INVESTMENT MANDATES AND THE DOWNSIDE OF PRECISE CREDIT RATINGS*

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September 2013

Abstract

Expert portfolio managers hold assets on behalf of inexpert clients, but managers' incentives differ from clients'. Managers write restrictive covenants into their contracts, typically including investment mandates that make contracts contingent on assets' credit ratings. In an economy in which competitive, risk-averse informed agents offer contracts to a risk-averse, uninformed investor, the equilibrium contract is affine in wealth and depends on credit ratings, like most real-world delegated investment contracts. It implements the first-best allocation, but the inclusion of credit ratings shuts down risk-sharing and makes everyone worse off. Credit ratings serve only to allow agents to compete more intensely by writing state-contingent contracts. We advocate the regulation of credit rating agencies to prohibit their publishing information in forms conducive to inclusion in rigid contracts. Further, our model provides a contract-based explanation for cyclical trends in mutual funds' flows.

*Thanks to Ron Anderson, Ulf Axelson, the late Sudipto Bhattacharya, Bruno Biais, Max Bruche, Jon Danielsson, Daniel Ferreira, Stéphane Guibaud, Francesco Nava, Paul Pfleiderer, Joel Shapiro, Balazs Szentes, Wei Xiong, and Jean-Pierre Zigrand for their input.

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1 Introduction

Delegated asset managers hold upwards of seventy percent of US publicly traded equity, assuming responsibility for private wealth management based on expertise they have and their clients lack. Unfortunately, finance professionals' incentives can never be perfectly aligned with the interests of their capital providers; the problem represents the theoretical trade-off between information and incentives for the economics of delegation and contracting.

Typical delegated asset management contracts constitute a lump-sum component plus a fee proportional to assets-under-management. In addition to such a profit-sharing rule, contracts frequently include investment mandates to limit risk-taking. According to the Bank for International Settlements (2003), investment mandates commonly include reference to credit ratings. Asset manager Threadneedle Investments provides an example of ratings-based investment mandates in its European Corporate Bond Fund prospectus:

> The portfolio will not be more than 25% invested in securities rated AAA (Standard & Poor's) or equivalent rating by another leading agency. A maximum of 10% of the portfolio can be invested in below investment grade securities.

Why do asset managers propose compensation schemes that depend on public information when clients employ them for their private knowledge? Why do they make their investments dependent on widely available announcements, killing their competitive edge? Who benefits from the current trend toward more detailed contracts? Do complex contracts mitigate incentive problems? How do contracts affect fund flows?

To address these questions, this paper presents a model of the portfolio management industry in which informed asset managers offer contracts competitively to an uninformed investor. Asset managers differ from investors in their risk aversion. In the model, the contract space, return distributions, and ratings categories are general, but the equilibrium contract, like real-world asset management contracts, is affine in wealth and dependent on future credit ratings, even when asset managers' information is strictly better than rating agencies'. Recent papers (Dybvig et al. (2010), He and Xiong (2013)) that study the moral hazard problem between investors and their clients find that investment mandates help to increase effort-provision. Our model, based on asymmetric information, suggests that a contract written on final wealth can already implement truthful revelation and efficient investment and that asset managers make use of credit ratings in their contracts only to compete more intensely with one another. With contractible ratings, an agent must offer a contract "rating-by-rating"; he must make zero profits for each realization of the credit rating. Otherwise, he would have to make positive profits for some rating and a competing agent could undercut him, offering a contract with favourable terms given that rating. Unfortunately, when agents break even state-by-state (viz. rating-by-rating) instead of on average, they and investors no longer share risk over ratings realizations, decreasing utilitarian welfare. The more informative are ratings, the more competition inhibits risk sharing. Since agents always just break even, decreasing ratings' precision is Pareto improving.

Finally, for a given realization of credit ratings, the riskier are assets, the lower is the performance compensation in the equilibrium contract. This result predicts flow from funds with low-powered contracts to funds with high-powered contracts in economic booms, in line with empirical evidence documenting the procyclicality of investments in equity funds and of withdrawals from money market funds.

In response to the questions above, our model says that asset managers refer to ratings only to help them compete; and ratings-based mandates shut down risk-sharing harming investors. Further, high-powered contracts induce procyclical fund flow.

Model and Results

The economy comprises an investor, at least two agents, a risk-free bond, a risky security with unknown distribution, and a public signal about its variance called a credit rating. All players have quadratic utility, but the investor's risk aversion differs from the agents'. The timing is as follows. Firstly, agents compete in contracts before learning their private information or observing the credit rating. They offer contracts to the investor that can depend on the realized return of the risky asset, the final wealth, and the credit rating. Secondly, the investor employs an agent to invest his wealth on his behalf. Thirdly, the credit rating is announced and the agent learns the true variance and invests. Finally, the return realizes and the investor and agent transfer wealth according to the agreed contract.

We firstly demonstrate that our extensive form game is equivalent to a family of principal-agent problems—one for each realization of the credit rating—in which the investor offers the contract take-it-or-leave-it to a single agent; then we apply the revelation principle before transforming the agency problems into social planners' problems for appropriate welfare weights. Since the efficient risk-sharing rule does not depend on the true state (the agent's type in the formalism) and the optimal investment does not depend on the welfare weight, the efficient sharing rule composed with the optimal investment implements the agent's truth-telling and thus efficiency. The contracts do depend on the credit rating, which proves to be a valuable tool for the agents to compete for the investor's business.

Since the equilibrium contract does not depend on the market return, the indirect mechanism (viz. the "observed" contract) coincides with the direct mechanism for each rating. If, as in reality, securities are rated in bands with a worse rating corresponding to higher variance, then the equilibrium contract is higher powered when ratings are good, which we interpret as an economic boom. The prediction jives with empirical evidence on fund flows: capital flows from money market funds to equity funds as economic conditions improve.

For our main result, we rank the ratings by informativeness according to the coarseness of the sigma algebras they generate and demonstrate that ex ante—namely, in expectation across the family of principal-agent problems—coarser ratings Pareto dominate finer ratings. Our proof uses the law of iterated expectations to show that one random variable second-order stochastically dominates another and then the result that the expectation of a concave function of a dominated random variable is less than the expectation of the function of a dominating one.

We also generalize the model in two ways. Firstly, we relax the assumption that asset managers learn the variance perfectly. If the asset managers' information is imperfect but still strictly superior to the credit rating agency's, then the analysis does not change substantively. If, however, the asset manager learns from the rating, the welfare consequences of increasing ratings' informativeness are ambiguous, because the allocative efficiency gained from fine ratings categories trades off against the risk-sharing benefits of coarse ones. Secondly, we include ratings changes after the investor has hired the agent and demonstrate that equilibrium contracts do not refer to them because agents contract on ratings only as an instrument of competition, a practice that loses efficacy after agents are employed.

Contracting Background

The literature on the misalignment of incentives between delegated portfolio managers and investors originates with Bhattacharya and Pfleiderer (1985), who consider the problem of an investor who must simultaneously screen talent and induce truth-telling. In their model, agents have CARA preferences and investors are approximately risk-neutral. Stoughton (1993) modifies the setting to include a moral hazard problem: managers take a costly action in order to become informed; he demonstrates the importance of nonlinear contracts. Palomino and Prat (2003) study the problem when the agent chooses the portfolio's riskiness unobservably and demonstrate that the optimal non-linear contract need not be complicated: his optimal contract is a bonus contract that pays a fixed fee above a threshold.

The economic spirit of our model resembles Palomino and Prat's, since we study agents' incentives to shift risk in an optimal contracting setting, but our structure is more truly a simplification of Bhattacharya and Pfleiderer's as we consider a problem of portfolio choice with hidden information but dispense with agents' heterogeneity. We add a public contracting variable that correlates with the agents' information and focus on risk-averse investors—thereby bringing risk-sharing to the foreground—and we solve for the optimal direct mechanism as a function of the players' risk aversions and agents' reservation payoff.

Notwithstanding our focus on contracting, our results are reminiscent of papers relating risk-sharing to truthful revelation of private information on the one hand and speculation in the presence of public information on the other. In a 1984 paper about information revelation and joint production given a social planner's sharing rule, Wilson demonstrates that private knowledge may not lead to inefficient risk-sharing. A similar result in our model obviates the usefulness of the public signal; in fact, decreasing the informativeness of public information leads to Pareto improvements. Hirshleifer applied his famous 1971 argument that traders may be uniformly better off if they agree not to obtain privately valuable information to a market setting very different from our model of strategic agency, but his economics are robust: public information destroys risk-sharing and, since it fails to mitigate the agency problem, it does only harm.

Policy Prescription

Our paper explains why even expert asset managers write contracts on signals that are to them uninformative: it gives them a competitive edge in boom times. Our results suggest that such mandates, ostensibly imposed to protect investors, only impair risk-sharing and thus welfare.

Both the global financial crisis that climaxed in 2008-2009 and the ensuing Eurozone sovereign debt crisis (the climax of which EU politicians continue to fight to deter/postpone) have brought scrutiny to the major credit rating agencies. Much academic attention has focused on the agencies' incentives and information-provision (notably, Mathis et al. (2009), Bolton et al. (2012), Skreta and Veldkamp (2009), and Doherty et al. (2012) among many others), but few papers have addressed the question of the effect of credit ratings on financial institutions and markets. Kurlat and Veldkamp (2011) do explore the problem in a two-asset rational expectations equilibrium and also rediscover some of Hirshleifer's reasoning: announcing credit ratings makes investors worse off, since more information about the payoff of the risky asset makes their securities too alike, thus preventing diversification—viz. public information impedes risk-sharing. Their paper uses a cardinal welfare measure to suggest that government enforcement of information disclosure may hurt investors. Given our model examines only a narrow channel of the effect of credit ratings, our regulatory advice is less bold: broaden ratings categories and focus on qualitative reporting, i.e. coarsen the contractible public information partition. Our suggestion jives with regulators' assertions that institutions should quit responding robotically to ratings, as rigid contingent contracts fine-tuned to CRA announcements force them to. For example, the Financial Stability Board, in 2010, told the G20 Finance Ministers that

> Investment managers and institutional investors must not mechanistically rely on CRA ratings for assessing the creditworthiness of assets. This principle applies across the full range of investment managers and of institutional investors, including money market funds, pension funds, collective investment schemes (such as mutual funds and investment companies), insurance companies and securities firms... [Investment managers should limit] the proportion of a portfolio that is CRA ratings-reliant.

2 Model

The model constitutes an extensive game of incomplete information in which agents first compete in contracts in the hope of being employed by a single investor and then invest his capital on his behalf in assets with exogenous returns. The solution concept is perfect bayesian equilibrium.

The Economy

The economy comprises a large number of agents, viewed as asset managers, with von Neuman–Morganstern utility u_{A} and outside option $\bar{u}$ as well as a single investor with von Neuman–Morgenstern utility u_{I} and one unit of initial wealth. There are two securities, a risk-free bond with gross return R_f and a risky asset with random gross return $\tilde{R}$; initially no one knows the distribution of $\tilde{R}$. Finally, a public signal $\tilde{\rho}$ is informative about the distribution of returns. Call $\tilde{\rho}$ the credit rating of the risky security.

Two key assumptions give the model structure. Firstly, all players have quadratic utility.

$$u_n(W) = -\frac{1}{2}\big(\alpha_n - W\big)^2 \tag{1}$$

for $n \in \{\mathrm{A}, \mathrm{I}\}$. The investor differs from the agents in his risk aversion (note that the coefficient of absolute risk aversion is $(\alpha_n - W)^{-1}$ so α_n represents risk tolerance).

Secondly, the mean return $\bar{R}$ of the risky asset is known. Since, with quadratic utility, players' expected utility depends only on the mean and variance of the distribution, summarize the unknown payoff-relevant component of the distribution with the random variance $\tilde{\sigma}^2$,

$$\sigma^2 := \operatorname{Var}\left[\tilde{R} \,\middle|\, \tilde{\sigma} = \sigma\right]. \tag{2}$$

With this notation the assumption that all players know the mean return of the risky asset reads

$$\mathbb{E}\left[\tilde{R} \,\middle|\, \tilde{\sigma} = \sigma\right] = \bar{R} \tag{3}$$

for each σ^2. Note that this assumption implies that the credit rating is informative only about the asset's risk and not about its expected return,

$$\mathbb{E}\left[\tilde{R} \,\middle|\, \tilde{\rho} = \rho\right] = \mathbb{E}\big[\tilde{R}\big] \tag{4}$$

but, in general,

$$\mathbb{E}\left[\tilde{\sigma}^2 \,\middle|\, \tilde{\rho} = \rho\right] \neq \mathbb{E}\left[\tilde{\sigma}^2\right]. \tag{5}$$

With these preferences, players' marginal utility is decreasing when their wealth is large. To prevent its unrealistic implications, we aim to restrict the set of possible realizations of final wealth so that

$$\operatorname{supp} \tilde{w} \subset [0, \alpha_{\mathrm{I}} + \alpha_{\mathrm{A}}), \tag{6}$$

which will ensure the equilibrium contract satisfies our feasibility conditions (cf. equation 11). To this end, make the technical assumption that

$$\big(\bar{R} - R_f\big)\big(R - \bar{R}\big) \leq \sigma^2 \tag{7}$$

for all pairs (σ, R).[1]

Actions and Contracts

The investor's only action is employing an agent a who then forms a verifiable portfolio with weight x in the risky asset and $1 - x$ in the bond with the investor's capital. The investor wishes to delegate investment to an agent because he is better informed but anticipates a misalignment of investment incentives since the investor's risk tolerance differs from the agents'.

Contracts attempt to align incentives to mitigate the downside of delegated asset management, allocating decision rights to the players with the most information. Critically, credit ratings are verifiable but agents' true information about the distribution of returns is not. Thus the contracting variables are credit rating ρ, the return on the risky asset R, and the final wealth, denoted

$$\tilde{w} \equiv w\big(x, \tilde{R}\big) := R_f + x\big(\tilde{R} - R_f\big). \tag{8}$$

Since for each realization $\tilde{R} = R$,

$$x = \frac{w - R_f}{R - R_f}, \tag{9}$$

[1] Condition 7, sufficient for condition 6, comes from solving the game assuming that the agent's participation constraint binds, then writing a sufficient condition for it to bind in light of the equilibrium.

it is without loss of generality to assume the portfolio weight x is contractible and omit R from the contract.

Assume that agents' contracts do not depend on other agents' contracts. Thus write that each agent a offers a contract

$$\Phi_a : \big(w, x, \rho\big) \mapsto \Phi_a\big(w, x, \rho\big), \tag{10}$$

but to economize on notation we often omit the contract's arguments and write $\Phi(\tilde{w})$ for $\Phi\big((w(x, \tilde{R}), x, \rho\big)$. A feasible contract is a Lebesgue measurable function such that

$$w - \alpha_{\mathrm{I}} < \Phi(w) < \alpha_{\mathrm{A}}, \tag{11}$$

which ensures marginal utility is positive.

There is full commitment.

The dependence of contracts on portfolio weights and credit ratings are the investment mandates in the model.

Timing

Aiming to understand why asset managers themselves use investment mandates in addition to or instead of performance incentives—contract on x and ρ and not just w—we have agents offer the contracts in our model. While the problem is ultimately equivalent to one in which the investor offers the contract to a single agent take-it-or-leave-it, we think that our set-up is important both to get the information structure right in the single-agent model and to understand applications and larger economics better.

After agents announce their contracts, the investor observes the credit rating and employs an agent who, knowing the true distribution of returns, goes on to form a portfolio with the investor's wealth. Finally, the assets pay off and players divide final wealth according to the initial contract. Formally, the timing is as follows:

1. Agents simultaneously offer contracts Φ_a.
2. The variance of the risky security realizes, $\tilde{\sigma}^2 = \sigma^2$ and ratings are released, $\tilde{\rho} = \rho$.

3. The principal observes the profile of contracts $\{\Phi_a\}_a$ and credit rating ρ and hires an agent a^*.

4. Agent a^* invests x^* in the risky asset.

5. The return of the risky asset realize, $\tilde{R} = R$, and final wealth

$$w = R_f + x^*(R - R_f) \tag{12}$$

   is distributed such that agent a^* is awarded $\Phi_{a^*}(w)$ and the investor keeps $w - \Phi_{a^*}(w)$.

Note that key to our timing is that players learn ratings after agents offer contracts but before investors have parted with their cash. Since employing lawyers to formalize the documents is both slow and costly for delegated asset managers, agents' fixing Φ before knowing ρ is consistent with our application. As the next section's results demonstrate, assuming ratings realize before, but do not change after, the investor's delegation decision is equivalent (in welfare and allocation terms) to the richer model in which credit ratings are also updated after the investor has committed to an agent.

3 Results

The main result that coarser credit ratings lead to Pareto improvements follows from first transforming the extensive form to look like a classical principal-agent problem and then rewriting it as a social planner's problem, where the challenge is to implement truth-telling and optimal risk-sharing simultaneously; the relationship of our result to Wilson's 1984 theorem on optimal sharing rules for joint production with dispersed information becomes apparent in this final formulation.

Competition Is Rating-by-Rating

The first lemma states that competition in contracts is Bertrand-like in the sense that the employed agent will receive his reservation utility conditional on any realization of the credit rating $\tilde{\rho}$; further the agents act so as to maximize the investor's expected utility conditional on every ρ subject to their participation constraints.

Lemma 3.1. *If Φ is the contract of the agent employed given rating $\hat{\rho}$ and there is another contract $\hat{\Phi}$ such that*

$$\mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w} - \hat{\Phi}\big(\tilde{w}\big)\Big) \mid \tilde{\rho} = \hat{\rho}\right] > \mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w} - \Phi\big(\tilde{w}\big)\Big) \mid \tilde{\rho} = \hat{\rho}\right], \tag{13}$$

then

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}\big(\tilde{w}\big)\Big) \mid \tilde{\rho} = \hat{\rho}\right] < \bar{u}. \tag{14}$$

Proof. Suppose, in anticipation of a contradiction, an equilibrium in which the employed agent offers contract Φ given credit rating $\hat{\rho}$ and there is another contract $\hat{\Phi}$ such that

$$\mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w} - \hat{\Phi}\big(\tilde{w}\big)\Big) \mid \tilde{\rho} = \hat{\rho}\right] > \mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w} - \Phi\big(\tilde{w}\big)\Big) \mid \tilde{\rho} = \hat{\rho}\right] \tag{15}$$

and

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}\big(\tilde{w}\big)\Big) \mid \tilde{\rho} = \hat{\rho}\right] \geq \bar{u}. \tag{16}$$

Suppose that agent $\hat{\mathrm{A}}$ offers the contract $\hat{\Phi}_\varepsilon$ constructed from $\hat{\Phi}$ given $\hat{\rho}$

$$\hat{\Phi}_\varepsilon(w, x, \hat{\rho}) := \alpha_{\mathrm{A}} - \sqrt{\big(\alpha_{\mathrm{A}} - \hat{\Phi}(w, x, \hat{\rho})\big)^2 - 2\varepsilon} \tag{17}$$

and that his action is according to the supposed equilibrium if $\rho \neq \hat{\rho}$. Note that

$$u_{\mathrm{A}}\big(\hat{\Phi}_\varepsilon\big) = u_{\mathrm{A}}\big(\hat{\Phi}\big) + \varepsilon \tag{18}$$

immediately by construction and the quadric form of the agents' utility. The contract does not change agents' incentives and the same portfolio weight x is chosen under either contract. Since x is unchanged and $u_{\mathrm{I}}(w - \hat{\Phi}_\varepsilon(w))$ is continuous in ε, for $\varepsilon > 0$ sufficiently small

$$\mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w} - \hat{\Phi}_\varepsilon\big(\tilde{w}\big)\Big) \mid \tilde{\rho} = \hat{\rho}\right] > \mathbb{E}\left[u_{\mathrm{I}}\Big(\tilde{w} - \Phi\big(\tilde{w}\big)\Big) \mid \tilde{\rho} = \rho\right]. \tag{19}$$

Thus the investor will employ agent $\hat{\mathrm{A}}$ who will receive utility greater than his utility at the supposed equilibrium given rating $\hat{\rho}$ where he was unemployed and obtaining $\bar{u}$ (and the same utility given all other ratings). Thus $\hat{\Phi}_\varepsilon$ is a profitable deviation for agent $\hat{\mathrm{A}}$ and Φ cannot be the contract of an agent employed at equilibrium given $\hat{\rho}$. □

Principal-Agent Formulation and Revelation Principle

Lemma 3.1 asserts that agents compete rating-by-rating, maximizing investor welfare subject to their participation constraints, that is to say that for every realization ρ of the credit ratings the contract of the employed agent and the corresponding portfolio weight solve the principal-agent problem:

$$\begin{cases} \text{Maximize } \mathbb{E}\left[u_{\mathrm{I}}\Big(w(x,\tilde{R}) - \Phi\big(w(x,\tilde{R}),x,\rho\big)\Big) \,\Big|\, \tilde{\rho}=\rho\right] \\ \text{subject to } \mathbb{E}\left[u_{\mathrm{A}}\Big(\Phi\big(w(x,\tilde{R}),x,\rho\big)\Big) \,\Big|\, \tilde{\rho}=\rho\right] \geq \bar{u} \text{ and} \\ x \in \arg\max\Big\{\mathbb{E}\left[u_{\mathrm{A}}\big(\Phi\big(w(\xi,\tilde{R}),\xi,\rho\big) \,\big|\, \tilde{\sigma}=\sigma\right] \;;\; \xi\in\mathbb{R}\Big\} \end{cases} \tag{P-A}$$

over all feasible contracts Φ. Applying the revelation principle allows us to restrict attention to direct mechanisms

$$\varphi\big(w\,;\,\hat{\sigma},\rho\big) := \Phi\big(w, x(\hat{\sigma}),\rho\big) \tag{20}$$

where x is an incentive compatible portfolio weight given Φ.

Replace the incentive compatibility of the portfolio allocation x with the truth-telling condition $\hat{\sigma} = \mathrm{Id}$:

$$\begin{cases} \text{Maximize } \mathbb{E}\left[u_{\mathrm{I}}\Big(W(\tilde{\sigma},\tilde{R}) - \varphi\big(W(\tilde{\sigma},\tilde{R}),\tilde{\sigma},\rho\big)\Big) \,\Big|\, \tilde{\rho}=\rho\right] \\ \text{subject to } \mathbb{E}\left[u_{\mathrm{A}}\Big(\varphi\big(W(\tilde{\sigma},\tilde{R}),\tilde{\sigma},\rho\big)\Big) \,\Big|\, \tilde{\rho}=\rho\right] \geq \bar{u} \text{ and} \\ \sigma \in \arg\max\Big\{\mathbb{E}\left[u_{\mathrm{A}}\big(\varphi\big(W(\hat{\sigma},\tilde{R}),\hat{\sigma},\rho\big) \,\big|\, \tilde{\sigma}=\sigma\right] \;;\; \hat{\sigma}\in\mathbb{R}\Big\} \end{cases} \tag{P-A(D)}$$

over all feasible contracts φ where W denotes the wealth as a function of the report $\hat{\sigma}$ rather than of the portfolio weight x directly,

$$W\big(\hat{\sigma},R\big) := w\big(x(\hat{\sigma}),R\big). \tag{21}$$

Note while the contract and wealth do not depend directly on the true variance σ^2, we already plugged $\hat{\sigma}(\sigma) = \sigma$ from the truth-telling condition into the statement of the problem.

Equilibrium Contract as the Solution of a Social Planner's Problem

Use the method of Lagrange multipliers to eliminate the participation constraint and say that the problem is to maximize

$$\mathbb{E}\left[u_{\mathrm{I}}\Big(W(\tilde\sigma,\tilde R\,)-\varphi\big(W(\tilde\sigma,\tilde R\,),\tilde\sigma,\rho\big)\Big)+\mu\left(u_{\mathrm{A}}\Big(\varphi\big(W(\tilde\sigma,\tilde R\,),\tilde\sigma,\rho\big)\Big)-\bar u\right)\,\Big|\,\tilde\rho=\rho\right] \tag{22}$$

subject to

$$\sigma\in\arg\max\left\{\mathbb{E}\left[u_{\mathrm{A}}\big(\varphi\big(W(\hat\sigma,\tilde R\,),\hat\sigma,\rho\big)\,\big|\,\tilde\sigma=\sigma\right]\ ;\ \hat\sigma\in\mathbb{R}\right\} \tag{23}$$

over feasible φ and $\mu\in\mathbb{R}$. Defining the social welfare given credit rating ρ (with weight one on the investor and μ on the agent) as

$$\begin{aligned}\mathcal{S}_{\mu,\rho}(x)[\varphi]:=\mathbb{E}\left[u_{\mathrm{I}}\Big(W(\tilde\sigma,\tilde R\,)-\varphi\big(W(\tilde\sigma,\tilde R\,),\tilde\sigma,\rho\big)\Big)\,\Big|\,\tilde\rho=\rho\right]+&\\+\,\mu\,\mathbb{E}\left[u_{\mathrm{A}}\Big(\varphi\big(W(\tilde\sigma,\tilde R\,),\tilde\sigma,\rho\big)\Big)\,|\,\tilde\rho=\rho\right],&\end{aligned} \tag{SW}$$

observe that (since lemma 3.1 says that the agent's participation constraint binds) the principal-agent problem is the social planner's problem $\mathrm{SP}(\mu,\rho)$ to maximize $\mathcal{S}$ given ρ subject to truth-telling whenever μ is the welfare weight such that the agent breaks even,

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\varphi_{\mu,\rho}\big(W(\tilde\sigma,\tilde R\,),\tilde\sigma,\rho\big)\Big)\,\Big|\,\tilde\rho=\rho\right]=\bar u, \tag{24}$$

where $\varphi_{\mu,\rho}$ is the solution to the problem.

Transforming the game into a social planner's problem combined with the fixed-point problem reveals that the task is to trade off efficient risk sharing with implementing truth-telling.

The Efficient Sharing Rule Implements Truth-telling

Step back from the game under scrutiny to observe that the optimal risk sharing rule is linear for all μ and ρ by maximizing

$$\mathbb{E}\left[u_{\mathrm{I}}\big(w-\phi(w)\big)+\mu\,u_{\mathrm{A}}\big(\phi(w)\big)\,\big|\,\tilde\sigma=\sigma\right] \tag{25}$$

unconstrained over all feasible ϕ, which immediately decouples into a family of one-dimensional optimization problems solvable by differentiation:

$$u_{\mathrm{I}}'\big(w - \phi_\mu(w)\big) = \mu\, u_{\mathrm{A}}'\big(\phi_\mu(w)\big) \tag{26}$$

or, plugging in quadratic utility,

$$w - \phi_\mu(w) - \alpha_{\mathrm{I}} = \mu\big(\phi_\mu(w) - \alpha_{\mathrm{A}}\big) \tag{27}$$

for all w. Thus the unconstrained efficient sharing rule is

$$\phi_\mu(w) = \alpha_{\mathrm{A}} + \frac{w - \alpha_{\mathrm{I}} - \alpha_{\mathrm{A}}}{1+\mu}, \tag{28}$$

which is feasible whenever $\mu > 0$ and assumption 6 holds. Since the standard deviation σ does not enter the expression, the social planner need not know the true variance to implement optimal risk sharing.

Given the optimal sharing rule, the expression for the corresponding optimal investment X_μ in the risky security will be useful. The social planner finds it by computing the maximum of

$$\begin{aligned} &\mathbb{E}\left[u_{\mathrm{I}}\left(R_f + x\big(\tilde{R} - R_f\big) - \phi_\mu\Big(R_f + x\big(\tilde{R} - R_f\big)\Big)\right) \,\middle|\, \tilde{\sigma} = \sigma\right] \\ &\qquad + \mu\mathbb{E}\left[u_{\mathrm{A}}\left(\phi_\mu\Big(R_f + x\big(\tilde{R} - R_f\big)\Big)\right) \,\middle|\, \tilde{\sigma} = \sigma\right], \end{aligned} \tag{29}$$

over all x. Mechanical computations collected in Appendix A.1 reveal that the optimal investment is

$$X_\mu(\sigma) \equiv X(\sigma) = \frac{\big(\bar{R} - R_f\big)\big(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f\big)}{\sigma^2 + \big(\bar{R} - R_f\big)^2}. \tag{30}$$

Note that the optimal investment does not depend on the welfare weight μ; in fact, given the optimal sharing rule ϕ_μ the investment X maximizes the agent's utility:

$$\begin{aligned} \mathbb{E}\left[u_{\mathrm{A}}\left(\phi_\mu\left(R_f + X(\sigma)\left(\tilde{R} - R_f\right)\right)\right) \Big| \tilde{\sigma} = \sigma\right] \\ \geq \mathbb{E}\left[u_{\mathrm{A}}\left(\phi_\mu\left(R_f + x\left(\tilde{R} - R_f\right)\right)\right) \Big| \tilde{\sigma} = \sigma\right] \end{aligned} \tag{31}$$

for all $x \in \mathbb{R}$, in particular for all $x \in \operatorname{Im} X$ so

$$\begin{aligned} \mathbb{E}\left[u_{\mathrm{A}}\left(\phi_\mu\left(R_f + X(\sigma)\left(\tilde{R} - R_f\right)\right)\right) \Big| \tilde{\sigma} = \sigma\right] \\ \geq \mathbb{E}\left[u_{\mathrm{A}}\left(\phi_\mu\left(R_f + X(\hat{\sigma})\left(\tilde{R} - R_f\right)\right)\right) \Big| \tilde{\sigma} = \sigma\right] \end{aligned} \tag{32}$$

for all $\hat{\sigma}$, which proves the following essential lemma.

Lemma 3.2. *The efficient sharing rule composed with the optimal investment $\phi_\mu \circ X$,*

$$\varphi_\mu\left(R_f + X(\hat{\sigma})\left(\tilde{R} - R_f\right), \hat{\sigma}, \rho\right) = \phi_\mu\left(R_f + X(\hat{\sigma})\left(\tilde{R} - R_f\right)\right), \tag{33}$$

implements the agent's truth-telling for any credit rating ρ.

Lemma 3.2 is closely related to Wilson's 1984 result on the "revelation of information for joint production", where he proves that when the efficient sharing rule is affine, truthful revelation is a Nash equilibrium. We import the methodology for connecting risk-sharing with implementation into the principal-agent setting, emphasizing the explicit (direct) implementation and, further, that the optimal sharing rule is the investor's optimal contract by the equivalence of the principal-agent problem and social planner's problem above. Note that Wilson's proof exploits that when the efficient sharing rule is affine its derivative is constant and cancels out of his problem's first-order condition; we instead use that in our case the optimal allocation is independent of the welfare weight.

The Break-even Welfare Weight and Ex Ante Utility

In order to characterize the employed agent's contract via the social planner's problem, determine the welfare weight μ_ρ given the credit rating ρ; thanks to truth-telling, the equilibrium allocation depends on the credit rating only via the participation constraint:

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\phi_{\mu_\rho}\Big(R_f + X(\tilde{\sigma})\big(\tilde{R} - R_f\big)\Big)\right) \Big| \, \tilde{\rho} = \rho\right] = \bar{u}, \tag{34}$$

which, via string of calculations employing the law of iterated expectations (cf. Appendix A.2), says

$$\big(1 + \mu_\rho\big)^2 = \frac{\big(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f\big)^2}{2\,|\bar{u}|}\,\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2}\,\middle|\, \tilde{\rho} = \rho\right]. \tag{35}$$

A tangential remark: the mapping

$$\tilde{\sigma}^2 \mapsto \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \big(\tilde{R} - R_f\big)^2} \tag{36}$$

under the expectation operator is concave, so that if the distribution of $\tilde{\sigma}^2$ spreads out (for example in the the second-order stochastic dominance sense) then μ_ρ decreases, suggesting that the more distribution risk the agent faces, the less the investor must compensate him despite his risk aversion, as captured by the social planner's lower welfare weight. The reason is that his investment decision comes after the realization of the variance, and thus the riskier decisions come with option value: when $\tilde{\sigma}^2$ is very low he will invest a lot in the risky asset, while when it is high he will invest relatively more in the riskless bond.

Further, the equilibrium welfare weight provides a handy formula for the investor's equilibrium expected utility given the rating ρ,

$$\mathbb{E}\left[u_{\mathrm{I}}\Big(W(\tilde{\sigma}, \tilde{R}) - \varphi\big(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho\big)\Big) \,\Big|\, \tilde{\rho} = \rho\right] = \bar{u}\,\mu_\rho^2 \tag{37}$$

(see Appendix A.3 for the short calculation) and thus his ex ante expected utility

$$\mathbb{E}\left[u_{\mathrm{I}}\Big(W(\tilde{\sigma}, \tilde{R}) - \varphi\big(W(\tilde{\sigma}, \tilde{R}), \tilde{\sigma}, \rho\big)\Big)\right] = \bar{u}\,\mathbb{E}\left[\mu_{\tilde{\rho}}^2\right]. \tag{38}$$

Direct Mechanism/Observed Contracts

The direct mechanism is

$$\Phi(w, x, \rho) = \alpha_{\mathrm{A}} + \frac{w - \alpha_{\mathrm{I}} - \alpha_{\mathrm{A}}}{1 + \mu_\rho} \tag{39}$$

where μ_ρ is defined in equation 35.

The compensation contract is affine in wealth, as are typical asset management contracts. For the next result (and the next result only), consider a simplified but realistic credit rating rule. Let $\tilde{\rho}$ define a partition of the realization of the variance $\sigma_0^2 < \sigma_1^2 < \cdots$, namely

$$\mathbb{P}\{\tilde{\sigma}^2 \in [\sigma_i^2, \sigma_{i+1}^2) \,|\, \rho_i\} = 1. \tag{40}$$

Proposition 3.1. *For $i < j$, $\mu_{\rho_i} < \mu_{\rho_j}$. Increases in the expected variance decrease the power of the contract, i.e. the slope in the wealth.*

Proof. Since

$$\frac{\sigma^2}{\sigma^2 + \left(\bar{R} - R_f\right)^2} \tag{41}$$

is increasing in σ^2,

$$\mathbb{E}\left[\left.\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2}\right| \rho_i\right] < \mathbb{E}\left[\frac{\sigma_{i+1}^2}{\sigma_{i+1}^2 + \left(\bar{R} - R_f\right)^2}\right] < \mathbb{E}\left[\left.\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2}\right| \rho_{i+1}\right]. \tag{42}$$

Immediately from equation 35, $\mu_{\rho_i} < \mu_{\rho_{i+1}}$ and by induction $\mu_{\rho_i} < \mu_{\rho_j}$ whenever $i < j$. Combined with equation 39, the result implies that lower expected variances correspond to steeper wealth compensation for agents. □

In the model, ratings describe the variance of the market portfolio. Define a "boom" a realization of $\tilde{\rho}$ implying low expected variance. With this interpretation, proposition 3.1 says that employed agents have higher powered contracts in booms than in busts. Since, almost uniformly, equity funds offer higher powered contracts than money market funds, the model predicts that the in-flows to equity funds relative to money market funds will be procyclical. Using a sample of US mutual fund data from 1991 to 2008, Chalmers et al. (2010) finds that investors direct funds away from money market funds towards equity funds when aggregate economic conditions improve, in line with our prediction.

Main Result: Coarser Credit Ratings Are Pareto-Improving

Since competition means that agents always receive their reservation utilities, the main result that coarsening credit ratings makes everyone better-off follows from directly comparing the ex ante expected utility of the investor across ratings systems, using the formula above combined with the connection between convex functions, second-order stochastic dominance, and the law of iterated expectations.

Proposition 3.2. *Coarser credit ratings Pareto-dominate finer ones: for any ratings $\tilde{\rho}_C$ and $\tilde{\rho}_F$ such that $\sigma(\tilde{\rho}_C) \subset \sigma(\tilde{\rho}_F)$, the ex ante equilibrium utility of all agencies is weakly higher given $\tilde{\rho}_C$ than $\tilde{\rho}_F$.*

Proof. Our proof has two main steps. Firstly to show that the investor's ex ante expected utility is minus the expectation of a convex function,

$$\bar{u}\,\mathbb{E}\left[\mu_{\tilde{\rho}}^2\right] = -c\,\mathbb{E}\left[f\Big(\mathbb{E}\left[Y \mid \tilde{\rho}\right]\Big)\right] \tag{43}$$

for $c > 0$, $f'' > 0$ and a random variable Y; and secondly to show that the expectation conditional on coarse ratings second-order stochastically dominates the expectation conditional on fine ratings,

$$\mathbb{E}\left[Y \mid \tilde{\rho}_C\right] \overset{\text{SOSD}}{\succ} \mathbb{E}\left[Y \mid \tilde{\rho}_F\right], \tag{44}$$

whence utility is greater under coarse ratings because minus a convex function is a concave function, and, à la risk aversion, the expectation of a concave function of a stochastically dominated random variable is greater than the expectation of the function of the dominated variable.

Step 1: Rewrite the investor's ex ante expected utility:

$$\begin{aligned}
\bar{u}\,\mathbb{E}\left[\mu_{\tilde{\rho}}^2\right] &= \bar{u}\,\mathbb{E}\left[\left(\sqrt{\frac{(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f)^2}{2|\bar{u}|}\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \,\middle|\, \tilde{\rho}\right]} - 1\right)^2\right] \\
&= \frac{\bar{u}(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f)^2}{\sqrt{2|\bar{u}|}}\mathbb{E}\left[\left[\sqrt{\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + (\bar{R} - R_f)^2} \,\middle|\, \tilde{\rho}\right]} - 1\right]^2\right] \\
&= -c\,\mathbb{E}\left[f\Big(\mathbb{E}\left[Y|\tilde{\rho}\right]\Big)\right]
\end{aligned} \tag{45}$$

where

$$c := \sqrt{|\bar{u}|/2}\,(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f)^2, \tag{46}$$

$$f(z) := \left(\sqrt{z} - 1\right)^2, \tag{47}$$

and

$$Y := \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2}. \tag{48}$$

Note that $c > 0$ and $f''(z) = z^{3/2}/2 > 0$.

Step 2: By definition,

$$\mathbb{E}\left[Y \mid \tilde{\rho}_C\right] \overset{\text{SOSD}}{\succ} \mathbb{E}\left[Y \mid \tilde{\rho}_F\right] \tag{49}$$

if there exists a random variable $\tilde{\varepsilon}$ such that

$$\mathbb{E}\left[Y \mid \tilde{\rho}_F\right] = \mathbb{E}\left[Y \mid \tilde{\rho}_C\right] + \tilde{\varepsilon} \tag{50}$$

and

$$\mathbb{E}\left[\tilde{\varepsilon} \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_C\right]\right] = 0. \tag{51}$$

For $\tilde{\varepsilon} = \mathbb{E}\left[Y \mid \tilde{\rho}_F\right] - \mathbb{E}\left[Y \mid \tilde{\rho}_C\right]$ from the above, the condition is

$$\mathbb{E}\left[\mathbb{E}\left[Y \mid \tilde{\rho}_F\right] - \mathbb{E}\left[Y \mid \tilde{\rho}_C\right] \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_C\right]\right] = 0 \tag{52}$$

or

$$\mathbb{E}\left[\mathbb{E}\left[Y \mid \tilde{\rho}_F\right] \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_C\right]\right] = \mathbb{E}\left[Y \mid \tilde{\rho}_C\right]. \tag{53}$$

Given the assumption $\sigma(\tilde{\rho}_C) \subset \sigma(\tilde{\rho}_F)$ and since conditioning destroys information—$\sigma\big(\mathbb{E}\left[Y \mid \tilde{\rho}_C\right]\big) \subset \sigma(\tilde{\rho}_C)$—apply the law of iterated expectations firstly to add and then to delete conditioning information to calculate that

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[Y \mid \tilde{\rho}_F\right] \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_C\right]\right] &= \mathbb{E}\left[\mathbb{E}\left[\mathbb{E}\left[Y \mid \tilde{\rho}_F\right] \,\middle|\, \rho_C\right] \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_C\right]\right] && (54)\\
&= \mathbb{E}\left[\mathbb{E}\left[Y \mid \tilde{\rho}_C\right] \,\middle|\, \mathbb{E}\left[Y \mid \tilde{\rho}_C\right]\right] && (55)\\
&= \mathbb{E}\left[Y \,\middle|\, \rho_C\right], && (56)
\end{aligned}$$

as desired. □

4 Extensions

Imperfect Private Information

Suppose that the agent receives an imperfect signal about the variance. Namely, he observes the realization of a random variable $\tilde{s}$ that is not independent of $\tilde{\sigma}$. Then, equation 30 for the portfolio allocation becomes

$$X(\rho, s) = \frac{\left(\bar{R} - R_f\right)\left(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f\right)}{\mathrm{Var}\left[\tilde{R} \,|\, \rho, s\right] + \left(\bar{R} - R_f\right)^2}. \tag{57}$$

The optimal contract is $\phi_\mu(R_f + X(\rho, s)(R - R_f))$ (where an equation analogue to 35 determines μ).

Clearly, whenever $\sigma(\tilde{\rho}) \subset \sigma(\tilde{s})$, then $X_\mu(\rho, s)$ does not depend on ρ and our main result remains unchanged. If, instead, $\sigma(\tilde{\rho}) \not\subset \sigma(\tilde{s})$ then a trade-off arises: finer credit ratings still shut down risk sharing, but they increase allocational efficiency, i.e. the portfolio weight is closer to first best. The net welfare effect is then ambiguous.

Our model and policy prescriptions therefore apply to markets in which delegated portfolio managers are better informed than credit rating agencies.

Additional Ratings' Changes

In our model, ratings realize once, after agents offer contracts but before the investor employs an agent. In reality, ratings upgrades and downgrades are frequent and investors and agents have long-term relationships. In the model, if ratings change after the investor has employed an agent, the optimal contract above still induces the agent to invest efficiently. The new rating influences the portfolio allocation only insofar as it improves the agent's information (cf. the preceding discussion of imperfect signals). Ratings changes after the investor and agent commit to a relationship never decrease efficiency and can be beneficial if they improve information. Our model therefore suggests that investment mandates matter only because funds are looking to attract new investors or because their current investors may withdraw their funds. The idea finds support in the observation that hedge funds, who raise money infrequently via long-term contracts do not use investment mandates.

5 Conclusion

In a simple but general model of delegated portfolio management, this paper characterizes the optimal contract between a risk-averse agent and a risk-averse investor (in the case of quadratic utility), finding a compensation scheme similar to typical real-world contracts: it is affine in wealth and makes reference to publicly available credit ratings.

Contracting on only final wealth and market returns achieves first best, but investment mandates allow agents to compete more intensely with one another by contracting on ratings state-by-state, thereby shutting down risk-sharing and moving them away from first best. Contracting on ratings makes everyone worse off, but agents always refer to them because they facilitate competition. The model identifies a downside of precise credit ratings: they inhibit risk-sharing between investors and asset managers.

Our main policy prescription is that credit rating agencies should provide information in forms prohibitive to their inclusion in rigid contracts. Our recommendation jives with the popular suggestion that markets should eliminate the mechanistic reliance on ratings, but casts doubt on the movement toward involved, hands-on risk regulation. Our model also suggests investment mandates may contribute to the cyclicality of mutual fund flows, providing further motivation for their abolition.

A Appendices

A.1 Computation of Optimal Investment

The problem stated in line 29 to find the optimal investment X_μ given the optimal sharing rule

$$\phi_\mu(w) = a + bw, \tag{58}$$

where the constants a and b are as in equation 28, is to maximize the expectation

$$-\frac{1}{2}\,\mathbb{E}\Bigg[\left(R_f + x(\tilde{R} - R_f) - a - b\left(R_f + x(\tilde{R} - R_f)\right) - \alpha_{\mathrm{I}}\right)^2 \\ + \mu\left(\left(a + b\left(R_f + x(\tilde{R} - R_f)\right) - \alpha_{\mathrm{A}}\right)^2\right)\,\Big|\,\tilde{\sigma} = \sigma\Bigg] \tag{59}$$

over all x. Thus the first-order condition says that for optimum X_μ

$$\mathbb{E}\Bigg[(1-b)(\tilde{R} - R_f)\left(R_f + X_\mu(\tilde{R} - R_f) - a - b\left(R_f + X_\mu(\tilde{R} - R_f)\right) - \alpha_{\mathrm{I}}\right) \\ + \mu\, b(\tilde{R} - R_f)\left(a + b\left(R_f + X_\mu(\tilde{R} - R_f)\right) - \alpha_{\mathrm{A}}\right)\,\Big|\,\tilde{\sigma} = \sigma\Bigg] = 0 \tag{60}$$

$$X_\mu = \frac{\left(\bar{R} - R_f\right)}{\mathbb{E}\left[(\tilde{R} - R_f)^2 \,\middle|\, \tilde{\sigma} = \sigma\right]}\left(\frac{(1-b)(a + \alpha_{\mathrm{I}}) - \mu\, b(a - \alpha_{\mathrm{A}})}{(1-b)^2 + b^2\mu} - R_f\right). \tag{61}$$

Substituting in for a and b from the expression in equation 28 gives that

$$(1-b)(a + \alpha_{\mathrm{I}}) - \mu b(a - \alpha_{\mathrm{A}}) = \frac{\mu\left(\alpha_{\mathrm{A}} + \alpha_{\mathrm{I}}\right)}{1+\mu} \tag{62}$$

and

$$(1-b)^2 + b^2\mu = \frac{\mu}{1+\mu} \tag{63}$$

therefore

$$X_\mu = \frac{\left(\bar{R} - R_f\right)\left(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f\right)}{\mathbb{E}\left[\left(\tilde{R} - R_f\right)^2 \,\middle|\, \tilde{\sigma} = \sigma\right]} \tag{64}$$

$$= \frac{\left(\bar{R} - R_f\right)\left(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f\right)}{\sigma^2 + \left(\bar{R} - R_f\right)^2}. \tag{65}$$

A.2 Computation of the Social Planner's Weight

Immediately from plugging in the expressions for u_{A}, ϕ_{μ_ρ}, and X into equation 34, observe that

$$\begin{aligned}
2|\bar{u}|\left(1 + \mu_\rho\right)^2 &= \mathbb{E}\left[\left(R_f + \frac{\left(\bar{R} - R_f\right)\left(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f\right)}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2}\left(\tilde{R} - R_f\right) - \alpha_{\mathrm{I}} - \alpha_{\mathrm{A}}\right)^2 \,\middle|\, \tilde{\rho} = \rho\right] \\
&= \left(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f\right)^2 \mathbb{E}\left[\left(\frac{\left(\bar{R} - R_f\right)\left(\tilde{R} - R_f\right)}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2} - 1\right)^2 \,\middle|\, \tilde{\rho} = \rho\right] \\
&= \left(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f\right)^2 \left\{1 - 2\mathbb{E}\left[\frac{\left(\bar{R} - R_f\right)\left(\tilde{R} - R_f\right)}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2} \,\middle|\, \tilde{\rho} = \rho\right] + \right. \\
&\quad \left. + \, \mathbb{E}\left[\left(\frac{\left(\bar{R} - R_f\right)\left(\tilde{R} - R_f\right)}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2}\right)^2 \,\middle|\, \tilde{\rho} = \rho\right]\right\}.
\end{aligned} \tag{66}$$

Applying the law of iterated expectations gives

$$\begin{aligned}
&1 - \frac{2|\bar{\mu}|\left(1 + \mu_\rho\right)^2}{\left(\alpha_{\mathrm{I}} + \alpha_{\mathrm{A}} - R_f\right)^2} \\
&= 2\mathbb{E}\left[\mathbb{E}\left[\frac{\left(\bar{R} - R_f\right)\left(\tilde{R} - R_f\right)}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2} \,\middle|\, \tilde{\sigma}\right] \,\middle|\, \tilde{\rho} = \rho\right] - \mathbb{E}\left[\mathbb{E}\left[\left(\frac{\left(\bar{R} - R_f\right)\left(\tilde{R} - R_f\right)}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2}\right)^2 \,\middle|\, \tilde{\sigma}\right] \,\middle|\, \tilde{\rho} = \rho\right] \\
&= 2\mathbb{E}\left[\frac{\left(\bar{R} - R_f\right)\mathbb{E}\left[\left(\tilde{R} - R_f\right) \,\middle|\, \tilde{\sigma}\right]}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2} \,\middle|\, \tilde{\rho} = \rho\right] + \mathbb{E}\left[\frac{\left(\bar{R} - R_f\right)^2 \mathbb{E}\left[\left(\tilde{R} - R_f\right)^2 \,\middle|\, \tilde{\sigma}\right]}{\left(\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2\right)^2} \,\middle|\, \tilde{\rho} = \rho\right]
\end{aligned} \tag{67}$$

and since

$$\mathbb{E}\left[\big(\tilde{R}-R_f\big)^2 \,\middle|\, \tilde{\sigma}\right] = \tilde{\sigma}^2 + \big(\bar{R}-R_f\big)^2 \tag{68}$$

we have

$$\begin{aligned}
&1-\frac{2|\bar{\mu}|\big(1+\mu_\rho\big)^2}{\big(\alpha_{\mathrm{I}}+\alpha_{\mathrm{A}}-R_f\big)^2}\\
&=\big(\bar{R}-R_f\big)^2\left\{\mathbb{E}\left[\frac{2}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\,\middle|\,\tilde{\rho}=\rho\right]-\mathbb{E}\left[\frac{1}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\,\middle|\,\tilde{\rho}=\rho\right]\right\} \qquad (69)\\
&=\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)^2}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\,\middle|\,\tilde{\rho}=\rho\right].
\end{aligned}$$

Finally, solve for $(1+\mu_\rho)^2$ and cross multiply to recover equation 35.

A.3 Computation of Expected Utility Given ρ

Plug in to equation 37 and compute, maintaining at first the shorthand

$$\tilde{w} = W(\sigma, R) = R_f + X(\sigma)\big(R-R_f\big), \tag{70}$$

that is:

$$
\begin{aligned}
&\mathbb{E}\left[u_{\mathrm{I}}\Big(W(\tilde{\sigma},\tilde{R})-\varphi\big(W(\tilde{\sigma},\tilde{R}),\tilde{\sigma},\rho\big)\Big)\;\Big|\;\tilde{\rho}=\rho\right]\\
&=-\frac{1}{2}\mathbb{E}\left[\Big(\alpha_{\mathrm{I}}-\tilde{w}+\phi_{\mu_\rho}(\tilde{w})\Big)^2\;\Bigg|\;\tilde{\rho}=\rho\right]\\
&=-\frac{1}{2}\mathbb{E}\left[\alpha_{\mathrm{I}}-\tilde{w}+\alpha_{\mathrm{A}}+\frac{\tilde{w}-\alpha_{\mathrm{I}}-\alpha_{\mathrm{A}}}{1+\mu_\rho}\;\Bigg|\;\tilde{\rho}=\rho\right]\\
&=-\frac{1}{2}\mathbb{E}\left[\alpha_{\mathrm{I}}-\tilde{w}+\alpha_{\mathrm{A}}+\frac{\tilde{w}-\alpha_{\mathrm{I}}-\alpha_{\mathrm{A}}}{1+\mu_\rho}\;\Bigg|\;\tilde{\rho}=\rho\right]\\
&=-\frac{1}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2\mathbb{E}\left[\big(\alpha_{\mathrm{I}}+\alpha_{\mathrm{A}}-\tilde{w}\big)^2\;\Big|\;\tilde{\rho}=\rho\right]\\
&=-\frac{1}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2\mathbb{E}\left[\Big(\alpha_{\mathrm{I}}+\alpha_{\mathrm{A}}-R_f-X(\tilde{\sigma})\big(\tilde{R}-R_f\big)\Big)^2\;\Bigg|\;\tilde{\rho}=\rho\right]\\
&=-\frac{1}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2\mathbb{E}\left[\left(\alpha_{\mathrm{I}}+\alpha_{\mathrm{A}}-R_f-\big(\alpha_{\mathrm{I}}+\alpha_{\mathrm{A}}-R_f\big)\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2\;\Bigg|\;\tilde{\rho}=\rho\right]\\
&=-\frac{\big(\alpha_{\mathrm{I}}+\alpha_{\mathrm{A}}-R_f\big)^2}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2\mathbb{E}\left[\left(1-\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2\;\Bigg|\;\tilde{\rho}=\rho\right].
\end{aligned}
\tag{71}
$$

Now, from equation 66 above,

$$
\mathbb{E}\left[\left(1-\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2\;\Bigg|\;\tilde{\rho}=\rho\right]=2|\bar{u}|\left(\frac{1+\mu_\rho}{\alpha_{\mathrm{I}}+\alpha_{\mathrm{A}}-R_f}\right)^2, \tag{72}
$$

so, finally,

$$
\mathbb{E}\left[u_{\mathrm{I}}\Big(W(\tilde{\sigma},\tilde{R})-\varphi\big(W(\tilde{\sigma},\tilde{R}),\tilde{\sigma},\rho\big)\Big)\;\Big|\;\tilde{\rho}=\rho\right]=\bar{u}\,\mu_\rho^2. \tag{73}
$$

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