THE PARADOX OF PLEDGEABILITY*

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Abstract

We develop a model in which collateral serves to protect creditors from the claims of other creditors. We find that, paradoxically, borrowers rely most on collateral when pledgeability is high, because this is when it is easy to take on new debt, which dilutes existing creditors. Creditors thus require collateral for protection against dilution—there is a "collateral rat race." But collateralized borrowing has a cost: it encumbers assets, constraining future borrowing and investment—there is a "collateral overhang." Our results suggest that policies aimed at increasing the supply of collateral may backfire, triggering an inefficient collateral rat race.

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1 Introduction

Collateral matters.[1] In much of the finance literature, collateral matters because it mitigates enforcement frictions between borrowers and creditors, i.e. "collateral pledging makes up for a lack of pledgeable cash" (Tirole (2006), p. 169). But collateral also plays another role, emphasized in the law literature:[2] collateral matters because it mitigates enforcement frictions among creditors, i.e. "a secured transaction [is] the protection...against the claims of competing creditors" (Kronman and Jackson (1979), p. 1143). These two roles of collateral correspond to the two components of property rights which accrue to secured creditors upon default: the "right of access"—a creditor's right to seize collateral—and the "right of exclusion"—a creditor's right to stop other creditors from seizing collateral (e.g., Hart (1995), Segal and Whinston (2012)).

In this paper, we present a finance model based on the latter role. We find that borrowers rely on collateral when pledgeability is high, not low—collateral does not make up for a lack of pledgeable cash. The reason is that high pledgeability makes it easy to take on new debt, which dilutes existing creditors. This leads existing creditors to require collateral for protection against possible dilution by collateralized debt—there is a collateral rat race. But collateralized borrowing has a cost: it encumbers assets, constraining future borrowing and investment—there is a collateral overhang. Further, the greater availability of collateral can have adverse effects—it can trigger an inefficient collateral rat race. In particular, policies aiming to increase the supply of collateral, such as expanding the set of assets that can be used as collateral, may backfire, as they can increase the demand for collateral. Likewise, upholding the absolute priority of secured debt can exacerbate the rat race. This finding gives support to arguments advanced in the law literature against this absolute priority rule (see Bjerre (1999) and Lubben (2016)).

Model preview. A borrower, B, has two riskless projects, Project 0 and Project 1, to finance sequentially. B finances Project 0 by borrowing from one creditor, C_0. After Project 0 is underway, B can finance Project 1 by borrowing from another creditor, C_1. Project 0's NPV is positive, but Project 1's NPV, which is revealed after Project 0 is underway, may be positive or negative. Thus, it is efficient for B to undertake Project 0 and to undertake Project 1 only in the event that its NPV is positive.

B's borrowing capacity is constrained by two frictions. First, *pledgeability* is limited.

[1] See, e.g., Aretz, Campello, and Marchica (2017), Benmelech and Bergman (2009, 2011), Rampini and Viswanathan (2013), and Rampini, Sufi, and Viswanathan (2014) for empirical evidence on the importance of collateral for borrowing.

[2] See, e.g., Bebchuk and Fried (1996), Hansmann and Kraakman (2002), Hansmann and Santilli (1997), Kronman and Jackson (1979), Schwarcz (1997), and Schwartz (1984).

Specifically, the total repayment from B to his creditors cannot exceed a fixed fraction θ of the final value of his projects, representing, e.g., the liquidation value of assets employed in the project or the cash flows that cannot be diverted (there is no formal difference between assets and cash flows; see footnote 9). Second, contracts are *non-exclusive* in that when B takes on debt to C_0, he cannot commit not to take on new debt to C_1.[3]

To finance a project, B can borrow via either secured (i.e. "collateralized") debt or via unsecured debt. If B borrows via secured debt, the secured creditor has an exclusive claim over the project(s) securing the debt. Thus, by borrowing secured, B "ring fences" his project(s) as collateral, protecting it from the claims of other creditors. If instead B borrows via unsecured debt, the creditor still has a claim on B's projects, but it is effectively junior to any new secured debt that B may take on and, hence, may be diluted with new secured debt. In summary, we focus on how collateral mitigates the non-exclusivity friction by establishing priority (for simplicity, we assume that it does not affect the limited-pledgeability friction). Our premise that collateral establishes priority among creditors echoes the law literature. Indeed, legally, "[t]he absolute priority rule describes the basic order of payment in bankruptcy. Secured creditors get paid first, unsecured creditors get paid next" (Lubben (2016), p. 581).

Results preview. Our first main result is that, paradoxically, if pledgeability θ is sufficiently high, B may not be able to borrow from C_0 unsecured, but only with collateral. To see why, suppose B finances Project 0 by borrowing from C_0 via *unsecured* debt. Because unsecured contracts are non-exclusive, B can borrow from another creditor, C_1, to finance Project 1. If B collateralizes both his projects to borrow from C_1, then C_1 is prioritized over C_0—the new secured debt dilutes the existing unsecured debt. As a result, C_0 may not lend unsecured in the first place and instead require collateral. However, dilution occurs only if B is able to borrow from C_1—i.e. if the value of B's pledgeable payoff exceeds his funding needs. In summary, if pledgeability is sufficiently high, then B would dilute C_0's unsecured debt with new secured debt to C_1 and, in anticipation, C_0 will not lend unsecured, but only with collateral. I.e., for high θ, there is a *collateral rat race*, by which collateralization is required as protection against future collateralization. Hence, contrary to common intuition, high pledgeability undermines unsecured credit.

Our second main result is that if B borrows from C_0 via secured debt, B may be unable to fund Project 1, even when it has positive NPV. This is because collateralizing Project 0 "uses up" pledgeable collateral, making it difficult for B to borrow to finance Project 1. Hence,

[3]Note that this assumption rules out covenants by which a borrower commits to one creditor not to borrow from new creditors in the future. As we discuss in Section 4, such covenants can mitigate the non-exclusive-contracting friction in reality, but rarely prevent a borrower from using collateral to borrow secured from new creditors, even if doing so violates a covenant.

collateralization effectively encumbers B's assets, i.e. it limits B's ability to use them to obtain funding to invest in Project 1. This *collateral overhang* problem resonates with practitioners' intuition that "encumbered assets are generally not available to obtain...liquidity" (Deloitte Blogs (2014)).

Next, we enrich our model by limiting the fraction of a project that can be collateralized. Hence, there may be some assets that are pledgeable—they can be seized in the future—but not collateralizable—they are hard to assign property rights to, e.g. they may not even exist at inception, but rather be built/acquired in the course of the project. Our third main result is that, although higher collateralizability can loosen borrowing constraints, it can also tighten them. This is because increasing collateralizability does *collateral damage:* it makes it easier to take on new debt from C_1, diluting C_0; this can trigger the collateral rat race. The more collateralizable Project 1 is, the more collateralizable Project 0 must be to provide protection against dilution. Hence, the more assets can be used as collateral at Date 1, the more assets are required as collateral at Date 0—increasing the supply of collateral increases the demand for collateral. And, thus, higher collateralizability can exacerbate the collateral overhang.

We also extend the model so that collateral plays two roles. It mitigates enforcement problems not only among creditors—establishing exclusivity as in our baseline model—but also between borrowers and creditors—increasing pledgeability as in most of the finance literature. We find that this classical role dominates for low pledgeability, when borrowers need collateral to get projects off the ground. But the new role we focus on dominates for high pledgeability, when creditors need collateral for protection against dilution.

Policy. Our analysis has implications for some public policies. First, some policy makers have aimed to increase the supply of collateral, having deemed it insufficient. For example, a number of countries expanded the set of movable/floating assets that can be used as collateral.[4] To the same end, some central banks committed to lend against illiquid financial securities at a pre-specified rate and haircut.[5] Our results suggest that such policies may backfire, since increasing the supply of collateral may increase the demand for collateral, triggering an inefficient rat race. Second, "[c]urrent law forces onto borrowers the power to defeat unsecured lenders by issuing secured debt" (Bjerre (1999), p. 308). Our analysis suggests that upholding the absolute priority of secured debt as such can lead to inefficient

[4] A number of European countries recently allowed movable assets to be used as collateral for the first time (see Calomiris, Larrain, Liberti, and Sturgess (2017), Campello and Larrain (2016), and Cerqueiro, Ongena, and Roszbach (2016)) as did Zimbabwe, where cows, sheep, and goats used as collateral are now recorded in a registrar at the central bank (Hawkins and Cotterill (2017)).

[5] To increase the supply of financial collateral, The European Central Bank enacted its Long-term Refinancing Operation and the Reserve Bank of Australia its Committed Liquidity Facility.

investment. This gives support to arguments advanced in the law literature against this absolute priority rule (see Bjerre (1999) and Lubben (2016)).

Contribution to the literature. To our knowledge, our model is the first to focus on the role that collateral can play in mitigating non-exclusivity, arguably its role legally (see, however, Ayotte and Bolton (2011)).[6,7] Our results suggest that (i) collateral may matter as much when pledgeability is high as when it is low and (ii) the ability to collateralize debt may not always be good for borrowers. These findings contrast with received theories of collateral (see, e.g., Hart (1995) and Tirole (2006)). But they still resonate with practice. For example, (i) collateral is essential in some of the world's most developed markets, such as interbank and syndicated loan markets, in which strong creditor rights, effective legal enforcement, intense regulatory supervision, and developed record-keeping technologies ensure that pledgeability is high. I.e., collateral matters even when it is not necessary "to make up for a lack of pledgeable cash." Further, (ii) lawyers observe that "borrowers would prefer to give up that power [to use collateral] in order to protect their unsecured lenders from the corresponding threat" (Bjerre (1999), p. 308). I.e., sometimes borrowers are better off without collateral.

Some of the mechanisms behind our results have parallels in finance papers that do not study collateral. Our "paradox of pledgeability"—increasing pledgeability undermines a borrower's ability to commit to future borrowing decisions—is a liabilities-side analog of Myers and Rajan's (1998) asset-side "paradox of liquidity"—increasing asset liquidity undermines a borrower's ability to commit to future investment decisions.[8] However, in our model, the borrower always wants to dilute, but cannot when pledgeability is low because creditors will not lend. In contrast, in Myers and Rajan (1998), the borrower does not always want to liquidate, and chooses not to when liquidity is low because it is not desirable.

In our "collateral rat race," collateral plays a similar role to short maturity in Brunnermeier and Oehmke's (2013) "maturity rat race." Unlike in that paper, too much protection against dilution can be inefficient, because it can induce the "collateral overhang."

This collateral overhang bears some similarity to Myers's (1977) "debt overhang," since debt in place prevents a borrower from financing positive-NPV projects. However, in the debt-overhang problem, a borrower will not raise capital because this would subsidize existing

[6]See Degryse, Ioannidou, and von Schedvin (2016) for evidence on collateral's role in mitigating non-exclusivity.

[7]A number of finance theory papers study non-exclusive contracting without collateral, e.g., Acharya and Bisin (2014), Admati, DeMarzo, Hellwig, and Pfleiderer (2013), Attar, Casamatta, Chassagnon, and Décamps (2015, 2017), Bisin and Gottardi (1999, 2003), Bisin and Rampini (2005), Bizer and DeMarzo (1992), DeMarzo and He (2016), Kahn and Mookherjee (1998), Leitner (2012), and Parlour and Rajan (2001).

[8]Donaldson and Micheler (2016) argue that increasing pledgeability can also paradoxically increase systemic risk, because it leads borrowers to favor non-resaleable over resaleable debt instruments (e.g., repos over bonds).

debt, whereas in the collateral-overhang problem, he cannot raise capital because existing debt is collateralized to prevent this. Further, the collateral-overhang problem arises even when the debt-overhang problem does not, i.e. when existing debt is riskless or can be renegotiated.

In our analysis of "collateral damage," we distinguish between pledgeable assets—those which can be seized by creditors ex post—and collateralizable assets—those which can be assigned to creditors ex ante. To our knowledge, this distinction is new to the theory literature, and hence so are the results that it generates.

Layout. The paper proceeds as follows. Section 2 presents the model. Section 3 contains the main results. Section 4 discusses the contracting environment. Section 5 concludes. The Appendix contains all proofs.

2 Model

2.1 Players and Projects

There is one good called cash, which is the input of production, the output of production, and the consumption good.[9] A borrower B lives for three dates, $t \in \{0, 1, 2\}$, and consumes at Date 2. B has no cash, but has access to two investment projects, Project 0 at Date 0 and Project 1 at Date 1. Both projects are riskless and pay off at Date 2, but the payoff of Project 1 is revealed only at Date 1. Specifically, Project 0 costs I_0 at Date 0 and pays off X_0 at Date 2. At Date 1 there are two states, $s \in \{L, H\}$, with $p := \mathbb{P}[s = H]$. In state s, Project 1 costs I_1^s at Date 1 and pays off X_1^s at Date 2. Everyone is risk neutral and there is no discounting or asymmetric information.

B can fund his projects by borrowing I_0 at Date 0 and I_1^s in state s at Date 1 from competitive credit markets: we assume that B makes a take-it-or-leave-it offer to borrow from creditor C_t at Date $t \in \{0, 1\}$.

[9] We chose a single-good set-up so that whether a good (or asset) serves as collateral depends only on the borrower's choice of debt instrument, not on the good itself. This allows us to focus on how collateral is used to establish priority, rather than to increase pledgeability, as in much of the finance literature (see, however, Subsection 3.5 and Subsection 3.6). In that literature, a pledgeable good (e.g., physical capital), is typically used as collateral to borrow a "divertable" good (e.g., cash).

Our perspective can cast light on situations in which whether a good is used as collateral seems not to depend on its intrinsic properties. For example, borrowers use securities as collateral to borrow cash in the repo market and use cash as collateral to borrow securities in the securities lending market—i.e. Good 1 is used as collateral to borrow Good 2 in some markets and Good 2 is used as collateral to borrow Good 1 in other markets. Borrowers are not using a "pledgeable" good as collateral to borrow a "divertable" good, rather, we suggest, they are using it as collateral to establish priority.

2.2 Pledgeability and Collateralizability

B must promise to repay his creditor(s) under two frictions. First, pledgeability is limited in that B may divert a fraction $(1-\theta)$ of projects' payoffs, leaving only the pledgeable fraction θ for his creditors. θ is the portion of a project's final value that creditors can seize. Second, contracts are non-exclusive in that if B borrows from C_0 at Date 0, he cannot commit not to borrow from C_1 at Date 1, potentially diluting C_0's initial claim.

The role of collateral in our model is to mitigate the effects of non-exclusive contracting: if a creditor's claim is collateralized (or "secured") by a fraction σ of a project with payoff X, that creditor has the exclusive right to the fraction σ of the project's pledgeable payoff, i.e. he has absolute priority over $\sigma\theta X$.[10] (In Subsection 3.5, we consider the possibility that not all of a project's pledgeable cash flow can be collateralized, i.e. there is an upper bound on σ.)

2.3 Borrowing Instruments

At Date t, B borrows the cost of Project t from C_t against the promise to repay the fixed amount F_t at Date 2. This promise can be secured, i.e. collateralized, or unsecured. If B collateralizes a fraction σ_0 of Project 0 to C_0, C_0 has priority over $\sigma_0\theta X_0$. This fraction of Project 0 cannot be collateralized again to C_1. However, anything that B has not collateralized to C_0 can be collateralized to C_1. Thus, B can give C_1 a senior claim on the fraction $(1-\sigma_0)$ of Project 0 and all of Project 1.

If there are multiple unsecured creditors, we assume that they are on equal footing in the event of B's default at Date 2, consistent with practice. We capture this by having unsecured creditors fifty-fifty Nash bargain at Date 2 over the residual value after the secured debt is paid.[11]

Our results are not sensitive to the fine details of the contracting environment. None of them depends on whether Date-2 repayments can be state-contingent and only the "collateral damage" results depend on the priority rule among unsecured creditors (Subsection 3.5).[12]

[10] A similar concept of securing assets away from third parties appears in Kiyotaki and Moore (2000, 2001).

[11] In practice, unsecured creditors are prioritized roughly according to the order in which they alert the court of a borrower's default at Date 2, i.e. the "first to file or perfect" is paid first (see, e.g., Picker (1999)). Our assumption of fifty-fifty bargaining is akin to assuming that that creditors are equally likely to win this race to alert the court. Our results also hold for general bargaining power. (For an influence-cost-based model that endogenizes creditors' bargaining positions given default, see Welch (1997).)

[12] That said, this priority rule among unsecured creditors is realistic and, moreover, contracting on seniority may be difficult or impossible, even in theory. The reason is that even if unsecured creditors attempt to contract on seniority, they may enter into contracts that contradict each other—e.g. each creditor has a contract that says it is senior. If contracts can be backdated, a court cannot prioritize them based on the order

Rather, our main results depend only on the assumptions that (i) secured debt is treated as senior, (ii) B cannot commit not to collateralize in the future, and (iii) B cannot make the fraction σ_0 of Project 0 he collateralizes depend on the Date-1 state. These assumptions reflect the real-world constraints that current law imposes on borrowers, as we explain in detail in Section 4.

2.4 Payoffs

We now give the players' terminal payoffs. First, define the indicator variable $\mathbb{1}_t$ as follows:

$$\mathbb{1}_t := \begin{cases} 1 & \text{if Project } t \text{ is undertaken,} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Thus, the total payoff W is given by

$$W = \mathbb{1}_0 X_0 + \mathbb{1}_1 X_1. \tag{2}$$

B's payoff is the sum of two terms: (i) the non-pledgeable part of the payoff and (ii) the residual of the pledgeable part of the payoff after repaying his debts F_t to C_t, i.e. B's payoff is $(1-\theta)W + \max\{\theta W - F_0 - F_1, 0\}$. If B does not default—i.e. $F_0 + F_1 \leq \theta W$—then creditor C_t gets F_t. If B does default—i.e. $F_0 + F_1 > \theta W$—then C_0 and C_1 divide θW according to priority.

2.5 Assumptions

We impose several restrictions on parameters, which serve to streamline the analysis by restricting attention to relevant cases.

Assumption 1. The pledgeable payoff of Project 0 in state L alone is worth more than I_0: $I_0 < (1-p)\theta X_0$.

This implies C_0 lends unless there is the risk of dilution in state L (see Lemma 2). This also implies that it is efficient to undertake Project 0, i.e. that $I_0 < X_0$.

Assumption 2. Project 1 has positive NPV in state H but negative NPV in state L: $I_1^H < X_1^H$ but $I_1^L > X_1^L$.

This implies it is efficient to undertake Project 1 in state H only.

in which they were written. Registries for secured debt exist to solve this problem. See Ayotte and Bolton (2011) for a model of priority based on the costs of checking for contradicting contracts.

Assumption 3. In both states, $s \in \{L, H\}$, the combined pledgeable cash flow from Project 0 and Project 1 is less than the investment cost: $\theta(X_0 + X_1^s) < I_0 + I_1^s$ for $s \in \{L, H\}$.

This implies that the limited pledgeability friction is severe enough that it may prevent B from investing even when it is efficient (i.e. in state H).

Assumption 4. The payoff of Project 1 is not too small: $(1-\theta)X_1^L > \theta X_0 - I_0$.

This more technical condition ensures that the payoff of Project 1 is always large enough that B has the incentive to undertake it (Lemma 1).[13]

Assumption 5. The cost of Project 1 is not too large: $I_1^H < \theta(X_0 + X_1^H)$.

This is another more technical condition. It ensures that the cost of Project 1 is not so large that B can never borrow from C_1 to finance it.

3 Results

3.1 Preliminaries

Before we get to our main results, we establish two preliminary lemmata that follow quickly from the assumptions.

Lemma 1. *B always borrows if it is feasible.*

Notably, this result implies that as long as C_1 is willing to lend at Date 1, B borrows to invest in Project 1, even in state L, when it has negative NPV. The reason is that borrowing from C_1 dilutes C_0's debt, subsidizing B's investment. Assumption 4 implies this subsidy is large enough that B always wants to borrow.

Lemma 2. *If B can commit not to borrow from C_1 in state L at Date 1, B can borrow from C_0 at Date 0.*

This result, follows immediately from Assumption 1 that B's pledgeable payoff in state L alone is worth more than I_0.

[13] Alternatively, one might assume that B gets private benefits from empire building and, therefore, always has the incentive to undertake Project 1, regardless of its NPV. In that case this assumption is unnecessary.

3.2 Paradox of Pledgeability

In the first-best outcome, all positive NPV projects are undertaken. Assumption 2 says that the first-best outcome is to undertake Project 0 at Date 0 and Project 1 at Date 1 in state H. We now show that if B borrows unsecured at Date 0, the first best is attained if pledgeability θ is sufficiently *low.* But, in contrast, if θ is high B cannot borrow unsecured at all—there is a paradox of pledgeability.

Proposition 1. (Paradox of Pledgeability) *Define*

$$\theta^* := \frac{I_1^L}{X_0 + X_1^L}. \tag{3}$$

If $\theta < \theta^*$, an equilibrium exists in which C_0 lends unsecured and the first best is attained, i.e. B borrows (secured) from C_1 in state H and does not borrow in state L.

If $\theta \geq \theta^*$, no equilibrium exists in which C_0 lends unsecured.

Low pledgeability prevents C_1 from lending to B in state L. Thus, it protects C_0 against dilution in state L, even though C_0's debt is unsecured. Since C_0 is repaid in state L, it accepts dilution in state H (Lemma 2). This makes it easier for B to borrow from C_1 in state H, allowing him to invest efficiently.[14]

An increase in pledgeability allows B to pledge more to C_1, making C_1 more willing to lend. Indeed, even though B will be unable to repay both creditors in full by Assumption 3, C_1 is still willing to lend via secured debt, since this new debt to C_1 is senior to B's existing debt to C_0. However, this dilution renders C_0 unwilling to lend in the first place—higher pledgeability makes it easier to borrow at Date 1 and, hence, paradoxically, makes it harder to borrow at Date 0. This follows from the non-exclusivity friction: when C_0 lends to B unsecured, it anticipates that B can borrow secured from C_1. Roughly, when pledgeability is high, B's ability to pledge collateral to C_1 allows him to supersede his non-exclusive relationship with C_0 with an exclusive relationship with C_1. When pledgeablity is low, this friction does not induce an inefficiency because B is too constrained to borrow from C_1 when $X_1 = X_1^L$—low pledgeability makes B's contract with C_0 effectively exclusive.

Note that our proof does *not* rely on the assumption that the repayment F_t is not contingent on the state s, since contingent contracts do not help B commit not to dilute C_0 in state L. We spell this out in the Appendix after the proof of the proposition.

Finally, note that, in general, very low pledgeability would prevent borrowing at Date 0. This inefficient outcome is ruled out by Assumption 1 (see, however, Subsection 3.6).

[14] Optimal "dilutable debt" also appears in Diamond (1993), Donaldson and Piacentino (2017), Hart and Moore (1995), and Stulz and Johnson (1985).

3.3 Collateral Rat Race

We now show that collateralization at Date 0 can protect against the inefficient dilution that occurs at Date 1 when θ is high (Proposition 1). I.e. collateralization today can protect against future collateralization. Indeed, an appropriately calibrated level of collateralization may give rise to the first-best outcome.

PROPOSITION 2. (RAT RACE) *Define*

$$I_1^* := I_1^L + \theta\big(X_1^H - X_1^L\big). \tag{4}$$

If $I_1^H < I_1^*$, an equilibrium exists in which B borrows (partially secured) from C_0 and the first best is attained, i.e. B borrows (secured) from C_1 in state H and does not in state L.

Given the assumptions of the proposition, the appropriate mix of secured and unsecured debt at Date 0 leads to efficient investment at Date 1, since B is too constrained to borrow from C_1 in state L but not in state H. In other words, if B borrows from C_0 via (only) partially secured debt, he can dilute the claim to a limited extent, not enough to borrow in state L but enough to borrow in state H.

This result says that although for high pledgeability B cannot invest efficiently if he borrows unsecured at Date 0 (Proposition 1), he can if he uses the appropriate amount of collateral. However, the only reason B needs to use collateral at Date 0 is that he cannot commit not to use it at Date 1—there is a collateral rat race in which creditors require collateral today to protect against dilution with collateral in the future.

Legal scholars have also observed that collateral is necessary to "protect lenders against dilution [with] secured debt" (Schwartz (1998), p. 1397) because it can serve to dilute existing creditors, given "[l]ate-arriving secured creditors can leapfrog earlier unsecured creditors, redistributing value to the benefit of the issuer and the secured creditor but to the detriment of unsecured creditors" (Listokin (2008), p. 1039).

This result suggests that borrowers may rely more on collateral when pledgeability is high than when it is low. As discussed in the Introduction, this contrasts with received theories, but still resonates with practice. Indeed, some of the worlds' most developed debt markets rely heavily on collateral. The analysis so far suggests that this race to collateralize is not always inefficient (the first best is attained). However, we now show that it can undermine borrowing and investment.

3.4 Collateral Overhang

Although collateralization protects C_0 from dilution, it can also prevent B from borrowing in the future, even to undertake efficient projects and even if B can renegotiate his debt with C_0. In other words, collateralization can encumber assets, leading to a collateral overhang problem.

Proposition 3. (Collateral overhang) *If $I_1^H > I_1^*$, in equilibrium, B borrows from C_0 at Date 0 secured by a fraction of Project 0 and B cannot borrow from C_1 at Date 1 in either state. Hence, there is inefficient underinvestment in state H.*

Given the assumptions of the proposition, in order to commit not to borrow in state L, B must also commit not to borrow in state H. This points to a downside of collateralization: whereas secured debt protects C_0 from dilution, it encumbers B's assets, preventing efficient investment in the future. Thus, the risk of future collateralization may lead to inefficient preemptive collateralization. Put differently, the ability to use collateral can create a friction when it allows a borrower to enter selectively into an exclusive contract. Further, ex interim renegotiation cannot resolve this inefficiency.

Corollary 1. *The equilibrium debt contract is renegotiation proof. I.e., B, C_0, and C_1 cannot renegotiate to undertake Project 1 in state H and thereby avoid the collateral overhang.*

Collateralization leads to inefficient asset encumbrance even if contracts can be renegotiated. This is because limited pledgeability implies that there is not enough pledgeable output to satisfy all creditors, no matter the division of surplus.[15]

This result also implies that the collateral overhang does not depend on our assumption that debt matures at Date 2, i.e. short-term debt does not help. We spell this out in the Appendix after the proof of the corollary.

Our results so far are closely in line with practitioners' intuition that "asset encumbrance not only poses risks to unsecured creditors"—collateralization dilutes unsecured creditors—"but also has wider...implications since encumbered assets are generally not available to obtain...liquidity"—collateralization leads to the collateral overhang (Deloitte Blogs (2014)). In other words, secured debt in place prevents a borrower from financing positive-NPV projects, like in Myers's (1977) "debt overhang." However, here the problem is not that a borrower chooses not to raise capital because the benefits of new investments go to existing

[15] This finding that the "collateral overhang" of secured credit cannot be resolved by renegotiation complements Bhattacharya and Faure-Grimaud's (2001) finding that when a firm's investments are non-contractible, renegotiation between borrowers and creditors may not resolve the debt-overhang problem.

debt holders, as in Myers (1977), but, rather, that existing debt is collateralized precisely to stop him from doing so. Further, the collateral-overhang problem arises even when the debt-overhang problem does not, i.e. when existing debt is riskless or can be renegotiated.

3.5 Collateral Damage

So far, we have assumed that all pledgeable assets can serve as collateral. As such, we study the role collateral plays in establishing priority under otherwise standard assumptions. In reality, however, some assets may be pledgeable—they can be seized in the future—but not collateralizable—they are hard to assign property rights to, e.g. they may not even exist at inception, but rather be built/acquired in the course of the project. Also, property rights on some existing assets, such as intellectual property, may be difficult to define legally. How do pledgeability and collateralizability interact?

To address this question, we suppose that B can collateralize at most a fraction μ_t of Project t at Date t, so B can divert $(1-\theta)X_t$ and collateralize $\theta\mu_t X_t$, but $\theta(1-\mu_t)X_t$ is neither divertable nor collateralizable. Since different projects may employ different types of assets, μ_t depends on the project.

We find that higher collateralizability can loosen borrowing constraints to help finance efficient investments. However, it can also increase the risk of dilution, and hence tighten borrowing constraints.

LEMMA 3. *Suppose B collateralizes a fraction σ_0 of Project 0 at Date 0. If*

$$\mu_1 \geq \frac{2I_1^s - \theta(1-\sigma_0)X_0}{\theta X_1^s} - 1, \tag{5}$$

B invests in Project 1 in state s (and C_0 is diluted).

This result implies that higher collateralizability can undermine efficiency by triggering a collateral rat race and leading to a collateral overhang.

PROPOSITION 4. (COLLATERAL DAMAGE) *Define*

$$\mu_1^* := \frac{2I_1^L - (1-\mu_0)\theta X_0}{\theta X_1^L} - 1 \tag{6}$$

and suppose p is not too large. If $\mu_1 \geq \mu_1^*$, B does not invest at Date 0 or Date 1.

This lower bound μ_1^* on μ_1 implies that improving collateralizability can have adverse effects. Further, observe that μ_1^* is increasing in μ_0. This says that the more collateralizable Project 1 is, the more collateralizable Project 0 needs to be in order to offer protection against dilution. This leads to the next corollary, which says that collateral demand at Date 0 may be increasing in collateral supply at Date 1.

COROLLARY 2. *Let σ_0^* denote C_0's demand for collateral, i.e. the smallest amount of collateral B can secure to C_0 so that C_1 prefers not to lend to B in state L:*

$$\sigma_0^* = \inf\left\{\sigma_0 \;\middle|\; \mu_1\theta X_1^L + \frac{1}{2}\Big((1-\sigma_0)\theta X_0 + (1-\mu_1)\theta X_1^L\Big) < I_1^L\right\}. \tag{7}$$

σ_0^* is increasing in μ_1.

Recently, governments have been "manufacturing quality collateral," because "there's still not enough of the quality stuff to go around...as quality collateral becomes impossible to find...[t]he crunch has further been heightened by the general trend towards collateralised lending and funding" (Kaminska (2011)). For example, several countries recently expanded the set of movable assets that can be used as collateral (see footnote 4). Moreover, in 2005, repo transactions backed by some assets became super senior in bankruptcy. In the context of our model, this corresponds to an increase in "collateralizability." Note that it did not necessarily affect pledgeability: repo borrowers could assign assets as collateral to specific repo creditors. Despite this effective increase in the supply of collateral, markets perceived a shortage of collateral, consistent with our idea that collateral supply creates collateral demand. As Caballero (2006) puts it, "The world has a shortage of financial assets. Asset supply is having a hard time keeping up with the global demand for...collateral" (p. 272); see also Di Maggio and Tahbaz-Salehi (2015).

3.6 The Two Roles of Collateral

So far, we have abstracted from how collateral mitigates enforcement problems between borrowers to focus on how it mitigates them among creditors. In reality, collateral gives creditors both (i) the "right to use" collateral—i.e. to seize the assets used as collateral—and (ii) the "right to exclude" others from collateral—i.e. to stop others from seizing the assets used as collateral (see, e.g., Hart (1995), Segal and Whinston (2012)). We now briefly discuss a version of our model in which collateral plays both roles. We show that the "right to use" dominates for low pledgeability, whereas the "right to exclude" dominates for high pledgeability.

Here, we assume that the proportion of a project that is pledgeable depends on whether debt is collatateralized or unsecured: it is $\theta_c := c\theta$ if B borrows collateralized and $\theta_u := u\theta$ if B borrows unsecured. We assume not only that collateralization establishes exclusivity, as in the baseline model, but also that collateralization increases pledgeability, i.e. $c > u$. We focus on the case in which B always has sufficient pledgeable cash flow to fund Project 0 via collateralized debt, i.e. $\theta_c X_0 > I_0$ and Assumption 5 holds with $\theta = \theta_c$, i.e. $I_1^H < \theta_c(X_0+X_1^H)$.

The next proposition says that the ability to borrow unsecured is hump-shaped in pledgeability, so increasing θ helps for small θ but hurts for high θ.

PROPOSITION 5. *Define*

$$\theta_u^* := \frac{I_0}{uX_0} \quad \text{and} \quad \theta_c^* := \frac{I_1^L}{c(X_0 + X_1^L)}. \tag{8}$$

If $\theta \in \big[\theta_u^*, \theta_c^*\big)$, an equilibrium exists in which C_0 lends unsecured and the first best is attained, i.e. B borrows (secured) from C_1 in state H and does not in state L.

If $\theta \notin \big[\theta_u^*, \theta_c^*\big)$, no equilibrium exists in which C_0 lends unsecured.

For low θ, B cannot borrow unsecured, but must use collateral to increase his pledgeable payoff—otherwise he could not borrow from C_0 to get Project 0 off the ground. For high θ, B also cannot borrow unsecured, but must use collateral to offer protection against the claims of other creditors—otherwise he could borrow from C_1 with collateral, diluting C_0's debt, as in the baseline model. To protect C_0 from this dilution, B must borrow secured at Date 0, which can lead to inefficient underinvestment at Date 1 due to the collateral overhang.

COROLLARY 3. *Define*

$$I_c^* := I_1^L + \theta_c\big(X_1^H - X_1^L\big). \tag{9}$$

If $I_1^H \geq I_c^*$, no equilibrium exists in which the first best is attained.

4 Discussion of Contracting Environment and Covenants

As we touched on in Subsection 2.3, the critical contracting assumptions are that (i) secured debt is treated as senior, (ii) borrowers cannot commit not to collateralize in the future, and (iii) borrowers cannot make collateralization contingent on future events. Here we argue that these assumptions reflect reality.

Secured debt is super senior. The seniority of secured debt is a basic feature of US bankruptcy law. Here, we take this as given. However, this is not an ad hoc policy of the courts. Rather, it reflects important constraints on the ability to establish priority. In general, unsecured debts cannot easily be prioritized temporally, since contracts can be backdated. In contrast, secured debts can be prioritized temporally via the physical transfer of collateral (or a deed to it) or via the public registration of a security interest in the collateral in a property registry.

Commitment not to collateralize/Negative pledge covenants. If they were perfectly enforced, covenants preventing future secured borrowing could protect creditors against dilution, limiting the need for collateral. Such covenants, called negative pledge covenants, exist in practice. However, their effectiveness is limited. This is because an unsecured creditor holds a claim against only the borrower, not against other creditors (cf. Ayotte and Bolton (2011) on contractual rights vs. property rights). Thus, an unsecured creditor cannot recover collateral that has been seized by a secured creditor. Bjerre (1999) describes these legal restrictions as follows:

> the negative pledge covenant [is a covenant] by which a borrower promises its lender that it will not grant security interests to other lenders. These covenants are common in unsecured loan agreements because they address one of the most fundamental concerns of the unsecured lender: that the borrower's assets will become unavailable to repay the loan, because the borrower will have both granted a security interest in those assets to a second lender and dissipated the proceeds of the second loan. Unfortunately, negative pledge covenants' prohibition of such conduct may be of little practical comfort, because as a general matter they are enforceable only against the borrower, and not against third parties who take security interests in violation of the covenant. Hence, when a borrower breaches a negative pledge covenant, the negative pledgee generally has only a cause of action against a party whose assets are, by hypothesis, already encumbered (pp. 306–307).

The effectiveness of negative pledge covenants in bankruptcy is especially limited for repo and derivatives liabilities, since these contracts are exempt from bankruptcy stays—i.e. creditors can liquidate collateral without the approval of the bankruptcy court, making it difficult or impossible for any third party to enforce a claim to the collateral.

Negative pledge covenants may still be useful outside bankruptcy. Since their violation constitutes a default, a borrower may adhere to the terms of covenants to avoid a default.[16] However, this may be insufficient to prevent a borrower from taking on debt in general. For example, a borrower in financial distress is likely to default anyway and is therefore willing to violate such covenants to gamble for resurrection by taking on new debt. More generally, it can be difficult to verify that a solvent firm has violated a covenant, especially for complex firms like banks, which may have thousands of counterparties. Indeed, banks effectively do

[16] Other theory papers have shown how such covenants can mitigate incentive problems in some contexts. E.g., Rajan and Winton (1995) show that they give creditors greater incentive to monitor and Gârleanu and Zwiebel (2009) show that they help to allocate decision rights efficiently given asymmetric information.

not have to disclose their short-term borrowing:

> There are no specific MD&A requirements to disclose intra-period short-term borrowing amounts, except for [some] bank holding companies [that must] disclose on an annual basis the average, maximum month-end and period-end amounts of short-term borrowings (Ernst & Young (2010)).

There is a another reason that banks in particular may not be able to promise not to dilute existing debt with new debt: the very business of banking constitutes maturity and size transformation, which requires frequent short-term borrowing from many creditors. If a bank agrees to covenants that restrict its ability to borrow in the future, it could undermine its ability to engage in these banking activities. As Bolton and Oehmke (2015) put it:

> debt covenants prohibiting the collateralization...are likely to be...costly to enforce...for financial institutions.... By the very nature of their business, financial institutions cannot assign...collateral to all depositors and creditors, because this would, in effect, erase their value added as financial intermediaries (p. 2356).

This emphasizes that non-exclusive contracting is an especially important friction for banks and, therefore, it may add credibility to our idea that non-exclusive contracting is an important reason that interbank markets are heavily reliant on collateral.

State-contingent collateralization. If it were possible, state-contingent collateralization could circumvent the inefficiencies in our model. At Date 0, B would commit to collateralize Project 0 to C_0 in state L but not in state H, thereby allowing B to take on new debt in state H, where it is efficient, but not in state L, where it is not. However, as in the baseline model, B would prefer to collateralize to C_1 in state L, reneging on his promise to collateralize to C_0. Thus, contingent collateralization effectively requires B to commit not to collateralize in the future, which we have argued may be impossible. Furthermore, even bilateral contingent contracting can be difficult in reality for a number of reasons established in the literature (see, e.g., Hart and Moore (1999), Segal (1999)).

Moreover, collateralization often requires a physical transfer of assets between the borrower and the creditor; in legal parlance, the secured debt is "possessory." In this case, state-contingent collateralization would require C_0 to be physically present at Date 1 to transfer possession, which could be costly or infeasible. This provides an additional rationale for our assumption that collateralization cannot be made state-contingent.

5 Conclusion

We have considered a model in which collateral serves to protect creditors against dilution with new debt. High pledgeability increases the risk of dilution, since it makes it easy to take on new secured debt and thus, paradoxically, makes creditors less willing to lend unsecured. Collateralization is required to protect against future collateralization—there is a collateral rat race.

This reliance on collateral leads to a collateral overhang problem, whereby collateralized assets are encumbered and cannot be used to raise liquidity. We find that decreasing the supply of collateral may mitigate this problem, by preventing the collateral rat race from getting started.

A Appendix

Proof of Lemma 1

To see that B always wants to borrow from C_0, observe that he gets zero if he does not. This is because if B does not invest in Project 0 he cannot invest in Project 1 either, since

$$\theta X_1^s < I_1^s \tag{10}$$

for $s \in \{L, H\}$, by the combination of Assumption 1 and Assumption 3.

To see that B always wants to borrow from C_1, suppose he has debt with face value F_0 to C_0. It must be that $F_0 \geq I_0$ by C_0's participation constraint. Thus, if B does not borrow from C_1, he gets at most $X_0 - I_0$. Now, if B borrows from C_1 in state s, he gets at least his default payoff of $(1-\theta)(X_0 + X_1^s)$. Thus, a sufficient condition for B to borrow is that

$$(1-\theta)\big(X_0 + X_1^s\big) \geq X_0 - I_0 \tag{11}$$

which reduces to $(1-\theta)X_1^s \geq \theta X_0 - I_0$, which is implied by Assumption 4. $\square$

Proof of Lemma 2

Suppose B does not borrow in state L, so his pledgeable cash flow is θX_0 in state L. Thus, the expected cash flow that B can pledge to C_0 is at least $\mathbb{P}[H] \times 0 + \mathbb{P}[L] \times \theta X_0 = (1-p)\theta X_0$. This is greater than I_0 by Assumption 1. Thus, B can pledge enough to C_0 to satisfy its participation constraint. $\square$

Proof of Proposition 1

We first show that if $\theta < \theta^*$, B dilutes C_0 if and only if $X_1 = X_1^H$. Suppose C_0 lends unsecured, so B can make C_1 senior by borrowing secured. Hence, C_1 lends if and only if

$$\theta\big(X_0 + X_1^s\big) \geq I_1^s \tag{12}$$

or

$$\theta \geq \frac{I_1^s}{X_0 + X_1^s}. \tag{13}$$

This is always satisfied for $s = H$ by Assumption 5, but it is not satisfied for $s = L$ if $\theta < \theta^*$. Thus, there is no dilution in state L. Hence, C_0 lends by Lemma 2.

Now, if $\theta \geq \theta^*$, inequality (13) is satisfied for $s = H$ by Assumption 5 and is also satisfied for $s = L$. Thus, B borrows from C_1 in both states $s \in \{L, H\}$ and there is always dilution. Thus, C_0 does not lend unsecured, since, by Assumption 3, if B undertakes both projects, there is never enough pledgeable cash flow to repay both creditors. $\square$

Contingent debt? Observe that the argument above for $\theta \geq \theta^*$ does not depend on our assumption that the repayments F_t do not depend on the state s. To see why contingent contracting cannot help, observe that in the first best C_0 must receive at least I_0 in state L, since it must get less than I_0 in state H. This is because the pledgeable payoffs are less than the project costs by Assumption 3 and C_1 must be guaranteed I_1^H in order to lend in state H (which is required in the first best). Thus, B must guarantee C_0 at least I_0 in state L. However, the argument above implies that, for high θ, B can never credibly promise I_0 to C_0 in state L, because he always dilutes C_0 with secured debt to C_1. $\square$

Proof of Proposition 2

We first show that if B collateralizes a fraction σ_0 to C_0, B dilutes C_0 if and only if $X_1 = X_1^H$. This implies that B promises C_0 more than the cost I_0.

Suppose C_0 collateralizes the fraction σ_0 of X_0 to C_0, so B can make C_1 senior on the fraction $(1 - \sigma_0)$ of Project 0 and all of Project 1. Hence, C_1 lends if and only if

$$\theta\big((1 - \sigma_0)X_0 + X_1^s\big)\big) \geq I_1^s. \tag{14}$$

The first best is attained if and only if this inequality is satisfied in state H and not in state L or

$$\frac{I_1^H - \theta X_1^H}{\theta X_0} \leq 1 - \sigma_0 < \frac{I_1^L - \theta X_1^L}{\theta X_0}. \tag{15}$$

Note that the left-most term is always less than one by Assumption 5 and the right-most term is always greater than zero by equation (10). Thus, there exists $\sigma_0 \in [0, 1]$ satisfying the condition as long as the left-most term is less than the right-most term, which holds as long as $I_1^H < I_1^*$ as in the hypothesis of the proposition. $\square$

Proof of Proposition 3

From the proof of Proposition 2 (equation (15)), we know that the first best is attained only if

$$\frac{I_1^H - \theta X_1^H}{\theta X_0} \leq 1 - \sigma_0 < \frac{I_1^L - \theta X_1^L}{\theta X_0}. \tag{16}$$

In order for there to exist σ_0 satisfying this condition it must be that the left-most term is less than the right-most term. But that reduces to the violation of the hypothesis of the proposition that $I_1^H > I_1^*$. Thus, there is no σ_0 that implements first best.

To borrow from C_0, B must commit not dilute in state L. But, by the argument above, B dilutes C_0 in state L whenever he does in state H for $I_1^H > I_1^*$. Thus, B must set σ_0 so high that he can never borrow from C_1. (Recall that B always prefers to borrow from C_0 than not to, by Lemma 1.) □

Proof of Corollary 1

Since the equilibrium is efficient for $I_1^H \le I_1^*$ (Proposition 2), we need to check for renegotiation proofness only when $I_1^H > I_1^*$, i.e. when the collateral overhang leads to inefficient underinvestment (Proposition 3). In this case, B must secure a large enough fraction σ_0 of Project 0 to C_0 that B cannot borrow from C_1 in state L, or

$$\theta\big((1-\sigma_0)X_0 + X_1^L\big) < I_1^L. \tag{17}$$

In order for renegotiation to be feasible, B, C_0, and C_1 must all be weakly better off. Thus, the combined payoff of C_0 and C_1 must weakly increase after renegotiation. Since they must invest I_1^H at Date 1, this implies the total repayment minus I_1^H must exceed the total payoff to C_0 absent renegotiation, or

$$\text{total repayment} - I_1^H \ge \sigma_0\theta X_0. \tag{18}$$

Now, the limited pledgeability friction implies that the total repayment is at most $\theta(X_0 + X_1^H)$. Thus, for renegotiation to be feasible it must be that

$$\theta(X_0 + X_1^H) - I_1^H \ge \sigma_0\theta X_0. \tag{19}$$

However, this cannot be satisfied together with equation (17) for $I_1^H > I_1^*$. Thus renegotiation is infeasible in this case. □

Short-term debt. In the baseline model we assume that B cannot borrow from C_0 via one-period debt and roll over. We now show that this is without loss of generality if we restrict attention renegotiation-proof debt.[17]

To consider short-term debt, we need to specify the sequence of moves at Date 1 and what happens if B defaults at Date 1. We assume that short-term debt matures after B has

[17]Note that if we do not require renegotiation-proofness, then short-term debt, combined with state-contingent repayments, can indeed help. It effectively plays the role of state-contingent collateralization (as discussed in Section 4): B could borrow from C_0 in exchange for repayments at Date 1 in state L and repayments at Date 2 in state H. If C_0 *commits* to liquidate when B defaults on its short-term debt, then B will not dilute in state L to avoid liquidation. (But he will still dilute efficiently in state H.)

had the opportunity to borrow from C_1 and invest in Project 1, without loss of generality.[18] And we assume that C_0 gets the right to liquidate B's projects, but that their liquidation value is zero. Alternatively, B and C_0 can renegotiate, rescheduling the debt.

PROPOSITION 6. *Renegotiation-proof short-term debt does not improve on the implementation of long-term contracts.*

Proof. The result follows immediately from the fact that B has no cash flows at Date 1, so C_0 has zero recovery value in the event of liquidation. As a result, C_0 always prefers to accept a rescheduling to Date 2 than to liquidate at Date 1 and hence B has incentive to dilute C_0's unsecured debt, even if it is short term.

(Note also that short-term *secured* debt leads to exactly the same collateral overhang as long-term secured debt: it prevents B from borrowing from C_1 in state H when dilution is efficient.) □

Proof of Lemma 3

Since B always borrows from C_1 if he can (Lemma 1), B borrows when he is unconstrained at Date 1, or whenever

$$\mu_1 \theta X_1^s + \frac{1}{2}\Big((1-\sigma_0)\theta X_0 + (1-\mu_1)\theta X_1^s\Big) \geq I_1^s. \tag{20}$$

To understand the expression on the left, recall that C_1 is senior on the collateralized portion of Project 1, which is at most $\mu_1 \theta X_1^s$, and that C_0 and C_1 are on equal footing for the uncollateralized portion of each project, i.e. C_0 and C_1 Nash bargain over $(1-\sigma_0)\theta X_0$ and $(1-\mu_1)\theta X_1^s$. The inequality reduces immediately to the condition in the proposition (equation (5)). □

Proof of Proposition 4

B borrows from C_1 in state L whenever

$$\mu_1 \theta X_1^L + \frac{1}{2}\Big((1-\mu_0)\theta X_0 + (1-\mu_1)\theta X_1^L\Big) \geq I_1^L. \tag{21}$$

Where the expression on the left is determined as in the proof of Lemma 3 with the maximum amount of collateralization of Project 0, i.e. $\sigma_0 = \mu_0$. The inequality reduces to $\mu_1 \geq \mu_1^*$, the condition in the proposition. Thus, C_0 gets repaid less than I_0 in state L by Assumption 3.

[18] This is without loss of generality because if, on the contrary, the debt matured before earlier, then B could not repay it since his projects do not payoff until Date 2.

This implies that C_0 does not lend if state L is sufficiently likely, i.e. if p is not too large, as assumed in the proposition. (B cannot borrow at Date 1 either by equation (10).) $\square$

Proof of Corollary 2

Immediately from the definition of σ_0^* in equation (7), we have that

$$\sigma_0^* = 1 - \frac{2I_1^L - \theta(1+\mu_1)X_1^L}{\theta X_0}, \tag{22}$$

which is increasing in μ_1. $\square$

Proof of Proposition 5

B can finance Project 0 only if his pledgeable cash flow exceeds I_0. B can borrow from C_0 via unsecured debt if (i) Project 0's unsecured pledgeable cash flows are sufficient to cover the investment and (ii) C_0 is not at risk of dilution by the new debt to C_1 in state L. Condition (i) says that

$$\theta_u X_0 \geq I_0 \tag{23}$$

and condition (ii) says that

$$\theta_c(X_0 + X_1^L) < I_1^L. \tag{24}$$

These conditions hold together if and only if $\theta \in [\,\theta_u^*\,,\,\theta_c^*\,)$, as required in the proposition. Thus, B borrows from C_0 and invests at Date 0 and does not borrow from C_1 and does not invest at Date 1 in state L. Assumption 5 implies that B does borrow from C_0 and does invest at Date 1 in state H. Thus, the first best is attained. $\square$

Proof of Corollary 3

The result follows from the same argument as Proposition 3. Specifically, set $\theta = \theta_c$ in equation (16) above. $\square$

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