

CREDIT MARKET COMPETITION, CORPORATE INVESTMENT AND INTERMEDIATION VARIETY*

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Abstract

How does competition among financiers affect the nature of borrowers' investments in the economy and how does this, in turn, affect which financial intermediaries arise as lenders in equilibrium? This paper develops a general equilibrium model to address this question. There are three main results. First, at low levels of credit market competition, firms invest excessively in (riskier) specialized projects, whereas at high levels of credit market competition, firms invest excessively in (safer) standardized projects. Efficient project choices arise in equilibrium for only intermediate levels of competition. Second, the emergence of relationship lending eliminates the inefficiency for low levels of competition, but not the inefficiency for high levels of competition. Third, this residual inefficiency encourages the emergence of specialized intermediaries that resemble private equity firms that arise only when credit market competition is sufficiently high, and which are shown to be highly levered.

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1 Introduction

This paper develops a theoretical model to address the question of how competition among potential lenders affects the *nature* of corporate investment, and how this effect leads endogenously to the emergence of relationship banking (see, for example, Boot and Thakor (2000) and Petersen and Rajan (1995) for research on relationship banking), and other forms of specialized lending (such as, for example, private equity). We conduct this analysis in the context of a general equilibrium model in which search fractions—akin to those in the labor market—and borrower-lender matching play an important role in determining equilibrium outcomes. The basic insight of the paper is that different degrees of credit market competition lead to different types of investment inefficiencies on the part of borrowers, and this generates incentives for the emergences of different types of financial intermediation to cope with these inefficiencies. The analysis sheds light on some existing evidence on relationship lending and private equity, and it produces new predictions.

The importance of the question we study is underscored by the fact that, in every country, banks play a dominant role in the allocation of credit, with significant consequences for corporate investment. Moreover, in large credit markets, like the US and the UK, the provision of financing by non-banks, such as private equity firms, is becoming increasingly important as credit markets become more competitive. It is widely recognized that credit market competition impinges on *how* lenders allocate credit and *how much* of it they allocate (see, for example, Cetorelli (2001), and Ratnovski (2013)). This makes credit market competition a central aspect of financial regulatory policy because of its potential implications for welfare. Not surprisingly, it is a topic that has received substantial research attention.

At first blush, one's economic intuition would say that higher credit market competition should increase welfare, since banks would pay higher deposit rates and charge lower loans rates, facilitating higher deposit inflow and more lending to consumers and firms, with a consequent increase in economic growth. While some theoretical papers have confirmed this insight in banking models, others have pointed out that this standard intuition is off the mark because it overlooks special features that distinguish banks from other firms and also the potentially subtle effects of credit market frictions.

Pagano (1993) develops a model in which lower interbank competition leads to lower equilibrium lending and lower economic growth, a conclusion verified by Guzman (2000) in a general equilibrium model of capital accumulation. However, arrayed against this conclusion are numerous other papers with different results. Shaffer (1997) shows that a more competitive banking system may end up funding a lower-quality borrower pool. Cao and Shi (2000) argue that higher competition would increase loan rates and reduce loan supply. Dell'Ariccia (2000) develops a

credit screening model to show that higher competition among banks may dilute their incentives to screen borrowers. Similarly, Manove, Padilla, and Pagano (1998) show that, compared to a competitive banking system, banks in a monopolistic system will screen borrowers more and accept less collateral. There are also numerous theoretical papers that have examined the effect of interbank competition on risk-taking. Some papers have formalized the intuition that, by diminishing the charter values of banks, increased interbank competition generates incentives for banks to take higher risk (e.g. Hellmann, Murdock, and Stiglitz (2000) and Repullo (2004)). However, some others have argued the opposite—by lowering the interest rates that banks charge their borrowers, increased interbank competition can induce *borrowers* to take less risk, thereby diminishing the default risk banks face (e.g. Boyd and De Nicoló (2005)). Martinez-Miera and Repullo (2010) extend this logic to show an inverse U-shaped relationship between bank competition and stability.

Our approach to the question of how competition affects the credit market is different in two significant respects. First, we are concerned with how competition impacts the *types* of projects firms invest in. Second, we also focus on how this impact is influenced by the *endogenous* emergence of alternatives to arm’s-length bank credit, such as relationship banking and private equity. This is important because, in an environment in which firms have the discretion to choose whether to invest in say well-established routine technologies or in specialized innovative technologies, it would be useful to know the impact of credit market competition on this choice, since this would determine the kinds of firms and technologies that emerge and thrive in the economy. In this sense, our paper is in the tradition of Petersen and Rajan (1995) who show that young firms receive more credit when the banking system is less competitive, and of Boot and Thakor (2000) who develop a model in which greater competition among banks causes banks to make *more* relationship loans, but each loan has lower added value for the borrower. What distinguishes our paper from these earlier analyses of relationship banking is that our paper models the endogenous coexistence of banks and other specialized intermediaries in a general equilibrium framework.

In our model, each firm can choose between a “standardized” (low-risk) project and a “specialized” (“differentiated”, higher-risk) project. Each creditor can become either a bank or a highly-levered non-bank. We will argue that these non-banks strongly resemble private equity firms and, therefore, we will refer to them with the label “PE”. A bank can choose between relationship lending and arm’s-length finance (or “transaction lending”, as in Boot and Thakor (2000)). In some states of nature the standardized project has higher social value and in others the specialized project has higher value. The creditor commits to the type of financing before the firm chooses its project, and the firm’s repayment obligation to the creditor is determined by bargaining over the division of the net surplus. In particular,

in the model, banks finance entrepreneurs using debt contracts and PEs finance entrepreneurs using equity. Thus, bargaining determines the face value of the debt that the firm owes the bank or the size of the equity stake the firm grants the PE. Competition among creditors affects both the creditor’s decision about whether to specialize in banking or private equity, and the bank’s decision about whether to make arm’s-length or relationship loans. Hence, credit market competition also affects the type of project the firm chooses.

Our first main result is that the general equilibrium involves the firm making the efficient project choice only for intermediate levels of credit market competition. When competition is sufficiently low, the specialized project is chosen excessively, relative to the first-best. When competition is sufficiently high, the standardized project is chosen excessively, relative to the first-best. Only for intermediate levels of competition does efficient project choice arise in equilibrium.

Our second main result is that this inefficiency creates an economic force for the emergence of a relationship lender who attenuates some of the inefficiencies in project choice. If a relationship bank extends credit, the inefficiency of excessive investment in the specialized project for low levels of competition disappears, but the inefficiency of excessive investment in the standardized project for high levels of competition remains.

The third main result is that this residual inefficiency encourages the emergence of highly-levered intermediaries. The key feature of these intermediaries is that they raise outside debt to finance the purchase of equity stakes in firms. Sufficiently high leverage makes them wish to fund only projects with sufficiently high upside, so high leverage emerges endogenously as a *commitment device* for these intermediaries to credibly commit to deny funding to entrepreneurs with standardized projects and to fund only differentiated projects. In other words, intermediary leverage serves to *discipline entrepreneurs*, preventing them from standardizing excessively.

In our final main result, we show that these highly levered intermediaries emerge as lenders only when credit market competition is sufficiently high. When they emerge, they induce an equilibrium separation in the credit market—they fund entrepreneurs with specialized projects and banks fund entrepreneurs with standardized projects. Further, we show that the proportion of finance provided by these intermediaries increases as credit competition increases. The rate of increase decreases, however, and they remain scarce even in the perfect competition limit. In equilibrium these intermediaries endogenously have the following properties: they are highly levered; they are scarce; they have high returns; and they invest in differentiated projects. These are all stylized facts about private equity firms (see, for example, Harris, Jenkinson, and Kaplan (2014)), Kaplan and Strömberg (2008) and Lerner, Sorensen, and Strömberg (2011); hence, we refer to these highly-levered intermediaries as PEs, although we are open to alternative interpretations of these

intermediaries.

To explain the core intuition of the baseline model and the first main result, we first develop a “toy version” of the model. This model strips away some of the richness of the actual model in order to get to the intuition directly. In particular, it does not explain the endogenous emergence of relationship banking and private equity, which are captured by the full model. After this, the actual model is presented in order to more fully explore the forces at work.

In contrast to theories in which monopolistic banking systems are the safest (e.g. Hellmann, Murdock, and Stiglitz (2000), Matutes and Vives (2000), and Repullo (2004)), in our analysis such banking systems are excessively risky since specialized projects are riskier than standardized projects. Our model predicts that the most competitive banking systems will involve the least risk, which sheds light on the country-level empirical evidence presented by Schaeck, Cihak, and Wolfe (2009) that more competitive banking systems are less prone to systemic crises. However, the caveat suggested by our theory is that this attainment of safety comes at the cost of over-investment in (safe) standardized projects. Our result is also consistent with the empirical finding that firms invest less in R&D-intensive projects when credit competition is high (e.g. Hombert and Matray (2013)).

Another interpretation of our main result is that bank loan portfolios will become most liquid when interbank competition is the highest. This is because a standardized loan is more easily transferable and hence more liquid than a specialized loan. This seems to accord well with casual observation, but we are not aware of existing empirical evidence on this implication.

In addition to the papers discussed above, our work is related to the vast literature on relationship banking (e.g. Berlin and Mester (1992), Boot and Thakor (1994, 2000), Inderst and Mueller (2004), Petersen and Rajan (1995), Rajan (1992), and Sharpe (1990)); see Boot (2000) for a review. It is also related to how search frictions in the credit market (e.g. Diamond (1990)) affect credit outcomes, as in Wasmer and Weil (2004)).

Our paper is also related to the literature on private equity. Campello and Rafael (2010), Chan (1983), Chan, Siegel, and Thakor (1990), Ueda (2004), and Winton and Yerramilli (2008) all provide theories of venture capital and its competitive role in credit allocation vis-à-vis banks. In a security design framework, Axelson, Strömberg, and Weisbach (2009) demonstrate the importance of leverage for private equity firms. Empirical evidence that private equity investments have outperformed public markets appears in Harris, Jenkinson, and Kaplan (2014). Metrick and Yasuda (2010) document that the buyout business in private equity is more scalable than the venture capital business. Metrick and Yasuda (2011) provide a review.

The rest of the paper is organized in four remaining sections. Section 2 introduces the “toy model.” Section 3 develops the actual model. Section 4 contains the

analysis. Section 5 develops the extension to analyze private equity. Section 6 concludes. All formal proofs are in the Appendix.

2 Toy Model

In this section we introduce a simplified version of the model that pins down the intuition of the main results. At the core of the model is the project choice of an entrepreneur who needs outside capital to fund his investment. The entrepreneur chooses between two projects, called *standardized* and *differentiated* (or “specialized”). Both projects cost I to implement. The standardized project is positive NPV with a deterministic cash flow. This project generates $V_s > I$ for sure. The differentiated project, in contrast, is both information-sensitive and risky.¹ To capture information sensitivity, we allow the differentiated project to be one of two types, which we refer to as “high” and “low”. Both the high-type and the low-type differentiated projects have binary risky cash flows. They pay off V_d when they succeed and zero otherwise. The difference between the types is the success probability. The high-type project yields V_d with probability p_h whereas the low-type project yields V_d with probability $p_\ell < p_h$. We assume that the high-type differentiated project has the highest NPV but the low-type differentiated project has the lowest NPV, namely

$$p_\ell V_d < V_s < p_h V_d.$$

This assumption implies that *efficient investment requires adapting to circumstances*, in the sense that an entrepreneur with a high-type differentiated project should undertake it, while an entrepreneur with a low-type differentiated project should undertake a standardized project. All agents are risk neutral and the riskless rate is zero.

The entrepreneur borrows from the creditor via a debt contract with face value F . Credit competition affects project choice via its affect on the terms of debt, i.e. on this face value F . For simplicity, we assume here that an entrepreneur and a creditor divide the net surplus of their relationship fifty-fifty, where the net surplus is the total value created by the funded project minus the values of the outside options of the two parties. Thus, the outside options will be essential in determining the terms of debt F . We make the following three assumptions about the players’ outside options, which we microfound in the full model.

TOY MODEL ASSUMPTION 1. *The creditor’s outside option is the value I of his capital.*

¹It is not crucial to our analysis that the standardized project is safe, what is important is that it is not informationally sensitive, and that it is less risky than the differentiated project.

The motivation for this assumption is that if the creditor does not lend, the amount I can be “stored” at a zero riskless rate.

TOY MODEL ASSUMPTION 2. *If the entrepreneur chooses the differentiated project, his outside option is zero, written $\pi_e^d = 0$.*

The motivation for this assumption is that the differentiated project is information-sensitive, and adverse selection in the credit market will make it impossible for the entrepreneur to find funding for his project elsewhere. Thus, if he chooses the differentiated project, he is captive to his creditor.

TOY MODEL ASSUMPTION 3. *If an entrepreneur chooses the standardized project, his outside option, labelled π_e^s , increases with the competitiveness of the credit market.*

The motivation for this assumption is as follows. Since the project is information-insensitive, adverse selection will not impede the funding of the entrepreneur’s project by the creditor. When the credit market is competitive, it will be easy to find another creditor to fund the project and the entrepreneur’s outside option will be high. When the credit market is not competitive, it will be difficult to find another creditor, and the entrepreneur’s outside option will be low. Here we take π_e^s to represent credit competition directly.²

The stage is now set to present the first main result: for only intermediate values of credit competition will the entrepreneur choose the efficient project. When credit competition is very low, the entrepreneur over-differentiates, choosing the differentiated project even when it is the low-type. When competition is very high, in contrast, the entrepreneur over-standardizes, choosing the standardized project even when a high-type differentiated project is available.

Before deriving the result explicitly in the toy model, we outline the intuition for each of these inefficiencies. First consider the entrepreneur with the low-type differentiated project. He wishes to standardize to increase the net surplus, but knows that by differentiating he will have an informational advantage, and may receive a cross-subsidy from the high-type. When competition is high, the entrepreneur’s strong bargaining position from standardization gives him the incentive not to choose the low-NPV differentiated project. For low levels of competition, however, the standardization incentive disappears, so the entrepreneur prefers to differentiate inefficiently. Now turn to the entrepreneur with the high-type differentiated project. He wishes to differentiate to increase the surplus, but he knows that by doing so he will lower his outside option, weaken his bargaining position and increase the cost of credit. For high levels of credit competition, the entrepreneur values his lower

²In the full model, we model credit market competition explicitly within a search framework and demonstrate this connection with π_e^s explicitly.

cost of borrowing from standardizing so much that he never differentiates. To summarize, credit competition mitigates the over-differentiation problem by increasing the value of the standardized project to the entrepreneur. However, for high levels of competition this same effect leads the entrepreneur not to differentiate enough.

We now derive the result in the toy model. Suppose an efficient equilibrium exists. We will show how too much or too little credit competition leads the entrepreneur to deviate from his strategy in this conjectured equilibrium. Recall that π_e^s proxies for competition. The first step is to calculate the face values of debt F_s and F_d that the entrepreneur must promise to repay in order to fund the standardized and differentiated projects respectively. Recall that the entrepreneur and the creditor divide the net surplus fifty-fifty. The net surplus from standardization is $V_s - I - \pi_e^s$, so F_s returns half the net surplus to the creditor on top of his initial investment I :

$$F_s = I + \frac{V_s - I - \pi_e^s}{2}.$$

The efficient outcome under consideration is separating, so only high-type entrepreneurs choose the differentiated project. Thus, the net surplus given the differentiated project is $p_h V_d - I - \pi_e^d = p_h V_d - I$ since $\pi_e^d = 0$ by assumption. Now the creditor must receive his investment I plus half the net surplus, so

$$p_h F_d = I + \frac{p_h V_d - I}{2}.$$

When is it incentive compatible for the entrepreneur with the high-type project to differentiate? He chooses the differentiated project as long as

$$p_h(V_d - F_d) \geq V_s - F_s$$

which can be rewritten as

$$p_h V_d \geq V_s + \pi_e^s$$

or, writing $\Delta V := p_h V_d - V_s$,

$$\pi_e^s \leq \Delta V$$

which says exactly that entrepreneurs with high-type projects differentiate only if credit competition does not exceed the surplus gains from differentiation. Thus, whenever competition is very high, entrepreneurs standardize inefficiently.

Next, when is it incentive compatible for the entrepreneur with the low-type project to standardize? He chooses the standardized project as long as

$$V_s - F_s \geq p_\ell(V_d - F_d)$$

which can be rewritten as

$$V_s - I + \pi_e^s \geq \frac{p_\ell}{p_h}(p_h V_d - I).$$

Here the right-hand side is the payoff of an entrepreneur with a low-type project from borrowing at the terms of the entrepreneur with a high-type project. When competition π_e^s is high, the inequality is satisfied because the gained bargaining position from standardization curbs the entrepreneur's incentive to over-differentiate. When competition is low, however, this mechanism is not in place; the inequality is violated and risk-shifting occurs.

Thus, we see two sides of credit market competition. Because higher competition encourages entrepreneurs with both high-type and low-type differentiated projects to standardize, increasing competition simultaneously curbs the incentive for entrepreneurs to over-differentiate when they have low-type projects and exacerbates the incentive for them to over-standardize when they have high-type projects. In particular, the equations above imply immediately that entrepreneurs with both types of projects invest efficiently for only intermediate levels of credit competition, i.e. only if

$$\frac{p_\ell}{p_h}(p_h V_d - I) - (V_s - I) \leq \pi_e^s \leq \Delta V. \quad (1)$$

Both these inefficiencies stem from the entrepreneur making inefficient project choices in order to benefit from better funding terms. In deriving this result we have assumed that the creditor cannot observe the type of the differentiated project. Can a creditor with the expertise to assess the quality of the entrepreneur's project mitigate the inefficiencies that result from too much or too little credit competition? To address this question we introduce a *relationship lender* who can observe the type of the entrepreneur's differentiated project. Our next main result is that the entrepreneur with the low-type differentiated project no longer over-differentiates when competition is low if he is financed by a relationship lender. The reason is that over-differentiation is driven by the entrepreneur's incentive to access cheap funding by choosing the differentiated project even though it is low-type. However, this funding advantage disappears when a creditor can observe the project type.

The argument above shows that relationship lending can mitigate the over-differentiation problem and thus increase efficiency when there is too little credit competition. However, the entrepreneur with the high-type differentiated project continues to over-standardize when competition is high, even when financed by a relationship lender. The reason is that the creditor's ability to observe the type of the entrepreneur's differentiated project does not diminish the incentive of the entrepreneur with the high-type differentiated project to choose standardized projects in order to avoid the relatively poor credit terms he obtains when he differentiates.

To summarize, relationship banking is valuable in credit markets with relatively low competition but fails to reduce the inefficient standardization that high competition causes.

For high competition the inefficiency of over-investment in standardized projects persists even with the introduction of relationship lending. Can other types of specialized finance mitigate this inefficiency? To address this question we allow the creditor to specialize in what we call private equity (PE). Unlike a bank, a PE borrows to fund an entrepreneur in exchange of an equity stake in the firm. Leverage is the key feature of a private equity firm, which induces it to fund only projects with sufficiently high upside. This leverage therefore acts as commitment device for the PE, who, unlike the bank, can commit not to fund standardized projects. In so doing, PEs effectively force entrepreneurs with high-type projects to differentiate, because these entrepreneurs know that if they standardize they will not obtain funding. For high levels of credit competition, banks make little profit because entrepreneurs standardize, thus a creditor may find it profitable to specialize in private equity—a PE can effectively force the entrepreneur with the high-type project to differentiate.

While the toy model brings out the mechanism behind the main results, it relies on a number of strong assumptions. To keep the analysis simple: (1) the creditor and entrepreneur split the surplus fifty-fifty, (2) the outside option of the entrepreneur with the differentiated project is zero, (3) the outside option of the entrepreneur with the standardized project proxies for credit market competition, (4) relationship lending is costless, and (5) a single creditor chooses whether to be a bank or a PE without taking into account the availability the types of credit to entrepreneurs. The full model is substantially richer in the sense that we micro-found or relax each of these assumptions using a dynamic search-and-matching framework. The complete model not only confirms the robustness of the intuition above, but it also has the benefit of providing additional results. For example, two results that are not encountered in the toy model but that emerge from our analysis of the full model are (1) that for very low levels of competition a creditor may be unwilling to make a positive NPV investment in a relationship lending technology due to a hold-up problem; and (2) for very high levels of credit market competition, some creditors will choose to specialize in private equity lending, but in equilibrium there will be a mix of banks and PE, in which banks fund standardized projects and PEs fund differentiated projects. Further, the proportion of creditors who specialize in PE is increasing in the competitiveness of the credit market, but banks remain present even in the perfect competition limit.

3 Model

3.1 Agents and Projects

There are two kinds of players, creditors and entrepreneurs. All players are risk-neutral and discount the future at net rate r equal to the return on the money market account. A creditor c provides start-up capital I to a penniless entrepreneur e to fund a project δ . Entrepreneur e has a choice between two projects, called *standardized*, $\delta = s$, and *differentiated*, $\delta = d$. A standardized project is information-insensitive and riskless. It pays off V_s for sure. In contrast, a differentiated project is information-sensitive—because its cash flow distribution is e 's private information—and risky—because its cash flow is random. Specifically, a differentiated project is one of two types, $\tilde{\tau} \in \{h, \ell\}$, which e will observe before choosing $\delta \in \{d, s\}$. The h -type differentiated project pays off V_d with probability p_h and zero otherwise, whereas the ℓ -type differentiated project pays off V_d with probability $p_\ell < p_h$ and zero otherwise. The probability that the project is type h is α . All random variables are independent.

Before granting a loan, a creditor chooses the type of credit η to provide. He either offers *relationship lending*, $\eta = r$, or *arms-length finance*, $\eta = a$. The difference between a relationship lender and an arm's-length lender is that the relationship lender can observe the differentiated entrepreneur's type, whereas an arm's length lender cannot.³ The creditor can always offer arm's-length finance at no cost, but to perform relationship lending he must pay a cost k . Finally, the cost k is entrepreneur-specific, in other words, it allows him to learn the type of only one entrepreneur's project.

We now impose restrictions on parameters to capture what we think are two key features of project choice and relationship banking. First, we note that efficient investment requires adapting to circumstances—what may be a good investment in one set of circumstances may be a poor choice in a different set of circumstances. To capture this in the model we impose conditions so that if an entrepreneur has an h -type differentiated project the efficient project is the differentiated project, whereas if an entrepreneur has an ℓ -type differentiated project the efficient project is the ℓ -type project. Second, we observe that one aspect of relationship lending is that it can increase the NPV of entrepreneurs' projects, i.e. banks can enhance the economic surplus related to their borrowers' projects through relationship lending. The notion that relationship lending adds value to the borrower is a consistent theme in the literature (see, for example, Boot (2000)). However, many different approaches have been used to model the way in which this value enhancement

³The idea that a relationship lender can obtain proprietary information about the borrower is well-established in the literature. See, for example, Rajan (1992), Sharpe (1990), Boot (2000).

occurs. For example, in Boot, Greenbaum, and Thakor (1993), it is the contractual flexibility of relationship banking that adds value. In Boot and Thakor (1994), the value enhancement comes from a repeated borrowing relationship that results in a reduction in the collateral the borrow must post after establishing a good credit record. In Petersen and Rajan (1995), relationship lending permits the design of intertemporal taxes and subsidies in loan contracts that generate higher surplus than possible with single-shot contracting.⁴ The approach we use is closest to that in Degryse and Van Cayseele (2000) who propose that the offering of multiple services by a bank to the the *same* borrower—such as letters of credit, deposits, check clearing and cash management—in addition to the loan expands the information about the borrower available to the bank and hence permits the bank to add more value to the relationship. Specifically, we capture this in reduced form by assuming that the proprietary information generated by the relationship permits the bank to help the entrepreneur implement a more efficient project choice, and that the efficiency gain attributable to this exceeds the cost of relationship lending. We now formalize these restrictions in the assumptions below.

ASSUMPTION 1. *An h -type differentiated project has the highest present value, but the standardized project has a higher present value than the ℓ -type differentiated project, which has negative NPV, or*

$$p_h V_d - I > V_s - I > 0 > p_\ell V_d - I. \quad (2)$$

ASSUMPTION 2. *A standardized project a has higher present value than the average differentiated project, or*

$$V_s > \alpha p_h V_d + (1 - \alpha) p_\ell V_d. \quad (3)$$

Finally, we also assume that the efficiency gains from the efficient project choice exceed the cost of relationship lending, i.e.

$$\alpha p_h V_d + (1 - \alpha) V_s - k > V_s, \quad (4)$$

which we state more succinctly in the next assumption with the notation $\Delta V := p_h V_d - V_s$.

ASSUMPTION 3.

$$k < \alpha \Delta V. \quad (5)$$

The motivation for this assumption is that a relationship lender's ability to observe the project type gives him the flexibility to adapt to the different circumstances

⁴This is not an exhaustive list of the ways in which relationship lending can add value. See Boot (2000) for more.

represented by the type of an entrepreneur's differentiated project. The benefit of creditors' flexibility is the ability to ensure that entrepreneurs invest in the most efficient project in every circumstance.⁵ This is a benefit that creditors who extend arm's length finance do not enjoy. The assumption above says that the benefit of flexibility outweighs the cost k of the relationship.

3.1.1 Search and Matching

Creditors and entrepreneurs find each other by searching in a decentralized market.

At time $t \in \{\dots, -1, 0, 1, \dots\}$ a set E_t of searching entrepreneurs matches with a set C_t of creditors with intensity $m(|E_t|, |C_t|)$. Define $\theta_t := |C_t|/|E_t|$, the credit market competition. Assume the probability that a creditor finds an entrepreneur at time t ,

$$q(\theta_t) := \frac{m(|E_t|, |C_t|)}{|C_t|}, \quad (6)$$

and the probability an entrepreneur finds a creditor at time t ,

$$Q(\theta_t) := \frac{m(|E_t|, |C_t|)}{|E_t|}, \quad (7)$$

depend only on θ_t (for which, for example, m being homogenous of degree one suffices).

Assume m is such that q and Q are differentiable with $q' < 0$, $Q' > 0$, $q(0) = 1$, $Q(0) = 0$, and with $q(\theta) \rightarrow 0$ and $Q(\theta) \rightarrow 1$ as $\theta \rightarrow \infty$. As credit competition increases the likelihood that a creditor finds an entrepreneur decreases and that an entrepreneur finds a creditor increases.

To make the model stationary, assume that each player that leaves the market is replaced by a player of the same type.

3.1.2 Stage Game Extensive Form

When an entrepreneur e is born, he learns his type. When $e \in E_t$ matches with a creditor $c \in C_t$ for the first time, they play the extensive form game defined by the timing below:

1. c chooses between relationship lending and arm's-length finance, $\eta \in \{r, a\}$. r costs k and enables the creditor to observe the type of the differentiated project.
2. If c has played $\eta = r$, he observes the type $\tilde{\tau} \in \{h, \ell\}$ of the differentiated project.

⁵Thus, in a sense our specification combines the approaches in Boot, Greenbaum, and Thakor (1993) and Degryse and Van Cayseele (2000).

3. e chooses between a differentiated project and a standardized project, $\delta \in \{d, s\}$. The choice is irreversible.
4. The face value of debt F is determined as follows
 - With probability β , c makes e a take-it-or-leave-it offer F_δ^c ; e accepts or rejects.
 - With probability $1 - \beta$, e makes c a take-it-or-leave-it offer F_δ^e ; c accepts or rejects.
5. The project pays off $V_\delta \in \{V_s, V_d\}$. e and c divide the surplus according to the agreed upon contract.

If the relationship between c and e breaks down, they search again in the market. Since e 's choice δ is irreversible, it remains to specify the game played between an entrepreneur committed to a project δ and his new potential creditor c' .

1. c' observes δ .
2. c' chooses between relationship lending and arm's-length finance, $\eta \in \{r, a\}$. r costs k and enables the creditor to observe the type of the differentiated project.
3. The face value of debt F is determined as follows
 - With probability β , c makes e a take-it-or-leave-it offer $F_\delta^{c'}$; e accepts or rejects.
 - With probability $1 - \beta$, e makes c' a take-it-or-leave-it offer F_δ^e ; c' accepts or rejects.

If c and e are searching but not matched at date t they search again.

Throughout the solution concept we use is Perfect Bayesian Equilibrium.

4 Results

This section establishes the main results (1) that the first-best is attainable for only intermediate levels of credit market competitiveness and (2) that relationship lending can mitigate the inefficiency but only for low levels of credit competition.

4.1 Background Mechanism

We establish the results first in partial equilibrium, taking the players' continuation values as given. This analysis resembles that of the toy model in Section 2. We then endogenize the continuation values and prove the results in terms of the ratio θ of creditors to entrepreneurs, which we call the credit market competition.

4.1.1 Out of Equilibrium Beliefs

Since this is a dynamic game of asymmetric information, off-the-equilibrium path beliefs about the type τ of an entrepreneur's project play a role in the analysis. Throughout, we focus on equilibria supported by the following beliefs: if a creditor matches with an entrepreneur who has already differentiated his project (necessarily because he failed to obtain funding from the creditor he was initially matched with), then the creditor believes the project is type ℓ . We summarize this with the following assumption.

ASSUMPTION 4. *If a creditor encounters an entrepreneur with an already differentiated project, he believes the project is type ℓ , written as $\mu(\tilde{\tau} = \ell | \delta = d) = 1$.*

We emphasize that this is not an assumption on primitives, but just a statement about which Perfect Bayesian Equilibria we focus on. With this restriction on the set of equilibria, we aim to capture the idea that creditors believe that if an entrepreneur has failed to find funding in the past, it is because previous creditors had private information about the quality of his project. In Appendix A.8 we use a game-theoretic refinement to show that this economic intuition is robust: if we impose a stronger equilibrium concept than Perfect Bayesian Equilibrium (which is akin to Sequential Equilibrium but still well-defined for games with infinite action spaces), then any efficient equilibrium *must* be supported by the beliefs in Assumption 4.

4.1.2 Continuation Values

When c and e are matched, their decisions about whether to engage in relationship lending and whether to undertake a differentiated project depend on the proportion of the total surplus from the match that they anticipate earning. For each player, a higher outside option leads to a greater share of the net surplus.

At the time at which the face value of debt is determined, these outside options are equal to the players' continuation values from searching again in the market. Denote the continuation value of the creditor by π_c and the continuation value of the entrepreneur with project δ by π_e^δ . We emphasize that because his project choice is irreversible, the continuation value of the entrepreneur depends on his project choice. Note that, in general, these values could depend on time, but we suppress this possibility since we will focus on stationary equilibria. Further, the continuation value of the entrepreneur with the differentiated project could also depend on the type τ of his project. However, since the entrepreneur with the differentiated project will have his credit completely rationed in the future, this will also not be the case, as the following lemma implies.

LEMMA 1. *The continuation value of the already differentiated entrepreneur is zero, i.e. $\pi_e^d = 0$.*

4.2 *First-Best*

The efficient outcome of this model involves the entrepreneur choosing the efficient project, namely $\delta = d$ when $\tau = h$ and $\delta = s$ when $\tau = \ell$ and the creditor playing $\eta = a$, avoiding the cost k of relationship lending. Our question is when is this outcome ($\eta = a, \delta_h = d, \delta_\ell = s$) implementable? Specifically, for which values of credit competition θ can it emerge in equilibrium? We now proceed to find conditions for it to be a stationary Perfect Bayesian Equilibrium of the model given the out-of-equilibrium beliefs $\mu(\ell | d) = 1$.

For this to be an equilibrium, e must self-select the efficient project without the discipline of his creditor— c cannot observe the type of the differentiated project since $\eta = a$. The main results of this section come from finding conditions for e 's incentive constraints to be satisfied.

Before finding these conditions, we emphasize the equilibrium beliefs for clarity. Since beliefs must be consistent in equilibrium, creditors who observe entrepreneurs choose d must believe they have h -type projects.

4.2.1 *Face Values*

The face value F of debt depends on which player is proposing the contract in round 4 of the stage game (Subsection 3.1.2).

The proposer always offers the face value that makes his opponent indifferent between accepting and rejecting. Thus there are four cases: (1) when e proposes and he has a standardized project, (2) when c proposes and e has a standardized project, (3) when e proposes and he has a differentiated project, and (4) when c proposes and e has a differentiated project.

We now compute each of these face values under the equilibrium beliefs. Note that when e chooses s , there is no asymmetric information, so the face value just serves as a means to divide surplus—it will not enter substantively in the analysis. When the project is differentiated, the face value of the debt contract will matter. Here the subscripts on the face values denote the project choice and the superscripts denote the proposer. When e proposes the face value with a standardized project is

$$F_s^e = \pi_c \quad (8)$$

and when c proposes and e has chosen a standardized project, the face value is given by

$$V_s - F_s^c = \pi_e^s. \quad (9)$$

When e proposes with a differentiated project c gets repaid with probability p_h so the face value is given by

$$p_h F_d^e = \pi_c \quad (10)$$

and when c proposes and e has chosen a differentiated project, the face value is given by

$$p_h(V_d - F_d^c) = \pi_e^d. \quad (11)$$

4.2.2 Incentive Constraints

If c believes that everyone is playing according to the profile $(\eta = a, \delta_h = d, \delta_\ell = s)$, he never has incentive to deviate to $\eta = r$ since it comes with a cost k and no informational benefit. Thus, to determine when this outcome is an equilibrium, we can focus entirely on e 's incentive constraints. We find conditions first for $\delta_\ell = s$ and then for $\delta_h = d$.

In order for $\delta_\ell = s$, the entrepreneur must prefer to standardize when his project is type ℓ . Recall that in round 4 of the stage game the creditor proposes with probability β . Thus with probability β e receives his outside option and with probability $1 - \beta$ e pushes c to his outside option. Hence for e to choose to standardize with the ℓ -type differentiated project, it must be that

$$\beta\pi_e^s + (1 - \beta)(V_s - F_s^e) \geq \beta\pi_e^d + (1 - \beta)p_\ell(V_d - F_d^e) \quad (12)$$

which, upon simplification, yields:

$$\beta\pi_e^s + (1 - \beta)(V_s - \pi_c) \geq (1 - \beta)\frac{p_\ell}{p_h}(p_h V_d - \pi_c). \quad (\text{IC}_\ell)$$

For e to choose to differentiate with the h -type differentiated project it must be that

$$\beta\pi_e^d + (1 - \beta)p_h(V_d - F_d^e) \geq \beta\pi_e^s + (1 - \beta)(V_s - F_s^e) \quad (13)$$

which, upon simplification, yields:

$$(1 - \beta)p_h V_d \geq \beta\pi_e^s + (1 - \beta)V_s. \quad (\text{IC}_h)$$

Combining the incentive constraints (IC_ℓ) and (IC_h) implies that the first best can be attained in equilibrium if and only if

$$\left(1 - \frac{p_\ell}{p_h}\right)\pi_c - V_s + p_\ell V_d \leq \frac{\beta}{1 - \beta}\pi_e^s \leq \Delta V, \quad (14)$$

recalling that $\Delta V := p_h V_d - V_s$. These inequalities are the analogue of the inequalities (1) in the toy model described in Section 2 above. Next, we find the values of π_c and π_e^s in terms of credit competition θ .

4.2.3 Value Functions

To show that real investment is choked off when credit competition is at the extremes (too low or too high), we compute the continuation values given that players believe $(\eta = a, \delta_h = d, \delta_\ell = s)$ is the stationary action profile of the stage game.

We look for stationary equilibria. The players' value functions from continuing to search determine the offers their opponents make in round 4 of the stage game. These offers determine the division of surplus. The creditor has a single continuation value π_c , whereas the entrepreneur's continuation value depends on his project choice, $\delta \in \{s, d\}$. Lemma 1 fixes $\pi_e^d = 0$. It remains to compute π_e^s and π_c . Note that π_c and π_e^s will be interdependent: a higher π_c lowers π_e^s because e anticipates having to give a larger share of the net surplus to c and vice versa.

On the $(\eta = a, \delta_h = d, \delta_\ell = s)$ equilibrium path, creditors always fund entrepreneurs the first time that they are matched. But it must be incentive compatible for the creditor to agree to fund a standardized entrepreneur rather than search again to wait for a differentiated entrepreneur. As long as players are sufficiently impatient this will always be the case. Specifically, to ensure that standardized projects are funded, we make the following assumption on parameters.

ASSUMPTION 5.

$$r > \frac{\alpha\beta\Delta V}{V_s - I}.$$

Note that we view the time between dates as relatively long since it is the time taken to develop a lending relationship and implement a project. Thus we do not consider the assumption that r is large to be overly restrictive.

We now proceed to compute the value functions maintaining the assumption that standardized entrepreneurs will find funding when they are matched. After computing the equilibrium value functions, observe that Assumption 5 suffices for this to be the case.

The players' value functions are their expected utilities today from continuing the game. Consider first the standardized entrepreneur. He will be matched tomorrow with probability $Q = Q(\theta)$ and will remain unmatched with probability $1 - Q$. If he is unmatched, he searches again. Since the equilibrium is stationary, he obtains π_e^s in this case. If he is matched, the creditor is the proposer with probability $1 - \beta$. In this case again the entrepreneur obtains π_e^s . With probability β , however, the entrepreneur is the proposer and in this case he obtains $(1 - \beta)(V_s - \pi_c)$ (see Section 4.2.2 above). This description summarizes all the possibilities and allows us to write a formula for π_e^s :

$$\pi_e^s = \frac{Q(\beta\pi_e^s + (1 - \beta)(V_s - \pi_c)) + (1 - Q)\pi_e^s}{1 + r},$$

or

$$\pi_e^s = \frac{(1-\beta)Q}{r + (1-\beta)Q} (V_s - \pi_c). \quad (15)$$

The explanation for the expression for the creditor's value function π_c resembles the explanation of the value function of entrepreneur who has a standardized project π_e^s represented in equation (15) above. It has two additional terms, however. The first additional term arises because the creditor with an entrepreneur who has one of two types: one with an h -type differentiated project (who plays $\delta = d$) and one with an ℓ -type differentiated project (who plays $\delta = s$). The second additional term arises because the creditor earns interest on the capital I that he has not invested. We now describe the terms that determine the creditor's value function. The creditor is matched with probability $q = q(\theta)$. With probability $1 - q$ he is unmatched and searches again to receive π_c , by stationarity. If he is matched, with probability α he is matched with an entrepreneur who has an h -type differentiated project and who chooses to do this differentiated project. In this case e proposes with probability $1 - \beta$, leaving c with π_c . With probability β c proposes and his utility is $p_h V_d$ since $\pi_e^d = 0$. Finally, consider the case in which c is matched with an entrepreneur who has an ℓ -type differentiated project. e then chooses to do the standardized project. Again, with probability $1 - \beta$, e proposes and c gets π_c . With probability β , c proposes and gets $V_s - \pi_e^s$. The final term is the interest rI that the creditor earns from holding his capital in the money-market account while searching. This description summarizes all the possibilities and allows us to write π_c as follows:

$$\pi_c = \frac{q \left[\alpha \left(\beta p_h V_d + (1-\beta) \pi_c \right) + (1-\alpha) \left(\beta (V_s - \pi_e^s) + (1-\beta) \pi_c \right) \right] + (1-q) \pi_c + rI}{1+r},$$

or

$$\pi_c = \frac{q \left(\alpha \beta p_h V_d + (1-\alpha) \beta (V_s - \pi_e^s) \right) + rI}{r + \beta q}. \quad (16)$$

We can now solve for the equilibrium value functions, which are the solution of the system of equations (15) and (16).

LEMMA 2. *In a stationary equilibrium with action profile $(\eta = a, \delta_h = d, \delta_\ell = s)$, the value functions are given by*

$$\pi_e^d = 0, \quad (17)$$

$$\pi_e^s = \frac{(1-\beta)Q}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)} \left(r(V_s - I) - \alpha\beta q \Delta V \right), \quad (18)$$

$$\pi_c = I + \frac{\beta q}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)} \left(r(\alpha \Delta V + V_s - I) + \alpha(1-\beta)Q(p_h V_d - I) \right). \quad (19)$$

One of the key shortcuts we took in the toy model of Section 2 was to assume that π_e^s proxied for competition. The next lemma says that π_e^s is strictly increasing in competition θ , so the shortcut is now micro-founded.

LEMMA 3. *In a stationary equilibrium with action profile $(\eta = a, \delta_h = d, \delta_\ell = s)$, π_e^s is increasing in θ .*

The key intuition for this result is that when θ increases, it decreases the time an entrepreneur expects to wait before he is matched with another creditor. This means that increasing θ improves the entrepreneur's opportunity to find outside funding and thus his bargaining position against his current creditor.

4.2.4 The Two Sides of Credit Market Competition

The inequalities (14) show that the efficient outcome can be supported in equilibrium for only intermediate values of π_e^s . Then, Lemma 3 shows that π_e^s indeed proxies for competition θ . This section shows that the intuition established in the toy model of Section 2 is robust: when competition θ is too high or too low, the efficient outcome is not an equilibrium.

PROPOSITION 1. *If*

$$\left(1 - \frac{p_\ell}{p_h}\right) \left(I + \frac{\beta}{\beta + r}\right) (\alpha \Delta V + V_s - I) > V_s - p_\ell V_d \quad (20)$$

and

$$\frac{\beta}{1 - \beta + r} (V_s - I) > \Delta V, \quad (21)$$

then there is a stationary equilibrium with action profile $(\eta = a, \delta_h = d, \delta_\ell = s)$ only if credit competition θ is neither too large nor too small.

In the toy model, we assumed that the entrepreneur's outside option was zero if he undertook the differentiated project, $\pi_e^d = 0$, and that the outside option π_e^s could proxy for competition θ (the Toy Model Assumptions 2 and 3). In the full model, we established these assumptions as results in Lemmas 1 and 3, respectively. Given these results, the intuition for why efficiency is lost in the face of extreme credit competition is the same as that presented in the analysis of the toy model.

4.3 Second Best

We now ask whether the creditor's option to develop a relationship with the entrepreneur can restore efficient project choice even when credit market competition is at one of the extremes. A glimpse of the answer to this question can be had by noting that since relationship lending entails an expense k for a creditor, efficiency

is unlikely to be restored fully. However, a welfare gain is likely with relationship lending since Assumptions 1, 2, and 3, say that

$$(\alpha p_h + (1 - \alpha)p_\ell)V_d < V_s < \alpha p_h V_d + (1 - \alpha)V_s - k, \quad (22)$$

which implies that the surplus gain from efficient project choice outweighs the cost of relationship lending. Thus, the second-best outcome is $(\eta = r, \delta_h = d, \delta_\ell = s)$. The question is: when does it constitute a stationary equilibrium?

4.3.1 Off-Equilibrium-Path Behavior

To find the conditions for this action profile to be part of an equilibrium, we must first specify the behavior off the equilibrium path. When c plays $\eta = r$, there is no asymmetric information between c and e on the equilibrium path, but there is asymmetric information off the equilibrium path when c plays $\eta = a$. In particular, if c plays $\eta = a$ and e plays $\delta = d$, what does c believe about the type of e 's project? We will focus on equilibria in which e plays s following c playing a . This is the unique off-path behavior if the entrepreneur with the h -type differentiated project always prefers to play s than to pool with the entrepreneur with the ℓ -type project.⁶ A sufficient condition for uniqueness is that the average NPV of differentiated projects is low, which we make precise with the next assumption.

ASSUMPTION 6.

$$\Delta V < \left(\frac{1}{\alpha p_h + (1 - \alpha)p_\ell} - 1 \right) I.$$

Further, we maintain our focus on equilibria in which a creditor believes that if he encounters an entrepreneur with a differentiated project, then he adopts the out-of-equilibrium belief that the project is type ℓ , $\mu(\tau = \ell \mid \delta = d) = 0$.

4.3.2 Face Values

As in Subsection 4.2.1 above, the face value of debt depends on who is the proposer in round 4 of the stage game and on the project choice. Since we are looking for an equilibrium in which $\eta = r$, the creditor also observes the type of the differentiated project. Note that the entrepreneur with the ℓ -type differentiated project will not obtain funding for it—it is negative NPV and the creditor observes the type τ . Thus, following $\eta = r$, e always plays $\delta = s$ if he has an ℓ -type differentiated project. Thus, again there are four face values to compute: (1) when e proposes and he has a standardized project, (2) when c proposes and e has a standardized project, (3) when e proposes and he has an h -type differentiated project, and (4) when c proposes and e has an h -type differentiated project. The expressions for

⁶If self-selected separation were possible we could implement first best anyway.

the face values are identical to those in Subsection 4.2.1. The key difference is that the continuation values π_e^s and π_c are different. Also, even though the notation is unchanged, it is useful to keep in mind that when the project is differentiated, the relationship lender observes whether $\tau = \ell$ or $\tau = h$ whereas this was not observed by the lender in an arm's-length transaction. To summarize, the face values are

$$F_s^e = \pi_c, \quad (23)$$

$$F_s^c = V_s - \pi_e^s, \quad (24)$$

$$F_d^e = \pi_c/p_h, \quad (25)$$

$$F_d^c = V_d. \quad (26)$$

4.3.3 *Entrepreneurs' Incentive Constraints*

As already mentioned in the previous section, if c has played $\eta = r$, then whenever e has an ℓ -type differentiated project, he plays $\delta = s$. Thus, to check whether there is a stationary equilibrium with action profile $(\eta = r, \delta_h = d, \delta_\ell = s)$, we need to check the incentive constraint for only the entrepreneur with the h -type project. As in the first-best, he plays $\delta = d$ whenever

$$(1 - \beta)p_h V_d \geq \beta\pi_e^s + (1 - \beta)V_s$$

or

$$\pi_e^s \leq \frac{1 - \beta}{\beta} \Delta V. \quad (27)$$

This is the incentive constraint (IC_h) already written down above, but recall that π_e^s depends on the equilibrium.

4.3.4 *Creditors' Incentive Constraint*

Unlike the first-best case, a creditor's incentive constraint now has bite. If he deviates to $\eta = a$ he saves the cost k of relationship lending, but forgoes the increased rents he gains when e has an h -type differentiated project. If he chooses $\eta = a$ he anticipates that the entrepreneur will standardize (see Assumption 6 and the discussion in Subsection 4.3.1). Therefore, his expected payoff is $\beta F_s^c + (1 - \beta)\pi_c$. If he chooses $\eta = r$, the expression for his payoff is more complicated for two reasons: (1) he must take into account the possibility that he is matched with an entrepreneur who has an ℓ -type differentiated project as well as the possibility that he is matched with an entrepreneur who has an h -type differentiated project; and (2) he must pay the cost k of relationship lending. If he is matched with an entrepreneur with an ℓ -type differentiated project, e standardizes and c 's payoff is again $\beta F_s^c + (1 - \beta)\pi_c$, whereas if c is matched with an entrepreneur with an h -type differentiated project,

e differentiates and c 's payoff is $\beta p_h F_d^c + (1 - \beta)\pi_c$. Thus, c plays $\eta = r$ whenever

$$\alpha \left(\beta p_h F_d^c + (1 - \beta)\pi_c \right) + (1 - \alpha) \left(\beta F_s^c + (1 - \beta)\pi_c \right) - k \geq \beta F_s^c + (1 - \beta)\pi_c.$$

This simplifies to

$$p_h F_d^c - \frac{k}{\alpha\beta} \geq F_s^c,$$

or

$$\pi_e^s \geq \frac{k}{\alpha\beta} - \Delta V. \quad (\text{IC}_c)$$

Note that the incentive constraints (IC_h) and (IC_c) combine to say that the second-best action profile $(\eta = r, \delta_h = d, \delta_\ell = s)$ is attainable if only if

$$\frac{k}{\alpha\beta} - \Delta V \leq \pi_e^s \leq \frac{1 - \beta}{\beta} \Delta V. \quad (28)$$

Thus, taking π_e^s again as a proxy for competition, we see that, even with relationship lending, efficient project choice is possible only for intermediate levels of credit competition. The inequalities above already convey the essence of our main results about relationship lending. First, relationship lending can mitigate the inefficiencies in the entrepreneur's project choice only for low levels of competition (the upper bound $(1 - \beta)\Delta V / \beta$ coincides with the upper bound in the inequalities (14)). Second, only if the cost k is small enough ($k \leq \alpha\beta\Delta V$), does relationship lending fully prevent the over-differentiation problem that was present for low levels of credit competition. If k is larger, the creditor's incentive constraint will be violated and he will prefer to perform arm's-length lending and save the cost k : there is a hold-up problem because even though the total surplus created $\alpha\Delta V$ exceeds the cost k (Assumption 3), the proportion of the increased surplus that c receives may not exceed it.

4.3.5 Value Functions

To express the range of competition for which $(\eta = r, \delta_h = d, \delta_\ell = s)$ is the stationary action profile in equilibrium, compute the players' continuation values as value functions. This is similar to the calculations in Subsection 4.2.3. In fact, the expression for π_e^s is identical:

$$\pi_e^s = \frac{Q(\beta\pi_e^s + (1 - \beta)(V_s - \pi_c)) + (1 - Q)\pi_e^s}{1 + r},$$

or

$$\pi_e^s = \frac{(1 - \beta)Q}{r + (1 - \beta)Q} (V_s - \pi_c). \quad (29)$$

The only difference between π_c in the efficient equilibrium (that with the stationary

action profile ($\eta = a, \delta_h = d, \delta_\ell = s$) above) and π_c in the equilibrium under consideration (with the stationary action profile ($\eta = r, \delta_h = d, \delta_\ell = s$)) is that when c is matched with an entrepreneur, he first pays k to invest in a relationship with e . Thus, his value function has an extra $-k$ term with probability q relative to equation (4.2.3) in Section 4.2.3:

$$\pi_c = \frac{q \left[\alpha \left(\beta p_h V_d + (1 - \beta) \pi_c \right) + (1 - \alpha) \left(\beta (V_s - \pi_e^s) + (1 - \beta) \pi_c \right) - k \right] + (1 - q) \pi_c + rI}{1 + r},$$

or

$$\pi_c = \frac{q \left(\alpha \beta p_h V_d + (1 - \alpha) \beta (V_s - \pi_e^s) - k \right) + rI}{r + \beta q}. \quad (30)$$

We can now solve for the equilibrium value functions, which are the solutions of the system of equations (29) and (30). The next lemma summarizes them.

LEMMA 4. *In stationary equilibrium with action profile ($\eta = a, \delta_h = d, \delta_\ell = s$), the value functions are given by*

$$\pi_e^d = 0, \quad (31)$$

$$\pi_e^s = \frac{(1 - \beta)Q}{r(r + \beta q) + (1 - \beta)Q(r + \alpha \beta q)} \left(r(V_s - I) + q(k - \alpha \beta \Delta V) \right), \quad (32)$$

$$\pi_c = I + \frac{\beta q \left(r(\alpha \Delta V + V_s - I) + \alpha(1 - \beta)Q(p_h V_d - I) - (r + (1 - \beta)Q)k \right)}{r(r + \beta q) + (1 - \beta)Q(r + \alpha \beta q)}. \quad (33)$$

4.3.6 When Can Relationship Lending Restore Efficiency?

This subsection presents our two main results about relationship lending. First, relationship lending completely solves the over-differentiation inefficiency that arises for low levels of competition as long as $k \leq \alpha \beta \Delta V$. However, when $k > \alpha \beta \Delta V$, the inefficiency persists for sufficiently low levels of competition. Second, relationship lending does not mitigate the over-standardization problem that arises for high levels of competition. The next lemma states the first of these results.

PROPOSITION 2. *Relationship lending restores efficiency for low levels of credit competition if and only if $k \leq \alpha \beta \Delta V$. That is, for low θ , there is a stationary equilibrium with action profile ($\eta = a, \delta_h = d, \delta_\ell = s$) if and only if $k \leq \alpha \beta \Delta V$.*

As we already touched on above, the result depends on two forces. First, a creditor will never fund an ℓ -type differentiated project when he observes the project's type because it has negative NPV by Assumption 1. Thus, the entrepreneur with the ℓ -type differentiated project knows that if he plays $\delta = d$, he will receive payoff zero,

which makes him prefer to standardize. However, the creditor must pay the entire cost k to perform relationship lending. Even though the total surplus gain is positive by Assumption 3, the proportion of this surplus gain allocated to the creditor may not suffice to induce the creditor to bear the cost *privately*. Due to this hold-up problem, there may still be inefficient project choice for low levels of competition. Note that this intuition is already present from the creditor's incentive constraint (IC_c) taking π_e^s as a proxy for competition. The proposition above combined with Lemma 4 simply confirms the intuition in general equilibrium.

The next main result relies more heavily on the general equilibrium framework provided by the search model. It says that the entrepreneur's incentive to over-standardize is stronger in the equilibrium with action profile $(\eta = r, \delta_h = d, \delta_\ell = s)$.

PROPOSITION 3. *Relationship lending never restores efficiency for high levels of credit competition. That is, if there is no stationary equilibrium with action profile $(\eta = r, \delta_h = d, \delta_\ell = s)$ for $\theta > \bar{\theta}$, then there is no stationary equilibrium with action profile $(\eta = a, \delta_h = d, \delta_\ell = s)$ for $\theta > \bar{\theta}$.*

Put simply, if, for any level of competition θ , the entrepreneur with the h -type differentiated project plays $\delta = s$ in the equilibrium with action profile $(\eta = a, \delta_h = d, \delta_\ell = s)$, then the entrepreneur with the h -type differentiated project plays $\delta = s$ in the equilibrium with action profile $(\eta = r, \delta_h = d, \delta_\ell = s)$. In other words, the incentive to standardize is even stronger when the creditor chooses relationship lending than when the creditor chooses arm's-length finance. The reason is that the creditor anticipates being in a weaker bargaining position with relationship lending. Because c bears the cost k , his outside option is relatively low, e takes advantage of c 's low outside option to negotiate better loan terms and capture more of the surplus. This means that standardization is even more attractive for e when c is playing $\eta = r$ than when c is playing $\eta = a$. This means that relationship lending can only exacerbate the over-standardization problem.

5 The Role for Non-bank Funding

The result that even relationship lending does not entirely eliminate investment inefficiencies creates a natural economic rationale for the emergence of other forms of intermediation to improve efficiency. In this section we extend the model to consider a class of investors who are levered and (in equilibrium) fund only h -type differentiated projects. We call these investors "private equity firms" or "PEs".

We use the label PEs because these intermediaries have a number of similarities with real-world private equity firms. Private equity firms are typically informed investors who use high leverage to acquire equity stakes in firms (see, for example, Kaplan and Strömberg (2008)). Our PEs share these properties: they learn the

type of the projects they invest in, they raise additional capital via debt, and they take equity stakes in entrepreneurs' projects. Further, in equilibrium, our model captures a number of stylized facts about PE behavior and returns.

The analysis shows that the emergence of PEs can improve efficiency, but cannot eliminate investment inefficiencies entirely. We find that PEs enter the market only for high levels of credit market competition. They fund only h -type differentiated projects, and, as a result, they restore efficiency when they are matched with entrepreneurs with h -type differentiated projects. However, they make entrepreneurs with ℓ -type differentiated projects wait inefficiently to be matched with banks in order to obtain funding for their standardized projects. Further, banks continue to exist in equilibrium, and some entrepreneurs with h -type differentiated projects still over-standardize because they are matched with banks.

The key mechanism that arises in equilibrium to improve efficiency is that PE leverage acts as a *commitment device* to fund only h -type differentiated projects. The reason is that, given PEs' debt, the standardized project becomes effectively negative NPV for the PE.

In our model, a difference between banks and PEs is that PEs are levered, whereas banks are not. Of course, in reality banks are also levered, but we think that this modeling choice captures a fundamental difference between the effect of leverage on banks' behavior and its effect on PEs' behavior. PE leverage induces PEs to choose investments with high upside even if they are not the highest NPV investments. Because a PE makes few investments,⁷ an equity stake in a project constitutes a large proportion of its assets.⁸ Therefore, the PE is likely to default when a project does not return high cash flows. Because the PE is highly levered and is protected by limited liability, its payoff depends only on the upside of the project—it is not exposed to the downside. Observe that this argument depends on the causal link from project failure to PE default, which, in turn, depends on the equity stake in the project being a significant proportion of the PE's investment portfolio. In contrast, leverage does not have the same effect on a bank's behavior. Even if we incorporated bank leverage into the model explicitly, banks would still choose the project with the highest NPV.⁹ Because a bank makes many investments, a loan to an entrepreneur constitutes only a small proportion of its assets. Therefore, the bank is unlikely to default because the entrepreneur's project does not return high cash flows. Despite its leverage and limited liability protection, the bank is still exposed to the downside risk of the entrepreneur's project. The reason

⁷Metrick and Yasuda (2010) shows that the median private equity firm makes a total of twelve investments over the entire life of the fund, which is typically ten years.

⁸In fact, the contemporary theory of banking relies on the intermediary making a large number of loans, each of which constitutes a very small fraction of its portfolio.

⁹To be precise, this holds as long as the size of the loan is small relative to the size of the bank's other assets, and the project's success is not correlated with the value of the bank's other assets.

is that when the entrepreneur defaults, the bank itself probably does not default and, hence, its limited liability protection does not bite. We can thus conclude that the role of leverage is fundamentally different across PEs and banks. The key role of intermediary leverage in inducing PEs to commit not to fund standardized projects is absent in the case of banks. Thus, our simplifying assumption that banks are unlevered is rather innocuous. Note, finally, that the difference between bank leverage and PE leverage is amplified by the difference between their funding contracts. Because PEs fund via equity, they are exposed to the upside of the projects they invest in. In contrast, because banks fund via debt, they are exposed only to the downside of the projects they invest in.

Despite parsimony in the assumptions about the nature of PEs, we can capture several salient stylized facts about them. In particular, in the model, PEs fund only specialized projects and they have high returns on investment but are still scarce, even in the perfect-competition limit. In our model, as in reality, PEs and banks fund very different types of projects: PEs fund (risky) specialized projects and banks fund (safe) standardized projects. In our model, this heterogeneity arises despite the absence of ex ante heterogeneity among either entrepreneurs or creditors—both banks and PEs emerge in equilibrium even though all creditors are ex ante identical. A key insight we offer is that the type of creditor an entrepreneur finds may be an important determinant of the type of investment the entrepreneur makes.

5.1 *Model with Private Equity*

In this section we outline how we model PEs. The entrepreneurs and their projects are identical to those in the baseline model. Creditors, on the other hand, make a one-time choice when they enter the market that was not in the baseline model. They can either remain traditional banks (and later give relationship or arm's-length loans to entrepreneurs) or they can specialize in private equity. If they remain banks, the model is identical to the model considered previously. If they specialize in private equity, they must borrow l from a competitive market. l will represent the operational cost of private equity; it helps us capture the important role of PE leverage. We call the face value of PE debt F_{PE} . A PE searches in the same way as a bank, but when matched with an entrepreneur e , the stage game a PE plays is different. It is summarized by the following timing:

1. e chooses between a differentiated and a standardized project, $\delta \in \{d, s\}$.
2. PE either borrows l and invests, or chooses to not invest and therefore keeps searching in the market.
 - If PE invests l he observes the type τ of the entrepreneur's differentiated project.

3. e and PE either negotiate the PE's equity stake γ , which is determined as follows:

- with probability β , PE makes e a take-it-or-leave-it offer $\gamma_\delta^{\text{PE}}$; e accepts or rejects
- with probability $1 - \beta$, e makes PE a take-it-or-leave-it offer γ_δ^e

or the PE diverts I .

4. The project succeeds or fails, with e and PE dividing the surplus according to the agreed equity stake. The PE repays his debt or defaults.

Note that because the project type is observable, we have $\gamma_\delta \in \{\gamma_s, \gamma_\ell, \gamma_h\}$. Note also that when negotiating the PE's equity stake, the PE's outside option is I , since he has the option to divert but, since his capital l is sunk, he does not have the option to keep searching. We justify the assumption that a PE can divert his capital with the following assumption on observability: if a PE's creditor observes that the project of the entrepreneur e that the PE funds does not pay off, the creditor does not know whether it failed randomly or whether it returned nothing because the PE withheld funds, i.e. diverted capital. This assumption serves to keep the PE's outside option sufficiently high at the point of bargaining over the equity stake.

Everything else, including the off-equilibrium beliefs and games played by off-equilibrium matches coincide with those in the baseline model.

To generate the results below, we focus on the high-competition region in which θ is large enough to induce entrepreneurs to over-standardize. Further, we make the following assumptions.

ASSUMPTION 7.

$$p_h V_d - I > l.$$

This assumption simply insures that h -type differentiated projects are a positive NPV net of the PE's monitoring cost l .

ASSUMPTION 8.

$$\frac{l}{p_h} > V_s - I.$$

This assumption serves to ensure that PEs require enough leverage— l is high enough—that their debt makes it undesirable for them to fund standardized projects at all.

ASSUMPTION 9.

$$r > \beta - \frac{\Delta V}{p_h V_d - I}.$$

This assumption serves to ensure that an entrepreneur always accepts funding from his first match, whether it is a bank or a PE. This assumption is not very restrictive, since we view the time between dates—the shortest possible time between matches—as relatively large and ΔV as relatively large. Further, β is less than one.

5.2 Private Equity Results

We now characterize an equilibrium of the extended model in which creditors may become either banks, denoted $c = b$, or private equity firms, denoted $c = \text{PE}$. Our main results of this section are the equilibrium characterization and comparative statics related to the proportion φ of PEs in the market.

5.2.1 Equity Stakes

The PE's equity stake γ depends on which player is proposing the contract in round 3 of the stage game (Subsection 3.1.2).

The proposer always offers a stake that makes his opponent indifferent between accepting and rejecting. There are four cases: (1) when e proposes and he has a standardized project, (2) when c proposes and e has a standardized project, (3) when e proposes and he has a differentiated project, and (4) when c proposes and e has a differentiated project. When e has a differentiated project, it can be either the h -type or the ℓ -type.

Note that whenever e and the PE negotiate the equity stake, the PE has paid l and has learnt e 's type, so there is no asymmetric information. Here the subscripts on the equity stakes denote the project choice, where the subscript τ indicates a differentiated project of type τ . The superscripts denote the proposer. When e proposes, the equity stake with a standardized project is given by

$$\gamma_s^e V_s = I$$

and when c proposes and e has chosen a standardized project, the equity stake is given by

$$(1 - \gamma_s^{\text{PE}}) V_s = \pi_e^s.$$

When e proposes with a differentiated project, the equity stake is given by

$$\gamma_\tau^e p_\tau V_d = I$$

and when c proposes and e has chosen a differentiated project, the equity stake is given by

$$(1 - \gamma_\tau^{\text{PE}}) p_\tau V_d = \pi_e^d,$$

where $\tau \in \{h, \ell\}$. Recall that the PE observes e 's type before negotiating the equity stake.

5.2.2 *Entrepreneurs' Incentive Constraints*

We look for equilibria in which PEs fund only h -type differentiated projects. In this section, we find e 's incentive constraint given that he believes that he will find funding from a PE only if he has an h -type differentiated project. We show in Section 5.2.3 that these beliefs are consistent in equilibrium.

First consider an entrepreneur who is matched with a PE and has an ℓ -type differentiated project. He knows that (1) he will not obtain funding from the PE and (2) if he searches and is matched with a bank, he is better-off if his project is standardized than if it is differentiated. Recall that we are focusing on the high competition region, in which entrepreneurs over-standardize as shown in Subsections 4.2 and 4.3. Thus, this entrepreneur with an ℓ -type differentiated project will always standardize.

Now consider an entrepreneur who is matched with a PE and has an h -type differentiated project. He either differentiates and gets funding from a PE or standardizes, does not obtain funding from a PE and continues searching for a bank. Thus, he differentiates whenever

$$\beta\pi_e^d + (1 - \beta)(1 - \gamma_d^e)p_h V_d \geq \pi_e^s$$

which simplifies to

$$(1 - \beta)(p_h V_d - I) \geq \pi_e^s. \quad (\text{IC}_h^{\text{PE}})$$

Finally, consider an entrepreneur who is matched with a bank. Since we are focusing on the high competition region, the entrepreneur will standardize as in Subsections 4.2 and 4.3. Recall that an entrepreneur who has differentiated but is still searching is believed to be bad by Assumption 4. This implies that an entrepreneur who is matched with a bank does not have the option of differentiating and obtaining funding from a PE in the future because the PE will observe that he failed to obtain funding in the past and will assume that it was because his project was the ℓ -type.

5.2.3 *PEs' Incentive Constraints*

In order to show that a PE funds only h -type differentiated projects, we must check that (1) he prefers to divert capital rather than fund an ℓ -type differentiated project and (2) he prefers to wait than to fund a standardized project. The first incentive constraint, that he prefers to divert capital when confronted with an ℓ -type differentiated project, is given by

$$\beta p_\ell (\gamma_d^{\text{PE}} V_d - F_{\text{PE}}) + (1 - \beta)I \leq I$$

or, simplifying,

$$p_\ell V_d - I \leq F_{\text{PE}},$$

which is always satisfied since $p_\ell V_d - I < 0$ and $F_{\text{PE}} \geq 0$.

The second incentive constraint, that the PE prefers to search again than to be matched with a standardized entrepreneur, is given by

$$\pi_{\text{PE}} \geq \beta(\gamma_s^{\text{PE}} V_s - F_{\text{PE}}) + (1 - \beta)I$$

which simplifies to:

$$\pi_{\text{PE}} \geq \beta(V_s - \pi_e^s - F_{\text{PE}} - I) + I. \quad (\text{IC}_s^{\text{PE}})$$

5.2.4 Value Functions

In this section we compute the value functions that will pin down players' continuation values in the incentive constraints demonstrated in the section above.

First, note that if e differentiates, his outside option vanishes as above due to Assumption 4. Recall that entrepreneurs believe that PEs fund only h -type differentiated projects. Thus, if e standardizes, he faces the new risk of being matched with a PE and being forced to search again. As in the baseline model, his value function is his expected utility before he searches again. If he is unmatched, which occurs with probability $1 - Q$, he searches again and therefore stationarity implies that he obtains π_e^s . If he is matched, a new term appears that is absent in the baseline model. With probability φ , he is matched with a PE. In this case, he does not obtain funding and searches again, obtaining π_e^s . With probability $1 - \varphi$, he is matched with a bank. In this case, he obtains his outside option π_e^s when the bank proposes the contract, which occurs with probability β . With complementary probability $1 - \beta$, e proposes the contract and gets the surplus $V_s - \pi_b$. Thus we can write the equation for e 's value function as

$$\pi_e^s = \frac{Q(\varphi\pi_e^s + (1 - \varphi)((1 - \beta)(V_s - \pi_b) + \beta\pi_e^s)) + (1 - Q)\pi_e^s}{1 + r}$$

which gives

$$\pi_e^s = \frac{(1 - \beta)(1 - \varphi)Q}{r + (1 - \beta)(1 - \varphi)Q}(V_s - \pi_b). \quad (34)$$

If a lender becomes a bank, $c = b$, then he funds only standardized projects in equilibrium, as derived in Subsections 4.2 and 4.3. Thus, his value function must take into account only the probability of finding a match, who proposes the contract, and the interest earned overnight. Specifically, the terms are as follows. b earns interest rI on his principal over the next period. If he does not find a match, which occurs with probability $1 - q$, he obtains π_b . If he does find a match, which

occurs with probability q , the entrepreneur proposes the contract and pushes him to his outside option π_b with probability $1 - \beta$. Finally, if the bank finds a match and proposes the contract, which occurs with joint probability βq , he obtains the surplus $V_s - \pi_e^s$. The equation for his value function is thus

$$\pi_b = \frac{q(\beta(V_s - \pi_e^s) + (1 - \beta)\pi_b) + (1 - q)\pi_b + rI}{1 + r},$$

which gives

$$\pi_b = I + \frac{\beta q(V_s - \pi_e^s - I)}{r + \beta q}. \quad (35)$$

We can now solve the system of equations (34) and (35) for π_e^s and π_b to obtain the value functions:

$$\pi_e^s = \frac{(1 - \beta)(1 - \varphi)Q}{r + q\beta + (1 - \beta)(1 - \varphi)Q}(V_s - I) \quad (36)$$

and

$$\pi_b = I + \frac{\beta q}{r + q\beta + (1 - \beta)(1 - \varphi)Q}(V_s - I). \quad (37)$$

We need to calculate the value function of the private equity lender, $c = \text{PE}$. As in the case in which $c = b$, he earns interest rI overnight. As usual, if the PE does not find an entrepreneur next period, he obtains π_{PE} by stationarity. This occurs with probability $1 - q$. If he does find a match, which occurs with probability q , there are several cases to consider. If he is matched with an entrepreneur with an ℓ -type differentiated project, which occurs with conditional probability $1 - \alpha$, the PE does not fund it and searches again, obtaining payoff π_{PE} . If he is matched with an entrepreneur with an h -type differentiated project. There are two cases: with probability $1 - \beta$ the entrepreneur proposes the equity stake, and with probability β the PE proposes the equity stake. If e is the proposer, he pushes the PE to his outside option I . Remember that his operational cost l is now sunk, so he does not have the option of searching again. Hence, his outside option if bargaining breaks down is to divert capital and obtain I . If, on the other hand, the PE makes the offer, he obtains the surplus but repays his own debt, F_{PE} . Thus, he obtains $p_h(V_d - F_{\text{PE}})$ because the entrepreneur's outside option is zero, $\pi_e^d = 0$. We can now write the equation for the PE's value function:

$$\pi_{\text{PE}} = \frac{q \left[\alpha \left(\beta p_h(V_d - F_{\text{PE}}) + (1 - \beta)I \right) + (1 - \alpha)\pi_{\text{PE}} \right] + (1 - q)\pi_{\text{PE}} + rI}{1 + r}.$$

Recall that the PE borrows from a competitive capital market. The market anticipates that the PE will fund only h -type entrepreneurs and repay if and only if these projects succeed (there is no diversion on the equilibrium path because ℓ -type

entrepreneurs standardize). Thus, the equilibrium face value of the PE's debt is

$$F_{\text{PE}} = \frac{l}{p_h}.$$

Hence, the PE's value function is

$$\pi_{\text{PE}} = I + \frac{q\alpha\beta(p_h V_d - l - I)}{r + \alpha q}, \quad (38)$$

having just simplified the equation above.

5.2.5 Equilibrium Characterization

The next proposition characterizes an equilibrium. The key features of the equilibrium are the following: (1) PEs enter the market for only sufficiently high levels of credit competition θ ; (2) PEs and banks coexist; (3) PEs fund only h -type differentiated projects; (4) PE returns are high; and (5) banks fund only standardized projects.

PROPOSITION 4. *For sufficiently high θ , there is an equilibrium characterized as follows*

- *All banks play $\eta = a$*
- *PEs invest l if and only if they observe entrepreneurs play $\delta = d$*
- *An entrepreneur with an h -type differentiated projects plays $\delta = d$ if and only if matched with a PE*
- *All entrepreneurs play $\delta = s$ when matched with a bank*
- *An entrepreneur with an h -type differentiated project obtains funding from the first creditor he is matched with; an entrepreneur with an ℓ -type differentiated project obtains funding from the first bank he is matched with*

We have already established the main elements of the proposition in the preceding subsections. The proof in the appendix simply verifies the incentive constraints $(\text{IC}_h^{\text{PE}})$ and $(\text{IC}_s^{\text{PE}})$, given the equilibrium value functions.

It is worth noting that the key difference between a PE and a bank is that PEs are highly levered and therefore fund only projects with sufficiently high upside. This is what allows the PE to improve efficiency—by forcing entrepreneurs not to over-standardize even though competition in the credit market is high. Thus, when an entrepreneur is matched with a PE, he anticipates that he will *not* obtain funding from the PE if he plays $\delta = s$.

5.2.6 Equilibrium Proportion of Private Equity

In sufficiently competitive markets, banks and PEs coexist. Coexistence requires creditors to be indifferent between devoting their capital to banking and devoting it to PE. They maintain indifference in equilibrium due to the presence of a *feedback loop*, which works as follows. Start by considering an equilibrium with only banks in the high competition region in which entrepreneurs are over-standardizing. As the credit market becomes more competitive, the rents from traditional banking decrease. When these rents are sufficiently low, the creditors benefit from specializing in private equity. But then since more intermediation services are devoted to private equity, the competition in the traditional banking sector decreases, making banking again more attractive. That is, the payoff from traditional banking is increasing in the proportion of creditors who specialize in private equity. This feedback loop is the key force in the model that allows for the coexistence of disparate types of credit.

The feedback mechanism appears directly from equating the value functions of the two types of creditors,

$$\pi_b(\theta, \varphi(\theta)) = \pi_{PE}(\theta)$$

or, simplifying:

$$I + \frac{\beta q}{r + \beta q + (1 - \beta)(1 - \varphi)Q}(V_s - I) = I + \frac{\alpha \beta q}{r + \alpha q}(p_h V_d - l - I).$$

Rearranging gives

$$\varphi = 1 - \frac{(r + \alpha q)(V_s - I) - \alpha(r + \beta q)(p_h V_d - l - I)}{\alpha(1 - \beta)Q(p_h V_d - l - I)}.$$

PROPOSITION 5. *As long as*

$$(r + \alpha)(V_s - I) > \alpha(r + \beta)(V_d - I - l)$$

and

$$V_s - I > \beta(V_d - I - l),$$

the proportion φ of PEs in the market is:

1. *positive only for sufficiently high credit competition θ ;*
2. *always less than one; and*
3. *increasing in credit market competition θ .*

The proposition above says that private equity emerges only in highly competitive credit markets, and it continues to become more important as credit markets become more competitive. However, private equity never eliminates banks. The

reason is, again, the feedback loop between competition in the banking market and the presence of private equity: the more capital that is devoted to private equity, the more attractive banking becomes. This general equilibrium effect is crucial for the coexistence of private equity and banking in the model.

6 Conclusions

This paper develops a general equilibrium model of competition in the credit market to investigate the question of how credit market competition affects corporate investment, and how the different investment inefficiencies at different levels of credit market competition generate incentives for the emergence of a variety of intermediaries. In the model entrepreneurs have two projects, a (safe) “standardized” project and a (risky) “differentiated” (or “specialized”) project. Which project is efficient to undertake depends on the state of nature, so efficient investment requires adapting to circumstances. The paper has three main results. The first is that project choice is inefficient when banking competition is at either extreme (too high or too low). Specifically, when competition in the banking market is very low, entrepreneurs over-differentiate, inefficiently forgoing the standardized project. When competition is very high, entrepreneurs over-standardize, inefficiently foregoing the differentiated project. The key force is that entrepreneurs with standardized projects can find funding more easily in the future, thus they can obtain better terms of debt from their creditors. This pushes entrepreneurs toward the standardized project, and the effect is strongest in the most competitive credit markets. The second main result is that relationship banking can mitigate the over-differentiation inefficiency that emerges for low levels of credit competition but not the over-standardization inefficiency that emerges for high levels of competition. The reason is that relationship banking allows banks to make more informed lending decisions, mitigating inefficiencies that arise from the asymmetric information that is associated with a specialized project. However, relationships with borrowers do not affect banks’ ability to fund standardized projects; relationship banking cannot force entrepreneurs not to standardize. Third, the residual inefficiency that relationship lending is unable to eliminate leads to the emergence of private equity (PE), but this happens only when the credit market is sufficiently competitive. A natural separation arises in general equilibrium as PE firms lend only to entrepreneurs who invest in h -type differentiated projects and banks lend only to those who invest in standardized projects.

To the best of our knowledge, this is the first paper that develops a theory in which arm’s-length or relationship banks coexist with PE firms, and this coexistence has ramifications for the nature of entrepreneurial investment. The paper thus exposes a natural economic link between credit market competition, investment

inefficiencies and intermediation variety, with general equilibrium effects playing a crucial role in the co-existence of various types of intermediaries. Our result that PE firms arise only when the credit market is very competitive accords well with the observation that PE is ubiquitous in the highly competitive credit market found in the US. Moreover, our theory also sheds light on why PE firms tend to be highly levered. High leverage is not just a tax-driven effect for PE firms. Rather, it is essential to the economic role they play.

A Appendix: Proofs

A.1 Proof of Lemma 1

Given the Assumption 4 on out-of-equilibrium beliefs, the creditor believes that he knows the quality of the project so he will not pay k to gain information via relationship lending. Further, given the Assumption 1 that the ℓ -type differentiated project has negative NPV, there is no face value that will deliver a positive expected payoff to the creditor. Since the creditor's outside option is positive, in fact $\pi_c \geq I$, no lending can take place.

A.2 Proof of Lemma 3

First recall the expression for π_e^s from Lemma 2:

$$\pi_e^s = \frac{(1-\beta)Q}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)} \left(r(V_s - I) - \alpha\beta q\Delta V \right).$$

First observe that the first term in the product,

$$f(\theta) := \frac{(1-\beta)Q}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)},$$

is increasing in θ . Compute the derivate with the quotient rule and group terms:

$$\begin{aligned} f'(\theta) &= \frac{\partial}{\partial \theta} \left(\frac{(1-\beta)Q}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)} \right) \\ &= \frac{Q'r(r+\beta q) - Qq'\beta(r+\alpha(1-\beta)Q)}{\left(r(r+\beta q) + (1-\beta)(r+\alpha\beta q)Q \right)^2}. \end{aligned}$$

To see that this expression is positive, recall the assumptions on the matching function from Subsection 3.1.1. Namely $q' < 0$ and $Q' > 0$. Thus $-q'$ is positive and so are all other terms, so $f' > 0$.

Now

$$\pi_e^s = f(\theta) \left(r(V_s - I) - \alpha\beta q\Delta V \right)$$

so

$$\frac{\partial \pi_e^s}{\partial \theta} = f'(\theta) \left(r(V_s - I) - \alpha\beta q\Delta V \right) - \alpha\beta q'\Delta V f(\theta).$$

Assumption 5 and the result above that $f' > 0$ imply that the first term is positive. f is positive because all its terms are positive, $\Delta V > 0$ and $-q' > 0$ as above. Thus, π_e^s is increasing in competition θ .

A.3 Proof of Proposition 1

First note that π_e^s and π_c as written in Lemma 2 are continuous in θ since q and Q are continuous in θ and the denominators are always positive. Thus, we must just show that an entrepreneur who has an ℓ -type differentiated project chooses $\delta = d$ when $\theta \rightarrow 0$ and that an entrepreneur who has an h -type differentiated project chooses $\delta = s$ when $\theta \rightarrow \infty$. That is to say that inequality (IC_ℓ) is violated for low θ and inequality (IC_h) is violated for high θ .

Before checking e 's incentive constraints, note the limits of the value functions from Lemma 2:

$$\begin{aligned}\lim_{\theta \rightarrow 0} \pi_c &= I + \frac{\beta}{r + \beta} (\alpha \Delta V + V_s - I), \\ \lim_{\theta \rightarrow 0} \pi_e^s &= 0, \\ \lim_{\theta \rightarrow \infty} \pi_e^s &= \frac{1 - \beta}{r + 1 - \beta} (V_s - I).\end{aligned}$$

Consider first the incentive constraint of the entrepreneur with an ℓ -type differentiated project. The constraint reads

$$p_\ell V_d - V_s + \left(1 - \frac{p_\ell}{p_h}\right) \pi_c \leq \frac{\beta}{1 - \beta} \pi_e^s$$

or, as $\theta \rightarrow 0$,

$$p_\ell V_d - V_s + \left(1 - \frac{p_\ell}{p_h}\right) \left(I + \frac{\beta}{\beta + r} (\alpha \Delta V + V_s - I)\right) \leq 0.$$

This is violated by the first condition in the statement of the proposition. Therefore there is no efficient equilibrium when θ is small.

Now consider the incentive constraint of the entrepreneur with an h -type differentiated project. The constraint reads

$$\frac{\beta}{1 - \beta} \pi_e^s \leq \Delta V$$

or, for $\theta \rightarrow \infty$,

$$\frac{\beta}{r + 1 - \beta} (V_s - I) \leq \Delta V.$$

This is violated by the second condition in the statement of the proposition. Therefore there is no efficient equilibrium when θ is large.

A.4 Proof of Proposition 2

From Lemma 4 observe that $\pi_e^s \rightarrow 0$ as $\theta \rightarrow 0$. Recall from the creditor's incentive constraint (IC_c) that he plays $\eta = r$ if and only if

$$\pi_e^s \geq \frac{k}{\alpha\beta} - \Delta V.$$

Thus, this holds as $\theta \rightarrow 0$ if and only if

$$0 \geq \frac{k}{\alpha\beta} - \Delta V.$$

That is to say that relationship lending restores efficiency for low θ —it eliminates the over-differentiation inefficiency—whenever $k \leq \alpha\beta\Delta V$.

A.5 Proof of Proposition 3

This proof involves comparing the incentive constraint (IC_h) of the entrepreneur with the high-type differentiated project across the equilibria described in Lemma 2 and Lemma 4. This is the incentive constraint that says the entrepreneur who has an h -type differentiated project chooses to play $\delta = d$. It reads always

$$\pi_e^s \leq \frac{1-\beta}{\beta} \Delta V,$$

but π_e^s depends on the equilibrium. Write $\pi_e^s|_{\eta=a}$ for the entrepreneur's value function as written in Lemma 2 and $\pi_e^s|_{\eta=r}$ for the entrepreneur's value function as written in Lemma 4.

Now, immediately from the expressions written in the lemmas,

$$\pi_e^s|_{\eta=r} - \pi_e^s|_{\eta=a} = \frac{(1-\beta)qQk}{r(r+\beta q) + (1-\beta)Q(r+\alpha\beta q)} > 0.$$

So immediately, $\pi_e^s|_{\eta=r} > \pi_e^s|_{\eta=a}$ which means that if

$$\pi_e^s|_{\eta=a} > \frac{1-\beta}{\beta} \Delta V$$

then

$$\pi_e^s|_{\eta=r} > \frac{1-\beta}{\beta} \Delta V.$$

That is to say if e over-standardizes given the $(\eta = a, \delta_h = d, \delta_\ell = s)$ equilibrium then he would also over-standardize given the $(\eta = r, \delta_h = d, \delta_\ell = s)$ equilibrium.

A.6 Proof of Proposition 4

The bulk of the argument lies in subsections preceding the statement of the proposition (Subsections 5.2.1, 5.2.2, 5.2.3, and 5.2.4). It remains to check the ICs stated in Subsections 5.2.2 and 5.2.3.

First, check inequality (IC_h^{PE}) , which says that the entrepreneur with an h -type project prefers to stay matched with a PE than to standardize and wait to be matched with a bank. Plugging in for π_e^s from equation (36), the inequality reads

$$(1 - \beta)(p_h V_d - I) \geq \frac{(1 - \beta)(1 - \varphi)Q}{r + q\beta + (1 - \beta)(1 - \varphi)Q}(V_s - I).$$

Note that the right-hand side of the inequality above is decreasing in φ , thus sufficient for the inequality above to be satisfied is to set $\varphi = 0$:

$$p_h V_d - I \geq \frac{Q}{r + q\beta + (1 - \beta)Q}(V_s - I). \quad (39)$$

Since the right-hand side is maximized when $q = 0$ and $Q = 1$ it suffices to show that

$$p_h V_d - I \geq \frac{1}{r + 1 - \beta}(V_s - I)$$

or, equivalently,

$$r \geq \beta - \frac{\Delta V}{p_h V_d - I}$$

which is guaranteed by Assumption 5.

It remains to check that inequality (IC_s^{PE}) is satisfied, namely that a PE prefers to search again than to fund a standardized entrepreneur. The constraint reads that

$$\pi_{PE} > \beta(V_s - \pi_e^s - F_{PE} - I) + I.$$

π_{PE} is always at least I , so the IC holds if

$$V_s - \pi_e^s - F_{PE} - I < 0. \quad (40)$$

Since in equilibrium $F_{PE} = l/p_h$, Assumption 8 implies that

$$F_{PE} > V_s - I$$

which suffices to guarantee that inequality (40) holds.

A.7 Proof of Proposition 5

Start with the expression for φ :

$$\varphi = 1 - \frac{(r + \alpha q)(V_s - I) - \alpha(r + \beta q)(p_h V_d - I - l)}{\alpha(1 - \beta)Q(p_h V_d - I - l)}$$

First observe that φ is increasing by direct differentiation. For simplicity use the shorthands $\xi_d := p_h V_d - I$ and $\xi_s := V_s - I$, to write

$$\varphi' = - \frac{\alpha(\xi_s - \beta(\xi_d - l))Qq' - (r(\xi_s - \alpha(\xi_d - l)) + \alpha(\xi_s - \beta(\xi_d - l))q)Q'}{\alpha(1 - \beta)(\xi_d - l)Q^2}.$$

Since $q' < 0$, $Q' > 0$ and $\xi_s - \beta(\xi_d - l) > 0$, by hypothesis, $\varphi' > 0$ as long as $\xi_s - \alpha(\xi_d - l) > 0$. This condition is implied by Assumption 2. Thus, we can conclude that $\varphi' > 0$.

Note that the numerator in the fraction in the expression for φ above is positive by hypothesis. Since $Q \rightarrow 0$ as $\theta \rightarrow 0$ the expression approaches minus infinity for low credit market competition, $\varphi \rightarrow -\infty$ as $\theta \rightarrow 0^+$. Since to be well-defined we must have $\varphi \in [0, 1]$ and φ is continuous and increasing, we see that $\varphi > 0$ for only sufficiently high θ .

Finally, to demonstrate that $\varphi < 1$ even in the limit as $\theta \rightarrow \infty$, we just compute the limit. Recall that $q \rightarrow 0$ and $Q \rightarrow 1$ as $\theta \rightarrow \infty$. Thus, as $\theta \rightarrow \infty$,

$$\varphi \rightarrow 1 - \frac{r(V_s - I) - \alpha r(p_h V_d - I - l)}{\alpha(1 - \beta)(p_h V_d - I - l)}$$

which is less than one since

$$r(V_s - I) > \alpha r(p_h V_d - I - l)$$

by the Assumption 2.

A.8 Equilibrium Refinement

In Subsection 4.1.1 we argued that the out-of-equilibrium beliefs we restricted attention to in Assumption 4 were the most economically reasonable. We now present a game-theoretic argument to justify this restriction on the out-of-equilibrium beliefs.

We consider a refinement akin to Sequential Equilibrium. Sequential Equilibrium is a well-defined solution concept for only games with finite action spaces (see Myerson and Reny (2014)), thus we impose the same refinement as Sequential Equilibrium only for the actions drawn from finite action spaces.

DEFINITION 1. *A Refined Perfect Bayesian Equilibrium is a Perfect Bayesian Equilibrium which is the limit of a sequence of strategy profiles and associated sensible*

beliefs in which the strategies are totally mixed whenever the action space is finite.

PROPOSITION 6. *In any efficient Refined Perfect Bayesian Equilibrium, the out-of-equilibrium beliefs specified in Assumption 4 are unique.*

Proof. Consider a sequence of assessments converging to the (efficient) stationary action profile $(\eta = a, \delta_h = d, \delta_\ell = s)$ that are totally mixed over the actions η and δ . In particular, suppose that the creditor plays r with probability ε^η , the entrepreneur with the h -type differentiated project plays s with probability ε^h , and the entrepreneur with the ℓ -type differentiated project plays d with probability ε^ℓ where $\varepsilon^\eta, \varepsilon^h, \varepsilon^\ell \rightarrow 0^+$.

We proceed to show that for $\varepsilon^\eta, \varepsilon^h, \varepsilon^\ell$ small and positive, only entrepreneurs with ℓ -type differentiated projects will fail to find funding from their initial creditors. This implies that if a creditor with an already-differentiated project is still searching it must be because he has an ℓ -type project and failed to find funding from his initial creditor. This will imply the desired result that the only reasonable out-of-equilibrium belief is that an entrepreneur's already-differentiated project is ℓ -type.

Since we are supposing that the equilibrium is efficient, if e plays s or e plays d and c plays a , c lends to e as in the equilibrium in Proposition 1. If, in contrast, c plays r and e plays d , c observes that e 's project is ℓ -type and therefore negative NPV. This occurs with probability $\varepsilon^\eta \varepsilon^\ell$. In this case c denies e credit and both parties search again. Since this is the only scenario in which c denies e credit, it must be believed that if an entrepreneur has already differentiated his project then it is type ℓ .

□

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