

DEADLOCK ON THE BOARD*

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Abstract

We develop a dynamic model of board decision-making. We show that a board could retain a policy all directors agree is worse than an available alternative. Thus, directors may retain a CEO they agree is bad—a deadlocked board leads to an entrenched CEO. We explore how to compose boards and appoint directors to mitigate deadlock. We find that board diversity and long director tenure can exacerbate deadlock. Moreover, we rationalize why CEOs and incumbent directors have power to appoint new directors: to avoid deadlock. Our model speaks to short-termism, staggered boards, and proxy access.

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1 Introduction

The board of directors is the highest decision-making authority in a corporation. But sometimes boards struggle to make decisions. In surveys, 67% of directors report the inability to decide about some issues in the boardroom. Moreover, 30% say they have encountered a boardroom dispute threatening the very survival of the corporation (IFC (2014), p. 2).¹ Such a “division among the directors” that “may render the board unable to take effective management action”—such deadlock on the board—can even lead directors to “vote wholly in disregard of the interests of the corporation” (Kim (2003), pp. 113, 120).² Deadlock on the board can be so costly to US corporations that most states have adopted deadlock statutes, which often give courts the power to dissolve a deadlocked corporation, a power they rarely have otherwise, except in the event of default or fraud. A substantial legal literature studies how corporations can resolve deadlock ex post.³ In this paper, we study how deadlock arises ex ante. And we ask whether it can, and should, be avoided. Can the right mix of directors ensure a board makes efficient decisions? And, if so, how should director elections be structured to help achieve the right board composition? Should director tenure be limited? And should shareholders have all the power to choose directors, or should executives and incumbent directors have some power as well?

To address these questions, we develop a dynamic model of board decision making. In the model, some directors refuse to replace a current policy with a new one just because

¹Further, from 2004–2006, 166 directors experienced disputes so severe that they publicly resigned from their boards at US public corporations, accepting potential damage to their careers (Marshall (2013); see also Agrawal and Chen (2017)).

²In the summer of 2017, deadlock on the board made it hard for Uber to appoint a CEO. According to Shervin Pishevar, an early shareholder in the company, there was one director, a representative of blockholder Benchmark Capital, who was “holding the company hostage and not allowing it to move forward in its critical executive search.... Benchmark has threatened to block investments...by bargaining over board seats” (Pishevar’s letter to Benchmark, August 15, 2017). Moreover, deadlock on the board led one frontrunner for the job, Meg Whitman, to withdraw her name from consideration, saying “it was becoming clear that the board was still too fractured to make progress on the issues that were important to me” (“Inside Uber’s Wild Ride in a Search of a New C.E.O.” *New York Times*, August 29, 2017). Whitman’s description of Uber’s board traces the dictionary definition of deadlock: “a situation, typically one involving opposing parties, in which no progress can be made” (*New Oxford American Dictionary*).

³See, e.g., Duke (1972), Howe (1967), Kim (2003), Landeo and Spier (2014a, 2014b), Lew (1980), and McDonald (1979).

they fear that other directors will refuse to replace the new policy in the future. There is complete deadlock: the board cannot move forward with any policy, even if all directors prefer it to the incumbent. Shareholders suffer, since a deadlocked board cannot remove low-quality policies or executives—a deadlocked board leads to an entrenched CEO.

The model gives a new perspective on boardroom diversity and director tenure, two hotly debated policy issues: more diverse opinions/preferences and longer tenures can exacerbate deadlock (their benefits notwithstanding).⁴ It also allows us to ask how the anticipation of deadlock can affect board composition via director elections. We find that shareholders should not have all the power over the future of the board. Indeed, shareholders themselves are better off if incumbent directors have some power over the election of new directors: with more power, directors anticipate fewer disagreements in the future, and hence are less likely to become deadlocked today. This result rationalizes why it is typically the CEO and incumbent directors who nominate new directors in practice.

Model preview. In the model, a board made up of multiple directors decides on a corporate policy at each date. The model is based on two key assumptions, reflecting how real-world boards operate. (i) The set of feasible policies changes over time. For example, in the context of CEO turnover decisions, the alternative candidates available to replace the incumbent CEO today may no longer be available in the future. (ii) Directors have different preferences over policies. We refer to these different preferences as “biases,” as they could reflect private benefits or misspecified beliefs. However, they could also reflect reasonable differences of opinion (see Section 6.1). For example, a director representing an activist investor could be biased toward an outside candidate with a history of asset divestitures, and an executive director could be biased toward an internal candidate with experience at the firm. We refer to a board as “aligned” if there is a majority of directors with the same bias, and as “diverse” otherwise.

⁴See Ferreira (2010) for a survey of the literature on boardroom diversity. And see, e.g., “Big Investors Question Corporate Board Tenures” (*Wall Street Journal*, March 23, 2016) and Katz and McIntosh (2014) for discussions of director tenure.

Results preview. Our first main result is that a diverse board can be deadlocked, in the sense that it retains even Pareto dominated policies—directors choose not to put an available policy in place, even when they all agree it is preferable to the incumbent. The reason is that each director knows that if the new incumbent appeals to other directors, they will want to keep it in place. Thus, she is worried that putting the new policy in place will prevent her from putting her own preferred policies in place when they become available. Hence, she votes down alternative policies just because they appeal to other directors, and so does the rest of the board.⁵ For example, despite having diverse opinions on many candidates, an activist representative and executive director could agree that an alternative CEO could run the firm better than the incumbent. But the directors both know that, given he can run the firm well, the alternative will be hard to replace in the future—it will be hard for the activist representative to appoint a divestiture-oriented CEO and hard for the executive director to appoint an internal candidate. As a result, they both want to keep the bad incumbent in place. Overall, deadlock arises because directors want to keep policies in place that will be easy to replace in the future.

In the context of CEO turnover, this result implies that a CEO can be so severely entrenched that he is not fired even if all directors prefer a replacement, and hence explains what seems to be a major source of corporate inefficiency (Taylor (2010)). This explanation of CEO entrenchment contrasts with those in the literature, which rely on a CEO’s actively entrenching himself (e.g., “invest[ing] in businesses related to their own background and experience”)⁶ or directors’ direct utility costs of voting against a CEO.^{7,8} We find that directors keep the bad incumbent CEO in place because they worry a good CEO will be too hard to replace.

This mechanism resonates with practice. For example, in 2017, Uber’s executive search

⁵ If a good policy is in place today, this same mechanism can lead to what we call “anti-deadlock,” in which policies are replaced when they should not be (see Corollary 2).

⁶Shleifer and Vishny (1989), p. 125. See also, e.g., Zwiebel (1996).

⁷See, e.g., Chemmanur and Fedaseyev (2018), Coles et al. (2014), Taylor (2010), and Warther (1998).

⁸One other paper in which directors choose not to fire a bad CEO is Dow (2013). In that paper, directors retain a bad CEO because they do not want to reveal that they made a mistake hiring him in the first place.

resulted in deadlock on the board, and the company stayed without a CEO for several months. One director was pushing for a weak CEO who would be easy to replace in the future. According to *Bloomberg*:

The company hopes to lock in a CEO by early September. The big question is whether the board can get on the same page. Getting a majority of the eight-person group to support a single candidate is looking to be difficult.... Some...have argued...that Kalanick [a current director and former CEO] would *prefer a weak CEO just to increase his chance of making a comeback* (“Behind Uber’s Messy CEO Search Is a Divided Boardroom,” *Bloomberg Technology*, July 28, 2017, emphasis added).

Since deadlock arises due to the dynamic linkage between present and future decisions, it arises only when directors care enough about the future. It does not arise in the benchmark in which directors vote only once, or, equivalently, if directors are myopic, in that they put a lot of weight on current decisions. In this case, a diverse board always acts in the interest of shareholders. Thus, our model provides a counterpoint to the broadly negative view of corporate short-termism, and speaks to the current debate over director tenure (Section 3.1).

To ask what board composition is best for shareholders, we explore how deadlock depends on whether the board is diverse or aligned. On a diverse board, deadlock arises. This is because, even if all directors agree on the best decision today, they anticipate disagreeing in the future. On an aligned board, in contrast, deadlock does not arise. This is because directors not only agree on the best decision today, they also know they will agree in the future. Thus, they have no incentive to block policies to ensure they get their way in the future—they will get their way anyway. Despite the absence of deadlock, an aligned board is not necessarily good for shareholders. Indeed, directors on an aligned board will put their preferred policies in place at shareholders’ expense. Thus, board composition presents a trade-off between the costs of deadlock on a diverse board and the tyranny of biases on an aligned board.

Our second main result is that a diverse board is better for shareholders if directors’ biases are small, but an aligned board is better if they are large. To see why, observe that

a diverse board acts in shareholders' interest all of the time if biases are small, but none of the time if they are large. The reason is that directors act against shareholders' interests only to preserve their option to get their way in the future, and this option is valuable enough to induce deadlock only if biases are large. In contrast, an aligned board acts against shareholders' interest some of the time, whether biases are small or large. The reason is that directors act against shareholders' interest only at times when their favorite policies are available.

Our third main result is an explanation of why shareholders should not have all the power to choose new board appointments. In fact, their power is limited in practice. As corporate governance advocate Adrian Cadbury puts it,

The classical theory of the board is that shareholders elect the directors.... In practice, however, the shareholders of most public companies have little say in the appointment of directors, other than to nod through the nominations presented by the current board..... The legitimacy of the board as the appointee of the shareholders is something of a fiction (Cadbury (2002), p. 66).

Although Cadbury seems to think giving incumbent directors this power is a bad idea, we argue that it can be optimal. The reason is that it can mitigate deadlock. If a director knows she will become part of a majority on the board if she appoints new directors, then she knows she will be able to put her favorite policies in place, regardless of the incumbent. Thus, she does not have to block good policies today to ensure that the incumbent is easy to replace in the future. However, shareholders should not cede all the power to appoint to incumbent directors; otherwise, as on an aligned board, incumbent directors' bias can take over the whole board. They should cede just enough power to prevent deadlock.

This result rationalizes the real-world institution by which incumbent directors nominate new directors and shareholders vote on them, so neither has all the power to appoint new directors. More generally, it points to a downside of full shareholder control, suggesting a heretofore overlooked cost of reforms like "proxy access," the controversial policy that shareholders be able to place their own director nominees directly on the ballot

(see, e.g., Akyol et al. (2012) and Bhandari et al. (2017)). The result also applies if there is an executive director on the board, and hence explains why the CEO often has influence over the appointment of new board members in practice (e.g., Coles et al. (2014), Hermalin and Weisbach (1998), and Shivdasani and Yermack (1999)).

We also ask how the power to appoint new directors should be distributed among the current directors. Should it be distributed equally among them? For example, should they all have seats on the nominating committee? We find that the answer is no. To prevent deadlock, it is enough to induce a majority of directors to vote to replace the incumbent.

Related literature. Dziuda and Loeper (2016) study political gridlock in a model with an endogenous status quo.⁹ In that paper, the same two policies are available at each date and voters with changing preferences vote on which to put in place. To generate gridlock, what matters is that the status quo stays in place when the voters cannot agree. In other words, each voter has veto power, and thus wants to lock in her preferred kinds of policies (e.g., left-wing policies) as the default outcome for voting in the future, even if she thinks others (e.g., right-wing policies) could be beneficial in the short term. In contrast, our results are unrelated to the default outcome. Indeed, voters in our model do not want to lock in a policy. Rather, they want to make it easy to replace.

Austen-Smith et al. (2019) build on Dziuda and Loeper’s (2016) framework to include three policies, and find that adding a mediocre policy can mitigate gridlock, preventing a bad policy from staying in place. As they explain (p. 11), their mechanism hinges on veto power, and is not at work in settings like ours in which multiple voters make decisions by majority voting. Our mechanism, in contrast, hinges on new policies becoming available over time. And, unlike theirs, it can generate too much policy change, not just too little (Corollary 2; see also footnote 5). We discuss these differences further in Section 3 and Section 5.

Although our main questions pertain to corporate boards—how should they be composed,

⁹Other papers in this literature include Duggan and Kalandrakis (2012), Dziuda and Loeper (2018), and Zápal (2012).

how should directors be appointed, and who should have power over the future of the board—they may also have analogs in some political situations. Hence, our findings could contribute to the political economy literature, which, to our knowledge, has yet to explore optimal committee composition and member appointments within this class of models.

A relatively small number of theory papers studies decision making by multiple directors on a corporate board.¹⁰ We contribute to this literature by including dynamic interactions, which are almost entirely absent from this literature, even though they alter behavior dramatically. Indeed, none of our results obtain with a one-shot decision since deadlock does not arise (Proposition 1). The only other papers that include dynamic decision making with multiple directors are Garlappi et al. (2017, 2019). In the 2017 paper, directors are not strategic. However, group dynamics still generate inefficiencies, because directors pass up an investment they all believe is good anticipating that it will not be managed the way they want after new information arrives. In the 2019 paper, directors are strategic, and if there is an even number of them, group dynamics generate inefficiencies because they anticipate disagreeing about when to exercise a real option.

We also add to the broader theory literature on boards.¹¹ Our finding that board diversity can exacerbate deadlock complements existing work on the downsides of director independence, since independent directors are likely to have different views than insiders on the board.¹²

Our result that it can be optimal to not give shareholders all of the control over director appointments, but to give some of it to incumbent directors, resonates with Burkart et al.’s (1997) idea that it can be optimal to not give shareholders complete control over a manager’s actions. In their model, limiting shareholder control encourages the manager to make firm-specific investments today, since he anticipates capturing the rents in the future. In ours, it

¹⁰See Chemmanur and Fedaseyeu (2018), Fluck and Khanna (2011), Harris and Raviv (2008), Levit and Malenko (2016), Malenko (2014), and Warther (1998); see also Baranchuk and Dybvig (2009) for a model with multiple directors, but not strategic decision making.

¹¹See Adams et al. (2010) for a survey.

¹²See Adams and Ferreira (2007), Kumar and Sivaramakrishnan (2008), Laux (2008), and Malenko (2014).

discourages directors from deadlocking the board today, since they do not anticipate getting deadlocked in the future. Overall, we give a new perspective on shareholder control based on directors' desire to maintain flexibility rather than managers' desire to expropriate rents.

Layout. Section 2 presents the model. Section 3 describes the baseline mechanism of deadlock on the board and entrenched policies. Section 4 studies optimal board composition. Section 5 examines who should have the power to appoint directors. Section 6 discusses robustness and presents several extensions. Section 7 discusses empirical implications and how to test them. Section 8 concludes.

2 Setup

There is a board comprising $N > 2$ directors, indexed by $i \in \{1, \dots, N\}$, who decide on a policy at each of two dates, $t \in \{1, 2\}$.¹³ At date t , there is an incumbent policy \hat{x}_{t-1} in place and a single alternative policy x_t available.¹⁴ The board decides whether to replace the incumbent with the alternative policy by strict majority voting: if (strictly) more than $N/2$ of the directors vote for the alternative x_t , then x_t becomes the incumbent policy, $\hat{x}_t = x_t$; otherwise, the incumbent policy stays in place, $\hat{x}_t = \hat{x}_{t-1}$.

Policies differ in two dimensions: in how much value they create for shareholders and in how much they appeal to biased directors. Specifically, a policy is characterized by its value v and type τ . And a director is characterized by her bias toward a preferred policy type τ . (So τ indicates both a type of policy and the type of directors it appeals to.) Whereas all directors get utility from v , a τ -biased director also gets additional benefit b from policies

¹³See Section 6.2 for an infinite-horizon extension.

¹⁴Thus, in each period, the board makes a decision between only two alternatives. This captures many issues for which boards choose between binary outcomes. Indeed, the most important board decisions are arguably whether to retain executives and whether to make an acquisition/divestment. Both of these are inherently binary. Other decisions are effectively binary, or close to it, because boards are unlikely to vote on the entire spectrum of possible alternatives, but only on shortlists proposed by executives, advisors, or one of the directors. Finally, many important board decisions can be meaningfully approximated by binary choices. For example, the decision of how much cash to pay out can be approximated by whether or not to cut a dividend. Moreover, restricting attention to binary decisions allows us to focus on a simple decision rule (majority voting) without running into the paradoxes inherent in preference aggregation (e.g., Arrow (1950), Gibbard (1973), and Satterthwaite (1975)).

that match her type: if a policy x of type τ_x is in place, a director i of type τ_i gets

$$b_i(x) = \begin{cases} b & \text{if } \tau_x = \tau_i, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

(We discuss different interpretations of directors' biases in Section 6.1.) Using the shorthand $\hat{v}_t \equiv v(\hat{x}_t)$, a director's payoff is

$$\hat{v}_1 + b_i(\hat{x}_1) + \delta(\hat{v}_2 + b_i(\hat{x}_2)),$$

where δ is directors' rate of time preference. (We allow δ to be larger than one, since date 2 may represent more calendar time than date 1.) Shareholders care about v , but not τ . Their payoff is $\hat{v}_1 + \delta_s \hat{v}_2$, where their rate of time preference δ_s need not coincide with directors'.¹⁵

We assume that the alternative policies' value and type are i.i.d. at date 1 and date 2, and we make the following assumptions on distributions. At each date t , the value v of alternative x_t is drawn from a uniform distribution on $[0, 1]$, with distribution function $F(v) \equiv \max\{0, \min\{1, v\}\}$. We assume that there are $N + 1$ bias types τ , allowing for one that appeals to each of the N directors and another that appeals to none of them. Those that appeal to directors are equally likely, each occurring with probability p , and the one that appeals to none thus occurs with probability $1 - Np$.

Solution concept. We solve for stage-undominated subgame perfect equilibria,¹⁶ characterized by sequentially rational strategies for each director i to vote for/against x_t at date $t \in \{1, 2\}$ given consistent beliefs, such that no one plays a weakly dominated strategy at any date.

¹⁵ v need not represent the common value of all shareholders, but could rather represent the average value of shareholders with heterogeneous biases, e.g., half of the shareholders could value policy x above $v(x)$ and half below. Thus, directors' biases could also reflect heterogeneous biases/preferences of individual shareholders. See Section 6.1 for further discussion.

¹⁶This refinement implies that directors act as if they have a non-zero probability of being pivotal. It serves just to rule out trivial equilibria. Without it, there would always be an equilibrium in which all directors voted against, for example. In this case, each director knows she is not pivotal, and that the incumbent will stay in place no matter how she votes. Hence, voting against and voting for are both weak best responses.

Board composition. We define the composition of the board by the distribution of bias types among its directors. Specifically, we refer to a board as *aligned* if there is a majority of directors with the same bias type, as *diverse* if there is not, and as *fully diverse* if there are no two directors who share the same bias (we use this stronger notion of diversity only in Section 5). At first, we assume that board composition stays the same at each date; later on, we allow it to change between date 1 and date 2.

3 Diversity and Deadlock

We begin by analyzing a diverse board. We solve the model backward. At the last date, the model is just a one-shot voting game. In a one-shot game, a diverse board always acts in the interest of shareholders, replacing \hat{x}_1 with x_2 if and only if $\hat{v}_1 < v_2$:

Proposition 1. (ONE-SHOT BENCHMARK.) *Suppose the board is diverse. At the last date, the incumbent policy \hat{x}_1 is retained if and only if it has higher value than the alternative x_2 , i.e., if and only if $\hat{v}_1 \geq v_2$.*

Intuitively, even though directors may have substantial biases, only a minority of them is biased toward \hat{x}_1 or x_2 on a diverse board. Thus, the pivotal director is effectively unbiased, $b_i(\hat{x}_1) = b_i(x_2) = 0$, and hence compares only \hat{v}_1 with v_2 . Thus, at the final date, she always votes for the policy shareholders prefer. The same intuition applies if directors are myopic ($\delta = 0$), because, for example, they serve only single-period terms, and therefore do not care about the future. Again, the pivotal director acts as if she is unbiased:

Corollary 1. *Suppose directors are myopic ($\delta = 0$). At any date, the statement of Proposition 1 applies.*

These results are useful benchmarks, emphasizing that if directors are not concerned about the future, then a diverse board is always good for shareholders. They also provide the first step in solving the dynamic model, in which we will see that a diverse board might

not be good for shareholders, because directors might not act in shareholders' interest at date 1.

Given the outcome at date 2, we can calculate any director's payoff from replacing/not replacing \hat{x}_0 with x_1 at date 1. Again, on a diverse board, only a minority of directors can be biased toward \hat{x}_0 or x_1 . So the pivotal director is not biased toward either policy that is available at date 1. She anticipates, however, that she may be biased toward the alternative policy that becomes available at date 2. Thus, when she votes to determine \hat{x}_1 , she takes into account that if her preferred type becomes available tomorrow, it will be put in place if and only if $v_2 > \hat{v}_1$ (Proposition 1). And between \hat{x}_0 and x_1 , she votes for the policy \hat{x}_1 with the value \hat{v}_1 that maximizes her forward-looking expected payoff:

$$i\text{'s payoff} \Big|_{\hat{x}_1} = \hat{v}_1 + \delta \left((1 - F(\hat{v}_1)) \mathbb{E} [v_2 + pb \mid v_2 \geq \hat{v}_1] + F(\hat{v}_1) \hat{v}_1 \right). \quad (2)$$

With this expression, we can already see that there is a trade-off in increasing \hat{v}_1 . On the one hand, high \hat{v}_1 is good: it increases the common-value part of her payoff both today and (in expectation) tomorrow. On the other hand, high v_1 is bad: it decreases the expected private benefit part of her payoff tomorrow. The reason is that if \hat{v}_1 is high, other directors will want to keep \hat{x}_1 in place tomorrow. Thus, they are likely to block her preferred policy tomorrow, decreasing her chance of getting her associated private benefits b . Intuitively, since other directors do not care about her private benefits, she wants to make the incumbent policy unattractive to them, so that they want to replace it with her preferred policy when it is available.

Every director thus has the incentive to keep a low-quality policy in place today to preserve the option to get her way in the future, when her preferred policy could become available. This incentive can be strong enough that all directors vote to keep \hat{x}_0 in place. To see this, substitute in for the uniform distribution, $F(\hat{v}_1) = \hat{v}_1$, and compare director i 's

payoff with $\hat{v}_1 = v_1$ and with $\hat{v}_1 = \hat{v}_0$. She votes to keep \hat{x}_0 if

$$\hat{v}_0 + \delta \left(\frac{1 + \hat{v}_0^2}{2} + (1 - \hat{v}_0)pb \right) \geq v_1 + \delta \left(\frac{1 + v_1^2}{2} + (1 - v_1)pb \right). \quad (3)$$

Under the assumption that $\hat{v}_0 < v_1$, we can divide by $v_1 - \hat{v}_0$ to get our first main result:

Proposition 2. (DEADLOCK.) *Suppose the board is diverse and $\hat{v}_0 < v_1$. The board retains \hat{x}_0 if and only if*

$$pb \geq \frac{1}{\delta} + \frac{1}{2}(\hat{v}_0 + v_1). \quad (4)$$

This result captures what we refer to as deadlock: the board keeps a bad policy in place, even though all directors individually could prefer the alternative (since $v_1 > \hat{v}_0$). The reason is that when the director votes at date 1, when her preferred policy is not available, she wants to preserve the option to get her way at date 2, when her preferred policy might be available. But she knows that at date 2, the majority of directors will not care that it is her preferred policy, and thus will vote to put it in place only if its quality exceeds that of the incumbent, or $v_2 > \hat{v}_1$. Thus, she has the incentive to keep \hat{v}_1 low to preserve her option to get her way in the future.¹⁷

This incentive is strong enough to outweigh the benefits of having a higher quality policy in place today, and thus a higher expected quality policy in place tomorrow, whenever condition (4) is satisfied. That is, a director keeps the incumbent in place despite its low value whenever she has a strong bias (b is large), she is likely to have the option to put her policy in place (p is large), or she values the future highly relative to the present (δ is large). But she replaces it with the alternative whenever the value of the alternative is high enough (v_1 is large) or the value of the incumbent is close to that of the alternative (given $\hat{v}_0 < v_1$, increasing \hat{v}_0 makes it closer to v_1 ; this decreases the option value created by replacing it).

Observe that if biases are negative, $b < 0$, condition (4) is never satisfied, and there is no

¹⁷This extends the real options intuition that it can be optimal to delay decisions that are hard to undo (see, e.g., Dixit and Pindyck (1994)). Here, decisions today determine the preferences of the pivotal director tomorrow, creating an endogenous reason that they can be hard to undo.

deadlock. The reason is that for negative biases, a director no longer wants to preserve the possibility that her favorite policy is put in place in the future. Rather, she wants to avoid the possibility that her least favorite policy is put in place in the future. And the way to do that is to make the incumbent policy as attractive as possible to other directors, i.e., to maximize the common value of the incumbent—to vote in shareholders’ interest.

The same mechanism that leads to deadlock—keeping a bad incumbent policy in place when a good one is available—can lead to what we refer to as “anti-deadlock”—replacing a good incumbent with a bad one. The mechanism is the same: to make it easier to get her preferred policy in place in the future, a director may prefer to replace a high-quality incumbent with a low-quality alternative.

Corollary 2. (ANTI-DEADLOCK.) *Suppose the board is diverse and $\hat{v}_0 \geq v_1$. The board replaces \hat{x}_0 with x_1 (inefficiently) if and only if condition (4) is satisfied.*

This result contrasts with Dziuda and Loeper’s (2016) political economy model, in which dynamic incentives always lead to too little change, and never too much. “Anti-deadlock” arises when keeping the incumbent in place would be efficient. Thus, our model suggests that slow-moving institutions, which make policies hard to replace, could be good. They could prevent directors from putting bad policies in place for purely strategic reasons. Thus, our model provides a rationale for the checks and balances expressly intended to create so-called structural gridlock, and slow down political reform (veto rules, super-majority rules, and judicial review are examples of such policies; see, e.g., Gerhardt (2013)).

3.1 Director Tenure

In our model, deadlock is the result of directors’ voting strategically to increase their chances of implementing their preferred policies later on. As the one-shot and myopic-director benchmarks illustrate, directors on a diverse board always act in the interest of shareholders if they do not care about future periods. Indeed, if directors’ rate of time preference, δ , is

sufficiently small, condition (4) is never satisfied, and directors always vote for shareholders' preferred policies:

Corollary 3. (TENURE.) *Suppose that $b > 0$ and the board is diverse. Directors always act in the interest of shareholders if and only if δ is sufficiently small, $\delta < \frac{1}{pb}$.*

In some circumstances, δ can be viewed as a measure of directors' remaining tenure, and thus our model can speak to the current debate on tenure limits and director elections. Formally, δ captures how much directors care about what policies are put in place in the future relative to the present. Thus, if directors care mainly about policies implemented while they are on the board—e.g., with career-concerns they care about how a company performs before they leave and enter the labor market for directors¹⁸—then it captures their expected remaining tenure.¹⁹ Thus, if directors' tenure is formally limited by regulation or company bylaws, then δ can be interpreted directly as the time to the end of their term; if their tenure is determined by reelection (as studied in Section 5), then δ can be interpreted as the time to the next election or the probability of being reappointed in it. With either interpretation, our analysis points to a positive side of shorter tenure, suggesting term limits and frequent elections could be good things (notwithstanding the benefits of long-termism, such as the potential to support cooperation; see, e.g., Fudenberg and Maskin (1986)).

The suggestion that director tenure should be limited resonates with current market sentiment. Deeming director tenures too long, a number of institutional investors are now voting against reappointments,²⁰ leading commentators to suggest that director tenure is “the next boardroom battle” (Libit and Freier (2016), p. 5; Francis and Lublin (2016)). The

¹⁸Indeed, evidence in Fos et al. (2018) suggests that directors' incentive to boost current performance is strongest near the ends of their terms.

¹⁹Note that directors may care about policies implemented even after they are not on the board. For example, if a director wants to implement CSR policies only for the social good they create, she will still value having them implemented after she is no longer on the board. In this case, the effect of tenure is attenuated, but can still hold as long as she cares more about policies implemented during her tenure than afterward.

²⁰For example, BlackRock and State Street have voted against reappointments ” (Libit and Freier (2016), p. 5; Francis and Lublin (2016)). Moreover, many countries, such as the UK, France, Spain, Hong Kong, and Singapore, have adopted some form of term limits for independent directors.

argument for shorter tenures has centered around the idea that after a long tenure, a director may become too close to management and may also lack fresh ideas about the business (e.g., Katz and McIntosh (2014)). Thus, our analysis offers a new, complementary perspective on the downside of long tenures: in anticipation of a long tenure, directors behave strategically, creating deadlock. More generally, our analysis uncovers a cost of long-termism: it can incentivize strategic voting, exacerbating deadlock. This provides a counterpoint to the broadly negative view of corporate short-termism; see, e.g., the former Vice President Joe Biden’s opinion that short-termism “saps the economy” (“How Short-Termism Saps the Economy,” *Wall Street Journal*, September 27, 2016).

4 Alignment and Optimal Board Composition

We now turn to an aligned board, one in which a majority of directors share the same bias. Unlike on a diverse board, deadlock does not arise. Rather, the board retains the current policy if and only if it is preferred by the majority of directors:

Proposition 3. (ALIGNED BOARD.) *Suppose the board is aligned with preferred policy τ^* . It retains an incumbent \hat{x}_{t-1} with type $\tau_{\hat{x}_{t-1}}$ rather than replaces it with an alternative x_t with type τ_{x_t} whenever*

$$\hat{v}_{t-1} + \mathbf{1}_{\{\tau_{\hat{x}_{t-1}}=\tau^*\}}b \geq v_t + \mathbf{1}_{\{\tau_{x_t}=\tau^*\}}b. \quad (5)$$

This result captures the costs and benefits of alignment. On the one hand, an aligned board always acts with bias. For example, it replaces high-quality policies that are not the majority’s preferred type with low-quality policies that are. On the other hand, unlike a diverse board, it does not suffer from deadlock due to strategic voting (cf. Proposition 2). The reason is that, as long as board composition does not change over time, directors that are in the majority today know they will also be in the majority in the future. Thus, they have no incentive to vote strategically to change the incentives of future pivotal directors, given they know they will be pivotal themselves.

Given this trade-off, could shareholders be better off with an aligned board than a diverse one? Could the bias be a small price to pay for avoiding deadlock? The next proposition says that the answer is yes if and only if directors are sufficiently biased.

Proposition 4. (SHAREHOLDER OPTIMAL BOARD COMPOSITION.) *For any initial incumbent policy \hat{x}_0 , shareholders are better off with a diverse board if directors' biases are sufficiently small and with an aligned board if they are sufficiently large (and \hat{x}_0 is not the preferred type of the majority of directors on the aligned board).*

If biases are sufficiently small, then directors on a diverse board vote in shareholders' interest (condition (4) is never satisfied). Intuitively, the option to get their way in the future cannot be appealing enough to outweigh the cost of keeping a bad policy in place today. In contrast, directors on an aligned board do not act in shareholders' interest even if biases are small. Their choices are always somewhat distorted by their biases. Hence, for small biases, board diversity is always a good thing.

If biases are large, then directors on a diverse board vote against shareholders' interest (condition (4) is satisfied). Intuitively, getting their way in the future is so appealing that they are willing to pay a high cost to preserve the option to do so. Hence, they are willing to keep even the lowest quality policies in place today, no matter what the alternative is. But increasing directors' biases on an aligned board does not affect their behavior as severely. The reason is that their behavior is distorted only if a policy of their preferred type is available. Since they know they will always be able to get their way in the future, they do not strategically manipulate other policies. Hence, for large biases, board diversity can be a bad thing, and even be worse than completely biased alignment.

The relative costs of diversity are greatest when directors on a diverse board have the strongest incentive to vote strategically in anticipation of future votes, namely when they care the most about the future, or δ is high. Given δ can be interpreted as director tenure (subject to the caveats discussed above), we have our next result:

Lemma 1. *Increasing “director tenure” δ increases the relative benefit of board alignment.*

Our analysis so far underscores the trade-off between diverse and aligned boards. On a diverse board the pivotal director is not biased toward either the incumbent or alternative, but she votes strategically in case she might be biased toward one or the other in the future. In contrast, on an aligned board, directors are all biased, but do not vote strategically. Is there a way to have the best of both worlds, and reduce both bias and strategic voting? In the next section, we examine a practical institution that partially achieves this goal: giving incumbent directors the power to appoint new directors.

5 Who Should Appoint Directors?

In practice, board composition is not constant. It changes as new directors join the board. How does this affect how current directors vote? Does it mitigate or exacerbate deadlock? Does the answer depend on how directors are appointed? In particular, does it depend on who has the power to appoint new directors, shareholders or incumbent directors?

With our model, we are poised to address these questions. We need only to incorporate the possibility that some new directors join the board (with everything else identical to the baseline model). To do so, we consider a fully diverse board after the date-1 vote on policies. To capture the possibility of both shareholder and incumbent director power to appoint new directors, we assume that, at date 2, the board acts in the interest of shareholders with probability $1 - \pi$ (e.g., because it stays diverse; see Proposition 1) and acts in the interest of some directors with probability π (e.g., because it becomes aligned; see Proposition 3). Thus, we interpret $\pi \in [0, 1]$ as directors’ power to appoint new directors. If π is strictly between zero and one, then shareholders and directors share power, as in the current institution in which incumbent directors nominate new ones and shareholders vote on the nominations. Finally, we assume that, when directors have power to appoint, new directors are aligned with incumbent director i with probability α_i , where $\sum_{i=1}^N \alpha_i = 1$. If all directors have the

same power, then $\alpha_i = 1/N$. But in practice, directors typically do not have the same power, for example, because some are on the nominating committee that proposes new directors and some are not, so $\alpha_i \neq \alpha_j$ for $i \neq j$. In this case, suppose, without loss, that the ranking of power coincides with the indexing of directors, i.e., $\alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_N$. We denote the lower median by $\alpha^m := \alpha_{\lfloor \frac{N+1}{2} \rfloor}$ (which will be the power of the pivotal director).²¹

To make our interpretation concrete, suppose, for a moment, that there are three directors, $i \in \{1, 2, 3\}$, each of whom prefers a different type of policy. At date 1, they vote on whether to replace \hat{x}_0 with x_1 . And then two new directors join the board. With probability $1 - \pi$, shareholders choose these new directors, so the board stays diverse and maximizes shareholders' interests at date 2. With probability π , the incumbent board chooses new directors. In this case, director i chooses new directors with probability α_i , and thus optimally appoints directors of the same type as himself. Thus, with probability $\pi\alpha_i$ a majority of directors (three out of five) have the same preferences as director i , so the board is aligned at date 2.

We now proceed to analyze how deadlock depends on directors' power to appoint. We then ask whether it is optimal for shareholders to cede power to appoint to directors. Finally, we investigate how shareholders should optimally allocate power among directors.

To begin, suppose that policy \hat{x}_0 is in place and x_1 is available, where neither is any director's preferred type, and x_1 is better than \hat{x}_0 in the sense that $v_1 > \hat{v}_0$. Does giving the board power to appoint affect how directors vote? We find that the answer is yes. Giving the board more power to appoint can mitigate deadlock, inducing directors to replace \hat{x}_0 . Specifically, given a fixed distribution of power among directors (i.e., given a fixed $\{\alpha_i\}_i$), giving more power to appoint to the board (i.e., increasing π) can make x_1 more attractive for directors:

Lemma 2. *Suppose $b > v_1 > \hat{v}_0$ and $\alpha^m - p + \frac{1}{\delta b} > 0$. Increasing power to appoint π reduces deadlock.*

²¹ $\lfloor x \rfloor$ is the “floor” of x , i.e. greatest integer less than or equal to x . Likewise, $\lceil x \rceil$ is the “ceiling” of x , i.e. the least integer greater or equal to x .

Intuitively, a director only deadlocks the board to make it easier to get her way in the future. Thus, the intuition is akin to that in Proposition 3 and Proposition 4: knowing the board could be aligned with her preference—knowing that she could get her way regardless of the policy that is in place at date 2—weakens her incentive to keep an easily replaceable policy in place. However, this comes with the cost of an aligned board, namely worse outcomes at date 2. The next result summarizes how much power shareholders should optimally give to the incumbent directors to manage the trade-off between avoiding deadlock at date 1 and not getting their preferred policy at date 2, for a fixed distribution of director power: they should either set $\pi = \pi^*$ or $\pi = 0$.

Proposition 5. (POWER TO APPOINT.) *Suppose $b > v_1 > \hat{v}_0$, $\alpha^m - p + \frac{1}{\delta b} > 0$, and*

$$\pi^* := \frac{1}{p} \left(1 - \frac{2b(\alpha^m - p + \frac{1}{\delta b})}{2\alpha^m b - \hat{v}_0 - v_1} \right) \in (0, 1). \quad (6)$$

It is optimal for shareholders to give incumbent directors positive power to appoint π^ whenever*

$$v_1 + \frac{\delta_s}{2} \left(1 + (1 - p\pi^*)v_1^2 \right) \geq \hat{v}_0 \frac{\delta_s}{2} \left(1 + \hat{v}_0^2 \right) \quad (7)$$

and no power otherwise.

This result implies that (under the conditions of the proposition) shareholders should not give incumbent directors full power over new appointments. Rather, shareholders should give them just enough power over the future to prevent deadlock today, and still retain some power to prevent incumbent directors from implementing low-quality policies in the future. The reason is that, by ceding power to the incumbent director, shareholders can overcome a commitment problem: they effectively commit not to block her preferred policies at date 2, which can improve date 1 outcomes. As a result, they can enjoy the benefit of a fully biased board without bearing all of the cost.

Should the power to appoint be distributed among all directors? Or concentrated within a subset of them? So far, we have taken the distribution of $\{\alpha_i\}_i$ as given. Now, we show that

if shareholders can choose π and α_i jointly, they want to concentrate the power to appoint with half of the directors.

Proposition 6. (DIRECTOR POWER.) *Suppose the conditions of Proposition 5 are satisfied with $\alpha^m = (\lceil \frac{N+1}{2} \rceil)^{-1}$. The optimal distribution of director power is to give no power to directors $1, \dots, \lfloor \frac{N-1}{2} \rfloor$ and an equal amount of power to each of the others.*

Intuitively, shareholders want to give directors just enough power to prevent them from voting strategically. To do so, they do not need to incentivize all directors, but only a majority of them. Hence, they optimally give half the directors enough power to induce them not to deadlock the board, and no power to the others.

Discussion. Overall, the results in this section provide a counterpoint to Adrian Cadbury’s views quoted in the Introduction. They imply that the institution by which shareholders cede some power to incumbent directors can be optimal in some circumstances, and hence offer an explanation of why we see it in practice, even though Cadbury and others seem to find it perverse.

Still, granting directors the power to appoint could seem like a costly way to resolve deadlock. For example, would it be cheaper for shareholders to use a compensation contract to induce directors to do the right thing, rather than to give them the power to appoint new directors, who could make value-destroying decisions? If shareholders knew that directors would not act in their interest, then a compensation contract indeed might be preferable. However, as we discuss in Section 6.1, directors’ biases could reflect shareholders’. Thus, although giving one director control over the board is bad for some shareholders, it could still be good for others. More generally, if shareholders knew directors’ preferences and beliefs, they might not need to allocate control over policies to them in the first place. They might be better off contracting over specific policy decisions. However, in reality, directors have relatively simple compensation contracts (typically just cash and equity), and make decisions by voting in board meetings. Presumably, directors are allocated control rights precisely because frictions such as asymmetric information, unforeseen contingencies,

imperfect contractual enforcement, legal uncertainty, and bounded rationality make efficient contracting infeasible, i.e., the Coase theorem does not apply.

Our results also give new perspectives on two important policy issues. The first is staggered director elections. Our results could point to a possible benefit of this policy, which prevents shareholders “from replacing a majority of the board of directors without the passage of at least two annual elections” (Bebchuk and Cohen (2005), p. 410). Since this is a question about multiple strategic directors interacting over time, something that has not been modeled, the literature has been largely silent on it.²² But, with fixed π , as in Lemma 2, our model may be able to say something about it. The reason is that we model incumbent directors who anticipate being joined on the board by new directors with uncertain biases. This is analogous (and sometimes equivalent) to modeling the key feature of staggered boards, namely that only part of a staggered board is up for election at a time. With this interpretation, our findings imply that uncertainty about the future of the board can mitigate deadlock on a diverse board, because directors who are not up for election are less likely to block policies today if they anticipate being joined by others with common preferences after the elections. The second relevant policy issue is proxy access. Our results could point to a possible cost of this policy, which increases shareholder power over the composition of the board (see, e.g., Akyol et al. (2012) and Bhandari et al. (2017)). Furthermore, our results apply if there is an executive director on the board, and hence explain why CEOs often exert influence over the appointment of new board members in practice (e.g., Hermalin and Weisbach (1998), Shivdasani and Yermack (1999)).

But, in practice, when the CEO chooses directors, he is effectively hiring someone who could fire him in the future. If he vies for directors who will not fire him in the future, what

²²Much of the literature on staggered boards focuses on their effect of lengthening directors’ terms, and the associated costs and benefits. Notably, it emphasizes that, on the one hand, staggered boards can be harmful, by preventing efficient takeovers and proxy fights and entrenching managers (e.g., Bebchuk et al. (2002)). But, on the other hand, they can be beneficial, by allowing to extract a higher offer price from potential bidders (e.g., Stulz (1988)) and by encouraging long-term investments by the firm’s managers and relationship-specific investments by the firm’s stakeholders (e.g., Cremers et al. (2017)). We abstract from this effect and zero in on a different, less-discussed aspect: whether the board is staggered or not affects directors’ strategic interaction over time.

board composition does he prefer? Does he always prefer directors who are biased toward him? Our analysis suggests that a low-quality CEO might actually prefer directors who are biased toward other types of CEOs. The reason is that adding such directors to the board can make it more diverse, which can create deadlock and thus decrease the likelihood that the low-quality CEO is replaced. Indeed, a board that is aligned toward his type would unilaterally decide to replace him whenever the available alternative is better,²³ whereas a diverse board could keep him in place exactly because the alternative is better (Proposition 2).

Finally, the results above provide a stark example of how our mechanism contrasts with that in Austen-Smith et al. (2019) and Dziuda and Loeper (2016). To see why, suppose that the board is diverse today, but there is one director, say i , who will be part of a majority for sure in the future, and hence an effective dictator (i.e., $\pi = 1$ and $\alpha_i = 1$). In our model, deadlock can occur, because the pivotal director today can manipulate director i 's alternatives in the future: by keeping a weak policy in place, today's pivotal director makes future alternatives of her own preferred type relatively more attractive to director i . But in Austen-Smith et al.'s and Dziuda and Loeper's models, deadlock cannot occur, because the same policies are available at each date. Hence, the pivotal director today cannot manipulate director i 's alternatives in the future: no matter what policy is chosen today, the same choice set is available to director i in the future (all the pivotal director can affect is the default outcome in the event of a fifty-fifty split, which is irrelevant if i becomes a dictator).

6 Robustness and Extensions

6.1 Interpretation of Biases and Diversity

Heterogeneous biases and diversity. Heterogeneous director biases are the key driver of our results. These biases capture realistic heterogeneity among directors. For example,

²³To be precise, “better” here means “preferred by the aligned board” in the sense of Proposition 3.

the interests of inside directors can be different from those of outside directors. In start-ups, founding entrepreneurs often sit on boards beside capital providers like VCs, which have different objectives for the corporation. Indeed, in 2017 deadlock was so severe on the board of Applied Cleantech, a technology start-up, that the investors on the board sued the founder for control. In mature firms, equity blockholders often sit on the board. These could be heirs to family firms, with an interest in preserving their legacies, or activist investors, with interests in preserving their reputations for fast value-enhancement. Other kinds of director heterogeneity are common. For example, in Germany it is common for directors to represent stakeholders such as bank creditors or employees/unions. Moreover, even independent directors, those without a material relationship to the corporation, have their own opinions and conflicts of interest, e.g., due to connections with the CEO²⁴ or their own career concerns.²⁵

Director heterogeneity can also reflect heterogeneity among shareholders themselves, who can have different preferences, e.g., due to different beliefs and portfolio positions. In close corporations, diverse shareholders sit directly on the board. And even in public corporations, diverse shareholders appoint directors to represent their diverse interests. In this case, the quantity $v(x_t)$ should not be interpreted as the value of the firm's shareholder equity. Rather, it can be interpreted as the average value of all shareholders, including any private benefits they might get, e.g., due to control or perks. For example, suppose one group of shareholders values a policy x at 5 and another at 10. In this case, they could be represented by two directors, $i \in \{1, 2\}$, on an aligned board, each of whom has common value $v(x) = 5$ and common bias $b_i(x) = 2.5$.

But, if shareholders themselves have heterogeneous valuations, there is another interpretation of director preferences, in which director biases reflect shareholder biases. In this

²⁴Independent directors can be connected to the CEO because, for example, the CEO appointed them (Coles et al. (2014)), they have overlapping social networks (Kramarz and Thesmar (2013)), or they serve on interlocking boards with the CEO (Haddock (1997)).

²⁵Fos et al. (2018) show that time to re-election affects directors' decisions, implying they care about their own careers, not just maximizing shareholder value.

case, the two groups of shareholders in the example above would be represented by two directors on a diverse board, with biases corresponding to the shareholders', i.e. they could have common value $v(x) = 5$ and heterogeneous biases $b_1(x) = 0$ and $b_2(x) = 5$. Our analysis suggests that these two ways of representing shareholders are not equivalent. If they are represented by an aligned board, there is no deadlock, whereas if they are represented by a diverse board, there could be (see Section 4). This suggests that an aligned board can be efficient if directors have the appropriate bias. But directors with this bias may not exist, or be impossible to find. It may be easier to find directors with small, but different biases, in which case a diverse board is efficient (Proposition 4).

Overall, our results highlight novel costs and benefits of diversity, which shareholders and policy makers can take into account when they determine how board composition should be chosen or regulated. But our set-up, in which diversity represents heterogeneity in directors' biases, could understate the benefits of diversity. Although Proposition 4 implies that diversity helps to keep (small) biases in check in our model, it could have other benefits in reality. For example, directors with diverse backgrounds, experience, and expertise could have more innovative ideas for new proposals or better information about existing ones. Thus, diversity could help boards implement better policies for reasons outside our model.

Preferences vs. beliefs. We have described directors' biases as reflecting differences in their preferences (i.e., tastes) over policies. But they can also reflect differences in beliefs. To see why, consider the following setup, which is equivalent to ours. At the end of each date, the policy x_t either "succeeds," generating value V , or "fails," generating zero. If the policy is type τ , a director who is not τ -biased believes it succeeds with probability $\pi_0(x_t)$, such that her value of the policy coincides with shareholder value, i.e., $\pi_0(x_t)V = v(x_t)$, so $\pi_0(x_t) = v(x_t)/V$. If the director is τ -biased, she believes it succeeds with probability $\pi_\tau(x_t) > \pi_0(x_t)$, such that her value of the policy is $v(x_t) + b$, i.e., $\pi_\tau(x_t)V = v(x_t) + b$ or

$$\pi_\tau(x_t) = \pi_0(x_t) + \frac{b}{V}. \quad (8)$$

6.2 Infinite Horizon

We now ask whether our results are specific to the two-date setup. We show that a version of deadlock can arise even if there is no final date in which directors anticipate that their preferred policies will probably get through. Even with an infinite horizon, directors do strategic blocking, keeping Pareto-dominated policies in place, as long as it improves their chances to get their way in the future.

We focus on the special case of our baseline setup in which there are only two directors and the initial incumbent \hat{x}_0 is “very bad,” in the sense that $\hat{v}_0 = 0$ and it is neither of the directors’ preferred type. We also assume there are only two types of alternatives, one appealing to each director, so that $p = 1/2$. Finally, we require $\delta \in (0, 1)$ to ensure that directors’ value functions are well defined.

We now show that, if biases are sufficiently large, a form of deadlock arises. Specifically, there is no Markov equilibrium in which \hat{x}_0 is always replaced immediately, even though it is Pareto dominated by any alternative:

Proposition 7. (INFINITE HORIZON.) *Suppose $\delta \in (0, 1)$, there are two directors, two types of alternatives, and \hat{x}_0 is “very bad” as described above. If*

$$b > \frac{1}{1 - \delta}, \tag{9}$$

then any Markov equilibrium is defined by a cutoff $\bar{V} \in (0, 1)$ such that

1. \hat{x}_0 stays in place until $v_t > \bar{V}$, in which case it is replaced;
2. after \hat{x}_0 is replaced, policies are replaced if and only if the alternative is a higher value policy of the same type.

7 Empirical Implications

Turning to our model’s empirical content, we discuss empirical proxies for our model’s key quantities and empirical predictions corresponding to its main results.

Proxies. Boards meet in the privacy of the boardroom without disclosing their minutes. Hence, in most countries deadlock is rarely revealed publicly outside of extreme cases, such as those that wind up in court or result in director resignations.²⁶ One exception is China, where firms must publicly disclose if their independent directors vote in dissent (e.g., Jiang et al. (2016)).²⁷

Testing for deadlock directly requires the kind of detailed data available in China. But even absent such data, our model suggests a way to test for deadlock indirectly: deadlock is manifested in boards’ retaining incumbent policies, even when superior alternatives are available (Proposition 2). Applied to boards’ key decisions, CEO turnover and corporate strategy, deadlock can be measured/proxied for by the following:

- (i) longer CEO tenure;
- (ii) longer periods to appoint a new CEO after a CEO is terminated (as with Uber’s deadlocked board);
- (iii) slow changes in strategy in response to a changing environment, even at the expense of the firm’s competitiveness (as is common in corporations, Hannan and Freeman (1984), Hopkins et al. (2013));
- (iv) more persistent corporate policies.

A number of our predictions require proxies not only for deadlock, but also for directors’ “biases” b_τ representing their preferences/private benefits or beliefs (Subsection 6.1). Proxies

²⁶Translation company Transperfect and startup Applied Cleantech are examples of deadlock cases that have gone all the way to court. Agrawal and Chen (2017) and Marshall (2013) analyze director resignations resulting from board disputes, which US companies must disclose by a 2004 SEC law.

²⁷Of course, companies typically want to keep such disagreements private, so boards that disclose their directors’ voting in dissent should make up only a fraction of deadlocked boards.

for directors’ preferences include the stakeholders they represent—directors could represent employee unions, outside creditors, corporate executives, and a variety of equity blockholders, such as VC investors, activists, and founding families; these diverse stakeholders are likely all to have different preferences over/private benefits from different company policies. Proxies for directors’ beliefs include diversity in directors’ experience, expertise, backgrounds, or skills, all of which are likely to lead to different views on the best policy for a company.

Predictions. Our main results correspond to testable predictions on the determinants of deadlock. In our model, deadlock is entirely the result of dynamic interactions among directors. It does not occur in a one-shot setting (cf. Proposition 1). Thus, deadlock is relatively more likely to arise over policies with long durations, such as long-term investment projects or employment contracts.

Prediction 1. *All else equal, deadlock is more likely in decisions over longer-term policies.*

Similarly, deadlock can only arise when directors remaining tenures are sufficiently long, so that they vote strategically in anticipation of future votes (cf. Corollary 3).

Prediction 2. *All else equal, deadlock is more likely when directors’ remaining tenures are longer.*

In contrast to much of the literature, which focuses on directors’ past tenure, this prediction underscores the costs and benefits of directors’ future tenure. Hence, our model predicts that deadlock is less likely to arise if many directors are likely to leave the board soon, e.g., because they are nearing retirement or they are reaching the legal maximum tenure in jurisdictions where such a maximum exists, such as the UK, France, Spain, Hong Kong, and Singapore (e.g., Katz and McIntosh (2014)).

Our analysis also shows that deadlock is more likely to arise on a diverse board because on an aligned board, directors do not have incentives to vote strategically (cf. Propositions 2 and 3). This result is consistent with the following empirical findings.²⁸

²⁸In addition to the finance papers listed below, a number of papers in the management literature find

- (i) Bernile et al. (2018) construct a board diversity index and show that a high index is associated with persistent corporate policies.
- (ii) Goodstein et al. (1994) show that diversity in directors' occupational or professional backgrounds is associated with less strategic change, such as fewer divestitures and reorganizations.
- (iii) Knyazeva et al. (2013) and Adams et al. (2018) find that diversity in directors' skills, expertise, and incentives is associated with lower firm value.
- (iv) Volkova (2018) finds that blockholder diversity has a negative influence on company value and operations, since proposed policy changes receive little support (if blockholders have their representatives on the board, diversity in blockholders' preferences will translate into diversity in directors' preferences).

Some other models could be consistent with the above evidence as well. For example, in a behavioral model, disagreement alone could lead directors to keep Pareto-dominated policies in place—they might feel aggrieved if they do not get their way, and thus want to spite other directors, analogously to the aggrieved contracting parties in Hart and Moore (2008). In contrast, in our rational model, disagreement alone is not enough: directors are strategic only because they might disagree (due to diversity) in the future. Hence, the distinguishing prediction of our model is that it is the interaction of board composition and measures of dynamic interactions between directors that gives rise to deadlock. Specifically:

Prediction 3. *All else equal, board diversity makes deadlock more likely, and the effect of diversity is stronger when directors' remaining tenures are longer and decisions involve longer-term policies.*

In our model, directors strategically block policies preferred by other directors to improve their future bargaining positions. Hence, given data on individual director voting (as is

that there are costs to diversity. See, e.g., Adams (2016), Knight et al. (1999), Lau and Murnighan (1998), and Milliken and Martins (1996).

available for Chinese firms), we have the following testable prediction:

Prediction 4. *All else equal, a director is more likely to vote against a policy if*

- (i) *there are other directors on the board who especially favor this alternative;*
- (ii) *these other directors have longer expected remaining tenure;*
- (iii) *the director himself has longer expected remaining tenure.*

This prediction reflects the real-options intuition at the core of our model: to preserve the option to get their way in the future, directors want to ensure that other directors are dissatisfied with the status quo. This prediction contrasts with other theory models of boards, making it useful to distinguish our model from them.

Practically, it suggests that a director is relatively likely to vote against a CEO candidate nominated by an influential blockholder on the board, since the blockholder is likely to nominate someone she is biased toward. For example, hedge fund activist campaigns are increasingly likely to include the demand to replace the incumbent CEO. Our model suggests that directors on the board are relatively likely to vote against the activist’s candidate if the activist has (or will get) board representation. Something like this happened at Uber during its CEO search in the summer of 2017, when some directors were opposed to hiring Meg Whitman because they viewed her as “potentially compromised by her strong affiliation with Benchmark,” a VC blockholder with a seat on the board.²⁹

Our model also speaks to when deadlock is most costly. Almost by definition, the costs of deadlock—of keeping policy \hat{x}_0 in place—are highest when the incumbent policy is the worst. As a result, combatting deadlock by making the board less diverse should be most beneficial when the value of replacing the incumbent \hat{x}_0 is the highest. This could be when firms have made poor decisions in the past, so \hat{x}_0 is a truly damaging policy for them, or when competition is high, so firms need to respond quickly to environmental changes, since having a bad policy \hat{x}_0 in place can quickly decrease their market share.

²⁹ “Inside Uber’s Wild Ride in a Search of a New C.E.O.” *New York Times*, August 29, 2017.

Prediction 5. *All else equal, the more costly it is to keep the incumbent policy in place, the more the adverse effect of diversity decreases firm value.*

Two things that are useful to test this prediction are stock price reactions to director appointments and shareholder support in director elections. Indeed, Cai et al. (2019) find the following consistent evidence: in more competitive environments, in which the cost of keeping a bad x_0 in place is especially high, prices react positively when new directors are connected to the incumbent board and shareholders are more likely to support connected directors.

Finally, a 2017 shake-up on General Electric’s board also resonates with this prediction. The company massively reshuffled its board to create an aligned group focused on its core growth areas. In line with our model, its rationale was to streamline decision making to get out of the trouble it was in. Indeed, according to the *Wall Street Journal*,

A housecleaning at General Electric Co.’s board...aims to create a board that is more closely aligned with CEO John Flannery’s strategy. [...] The unusual shake-out...shows the depths of the problems that developed on the board’s watch...shares of the one-time industrial bellwether have plunged 42% this year. Last week, the company slashed profit targets and cut its dividend by half (“GE Housecleaning Will Alter Board’s Makeup,” *Wall Street Journal*, November 19, 2017).

8 Conclusion

We presented a model of a corporate board comprising multiple, heterogenous directors interacting over time. Despite its realism, modeling such dynamic group decision making is new to the literature on corporate boards. And it leads to new, empirically relevant results. It generates deadlock on the board, which causes pervasive entrenchment, and hence explains why corporations are often too slow to turn over their top management and to adapt their strategies to a changing competitive environment. And it gives a new take on board composition, director appointments, director tenure, staggered boards, and proxy access.

A Proofs

A.1 Proof of Proposition 1

First, recall that the definition of a diverse board is that no majority is biased toward any one policy.

Now, the result follows from the fact that the majority of directors is not biased toward either the incumbent \hat{x}_1 or the alternative x_2 . Thus, the majority, which controls the decision, cares only about the common values \hat{v}_1 and v_2 , and votes accordingly. \square

A.2 Proof of Corollary 1

Given $\delta = 0$, directors in any period act as if it is the last period. Hence, the result follows immediately from Proposition 1.

A.3 Proof of Proposition 2

The derivation is in the text. \square

A.4 Proof of Corollary 2

The derivation coincides with that of Proposition 2. In this case, the board makes an inefficient decision if (3) is violated. Dividing by $v_1 - \hat{v}_0 < 0$ gives (4). \square

A.5 Proof of Corollary 3

The result follows immediately from the condition of Proposition 2 with $\hat{v}_0 = v_1 = 0$. \square

A.6 Proof of Proposition 3

On an aligned board, there is a majority of directors who know they will always be in the majority. Thus, they act like a (biased) unitary decision maker maximizing her expected

payoff at each date, as per the expression in the proposition. \square

A.7 Proof of Proposition 4

A.7.1 Aligned board

Consider an aligned board in which a majority of directors are τ -biased. We consider two cases, first that x_1 is not type τ and second that it is.

Aligned board, case 1. x_1 is not type τ . In this case, x_1 is put in place at Date 1 if and only if $v_1 \geq \hat{v}_0$. If x_1 is put in place, $\hat{v}_1 = v_1$; otherwise, $\hat{v}_1 = \hat{v}_0$.

Following the board's choice at Date 1, there are two possibilities at Date 2: (i) x_2 is type τ , in which case x_2 is put in place if and only if $b + v_2 > \hat{v}_1$. And, (ii) x_2 is of another type τ' , in which case x_2 is put in place if and only if $v_2 > \hat{v}_1$.

Now we compute shareholders' expected payoff conditional on x_1 not being type τ .

$$\begin{aligned}
\mathbb{E} \left[\text{payoff} \mid x_1 \text{ is type } \tau' \right] = & \left(1 - F(\hat{v}_0) \right) \mathbb{E} \left[v_1 + \delta_s \left\{ (1 - p) \left[(1 - F(v_1)) \mathbb{E}[v_2 \mid v_2 > v_1] + F(v_1)v_1 \right] \right. \right. \\
& \left. \left. + p \left[(1 - F(v_1 - b)) \mathbb{E}[v_2 \mid v_2 + b > v_1] + F(v_1 - b)v_1 \right] \right\} \mid v_1 > \hat{v}_0 \right] \\
& + F(\hat{v}_0) \mathbb{E} \left[\hat{v}_0 + \delta_s \left\{ (1 - p) \left[(1 - F(\hat{v}_0)) \mathbb{E}[v_2 \mid v_2 > \hat{v}_0] + F(\hat{v}_0)\hat{v}_0 \right] \right. \right. \\
& \left. \left. + p \left[(1 - F(\hat{v}_0 - b)) \mathbb{E}[v_2 \mid v_2 + b > \hat{v}_0] + F(\hat{v}_0 - b)\hat{v}_0 \right] \right\} \mid v_1 < \hat{v}_0 \right]
\end{aligned} \tag{10}$$

Calculating, we get

$$\mathbb{E} \left[\text{payoff} \middle| x_1 \text{ is type } \tau' \right] = \begin{cases} \frac{1 + \hat{v}_0^2}{2} + \frac{\delta_s}{6} \left(4 + 2(1 - p)\hat{v}_0^3 - pb^2(3 - 2b) \right) & \text{if } \hat{v}_0 < b \\ \frac{1 + \hat{v}_0^2}{2} + \frac{\delta_s}{6} \left(4 - 3pb^2 + 2\hat{v}_0^3 \right) & \text{if } \hat{v}_0 > b \end{cases} \quad (11)$$

Aligned board, case 2. x_1 is type τ . In this case, x_1 is put in place at Date 1 if and only if $v_1 + b \geq \hat{v}_0$. If x_1 is put in place, $\hat{v}_1 = v_1$; otherwise, $\hat{v}_1 = \hat{v}_0$.

Following the board's choice at Date 1, there are four possibilities at Date 2: (i) x_1 is put in place and x_2 is type τ , in which case x_2 is put in place if and only if $v_2 > v_1$. (ii) x_1 is put in place and x_2 is not type τ , in which case x_2 is put in place if and only if $v_2 > v_1 + b$. (iii) x_0 is retained and x_2 is type τ , in which case x_2 is put in place if and only if $v_2 + b > \hat{v}_0$. And, (iv) x_0 is retained and x_2 is not type τ , in which case x_2 is put in place if and only if $v_2 > \hat{v}_0$.

Now we compute shareholders' expected payoff conditional on x_1 being type τ .

$$\begin{aligned} \mathbb{E} \left[\text{payoff} \middle| x_1 \text{ is type } \tau \right] = & \left(1 - F(\hat{v}_0 - b) \right) \mathbb{E} \left[v_1 + \delta_s \left\{ p \left[(1 - F(v_1)) \mathbb{E}[v_2 | v_2 > v_1] + F(v_1)v_1 \right] \right. \right. \\ & \left. \left. + (1 - p) \left[(1 - F(v_1 + b)) \mathbb{E}[v_2 | v_2 > v_1 + b] + F(v_1 + b)v_1 \right] \right\} \middle| v_1 > \hat{v}_0 - b \right] \\ & + F(\hat{v}_0 - b) \mathbb{E} \left[\hat{v}_0 + \delta_s \left\{ p \left[(1 - F(\hat{v}_0 - b)) \mathbb{E}[v_2 | v_2 > \hat{v}_0 - b] + F(\hat{v}_0 - b)\hat{v}_0 \right] \right. \right. \\ & \left. \left. + (1 - p) \left[(1 - F(\hat{v}_0)) \mathbb{E}[v_2 | v_2 > \hat{v}_0] + F(\hat{v}_0)\hat{v}_0 \right] \right\} \middle| v_1 < \hat{v}_0 - b \right]. \end{aligned} \quad (12)$$

Calculating, we get

$$\mathbb{E} \left[\text{payoff} \middle| x_1 \text{ is type } \tau \right] = \begin{cases} \frac{1}{2} + \frac{\delta_s}{6} (4 - (1-p)(3-2b)b^2) & \text{if } \hat{v}_0 < b \\ \frac{1}{2} (1 + \hat{v}_0^2 - b^2) + \frac{\delta_s}{6} (4 + 2\hat{v}_0^3 + 4pb^3 - 3b^2 + 3pb^2 - 6pb^2\hat{v}_0) & \text{if } \hat{v}_0 > b \end{cases} \quad (13)$$

Now, we have to take the expectation over the two cases to get shareholders' ex ante expected payoff:

$$\begin{aligned} \mathbb{E} [\text{payoff}] &= (1-p)\mathbb{E} [\text{payoff} \middle| x_1 \text{ is type } \tau'] + p\mathbb{E} [\text{payoff} \middle| x_1 \text{ is type } \tau] \\ &= \begin{cases} \frac{1 + (1-p)\hat{v}_0^2}{2} + \frac{\delta_s}{6} (4 - 2p(1-p)(3-2b)b^2 + 2(1-p)^2\hat{v}_0^3) & \text{if } \hat{v}_0 < b \\ \frac{1 + \hat{v}_0^2 - pb^2}{2} + \frac{\delta_s}{6} (4 + 4p^2b^3 - 6pb^2 + 6p^2b^2 - 6p^2b^2\hat{v}_0 + 2\hat{v}_0^3) & \text{if } \hat{v}_0 > b \end{cases} \end{aligned} \quad (14)$$

A.7.2 Diverse board

Define

$$v_1^* := 2pb - \frac{2}{\delta} - \hat{v}_0, \quad (15)$$

as the value of v_1 that makes inequality (4) bind. Recall from Proposition 2 that if $v_1 - \hat{v}_0 \geq 0$, then x_0 is (inefficiently) retained if $v_1 \leq v_1^*$. Similarly, recall from Corollary 2 that if $v_1 - \hat{v}_0 < 0$, then x_0 is (efficiently) retained if $v_1 > v_1^*$.

Notice that this implies that if v_1^* is small, the board always acts in the interest of shareholders, putting x_1 in place if and only if $v_1 > \hat{v}_0$ and, in contrast, if v_1^* is large, the board always acts against the interest of shareholders, putting x_1 in place if and only if $v_1 \leq \hat{v}_0$. Intuitively, v_1^* is high when directors' biases b are high, and when b is high, the board is deadlocked or "anti-deadlocked."

We proceed to calculate shareholders' ex ante payoff. The payoff depends on the region that v_1^* is in. In particular, we have four cases:

- (1) $v_1^* < 0$,
- (2) $v_1^* \in [0, \hat{v}_0)$,
- (3) $v_1^* \in [\hat{v}_0, 1)$,
- (4) $v_1^* \geq 1$.

Case (1): $v_1^* < 0$. In this case, Proposition 2 and Corollary 2 imply that the board retains \hat{x}_0 if and only if it is optimal for shareholders, or $\hat{v}_0 \geq v_1$. The expected payoff is the average over two subcases depending on whether x_0 is retained or not at Date 1:

(1a) $v_1 \leq \hat{v}_0$: In this case, x_0 is retained. The expected shareholder payoff as of Date 1 is

$$\begin{aligned} \mathbb{E}[\text{payoff}] &= \hat{v}_0 + \delta_s \left[(1 - F(\hat{v}_0)) \mathbb{E}[v_2 | v_2 \geq \hat{v}_0] + F(\hat{v}_0) \hat{v}_0 \right] \\ &= \hat{v}_0 + \delta_s \frac{1 + \hat{v}_0^2}{2}. \end{aligned} \tag{16}$$

(1b) $v_1 > \hat{v}_0$: In this case, x_1 is put in place. The expected payoff as of Date 1 is, similarly,

$$v_1 + \delta_s \frac{1 + v_1^2}{2}.$$

Taking the weighted average over these two cases gives the ex ante expected payoff:

$$\begin{aligned} \mathbb{E}[\text{payoff}] &= \hat{v}_0 \mathbb{E} \left[\hat{v}_0 + \delta_s \frac{1 + \hat{v}_0^2}{2} \mid v_1 \leq \hat{v}_0 \right] + (1 - \hat{v}_0) \mathbb{E} \left[v_1 + \delta_s \frac{1 + v_1^2}{2} \mid v_1 > \hat{v}_0 \right] \\ &= \frac{1}{2} (1 + \hat{v}_0^2) + \frac{\delta_s}{6} (4 + 2\hat{v}_0^3). \end{aligned} \tag{17}$$

Case (2): $\hat{v}_0 > v_1^* > 0$. In this case, the expected payoff is the average over three subcases depending on whether x_0 is retained or not at Date 1:

2a) $v_1 \leq v_1^*$: In this case, $v_1 < \hat{v}_0$, so x_1 is put in place. The expected payoff as of Date 1 is $v_1 + \delta_s \frac{1 + v_1^2}{2}$.

(2b) $v_1 \in [v_1^*, \hat{v}_0]$: In this case, x_0 is retained. The expected payoff as of Date 1 is $\hat{v}_0 + \delta_s \frac{1 + \hat{v}_0^2}{2}$.

(2c) $v_1 > \hat{v}_0$: In this case, $v_1 > v_1^*$, so x_1 is put in place. The expected payoff as of Date 1 is $v_1 + \delta_s \frac{1 + v_1^2}{2}$.

Taking the weighted average over these three cases gives the ex ante expected payoff:

$$\begin{aligned} \mathbb{E}[\text{payoff}] &= v_1^* \mathbb{E} \left[v_1 + \delta_s \frac{1 + v_1^2}{2} \mid v_1 < v_1^* \right] + (\hat{v}_0 - v_1^*) \mathbb{E} \left[\hat{v}_0 + \delta_s \frac{1 + \hat{v}_0^2}{2} \mid v_1^* \leq v_1 \leq \hat{v}_0 \right] \\ &\quad + (1 - \hat{v}_0) \mathbb{E} \left[v_1 + \delta_s \frac{1 + v_1^2}{2} \mid v_1 > \hat{v}_0 \right] \\ &= \frac{1}{2} (1 + (\hat{v}_0 - v_1^*)^2) + \frac{\delta_s}{6} (4 + 2\hat{v}_0^3 - 3\hat{v}_0^2 v_1^* + v_1^{*3}). \end{aligned} \quad (18)$$

Case (3): $\hat{v}_0 < v_1^* < 1$. In this case, the expected payoff is the average over three subcases depending on whether x_0 is retained or not at Date 1:

(3a) $v_1 < \hat{v}_0$: In this case, $v_1 < v_1^*$, so x_1 is put in place. The expected payoff as of Date 1 is $v_1 + \delta_s \frac{1 + v_1^2}{2}$.

(3b) $v_1 \in [\hat{v}_0, v_1^*]$: In this case, x_0 is retained. The expected payoff as of Date 1 is $\hat{v}_0 + \delta_s \frac{1 + \hat{v}_0^2}{2}$.

(3c) $v_1 > v_1^*$: In this case, $v_1 > \hat{v}_0$, so x_1 is put in place. The expected payoff as of Date 1 is $v_1 + \delta_s \frac{1 + v_1^2}{2}$.

Taking the weighted average over these three cases gives the ex ante expected payoff:

$$\begin{aligned} \mathbb{E}[\text{payoff}] &= \hat{v}_0 \mathbb{E} \left[v_1 + \delta_s \frac{1 + v_1^2}{2} \mid v_1 < \hat{v}_0 \right] + (v_1^* - \hat{v}_0) \mathbb{E} \left[\hat{v}_0 + \delta_s \frac{1 + \hat{v}_0^2}{2} \mid \hat{v}_0 \leq v_1 \leq v_1^* \right] \\ &\quad + (1 - v_1^*) \mathbb{E} \left[v_1 + \delta_s \frac{1 + v_1^2}{2} \mid v_1 > v_1^* \right] \\ &= \frac{1}{2} (1 - (v_1^* - \hat{v}_0)^2) + \frac{\delta_s}{6} (4 - 2\hat{v}_0^3 + 3\hat{v}_0^2 v_1^* - v_1^{*3}). \end{aligned} \quad (19)$$

Case (4): $v_1^* \geq 1$. In this case, the expected payoff is the average over two subcases depending on whether x_0 is retained or not at Date 1:

(4a) $v_1 < \hat{v}_0$: In this case, $v_1 < v_1^*$, so x_1 is put in place. The expected payoff as of Date 1 is $v_1 + \delta_s \frac{1+v_1^2}{2}$.

(4b) $v_1 \geq \hat{v}_0$: In this case, x_0 is retained. The expected payoff as of Date 1 is $\hat{v}_0 + \delta_s \frac{1+\hat{v}_0^2}{2}$.

Taking the weighted average over these two cases gives the ex ante expected payoff:

$$\begin{aligned} \mathbb{E}[\text{payoff}] &= \hat{v}_0 \mathbb{E} \left[v_1 + \delta_s \frac{1+v_1^2}{2} \mid v_1 < \hat{v}_0 \right] + (1 - \hat{v}_0) \mathbb{E} \left[\hat{v}_0 + \delta_s \frac{1+\hat{v}_0^2}{2} \mid v_1 \geq \hat{v}_0 \right] \\ &= \frac{1}{2} \left(1 - (1 - \hat{v}_0)^2 \right) + \frac{\delta_s}{6} (3 - 2\hat{v}_0^3 + 3\hat{v}_0^2). \end{aligned} \quad (20)$$

A.7.3 Comparing aligned and diverse board

Small biases. Suppose biases are small enough, such that $v_1^* \equiv 2pb - \frac{2}{\delta} - \hat{v}_0 < 0$. Then, a diverse board always acts in the interest of shareholders (Case (1) above), but an aligned board does not (Proposition 3). Hence a diverse board is better for shareholders than an aligned board.

Large biases. Suppose that biases are large enough, such that $v_1^* \equiv 2pb - \frac{2}{\delta} - \hat{v}_0 \geq 1$, as in Case (4) above. Then, shareholders' expected payoff with a diverse board is as in equation (20). In addition, $v_1^* \geq 1$ implies $2pb \geq 1 + \frac{2}{\delta} + \hat{v}_0 > 2p$ (since $p \leq 1/3$ by the assumption that there are at least three directors), and hence $b > 1 \geq \hat{v}_0$, so shareholders' expected payoff with an aligned board is given by the case $\hat{v}_0 < b$ in equation (14). Comparing these two equations, we find that shareholders prefer an aligned board if

$$\frac{1 + (2-p)\hat{v}_0^2 - 2\hat{v}_0}{2} + \frac{\delta_s}{6} (1 - 2p(1-p)(3-2b)b^2 + 2(1-p)^2\hat{v}_0^3 + 2\hat{v}_0^3 - 3\hat{v}_0^2) > 0$$

Note also that $(3-2b)b^2$ is decreasing in b for $b > 1$, and since, as shown above, $v_1^* \geq 1$ implies $b > 1$, it is sufficient to show that

$$\frac{1 + (2-p)\hat{v}_0^2 - 2\hat{v}_0}{2} + \frac{\delta_s}{6} (1 - 2p(1-p) + 2(1-p)^2\hat{v}_0^3 + 2\hat{v}_0^3 - 3\hat{v}_0^2) > 0. \quad (21)$$

To proceed, we rewrite this inequality, defining

$$f_1(\hat{v}_0) := 1 + (2 - p) \hat{v}_0^2 - 2\hat{v}_0 \quad (22)$$

and

$$f_2(\hat{v}_0) := 1 - 2p(1 - p) + 2(1 - p)^2 \hat{v}_0^3 + 2\hat{v}_0^3 - 3\hat{v}_0^2. \quad (23)$$

Now, equation (21) says $\frac{1}{2}f_1(\hat{v}_0) + \frac{\delta_s}{6}f_2(\hat{v}_0) > 0$. We will show that both terms are positive.

Starting with f_1 , we first note that f_1 is positive on the boundaries of $[0,1]$: $f_1(0) > 0$ and $f_1(1) = 1 - p > 0$. Thus, given f_1 is continuous, it is positive on all of $[0,1]$ if it is positive for any local extremum in $[0,1]$. This is the case. Indeed, it has a unique local extremum: $f_1'(\hat{v}_0) = 0$ if and only if $\hat{v}_0 = 1/(2 - p)$. And, indeed, $f_1\left(\frac{1}{2-p}\right) = \frac{1-p}{2-p} > 0$, as needed.

Next, turn to f_2 . As above, we first note that f_2 is positive on the boundary of $[0,1]$. To show this, we split up the argument in two steps:

- $f_2(0) = 1 - 2p(1 - p) > 0$.
- $f_2(1) \equiv f_2(1; p) = 2(2p^2 - 3p + 1)$. To see that $f_2(1; p)$ is positive, it is enough to show that it is positive for all $p \leq 1/3$ (given $p \leq 1/3$ by the assumption that there are at least three directors). To show this, we show that $f_2(1; p)$ is positive at $p = 0$ and $p = 1/2$ and decreasing in between. Indeed:

1. $f_2(1; 1/2) = 2$ and $f_2(1; 1/2) = 0$, and
2. $f_2(1; p)$ is decreasing for $p \in [0, 1/2]$ because $\frac{\partial}{\partial p}f_2(1; p) = 2(4p - 3)$.

Thus, given f_2 is continuous, it is positive on all of $[0,1]$ if it is positive for any local extremum in $[0,1]$. This is the case. Indeed, $f_2(\hat{v}_0)$ has a unique local extremum: $f_2'(\hat{v}_0) = 0$ if and only if $\hat{v}_0 \equiv \hat{v}_0^* = \frac{1}{1+(1-p)^2}$. And, at the extremum,

$$f_2\left(\frac{1}{1+(1-p)^2}\right) \equiv f_2(\hat{v}_0^*; p) = 1 - 2p(1 - p) - \frac{1}{(1+(1-p)^2)^2}. \quad (24)$$

This function is decreasing on $p \in [0, \frac{1}{3}]$. In addition, at $p = \frac{1}{3}$, it equals $\frac{5}{9} - \left(\frac{9}{13}\right)^2 > 0$, and hence it is positive for all $p \in [0, 1/3]$.

Hence, both f_1 and f_2 are positive for all $\hat{v}_0 \in [0, 1]$. So shareholders prefer an aligned board to a diverse board when biases are large. \square

A.8 Proof of Lemma 1

According to the proof of Proposition 4, shareholders' expected payoff with an aligned board does not depend on δ , but it does with a diverse board. It is thus sufficient to show that shareholders' expected payoff with a diverse board is decreasing in δ .

First, note that shareholders' expected payoff with a diverse board depends on δ via v_1^* , which is defined in equation (15), and that $\frac{\partial v_1^*}{\partial \delta} = \frac{2}{\delta^2} > 0$.

As in the proof of Proposition 4, there are four cases:

Case (1): $v_1^* < 0$. In this case, the board acts in shareholders' interest for all δ , as shown above. Hence, shareholders' payoff does not depend on δ .

Case (2): $0 < v_1^* < \hat{v}_0$. Shareholders' expected payoff is as in equation (18). Differentiating this with respect to δ , we get

$$\frac{\partial}{\partial \delta} \mathbb{E}[\text{payoff}] = \left(-\frac{1}{2} 2(\hat{v}_0 - v_1^*) - \frac{\delta_s}{2} \hat{v}_0^2 + \frac{\delta_s}{2} v_1^{*2} \right) \frac{\partial v_1^*}{\partial \delta}, \quad (25)$$

which is negative because $\frac{\partial v_1^*}{\partial \delta} > 0$ and because, in this case, $\hat{v}_0 > v_1^* > 0$ by hypothesis.

Case (3): $\hat{v}_0 < v_1^* < 1$. Shareholders' expected payoff is as in equation (19). Differentiating this with respect to δ , we get

$$\frac{\partial}{\partial \delta} \mathbb{E}[\text{payoff}] = \left(-\frac{1}{2} 2(v_1^* - \hat{v}_0) + \frac{\delta_s}{2} \hat{v}_0^2 - \frac{\delta_s}{2} v_1^{*2} \right) \frac{\partial v_1^*}{\partial \delta}, \quad (26)$$

which is negative because $\frac{\partial v_1^*}{\partial \delta} > 0$ and because, in this case, $\hat{v}_0 < v_1^*$ by hypothesis and $\hat{v}_0 \geq 0$.

Case (4): $v_1^* > 1$. By equation (20), shareholders' payoff does not depend on v_1^* (and

hence on δ). □

A.9 Proof of Lemma 2

To show that increasing power to appoint π decreases deadlock, we proceed in three steps. First, we compute a director's payoff as a function of the date-2 incumbent value \hat{v}_1 . Second, we compare her payoff for $\hat{v}_1 = \hat{v}_0$ and $\hat{v}_1 = v_1$. We find that there is deadlock if v_1 is below a cutoff v_1^* , which depends on π . Finally, third, we show that this cutoff v_1^* is decreasing in π , implying that the more power to appoint directors have, the more likely it is that new high-quality policies are put in place, i.e., the less deadlock there is.

A.9.1 Expected payoff at date 1

When director i votes at date 1, she takes into account three cases at date 2. We refer to these as Case (1), Case (2), and Case (3) below.

Case (1): With probability $\pi\alpha_i$, she effectively controls the board at date 2. In this case, there are two subcases to consider:

1. The alternative x_2 is director's i preferred type (this occurs with probability p). Recall that the incumbent policy \hat{x}_1 is not any director's preferred type. Then, the director replaces the current policy if $v_2 + b > \hat{v}_1$ and gets

$$(1 - F(\hat{v}_1 - b))\mathbb{E}[v_2 + b \mid v_2 \geq \hat{v}_1 - b] + F(\hat{v}_1 - b)\hat{v}_1. \quad (27)$$

2. The alternative x_2 is another type (this occurs with probability $1 - p$). Then, she replaces the current policy if $v_2 > \hat{v}_1$ and gets

$$(1 - F(\hat{v}_1))\mathbb{E}[v_2 \mid v_2 \geq \hat{v}_1] + F(\hat{v}_1)\hat{v}_1. \quad (28)$$

Taking the expectation over these two cases, we get:

$$\begin{aligned}
U_2^{(1)}(\hat{v}_1) &= p \left\{ (1 - F(\hat{v}_1 - b)) \mathbb{E}[v_2 + b \mid v_2 \geq \hat{v}_1 - b] + F(\hat{v}_1 - b) \hat{v}_1 \right\} \\
&\quad + (1 - p) \left\{ (1 - F(\hat{v}_1)) \mathbb{E}[v_2 \mid v_2 \geq \hat{v}_1] + F(\hat{v}_1) \hat{v}_1 \right\} \\
&= \begin{cases} p \left(\frac{1}{2} + b \right) + (1 - p) \frac{1 + \hat{v}_1^2}{2} & \text{if } \hat{v}_1 < b \\ \frac{1 + \hat{v}_1^2 + pb^2}{2} + (1 - \hat{v}_1)pb & \text{if } \hat{v}_1 > b. \end{cases} \tag{29}
\end{aligned}$$

Case (2): With probability $\pi(1 - \alpha_i)$ another director effectively controls the board at date 2. In this case, there are two subcases to consider:

1. The alternative x_2 is this other director's preferred type (this occurs with probability p). Then, the current policy is replaced if $v_2 + b > \hat{v}_1$. Director i gets

$$(1 - F(\hat{v}_1 - b)) \mathbb{E}[v_2 \mid v_2 \geq \hat{v}_1 - b] + F(\hat{v}_1 - b) \hat{v}_1. \tag{30}$$

2. The alternative x_2 is not this other director's preferred type (this occurs with probability $1 - p$). Then, she replaces the current policy if $v_2 > \hat{v}_1$ and gets

$$(1 - F(\hat{v}_1)) \mathbb{E} \left[v_2 + \frac{p}{1 - p} b \mid v_2 \geq \hat{v}_1 \right] + F(\hat{v}_1) \hat{v}_1. \tag{31}$$

Taking the expectation over these two cases, we get:

$$\begin{aligned}
U_2^{(2)}(\hat{v}_1) = & p \left\{ (1 - F(\hat{v}_1 - b)) \mathbb{E}[v_2 \mid v_2 \geq \hat{v}_1 - b] + F(\hat{v}_1 - b) \hat{v}_1 \right\} + \\
& + (1 - p) \left\{ (1 - F(\hat{v}_1)) \mathbb{E} \left[v_2 + \frac{p}{1-p} b \mid v_2 \geq \hat{v}_1 \right] + F(\hat{v}_1) \hat{v}_1 \right\} \\
= & \begin{cases} \frac{1}{2} + bp(1 - \hat{v}_1) + \frac{(1-p)\hat{v}_1^2}{2} & \text{if } \hat{v}_1 < b \\ \frac{1 + \hat{v}_1^2 - pb^2}{2} + pb(1 - \hat{v}_1) & \text{if } \hat{v}_1 > b. \end{cases} \tag{32}
\end{aligned}$$

Case (3): With probability $1 - \pi$, the board is diverse at date 2 (this, it turns out, is like a hypothetical unbiased director controlling the board at date 2). In this case, the current policy is replaced if $v_2 \geq \hat{v}_1$, and director i gets

$$\begin{aligned}
U_2^{(3)}(\hat{v}_1) = & (1 - F(\hat{v}_1)) \mathbb{E}[v_2 + pb \mid v_2 \geq \hat{v}_1] + F(\hat{v}_1) \hat{v}_1 \\
= & \frac{1 + \hat{v}_1^2}{2} + (1 - \hat{v}_1) pb. \tag{33}
\end{aligned}$$

Expected Payoff at Date 1: Since we assumed that neither \hat{x}_0 nor x_1 is the director's preferred type, her expected payoff at date 1 is

$$U(\hat{v}_1) = \hat{v}_1 + \delta \left[\pi \left(\alpha_i U_2^{(1)}(\hat{v}_1) + (1 - \alpha_i) U_2^{(2)}(\hat{v}_1) \right) + (1 - \pi) U_2^{(3)}(\hat{v}_1) \right], \tag{34}$$

and she chooses to keep x_0 in place if

$$U(\hat{v}_0) > U(v_1). \tag{35}$$

A.9.2 Comparison: Is \hat{x}_0 replaced with x_1 ?

Now, we use the assumption that $b > v_1 > \hat{v}_0$ to simplify equation (34) to

$$U(\hat{v}_1) = \hat{v}_1 + \delta \left(\frac{1 + \hat{v}_1^2}{2} + (1 - \hat{v}_1)pb + \frac{p\pi(2\alpha_i b - \hat{v}_1)\hat{v}_1}{2} \right). \quad (36)$$

Taking the difference of the expression above with $\hat{v}_1 = \hat{v}_0$ and $\hat{v}_1 = v_1$ gives

$$U(\hat{v}_0) - U(v_1) = \delta(1 - \pi p) \frac{(\hat{v}_0 - v_1)(\hat{v}_0 + v_1)}{2} + (1 - (1 - \alpha_i \pi)\delta p b)(\hat{v}_0 - v_1). \quad (37)$$

This is positive if

$$v_1 < \frac{2\delta(1 - \alpha_i \pi)bp - 2 - \delta(1 - p\pi)\hat{v}_0}{\delta(1 - \pi p)}. \quad (38)$$

Note that the higher is α_i , the lower is this cutoff, suggesting that directors with higher α 's are more likely to vote to replace \hat{x}_0 than directors with lower α 's.

There can be deadlock if this condition holds for the *median director*, who is pivotal in the date-1 vote. I.e., there is deadlock if

$$\begin{aligned} v_1 < v_1^* &:= \text{median} \left\{ \frac{2\delta(1 - \alpha_i \pi)bp - 2 - \delta(1 - p\pi)\hat{v}_0}{\delta(1 - \pi p)} \right\}_i \\ &= \frac{2\delta(1 - \alpha^m \pi)bp - 2 - \delta(1 - p\pi)\hat{v}_0}{\delta(1 - \pi p)}, \end{aligned} \quad (39)$$

where α^m is the power to appoint of the median director. (Formally, this is the lower median, $\lfloor \frac{N+1}{2} \rfloor$. If N is even, then the policy stays in place unless strict majority, i.e., $N/2 + 1$ of the directors vote to replace it. Since directors with higher α 's are more likely to vote to replace, the pivotal director is the $(N/2 + 1)$ st *from the top*, or the lower median.)

A.9.3 v_1^* is decreasing in π

To see that deadlock, as captured by the cutoff v_1^* , is decreasing in power to appoint, π , we differentiate v_1^* in equation (39) above:

$$\frac{\partial v_1^*}{\partial \pi} = -\frac{2(1 - \delta(p - \alpha^m)b)p}{\delta(1 - \pi p)^2}. \quad (40)$$

This is negative if $1 - \delta(p - \alpha^m)b > 0$, which is the condition in the lemma. \square

A.10 Proof of Proposition 5

We identify sufficient conditions for there to be an interior-optimal power to appoint π for a given \hat{v}_0 and v_1 .

We start with shareholders' payoff $U^S(\hat{v}_1, \pi)$ at date 1, given policy in place \hat{v}_1 and power to appoint π . At date 2, the board is either diverse (which occurs with probability $1 - \pi$) or aligned (which occurs with probability π), and shareholders' payoff in these two cases is

$$U^S|_{\text{diverse}} = (1 - F(\hat{v}_1))\mathbb{E}[v_2|v_2 > \hat{v}_1] + F(\hat{v}_1)\hat{v}_1 = \frac{1 + \hat{v}_1^2}{2}, \quad (41)$$

and

$$\begin{aligned} U^S|_{\text{aligned}} &= p((1 - F(\hat{v}_1 - b))\mathbb{E}[v_2|v_2 + b > \hat{v}_1] + F(\hat{v}_1 - b)\hat{v}_1) + (1 - p)\left(\frac{1 + \hat{v}_1^2}{2}\right) \\ &= p\frac{1}{2} + (1 - p)\left(\frac{1 + \hat{v}_1^2}{2}\right) = \frac{1 + (1 - p)\hat{v}_1^2}{2}. \end{aligned} \quad (42)$$

Thus,

$$U^S(\hat{v}_1, \pi) = \hat{v}_1 + \delta_s((1 - \pi)U^S|_{\text{diverse}} + \pi U^S|_{\text{aligned}}) \quad (43)$$

$$= \hat{v}_1 + \delta_s\left(\frac{1 + (1 - p\pi)\hat{v}_1^2}{2}\right). \quad (44)$$

From condition (39), we know that the board retains \hat{x}_0 (inefficiently) at date 1 if $v_1 < v_1^*$. From Lemma 2, we know that if $\alpha^m > p - \frac{1}{\delta b}$, then higher π reduces deadlock. Thus, intuitively, there is a trade-off of increasing π : on the one hand, it increases the distortion at date 2, but, on the other hand, it reduces deadlock. Hence, the optimal power to appoint given to directors is either zero or the minimum positive π that prevents deadlock.

To proceed, we compare the expected payoff to shareholders given $\pi = 0$ (so there is no distortion at date 2) with their payoff given the minimum π such that x_1 is put in place, or π^* that makes the deadlock condition in equation (39) hold with equality,

$$\pi^* := \frac{1}{p} \left(1 - \frac{2b(\alpha^m - p + \frac{1}{\delta b})}{2\alpha^m b - \hat{v}_0 - v_1} \right). \quad (45)$$

If $\pi^* > 1$, there is deadlock even if directors have full power to appoint, so the optimal $\pi = 0$. Similarly, if $\pi^* \leq 0$, there is no deadlock even if shareholders have full power to appoint, so the optimal $\pi = 0$. If $\pi^* \in (0, 1)$, the optimal power to appoint is either π^* or 0, and setting $\pi = \pi^*$ is better than setting $\pi = 0$ if

$$v_1 + \delta_s \left(\frac{1 + (1 - p\pi^*)v_1^2}{2} \right) \geq \hat{v}_0 + \delta_s \left(\frac{1 + \hat{v}_0^2}{2} \right), \quad (46)$$

which is the condition in the proposition. □

A.11 Proof of Proposition 6

The structure of the proof is as follows.

Step 1. We show that if it is optimal to give directors positive power to appoint, $\pi > 0$, then the optimal power to appoint is the minimal π that prevents deadlock (no matter the distribution of power to appoint $\{\alpha_i\}_i$).

Step 2. We show that for any interior optimum of power to appoint π , the optimum is decreasing in α^m . Thus, it is optimal to maximize α^m subject to preventing deadlock.

Step 3. We show that the maximal α^m that prevents deadlock is $\alpha^m = (\lceil \frac{N+1}{2} \rceil)^{-1}$.

We now implement these steps.

Step 1. Immediately from the expression for shareholders' payoff in equation (44), i.e., $U^S(\hat{v}_1, \pi) = \hat{v}_1 + \delta_s \left(\frac{1+(1-p\pi)\hat{v}_1^2}{2} \right)$, we see that shareholders are better off if π is smaller, and that their payoff does not depend on $\{\alpha_i\}_i$, i.e., how the power is distributed among directors. (The reason is that all that matters to shareholders is the total power given to directors, $\sum_{i=1}^N \pi \alpha_i = \pi$.)

Step 2. Given the conditions of Proposition 5 are satisfied with $\alpha^m = (\lceil \frac{N+1}{2} \rceil)^{-1}$, we know that if $\alpha^m = (\lceil \frac{N+1}{2} \rceil)^{-1}$, then the optimal π^* , as defined in equation (6), is interior. Thus, we can differentiate this optimal π^* with respect to α^m :

$$\frac{\partial \pi^*(\alpha^m)}{\partial \alpha^m} = - \frac{2b(\delta(2pb - \hat{v}_0 - v_1) - 2)}{\delta p(2\alpha^m b - \hat{v}_0 - v_1)^2} \quad (47)$$

$$= - \frac{2b\pi^*(\alpha^m)}{2\alpha^m b - \hat{v}_0 - v_1} < 0, \quad (48)$$

because $\pi^*(\alpha^m) > 0$ and $2\alpha^m b - \hat{v}_0 - v_1 > 0$, again by the conditions in Proposition 5 (if $2\alpha^m b - \hat{v}_0 - v_1 < 0$, then $\alpha^m - p + \frac{1}{\delta b} > 0$ implies $\pi^* > \frac{1}{p} > 1$, i.e., a violation of the conditions in Proposition 5).

In summary, π^* is a strictly decreasing function of α^m , whenever π^* is positive. Intuitively, the higher is the power α^m allocated to the pivotal director, the lower is the π that is needed to prevent this pivotal director from deadlocking the board. Thus, to minimize π^* , we maximize α^m .

Step 3. Finally, because $\alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_{\lfloor \frac{N-1}{2} \rfloor} \leq \alpha_{\lfloor \frac{N+1}{2} \rfloor} = \alpha^m \leq \dots \leq \alpha_N$, then to maximize α^m , we set $\alpha_1 = \alpha_2 = \dots = \alpha_{\lfloor \frac{N-1}{2} \rfloor} = 0$ and $\alpha^m = \dots = \alpha_N > 0$, implying $\alpha^m = (\lceil \frac{N+1}{2} \rceil)^{-1}$. \square

A.12 Proof of Proposition 7

For the proof, we introduce some notation for directors' value functions as functions of the policy in place (which is the Markov state):

- Let $U_{\tau_1}(v, \tau_2)$ denote the value function of the τ_1 -director after the date- t vote that put (or kept) a policy with value v and type τ_2 in place, but before the realization of date- t flow utilities from this policy.
- Let $U_{\tau_1}^{\text{ex}}(v, \tau_2)$ denote $U_{\tau_1}(v, \tau_2)$ minus the current-period flow utility, i.e., the value function of the τ_1 -director after the date- t vote that put (or kept) a policy with value v and type τ_2 in place, but *after* the realization of date- t flow utilities from this policy.
- Let U_{τ}^0 denote the value function of the τ -director after the date- t vote that kept the initial policy \hat{x}_0 in place, but before the realization of date- t flow utilities from this policy.
- Let $U(v)$ denote the component of a director's value function reflecting *only the v part* (so not the b part) of her flow utilities, after the date- t vote that put (or kept) a policy different from x_0 with value v in place, but before the realization of date- t flow utilities from this policy. (It turns out that this will not depend on a director's bias or the type of the policy.)

Before we present the main proof, we formulate and prove an auxiliary lemma.

Lemma 3. *Suppose the conditions of the proposition hold and a policy with type τ is in place. In any Markov equilibrium, a τ -biased director votes against any policies that are not of type τ .*

Proof: Let \hat{v}_t denote the value of the policy in place and suppose (in anticipation of a contradiction) that a τ -biased director votes for an alternative with value v_t and type $\tau' \neq \tau$.

Given the definition of U^{ex} above, we can write a condition for this vote to be profitable as

$$\hat{v}_t + b + U^{\text{ex}}(\hat{v}_t, \tau) \leq v_t + U^{\text{ex}}(v_t, \tau') \quad (49)$$

or

$$b \leq U^{\text{ex}}(v_t, \tau') - U^{\text{ex}}(\hat{v}_t, \tau) + v_t - \hat{v}_t. \quad (50)$$

Given that the most a director can get in any period is $1 + b$, it must be that $U^{\text{ex}}(v_t, \tau')$ is bounded above as follows:

$$U^{\text{ex}}(v_t, \tau') \leq \sum_{t=1}^{\infty} \delta^t (1 + b) = \frac{\delta}{1 - \delta} (1 + b). \quad (51)$$

And, given that the τ -director has the option of keeping the incumbent policy with value \hat{v}_t and type τ in place forever and getting $\hat{v}_t + b$ in every period (by voting against any alternatives), it must be that $U^{\text{ex}}(\hat{v}_t, \tau)$ is bounded below as follows:

$$U^{\text{ex}}(\hat{v}_t, \tau) \geq \sum_{t=1}^{\infty} \delta^t (\hat{v}_t + b) = \frac{\delta}{1 - \delta} (\hat{v}_t + b). \quad (52)$$

We also know that $\hat{v}_t \geq 0$ and $v_t \leq 1$. Together, substituting these bounds into equation (50), we get the following necessary condition on b :

$$b \leq \frac{\delta}{1 - \delta} (1 + b) - \frac{\delta}{1 - \delta} b + 1 = \frac{1}{1 - \delta}. \quad (53)$$

This contradicts the assumption in equation (9). □

Given our focus on stage-undominated equilibria, the lemma implies that once a τ -type policy is in place, it is replaced if and only if an alternative policy has a higher quality and is of the same type.

Next, suppose that the incumbent policy \hat{x}_0 is in place. Since \hat{x}_0 is “very bad,” it follows immediately that directors vote for policies of their preferred type. In the remainder of the

proof, we show that directors vote for policies that are not of their preferred type if and only if their values are above a threshold, denoted \bar{V} .

The “only if” part is immediate: if a τ -biased director wants to put a τ' -policy in place with value v_1 , she also wants to put one in place with value $v_2 > v_1$.

We prove the “if” part by showing no-deviation conditions under these strategies.

Given the lemma, after $(\hat{v}, \hat{\tau})$ is chosen, it will only be replaced by a policy of the same type $\hat{\tau}$ and only if its value is higher than \hat{v} . Hence, the value component $U(\hat{v})$ is the same for both directors, only depends on \hat{v} , and satisfies

$$U(\hat{v}) = \hat{v} + \delta \left(\Pr(\tau = \hat{\tau}, v > \hat{v}) \mathbb{E}[U(v) | v > \hat{v}] + (1 - \Pr(\tau = \hat{\tau}, v > \hat{v})) U(\hat{v}) \right) \quad (54)$$

$$= \hat{v} + \delta \left(p(1 - \hat{v}) \mathbb{E}[U(v) | v > \hat{v}] + (1 - p(1 - \hat{v})) U(\hat{v}) \right). \quad (55)$$

Given that a director gets b in each period if and only if a policy of her preferred type is put in place, we have the following value functions given that the initial incumbent is replaced:

$$U_\tau(\hat{v}, \tau) = U(\hat{v}) + \frac{b}{1 - \delta} \quad (56)$$

$$U_\tau(\hat{v}, \tau') = U(\hat{v}). \quad (57)$$

Now we turn to directors' value functions given the initial incumbent is still in place.

$$U_\tau^0 = \delta \Pr(v < \bar{V}) U_\tau^0 + \delta \Pr(v > \bar{V}) \left(\mathbb{E}[U(v) | v > \bar{V}] + p \frac{b}{1 - \delta} \right) \quad (58)$$

$$= \delta \bar{V} U_\tau^0 + \delta (1 - \bar{V}) \left(\mathbb{E}[U(v) | v > \bar{V}] + p \frac{b}{1 - \delta} \right) \quad (59)$$

and, hence,

$$U_\tau^0 = \frac{\delta(1 - \bar{V})}{1 - \delta \bar{V}} \left(\mathbb{E}[U(v) | v > \bar{V}] + p \frac{b}{1 - \delta} \right). \quad (60)$$

In order for the strategies described in the proposition to be an equilibrium, two conditions must be satisfied:

1. It must be that a τ -biased director prefers a τ' -policy to \hat{x}_0 if its value v is greater than the cutoff \bar{V} :

$$U_\tau^0 \leq U(v) \text{ for } v \geq \bar{V}. \quad (61)$$

2. It must be that a τ -biased director prefers \hat{x}_0 to a τ' -policy if its value v is less than the cutoff \bar{V} :

$$U_\tau^0 \geq U(v) \text{ for } v < \bar{V}. \quad (62)$$

Since the value functions are continuous and monotone in v , these conditions imply that a τ -biased director must be indifferent between the initial incumbent \hat{x}_0 and a τ' -policy with value equal to the cutoff \bar{V} , i.e., $U_\tau^0 = U(\bar{V})$, or

$$\frac{\delta(1-\bar{V})}{1-\delta\bar{V}} \left(\mathbb{E}[U(v) | v > \bar{V}] + p \frac{b}{1-\delta} \right) = U(\bar{V}). \quad (63)$$

We now re-write the equation (55) evaluated at $\hat{v} = \bar{V}$ as

$$(1-\bar{V}) \mathbb{E}[U(v) | v > \bar{V}] = \frac{U(\bar{V}) - \bar{V}}{\delta p} - \frac{1}{p} [1 - p(1-\bar{V})] U(\bar{V}) \quad (64)$$

to substitute into the indifference condition in equation (63) and get

$$U(\bar{V})(1-\delta) \left(1 - \frac{1}{p} \right) = \delta p \frac{b}{1-\delta} - \bar{V} \left(\delta p \frac{b}{1-\delta} + \frac{1}{p} \right). \quad (65)$$

Recalling the assumption that there are only two types of alternatives, we can substitute $p = 1/2$ to write this as

$$G(\bar{V}) := \bar{V} \left(2 + \frac{\delta}{2(1-\delta)} b \right) - U(\bar{V})(1-\delta) - \frac{\delta}{2(1-\delta)} b = 0,$$

where G is a continuous function.

To prove that $\bar{V} \in (0, 1)$, we show that $G(0) < 0$ and $G(1) > 0$, so the result follows from

the intermediate value theorem:

$$G(0) = -U(0)(1 - \delta) - \frac{\delta}{2(1 - \delta)}b < 0, \quad (66)$$

since $U(v) \geq 0$, and

$$G(1) = 2 - U(1)(1 - \delta) > 0, \quad (67)$$

since $U(1) \leq 1 + \delta + \dots = \frac{1}{1 - \delta}$.

□

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