liquidity dry-up, B's creditor is suddenly unable to trade when he is hit by a liquidity shock and, thus, he must demand redemption from B. We refer to this run as a "money run" because it is the result of the failure of B's debt to function as a liquid money in the secondary market.

These first two results imply that demandability cuts both ways. Indeed, a high redemption value comes with a benefit: it helps $C_0$ to extract a high price from $C_1$, which makes $C_0$ more willing to lend. But this high price also has a cost: it increases the likelihood that $C_1$ chooses not to trade, which makes $C_0$ more likely to run—$C_0$'s option to redeem on demand can undermine itself, putting him in such a strong bargaining position that he has no willing counterparty. But B may still set the maximum possible redemption value (equal to the total liquidation value of its investment). Why? Because B gets the full benefit of cheaper borrowing, but does not bear the full cost of money runs. Hence, although financial fragility may be necessary—B must make its debt demandable to invest efficiently—it can also be excessive—B makes the redemption value too high, exposing itself to more runs than necessary.

For our third main result, we suppose that the horizon mismatch is so severe that B cannot fund its investment, even via a banknote. In this case, direct finance is not possible. But perhaps a form of intermediated finance is? To address this question, we consider $N$ parallel versions of the model—$N$ parallel borrowers borrow from $N$ parallel creditors who trade in $N$ parallel OTC markets. We assume that both borrowers' investments and creditors' liquidity shocks are perfectly correlated, so there is no possibility of diversification. But we find that the borrowers can still benefit from pooling their investments. They can issue $N$ banknotes, each backed by the entire pool. Why does each creditor have a claim on the liquidation value of the pool, rather than just on a fraction $1/N$ of it? Because in an equilibrium in which banknotes circulate, no one redeems on the equilibrium path; thus, if one creditor deviates, he is the only one redeeming, and he has a first claim on all of the assets. This option to redeem off equilibrium is enough to increase the banknotes' secondary market price on equilibrium, and hence to boost debt capacity. There is a money-creation rationale for banking: borrowers form a "bank" only to create demandable debt, or "money"; they endogenously transform liquidity and maturity, pool assets, and have dispersed creditors. And, like a bank, they are fragile. Given the high redemption values, prices are high and counterparties are reluctant to trade. Thus the bank is vulnerable to money runs, which can trigger liquidation of the whole pool of investments.

These results on bank money creation do not only speak to historical bank liabilities and the foundations of banking. They matter for contemporary policy. Today, private banks create 97% of broad money, which includes deposits, repos, and money market mutual fund shares—all of which are tradable, demandable, and run-prone (McLeay, Radia, and Thomas (2014); see Subsection 5.1). Broadly, our findings stress how the structure of the secondary market for bank debt jointly determines banks' choice of funding instruments and financial stability. And they point to how open market operations, which prop up liquidity in the secondary market, can have unexplored benefits, substituting for more standard central bank policies, which provide liquidity directly to banks—a market maker of last resort could provide banks with more funding liquidity than a lender of last resort. And this perspective casts new light on how regulatory interventions, like asset purchase programs, capital requirements, and suspension of convertibility affect financial stability and credit supply (Subsection 5.2).

**Further results.** We construct an equilibrium in which money runs happen on the equilibrium path due to "confidence crises" that occur with sunspot probability $\lambda$. Under the assumption that confidence crises happen only if B's debt is demandable, we ask: what is the largest $\lambda$ for which B still makes its debt demandable? Our model is tractable enough to admit a closed-form expression for this number. For "reasonable" parameters, we find that it is large (about 14%), suggesting that our model can plausibly explain why banks choose run-prone instruments even though doing so exposes them to costly liquidation.

We also explore three extensions. (i) We show that if B can choose its investment, its choice can be distorted toward high-liquidation-value investments, which facilitate its issuing demandable debt. (ii) We study a version of the model with a continuum of creditors in which debt can be rolled over as well as traded. We show that the results of our baseline model are robust. (This setup also has the attractive feature that not every withdrawal is a run.) (iii) We add random trading costs and show that our results generalize to this setup.

**Layout.** The rest of the Introduction includes a discussion of related literature. Section 2 presents the model. Section 3 analyzes benchmarks. Section 4 includes our main results. Section 5 discusses policy, applications, and empirical content. Section 6 includes a discussion of our assumptions and some extensions. Section 7 is the conclusion.

## 1.1 Related Literature

We make four main contributions to the literature.

First, we offer a new rationale for demandable debt. This adds to the literature in two ways. (i) It complements the literature that shows how demandability can help to mitigate moral hazard problems (Calomiris and Kahn (1991) and Diamond and Rajan (2001a, 2001b)).[3] In particular, we show how demandability can help to increase the value of bank debt as "private money." Thus, our model connects two of the main features of bank liabilities: they circulate as money and are redeemable on demand. (ii) It provides a counterpoint to the literature that suggests that tradeability can substitute for demandability. Notably, Jacklin (1987) shows that, in Diamond and Dybvig's (1983) environment, you do not need to redeem debt on demand if you can just trade it in the secondary market.[4,5] We show

[3]In their conclusion, Diamond and Rajan (2001a) make the link between demandability and circulating banknotes informally, saying that

> deposits are readily transferable, and liquid, because buyers of deposits have no less ability to extract payment than do sellers of deposits. Thus, the deposits can serve as bank notes or checks that circulate between depositors. This could explain the special role of banks in creating inside money (p. 425).

We make this link formally in this paper.

[4]However, Jacklin (1987) does point out that tradeable debt can have one disadvantage relative to demandable debt: investments at the initial date can be distorted in anticipation of trading later on (see also Allen and Gale (2004), Farhi, Golosov, and Tsyvinski (2009), and Kučinskas (2017)).

[5]Other papers show that there may still be a role for demandability if tradeability is limited (Allen and

that if bank debt is traded in an OTC market, like banknotes, deposits, and repos are, then demandability complements tradeability by increasing the price at which it trades.

Second, we uncover a new kind of bank run. By connecting the fragility of money to the fragility of banks, this adds both to the literature on coordination-based bank-run models following Diamond and Dybvig (1983) and to the literature on search-based money models following Kiyotaki and Wright (1989, 1993). In these money models, monetary exchange is fragile since trade is self-fulfilling. Similarly, in the bank run models, bank deposits are fragile since withdrawals are self-fulfilling. To the best of our knowledge, we are the first to show that such bank fragility follows immediately from such monetary fragility[6] and, hence, coordination-based bank runs can occur even with a single depositor—i.e. without multiple depositors racing to withdraw from a common pool of assets.[7,8] This helps to explain how runs can occur on collateral-backed debt, complementing the existing literature (see Kuong (2015) and Martin, Skeie, and von Thadden (2014a, 2014b)). Likewise, it helps to explain why private money is fragile, despite being backed by assets: to maximize its debt capacity, a bank sets the redemption value so high that there is always an equilibrium with a run, even if trading/entry costs become vanishingly small.

Third, we show that the need to create circulating demandable debt gives rise to numerous other banking actives. This adds to the literature on the foundations of banking, connecting pooling assets (e.g., Boyd and Prescott (1986), Diamond (1984), Diamond and Dybvig (1983), and Ramakrishnan and Thakor (1984)) with money creation (e.g., Gu, Mattesini, Monnet, and Wright (2013) and Donaldson, Piacentino, and Thakor (2018)). Notably, in contrast to papers that emphasize how pooling helps banks meet redemptions in equilibrium via diversification, we show that pooling improves creditors' option to redeem off equilibrium even absent diversification.

Fourth, by studying security design when securities are traded OTC in the secondary mar-

Gale (2004), Antinolfi and Prasad (2008), Diamond (1997), and von Thadden (1999)). In these models, banks issue demandable debt *in spite of* trade in secondary markets, e.g., to overcome trading frictions, such as limited market participation. In our model, banks issue demandable debt *because of* trade in secondary markets—the option to redeem on demand improves the terms of trade in the secondary market.

[6]A number of papers study bank money creation independently of financial fragility (e.g., Donaldson, Piacentino, and Thakor (2018), Gu, Mattesini, Monnet, and Wright (2013), Kiyotaki and Moore (2001, 2002, 2005)) and some others embed Diamond–Dybvig runs in economies with private money (e.g., Champ, Smith, and Williamson (1996) and Sanches (2015); see also Sultanum (2018)). Relatedly, Sanches (2016) argues that banks' inability to commit to redeem deposits can make private money unstable.

[7]Our focus on runs that result from dynamic coordination failures among counterparties in the secondary market complements models that focus on runs that result from dynamic coordination failures among depositors in the primary market (the dynamic analog of Diamond–Dybvig-type runs), such as He and Xiong (2012); see also Qi (1994).

[8]Bond and Rai (2009) uncover another kind of run that can occur with a single depositor, or even with no depositors whatsoever: a "borrower run."

ket, we add to the literature in three ways. (i) It complements the search-based money literature which analyzes which type of asset is the socially optimal medium of exchange for trade in the secondary market (e.g., Kiyotaki and Wright (1989) and Burdett, Trejos, and Wright (2001)). We analyze which type of contract is the privately optimal circulating instrument for funding in the primary market. (ii) It extends results in the literature on corporate bonds that suggest short-maturity bonds can have the benefit of high resale prices in the secondary market, but the cost of frequent debt issuances (Bruche and Segura (2016) and He and Milbradt (2014)). These papers restrict attention to debt contracts as in Leland and Toft (1996). We point out that with more general contracts, the benefit can come without the cost: demandable debt props up the secondary market price by giving sellers the option to redeem on demand, an option that need never be exercised.[9] (iii) It provides a counterpoint to the literature that suggests that security design may prevent bank runs (e.g., Andolfatto, Nosal, and Sultanum (2018), Green and Lin (2003), and Peck and Shell (2003)). This literature suggests that if the space of securities is rich enough, then bank runs do not arise in Diamond and Dybvig's (1983) environment. Our analysis suggests that the security designs proposed in this literature may not prevent all kinds of bank runs. This is because, in our environment, it is exactly the possibility of a run, i.e. the option to redeem on demand, that makes the banknote the optimal funding instrument.

More broadly, this paper complements the related line of research that focuses on information, rather than OTC trading frictions, in secondary-market trade (Gorton and Pennacchi (1990), Dang, Gorton, and Hölmstrom (2015a, 2015b), Dang, Gorton, Holmström, and Ordoñez (2017), Gorton and Ordoñez (2014), Jacklin (1989), Chemla and Hennessy (2014), and Vanasco (2016)). This literature generally focuses on fundamental risk, and suggests that information frictions in the secondary market lead banks to do risk transformation, and this improves social efficiency. We focus on coordination risk, and suggest that OTC trading frictions in the secondary market lead banks to do liquidity transformation but that this can decrease social efficiency.

# 2 Model

In this section, we present the model.

[9] In an extension, Bruche and Segura (2016) do consider a version of puttable debt. However, they effectively assume it is not tradeable, which shuts down the interaction of demandability and tradeability that is critical to our results.

## 2.1 Players, Dates, and Technologies

There is a single good, which is the input of production, the output of production, and the consumption good. Time is discrete and the horizon is infinite, $t \in \{0, 1, ...\}$.

There are two types of players, a borrower B and infinitely many deep-pocketed creditors $C_0$, $C_1$, ..., where $C_t$ is "born" at Date $t$. Everyone is risk-neutral and there is no discounting. B is penniless but has a positive-NPV investment. The investment costs $c$ at Date 0 and pays off $y > c$ at a random time in the future, which arrives with intensity $\rho$. Thus, the investment has NPV $= y - c > 0$ and expected horizon $1/\rho$. B may also liquidate the investment before it pays off; the liquidation value is $\ell < c$.

B can fund its project by borrowing from a creditor. However, there is a horizon mismatch similar to that in Diamond and Dybvig (1983): creditors may need to consume before B's investment pays off. Specifically, creditors consume only if they suffer "liquidity shocks," which arrive at independent random times with intensity $\theta$ (after which they die). Hence, a creditor's expected "liquidity horizon" is $1/\theta$.

For now, we focus on a single borrower funding a single investment with debt to a single creditor; this helps us to distinguish the forces in our model from those in the literature.[10] Later, we include multiple borrowers funding multiple investments from multiple creditors; this allows us to show how the forces in our model give rise to something that looks like real-world banking.

## 2.2 Borrowing Instruments

At Date 0, B borrows the investment cost $c$ from its initial creditor $C_0$ via an instrument with terminal repayment $R \leq y$, made when the investment pays off, and redemption value $r \leq \ell$, made if the instrument is redeemed earlier. Creditors can exchange the instrument among themselves and B must repay whichever creditor holds it. Hence, the instrument is tradeable demandable debt, and we refer to it as a "banknote," although it also resembles a bank deposit or a repo. We let $v_t$ denote the Date-$t$ value of B's debt to a creditor not hit by a liquidity shock.

As benchmarks, we consider instruments that may not be tradeable (so B has to repay $C_0$) and/or may not be demandable, but may be "long-term" (so B makes only the terminal repayment). So, we allow B to borrow via the banknote or one of the following debt instruments: (i) non-tradeable long-term debt, which we refer to as a "loan," (ii) non-tradeable

[10] For example, there is no coordination problem among multiple creditors (but we show there can be a different coordination problem with a single creditor) and there is no possibility to pool multiple investments (but we show a new reason to pool investments in an enriched environment (Subsection 4.4)).

demandable debt, which we refer to as a "puttable loan"; and (iii) tradeable long-term debt, which we refer to as a "bond" (although it also resembles an equity share). These instruments are summarized in Figure 1. They constitute all of the feasible Markovian instruments in the sense that they are all transfers from B to the debtholder that can depend on the state of B's investment at Date $t$ (but not on the date itself) and do not violate B's limited-liability constraints.

Figure 1: Debt Instruments

| | not demandable | demandable |
|---|---|---|
| non-tradeable | "loan" | "puttable loan" |
| tradeable | "bond" | "banknote" (deposits or repos) |

## 2.3 Secondary Debt Market: Entry, Bargaining, and Settlement

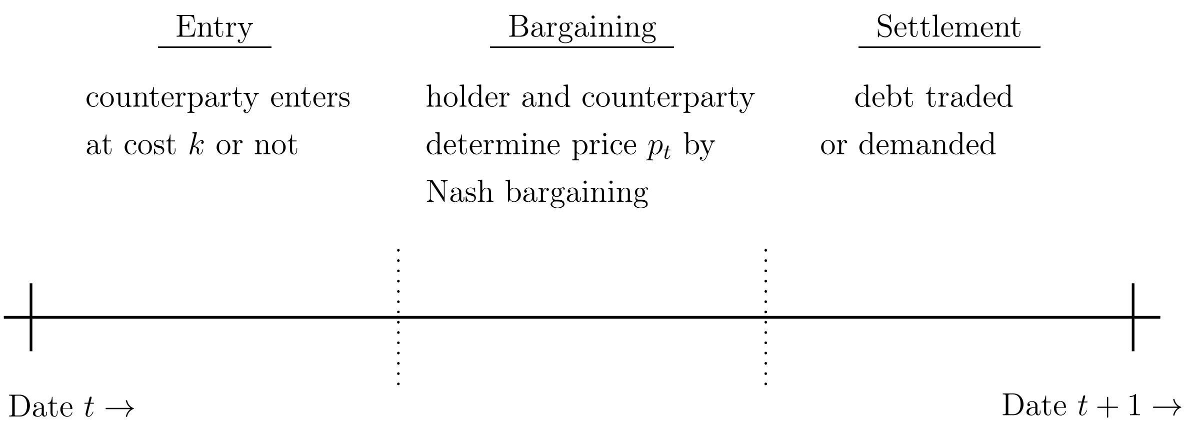
If B has borrowed via tradeable debt, then creditors can trade it bilaterally in an OTC market. At each Date $t$, $\mathrm{C}_t$ is the single (potential) counterparty with whom the debtholder, denoted by $\mathrm{H}_t$, can trade B's debt. If $\mathrm{C}_t$ pays an entry cost $k$,[11] he meets $\mathrm{H}_t$. Then, $\mathrm{C}_t$ and $\mathrm{H}_t$ determine the price $p_t$ via generalized Nash bargaining. $\mathrm{H}_t$'s bargaining power is denoted by $\eta$. If $\mathrm{C}_t$ and $\mathrm{H}_t$ agree on a price, then trade is settled: $\mathrm{C}_t$ becomes the debtholder in exchange for $p_t$ units of the good. Otherwise, $\mathrm{H}_t$ retains the debt. If the debt is demandable, $\mathrm{H}_t$ can demand redemption from B or he can remain the debtholder at Date $t + 1$. This sequence of entry, bargaining, and settlement is illustrated in Figure 2.[12] (The entry and bargaining stages are standard in the literature; the settlement stage is our addition to model demandable debt.)

We let $\sigma_t$ denote $\mathrm{C}_t$'s mixed strategy if $\mathrm{H}_t$ is hit by a liquidity shock, so $\sigma_t = 1$ means that $\mathrm{C}_t$ enters for sure and $\sigma_t = 0$ means that $\mathrm{C}_t$ does not enter. Thus, $\sigma_t$ also represents the probability that $\mathrm{H}_t$ finds a counterparty when hit by a liquidity shock. Observe that we restrict attention to $\mathrm{C}_t$'s strategy given $\mathrm{H}_t$ is hit by a liquidity shock without loss of

[11] This entry cost has a variety of interpretations, which we discuss in Subsection 6.1. However, our results hold even if it is arbitrarily small (see Subsection 4.4). All that matters is that $\mathrm{C}_t$ bears some fixed cost to get B's debt from $\mathrm{H}_t$, and that this cost is sunk, so cannot be shared in bargaining. It does not matter whether $\mathrm{H}_t$ bears a (possibly larger) cost too.

[12] By separating bargaining and settlement, we zero in on tradability and demandability—$\mathrm{H}_t$ agrees to trade with $\mathrm{C}_t$ or not at the bargaining stage and then demands redemption from B or not at the settlement stage. This structure precludes other arrangements, e.g., in which B intermediates trades between $\mathrm{H}_t$ and $\mathrm{C}_t$. We discuss such "rollover" arrangements in Subsection 6.1 and modify our set-up to speak to them explicitly in Subsection 6.3.

Figure 2: Secondary-market Trade




generality.[13]

## 2.4 Timeline

First, B makes $C_0$ a take-it-or-leave-it offer of a repayment and a redemption value, as described in Subsection 2.2 above. Then, if $C_0$ accepts, he becomes the initial debtholder $H_1$. The debtholder may redeem on demand or may trade in the secondary market, as described in Subsection 2.3 above. Formally, the extensive form is as follows.

Date 0 — B offers $C_0$ a repayment $R$ and a redemption value $r$.

If $C_0$ accepts, then B invests $c$. $C_0$ is the initial debtholder, $H_1 = C_0$.

Date $t > 0$ — If B's investment pays off: B repays $R$ to $H_t$ and B consumes $y - R$.

If B's investment does not pay off: there is entry, bargaining, and settlement as described in Subsection 2.3.

> If there is trade, $C_t$ becomes the new debtholder, $H_{t+1} = C_t$.
>
> If there is no trade, $H_t$ either holds the debt, $H_{t+1} = H_t$, or redeems on demand, in which case B liquidates its investment, repays $r$ to $H_t$, and consumes $\ell - r$.

[13] The reason that this is without loss of generality is that $C_t$ would never enter if $H_t$ were not shocked: if $H_t$ is not shocked, $H_t$ and $C_t$ are identical and there are no gains from trade, so it is never worth it to pay the entry cost $k$ for the opportunity to trade.

### 2.5 Equilibrium

The solution concept is subgame perfect equilibrium. An equilibrium constitutes (i) the repayments $R$ and $r$, (ii) the price of debt in the secondary market $p_t$ at each date, and (iii) the entry strategy $\sigma_t$ of the potential counterparty $\mathrm{C}_t$ such that B's choice of instrument and $\mathrm{C}_t$'s choice to enter are sequentially rational, $p_t$ is determined by Nash bargaining, and each player's beliefs are consistent with other players' strategies and the outcomes of Nash bargaining.

For most of the paper, we focus on stationary equilibria, i.e. $\sigma_t \equiv \sigma$ and $p_t \equiv p$.

## 3 Benchmarks

To begin, we consider three benchmark instruments, the loan, the puttable loan, and the bond. We verify two results in the literature in our environment: (i) demandability can increase debt capacity as in Calomiris and Kahn (1991) and (ii) tradeability can substitute for demandability as in Jacklin (1987).

### 3.1 Loan

First, we consider a loan, i.e. non-tradeable long-term debt. At Date $t$, the value $v_t$ of the loan with face value $R$ can be written recursively:

$$v_t = \rho R + (1-\rho)(1-\theta)v_{t+1}. \tag{1}$$

The terms are determined as follows. With probability $\rho$, B's investment pays off and B repays $R$. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder $\mathrm{H}_t$ is hit by a liquidity shock. Since the loan is neither tradeable nor demandable, $\mathrm{H}_t$ gets zero. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and $\mathrm{H}_t$ is not hit by a liquidity shock. $\mathrm{H}_t$ retains B's debt at Date $t+1$, which has value $v_{t+1}$ at Date $t$ since there is no discounting.[14] By stationarity ($v_t = v_{t+1} \equiv v$), equation (1) gives

$$v = \frac{\rho R}{\rho + (1-\rho)\theta}. \tag{2}$$

Even though B will always repay eventually, the loan's value $v$ is less than its face value

[14] Formally, the value of holding B's debt is the Date-$t$ *expected* value of B's debt at Date $t+1$, i.e. we should write $\mathbb{E}_t[v_{t+1}]$ instead of $v_{t+1}$. For now, we focus on deterministic equilibria. Thus, this difference is immaterial and we omit the expectation operator for simplicity. (In Subsection 4.3, we do keep track of the expectation operator.)

$R$. The loan is discounted because, without the option to demand debt or trade it, $\mathrm{H}_t$ gets nothing in the event of a liquidity shock. Hence, the discount vanishes as shocks become unlikely, $v \to R$ as $\theta \to 0$. For $\theta > 0$, demandability and tradeability can help to reduce the discount, as we see next.

### 3.2 Puttable Loan

Now we consider a puttable loan, i.e. non-tradeable demandable debt. At Date $t$, the value $v_t$ of the puttable loan can be written recursively:

$$v_t = \rho R + (1-\rho)\Big(\theta r + (1-\theta) v_{t+1}\Big). \tag{3}$$

The terms are determined as follows. With probability $\rho$, B's investment pays off and B repays $R$. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder $\mathrm{H}_t$ is hit by a liquidity shock. Since the loan is demandable, but not tradeable, $\mathrm{H}_t$ redeems on demand and gets $r$. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and $\mathrm{H}_t$ is not hit by a liquidity shock. $\mathrm{H}_t$ retains B's debt at Date $t+1$, which has value $v_{t+1}$ at Date $t$ since there is no discounting. By stationarity ($v_t = v_{t+1} \equiv v$), equation (3) gives

$$v = \frac{\rho R + (1-\rho)\theta r}{\rho + (1-\rho)\theta}. \tag{4}$$

We now compare the puttable loan's debt capacity with the loan's, where "debt capacity" refers to the maximum B can borrow given limited liability. I.e. we compare equation (4) with $R = y$ and $r = \ell$ and equation (2) with $R = y$:

Proposition 1. (Benchmark: benefit of demandability.) *If*

$$\frac{\rho y}{\rho + (1-\rho)\theta} < c \le \frac{\rho y + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta}, \tag{5}$$

*then B can fund itself with a puttable loan but not with a loan.*

The analysis so far already points to one rationale for demandable debt. As in Calomiris and Kahn (1991), the option to liquidate insures $\mathrm{C}_0$ against bad outcomes, making him more willing to lend.[15] Thus, by issuing demandable debt, B expands its debt capacity.

[15] In Calomiris and Kahn (1991), "bad outcomes" are associated with moral hazard problems, rather than liquidity shocks.

### 3.3 Bond

Now we consider a bond, i.e. tradeable long-term debt.(This instrument can also represent an equity claim; debt and equity have equivalent payoffs, since the terminal payoff $y$ is deterministic.) At Date $t$, the value $v_t$ of the bond can be written recursively:

$$v_t = \rho R + (1-\rho)\Big(\theta\sigma_t p_t + (1-\theta)v_{t+1}\Big). \tag{6}$$

The terms are determined as follows. With probability $\rho$, B's investment pays off and B repays $R$. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder $\mathrm{H}_t$ is hit by a liquidity shock. Since the bond is tradeable, but not demandable, $\mathrm{H}_t$ gets $p_t$ if he finds a counterparty, which happens with probability $\sigma_t$, and nothing otherwise. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and $\mathrm{H}_t$ is not hit by a liquidity shock. $\mathrm{H}_t$ retains B's debt at Date $t+1$, which has value $v_{t+1}$ at Date $t$ since there is no discounting.

To solve for the value $v_t$, we must first find the secondary-market price of the bond $p_t$.

Lemma 1. *The secondary-market price of the bond is $p_t = \eta v_t$.*

The bond price splits the gains from trade between $\mathrm{H}_t$ and $\mathrm{C}_t$ in proportions $\eta$ and $1-\eta$. Since $\mathrm{H}_t$ has value zero in this case ($\mathrm{H}_t$ dies at the end of the period and the bond is not demandable), the gains from trade are just the value $v_t$ of the bond to the new debtholder $\mathrm{C}_t$.

By stationarity ($v_t = v_{t+1} \equiv v$ and $\sigma_t \equiv \sigma$) and the preceding lemma ($p_t \equiv p \equiv \eta v$), equation (6) gives

$$v = \frac{\rho R}{\rho + (1-\rho)\theta(1-\eta\sigma)}. \tag{7}$$

We now compare the bond's debt capacity (equation (7) with $R = y$ and $\sigma = 1$)[16] to the puttable loan's (equation (4) with $R = y$ and $r = \ell$):

Proposition 2. (Benchmark: tradeability substitutes demandability.) *Suppose the bond circulates in equilibrium ($\sigma = 1$).*[17] *If*

$$\frac{\rho y + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta} < c \leq \frac{\rho y}{\rho + (1-\rho)\theta(1-\eta)}, \tag{8}$$

[16] The debt capacity of a tradeable instrument refers to the maximum B can borrow *if* it circulates, or $\sigma = 1$. Thus, since $\sigma$ is chosen by $\mathrm{C}_t$, the debt capacity is an upper bound on what B can borrow. I.e. the condition that the debt capacity exceeds $c$ is necessary but not sufficient for B to invest.

[17] As we will see below (setting $r = 0$ in equation (13)), there is an equilibrium in which the bond circulates as long as $\mathrm{C}_t$'s entry cost $k$ is sufficiently small.

*then B can fund itself with a bond but not with a puttable loan (or a loan).*

If the bond circulates, B can borrow against the full value $y$ whenever trading frictions vanish (in the sense that $\mathrm{H}_t$ gets the bargaining power). I.e. if $\sigma = 1$, then there is no role for demandability whenever $\eta \to 1$. Hence, the analysis so far supports Jacklin's (1987) intuition that tradeability substitutes for demandability. If $\mathrm{C}_0$ is hit by a liquidity shock, he can trade B's debt in the market, rather than die with it. In other words, like the option to demand, the option to trade insures $\mathrm{C}_0$ against bad outcomes, making him more willing to lend. Indeed, absent trading frictions ($\eta \to 1$), B can expand its debt capacity more by issuing tradeable debt (a bond) than by issuing demandable debt (a puttable loan). However, we will see next that with trading frictions ($\eta < 1$), there is a role for demandability, even if debt is never redeemed in equilibrium (Proposition 3).

# 4 Banknote and Banking

In this section, we analyze the banknote and present our main results.

## 4.1 Bright Side of Demandable Debt

Now we consider a banknote, i.e. tradeable, demandable debt. At Date $t$, the value $v_t$ of the banknote can be written recursively:

$$v_t = \rho R + (1-\rho)\Big(\theta\big(\,\sigma_t p_t + (1-\sigma_t) r\,\big) + (1-\theta) v_{t+1}\Big). \tag{9}$$

The terms are determined as follows. With probability $\rho$, B's investment pays off and B repays $R$. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder $\mathrm{H}_t$ is hit by a liquidity shock. Since the banknote is both tradeable and demandable, $\mathrm{H}_t$ gets $p_t$ if he finds a counterparty, which happens with probability $\sigma_t$, and otherwise redeems on demand and gets $r$. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and $\mathrm{H}_t$ is not hit by a liquidity shock. $\mathrm{H}_t$ retains the banknote at Date $t+1$, which has value $v_{t+1}$ at Date $t$ since there is no discounting. (NB: if $r = 0$ this corresponds to the value of the bond in equation (6).)

To solve for the value $v_t$, we must first give the secondary-market price of the banknote $p_t$.

LEMMA 2. *The secondary-market price of the banknote is $p_t = \eta v_t + (1-\eta) r$.*

The price of the banknote splits the gains between $\mathrm{H}_t$ and $\mathrm{C}_t$ in proportions $\eta$ and $1-\eta$. Since $\mathrm{H}_t$ has value $r$ ($\mathrm{H}_t$ redeems on demand and gets $r$ if he does not trade with $\mathrm{C}_t$), the gains from trade are $v_t - r$, the value to the new debtholder $\mathrm{C}_t$ minus the value to the current debtholder $\mathrm{H}_t$. The price that splits these gains is $p_t = r + \eta(v_t - r) = \eta v_t + (1-\eta)r$.[18]

By stationarity ($v_t = v_{t+1} \equiv v$ and $\sigma_t \equiv \sigma$) and the preceding lemma ($p_t \equiv p = \eta v + (1-\eta)r$), equation (9) gives

$$v = \frac{\rho R + (1-\rho)\theta\big(1-\eta\sigma\big)r}{\rho + (1-\rho)\theta\big(1-\eta\sigma\big)}. \tag{10}$$

We now compare the banknote's debt capacity ($v$ with $R = y$, $r = \ell$, and $\sigma = 1$) to the benchmark instruments' (Section 3). We find that B can borrow more via a banknote than via any other instrument.

Proposition 3. (Bright side.) *Suppose the banknote circulates ($\sigma = 1$).*[19] *If*

$$\max\left\{\frac{\rho y + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta}, \frac{\rho y}{\rho + (1-\rho)\theta(1-\eta)}\right\} < c \le \frac{\rho y + (1-\rho)\theta(1-\eta)\ell}{\rho + (1-\rho)\theta(1-\eta)}, \tag{11}$$

*then B can fund itself only with the banknote.*

Unlike the puttable loan, the banknote need not be redeemed in equilibrium. Like the bond, it can circulate in the secondary market until maturity. But it is still more valuable than the bond. The reason is that just the option to redeem the banknote on demand (off equilibrium) puts the debtholder in a strong bargaining position in the secondary market, increasing its price. Thus, given secondary market trading frictions ($\eta < 1$), demandability *complements* tradability: your option to demand debt increases the price you trade at. This high price leads to a high debt capacity: in anticipation of being able to sell at a high price in the secondary market, $\mathrm{C}_0$ is willing to pay a high price in the primary market.

What kind of borrower needs to issue the banknote? To answer, we rewrite the condition of Proposition 3. From the left term of equation (11):

$$\frac{1}{\rho} > \frac{1}{\theta} \cdot \frac{y-c}{(1-\rho)(1-\eta)c}. \tag{12}$$

This says that creditors' expected liquidity horizon $1/\theta$ is small relative to B's expected investment horizon $1/\rho$. Hence, B's debt is a kind of inside money, since a creditor generally does not hold it for its entire maturity; rather he holds it for a short time and then uses it to get liquidity from another creditor—as Kiyotaki and Moore (2001) put it, "[w]henever

[18] This result depends on how outside options determine the division of surplus in bargaining. See Subsection 6.1 for a discussion.

[19] We will see below (equation (13)) that there is an equilibrium in which the banknote circulates as long as $\mathrm{C}_t$'s entry cost $k$ is sufficiently small.

paper circulates as a means of short-term saving (liquidity), it can properly be considered as money, or a medium of exchange, because agents hold it not for its maturity value but for its exchange value" (p. 1). Moreover, it implies that B intermediates between short-horizon creditors and a long-horizon investment. Hence, B is starting to resemble a bank, as maturity transformation is one of banks' defining features. But this is just the first step in our argument that B is a bank. Below, we will see that B will endogenously look a lot like a real-world bank: it will not only transform maturity, but pool assets and engage in other canonical banking activities as well, all to create valuable money (Subsection 4.4).

## 4.2 Money Runs

Having established how a banknote helps B raise funds in the primary market, we now turn to how it trades in the secondary market, and whether it could be in fact redeemed early. In other words, does the banknote always circulate ($\sigma = 1$), as we assumed above? To answer, we assume that B has issued a banknote at Date 0 with terminal repayment $R$ and redemption value $r$, and we look at the equilibria of the subgames for $t > 0$. (We determine $R$ and $r$ in equilibrium in Proposition 5.)

First, observe that B's banknote circulates as long as $\sigma_t = 1$ is a best response to the belief that $\mathrm{C}_{t'}$ plays $\sigma_{t'} = 1$ for all $t' > t$. This is the case as long as $\mathrm{C}_t$ is willing to pay the entry cost $k$ to gain the surplus $v - p$ given $\sigma = 1$, or

$$k \;\leq\; v - p\,\Big|_{\sigma=1} = \frac{\rho(1-\eta)(R-r)}{\rho + (1-\rho)\theta(1-\eta)}, \tag{13}$$

having substituted in from Lemma 2 and equation (10).

But there may also be another equilibrium in which B's banknote does not circulate. B's banknote does not circulate as long as $\sigma_t = 0$ is a best response to the belief that $\mathrm{C}_{t'}$ plays $\sigma_{t'} = 0$ for all $t' > t$. This is the case as long as $\mathrm{C}_t$ is *not* willing to pay the entry cost $k$ to gain the surplus $v - p$ given $\sigma = 0$, or

$$k \geq v - p\,\Big|_{\sigma=0} = \frac{\rho(1-\eta)(R-r)}{\rho + (1-\rho)\theta}, \tag{14}$$

again having substituted in from Lemma 2 and equation (10). If $r$ were fixed, this "bad" equilibrium would arise only for sufficiently high $k$. But $r$ is endogenous, not fixed. We show below that it can increase if $k$ decreases, so that this equilibrium can arise even for arbitrarily small $k > 0$ (see Subsection 4.4).[20]

[20]See Rocheteau and Wright (2013) for a model in which multiple (non-steady state) equilibria arise in a decentralized market without a fixed cost.

Proposition 4. (Money runs.) *Suppose that B borrows via a banknote with terminal repayment $R$ and redemption value $r$. If the entry cost $k$ is such that*

$$\frac{\rho(1-\eta)(R-r)}{\rho+(1-\rho)\theta} \leq k \leq \frac{\rho(1-\eta)(R-r)}{\rho+(1-\rho)\theta(1-\eta)}, \tag{15}$$

*then the $t > 0$ subgame has both an equilibrium in which B's debt circulates ($\sigma = 1$) and there is no early liquidation and an equilibrium in which B's debt does not circulate ($\sigma = 0$) and there is early liquidation. There is also a mixed equilibrium, with*

$$\sigma = \frac{1}{\eta}\left(1 - \frac{\rho}{(1-\rho)\theta k}\Big((1-\eta)\big(R-r\big)-k\Big)\right). \tag{16}$$

If a counterparty $C_t$ doubts future liquidity, i.e. he doubts that he will find a counterparty in the future, then $C_t$ will not enter. As a result, the debtholder $H_t$ indeed will not find a counterparty. There is a self-fulfilling dry-up of secondary-market liquidity. With demandable debt, this has severe real effects: unable to trade, $H_t$ redeems his debt on demand, leading to the costly liquidation of B's investment. In other words, a change in just the beliefs about future liquidity leads to the failure of B's debt as a medium of exchange in the secondary market—the failure of B's debt as money. As a result, there is sudden withdrawal of liquidity from B, i.e. a bank run, or a *money run.*

Corollary 1. *Suppose $k$ satisfies condition (15). If $C_t$'s beliefs change from $\sigma_{t'} = 1$ to $\sigma_{t'} = 0$ for $t' > t$, the debtholder $H_t$ "runs" on B, i.e. $H_t$ unexpectedly demands redemption of his debt, forcing B to liquidate its investment.*

The literature has stressed bank failures resulting from shocks to fundamentals (e.g., Allen and Gale (1998) and Gorton (1988)) or beliefs about primary market withdrawals (Diamond and Dybvig (1983)). Friedman and Schwartz (2008) emphasize that such bank failures, whatever their root cause, disrupt economic activity because banks create money—e.g., they issue banknotes—which facilitates trade. Our model also connects bank failure with money creation. But the chain of causation goes in the opposite direction: the banknote is redeemed only because it fails to circulate. Thus, a run can occur even with a single creditor, who redeems his debt when he cannot trade it. It need not be the result of many creditors racing to be the first to redeem from a common pool of assets. Thus, our model explains runs on repos and nineteenth-century banknotes, which are individually collateralized, not backed by common assets (see Subsection 5.1 and footnote 1).

In our model, financial fragility is a necessary evil. It is necessary because B must issue a fragile instrument—the banknote—to fund itself (Proposition 3). And it is evil because money runs lead to inefficient liquidation. This contrasts with the literature on the necessity of financial fragility, which stresses its virtue, not evil (Allen and Gale (1998), Diamond and Rajan (2001a, 2001b)).

Although financial fragility is necessary in our model, it can still be excessive. To see why, first observe that increasing the redemption value $r$ makes runs "more likely": high $r$ puts $\mathrm{H}_t$ in a strong bargaining position, increasing the price $\mathrm{C}_t$ pays. This makes it less attractive for him to enter. And if $\mathrm{C}_t$ does not enter, $\mathrm{H}_t$ is unable to trade and must redeem early—must run.

COROLLARY 2. *Increasing the redemption value $r$ makes the banknote less likely to circulate in the following senses:*

*(i) each counterparty $C_t$ enters only for lower entry cost $k$ (given the strategy of other counterparties);*

*(ii) $\sigma = 1$ is an equilibrium of the $t > 0$ subgame only for lower $k$;*

*(iii) $\sigma = 0$ is not an equilibrium of the $t > 0$ subgame for lower $k$.*

Hence, demandability cuts both ways. It is both the thing that allows B to fund itself and the thing that exposes B to runs. It increases B's debt capacity, since it props up the price of B's debt (Proposition 3). But it also increases B's liquidation risk, since it makes $\mathrm{C}_t$ reluctant to enter.

Does B internalize the full cost of liquidation risk? The next result says that the answer is no.

LEMMA 3. *Suppose that the probability that each counterparty enters is an increasing function $f$ of $(R - r)$.*[21] *As long as the derivative $f'$ is not too large, B sets the maximum redemption value, $r = \ell$.*

Intuitively, there is a benefit to B of increasing the redemption value $r$: $\mathrm{C}_0$ requires less compensation for the risk of having to sell at a discount in the secondary market. This benefit is a cost to $\mathrm{C}_0$'s future counterparty, who pays a high price for the banknote. But he

[21] To be clear, for this result, we consider B's best response to counterparties' strategies $\sigma = f(R - r)$, where $f$ depends on $R - r$, as motivated by $\mathrm{C}_t$'s entry condition (equation (42)). For now, we do not require these strategies to be consistent, as in equilibrium. However, we show in Subsection 6.4 below that if the entry cost is a random variable $\tilde{k}_t$, such strategies arise in equilibrium (and modifying the distribution of $\tilde{k}_t$ gives us some freedom to generate different functions $f$).

The direct effect of increasing $r$ is to make $\mathrm{C}_t$ less likely to enter, as stated in Corollary 2. We capture this by assuming that $f' > 0$. But we should point out that increasing $r$ can also have an indirect effect, since it can change counterparties' equilibrium strategies. Indeed, in some circumstances this indirect effect can be important (e.g., in the mixed equilibrium in Proposition 4).

is not there at Date 0, when B and $C_0$ are bargaining. Hence, although B and $C_0$ maximize their joint surplus, they do not fully internalize this cost, and B continues to increase $r$ even when it has no social benefit (cf. Subsection 4.4, in which we model a coalition of borrowers that makes the redemption value so large that counterparties are indifferent between entering and staying out).

### 4.3 Equilibrium Runs

We now turn to characterizing an equilibrium in which B borrows via a banknote and money runs arise on the equilibrium path. To do this, we introduce a "sunspot" coordination variable at each date, $s_t \in \{0, 1\}$. We will interpret $s_t = 1$ as "normal times" and $s_t = 0$ as a "confidence crisis," since the sunspot does not affect economic fundamentals, but serves only as a way to coordinate beliefs. We assume that $s_0 = 1$, that $\mathbb{P}\left[s_{t+1} = 0 \,|\, s_t = 1\right] =: \lambda$, and that $\mathbb{P}\left[s_{t+1} = 0 \,|\, s_t = 0\right] = 1$, where we think about $\lambda$ as a small number. In words: the economy starts in normal times and a permanent[22] confidence crisis occurs randomly with small probability $\lambda$.

We now look for a Markov equilibrium, i.e. an equilibrium in which the sunspot (rather than the whole history) is a sufficient statistic for $C_t$'s action:

$$\sigma_t = \begin{cases} \sigma^1 & \text{if } s_t = 1, \\ \sigma^0 & \text{if } s_t = 0. \end{cases} \tag{17}$$

We can now write the banknote's value $v$ as a function of $s_t$ (cf. the analogous equation for the stationary case in equation (9)):

$$v^0 = \rho R + (1-\rho)\Bigg(\theta\Big(\sigma^0 p^0 + \big(1-\sigma^0\big) r\Big) + (1-\theta) v^0\Bigg), \tag{18}$$

$$v^1 = \rho R + (1-\rho)\Bigg(\theta\Big(\sigma^1 p^1 + \big(1-\sigma^1\big) r\Big) + (1-\theta)\Big(\lambda v^0 + (1-\lambda) v^1\Big)\Bigg). \tag{19}$$

The next proposition characterizes an equilibrium in which the "confidence crisis" induces a money run.

Proposition 5. (Equilibrium with sunspot runs.) *Suppose that the condition in equation (11) is satisfied strictly. As long as $\lambda$ is sufficiently small, there exists $k$ such that B can fund its investment only with tradeable, demandable debt (a banknote), even though*

[22]We assume that the crisis is an absorbing state just to keep the model tractable.

*it admits a money run when $s_t = 0$. Specifically, $C_t$ plays $\sigma_t = s_t$, and the value of the banknote when $s_t = 0$ is*

$$v^0 = \frac{\rho R + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta}, \tag{20}$$

*the value of the banknote when $s_t = 1$ is*

$$v_1 = \frac{\Big(\rho + (1-\rho)\big(\lambda(1+\theta\eta) + (1-\lambda)\theta\big)\Big)c - (1-\rho)\lambda\theta\eta\ell}{\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)}, \tag{21}$$

*the repayment $R$ is*

$$R = c + \frac{(1-\rho)\theta\Big(\rho\big(\lambda + (1-\lambda)(1-\eta)\big) + (1-\rho)\big(\lambda + (1-\lambda)\theta(1-\eta)\big)\Big)}{\rho\Big(\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)\Big)}\Big(c - \ell\Big), \tag{22}$$

*and the redemption value is $r = \ell$.*

With these closed-form expressions, it is easy to see how the price of debt depends on parameters.

Corollary 3. (Comparative statics.) *The (net) interest rate $(R-c)/c$ is*

*(i) decreasing in the liquidation value $\ell$;*

*(ii) decreasing in debtholders' bargaining power $\eta$;*

*(iii) decreasing in creditors' liquidity horizon $1/\theta$;*

*(iv) increasing in the probability of a confidence crisis $\lambda$;*

*(v) increasing in the investment size $c$;*

*(vi) increasing in the investment horizon/expected maturity $1/\rho$. Moreover, the term structure is upward sloping, in the sense that the yield[23] $\rho(R-c)/c$ is also increasing in $1/\rho$.*

[23] This uncompounded yield is approximately equal to the continuously compounded yield, which you might be more used to. For a zero-coupon instrument:

$$\text{continuously compounded yield} \equiv \rho \log\frac{R}{c} = \rho\log\left(1 + \frac{R-c}{c}\right) \approx \rho\frac{R-c}{c}, \tag{23}$$

for small $(R-c)/c$ (given the Taylor expansion of $\log(1+x)$).

In our model, the interest rate is compensation for liquidity risk. The results (i)–(iv) capture that increasing $\ell$ and $\eta$ decrease liquidity risk and increasing $\theta$ and $\lambda$ increase it. (v) says that bigger investments are effectively riskier (all else equal). The reason is that, for fixed liquidation value $\ell$, they are liquidated at a larger discount in a confidence crisis. (vi) says that longer maturity investments demand not only higher repayments, but also higher per-period interest rates, even though there is no discounting in preferences. The reason is that as maturity increases both the probability that $C_0$ has to trade at a discount before maturity *and* the size of the discount he trades at increase. So illiquidity in the OTC market generates the term structure.[24]

**A word on welfare and a numerical example.** Given its multiple equilibria, our model does not admit a general welfare analysis. To speak to welfare, we make the following assumption, motivated by the idea that confidence crises are likely only if there is the risk of early redemption: confidence crises can occur if B borrows via the banknote, but not if B borrows via the bond. We ask: if both the banknote and the bond are feasible, how small does the probability $\lambda$ of a confidence crisis have to be for B to prefer the banknote?

Proposition 6. (Confidence crisis probability.) *Suppose confidence crises can occur only if B borrows via the banknote. If the bond is feasible, B still borrows via the banknote whenever the probability $\lambda$ of a confidence crisis is below the threshold $\lambda^*$,*

$$\lambda^* = \frac{\rho\big(\rho + (1-\rho)\theta\big)(1-\eta)\ell}{\rho\big(y-\ell\big) + \big(\rho + (1-\rho)\theta\big)(1-\eta)(\rho\ell - c)}, \tag{24}$$

*and borrows via the bond otherwise.*

For example, if $\theta = 1/4$, $\rho = 1/10$, $y = 20$, $c = 10$, $\ell = 8$, and $\eta = 3/4$, B chooses the banknote whenever $\lambda \leq \lambda^* \approx 14.4\%$.[25] This points to a potentially attractive feature of our model: unlike in many quantitative bank run models, a borrower chooses the run-prone instrument for "reasonable" parameters even when the probability of a run is relatively high. This seems consistent with banks' behavior historically. In the nineteenth-century US, banks issued banknotes despite ubiquitous runs.

[24] See Kozlowski (2017) for a macroeconomic model in which trading frictions generate the yield curve.

[25] We think about these as annual numbers. $\theta = 1/4$, the number used in Ennis and Keister (2003), implies creditors suffer liquidity shocks on average once every four years. $\rho = 1/10$ implies the investment is long-term, taking ten years to complete on average. Given this maturity, $y = 20$ and $c = 10$ imply the investment has annual return of 7.2%. $\ell = 8$ implies the investment has a 20% liquidation discount relative to its book value. $\eta = 3/4$ implies that debtholders get most of the surplus, but far from all of it; this is intended to capture some degree of competition among counterparties.

## 4.4 Banking

We now suppose that the horizon mismatch (equation (11)) is so severe that the borrower cannot raise $c$ to fund its investment, not even via a banknote. In this case, direct finance is not possible. But perhaps a form of intermediated finance is?

To address this question, we now consider $N$ parallel versions of our baseline model: $N$ identical borrowers $\mathrm{B}^1, ..., \mathrm{B}^N$ can do parallel investments at Date 0 and $N$ identical creditors $\mathrm{C}_t^1, ..., \mathrm{C}_t^N$ can enter parallel markets at each Date $t > 0$. At Date 0, the borrowers can issue mutualized instruments, backed by the whole pool of their investments. Redeeming creditors are paid first come, first served à la Diamond and Dybvig (1983). At each subsequent date, each version of the model proceeds exactly as in the baseline model, as described in Section 2. Note that we assume that the parallel versions of the model are identical in every state, i.e. investments/liquidity shocks are perfectly correlated across borrowers/creditors. Thus, there are no diversification benefits from pooling loans/deposits as in Diamond (1984)/Diamond and Dybvig (1983).

Even absent diversification, the borrowers can benefit from pooling their investments to increase their debt capacity and raise $c$. The reason is that pooling allows borrowers to increase the redemption value $r$ of each banknote up to $N\ell$, rather than just up to $\ell$.

Why does each creditor have a claim on the whole liquidation value $N\ell$ rather than just on a fraction $1/N$ of it? The answer is that in an equilibrium in which banknotes circulate, no one redeems on the equilibrium path; thus, if one creditor deviates, he is the only one redeeming, and can get paid up to $N\ell$. Now the redemption value $r$ can become arbitrarily large as the number of borrowers $N$ pooling assets increases. The option to redeem for high $r$ off equilibrium increases the secondary-market price of debt on equilibrium, which can, in turn, boost debt capacity.

But that does not mean that borrowers should make $r$ arbitrarily large. If it is too large, banknotes do not circulate, viz. creditors do not enter if they anticipate being in a weak bargaining position. Their entry condition (equation (13)) puts an upper bound $r^{\max}$ on $r$:

$$r \leq r^{\max} := R - \frac{\rho + (1-\rho)\theta(1-\eta\sigma)}{\rho(1-\eta)}k. \tag{25}$$

Now, to find the debt capacity of a banknote, we substitute $r = r^{\max}$, $R = y$, and $\sigma = 1$

into the value of the banknote (equation (10)), to get

$$\max v = \left.\frac{\rho R + (1-\rho)\theta\big(1-\eta\sigma\big)r}{\rho + (1-\rho)\theta\big(1-\eta\sigma\big)}\right|_{r=r^{\max},\,R=y,\,\sigma=1} \tag{26}$$

$$= y - \frac{(1-\rho)\theta}{\rho}\,k. \tag{27}$$

Given borrowers can undertake an investment only if its debt capacity exceeds its cost ($\max v > c$), equation (27) implies that the borrowers can undertake investments if and only if the NPV, $y - c$, exceeds creditors' total expected entry costs, $\frac{(1-\rho)\theta}{\rho}\,k$.[26] Thus, by forming a "bank," the borrowers can issue banknotes to fund all (and only) investments with positive total surplus. There is a money-creation rationale for banking:

PROPOSITION 7. (BANKING.) *Suppose*

$$N \geq \frac{1}{\ell}\left(y - \frac{\rho + (1-\rho)(1-\eta)}{\rho(1-\eta)}k\right). \tag{29}$$

*There is an equilibrium in which borrowers successfully fund all investments, raising $c$ by issuing a banknote to each of the Date-0 creditors, if and only if the investments have positive total surplus, i.e. the NPV is higher than the total expected entry costs, or*

$$y - c \geq \frac{(1-\rho)\theta}{\rho}k. \tag{30}$$

To fund all positive-surplus investments, borrowers have to set $r$ so high that counterparties are indifferent between entering and staying out. This makes them especially susceptible to runs, since an arbitrarily small change in a counterparty's belief about others' strategies makes him stay out, leading to a money run. Moreover, unlike in the baseline model, a run can occur no matter how small the entry cost $k$ is. If $k$ is small, the borrowers make $r$ high, so counterparties are still indifferent between entering and staying out (cf. equation (14)).

And now a money run has severe consequences. As in a real-world bank run, there is mass liquidation: with $r > \ell$, multiple investments need to be liquidated to redeem each banknote. In addition to this fragility, the coalition of borrowers has other defining features of a real-world bank.

[26] This expression for the expected entry costs can be understood as follows: from Date 1 onward, creditors pay $k$ at Date $t$ if debtholders are shocked while investments are still underway, which occurs with probability $(1-\rho)^t\theta$. Hence,

$$\text{total expected entry costs } = \sum_{t=1}^{\infty}(1-\rho)^t\theta k = \frac{(1-\rho)\theta}{\rho}k. \tag{28}$$

1. **Liquidity tranformation.** The bank funds illiquid assets (non-tradeable investments that are costly to liquidate early) with liquid liabilities (circulating demandable debt).
   - Issuing liquid (tradeable) liabilities gives creditors insurance against liquidity shocks.
2. **Maturity transformation.** The bank funds long-term investments with short-term (demandable) liabilities.
   - Issuing demandable liabilities allows creditors to trade at a high price given liquidity shocks.
3. **Asset pooling.** The bank pools borrowers' investments, reusing their liquidation value to back demandable debt.
   - Issuing debt backed by a pool of assets gives creditors a high redemption value.
4. **Dispersed depositors (creditors).** The bank borrows from a large number of dispersed creditors.
   - Issuing debt to many creditors gives them the option to redeem against the same assets (hence dispersed creditors are necessary for asset pooling to help).
5. **Fragility.** The bank borrows via debt that is susceptible to runs, and runs force early liquidation of multiple investments.
   - Issuing run-prone debt, i.e. demandable debt with high redemption value, is necessary to make the secondary market price high enough that the bank can fund efficient investments.

In the banking equilibrium, financial fragility is not necessarily the result of monetary fragility. With dispersed creditors, there is a common pool problem, which makes creditors want to redeem if they believe others are going to. Thus, not all runs need be money runs; there can be Diamond–Dybvig runs too, and these different types of runs could exacerbate each other.

The banking equilibrium also makes it easier to apply our model to contemporary deposit markets, in which entry costs seem likely to be small, deposits are redeemable at par, and discounts on them are negligible:

COROLLARY 4. *Consider the banking equilibrium in Proposition 7 in which $r = r^{\max}$. As entry costs become small, i.e. $k \to 0$, the redemption value and secondary market price approach the face value, i.e. $r \to R$ and $p \to R$.*

# 5 Policy, Applications, and Empirical Content

We now turn to our model's contemporary applications, policy implications, and empirical content.

## 5.1 Contemporary Applications

Our model applies well to the nineteenth-century US, when you would trade banknotes OTC to get beer from the barman, and bank runs were ubiquitous. But it applies to contemporary economies too, since much of the money we use today—like repos, deposits, and money market mutual fund shares—is also bank debt, and bank runs remain a major policy concern.

Today, you exchange repos in an OTC market to get liquidity from a financial counterparty. Even though repo contracts do not trade per se—they are formally bilateral agreements, not a tradeable instruments—the collateral underlying them does.[27] As Gorton and Metrick (2010) put it,

> [An] important feature of repos is that the...collateral can be "spent"...used as collateral in another, unrelated, transaction.... This...means that there is a money velocity associated with the collateral. In other words, the same collateral can support multiple transactions, just as one dollar of cash can lead to a multiple of demand deposits at a bank. The collateral is functioning like cash (p. 510).

Repos are also effectively redeemable on demand, since repo positions are typically left open until creditors demand they be closed; other short-term debt positions (e.g., commercial paper), in contrast, are closed unless borrowers roll them over successfully. In this sense, repos are more like banknotes—perpetual debt that can be redeemed on demand—than like other short-term debt. Thus, "repo runs," salient events of the 2008–2009 financial crisis, could be money runs. As such, our framework casts light on the puzzle of how runs arise even though each repo is individually collateralized, and the common pool problem necessary to generate Diamond–Dybvig runs is absent (see Gorton and Metrick (2010, 2012) and Krishnamurthy, Nagel, and Orlov (2014)).

Bank deposits are also private money. Debit payments and bank transfers are bilateral (OTC) transfers of bank deposits made at par (see Corollary 4). Of course, most deposits are redeemable on demand. And runs on deposits, although no longer commonplace, do occur

[27] See also Donaldson, Lee, and Piacentino (2018), Donaldson and Micheler (2018), Singh (2010), and Singh and Aitken (2010).

in times of crisis (e.g., on Countrywide, IndyMac, and Northern Rock in the 2008 financial crisis; see, e.g., Iyer and Puri (2012)).

Money market mutual fund shares are money like too. They are tradeable and demandable and vulnerable to runs (see Schmidt, Timmermann, and Wermers (2016)).[28]

## 5.2 Policy

Our analysis stresses how the structure of the secondary market for bank debt determines banks' debt structure and creates financial fragility. Indeed, given bank debt is traded OTC, our results suggest that financial fragility may be a necessary evil: money runs are the cost of the debt capacity afforded by demandable debt. However, decreasing secondary-market trading frictions can make banks less reliant on demandable debt, decreasing the likelihood of runs. Thus, to improve bank stability, a policy maker may be better off intervening in the secondary market than regulating banks directly. Broadly, this suggests that open market operations, which prop up liquidity in the secondary market, can substitute for direct liquidity provision measures such as TAF, TSLF, TALF, etc.[29] More specifically, the model gives the following new perspectives on regulation:

1. **Asset purchase guarantees.** In 2008 the US Treasury opened its Temporary Guarantee Program, in which it promised to buy the shares of money market mutual funds at a guaranteed price. This off-equilibrium promise to buy bank debt could eliminate the "bad" equilibrium, in which counterparties do not enter the secondary market fearing it will dry up in the future.

2. **Exchanges for non-demandable debt.** Centralized exchanges and clearing houses for bank bonds, like those for stocks, could decrease trading frictions, leading banks to issue more bonds and become less reliant on demandable debt.

3. **Capital requirements.** In our model, capital requirements are a double-edged sword. They can help, by curbing banks' incentive to use too much demandable debt (Corollary 3). But they can also hurt, by inefficiently constraining investment (Proposition 3).

4. **Narrow banking.** In our model, a bank can fund all worthwhile investments if it can pool them and issue demandable debt backed by the whole pool (Proposition 7). This suggests a downside to the idea of narrow banking, which suggests that

[28]This empirical work notwithstanding, some still question the importance of liquidity risk in the 2008–2009 financial crisis, stressing solvency risk. See, e.g., Thakor (2018).

[29]See https://www.federalreserve.gov/monetarypolicy/bst_crisisresponse.htm for a summary of such policies implemented during the financial crisis.

real investments should be separate from deposit-taking (its financial stability benefits notwithstanding).

5. **Suspension of convertibility.** Unlike in Diamond and Dybvig's (1983) model of bank runs, in which suspension of convertibility restores efficiency, in our model it may have an adverse effect. Since it prevents creditors from redeeming on demand to meet their liquidity needs, it leads to lower secondary-market debt prices and, hence, to constrained bank borrowing and potentially to underinvestment.

### 5.3 Empirical Content

As mentioned above, our model is motivated by empirical observations about financial fragility and circulating bank debt, such as banknotes, deposits, and repos. In particular, our model offers an explanation of the following facts: (i) runs on bank debt are relatively common, even when the debt is backed by collateral; (ii) runs are often precipitated by the failure of debt to circulate in secondary markets; and (iii) banks choose to borrow via demandable debt even though it exposes them to costly runs.

Our model also casts light on several other stylized facts. (i) Demandable bank instruments, such as banknotes, deposits, and repos, are more likely to serve as media of exchange than other negotiable instruments, such as bonds and shares. In the model, this is because the option to redeem on demand props up the secondary-market price of bank debt. Thus, if you hold a variety of instruments, you prefer to use demandable instruments to raise liquidity in the secondary market and to hold long-term instruments until maturity. (ii) Our model casts light on why bank debt is more likely to be demandable than corporate debt. Indeed, the model suggests that banks emerge to create demandable debt, since pooling assets allows them to increase redemption values (Proposition 7). (iii) Our model casts light on why nineteenth-century banknotes traded at a greater discount in markets farther away from the issuing bank: distance from the issuer made the notes harder to redeem on demand, weakening note holders' bargaining positions in the secondary market and decreasing the price of banknotes (see Gorton (1996)). (iv) Our model generates runs even with a single depositor, consistent with the fact that many runs are not market-wide, but rather occur in isolation. Indeed, Krishnamurthy, Nagel, and Orlov (2014) find that repo runs occurred in relative isolation during the financial crisis.

## 6 Discussion of Assumptions and Extensions

In this section, we discuss some of our key assumptions and then analyze extensions.

## 6.1 Discussion of Assumptions

**Entry costs.** As we stress in Subsection 4.4, money runs can arise no matter how small a creditor's entry cost $k$ is. Such a small $k$ could be realistic for some contemporary markets, like retail markets in which consumers trade deposits for goods via debit cards. But a larger entry cost also has natural interpretations. Historically, it could represent the physical cost of coming to market or, alternatively, of acquiring the expertise/technology to check for counterfeit instruments. Today, it could represent the cost of setting up a trading desk to participate in a specific market (e.g., the repo market) or, alternatively, of establishing the legal infrastructure to handle certain instruments (e.g., the GMRA master agreement for repos). More generally, it could represent any cost of searching for a counterparty as in the search money literature, of trading/transacting as in the finance literature, or of posting a vacancy as in the labor literature. Any cost sunk before counterparties meet suffices for our results.

**Rollover.** To focus on trade in the secondary market, we want to abstract from rollover in the primary market (its practical importance notwithstanding). Indeed, the entry-bargaining-settlement setup in Subsection 2.3 deliberately precludes strategies in which B borrows via one-period contracts, and issues new debt to $\mathrm{C}_t$ to settle its existing debt with $\mathrm{H}_t$ at each date: since B would have to settle first with $\mathrm{C}_t$ and then with $\mathrm{H}_t$, this would require an additional settlement stage. Moreover, such a one-period rollover strategy would typically be less desirable than demandable debt in our baseline environment anyway: someone would have to pay the cost $k$ to enter and buy the new issue in every period, rather than to enter and trade existing debt only in periods in which $\mathrm{H}_t$ is hit by a liquidity shock. More practically, secondary market trade allows the borrower to avoid floatation costs, which could be prohibitive if borne in every period in the rollover strategy.

That said, below we include rollover in our environment (under some additional assumptions), and show that money runs can still occur (Subsection 6.3).

**Bargaining protocol.** In our model, demandability matters because the redemption value $r$ serves as the outside option in bargaining. Thus, security design can substitute for market design: the borrower can adjust the terms of trade in the secondary market, choosing $r$ to calibrate the division of surplus between counterparties, even though the bargaining power $\eta$ is immutable.

Our results hold for bargaining protocols, like Nash bargaining, in which the outside option determines the division of the surplus. Not every non-cooperative bargaining game has this feature in equilibrium (Sutton (1986)). But many do. Indeed, the Nash outcome coincides with the equilibrium of a game in which bargainers either (i) risk having the bargaining process suddenly break down or (ii) have the ability to make take-it-or-leave-it offers (see, e.g., Binmore, Rubinstein, and Wolinsky (1986)). Within our model, the risk of a breakdown could reflect the probability that a counterparty abandons the negotiation because he is hit by a liquidity shock himself or because he finds another, more profitable trade to execute. And the ability to make take-it-or-leave-it offers could reflect the situation in modern "'hi-tech' markets [like the repo market] in which binding deals are made quickly over the telephone [or Bloomberg chat]" (Binmore, Osborne, and Rubinstein (1992)), p. 190; see Shaked (1994)).

**Infinite horizon.** Money runs arise due to dynamic coordination—a counterparty enters if he believes his future counterparty will, who enters if he believes his future counterparty will.... Thus, if it is common knowledge that any counterparty is the last one, he will never enter, and the "good" equilibrium would unravel by backward induction. We avoid this by assuming that the horizon is infinite, so every counterparty has a future counterparty. Indeed, there is no date at which the banknote expires for sure; as such, tradable, demandable debt may have more in common with perpetual debt than with short-term debt.

The infinite horizon is one way to capture the idea that each counterparty believes that an instrument could continue for one more period with positive probability. It is the way used in the new monetarist literature, following Kiyotaki and Wright (1989, 1993), but it is not the only way; for example, counterparties could trade an infinite number of times in a bounded time or be uncertain about their position in a finite trading sequence (see, e.g., Moinas and Pouget (2013)).

## 6.2 Asset Choice

What if a single borrower B chooses the type of its investment before borrowing from $C_0$? Do frictions in the secondary market distort its choice? Yes, toward high-liquidation-value investments:

Proposition 8. (Excessive liquidity.) *Suppose that B can choose between an investment with payoff $y$ and liquidation value $\ell$ and another investment that is otherwise identical but has lower payoff $y' < y$ and higher liquidation value $\ell'$, where*

$$\ell' > \ell + \frac{\rho}{(1-\rho)\theta(1-\eta)}\big(y - y'\big). \tag{31}$$

*There exists an investment cost $c$ such that in any equilibrium in which investment occurs, B chooses the low-NPV, high-liquidation-value investment $(y', \ell')$.*

Intuitively, with a high-liquidation-value investment, B can issue a high-redemption-value banknote and borrow more. Thus, to make its debt money-like, B chooses to increase its liquidation value even at the expense of NPV.

## 6.3 Partial Rollover

We now turn to a version of the model in which counterparties are matched with debtholders in a single market via a homogenous matching technology, unlike in Subsection 4.4 in which they trade in parallel markets. This set-up allows us to show that money runs can occur even if (i) there are no aggregate shocks to liquidity and (ii) B can raise money from new creditors at the beginning of each date, thereby rolling over its debt to meet redemptions.[30] Moreover, unlike in the baseline model, not every withdrawal is a run. Rather, some debtholders redeem at each date.

Here we do not model funding/investment, but focus on the secondary market, assuming that banknotes are held by a unit continuum of debtholders, a fraction $\theta$ of which needs liquidity at each date. Counterparties can enter at cost $k$, in which case they are matched with debtholders via a homogenous matching function. Thus the probability $\sigma_t$ with which a debtholder meets a counterparty depends on the number of counterparties that enter. The fraction $\theta\sigma_t$ of debtholders that meet counterparties trade in the secondary market. The remaining $\theta(1-\sigma_t)$ redeem for $r$. We assume that B issues new identical banknotes to raise exactly enough to meet these redemptions at the beginning of each date.[31]

The next result says that this set-up has multiple steady states. Indeed, there is a "good equilibrium," in which many counterparties enter and few debtholders are left unmatched. In this equilibrium, there are relatively few withdrawals at each date, so B chooses its rollover strategy to raise a relatively small amount of liquidity. But there is also a "bad equilibrium," in which few counterparties enter and many debtholders are left unmatched. In this equilibrium, there are more withdrawals at each date, so B has to choose a rollover strategy to raise more liquidity. Thus, a change in beliefs can lead to a money run analogous to that in Corollary 1: if counterparties today believe that few of their future counterparties will enter, then few of them enter today; this leads to an unexpectedly high number of withdrawals—a money run.

Proposition 9. (Money runs with partial rollover.) *Let the matching technology*

[30]This distinguishes our run risk from rollover risk, where we use "run risk" to mean the risk of an unexpectedly large number of withdrawals. In contrast, we use "rollover risk" to mean the risk that B attempts to raise new debt and fails. Below, we assume B can roll over costlessly—there is no rollover risk—but B cannot go back to the market to meet a large number of withdrawals without some delay—there is run risk.

[31]Assuming that B decides how much to raise at the beginning of the date makes runs possible and assuming that all banknotes are identical (with face value $R$ and redemption value $r$) keeps the model stationary.

*be given by $\sigma = \mu\sqrt{q}$, where $q$ is the number of counterparties that enter and $\mu > 0$ is a parameter. Suppose that B borrows via banknotes from a continuum of creditors. The $t > 0$ subgame has two stationary equilibria, one in which many counterparties enter,*

$$\sigma = \frac{k\big(\rho + (1-\rho)\theta\big) + \sqrt{k^2\big(\rho + (1-\rho)\theta\big)^2 - 4\mu^2 k\rho(1-\rho)(R-r)\theta\eta(1-\eta)}}{2k(1-\rho)\theta\eta} =: \sigma_+ \tag{32}$$

*—banknotes are liquid—and another in which few counterparties enter,*

$$\sigma = \frac{k\big(\rho + (1-\rho)\theta\big) - \sqrt{k^2\big(\rho + (1-\rho)\theta\big)^2 - 4\mu^2 k\rho(1-\rho)(R-r)\theta\eta(1-\eta)}}{2k(1-\rho)\theta\eta} =: \sigma_- \tag{33}$$

*—banknotes are illiquid—as long as $\sigma_+$ and $\sigma_-$ above are well-defined probabilities.*

This result implies that money runs can occur even with no aggregate risk, no rollover risk, and no sequential-service constraint. This affirms that money runs result only from intertemporal coordination in the secondary market and helps distinguish our model of bank fragility from models of rollover risk (e.g., Acharya, Gale, and Yorulmazer (2011) and He and Xiong (2012)).

## 6.4 Random Entry Costs

Now we modify the baseline model so that $C_t$'s entry cost is a random variable $\tilde{k}_t \sim \mathrm{U}\left[0, \bar{k}\right]$. We find that the $t > 0$ subgame has multiple symmetric cut-off strategy equilibria.

PROPOSITION 10. (CUT-OFF EQUILIBRIA.) *Define*

$$k^{\pm} = \frac{\bar{k}}{2(1-\rho)\theta\eta}\left(\rho + (1-\rho)\theta \pm \sqrt{\big(\rho + (1-\rho)\theta\big)^2 - \frac{4\rho(1-\rho)\theta\eta(1-\eta)\big(R-r\big)}{\bar{k}}}\right). \tag{34}$$

*If $k^{\pm} \in [0,1]$, then the $t > 0$ subgame has equilibria in which $C_t$ enters whenever his entry cost is below $k^*$ for $k^* = k^-$ and $k^* = k^+$.*

Given $\tilde{k}_t \sim \mathrm{U}\left[0, \bar{k}\right]$, the probability a creditor enters is $k^*/\bar{k}$. In Lemma 3, we assume this probability is an increasing function $f$ of $R - r$. From equation (34), we see that this assumption holds in the $k^-$-equilibrium. Hence, this extension gives an equilibrium foundation for our assumption. (The assumption does not hold in the $k^+$ equilibrium. Although $f$ is still a function of $R - r$, it is decreasing, as in the mixed equilibrium; cf. the discussion in footnote 21.)

# 7 Conclusion

What is a bank? A bank is something that creates money, i.e. debt that facilitates trade in decentralized markets. By thinking about a bank this way, we found a new rationale for demandable debt, a new type of bank run—a "money run"—and a new explanation for the other quintessential things banks do, such as pooling assets and maturity/liquidity transformation. In contrast to the literature, our results suggest that financial fragility may be a necessary evil and that regulating markets may help bank stability just as much as regulating banks themselves.

# A Proofs

## A.1 Proof of Proposition 1

For an instrument $i$, let $\max v_i$ be an instrument's debt capacity, i.e. its maximum value over any $R$, $r$, and $\sigma$:

$$\max v_i := \sup \left\{ v_i \,\middle|, r \leq \ell \,,\, R \leq y \,, \sigma \in [0,1] \right\}. \tag{35}$$

So, $\mathrm{C}_0$ lends against instrument $i$ only if $\max v_i \geq c$. Hence, B can fund itself with the puttable loan but not with the loan if and only if

$$\max v_{\text{loan}} < c \leq \max v_{\text{putt. loan}}. \tag{36}$$

Substituting $r = \ell$ and $R = y$ into the expressions for their values in equations (2) and (4) gives the condition in the proposition. □

## A.2 Proof of Lemma 1

When $\mathrm{C}_t$ and $\mathrm{H}_t$ are matched, $\mathrm{H}_t$ has been hit by a liquidity shock. Thus, $\mathrm{C}_t$'s value of the bond is $v_t$ and $\mathrm{H}_t$'s value of the bond is zero (since $\mathrm{H}_t$ consumes only at Date $t$ and the bond is not demandable). The total surplus is thus $v_t$, which $\mathrm{C}_t$ and $\mathrm{H}_t$ split in proportions $1-\eta$ and $\eta$, in accordance with the Nash bargaining solution. Thus the price is $p_t = \eta v_t$. □

## A.3 Proof of Proposition 2

The proof is analogous to that of Proposition 1. B can borrow via a bond but not with a puttable loan if and only if

$$\max v_{\text{putt. loan}} < c \leq \max v_{\text{bond}}, \tag{37}$$

where $\max v$ is as defined in equation (35). Substituting $r = \ell$, $R = y$, $\sigma = 1$ into the expressions for their values in equations (4) and (7) gives the condition in the proposition. □

## A.4 Proof of Lemma 2

When $\mathrm{C}_t$ and $\mathrm{H}_t$ are matched $\mathrm{H}_t$ has been hit by a liquidity shock. Thus, $\mathrm{C}_t$'s value of the banknote is $v_t$ and $\mathrm{H}_t$'s value of the banknote is $r$ (since $\mathrm{H}_t$ consumes only at Date $t$, it redeems on demand if it does not trade). The gains from trade are thus $v_t - r$, which $\mathrm{C}_t$ and

$\mathrm{H}_t$ split in proportions $1-\eta$ and $\eta$, in accordance with the Nash bargaining solution, i.e. $p_t$ is such that

$$\mathrm{H}_t \text{ gets} \quad \eta\big(v_t - r\big) + r = p_t, \tag{38}$$

$$\mathrm{C}_t \text{ gets} \quad (1-\eta)\big(v_t - r\big) = v_t - p_t, \tag{39}$$

or $p_t = \eta v_t + (1-\eta) r$. $\square$

## A.5 Proof of Proposition 3

The proof is analogous to those of Proposition 1 and Proposition 2. B can borrow via a banknote but not with a puttable loan or a bond if and only if

$$\max\Big\{ \max v_{\text{putt. loan}} \,,\, \max v_{\text{bond}} \Big\} < c \le \max v_{\text{b.note}}, \tag{40}$$

where $\max v$ is as defined in equation (35). Substituting $r = \ell$, $R = y$, $\sigma = 1$ into the expressions for their values in equations (4), (7), and (10) gives the condition in the proposition. $\square$

## A.6 Proof of Proposition 4

For the pure equilibria, the argument is in the text (see equations (13) and (14)).

For the mixed equilibrium, $\mathrm{C}_t$ must be indifferent between entering and staying out, $k = v - p$, or

$$k = \frac{\rho(1-\eta)\big(R-r\big)}{\rho + (1-\rho)\theta(1-\eta\sigma)}. \tag{41}$$

Solving for $\sigma$ gives the expression in the proposition. $\square$

## A.7 Proof of Corollary 1

The result follows immediately from Proposition 4. $\square$

## A.8 Proof of Corollary 2

We prove points (i)–(iii) in turn.

**(i)** Consider $\mathrm{C}_t$'s best response given other counterparties play $\sigma$. $\mathrm{C}_t$ enters if

$$k \le v - p = \frac{\rho(1-\eta)\big(R-r\big)}{\rho + (1-\rho)\theta(1-\eta\sigma)}. \tag{42}$$

The RHS above is decreasing in $r$ (for fixed $\sigma$).

**(ii)** The result follows immediately from equation (13).

**(iii)** The result follows immediately from equation (14). $\square$

### A.9 Proof of Lemma 3

We prove the result by setting up B's maximization problem over $R$ and $r$ given $\sigma = f(R-r)$ and showing that B optimally sets $r = \ell$. We proceed in the following steps.

(i) We write down B's utility as a function of $R$ and $r$.

(ii) We set up the constrained maximization problem to find $R$ and $r$.

(iii) We show that the constraint in the maximization problem binds.

(iv) We show that the objective in the maximization problem is increasing in $r$ given the constraint binds.

(v) We conclude that $r = \ell$, its maximum possible value.

**B's utility.** Let $u$ denote B's expected utility, which can be written recursively as

$$u_t = \rho(y - R) + (1 - \rho)\Big(\theta\big(\sigma_t u_{t+1} + (1 - \sigma_t)(\ell - r)\big) + (1 - \theta)u_{t+1}\Big) \tag{43}$$

The terms are determined as follows. With probability $\rho$, B's investment pays off and B repays $R$, keeping $y - R$. With probability $(1 - \rho)\theta$, B's investment does not payoff and the debtholder $\mathrm{H}_t$ is hit by a liquidity shock. With conditional probability $\sigma_t$, $\mathrm{H}_t$ finds a counterparty and B continues its investment, getting $u_{t+1}$, since there is no discounting. Otherwise, with conditional probability $1 - \sigma_t$, $\mathrm{H}_t$ does not find a counterparty and redeems on demand. B must liquidate its investment and repay $r$, so it gets $\ell - r$. With probability $(1 - \rho)(1 - \theta)$, B's investment does not pay off and $\mathrm{H}_t$ is not hit by a liquidity shock. Again, B continues and gets $u_{t+1}$. Given ($u_t = u_{t+1} \equiv u$), substituting $\sigma_t \equiv f \equiv f(R - r)$ in accordance with the hypothesis of the proposition and solving for $u$ gives

$$u = \frac{\rho(y - R) + (1 - \rho)\theta(1 - f)(\ell - r)}{\rho + (1 - \rho)\theta(1 - f)}. \tag{44}$$

**B's maximization problem.** B will choose $R$ and $r$ to maximize $u$ subject to the constraint that $v \geq c$ (so $\mathrm{C}_0$ lends). Substituting for $u$ form equation (44) and for $v$ from equation (10) with $\sigma = f(R-r)$, this reads:

$$\begin{cases} \text{maximize} & \dfrac{\rho\big(y-R\big)+(1-\rho)\theta(1-f)(\ell-r)}{\rho+(1-\rho)\theta(1-f)} \\ \text{s.t.} & \dfrac{\rho R+(1-\rho)\theta(1-\eta f)r}{\rho+(1-\rho)\theta(1-\eta f)} \geq c. \end{cases} \tag{45}$$

**Constraint binds.** To show that the constraint binds, we show that decreasing $R$ (i) increases the objective and (ii) tightens the constraint:

(i) By differentiation, $\frac{\partial u}{\partial R} < 0$ as long as

$$f'\Big[(1-\rho)\theta\big(y-R-(\ell-r)\big)\Big] < \rho+(1-\rho)\theta(1-f) \tag{46}$$

If the term in square brackets is negative, this is always satisfied, since $f' > 0$. If it is positive, then it is satisfied as long as $f'$ is sufficiently small, which is required by hypothesis.

(ii) By differentiation, $\frac{\partial v}{\partial R} > 0$ as long as

$$f' > -\frac{\rho+(1-\rho)\theta(1-\eta f)}{(1-\rho)\theta\eta(R-r)}. \tag{47}$$

This is always satisfied given $f' > 0$.

**Optimal** $r$**.** To show that $r = \ell$, we show that $u$ is increasing in $r$ given the constraint binds. To see this, compute the total derivative of $u = u\big(r, f(R-r), R(r)\big)$ "along the constraint":

$$\frac{du}{dr} = \frac{\partial u}{\partial r} + \frac{\partial u}{\partial f}\frac{df}{dr} + \frac{\partial u}{\partial R}\frac{dR}{dr} \tag{48}$$

$$= \frac{\partial u}{\partial r} + \frac{\partial u}{\partial f}f'\left(\frac{dR}{dr}-1\right) + \frac{\partial u}{\partial R}\frac{dR}{dr}, \tag{49}$$

where $dR/dr$ comes from differentiating the constraint (given it binds) and the partial derivatives follow from direct computation:

$$\frac{dR}{dr} = -\frac{\theta(1-\rho)\Big((1-\eta f)(\rho+\theta(1-\rho)(1-\eta f)) - \eta\rho(R-r)f'\Big)}{\rho\left(\rho+\theta(1-\rho)(1-\eta f)+\eta\theta(1-\rho)(R-r)f'\right)}, \tag{50}$$

$$\frac{\partial u}{\partial R} = -\frac{\rho}{\rho+(1-\rho)\theta(1-f)}, \tag{51}$$

$$\frac{\partial u}{\partial f} = \frac{(y-R-(\ell-r))\theta(1-\rho)\rho}{(\rho+(1-\rho)\theta(1-f))^2}, \tag{52}$$

$$\frac{\partial u}{\partial r} = -\frac{(1-\rho)\theta(1-f)}{\rho+(1-\rho)\theta(1-f)}. \tag{53}$$

Substituting equations (50), (51), (52), and (53) into equation (49) and manipulating, we see that the derivative $du/dr > 0$ as long as so long as

$$\begin{aligned}\Big[\big(\rho+\theta(1-\rho)(1-\eta f)\big)^2\big(y-\ell-(R-r)\big)+\eta\big(\rho+(1-\rho)\theta(1-f)^2\big(R-r\big)\Big]f' < \\ < (1-\eta)f\big(\rho+(1-\rho)\theta(1-\eta f)\big)\big(\rho+(1-\rho)\theta(1-f)\big).\end{aligned} \tag{54}$$

If the term in square brackets is negative, this is always satisfied, since $f' > 0$. If it is positive, then it is satisfied as long as $f'$ is small, which is required by hypothesis. □

## A.10 Proof of Proposition 5

We first solve for the values $v^0$ and $v^1$ in terms of $r$ and $R$ given the strategies $\sigma^0 = 0$ and $\sigma^1 = 1$. We then show that these strategies are indeed best responses (for some $k$). Finally, we argue that $r = \ell$ and compute the repayment $R$. Finally, we substitute $r$ and $R$ back into the values to get the expressions in the proposition. Then, the fact that B can borrow via the banknote and only via the banknote for $\lambda$ sufficiently small follows immediately from Proposition 3 and the continuity of $v^1$ in $\lambda$.

**Values.** From equation (18) with $\sigma^0 = 0$, we have immediately that

$$v^0 = \frac{\rho R + (1-\rho)\theta r}{\rho + (1-\rho)\theta} \tag{55}$$

(this is just the value of the puttable loan in equation (4)). From Lemma 2 (the logic of which is not affected by the presence of sunspots), we have the prices

$$p^0 = \eta v_0 + (1-\eta)r, \tag{56}$$

$$p^1 = \eta\left(\lambda v^0 + (1-\lambda)v^1\right) + (1-\eta)r. \tag{57}$$

Thus, equation (19) with $\sigma^1 = 1$ reads

$$v^1 = \rho R + (1-\rho)\left(\theta\left(\eta\left(\lambda v^0 + (1-\lambda)v^1\right) + (1-\eta)r\right) + (1-\theta)\Big(\lambda v^0 + (1-\lambda)v^1\Big)\right), \quad (58)$$

so

$$v^1 = \frac{\rho R + (1-\rho)\Big(\lambda\big(1-\theta(1-\eta)\big)v_0 + \theta(1-\eta)r\Big)}{\rho + (1-\rho)\Big(\lambda\big(1-\theta(1-\eta)\big) + \theta(1-\eta)\Big)}. \quad (59)$$

**Best responses.** $\sigma^1 = 1$ and $\sigma^0 = 0$ are best responses if

$$v^0 - p^0 \ \leq\ k\ \leq\ v^1 - p^1 \quad (60)$$

or

$$v^0 - \eta v^0 - (1-\eta)r \ \leq\ k\ \leq\ v^1 - \eta\left(\lambda v^0 + (1-\lambda)v^1\right) - (1-\eta)r. \quad (61)$$

This is satisfied for some $k$ as long as $v^1 \geq v^0$, which is the case as long as $R \geq r$, which must be the case since $R > c > \ell \geq r$.

**Repayments.** $r = \ell$ since $v^1$ is (uniformly) increasing in $r$ but, for $\lambda$ small, B's payoff does not depend on $r$ (directly).[32],[33]

Now, the repayment $R$ is determined by solving

$$c = \lambda v^0 + (1-\lambda)v^1. \quad (62)$$

Substituting in for $v^0$ and $v^1$ from equations (55) and (59) and solving for $R$, we find

$$R = c + \frac{(1-\rho)\theta\Big(\rho\big(\lambda + (1-\lambda)(1-\eta)\big) + (1-\rho)\big(\lambda + (1-\lambda)\theta(1-\eta)\big)\Big)}{\rho\Big(\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)\Big)}\Big(c - \ell\Big), \quad (63)$$

as expressed in the proposition.

[32]Intuitively, if you are "close" to the good equilibrium (so the banknote almost always circulates), you get all of the benefit increasing $r$ (via the increased price), but almost none of the cost (via the increased payout given early redemption). Formally, $\frac{\partial v}{\partial r} > 0$ uniformly in $\lambda$, but $\frac{\partial u}{\partial r} \to 0$ as $\lambda \to 0$ (see the expressions for B's payoffs in equations (69) and (70)).

[33]Note that we are calculating the optimal values of $R$ and $r$ as if they do not affect the equilibria of the $t > 0$ subgames. I.e. counterparties enter in state $s_t = 1$ and not in state $s_t = 0$, as described in the proposition, off the equilibrium path as well as on it. However, other equilibria are possible too, supported by different off-equilibrium behavior.

We can then use the expressions for $R$ and $v^0$ above and substitute them into $v_1$ to find

$$v^1 = \frac{\Big(\rho + (1-\rho)\big(\lambda(1+\theta\eta) + (1-\lambda)\theta\big)\Big)c - (1-\rho)\lambda\theta\eta\ell}{\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)}, \tag{64}$$

as expressed in the proposition. $\square$

## A.11 Proof of Corollary 3

The results follow directly from differentiation given the expression for $R$ in equation (22). $\square$

## A.12 Proof of Proposition 6

We first solve for B's Date-0 utility if it issues a bond, which we label $u|_{\text{bond}}$. Then we solve for B's utility if it issues a banknote, which we label $u|_{\text{b.note}}$. Then we show $u|_{\text{b.note}} \geq u|_{\text{bond}}$ whenever $\lambda \leq \lambda^*$.

**Bond.** Suppose B issues a bond. By assumption, the bond always circulates. Hence, B never liquidates early and eventually gets $y$ and repays $R$. Since there is no discounting, B's utility is $u = y - R$. Since the bond is like a banknote that always circulates with redemption value zero, $R$ is given by equation (22) with $\lambda = 0$ and $\ell$ replaced by zero (since $r = 0$ instead of $r = \ell$). We have

$$u = y - \frac{\rho + (1-\rho)\theta(1-\eta)}{\rho}c =: u\,\Big|_{\text{bond}}. \tag{65}$$

**Banknote.** Suppose B issues a banknote. Denote B's utility in state $s_t$ by $u^{s_t}$. First, consider $s_t = 0$. $u^0$ solves

$$u^0 = \rho(y - R) + (1-\rho)\big(\theta(\ell - r) + (1-\theta)u^0\big), \tag{66}$$

where the terms are determined as follows. With probability $\rho$, B's investment pays off and B repays $R$, keeping $y - R$. With probability $(1-\rho)\theta$, B's investment does not payoff and the debtholder $\mathrm{H}_t$ is hit by a liquidity shock. Since $s_t = 0$, $\sigma_t = 0$ and $\mathrm{H}_t$ redeems on demand and B must liquidate its investment and repay $r$, getting $r - \ell$. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and $\mathrm{H}_t$ is not hit by a liquidity shock. B gets $u^0$, since $s_{t+1} = 0$ given $\mathbb{P}\left[s_{t+1} = 0 \,|\, s_t = 0\right] = 1$. Solving for $u^0$ with $r = \ell$ gives

$$u^0 = \frac{\rho(y - R)}{\rho + (1-\rho)\theta}. \tag{67}$$

Now, consider $s_t = 1$. $u^1$ solves

$$u^1 = \rho(y - R) + (1 - \rho)\big(\lambda u^0 + (1 - \lambda)u^1\big), \tag{68}$$

where the terms are determined as follows. With probability $\rho$, B's investment pays off and B repays $R$, keeping $y - R$. With probability $1 - \rho$, B's investment does not payoff. In this case, B continues its investment to the next date (it does not matter if $\mathrm{H}_t$ is shocked, since B's debt always circulates given $s_t = 1$). Hence, with conditional probability $\lambda$, $s^{t+1} = 0$ and B gets $u^0$ and, with conditional probability $1 - \lambda$, $s^{t+1} = 1$ and B gets $u^1$. Solving for $u^1$ gives

$$u^1 = \frac{\rho(y - R) + (1 - \rho)\lambda u^0}{\rho + (1 - \rho)\lambda}. \tag{69}$$

B's Date-0 utility is thus

$$u_{\text{b.note}} = \lambda u^0 + (1 - \lambda)u^1. \tag{70}$$

Substituting for $u^0$, $u^1$, and $R$ from equations (67), (69), and (22) and differentiating immediately gives the following lemma, which is useful below.

Lemma 4. *$u\big|_{\text{b.note}}$ is continuously decreasing in $\lambda$.*

*Proof.* Direct computation gives

$$\frac{\partial}{\partial\lambda}\Big(u\,\big|_{\text{b.note}}\Big) = -\frac{(1-\rho)\theta\Big(\rho y - \big(\rho + (1-\rho)\theta(1-\eta)\big)c + \rho\eta(c - \ell) + (1-\rho)\theta(1-\eta)\ell\Big)}{\big(\rho + (1-\rho)\theta\big)\big(\lambda + (1-\lambda)\rho\big)^2}. \tag{71}$$

This is negative since each term in the numerator is positive, given $\rho y - (\rho + (1-\rho)\theta(1-\eta))c \geq 0$ by the assumption that the bond is feasible (equation (8)).

□

**Comparison.** B prefers to issue a banknote than a bond whenever $u|_{\text{b.note}} \geq u|_{\text{bond}}$. From the expressions above, equality holds if

$$\lambda^* = \frac{\rho\big(\rho + (1-\rho)\theta\big)(1-\eta)\ell}{\rho(y - \ell) + \big(\rho + (1-\rho)\theta\big)(1-\eta)(\rho\ell - c)}. \tag{72}$$

And given $u|_{\text{bond}}$ does not depend on $\lambda$ and $u|_{\text{b.note}}$ is increasing in $\lambda$ (Lemma 4), $u|_{\text{b.note}} \geq u|_{\text{bond}}$ exactly when $\lambda \leq \lambda^*$. □

## A.13 Proof of Proposition 7

Most of the argument is in the text preceding the proposition. It remains only to show that creditors' entry condition puts a tighter bound on the redemption value than the liquidation value does, i.e. $r \leq r^{\max}$ is a tighter constraint than $r \leq N\ell$. And, indeed, the assumption in equation (29) says exactly that $r^{\max} \leq N\ell$. $\square$

## A.14 Proof of Corollary 4

The result follows immediately from the expressions for $r^{\max}$ in equation (25) and $p$ in Lemma 2 (given the expression for $v$ in equation (10)). $\square$

## A.15 Proof of Proposition 8

By Proposition 3, B can invest in $(y', \ell')$ but not in $(y, \ell)$ if and only if

$$\max v_{\text{b.note}}\big|_{(y,\ell)} < c \leq \max v_{\text{b.note}}\big|_{(y',\ell')}, \tag{73}$$

where $\max v$ is as defined in equation (35). Substituting for $R$, $r$ and $\sigma$ in the value of the banknote (equation (10)), this says that

$$\frac{\rho y + (1-\rho)\theta(1-\eta)\ell}{\rho + (1-\rho)\theta(1-\eta)} < c \leq \frac{\rho y' + (1-\rho)\theta(1-\eta)\ell'}{\rho + (1-\rho)\theta(1-\eta)}. \tag{74}$$

There exists $c$ satisfying the above inequalities whenever the left-most term is less than the right-most term. This reduces to the condition in the proposition (equation (31)). $\square$

## A.16 Proof of Proposition 9

Observe first that the value of the banknote is given by the same expression as in the baseline model (equation (10)). But now an interior value of $\sigma$ is determined by counterparties' entry condition. Recall that the matching function is homogenous, so each counterparty is matched with a debtholder with probability $\sigma/q$. Counterparties' entry condition is thus

$$\frac{\sigma}{q}\big(v - p\big) \geq k, \tag{75}$$

where $q$ represents the steady-state mass of counterparties entering at each date. Since each counterparty is small, the inequality above will bind. Substituting in for $v$ and $p =$

$\eta v + (1-\eta)r$, we have

$$\frac{\sigma}{q}\left(\frac{\rho(1-\eta)\big(R-r\big)}{\rho+(1-\rho)\theta(1-\eta\sigma)}\right) = k. \tag{76}$$

With $\sigma = \mu\sqrt{q}$, this can be re-written as

$$\mu k(1-\rho)\theta\eta q - k\big(\rho+(1-\rho)\theta\big)\sqrt{q} + \mu\rho(1-\eta)(R-r) = 0. \tag{77}$$

This is a quadratic equation in $\sqrt{q}$. It has the two solutions, i.e. there are two steady states,

$$\sqrt{q}_{\pm} = \frac{k\big(\rho+(1-\rho)\theta\big) \pm \sqrt{k^2\big(\rho+(1-\rho)\theta\big)^2 - 4\mu^2 k\rho(1-\rho)(R-r)\theta\eta(1-\eta)}}{2\mu k(1-\rho)\theta\eta}. \tag{78}$$

Substituting $\sigma_{\pm} = \mu\sqrt{q_{\pm}}$ gives the expressions in the proposition. □

## A.17 Proof of Proposition 10

Given $\tilde{k}_t \sim \mathrm{U}\left[0, \bar{k}\right]$, we can replace $\sigma$ in $\mathrm{C}_t$'s entry condition (equation (42)) by $\mathbb{P}\left[\sigma_t = 1\right] = k^*/\bar{k}$, . $\mathrm{C}_t$ must be indifferent at the cut-off $k^*$:

$$k^* = \frac{\rho(1-\eta)\big(R-r\big)}{\rho+(1-\rho)\theta\big(1-\eta k^*/\bar{k}\big)}. \tag{79}$$

Rearranging gives a quadratic equation which has the two solutions in equation (34), as long as they are well defined, i.e. $k^{\pm} \in [0,1]$, as required by the proposition. □

# B Table of Notations

| | |
|---|---|
| **Players and Indices** | |
| $t$ | time index |
| B | borrower or "bank" |
| $\mathrm{C}_t$ | (potential) creditor/counterparty at Date $t$ |
| $\mathrm{H}_t$ | debtholder at Date $t$ |
| **Technologies and Preferences** | |
| $y$ | payoff of B's investment |
| $c$ | cost of B's investment |
| $\ell$ | liquidation value of B's investment |
| $\rho$ | probability B's investment pays off each date |
| $\theta$ | probability $\mathrm{C}_t$ is hit by liquidity shock at each date |
| $u$ | B's utility (used only in the Appendix) |
| **Prices, Values, and Strategies** | |
| $R$ | terminal repayment (face value of debt) |
| $r$ | redemption value |
| $v_t$ | value of B's debt to a creditor at Date $t$ |
| $p_t$ | secondary-market price of B's debt at Date $t$ |
| $\sigma_t$ | mixed strategy of counterparty $\mathrm{C}_t$ |
| **Other Variables** | |
| $s_t$ | sunspot at Date $t$ (Subsection 4.3) |
| $\lambda$ | $\mathbb{P}\left[s_{t+1}=0 \mid s_t=1\right]$, "confidence crisis" probability (Subsection 4.3) |
| max $v$ | debt capacity/maximum value of an instrument (equation (35)) |
| $r^{\max}$ | maximum redemption value s.t. $\mathrm{C}_t$ enters (equation (25)) |
| $\mu$ | matching parameter in Subsection 6.3 |

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