

CONFLICTING PRIORITIES: A THEORY OF COVENANTS AND COLLATERAL*

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PRELIMINARY

Abstract

Debt secured by collateral has absolute priority in the event of default—it is paid ahead of unsecured debt, even if unsecured debt is protected by negative pledge covenants prohibiting new secured debt. We develop a model of how this priority rule leads to conflicts among creditors, but can be optimal nonetheless: borrowers’ option to use collateral in violation of covenants allows for the dilution of existing debt, and hence prevents under-investment, whereas creditors’ option to accelerate debt following a covenant violation deters dilution, and hence prevents over-investment. The optimal investment policy is implementable via a mix of different types of debt, including secured and unsecured debt with tight and loose covenants. The model is consistent with a number of stylized facts about debt structure, covenants, and their violations.

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1 Introduction

Firms finance themselves mainly with debt.¹ They often combine several types of debt, including debt protected by covenants and debt backed by collateral (see Barclay and Smith (1995), Rauh and Sufi (2010), and Colla, Ippolito, and Li (2013)). But these debts are not treated equally in the event of a default—they are prioritized. The absolute priority rule dictates that debt secured by collateral be paid in full before the collateral can be used to make any other payments.² Hence, new secured debt may “leapfrog” existing unsecured debt. For protection, unsecured debt can include so-called anti-dilution covenants. Indeed, the explicit promise not to take on new secured debt, called a negative pledge covenant, is among the most common covenants.³ Negative pledge covenants give unsecured creditors the right to accelerate their debt if the borrower takes on new secured debt. Yet, this secured debt retains its absolute priority even if issued in violation of the covenant, leaving unsecured creditors little more than the right to demand repayment from someone who has already pledged his assets elsewhere. As a result, legal scholars doubt whether negative pledge covenants are of any use at all:

The covenant does not prevent third parties from acquiring a security interest, but [is] merely...a hollow promise, for in the very act of breaching the covenant, the borrower places its assets out of reach of the negative pledgee and into the hands of the very third party against which the negative pledgee seeks protection (Bjerre (1999)).

But if negative pledge covenants cannot enforce priority, why do borrowers rely on them so much, rather than simply using secured debt? Why do borrowers use secured debt as part of a multi-layered debt structure, which includes unsecured debts with and without negative pledge covenants as well? Moreover, why is one type of debt given absolute priority, so that it can expressly undermine other contracts?

Model preview. To address these questions, we present a model in which a borrower, B, finances two projects sequentially, subject to the following two frictions. First, pledgeability is limited, so B cannot borrow against the full present value of his projects. As a result,

¹For example, debt, including convertibles, public bonds, private loans, and private debt placements accounts for 95.6% of financing in Erel, Julio, Kim, and Weisbach’s (2012) sample of public firms.

²Although the absolute priority of secured debt over unsecured debt is not always upheld in bankruptcy, violations are rare; occurring in none of the Ch. 7 and in only 11% of the Ch. 11 bankruptcies in Bris, Welch, and Zhu (2006). Violations of the APR between unsecured debt and equity are somewhat more common (see Eberhart, Moore, and Roenfeldt (1900), Franks and Torous (1989), and Weiss (1990).)

³For example, negative pledge covenants are included in 44% of the debt in Billett, King, and Mauer’s (2007) sample and 92% in Ivashina and Vallée’s (2018) sample.

B can be financially constrained, i.e. unable to finance some valuable projects. Second, contracts are non-exclusive, so B cannot commit not to borrow from different creditors in the future. As a result, B can enter into conflicting contracts. In particular, if B signs one contract promising not to sign any other contracts, nothing stops him from breaking the promise. Hence, there must be rules for how conflicts among contracts are resolved. We assume that, as in practice, collateral serves to establish priority among debt contracts: debt that is secured by collateral trumps debt that is not. Hence, secured debt always has the first claim on the collateral, even if taken on in violation of a covenant and even if unsecured debt is accelerated in response to such a violation.

Results preview. We first ask what happens if B finances his first project via unsecured debt without covenants. Our first main result is that this can lead to over-investment. The reason is that B can finance his second project via secured debt, diluting the existing unsecured debt. This effectively forces part of the project’s cost onto existing creditors, so that B finds it optimal to invest even in some negative-value projects. Thus, dilution can be bad, because it can induce over-investment.

We then ask what happens if B finances his first project entirely via secured debt. Our second main result is that this avoids over-investment but can lead to under-investment. Since secured debt has absolute priority, it cannot be diluted. This can prevent inefficient dilution, limiting over-investment. However, some dilution may be necessary to loosen financial constraints stemming from limited pledgeability—dilution can be good, because it can prevent under-investment. Thus, by blocking dilution, secured debt can cause a “collateral-overhang”—a problem that we show cannot be resolved through financial restructuring (by renegotiation). This resonates with practitioners’ intuition that secured borrowing “encumbers assets”:

Asset encumbrance not only poses risks to unsecured creditors...but also has wider...implications since encumbered assets are generally not available to obtain...liquidity (Deloitte Blogs (2014)).

B can mitigate this inefficiency by financing his first project via a mix of secured and unsecured debt, hence allowing for some dilution. Indeed, if little dilution is needed to finance valuable projects, B can choose a fraction of secured debt that allows him to undertake them, but still prevents him from doing negative-value projects. However, if much dilution is needed to finance valuable projects, under-investment persists.

Hence, we ask what happens if B finances his first project via unsecured debt with negative pledge covenants, i.e. borrowing without collateral but promising not to borrow with collateral in the future. Although this unsecured debt can be diluted by new secured

debt, the threat of acceleration could deter dilution, since demanding early repayment could force B to liquidate, destroying part of his projects' value. However, if all existing debt has negative pledge covenants, the acceleration threat is not credible: creditors have nothing to gain from acceleration—they are paid after the new secured debt whether they accelerate or not. Hence, negative pledge covenants are not effective in curbing B's over-investment.

Last we ask what happens if B finances his first project via unsecured debt, some of which has negative pledge covenants, but some of which does not. Our third main result is that, in this case, the acceleration threat can be credible and thus can deter dilution. The reason is that now unsecured creditors with negative pledge covenants have something to gain from acceleration: they get paid ahead of unsecured creditors without negative pledge covenants. There is yet another side of dilution: to make the acceleration threat credible.

The acceleration threat can also lead to under-investment, deterring not only inefficient dilution, but also efficient dilution. Like with secured debt, B chooses the fraction of debt with negative pledge covenants to mitigate this inefficiency. Unlike with secured debt, B can find this fraction when a lot of dilution is needed to finance valuable projects (not little dilution as for secured debt). B can finance these projects knowing that creditors will waive any covenants he violates. To see why, observe that if B issues new secured debt, his existing unsecured debt is ipso facto junior. It is paid after new secured debt, but ahead of equity. And the more it is diluted, the closer it is to a residual claim—the more it resembles equity, which is a call option on B's assets that creditors are reluctant to exercise early by accelerating. Hence, although covenants deter B from undertaking projects if only a little of dilution is needed for financing, they cannot if a lot is.

Our fourth main result is that B can choose a debt structure committing him to the efficient investment policy. The optimal debt structure typically mixes different types of debt, including debt with negative pledge covenants, secured debt issued in violation of those covenants, and unsecured debt without negative pledge covenants.

This result rationalizes the absolute priority of secured debt: understanding this priority rule, B can choose debt instruments appropriately to commit to financing all and only efficient investments—using only non-state-contingent (debt) instruments, he implements the optimal state-contingent policy. To do so, he uses non-exclusivity to his advantage, exploiting the option to dilute unsecured debt with new secured debt. Absolute priority is useful: its power to defeat other claims, even those with negative pledge covenants, facilitates contingent dilution.

Policy. Our results speak to the role of the absolute priority rule. The rule is a subject of debate in the law literature; e.g., Bebchuk and Fried (1996) challenge

the desirability of a fundamental and longstanding feature of bankruptcy law: the principle that a secured creditor is entitled to receive the entire amount of its secured claim...before any unsecured claims paid (p. 859),

arguing that the absolute priority of secured debt facilitates dilution. While our model affirms this conclusion, it reveals that (i) relaxing the absolute priority rule could block dilution too much, reducing financial flexibility and leading to under-investment, and that (ii) given the current priority rules, borrowers may be able to structure their debt to block inefficient dilution but allow for efficient dilution.

Realism. The optimal debt structure in our model resembles real-world debt structure. B uses a mix of simple instruments—B could not do better using complete contingent contracts. Moreover, our model explains the following facts:

- (i) Borrowers frequently use negative pledge covenants despite their weakness, when they could simply rely on secured debt (e.g., Billett, King, and Mauer (2007) and Ivashina and Vallée (2018)). Thus we respond to the puzzle stressed by, e.g., Bjerre (1999):

Some may wonder why, given their weakness, costs, and difficulties, lenders bother with negative pledge covenants at all.... [B]orrowers have strong incentives to breach the covenant if necessary financing is available only on a secured basis. [...] The foregoing simply raises, however, the broader question of why lenders ever agree to lend on an unsecured basis, with or without a negative pledge covenant, if collateral is available (pp. 338–339).

- (ii) Debt secured by collateral and debt protected by covenants can coexist as part of a multi-layered debt structure (Rauh and Sufi (2010)).
- (iii) Only a fraction of available assets is used to secure debt and not all unsecured debt has tight covenants—there is not a pecking order of debt structure, in which borrowers use collateral first, then covenants; rather, they mix different instruments, possibly exploiting complementarities among them (Rauh and Sufi (2010) and Ivashina and Vallée (2018)).
- (iv) Covenants are frequently violated (e.g., Chava and Roberts (2008), Dichev and Skinner (2002), and Roberts and Sufi (2009)).
- (v) Following violations, covenants are typically waived and debt is rarely accelerated (e.g., Beneish and Press (1993, 1995), Gopalakrishnan and Prakash (1995), Nini, Smith, and Sufi (2012), and Sweeney (1994)).

- (vi) Borrowers have public and private debt at the same time, and private debt has tighter covenants than public debt (Gopalakrishnan and Prakash (1995)).

Good dilution may be most important for growth firms, which have substantial investment opportunities but limited pledgeable assets/cash flow, whereas bad dilution may be most important for distressed firms, which have incentive to undertake even bad investment opportunities, e.g., tunneling, asset stripping, or risk shifting. If so, our model also explains the following:

- (v) Covenant use increases in growth opportunities (Billett, King, and Mauer (2007)).
- (vi) Firms do “priority spreading,” using secured and subordinated debt, when they near distress (Badoer, Dudley, and James (2018) and Rauh and Sufi (2010)).

Literature. Our paper contributes to the large finance theory literature on collateral and the small one on covenants (e.g., Gârleanu and Zwiebel (2009), Park (2002), Rajan and Winton (1995)). In this literature, covenants and collateral typically mitigate conflicts of interest between borrowers and creditors.⁴ We focus on how they mitigate conflicts of interest among creditors, which is arguably the main legal role of collateral and the express intention of anti-dilution covenants.⁵ Bolton and Oehmke (2015) and Donaldson, Gromb, and Piacentino (2018) do explore how collateral establishes priority among creditors, but both papers rule out negative-pledge covenants, our main focus here.⁶ Ayotte and Bolton (2011) do not. Hence, this is probably the closest paper to ours. Unlike us, however, they do not allow for good dilution, and they do not rationalize covenant violations (and subsequent waivers) or the existing priority structure. They also abstract from acceleration and renegotiation, two of the most important features of covenants, both in our model and in practice.

Our paper is also related to the law literature on secured debt and priority (e.g., Bebchuk and Fried (1996), Hansmann and Kraakman (2002), Hansmann and Santilli (1997), Kronman and Jackson (1979), Schwarcz (1997), and Schwartz (1984, 1994, 1997). And to papers on contracting subject to legal rules (e.g., Aghion and Hermalin (1990) and Gennaioli (2006)).

Layout. Section 2 presents the model. Section 3 presents the first- and second-best benchmarks. Section 4 contains the analysis of unsecured debt and secured debt. Section 5

⁴See, e.g., Tirole (2006) on collateral and Smith (1993) on covenants.

⁵Attar, Casamatta, Chassagnon, and Décamps (2015) show, however, that some covenants can help creditors to collude.

⁶Indeed, they use lawyers’ argument that negative pledge covenants are futile to justify abstracting from these covenants altogether.

contains the analysis of negative pledge covenants. Section 6 includes a full characterization of the equilibrium and a discussion of our results. Section 7 is the Conclusion. All proofs are in the Appendix.

2 Model

We consider a model in which a borrower B finances two projects sequentially subject to frictions. The model has one good, “cash”; three dates $t \in \{0, 1, 2\}$; universal risk neutrality; and no discounting.

2.1 Projects

B is penniless, but has access to two investment projects, Project 0 and Project 1.

Project 0 costs I_0 at Date 0 and generates a risky payoff at Date 2 when B consumes: with probability p , the project succeeds and pays off $X_0 + Y_0$, where $X_0 \geq 0$ is pledgeable and $Y_0 \geq 0$ is not; otherwise, it fails and pays nothing. Pledgeable cash flow can be promised to third parties (viz. creditors); non-pledgeable cash flow cannot be.

Project 1 can be high or low quality, where its quality $Q \in \{H, L\}$ is revealed at Date 1, with $\mathbb{P}[Q = H] =: q$. The project costs I_1 at Date 1 and pays off at Date 2, when it succeeds or fails with Project 0. If it succeeds, it pays off $X_1^Q + Y_1^Q$, where $X_1^Q \geq 0$ is pledgeable and $Y_1^Q \geq 0$ is not. If it fails, it pays nothing.

We use the notation $X_{\text{tot.}}$ for the total pledgeable cash flow if all projects undertaken succeed:

$$X_{\text{tot.}} := \mathbb{1}_0 X_0 + \mathbb{1}_1 X_1^Q, \quad (1)$$

where $\mathbb{1}_t$ is the indicator variable,

$$\mathbb{1}_t := \begin{cases} 1 & \text{if Project } t \text{ is undertaken,} \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

Projects mature at Date 2 but can be liquidated early, before they mature at Date 2, for the expected value of their pledgeable cash flows $pX_{\text{tot.}}$. Observe that liquidation is inefficient in that it destroys all (but only) non-pledgeable cash flow. This reflects any cost of terminating a project before completion.

2.2 Financing

2.2.1 Frictions

At Date $t \in \{0, 1\}$, B can borrow from competitive creditors under two frictions.

First, cash flow pledgeability is limited so B cannot borrow against its projects' full value. This implies that B could be unable to finance positive NPV projects.

Second, contracts are non-exclusive: B cannot commit at Date 0 not to contract with new creditors at Date 1.

2.2.2 Instruments

We focus on three debt instruments: secured and unsecured debt with or without negative pledge covenants. These instruments reflect practice and we will show that, in our model, restricting attention to them is without loss of generality.

1. **Secured debt** is a promise to repay a fixed face value at Date 2 secured by pledgeable cash flow as collateral. (The role of the collateral depends on the priority rule, as described below.)
2. **Unsecured debt** is a promise to repay a fixed face value which is not backed by collateral.
3. **Unsecured debt with negative pledge covenants** is unsecured debt that comes with the option to accelerate, i.e. to demand repayment at Date 1, if the borrower takes on new secured debt (i.e. if he violates the negative pledge covenant), although this is only an option (i.e. covenants can be waived).

2.2.3 Priority Rules

Contracts being non-exclusive, B can enter into different contracts with different creditors that need not be consistent: they can conflict. In particular, B can take on debt at Date 1 promising new creditors repayments he cannot make unless he defaults on payments promised to existing creditors (i.e. on debt taken at Date 0), or he may even violate negative pledge covenants. As such, there must be rules specifying how to resolve potential conflicts among mutually inconsistent contracts. We consider the following priority rules.

1. **Secured debt vs. unsecured debt.** Secured debt is paid ahead of unsecured debt (up to the value of collateral), irrespective of acceleration.

2. **Secured debt vs. secured debt.** Secured debt is paid in order of issuance: secured debt issued at Date 0 is prioritized ahead of secured debt issued at Date 1.
3. **Unsecured debt vs. unsecured debt.** Here there are two cases. If debt is not accelerated, unsecured debt is paid pro rata in the event of default. If debt is accelerated, its maturity is shortened so that it can be repaid before default. Hence, accelerated debt has effective priority over other unsecured debt (but not over secured debt).

These priority rules reflect practice; as Schwartz (1989) puts it,

Current law regulating these priorities rests on three “priority principles”: First, if the first creditor to deal with the debt makes an unsecured loan, it shares pro rata with later unsecured creditors in the debtor’s assets on default. Second, if this initial creditor makes an unsecured loan and a later creditor takes security, the later creditor has priority over the initial creditor in the assets subject to the security interest. Third, if the initial creditor makes a secured loan, it generally has priority over later creditors in the assets in which it has security (p. 209);

see also Barclay and Smith (1995). Hahn (2010) details how acceleration can dilute unsecured debt but not secured debt:

[Acceleration] facilitates collection by the speedy...creditors [who accelerate their debt] with the potential of harming the less fortunate ones [who do not].... Moreover, in the case of a debtor who is also indebted to secured creditors acceleration by unsecured creditors...seems somewhat futile (p. 240).

Beyond being realistic, these priority rules are (weakly) optimal in our model (see Proposition 4 below).

2.3 Timeline

The timeline is as follows:

Date 0: B funds Project 0 from competitive creditors or does not.

Date 1: The quality Q of Project 1 is revealed.

B funds Project 1 from competitive creditors or does not.

If a covenant is violated, creditors accelerate (causing liquidation) or do not.

Date 2: If not liquidated at Date 1, projects succeed or fail (together) with probability p , and B makes repayments or defaults.

2.4 Assumptions

We impose three restrictions on parameters.

Assumption 1. Project 0 is efficient and Project 1 is efficient if and only if it is high quality:

$$p(X_0 + Y_0) > I_0, \quad (3)$$

$$p(X_1^H + Y_1^H) > I_1 > p(X_1^L + Y_1^L). \quad (4)$$

The next assumption implies that there is enough pledgeable cash flow for the efficient strategy to be implementable with an exclusive contract. This ensures that our results are driven by non-exclusivity, not limited pledgeability (see Section 3).

Assumption 2. If B undertakes undertaking Project 0 and undertakes Project 1 only if it is high quality, the expected pledgeable cash flow exceeds the expected investment cost:

$$pX_0 - I_0 + q(pX_1^H - I_1) \geq 0. \quad (5)$$

We focus on the case in which the value of collateral always exceeds the face value of the debt it secures. To do so, we assume that the liquidation value of B's projects is sufficiently high.

Assumption 3. Irrespective of Project 1's quality, the total liquidation value of Project 0 and Project 1 exceeds the face value needed to finance Project 1, i.e. for $Q \in \{H, L\}$,

$$p(X_0 + X_1^Q) > \frac{I_1}{p}. \quad (6)$$

To see that $F_1 = I_1/p$ is the face value of debt B must take on to finance Project 1, observe that creditors' break-even condition is just $pF_1 = I_1$, given projects succeed with probability p and pay zero otherwise.

3 First Best and Second Best

The first-best strategy follows immediately from Assumption 1.

Lemma 1. (First best) *The efficient strategy is to undertake Project 0 and to undertake Project 1 if and only if it is high quality.*

We define the second-best outcome as the best outcome implementable with exclusive contracts, i.e. the outcome that maximizes net output subject only to the limited pledgeability constraint.

Lemma 2. (Second best) *The first best is implementable with exclusive contracts.*

The second-best outcome is (first-best) efficient if B and a single creditor can commit to an exclusive contract at Date 0 with Date-2 repayments F^H given success if $Q = H$ and F^L given success if $Q = L$ such that:

1. Irrespective of Project 1's quality, B's pledgeable cash flow suffices to meet the promised repayments given success at Date 2 (under the first-best strategy):

$$X_0 + X_1^H \geq F^H, \quad (7)$$

$$X_0 \geq F^L. \quad (8)$$

2. Given B's repayments F^H and F^L , the creditor is willing to participate at Date 0, i.e. her expected repayment exceeds her expected investment costs (under the first-best strategy):

$$p(qF^H + (1 - q)F^L) \geq I_0 + qI_1. \quad (9)$$

Assumption 2 implies that these inequalities are satisfied for $F^H = X_0 + X_1^H$ and $F^L = X_0$. Thus, borrowing with exclusive, state-contingent contracts yields the first best outcome. With non-exclusive contracts, however, this need not be the case, as we study next.

4 Unsecured and Secured Debt

In this section, we study how the non-exclusivity friction affects financing and, ultimately, investment. We start by considering the case in which B borrows with unsecured and secured debt without covenants. We find that the first best may not be implementable with only these instruments.

4.1 Unsecured Debt and Over-investment

We begin by asking whether B can implement the efficient investment policy by borrowing via (only) unsecured debt at Date 0 without covenants. Thus, unlike in Lemma 2 above, B's contract is neither state-contingent nor exclusive. Can B still commit to follow the efficient investment strategy? I.e. can he satisfy the following two necessary conditions for efficiency?

- (i) *B undertakes Project 1 if $Q = H$.* Since existing debt is unsecured, B can secure all of his pledgeable cash flow to Date-1 creditors. Hence, B is able to finance Project 1 if

and only if this total pledgeable cash flow exceeds the cost of Project 1, i.e.

$$p(X_0 + X_1^H) \geq I_1. \quad (10)$$

By Assumption 3, this condition holds and B is able to fund Project 1 if $Q = H$. B is also willing to fund Project 1 if his payoff from doing so exceeds that without Project 1, i.e.

$$p\left(Y_0 + Y_1^H + \max\left\{0, X_0 + X_1^H - F_0 - \frac{I_1}{p}\right\}\right) \geq p\left(Y_0 + \max\{0, X_0 - F_0\}\right), \quad (11)$$

where F_0 is the face value of unsecured debt needed to fund Project 0 and I_1/p is the face value of secured debt needed to fund Project 1. This can be simplified as

$$Y_1^H + \max\left\{0, X_0 + X_1^H - F_0 - \frac{I_1}{p}\right\} \geq \max\{0, X_0 - F_0\}, \quad (12)$$

which is satisfied (by Assumption 1 with $Q = H$). This simply reflects that Project 1 has positive NPV if $Q = H$ —B captures at least the NPV, and may also benefit from dilution.

- (ii) *B does not undertake Project 1 if $Q = L$.* Again, since the existing debt is unsecured, B can secure all of his pledgeable cash flow to Date-1 creditors to fund Project 1. Hence, as above, B is able to finance Project 1 (by Assumption 3 with $Q = L$). Thus, he chooses not to invest in Project 1 only if funding it via secured debt would (weakly) decrease his payoff, or

$$Y_1^L + \max\left\{0, X_0 + X_1^L - F_0 - \frac{I_1}{p}\right\} \leq \max\{0, X_0 - F_0\}. \quad (13)$$

This says that as long as Project 1 is sufficiently pledgeable (i.e. Y_1^L is low), then B does not invest in it. Otherwise, he over-invests, since old creditors bear the cost of investment, but B captures (at least) the entire non-pledgeable part of it Y_1^L —dilution is effectively a tax imposed on old debt that subsidizes new financing/investment.

After solving for the equilibrium face value F_0 , we find that the conditions above can be satisfied together whenever Y_1^L is sufficiently small.

Proposition 1. (Unsecured debt) *B can implement the efficient outcome borrowing un-*

secured (without covenants) at Date 0 if and only if

$$Y_1^L \leq \min \left\{ X_0 - \frac{I_0}{p}, X_0 - \frac{I_0}{p} + \frac{q}{1-q} \left(X_0 + X_1^H - \frac{I_0 + I_1}{p} \right) \right\}. \quad (14)$$

When Y_1^L is high, B has the incentive to take advantage of dilution, leading to over-investment in the low-quality project. He would better off committing not to, and hence borrow at better terms at Date 0. He may be able to use secured debt to do so, as we turn to next.

4.2 Secured Debt and Under-investment

Here we ask whether B can implement efficiency by borrowing secured at Date 0. Since secured debt is time prioritized (cf. Subsection 2.2.3), B cannot dilute it at all. This can prevent the dilution that could be necessary to fund high-quality projects at Date 1. But if only a fraction of B's debt is secured at Date 0, he can still dilute the fraction that is unsecured: the secured debt F_0^σ that B takes on at Date 0 is a cap on dilution. If B can keep this cap loose enough to allow dilution to fund the high-quality project at Date 1, while still keeping it tight enough to prevent funding the low-quality project, he can satisfy the two necessary conditions for efficiency:

- (i) *B undertakes Project 1 if $Q = H$.* B can invest in the high-quality project as long as he has enough financial flexibility, or the cost of investment is less than what he can promise to repay Date-1 creditors, i.e. less than his “unencumbered pledgeable cash flow”:

$$p(X_0 + X_1^H - F_0^\sigma) \geq I_1. \quad (15)$$

- (ii) *B does not undertake Project 1 if $Q = L$.* B will not invest in the low-quality project as long as he does not have too much financial flexibility:⁷

$$p(X_0 + X_1^L - F_0^\sigma) < I_1. \quad (16)$$

These conditions are satisfied together whenever X_1^L is sufficiently small.

Proposition 2. (Secured debt) *B can implement the efficient outcome via a mix of secured and unsecured debt at Date 0 if*

$$X_1^L < X_1^H. \quad (17)$$

⁷He also will not invest if Y_1^L is small enough that he has no incentive to invest; we do not focus on this case here, since we already showed that B can implement the first best with unsecured debt in that case anyway (Proposition 1).

X_1^Q is the pledgeable cash flow created by funding Project 1. If it is low, then B can fund Project 1 only by diluting existing debt, and the lower it is, the more dilution is needed to fund the project. If $X_1^L < X_1^H$, more dilution is needed to finance the low-quality project than the high-quality one. Hence, B can choose an amount of secured debt that ensures he can dilute existing debt enough to finance the high-quality project, but not to finance low-quality one. If $X_1^L > X_1^H$, however, B cannot use secured debt to constrain financing of the low-quality project without constraining financing of the high-quality one too.

Corollary 1. (Collateral overhang) *Suppose*

$$X_1^L \geq X_1^H \tag{18}$$

and B secures a fraction of its cash flow to his Date-0 creditors so that he cannot finance the low-quality project at Date 1. He cannot finance the high-quality project either (even if his Date-0 debt can be renegotiated).

This is a manifestation of the “collateral overhang problem” in Donaldson, Gromb, and Piacentino (2018): whereas secured debt prevents B from diluting Date-0 creditors to fund an inefficient investment, it also prevents him from diluting them to fund an efficient investment—collateralization encumbers B’s assets. Perhaps, then, negative pledge covenants, which do not necessarily prevent dilution, can help? This is what we turn to next.

5 Negative Pledge Covenants

In this section, we ask whether negative pledge covenants can help B to implement efficiency. We consider first Date-0 financing entirely with debt with covenants, and then with a mix of debt with and without covenants.

5.1 Financing Entirely via Unsecured Debt with Covenants

Here we suppose that B finances his first project entirely via unsecured debt with negative pledge covenants to a single creditor. Given the debt is unsecured, B can always dilute it with new secured debt, even in violation of its explicit covenants. However, the creditor has the right to accelerate her debt following the violation. Since acceleration forces liquidation, the threat of acceleration could deter dilution, and potentially even lead B to invest efficiently. The acceleration threat must be credible, however. And it is only credible if the creditor has something to gain from accelerating her debt.

But what can she gain by accelerating once a violation has already taken place, given the violation itself entailed prioritizing new debt? In this case, nothing.

Lemma 3. *Suppose B finances his first project entirely via unsecured debt with negative pledge covenants to a single creditor. The creditor never accelerates. Hence, the outcome coincides with that in which B finances the project entirely via unsecured debt (Proposition 1).*

To understand the result, suppose that B cannot fully repay his creditor even in success (this is necessary for dilution, hence without loss). If the creditor accelerates, she has a junior claim on B's assets in liquidation, and gets $pX_{\text{tot.}} - F_1$, where F_1 is the face value of the secured debt that B took on to finance Project 1. If she does not accelerate, she has a junior claim on B's assets at maturity, and gets $p(X_{\text{tot.}} - F_1)$. Since

$$pX_{\text{tot.}} - F_1 < p(X_{\text{tot.}} - F_1), \quad (19)$$

the creditor never accelerates, and B is not deterred from taking on new secured debt. The reason is that liquidation subsidizes secured debt, since it makes it less risky: it is repaid F_1 for sure, not just with probability p . This subsidy is a tax on unsecured debt. To avoid it, the unsecured creditor does not accelerate.

5.2 Financing with a Mix of Unsecured Debt with and without Covenants

We now suppose that B finances his first project via a fraction ϕ of unsecured debt with negative pledge covenants to one creditor and a fraction $(1 - \phi)$ of unsecured debt without covenants to other creditors. Ironically, having less debt with negative pledge covenants can make the covenants more effective.

The reason is that the creditor with covenants now has more to gain from acceleration, so her acceleration threat could be credible. Although acceleration does nothing to reverse the dilution imposed on her via the new secured debt, it now has a benefit: it allows her to dilute the fraction $(1 - \phi)$ of unsecured debt without covenants, getting paid before B defaults on his other unsecured debt at maturity. Here is yet another side of dilution: the option to dilute other unsecured debt (through acceleration) creates a credible threat to deter dilution with secured debt (through priority).

The fraction ϕ of debt with covenants determines the strength of the acceleration threat—the smaller ϕ is, the more other debt there is to dilute, and the more there is to gain from accelerating. Thus, B may be able to choose ϕ to make the threat credible at the right time,

detering Date-1 investment in the low-quality project, but not the high-quality project, i.e. satisfying the two necessary conditions for efficiency:

- (i) *B undertakes Project 1 if $Q = H$.* B finances the high-quality project, borrowing secured in violation of covenants, only if he anticipates that the creditor with covenants will not accelerate afterward, i.e. if she prefers to get paid given success at maturity, behind the secured debt F_1 but pari passu with other unsecured debt, than to accelerate and force liquidation to get paid for sure today, still behind secured debt but now ahead of other unsecured debt:

$$p(X_0 + X_1^H) - F_1 \leq p\phi(X_0 + X_1^H - F_1). \quad (20)$$

(We have supposed for simplicity that dilution is severe enough that B cannot repay the debt with negative pledge covenants in full given acceleration. This turns out to be without loss of generality; see the proof of Proposition 3.)

- (ii) *B does not undertake Project 1 if $Q = L$.* B does not finance the low-quality project if he anticipates that creditors will accelerate afterward, or, analogously to the previous case,

$$p(X_0 + X_1^L) - F_1 \geq p\phi(X_0 + X_1^L - F_1). \quad (21)$$

There is a fraction of debt ϕ with negative pledge covenants such that these conditions are satisfied together whenever X_1^L is sufficiently large:

Proposition 3. (Covenants) *B can implement the efficient outcome via a mix of unsecured debt with and without negative pledge covenants at Date 0 if*

$$X_1^L \geq X_1^H \quad (22)$$

(even if his Date-0 debt can be renegotiated).

Recall that X_1 reflects how much B must dilute existing debt to finance Project 1 (see Subsection 4.2). Hence, condition (22) in the proposition says that we can implement the first best with covenants exactly when we might not be able to with secured debt (Proposition 2): if financing the low-quality project dilutes less than financing the high-quality project. Otherwise, there is no ϕ such that the acceleration threat deters financing the low-quality project without deterring financing the high-quality project too.

The reason is that there is more to gain from acceleration when dilution is less severe, making the threat credible when dilution is relatively small, but not when it is large. To see

why, observe that if B violates a covenant by financing a new project with secured debt, the existing unsecured debt is ipso facto junior. It is paid after new secured debt, but ahead of equity. Hence, it is both debt-like and equity-like. And the more it is diluted, the closer it is to a residual claim—the more it resembles equity, a call option on B’s assets that creditors are reluctant to exercise early—and the less credible the acceleration threat is. When dilution is large, it is better not to accelerate, but to “gamble for resurrection” as in the prototypical problem of a firm in distress.

Unlike in the prototypical problem, however, this gambling incentive is exactly what leads to the efficient action: it makes the acceleration threat credible at the right time, and hence covenants allow for some dilution—good dilution—despite their stated intention not to.

It is worth stressing that although liquidation is inefficient, B cannot renegotiate with his creditors to bribe them not to accelerate, which would undermine the liquidation threat. The reason is that continuation only produces extra non-pledgeable cash flow, and B cannot credibly promise to give it to his creditors. Hence, creditors (weakly) prefer just to liquidate and seize B’s assets at Date 1.

6 Equilibrium Characterization and Discussion

6.1 Characterization

Our analysis above implies that B can always find a debt structure to implement the first best, but how the structure looks depends on parameters.

Proposition 4. (Characterization) *The equilibrium is (first-best) efficient and can be implemented as follows. At Date 0, B finances Project 0 by borrowing I_0 via debt with total face value*

$$F_0 = \frac{I_0}{p} + \max \left\{ 0, \frac{q}{1-q} \left(\frac{I_0}{p} + \frac{I_1^H}{p} - X_0 - X_1^H \right), \frac{1-q}{q} \left(\frac{I_0}{p} - X_0 \right) \right\}, \quad (23)$$

where the proportions of this debt that are unsecured without covenants, secured, and unsecured with covenants depend on parameters as follows:

- If $Y_1^L \leq \min \left\{ X_0 - \frac{I_0}{p}, X_0 - \frac{I_0}{p} + \frac{q}{1-q} \left(X_0 + X_1^H - \frac{I_0+I_1}{p} \right) \right\}$, the debt is all unsecured without covenants.
- Otherwise, if $X_1^H > X_1^L$, an amount $F_0^\sigma \in \left(X_0 + X_1^L - \frac{I_1}{p}, X_0 + X_1^H - \frac{I_1}{p} \right]$ is secured.

- Otherwise, the debt is unsecured, and a fraction $\phi \in \left[\frac{p(X_0 + X_1^H) - I_1/p}{p(X_0 + X_1^H - I_1/p)}, \frac{p(X_0 + X_1^L) - I_1/p}{p(X_0 + X_1^L - I_1/p)} \right]$ has negative pledge covenants.

At Date 1 *B* finances Project 1 by borrowing I_1 via secured debt with face value $F_1^H = I_1/p$ if $Q = H$, and does not finance it if $Q = L$.

This result rationalizes the real-world priority structure, in the sense that it allows *B* to use the instruments at his disposal to implement the first-best outcome. The way he uses the instruments also reflects practice, as we discuss next.

6.2 Discussion

Covenants vs. collateral. The literature stresses the substitutability of covenants and collateral; for example, Schwartz (1989) says that

Secured debt and covenants are substitutes (both are issued to protect against dilution) (p. 1418).

Indeed, this is true in our model. But we show that there is also complementarity between covenants and collateral: covenants can implement efficiency only in conjunction with collateral. Although you need covenants to promise not to use collateral—not to dilute unsecured debt inefficiently—you also need collateral to break that promise—to dilute efficiently.

Maturity vs. collateral. Folk wisdom suggests that maturity and collateral are substitutes.⁸ Indeed, shortening maturity and pledging collateral are two ways to establish priority in our model. But they can still be complements: shortening maturity via acceleration is not only a way for unsecured creditors to get priority, it is also a way for them to prevent secured creditors from getting priority, since the acceleration threat makes it unattractive for the borrower to pledge collateral to new creditors.

Debt vs. debt. The literature stresses how covenants address conflicts between debt and equity. Notably, Smith and Warner (1979) say

In this paper, we examine how debt contracts are written to control the bondholder-stockholder conflict. We investigate the various kinds of bond covenants which are included in actual debt contracts (p. 117).

⁸This folk wisdom seems to come from a combination of theories; for example, Hertzberg, Liberman, and Paravisini (2018) say “In theory, lenders can partially mitigate these inefficiencies by using contract terms...such as high collateral (Bester (1985), short maturity (Flannery (1986)), or strict covenants (Levine and Hughes (2005)).”

Our analysis suggests that conflicts among different debts could be as important as conflicts between debt and equity—indeed, negative pledge covenants need not exist at all in our model if creditors did not have conflicting priorities.

Creditors vs. creditors. In our model, the threat of acceleration helps to mitigate conflicts among debts. But to make the acceleration threat credible, creditors are pitted against each other—one creditor has the incentive to accelerate only to dilute another’s debt. Thus, efficiency relies on how some conflicts among multiple creditors mitigate others. One creditor, with negative pledge covenants, must act strategically, deciding whether to accelerate its debt or waive a covenant violation. Such a large, strategic creditor could represent a bank. Other creditors, without negative pledge covenants, are passive by comparison. Whether they are diluted or not depends on what the borrower and the bank do. These creditors could represent bondholders. Indeed, in practice, bank debt is concentrated and relatively covenant heavy, whereas bonds are dispersed and relatively covenant lite.

Dilution vs. dilution. Debt dilution is largely viewed as a “serious danger” for firms (Schwartz (1997)) and, likewise, a “major problem” for countries (Eyigungor (2013)). Indeed, dilution can be bad in our model: dilution via collateral can lead to over-investment and dilution via acceleration can lead to inefficient liquidation. But it can also be good: dilution via collateral can prevent under-investment and dilution via acceleration creates a threat that deters other, inefficient dilution. The optimal debt structure—the amount of secured debt and the amount of unsecured debt with negative pledge covenants—allows for good dilution while preventing bad dilution.

Contingent outcomes vs. non-contingent contracts (and contingent debt structure). The literature has paid a lot of attention to contingent contracting. In corporate finance, it has also focused a lot on the debt vs. equity decision, and explored how contingent contracts can be implemented via a mix of debt and equity, as well as some other instruments, such as credit lines. Our model is about implementing a contingent contract too; for the equilibrium to be efficient, B should invest if $Q = H$ but not if $Q = L$. But we focus on the debt vs. debt decision, and show that the efficient strategy can be implemented with a variety of debt contracts that are not contingent at all. Rather, contingencies are implemented via contingent dilution, which itself is implemented by mixing debts with different covenants and priorities. The mix of debt contracts B uses resembles firms’ real-world funding structure: it is almost all debt, but debt is heterogeneous.

Absolute vs. partial priority. The absolute priority rule dictates secured debt is paid in full before anyone else is paid anything. Bebchuk and Fried (1996) argue that such absolute priority of secured debt can create inefficiencies, because it gives secured debt the power to defeat other claims. We argue that this is not always a bad thing, because dilution

can be good, helping to overcome limited pledgeability. Moreover, we show how borrowers can use a mix of different types of (non-contingent) debt to allow for contingent dilution, allowing efficient dilution, but still preventing inefficient dilution.

The price of debt with vs. without covenants. How do covenants affect debt pricing? Their being prevalent in contracts suggests they might matter a lot.⁹ But their being enforced seldom could suggest they might not. In our model, debt with covenants has the same price as debt without, even when covenants are effective (see Lemma 4 in the Appendix). Indeed, covenants are effective exactly because there is debt without covenants that can be diluted—it is this option to dilute that makes the acceleration threat credible. However, all debt, not just that with covenants, is more valuable because some of it has covenants (which discipline the borrower through the acceleration threat) and some of it does not (which makes this threat credible). This is consistent with evidence in Bradley and Roberts (2015) which finds that firms’ bonds have lower yields when their loans have more covenants.

Flexibility vs. rigidity. In many models, covenants are hard restrictions, and hence impose the cost of limited flexibility. In ours, in contrast, covenants can be violated, and indeed bring the benefit of increased flexibility with respect to secured debt.

7 Conclusion

We present a model of financial contracting in which contracts are non-exclusive, and hence can conflict: contracts may contain covenants putting restrictions on other contracts, but these covenants can be violated. In this case, a priority rule is needed to resolve conflicts among contracts. Hence, contracts are meaningful only with respect to the priority rule.

In practice, secured debt has absolute priority. This creates the risk of dilution: new secured debt overrides existing unsecured debt. Given this priority rule, negative pledge covenants restricting new secured debt might seem futile—they can be overridden by the very dilution they are supposedly there to prevent. But we show that this can be a good thing. The reason is that in addition to the usual bad side of dilution (it leads to over-investment), there are good sides as well. First, it can loosen borrowing constraints that could be too tight due to limited pledgeability, and hence prevent over-investment. Second, it subsidizes accelerating creditors, hence making their threat credible and preventing bad dilution. In our environment, a borrower who understands the existing priority structure can choose his debt structure to get the good sides of dilution without the bad, and hence implement the efficient investment strategy. Hence, our model rationalizes the existing priority rules.

⁹Matvos (2013) and Green (2018) use structural models to argue that covenants are economically valuable.

A Proofs

A.1 Proof of Lemma 1

The result follows immediately from Assumption 1. \square

A.2 Proof of Lemma 2

Assumption 2 and Assumption 3 imply these inequalities can be satisfied.

One easy way to see this is to make the first two bind, so $F_0^H = X_0 + X_1^H$ and $F_0^L = X_0$. In this case, the third (inequality (9)) reduces to Assumption 2. \square

A.3 Proof of Proposition 1

To prove the proposition, we consider face value F_0 if B follows the efficient strategy and determine when B has no incentive to deviate and invest if $Q = L$. (We know B will invest if $Q = H$ irrespective of F_0 .) If $Q = H$, B can borrow I_1 with secured debt with face value F_1 such that $p \min\{X_0 + X_1^H, F_1\} = I_1$. Thus, by Assumption 3, $F_1 = I_1/p$.

Case 1: $X_0 + X_1^H \geq I_0/p + I_1/p$ and $X_0 \geq I_0/p$.

In this case, if the projects succeed, B is able to pay I_0/p to Date-0 creditors irrespective of Q and so

$$F_0 = I_0/p. \quad (24)$$

Condition (13) becomes

$$Y_1^L + \max\left\{0, X_0 + X_1^L - \frac{I_0 + I_1}{p}\right\} \leq X_0 - \frac{I_0}{p}. \quad (25)$$

There are two subcases, depending on whether B defaults on Date-0 creditors if he invests when $Q = L$ and the projects succeed.

Subcase 1.1 $X_0 + X_1^L > I_0/p + I_1/p$.

In this case, B does not default. As a result, he would bear the full negative value of Project 1 when $Q = L$ and so does not undertake it in that case.

Subcase 1.2 $X_0 + X_1^L < I_0/p + I_1/p$.

In this case, if B invests in Project 1 when $Q = L$ and the projects succeed, he defaults on Date-0 creditors. Hence, condition (13) becomes

$$Y_1^L \leq X_0 - \frac{I_0}{p}. \quad (26)$$

Summing up, B will undertake Project 1 when $Q = L$ if

$$X_0 + X_1^L < \frac{I_0 + I_1}{p} \quad \text{and} \quad Y_1^L > X_0 - \frac{I_0}{p}. \quad (27)$$

By Assumption 1, $Y_1^L < I_1/p - X_1^L$, so one condition implies the other: in this case, there is over-investment if and only if $Y_1^L > X_0 - I_0/p$, and conversely, B will not undertake Project 1 when $Q = L$ if and only if

$$Y_1^L \leq X_0 - I_0/p. \quad (28)$$

Case 2: $X_0 + X_1^H < I_0/p + I_1/p$ and $X_0 \geq I_0/p$.

In this case, if B undertakes Project 1 and the projects succeed, he defaults on his Date-0 debt for $Q = H$ but not for $Q = L$. Hence, F_0 is given by the following break-even condition for Date-0 creditors:

$$I_0 = p \left(q \left(X_0 + X_1^H - \frac{I_1}{p} \right) + (1 - q)F_0 \right) \quad (29)$$

so

$$F_0 = \frac{I_0/p - q(X_0 + X_1^H - I_1/p)}{1 - q}. \quad (30)$$

Note that given $X_0 + X_1^H < I_0/p + I_1/p$, Assumption 2 implies $F_0 \leq X_0$, so B does not default if $Q = L$. Thus, condition (13) becomes

$$Y_1^L + \max \left\{ 0, X_0 + X_1^L - F_0 - \frac{I_1}{p} \right\} \leq X_0 - F_0 \quad (31)$$

There are two subcases, depending on whether B defaults if B undertakes Project 1 when $Q = L$ and the projects succeed.

Subcase 2.1: $X_0 + X_1^L \geq F_0 + I_1/p$.

In that case, B would not default and so would bear the full negative value of Project 1. Hence, he does not undertake Project 1 if $Q = L$.

Subcase 2.2: $X_0 + X_1^L < F_0 + I_1/p$. In that case, B would default and condition (13) becomes

$$Y_1^L \leq X_0 - F_0, \quad (32)$$

which, by Assumption 1, implies the subcase's condition, i.e.

$$X_0 + X_1^L < F_0 + \frac{I_1}{p}. \quad (33)$$

Hence, B does not undertake Project 1 when $Q = L$ if and only if condition (32) holds which, plugging in for F_0 , can be rewritten as

$$(1 - q)Y_1^L \leq X_0 - \frac{I_0}{p} + q \left(X_1^H - \frac{I_1}{p} \right). \quad (34)$$

Case 3: $X_0 < I_0/p$. In this case, B defaults if $Q = L$ but not if $Q = H$ and the projects succeed. Thus, Date-0 creditors' break-even condition is

$$I_0 = p(qF_0 + (1 - q)X_0) \quad (35)$$

so

$$F_0 = \frac{I_0/p - (1 - q)X_0}{q}. \quad (36)$$

Note that given $X_0 < I_0/p$ in this case, Assumption 2 implies that $F_0 + I_1/p \leq X_0 + X_1^H$, so B does not default if $Q = H$ and the projects succeed. In this case B always defaults if $Q = L$. Hence, inequality (13) reduces to $Y_1^L \leq 0$, which is never satisfied.

Efficiency conditions. In summary, efficient investment requires that $X_0 - I_0/p \geq 0$ (from Case 3) and that (from Case 1)

$$Y_1^L \leq X_0 - \frac{I_0}{p} \quad \text{if} \quad X_0 + X_1^H - \frac{I_0 + I_1}{p} \geq 0 \quad (37)$$

and (from Case 2)

$$Y_1^L \leq X_0 - \frac{I_0}{p} + \frac{q}{1 - q} \left(X_0 + X_1^H - \frac{I_0 + I_1}{p} \right) \quad \text{if} \quad X_0 + X_1^H - \frac{I_0 + I_1}{p} < 0. \quad (38)$$

Taken together, equations (37) and (38) can be written as condition (14) in the proposition.

(Finally, note that we can omit the condition that $X_0 \geq I_0/p$, since it is implied by the condition that $Y_0 \leq X_0 - I_0/p$.) \square

A.4 Proof of Proposition 2

Immediately from equations (15) and (16), efficiency is implementable whenever there is a face value F_0^σ such that

$$X_0 + X_1^L - \frac{I_1}{p} \leq F_0^\sigma < X_0 + X_1^H - \frac{I_1}{p} \quad (39)$$

The RHS is positive by Assumption 3; hence, F_0^σ exists whenever the LHS is less than the RHS, or $X_1^H > X_1^L$, which is the condition in the proposition. \square

A.5 Proof of Corollary 1

The baseline result follows from the observation that the inequalities (15) and (16) cannot be satisfied at once if $X_1^L \geq X_1^H$, which is the condition in the corollary (cf. the proof of Proposition 2).

Renegotiation proofness. First, observe that, by hypothesis, the L -quality project cannot be financed, or

$$p(X_0 + X_1^L - F_0^\sigma) < I_1 \quad (40)$$

and, also by hypothesis, $X_1^H < X_1^L$, so

$$p(X_0 + X_1^H - F_0^\sigma) < I_1. \quad (41)$$

Now suppose (in anticipation of a contradiction) that B can renegotiate with his creditors to do the H -quality project at Date 1, i.e. that he can reallocate cash flow to make everyone strictly better off (and hence agree to renegotiation). This requires that Date-0 creditors get at least pF_0^σ (which they get if they do not renegotiate) and Date-1 creditors get at least I_1 (which they pay to invest). Since B can promise creditors only the pledgeable cash flow, it must be that there is enough pledgeable cash flow to make all creditors better off, or

$$p(X_0 + X_1^H) > pF_0^\sigma + I_1, \quad (42)$$

which contradicts the inequality (41). Hence, renegotiation is not feasible.

A.6 Proof of Lemma 3

The argument for why the single creditor never accelerates is in the text. Without the acceleration threat, unsecured debt with negative pledge covenants is equivalent to unsecured debt. Hence, the outcome is that described in Proposition 1.

A.7 Proof of Proposition 3

Before starting the proof, we write down creditors' payoffs from accelerating or not. First, observe that if B borrows at Date 1, he always borrows fully secured, to maximize the benefit of dilution. Hence, from Date-1 creditors' break-even condition, the face value of Date-1 debt

is

$$F_1 = \frac{I_1}{p} \quad (43)$$

Now, we denote the total face value of Date-0 debt with and without covenants F_0^c and F_0^{nc} respectively, with $F_0^c + F_0^{nc} \equiv F_0$, and, likewise, amount borrowed with and without covenants by I_0^c and I_0^{nc} respectively, with $I_0^c + I_0^{nc} \equiv I_0$, and, by the definition of ϕ , $I_0^c \equiv \phi I_0$. There are three relevant cases:¹⁰

1. **B does not borrow at Date 1.** In this case, B repays in full at Date 2 if $X_{\text{tot.}} \geq F_0$ and defaults otherwise, in which case creditors are paid pro rata:
 - Unsecured creditors with covenants get $p \min\{F_0^c, \phi X_{\text{tot.}}\}$.
 - Unsecured creditors without covenants get $p \min\{F_0^{nc}, (1 - \phi)X_{\text{tot.}}\}$.
2. **B borrows secured at Date 1, but debt is not accelerated.** In this case, B repays in full at Date 2 if $X_{\text{tot.}} \geq F_0 + F_1$ and defaults otherwise, in which case he repays the secured debt first and the unsecured debt pro rata:
 - Secured creditors break even, getting F_1 with probability p (recall that $F_1 = I_1/p$ from equation (43)).
 - Unsecured creditors with covenants get $p \min\{F_0^c, \phi(X_{\text{tot.}} - F_1)\}$.
 - Unsecured creditors without covenants get $p \min\{F_0^{nc}, (1 - \phi)(X_{\text{tot.}} - F_1)\}$.
3. **B borrows secured at Date 1, and debt is accelerated.** In this case, B repays in full at Date 1 if $pX_{\text{tot.}} \geq F_0 + F_1$ and defaults otherwise, in which case he repays secured debt first, the accelerating unsecured creditors (those with covenants) next, and other unsecured creditors last:
 - Secured creditors get F_1 (given $pX_{\text{tot.}} \geq F_1$ by Assumption 3).
 - Unsecured creditors with covenants get $\min\{F_0^c, pX_{\text{tot.}} - F_1\}$.
 - Unsecured creditors without covenants get either the smaller of their face value and the assets remaining after all other creditors have been repayed: $\min\left\{F_0^{nc}, pX_{\text{tot.}} - F_1 - \min\{F_0^c, pX_{\text{tot.}} - F_1\}\right\}$.

Before moving on the main argument, we prove a lemma that said that the interest rates on debt with and without covenants are the same under the efficient strategy.

¹⁰We omit cases in which B takes on new unsecured debt at Date 1, because it is easy to show that doing so is dominated by taking on new secured debt at Date 1.

Lemma 4. *If B does the first-best strategy, then the interest rates on debt with and without covenants coincide: $F^c/I_0^c = F_0^{nc}/I_0^{nc}$. Hence $I_0^c = \phi I_0$ and $I_0^{nc} = (1 - \phi)I_0$.*

Proof. There are three cases.

Case 1: $p(X_0 + X_1^H) > I_0 + I_1^H$ **and** $pX_0 \geq I_0$. In this case, all debt is repaid in full in the event of success and repaid nothing otherwise. Thus, $F_0^c = I_0^c/p$ and $F_0^{nc} = I_0^{nc}/p$. Hence $F_0 = I_0/p$ which implies that $F_0^c = \phi I_0$ and $F_0^{nc} = (1 - \phi)F_0 = (1 - \phi)I_0$.

Case 2: $p(X_0 + X_1^H) < I_0 + I_1^H$ **and** $pX_0 \geq I_0$. In this case, B following success if $Q = H$, but not if $Q = L$. Using $I_0^c = \phi I_0$ and $I_0^{nc} = (1 - \phi)I_0$, creditors' break-even conditions are

$$\phi I_0 = p \left(q \phi \left(X_0 + X_1^H - \frac{I_1}{p} \right) + (1 - q) F_0^{nc} \right), \quad (44)$$

$$(1 - \phi) I_0^{nc} = p \left(q(1 - \phi) \left(X_0 + X_1^H - \frac{I_1}{p} \right) + (1 - q) F_0^{nc} \right), \quad (45)$$

having used $F_1 = I_1/p$. Solving for F_1^c and F_1^{nc} above gives the result.

Case 3: $pX_0 < I_0$. In this case, B defaults given success if $Q = L$ but not if $Q = H$. Again, we use $I_0^c = \phi I_0$ and $I_0^{nc} = (1 - \phi)I_0$ to write creditors' break-even conditions:

$$\phi I_0 = p \left(q F_0^c + (1 - q) \phi X_0 \right), \quad (46)$$

$$(1 - \phi) I_0 = p \left(q(1 - \phi) F_0^{nc} + (1 - q)(1 - \phi) X_0 \right). \quad (47)$$

Again, solving for F_0^c and F_0^{nc} gives the result. □

We now turn to the proof of the proposition. As we argued in the text, acceleration must be incentive compatible following a covenant violation if $Q = L$ but not if $Q = H$. Rather than the conditions (20) and (21), in which we assumed that B never repaid in full, we now have

$$\min \left\{ \phi F_0, p(X_0 + X_1^H) - \frac{I_1}{p} \right\} \leq p \min \left\{ \phi F_0, \phi \left(X_0 + X_1^H - \frac{I_1}{p} \right) \right\}, \quad (48)$$

$$\min \left\{ \phi F_0, p(X_0 + X_1^L) - \frac{I_1}{p} \right\} \geq p \min \left\{ \phi F_0, \phi \left(X_0 + X_1^L - \frac{I_1}{p} \right) \right\}. \quad (49)$$

Before launching into the main argument, we can dispense with a few cases relatively easily:

1. We can focus on cases in which there is dilution if $Q = L$, or $F_0 > X_0 + X_1^L - I_1/p$. Otherwise, B will not finance Project 1 in this case, since it has negative NPV.
2. By implication, we can focus on cases in which there is dilution if $Q = H$ as well, since $X_1^H < X_1^L$ by hypothesis.
3. We can focus on cases in which the accelerated debt is not paid in full if $Q = L$, since we are looking only for a sufficient condition (and otherwise acceleration is always IC).

Now, the ICs can be simplified to read

$$\min \left\{ \phi F_0, p(X_0 + X_1^H) - \frac{I_1}{p} \right\} \leq q\phi \left(X_0 + X_1^H - \frac{I_1}{q} \right), \quad (50)$$

$$p(X_0 + X_1^L) - \frac{I_1}{p} \geq p\phi \left(X_0 + X_1^L - \frac{I_1}{p} \right). \quad (51)$$

To get sufficient conditions, we can split (50) in two, and write

$$p(X_0 + X_1^H) - \frac{I_1}{p} \leq p\phi \left(X_0 + X_1^H - \frac{I_1}{p} \right), \quad (52)$$

$$p(X_0 + X_1^H) - \frac{I_1}{p} \leq \phi F_0, \quad (53)$$

$$p(X_0 + X_1^L) - \frac{I_1}{p} \geq p\phi \left(X_0 + X_1^L - \frac{I_1}{p} \right). \quad (54)$$

Combining the above, we have

$$\max \left\{ \frac{p(X_0 + X_1^H) - I_1/p}{F_0}, \frac{p(X_0 + X_1^H) - I_1/p}{p(X_0 + X_1^H - I_1/p)} \right\} \leq \phi \leq \frac{p(X_0 + X_1^L) - I_1/p}{p(X_0 + X_1^L - I_1/p)}. \quad (55)$$

Now, we can do away with the max above. Recall that we are focused on a case in which there is dilution (hence default) given success if $Q = H$. It follows that $F_0 > X_0 + X_1^H - I_1/p > p(X_0 + X_1^H - I_1/p)$ and hence

$$\max \left\{ \frac{p(X_0 + X_1^H) - I_1/p}{F_0}, \frac{p(X_0 + X_1^H) - I_1/p}{p(X_0 + X_1^H - I_1/p)} \right\} = \frac{p(X_0 + X_1^H) - I_1/p}{p(X_0 + X_1^H - I_1/p)}. \quad (56)$$

So we can implement the first best if we can find ϕ satisfying

$$\frac{p(X_0 + X_1^H) - I_1/p}{p(X_0 + X_1^H - I_1/p)} \leq \phi \leq \frac{p(X_0 + X_1^L) - I_1/p}{p(X_0 + X_1^L - I_1/p)}. \quad (57)$$

Since the LHS is always less than one and the RHS is greater than zero by Assumption 3, such a ϕ exists whenever the LHS is less than the RHS, or $X_1^H \leq X_1^L$, which is the condition in the proposition.

Renegotiation proofness. This argument hinges on acceleration being a credible threat when $Q = L$, even though liquidation is inefficient. To complete the proof, we show that this is robust to the possibility of renegotiation. For renegotiation to be feasible, all parties, i.e. (i) B, (ii) Date-1 secured creditors, (iii) Date-0 creditors, both protected by covenants and not, must be strictly better off. However, if B avoids liquidation and continues, the most he can promise his creditors is $p(X_0 + X_1^L)$. But this is only equal to the liquidation value that creditors are already dividing up among themselves. Hence, there is no way to make them collectively better off.

A.8 Proof of Proposition 4

The expression face value F_0 follows from equations (24), (30), and (36) in the proof of Proposition 1. The regions in which B uses secured debt or covenants and the ranges of σ_0 and ϕ follow from Proposition 2 and Proposition 3 (and their proofs).

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