THE DOWNSIDE OF PRECISE PUBLIC INFORMATION FOR DELEGATED ASSET MANAGEMENT*

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Abstract

We theoretically investigate the effect of public information—such as credit ratings and securities analysts' reports—on investor welfare in the context of delegated asset management. Specifically, we ask: does more precise public information increase investor welfare by decreasing an asset manager's informational advantage and, thereby, mitigating the agency problem between him and his client? We show, first, that public information does not align incentives and, second, that decreasing the precision of the information is Pareto improving. Interpreting public information as credit ratings, this suggests that widening ratings categories makes everyone better off. Our results are consistent with some empirical facts about asset management fees.

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1 Introduction

Delegated asset managers now hold the majority of securities issued by US firms—they are estimated to hold upwards of seventy percent both of publicly traded corporate debt and of publicly traded equity.[1] As in all delegation relationships, the relationship between delegated asset managers and their clients creates the potential for agency conflicts. For example, asymmetric information between delegated asset managers and clients may allow asset managers to make investments in their own interest rather than their clients' interests. However, in financial markets public information is created that may reduce this asymmetry of information. For example, credit ratings agencies announce bond ratings and securities analysts publish reports on equity. These public announcements provide clients with information about the assets that delegated asset managers invest in. It is an open question how this public information affects welfare and, thus, whether it should be regulated. We address these questions within the specific context of delegated asset management.

Delegated asset managers (e.g., mutual funds and pension funds) invest on behalf of their clients, but the incentives of these asset managers may not always be aligned with the incentives of their clients. Such a misalignment of incentives between an asset manager and his clients may create an agency problem, potentially most severe when clients are uninformed. Since public signals such as credit ratings and analysts' reports are intended to provide public information to investors, it seems plausible to conjecture that the presence of such public information can mitigate this agency problem. This, in turn, suggests that providing investors with more precise public information should increase their welfare. However, the theoretical model we develop to address this issue indicates that this intuition is not right. More precise public information can actually lower welfare. Our model demonstrates a new economic inefficiency caused by public information, which is present *even when the information is unbiased.* In our model, *increasing* the precision of unbiased public information makes all parties *worse* off.

To study the role of public information in delegated asset management, we set up an optimal contracting model in which a risk-averse principal (the client) delegates a decision to a risk-averse, informed agent (the asset manager). There are two key frictions in the model: first, agents have private information and, second, the principal and the agents differ in their attitudes toward risk. The agents' private information creates the motive for delegation and the difference in risk attitudes creates the misalignment of incentives.

[1]See Cai, Han, and Li (2012) for estimates for corporate debt and French (2008) for estimates for equity.

Below we study a general model first, in which players are risk averse[2] with general preferences (within the same HARA class) and the agent's action is also general, not restricted to a portfolio choice decision. In this setting, we show the following two main results: first, that a contract (i.e. asset management fees) written on final wealth alone can implement the efficient action for each realization of the credit rating and, second, that increasing the precision of the public signal makes everyone worse off.

We subsequently study a delegated portfolio choice problem in which players have more restricted utility functions, which is a special case of our general model. In this analysis we interpret the public information as a credit rating. The results in this section suggest that a policy maker concerned about the use of credit ratings in delegated asset management should consider regulating credit ratings to be less precise, for example by broadening ratings categories. Further, this example generates empirical predictions on both the form of delegated asset management fees and on the fluctuations of investors' expenses which are broadly consistent with those observed in practice. The assumptions in this section make our application to delegated asset management more concrete and allow us to make both a policy prescription and to generate some empirical content. Moreover, this specific example serves two other purposes. First, it allows us to find a closed-form solution for the optimal contract and, second, it allows us to write constructive proofs, in which we derive our results by explicit calculation.

The timing of the model is as follows: first, the public signal is realized and identical agents competitively offer contracts (i.e. asset management fees). Each agent's contract can depend on the final wealth and the agent's action, but not on the agent's private information—the agents offer the contracts before they learn their private information. Second, the principal decides which agent to employ to invest on his behalf. Third, the employed agent learns his private information and takes an action. Formally, the agent's private information pertains to the conditional distribution of final wealth given each possible action and is at least as informative as the public signal.[3] To be the portfolio choice application, the agent's private information pertains to his superior knowledge of the return distribution of a risky security and his action is the proportion of the principal's wealth invested in that risky security. Finally, wealth is realized and the principal and agent divide it according to the initial contract.

[2]We were motivated to focus on the problem in which the principal is risk-averse by our interest in the delegated asset management problem. In this application, clients may be relatively small and the investment they delegate may constitute a large fraction of their wealth. For this reason, the classical case of a risk-neutral principal is not appropriate for this application. Our focus on the case in which both the principal and the agents are risk averse is the source of our novel theoretical results.

[3]In an extension in Subsection 4.3.3, we relax this assumption, allowing the agent to learn from the public signal.

The first main result is that the contract that depends on final wealth alone solves the incentive problem. Under this contract, the agent takes the efficient action and provides efficient insurance to the principal, for each realization of the public signal. The reason that the optimal contract solves the incentive problem is that the contract that implements efficient risk sharing makes the principal and agent equally sensitive to the final payoff; since the only incentive problem comes from the difference in risk aversion, this optimal sharing rule aligns the agent's incentives with the principal's. Therefore, the principal can delegate the decision to the agent knowing that the agent will act in their joint interest given the contract is the efficient sharing rule. Put differently, the first-best action is incentive compatible. Thus, the public signal does not mitigate the incentive problem, because it is already solved by a contract written on final wealth alone.

Our second main result says that decreasing the precision of the public signal is Pareto improving; this implies that there is an inefficiency associated with precise public information. How can we reconcile this with the result described above that suggests that the agent never takes an inefficient action? We now explain that there is in fact no contradiction between these two results. We begin with a lemma that is a key intermediate step to understanding the mechanism at work behind our second main result. The lemma says that agents must break even for each realization of the public signal. The reason is simple: given the realization of the public signal, agents are effectively Bertrand competing in their contracts. Thus, because the agents offer their contracts after the signals are observed, it is impossible for an agent to break even only ex ante—on average across possible realizations of the public signal—but not ex post—for every realization of the public signal. If an agent were to offer such a contract, where he took a loss for some realization with an off-setting gain for another realization, another agent could undercut him when he made the gains, leaving him unemployed and making losses on average.

The second main result is that decreasing the precision of the public signal is Pareto improving. Since, by the lemma described above, agents receive the same payoff (their reservation utility) for each realization of the public signal, they do not bear any risk over the realization of the public signal. Therefore, the principal bears all of the risk associated with the public signal. In other words, the agents' competition prevents them from providing insurance to the principal. But, decreasing the precision of the public signal attenuates the negative welfare effects that result from the failure of insurance.

Typically, there is a trade-off between risk-sharing and incentives in principal-agent problems. In our set-up, the optimal contract, which depends on final wealth alone, solves both the risk-sharing and incentive problems given each realization of the public signal. However, since the agents offer contracts given the public signal, the optimal contract does not implement efficient risk-sharing over the realizations of the public signal. This is the Hirshleifer (1971) effect by which better public information prevents efficient risk-sharing. In this paper, we want to emphasize the following two insights. First, precise public information generates a Hirshleifer effect in delegated asset management, preventing asset managers from providing insurance to their clients. And, second, that optimal contracting on final wealth alone may mitigate the incentive problems between asset managers and their clients, suggesting that even unbiased public information may have a mainly negative role for delegated asset management, preventing insurance without mitigating incentive problems.

Our model may provide some useful insight into the specific role of credit ratings in the delegated asset management industry. We make this application concrete by considering a two-asset world with a riskless bond and a risky security. We interpret the agent's action as his portfolio choice decision between the two assets, the agent's private information as his knowledge of the distribution of the return of the risky security, and the public signal as the credit rating of the risky security. For this part of the paper, we restrict attention to the case in which both the principal and agent have quadratic utility (but still differ in their aversion to risk). In this setting we can solve not only for the optimal contract but also for the equilibrium action/portfolio weight in closed form. This allows us to establish the main results via explicit calculation. In particular, to show that decreasing the precision of the credit rating improves welfare, we write down the players' indirect utilities explicitly and compare them across different ratings partitions. Our application is more than an illustration of our theoretical analysis. It comes with a policy prescription: broaden ratings categories to improve risk sharing. Since contracting on credit ratings prevents the asset manager from bearing the risk in the ratings' realizations, this prescription allows portfolio managers to provide insurance as well as expertise to their clients.

The observed contract in our model is affine in final wealth. This is consistent with almost all mutual fund management fees, which are typically simple percentages of assets under management at each date (see, for example, Elton, Gruber, and Blake (2003)). One can interpret the constant term in the affine contract as the amount of initial wealth paid to the asset manager and the coefficient of the linear term as the percentage of portfolio returns that the asset manager charges his clients.

Further, in an extension we impose some additional structure on our model to try to generate further empirical content. In this extension, we interpret fund managers' fees more broadly to include not only asset management fees but also general operating expenses charged to investors. With this additional structure, our model suggests that the total fees paid from clients to asset managers are higher when the expected variance of the risky security is higher—which we argue corresponds to a recession.[4] This is consistent with the fact that bond funds have countercyclical expense ratios, which corresponds to total fund fees as a fraction of AUM being higher in recessions.

Our results stand in contrast to the findings in several papers that suggest that public signals are unambiguously welfare-improving in principal-agent settings with asymmetric information, notably Nalebuff and Scharfstein (1987), Cremer and McLean (1988), Riordan and Sappington (1988). These papers rely on large punishments (and, therefore, they rely implicitly on risk-neutrality) to implement the agent's truth-telling. Kessler, Lülfesmann, and Schmitz (2005) question these findings by including limited liability with endogenous punishments; they find that public information can decrease efficiency in some cases. We alter the set-up in a different way—in our model players are risk-averse and public information is verifiable ex interim rather than ex post—and we find that better public information is always welfare-decreasing.

In addition to the literature on contracting in the presence of a public signal, our paper relates to the literature on socially optimal group decision making (Amershi and Stoeckenius (1983), Pratt and Zeckhauser (1989), Pratt (2000), Wilson (1968)). This work typically does not study strategic behaviour. One exception is Wilson (1984), which studies a social planner who must induce agents to reveal private information. Wilson's theoretical insights guided us toward the solution for the optimal contract in our model, but he does not study endogenous contracting or public information. In early work on the principal-agent relationship, Ross (1973) uses similar techniques in a model in which the agent does not have private information. Ross (1973) explores the agency problem further; this paper includes an analysis of the problem in which the agent does have private information but the principal is not an expected utility maximizer, but rather has a maximin objective function (as in recent work on robust contracting, see, e.g., Carroll (2015) and Chassang (2013)).

Our application to asset management is related to the literature on delegated portfolio choice (Bhattacharya and Pfleiderer (1985), Dybvig, Farnsworth, and Carpenter (2010), Palomino and Prat (2003), Stoughton (1993)). None of these papers considers the role of public information, but Admati and Pfleiderer (1997) and He and Xiong (2013) do. Admati and

[4]We make this argument by mapping the expected variance conditional on the public information to the VIX index, which is strongly countercyclical.

Pfleiderer (1997) studies the role of performance benchmarks in a classical delegated investment setting and He and Xiong (2013) studies the role of penalties based on publicly observed market quantities (mainly based on tracking error) when the agent is a portfolio manager and the principal is a fund family.

There is also an active theory literature studying credit rating agencies (Bolton, Freixas, and Shapiro (2012), Bar-Isaac and Shapiro (2013), Boot, Milbourn, and Schmeits (2006), Fulghieri, Strobl, and Xia (2013), Kurlat and Veldkamp (2011), Manso (2014), Mathis, McAndrews, and Rochet (2009), Opp, Opp, and Harris (2013), Sangiorgi and Spatt (2014), Skreta and Veldkamp (2009)). The most related paper in this literature is Parlour and Rajan (2014) that suggests that contracting on credit ratings may mitigate the incentive problem between an asset manager and his client when contracts are incomplete. They go a step further than we do and find the returns on assets in a market equilibrium setting. Other papers study the coarseness of credit ratings partitions in strategic information transmission settings (Donaldson and Piacentino (2012), Goel and Thakor (2014), Doherty, Kartasheva, and Phillips (2012), Kartasheva and Yilmaz (2013), Kovbasyuk (2013)). These papers emphasize the incentives of credit ratings agencies. Our paper shows the normative benefit of coarse credit ratings, without addressing the source of the ratings; papers such as Lizzeri (1999) and Goel and Thakor (2014), in contrast, provide a positive rationale for why CRAs may issue coarse ratings in equilibrium. Our paper complements these findings by suggesting that the incentive distortions that lead to coarse ratings may in fact have a positive effect, at least for the specific case of delegated asset management we analyze.

Finally, we distinguish our paper from papers that study contracting with moral hazard, which often draw contrasting conclusions about the welfare value of public information. For example, Chaigneau, Edmans, and Gottlieb (2014a, 2014b) provide a detailed study of the costs and benefits of public information for contracting within a setting similar to Innes (1990).

The paper proceeds as follows. In Section 2 we introduce the general model and in Section 3 we present the results of the general model. Section 4 presents the portfolio choice application and discusses some extensions, policy prescriptions, and empirical predictions. Section 5 concludes.

2 Model

The model constitutes an extensive game of incomplete information in which agents first compete in contracts in the hope of being employed by a single principal and then the employed agent takes an action of behalf of the principal.

2.1 Players

There is a single principal with a unit wealth and von Neumann–Morgenstern utility u_{P} and at least two competitive agents with von Neumann–Morgenstern utility u_{A} and outside option $\bar{u}$. The principal and the agents differ in their risk aversion. We make no assumption as to whether the principal or the agent is more risk averse, but, for the proof of our main result, we require that both utility functions are in the same class of hyperbolic absolute risk-aversion. Specifically, their absolute risk tolerances are affine with the same slope,

$$-\frac{u'_{\mathrm{P}}(w)}{u''_{\mathrm{P}}(w)} = a_{\mathrm{P}} + bw \tag{1}$$

and

$$-\frac{u'_{\mathrm{A}}(w)}{u''_{\mathrm{A}}(w)} = a_{\mathrm{A}} + bw \tag{2}$$

for $a_i > -bw$ for all w and for $i \in \{\mathrm{P}, \mathrm{A}\}$.[5] Note that this assumption imposes no restriction on the magnitude of the difference between the principal's and agent's risk aversions. When we consider the application to delegated asset management (Section 4) we assume that players have quadratic utility; quadratic utility satisfies conditions (1) and (2) with $b = -1$.

Agents have private information, captured by their "type" σ. A public signal ρ conveys information about σ. In the application to delegated asset management, σ represents agents' expert knowledge about the risk of the market securities they invest in and ρ represents, for example, the securities' credit ratings or an equity analysts' reports.

2.2 Actions and Contracts

The principal wishes to delegate investment to an agent because he is better informed. The agent will take an action x that affects the distribution of wealth $\tilde{w}(x)$ that the principal and agent will divide ex post. However, the principal anticipates a misalignment of investment incentives since his risk aversion differs from the agents'.

Contracts attempt to align incentives to mitigate the downside of delegating investment. Each agent a offers contract Φ_a which may depend on the final wealth w, and his action x.

[5] For example, when $b = 0$ conditions (1) and (2) imply that the principal and the agents have exponential utility with constant coefficients of absolute risk aversion a_{P}^{-1} and a_{A}^{-1}.

The agent chooses x after he has entered the contract. The action choice affects only the distribution of the final wealth $\tilde{w}(x)$. We assume that $\tilde{w}$ is a concave function of x for every state of the world. In our portfolio management application in Section 4, we interpret x as the proportion of wealth invested in an asset. Note that the agent's type σ does not enter the contract because it is not verifiable; however, ρ may enter the contract as a proxy.

2.3 Timing

After ρ is realized, the agent announces the contract and the the principal employs an agent who chooses x after learning σ. Then, wealth realizes and players divide it according to the initial contract. Formally, the timing is as follows:

1. ρ is realized.
2. Agents simultaneously offer contracts Φ_a.
3. σ is realized.
4. The principal observes ρ and the profile of contracts $\{\Phi_a\}_a$ and hires an agent a^*.
5. Agent a^* chooses x.
6. Final wealth realizes and it is distributed such that agent a^* is awarded $\Phi_{a^*}(w, x)$ and the investor keeps $w - \Phi_{a^*}(w, x)$.

Note that key to our timing is that players learn ρ before the agent offers the contract and the principal employs an agent. In Section 4.3.4, we demonstrate that our results are robust to the inclusion of a second public signal that realizes after the agent has been employed. Note that since no agent takes an action between when the contracts Φ_a are offered and when the investment x is chosen, the model is equivalent to one in which the agent learns his type only after having been employed by the principal. In fact, we will show below that without the public signal, this game is equivalent to a classical principal-agent problem for each realization of the public signal. It is, however, important that the agent learns his private information after contracting; this allows us to abstract from the possibility that agents use contracts as signals of their private information.

In a previous version of this paper we considered the alternative timing in which ρ was realized after the agent offered the contract and the contract could be contingent on ρ. That version yielded effectively identical results and is available upon request. We believe the current timing to be more realistic since credit ratings and analysts' reports are always available, including at the time in which asset managers set their fees.

2.4 Note on Notation

We frequently omit the arguments of variables. The contract Φ always depends on wealth w and the agent's action x, as well as the offering agent a, but we frequently write just $\Phi(w)$. The agent chooses the action given his type σ, but we usually write just x for $x(\sigma)$. Later we will introduce a social planner's problem, in which the welfare function places weight μ_ρ on the agent given the realization ρ of the public signal. We sometimes suppress this dependence and write μ for μ_ρ. Finally, the social planner's sharing rule φ depends on final wealth directly and on the public signal indirectly via the welfare weight. While we sometimes write formally $\varphi_{\mu_\rho}(w)$, we frequently abbreviate to $\varphi_\mu(w)$ or even just $\varphi(w)$. Also, let us emphasize that for a random variable $\tilde{\rho}$, $\sigma(\tilde{\rho})$ indicates the sigma-algebra generated by $\tilde{\rho}$. This should not be confused with the agent's type σ.

3 Results

3.1 Agents Break Even for Every Realization of the Public Signal

We first show that agents must break even for every realization of the public signal. This will allow us to transform our game into a family of principal-agent problems, one for each realization of the public signal. That is to say that for every realization of the public signal the agent must offer the contract that maximizes the principal's utility and assures him at least his reservation payoff.

LEMMA 3.1. *The employed agent a^* breaks even for each realization ρ of the public signal, or*

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\Phi_{a^*}(\tilde{w})\Big)\,\Big|\,\tilde{\rho}=\rho\right]=\bar{u}$$

for all ρ.

Proof. The proof is in Appendix A.1.1. □

That agents receive their reservation utility in equilibrium is unsurprising because they are competitive. The takeaway from Lemma 3.1 above is that agents receive their reservation utility for *every realization of the public signal*. There cannot be an equilibrium in which agents break even in expectation over all possible realizations unless they break even for every realization. In fact, if that is the case, then an agent who receives less than his reservation utility for some realization of the public signal must receive in excess of his reservation utility for another realization. But since the agent is getting strictly in excess of his reservation utility for this realization, another agent can undercut him by offering a contract that grants him more than his reservation utility and allocates more of the surplus to the principal.

The proof is by contradiction. It is standard except for one subtlety. We first suppose that an agent receives strictly in excess of his reservation utility for some realization of the signal. This agent must therefore be employed given this realization. But then another agent, otherwise unemployed and receiving his reservation utility, would undercut the employed agent for this realization of the signal. Therefore, à la Bertrand competition, the agents must break even given this realization. The only subtlety of the proof is that agents' contracts affect their incentives and hence their actions. Thus, when a deviant agent offers the principal a contract, the principal must take the effect of this contract on the agent's action into account. Our proof circumvents this issue by constructing a deviation that preserves the incentives of the originally employed agent while allocating more surplus to the principal. Specifically, if the supposed equilibrium contract is Φ, then the deviation $\Phi_\varepsilon(w) := u_{\mathrm{A}}^{-1}\big(u_{\mathrm{A}}\big(\Phi(w)\big) - \varepsilon\big)$ preserves the employed agent's incentives and allocates more of the surplus to the principal.

The argument in the proof of Lemma 3.1 also implies that the contract must maximize the principal's utility for every realization of the signal ρ as is summarized in Corollary 3.1 below. The reason is that if the employed agent does not maximize the principal's utility, then another agent can deviate to a contract more favorable to the principal that also leaves him a small surplus above $\bar{u}$.

COROLLARY 3.1. *If Φ_{a^*} is the contract of the employed agent a^* given realization $\hat{\rho}$ and there is another contract $\hat{\Phi}$ such that*

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w} - \hat{\Phi}\big(\tilde{w}\big)\Big) \mid \tilde{\rho} = \hat{\rho}\right] > \mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w} - \Phi_{a^*}\big(\tilde{w}\big)\Big) \mid \tilde{\rho} = \hat{\rho}\right],$$

then it must be that

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}\big(\tilde{w}\big)\Big) \mid \tilde{\rho} = \hat{\rho}\right] < \bar{u}.$$

3.2 Principal-Agent Formulation

Lemma 3.1 and Corollary 3.1 taken together say that the principal chooses the contract that maximizes his expected utility subject to the constraint that the agent receives his reservation utility for every realization of the signal ρ. That is to say that the equilibrium contract solves the principal-agent problem for every ρ. The twist on a standard principal-agent problem is that the agent's participation constraint depends on the public signal.

PROPOSITION 3.1. *For each realization ρ of the public signal, the contract of the employed agent $a^*(\rho)$ solves the following principal-agent problem:*

$$\begin{cases} \quad \textit{Maximize} \;\; \mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(x) - \Phi\big(\tilde{w}(x), x\big)\Big) \,\big|\, \tilde{\rho} = \rho\right] \\[2ex] \quad \textit{subject to} \;\; \mathbb{E}\left[u_{\mathrm{A}}\Big(\Phi\big(\tilde{w}(x), x\big)\Big) \,\big|\, \tilde{\rho} = \rho\right] = \bar{u} \quad \textit{and} \\[2ex] x \in \arg\max\left\{\mathbb{E}\left[u_{\mathrm{A}}\big(\Phi\big(\tilde{w}(\xi), \xi\big) \,\big|\, \tilde{\sigma} = \sigma\right] \;\; ; \;\; \xi \in \mathbb{R}\right\} \end{cases} \tag{3}$$

over contract Φ.

3.3 Equilibrium Contract as the Solution of a Social Planner's Problem

For each realization of the public signal ρ, we transform the principal-agent problem into a social planner's problem. The social planner will maximize social welfare subject to the agent's incentive compatibility constraint. Call the agent's welfare weight μ_ρ for a given ρ. This will coincide with the Lagrange multiplier on the agent's participation constraint in the principal-agent problem for a given ρ. This approach allows us to eliminate the agent's participation constraints temporarily to focus on incentive compatibility.

Now use the method of Lagrange multipliers to eliminate the participation constraint and say that the problem is to maximize

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(x) - \Phi\big(\tilde{w}(x), x\big)\Big) + \mu_\rho\Big[u_{\mathrm{A}}\Big(\Phi\big(\tilde{w}(x), x\Big) - \bar{u}\Big] \,\Bigg|\, \tilde{\rho} = \rho\right]$$

subject to

$$x \in \arg\max\left\{\mathbb{E}\left[u_{\mathrm{A}}\Big(\Phi\big(\tilde{w}(\xi), \xi\Big) \,\Big|\, \tilde{\sigma} = \sigma\right] \;\; ; \;\; \xi \in \mathbb{R}\right\}$$

over contract Φ.

For any Lagrange multiplier μ_ρ, the problem is equivalent to the social planner's problem with welfare weight μ_ρ associated with the agent. That is to say that, for given μ_ρ, we can omit the agent's outside option $\bar{u}$ and solve the following social planner's problem for Φ and x:

$$\begin{cases} \text{Maximize} \;\; \mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(x) - \Phi\big(\tilde{w}(x), x\big)\Big) + \mu_\rho u_{\mathrm{A}}\Big(\Phi\big(\tilde{w}(x), x\big) \;\Big|\; \tilde{\rho} = \rho \right] \\ \\ \text{subject to} \;\; x \in \arg\max \left\{ \mathbb{E}\left[u_{\mathrm{A}}\Big(\Phi\big(\tilde{w}(\xi), \xi\big) \;\Big|\; \tilde{\sigma} = \sigma \right] \;;\; \xi \in \mathbb{R} \right\}. \end{cases} \tag{4}$$

Below we solve the problem for a generic Lagrange multiplier and only later we use the agent's binding participation constraint to solve for μ_ρ for each ρ. Transforming the game into a social planner's problem reveals that the task is to trade off efficient risk sharing against implementing efficient investment.

3.4 The Efficient Sharing Rule Implements Efficient Investment

We now find the contract that solves the social planner's problem. We do this by characterizing the first-best contract and action—i.e. those that the social planner would choose if he had perfect information. We then show that given the first-best contract the first-best action is incentive compatible, so the solution to the social planner's problem coincides with the first-best outcome. Thus, in fact, there is no tension between risk sharing and efficient investment in equilibrium.

PROPOSITION 3.2. *If the contract is the efficient sharing rule, then the incentive compatible action is the social optimum.*

Namely, if φ maximizes

$$u_{\mathrm{P}}(w - \varphi) + \mu_\rho u_{\mathrm{A}}(\varphi)$$

then

$$x \in \arg\max \left\{ \mathbb{E}\left[u_{\mathrm{A}}\Big(\varphi\big(\tilde{w}(\xi)\big)\Big) \;\Big|\; \tilde{\sigma} = \sigma \right] \;;\; \xi \in \mathbb{R} \right\}$$

implies

$$x \in \arg\max \left\{ \mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(\xi) - \varphi\big(\tilde{w}(\xi)\big)\Big) + \mu_\rho u_{\mathrm{A}}\Big(\varphi\big(\tilde{w}(\xi)\big)\Big) \;\Big|\; \tilde{\sigma} = \sigma \right] \;;\; \xi \in \mathbb{R} \right\}.$$

Proof. The proof is in Appendix A.1.2. □

The main takeaway of Proposition 3.2 is that for any ρ the efficient contract implements the efficient action.

In the proof we first find the efficient φ. We then demonstrate that, given this φ, the agent would choose the social optimum. That is to say that the action that the agent chooses coincides with the action a social planner would choose if he had the agent's private information.

To understand the connection between incentive alignment and risk sharing, recall that a sharing rule φ is efficient if it maximizes $u_{\mathrm{P}}(w - \varphi) + \mu_\rho u_{\mathrm{A}}(\varphi)$ for each realization of w or

$$u_{\mathrm{P}}'(w - \varphi(w)) = \mu_\rho u_{\mathrm{A}}'(\varphi(w)). \tag{5}$$

On the other hand, the sharing rule φ aligns the incentives of the principal and the agent globally if one's utility function is a positive affine transformation of the other's utility function given the sharing rule φ, or

$$u_{\mathrm{P}}(w - \varphi(w)) = \alpha u_{\mathrm{A}}(\varphi(w)) + \beta$$

for some $\alpha > 0$ and $\beta \in \mathbb{R}$. Differentiating this condition with respect to w gives

$$u_{\mathrm{P}}'(w - \varphi(w)) = \frac{\alpha\varphi'(w)}{1 - \varphi'(w)} u_{\mathrm{A}}'(\varphi(w)).$$

This last condition coincides with the condition above of efficient risk sharing (condition (5)) exactly when $\mu_\rho = \dfrac{\alpha\varphi'(w)}{1 - \varphi'(w)}$, which is possible if and only if φ' is a constant or φ is affine. The only remaining step in the argument is to show that the efficient sharing rule is affine for the preferences we consider, which we show in Lemma A.1 in Appendix A.1.2.

3.5 Coarser Public Signals Are Pareto-Improving

Proposition 3.2 shows that the optimal contract eliminates the incentive problem for every σ and the risk sharing problem for every ρ. The problem remains to share risk over realizations of the public signal. The next result states that less precise public signals Pareto dominate more precise public signals. The reason is that the public signal does not mitigate the incentive problem but only hinders risk sharing.

From now on, since the optimal contract solves the incentive problem, we omit incentive constraints and focus directly on the social planner's problem (with complete information) as per Proposition 3.2.

Proposition 3.3. *Coarser public signals Pareto-dominate finer ones: for any signal $\tilde{\rho}_c$ and $\tilde{\rho}_f$ such that $\sigma(\tilde{\rho}_c) \subset \sigma(\tilde{\rho}_f)$, the ex ante equilibrium utility of all players is weakly higher given $\tilde{\rho}_c$ rather than $\tilde{\rho}_f$.*

Proof. Below we omit the dependence of φ on x because, by Proposition 3.2, the efficient x is chosen for every σ independently of ρ. Below call $\varphi_{\mu_{\rho_f}}$ and $\varphi_{\mu_{\rho_c}}$ the efficient sharing rules associated with fine and coarse public signals respectively.

First, the agent's participation constraint given $\tilde{\rho}_f$ is

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\varphi_{\mu_{\rho_f}}(\tilde{w})\right) \,\middle|\, \tilde{\rho}_f\right] = \bar{u}.$$

Now, since $\sigma(\tilde{\rho}_c) \subset \sigma(\tilde{\rho}_f)$, use the law of iterated expectations and the condition above to observe that

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\varphi_{\mu_{\rho_f}}(\tilde{w})\right) \,\middle|\, \tilde{\rho}_c\right] = \mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{A}}\left(\varphi_{\mu_{\rho_f}}(\tilde{w})\right) \,\middle|\, \tilde{\rho}_f\right] \,\middle|\, \tilde{\rho}_c\right] = \mathbb{E}\left[\bar{u} \,\middle|\, \tilde{\rho}_c\right] = \bar{u}.$$

This says that $\varphi_{\mu_{\rho_f}}$ satisfies the participation constraint given ρ_c. Since $\varphi_{\mu_{\rho_c}}$ solves the principal-agent problem given ρ_c—viz. it maximizes the principal's utility given the agent's participation constraint—

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w} - \varphi_{\mu_{\rho_c}}(\tilde{w})\Big) \,\middle|\, \tilde{\rho}_c\right] \geq \mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w} - \varphi_{\mu_{\rho_f}}(\tilde{w})\Big) \,\middle|\, \tilde{\rho}_c\right].$$

Now we use the inequality above and we apply the law of iterated expectations again to prove that the principal is better off given the coarser public signal, namely

$$\begin{aligned}
\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w} - \varphi_{\mu_{\rho_c}}(\tilde{w})\Big)\right] &= \mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w} - \varphi_{\mu_{\rho_c}}(\tilde{w})\Big) \,\middle|\, \tilde{\rho}_c\right]\right] \\
&\geq \mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w} - \varphi_{\mu_{\rho_f}}(\tilde{w})\Big) \,\middle|\, \tilde{\rho}_c\right]\right] = \mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w} - \varphi_{\mu_{\rho_f}}(\tilde{w})\Big)\right].
\end{aligned}$$

Since agents always break even and the principal is better off with coarser public signals, $\tilde{\rho}_c$ Pareto dominates $\tilde{\rho}_f$. □

The main step of the proof is to show that a contract that is feasible given a fine signal structure is also feasible given a coarse signal structure. This follows directly from the law of iterated expectations. Since coarsening the signal structure expands the set of feasible contracts, it can only increase the principal's objective (recall that the incentive constraints are not binding, by Proposition 3.2). Since the agent always breaks even, increasing the principal's profits constitutes a Pareto improvement.

The intuition behind this result comes from Lemma 3.1. Because competition makes agents break even state-by-state, there is one participation constraint for each realization of the public signal. Thus, with a finer signal structure there are more realizations of the public signal and, thus, more constraints on the principal's objective. Because we know from Proposition 3.2 that the efficient action is always taken, these constraints restrict only risk sharing between the principal and the agent. A finer signal structure shuts down risk sharing and reduces welfare.

4 An Example: Portfolio Choice with Quadratic Utility

To fix ideas we consider the specific case of portfolio choice with quadratic utility. This example allows us to solve the model explicitly and thus it exposes the forces behind the more general proofs above.

4.1 Setup

The portfolio choice model has a risk-free bond with gross return R_f and a risky asset with random gross return $\tilde{R}$. The agent's private information σ will be the standard deviation of $\tilde{R}$ and ρ will be an imperfect public signal about this risk parameter. Call ρ the credit rating of the risky security.[6] The agent's action x represents the proportion of wealth invested in the risky security; therefore,

$$\tilde{w}(x) = R_f + x\big(\tilde{R} - R_f\big).$$

We assume that all players have quadratic utility,

$$u_i(w) = -\frac{1}{2}\big(a_i - w\big)^2$$

for $i \in \{\mathrm{A}, \mathrm{P}\}$. The principal differs from the agents in his risk aversion. Note that the coefficient of absolute risk tolerance is $a_i - w$, so these utility functions are in the same class

[6]In reality, credit ratings indicate the default probability of a bond. Typically, increasing the default probability will simultaneously increase the variance and decrease the expectation of returns. (In a model of a zero-coupon bond that pays off zero in the event of default, this statement is precise so long as the default probability is less than half.) To simplify the calculations, we assume that ratings are not informative about expected returns. This simplification is innocuous, because the full model above shows that all the qualitative results below hold very generally, including in situations in which ratings are informative, for example, about both the mean and variance. See footnote 8 for further discussion of this assumption.

of hyperbolic absolute risk aversion as defined in equations (1) and (2).[7]

We make some restrictions on the distribution of $\tilde{R}$ to simplify the belief updating. We assume that the mean return $\bar{R}$ of the risky asset is known and independent of the agent's private information σ.[8] In fact, since with quadratic utility players' expected utility depends on only the mean and variance of the distribution, all relevant asymmetric information is about the variance σ^2. Note that this assumption implies that the credit rating is informative only about the asset's risk and not about its expected return,

$$\mathbb{E}\left[\tilde{R} \,\middle|\, \tilde{\rho} = \rho\right] = \mathbb{E}\big[\tilde{R}\big].$$

With quadratic utility, players' marginal utility is decreasing when their wealth is large. We restrict parameters to ensure that players' wealth is not so large. In particular, it must be that the wealth of the principal and that of the agent are not too large, or, respectively,

$$w - \Phi(w) < a_{\mathrm{P}} \tag{6}$$

and

$$\Phi(w) < a_{\mathrm{A}}. \tag{7}$$

These conditions depend on endogenous quantities; however, the following technical assumption on primitives is a sufficient condition to guarantee they will hold in equilibrium:

$$\big(\bar{R} - R_f\big)\big(R - \bar{R}\big) \leq \sigma^2 \tag{8}$$

for all pairs (σ, R).[9]

[7]In some delegation contexts, the agent's limited liability is an important friction. In fact, some classes of delegated asset managers, such as hedge funds and private equity funds, are typically highly levered and, hence, limited liability protection may lead them to act as if they are risk loving, not risk averse. Mutual funds, however, are prohibited from issuing debt (for US regulations, see Section 18(f) of the Invest Company Act of 1940) and, thus, they do not benefit from limited liability because they have no debt upon which to default. Therefore, we think about the agents in the example as mutual funds or similarly unlevered investment funds.

[8]To clarify this assumption, consider a concrete example. It may be that all investors know that the risky security has normally distributed returns with expectation $\bar{R}$ of 5 percent, but the standard deviation σ may be unknown. For example, the standard deviation could be either 25 percent or 35 percent. The assumption we make is that learning whether the standard deviation is 25 percent or 35 percent does not change the expected return of 5 percent. We emphasize that all our main qualitative results hold much more generally, as derived above, so we make this assumption here only for computational convenience.

[9]Condition (8) comes from solving the game assuming that the agent's participation constraint binds, then writing a sufficient condition for it to bind in light of the equilibrium.

4.2 Results

4.2.1 Competition Is Rating-by-Rating

Lemma 3.1 implies that agents must break even for each realization of the credit rating. Recall that the reason is that competition in contracts is Bertrand-like in the sense that the employed agent will receive his reservation utility conditional on any realization of the credit rating $\tilde{\rho}$; further agents act so as to maximize the investor's expected utility conditional on every ρ subject to their participation constraints.

The proof of Lemma 3.1 is in Appendix A.1.1, but re-iterating the main argument with these specific utility functions can clarify the proof. Recall that the proof is by contradiction. Supposing an equilibrium in which an agent receives in excess of his reservation utility for some realization of the public signal, a deviating agent can undercut him. However, we must be careful to take into account the effect of the new contract on the agent's incentives. We construct a deviation that does not distort incentives. With the current utility specification we can write it explicitly. In particular, if the initial contract given the rating $\hat{\rho}$ is $\hat{\Phi}$, the contract for $\varepsilon > 0$ is

$$\hat{\Phi}_{\varepsilon}(w) := u_{\mathrm{A}}^{-1}\Big(u_{\mathrm{A}}\Big(\hat{\Phi}(\tilde{w}) - \varepsilon\Big)\Big) = a_{\mathrm{A}} - \sqrt{\big(a_{\mathrm{A}} - \hat{\Phi}(w)\big)^2 - 2\varepsilon},$$

which gives the agent identical incentives to Φ and allocates more surplus to the principal.

4.2.2 Principal-Agent Formulation and Social Planner's Problem

Lemma 3.1 asserts that agents compete rating-by-rating, maximizing investor welfare subject to their participation constraints. That is to say that for every realization ρ of the credit rating, the contract of the employed agent and the corresponding portfolio weight solve the principal-agent problem of Proposition 3.1. Using the method of Lagrange multipliers we can transform the principal-agent problem into the social planner's problem summarized by the system (4). Now, unlike in the general case, we can compute simple expressions not only for the optimal contract but also for the agent's action x and the Lagrange multiplier/welfare weight μ_{ρ}.

4.2.3 The Efficient Sharing Rule Implements Efficient Investment

First, find the optimal sharing rule using the first-order condition in equation (5),

$$u'_{\mathrm{P}}\big(w - \varphi_\mu(w)\big) = \mu\, u'_{\mathrm{A}}\big(\varphi_\mu(w)\big),$$

or, for quadratic utility,

$$w - \varphi_\mu(w) - a_{\mathrm{P}} = \mu\big(\varphi_\mu(w) - a_{\mathrm{A}}\big)$$

for all w. Thus the efficient sharing rule is

$$\varphi_\mu(w) = a_{\mathrm{A}} + \frac{w - a_{\mathrm{P}} - a_{\mathrm{A}}}{1+\mu}. \tag{9}$$

Observe that the standard deviation σ does not enter the expression, and thus that the social planner need not know the true variance to implement optimal risk sharing.

Given the optimal sharing rule, we now calculate the first-best investment in the risky security x^* in the sense that x^* is the investment that the social planner would make if he knew the standard deviation σ. The first-best will be useful in finding the solution to the second-best problem in which the social planner knows only ρ and the agent chooses x. This x in turn constitutes the equilibrium allocation of the model. The reason that it is useful to compute the first-best outcome is that we proceed to show that it is an attainable outcome of the second-best problem. In other words, we solve the game by showing that the social optimum is attainable.

The social planner finds x^* by computing the maximum of

$$\begin{aligned}\mathbb{E}\left[u_{\mathrm{P}}\left(R_f + x(\tilde{R} - R_f) - \varphi_\mu\Big(R_f + x(\tilde{R} - R_f)\Big)\right) \,\middle|\, \tilde{\sigma} = \sigma\right]& \\ + \mu\mathbb{E}\left[u_{\mathrm{A}}\left(\varphi_\mu\Big(R_f + x(\tilde{R} - R_f)\Big)\right) \,\middle|\, \tilde{\sigma} = \sigma\right],&\end{aligned} \tag{10}$$

over all x. Mechanical computations collected in Appendix A.2.1 reveal that the optimal investment is

$$x^*(\sigma) = \frac{\big(\bar{R} - R_f\big)\big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\big)}{\sigma^2 + \big(\bar{R} - R_f\big)^2}. \tag{11}$$

Note that the optimal investment does not depend on the welfare weight μ, thus the social planner chooses the same x^* for all μ, even as $\mu \to \infty$. But, now, in the limit as $\mu \to \infty$, since in this case the social planner puts all the weight on the agent, his objective coincides with the agent's. Put differently, if the contract is the efficient sharing rule, the agent always takes the socially optimal action. This observation implies Proposition 3.2 in the context of this example.

4.2.4 The Break-even Welfare Weight and Ex Ante Utility

Now we can characterize the employed agent's contract explicitly by finding the Lagrange multiplier μ_ρ for each ρ. For a given contract φ_{μ_ρ} the agent must break even, so we can determine μ_ρ directly from the agent's participation constraint:

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\varphi_{\mu_\rho}\left(R_f + x(\tilde{\sigma})\big(\tilde{R} - R_f\big)\right)\right) \middle| \tilde{\rho} = \rho\right] = \bar{u}. \tag{12}$$

This equation, combined with the closed-form expressions for φ_{μ_ρ} and $x^*(\sigma)$ above, allows us to compute μ_ρ in closed-form. A string of calculations employing the law of iterated expectations (cf. Appendix A.2.2), says

$$\big(1 + \mu_\rho\big)^2 = \frac{\big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\big)^2}{2\,|\bar{u}|}\,\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2} \middle| \tilde{\rho} = \rho\right]. \tag{13}$$

This formula will be useful to express the ex ante utility of the principal and then to see constructively how changing the coarseness of the ratings partition affects investor welfare. In particular, within the framework of the example, we will be able to provide a less abstract proof of Proposition 3.3.

Before we proceed to the welfare analysis, we highlight one insight that the expression for the Lagrange multiplier offers. The mapping

$$\tilde{\sigma}^2 \mapsto \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \big(\tilde{R} - R_f\big)^2}$$

under the expectation operator in equation (12) is concave, so that if the distribution of $\tilde{\sigma}^2$ spreads out (for example in the sense of second-order stochastic dominance), then μ_ρ decreases. This suggests that the more distribution risk the agent faces, the less the investor must compensate him despite his risk aversion, as captured by the social planner's lower welfare weight. This observation presents a puzzle: why would the agent, who is risk-averse, prefer a riskier distribution?

The puzzle finds its resolution in the observation that higher dispersion of the variance comes with option value, and thus convexity, making him risk-loving over this kind of risk. The reason is that his investment decision comes after the realization of the variance, and thus the riskier decisions come with option value allowing him to adjust his investment decision to market conditions: when σ^2 is very low he will invest a lot in the risky asset, while when it is high he will invest relatively more in the riskless bond.

Now return to the main analysis. To analyze welfare we use the equilibrium welfare weight to find a formula for the investor's equilibrium expected utility given the rating ρ,

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)\,\Big|\,\tilde{\rho}=\rho\right]=\bar{u}\,\mu_{\rho}^{2} \tag{14}$$

(see Appendix A.2.3 for the short calculation). Thus his ex ante expected utility:

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)\right]=\bar{u}\,\mathbb{E}\left[\mu_{\tilde{\rho}}^{2}\right]. \tag{15}$$

4.2.5 Coarser Credit Ratings Are Pareto-Improving

Since competition means that agents always receive their reservation utilities, the main result that coarsening credit ratings makes everyone better-off follows from comparing the ex ante expected utility of the investor across ratings systems.[10] To do this we use formula (15) above combined with the connection between convex functions, second-order stochastic dominance, and the law of iterated expectations.

Within the setting of the example, we can now provide a constructive proof for Proposition 3.3 above, which says that coarse credit ratings Pareto dominate finer ones.

Our proof has two main steps. We summarize these steps briefly before giving the full proof. The first step is to show that the investor's ex ante expected utility is minus the expectation of a convex function,

$$\bar{u}\,\mathbb{E}\left[\mu_{\tilde{\rho}}^{2}\right]=-c\,\mathbb{E}\left[f\Big(\mathbb{E}\left[Y\mid\tilde{\rho}\right]\Big)\right]$$

for (appropriately defined) $c>0$, $f''>0$ and a random variable Y. The second step is to show that the expectation conditional on coarse ratings second-order stochastically dominates

[10] Refinements of credit ratings partitions occur in reality. For example, in 1982 Moody's added numerical modifiers to its ratings, thereby refining its ratings partition. See Kliger and Sarig (2000) for analysis of this event.

the expectation conditional on fine ratings,

$$\mathbb{E}\left[Y \mid \tilde{\rho}_c\right] \overset{\text{SOSD}}{\succ} \mathbb{E}\left[Y \mid \tilde{\rho}_f\right].$$

Whence utility is greater under coarse ratings because minus a convex function is a concave function, and, à la risk aversion, the expectation of a concave function of a stochastically dominated random variable is greater than the expectation of the function of the dominated random variable.

Step 1: Rewrite the investor's ex ante expected utility:

$$\begin{aligned}
\bar{u}\,\mathbb{E}\left[\mu_{\tilde{\rho}}^2\right] &= \bar{u}\,\mathbb{E}\left[\left(\sqrt{\frac{(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f)^2}{2|\bar{u}|}\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+(\bar{R}-R_f)^2}\,\middle|\,\tilde{\rho}\right]}-1\right)^2\right] \\
&= \frac{\bar{u}(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f)^2}{\sqrt{2|\bar{u}|}}\mathbb{E}\left[\left[\sqrt{\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+(\bar{R}-R_f)^2}\,\middle|\,\tilde{\rho}\right]}-1\right]^2\right] \\
&= -c\,\mathbb{E}\left[f\Big(\mathbb{E}\left[Y|\tilde{\rho}\right]\Big)\right]
\end{aligned}$$

where

$$\begin{aligned}
c &:= \sqrt{|\bar{u}|/2}\,(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f)^2, \\
f(z) &:= \left(\sqrt{z}-1\right)^2,
\end{aligned}$$

and

$$Y := \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+\left(\bar{R}-R_f\right)^2}.$$

Note that $c>0$ and $f''(z) = z^{3/2}/2 > 0$.

Step 2: By definition,

$$\mathbb{E}\left[Y \mid \tilde{\rho}_c\right] \overset{\text{SOSD}}{\succ} \mathbb{E}\left[Y \mid \tilde{\rho}_f\right]$$

if there exists a random variable $\tilde{\varepsilon}$ such that

$$\mathbb{E}\left[Y \mid \tilde{\rho}_f\right] = \mathbb{E}\left[Y \mid \tilde{\rho}_c\right] + \tilde{\varepsilon}$$

and

$$\mathbb{E}\left[\tilde{\varepsilon}\,\middle|\,\mathbb{E}\left[Y \mid \tilde{\rho}_c\right]\right] = 0.$$

For $\tilde{\varepsilon} = \mathbb{E}[Y \mid \tilde{\rho}_f] - \mathbb{E}[Y \mid \tilde{\rho}_c]$ from the above, the condition is

$$\mathbb{E}\left[\mathbb{E}[Y \mid \tilde{\rho}_f] - \mathbb{E}[Y \mid \tilde{\rho}_c] \,\middle|\, \mathbb{E}[Y \mid \tilde{\rho}_c]\right] = 0$$

or

$$\mathbb{E}\left[\mathbb{E}[Y \mid \tilde{\rho}_f] \,\middle|\, \mathbb{E}[Y \mid \tilde{\rho}_c]\right] = \mathbb{E}[Y \mid \tilde{\rho}_c].$$

Given the assumption $\sigma(\tilde{\rho}_c) \subset \sigma(\tilde{\rho}_f)$ and since conditioning destroys information—$\sigma\big(\mathbb{E}[Y \mid \tilde{\rho}_c]\big) \subset \sigma(\tilde{\rho}_c)$—apply the law of iterated expectations firstly to add and then to delete conditioning information to calculate that

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}[Y \mid \tilde{\rho}_f] \,\middle|\, \mathbb{E}[Y \mid \tilde{\rho}_c]\right] &= \mathbb{E}\left[\mathbb{E}\left[\mathbb{E}[Y \mid \tilde{\rho}_f] \,\middle|\, \tilde{\rho}_c\right] \,\middle|\, \mathbb{E}[Y \mid \tilde{\rho}_c]\right] \\
&= \mathbb{E}\left[\mathbb{E}[Y \mid \tilde{\rho}_c] \,\middle|\, \mathbb{E}[Y \mid \tilde{\rho}_c]\right] \\
&= \mathbb{E}\left[Y \,\middle|\, \tilde{\rho}_c\right],
\end{aligned}$$

as desired.

4.3 Empirical Predictions and Extensions

Our main results focus on welfare and, as a result, are difficult to compare with empirical facts. However, in this section, we do our best to argue that our findings are consistent with some stylized facts about delegated asset management contracts. First, we argue that the contracts that agents offer in our model resemble real world asset management fees. We then impose more structure on the public information so that it more closely resembles an asset's credit rating; we then perform comparative statics on agents' fees as a function of the expected variance of the risky asset. This generates empirical predictions consistent with changes in bond funds' total fees over the business cycle.

At the end of this section, we consider two extensions to verify the robustness of our results. First, we relax the assumption that the asset manager has strictly superior information than that contained in the rating and, second, we allow for additional changes in ratings.

4.3.1 Asset Manager's Observed Contracts

The agent's equilibrium contract is

$$\varphi_{\mu_\rho}(w) = a_{\mathrm{A}} + \frac{w - a_{\mathrm{P}} - a_{\mathrm{A}}}{1 + \mu_\rho} \tag{16}$$

$$= \frac{\mu_\rho \, a_{\mathrm{A}} - a_{\mathrm{P}}}{1 + \mu_\rho} + \frac{1}{1 + \mu_\rho} \, w, \tag{17}$$

where μ_ρ is defined in equation (13). Observe that this contract is affine in wealth—it is a constant term plus a linear term in wealth.

Real-world mutual fund management fees are also typically affine in wealth. Specifically, they are almost always stated as a percentage of assets under management (AUM) per unit time.[11] While AUM fees are charged on a per-unit-time basis, our model has only two dates (one period). To see how our model aligns with real world fees, call the date when the principal employs the agent Date t and the date when the principal's portfolio return is realized Date $t+1$. For $\tau \in \{t, t+1\}$, call the percentage fee charged at Date τ α_τ and the assets under management at Date τ AUM_τ. Hence the observed fees in our model are

$$\varphi_{\text{observed}} = \alpha_t \, \mathrm{AUM}_t + \alpha_{t+1} \, \mathrm{AUM}_{t+1}. \tag{18}$$

From the point of view of Date t, this contract is affine. The reason is that the term $\alpha_t \, \mathrm{AUM}_t$ is constant from the point of view of Date t (since the client knows his Date t wealth at Date t) and $\alpha_{t+1} \, \mathrm{AUM}_{t+1}$ is proportional to the wealth at Date $t+1$. We conclude that real-world observed asset management fees are affine, as are the fees that come out of our model endogenously.

4.3.2 Fee Changes over the Business Cycle

In our analysis above on the form of the equilibrium contract, we interpreted the contract as the asset managers' management fee directly. We focus on these fees because they are the fees that receive the most attention in the literature. However, in this section we interpret the fees in our model more broadly. In practice, fees from clients to funds include not only management fees but also other operating fees, including distribution (12b-1) fees, other expenses, and total

[11] Elton, Gruber, and Blake (2003) show that in 1999 more than 98 percent of mutual funds charged their clients fee exclusively as a percentage of AUM.

annual operating expenses.[12] Each of these fees is typically either a flat fee or a proportion of AUM; thus, our prediction that the asset management contracts are affine is robust to this broader interpretation of the contract. The *expense ratio* of a fund summarizes these total fees; it is calculated annually by dividing a fund's operating expenses by its AUM. It is well-known that expense ratios vary countercyclically—they are high in recessions and low in booms. In this section, we argue that our findings about the equilibrium contract are consistent with this fact.

In order to perform the comparison between equilibrium contracts in our model and observed expense ratios, we need to impose a bit more structure on our model. First, we consider an explicit form of public information that resembles credit ratings. Then, we try to connect our model to business cycle fluctuations by using the realization of the public signal as a proxy for the macroeconomic state. The expected variance given the public signal emerges naturally in our model and it can be interpreted as the VIX index. Since the VIX index is strongly countercyclical, this provides a mapping from the realization of the public signal to the business cycle.

First, we impose more structure on the public information. For this subsection (and this subsection only), consider a simplified but realistic credit rating rule. Let $\tilde{\rho}$ define a partition of the realization of the variance $\sigma_0^2 < \sigma_1^2 < \cdots$, namely

$$\mathbb{P}\{\tilde{\sigma}^2 \in [\sigma_i^2, \sigma_{i+1}^2) \,|\, \rho_i\} = 1. \tag{19}$$

We think that this is a realistic reduced-form representation of ratings categories. The information structure above is a partition of the continuum of possible asset variances into discrete categories, just as are ratings categories in the real world; typical ratings categories correspond to ranges of default probabilities. Given the interpretation of the elements of the partition as ratings categories, if $i < j$, then the rating ρ_i corresponds to a *higher* rating than the rating ρ_j—this is because it corresponds to lower variance and, therefore, typically a lower default probability.

Below we show two results relating to the conditional expected variance. Note that, given the above structure on the public information, the conditional expected variance is lower given $\tilde{\rho} = \rho_i$ than $\tilde{\rho} = \rho_j$ if and only if $i < j$—in notation,

$$\mathbb{E}\left[\tilde{\sigma}^2 \,|\, \rho_i\right] < \mathbb{E}\left[\tilde{\sigma}^2 \,|\, \rho_j\right] \quad \text{iff} \quad i < j,$$

[12] An explanation of these fees is provided by the SEC: http://www.sec.gov/answers/mffees.htm.

which follows immediately from the definition in equation (19).

LEMMA 4.1. *For $i < j$, $\mu_{\rho_i} < \mu_{\rho_j}$.*

Proof. The proof is in Appendix A.2.4 $\square$

PROPOSITION 4.1. *For $i < j$,*

$$\varphi_{\mu_{\rho_i}}(w) < \varphi_{\mu_{\rho_j}}(w). \tag{20}$$

Proof. We first show that that φ is globally increasing in μ:

$$\frac{\partial \varphi_\mu(w)}{\partial \mu} = \frac{\partial}{\partial \mu}\left(a_\mathrm{A} + \frac{w - a_\mathrm{P} - a_\mathrm{A}}{1+\mu}\right) \tag{21}$$

$$= -\frac{w - a_\mathrm{P} - a_\mathrm{A}}{(1+\mu)^2} \tag{22}$$

$$> 0, \tag{23}$$

since $w - a_\mathrm{P} - a_\mathrm{A} < 0$ by the assumptions in equations (6) and (7). Now, since Lemma 4.1 says that $\mu_{\rho_i} < \mu_{\rho_j}$ whenever $i < j$, we have that $\varphi_{\mu_{\rho_i}} < \varphi_{\mu_{\rho_j}}$ as desired. $\square$

The result in Proposition 4.1 carries over to the *expense ratio*—a fund's expense ratio is the total fees it charges investors, divided by its AUM, which, in our model, we think corresponds to management fees divided by the wealth or, symbolically, calling the expense ratio ER,

$$\mathrm{ER} := \frac{\varphi(w)}{w}.$$

Now, the result says immediately that

$$\frac{\varphi_{\mu_{\rho_i}}(w)}{w} < \frac{\varphi_{\mu_{\rho_j}}(w)}{w}. \tag{24}$$

or, using subscripts to represent the associated public information,

$$\mathrm{ER}_{\rho_i} < \mathrm{ER}_{\rho_j}. \tag{25}$$

We now explain the empirical content of Proposition 4.1. We do this by relating our result to the variation of fund fees over the business cycle. In order to connect our result to business cycle fluctuations, observe that the conditional expected variance $\mathbb{E}\left[\sigma^2 \mid \rho_i\right]$, which is captured by ρ_i, strongly resembles the VIX index, a measure of the market's expectation of the variance on the S&P 500 index. The VIX is countercyclical, it is high in recessions and low in booms. Thus, we can interpret equation (25)—which says explicitly that fund expense ratios are relatively high when conditional expected variance is high—as saying that expense ratios are relatively high in recessions and low in booms, i.e. that fund fees are countercyclical. This is consistent with the empirical fact that the expense ratio of bond funds is countercyclical.[13] The standard explanation for the countercyclicality of expense ratios in the practitioner literature is based on the effect of wealth in the denominator of the ratio. It suggests that given a component of fund fees is fixed, increases in the denominator, i.e. in wealth or AUM, that occur in booms, lead to decreases in the ratio. Our model suggests an explanation for the countercyclicality of the expense ratio that works through contracts themselves, even absent any wealth effects, which we would expect to amplify our result.

4.3.3 Imperfect Private Information

Suppose that the agent receives an imperfect signal about the variance. Namely, he observes the realization of a random variable $\tilde{s}$, which we assume is not independent of $\tilde{\sigma}$. In this case, the agent may learn from the public signal ρ and use the additional information it provides in his portfolio choice decision. In this case, equation (11) for the portfolio allocation becomes

$$x(\rho, s) = \frac{\left(\bar{R} - R_f\right)\left(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\right)}{\mathrm{Var}\left[\tilde{R} \,|\, \rho, s\right] + \left(\bar{R} - R_f\right)^2}.$$

The optimal contract is $\varphi_\mu(R_f + x(\rho, s)(R - R_f))$ (where an equation analogous to (13) determines μ).

Clearly, whenever $\sigma(\tilde{\rho}) \subset \sigma(\tilde{s})$, $x(\rho, s)$ does not depend on ρ and our main result remains unchanged. If, instead, $\sigma(\tilde{\rho}) \not\subset \sigma(\tilde{s})$, then a trade-off arises: finer public information still shut down risk sharing, but they increase allocational efficiency, i.e. the portfolio weight is closer to first best, because the agent makes a more informed decision at the time of investment. The net welfare effect is then ambiguous.

4.3.4 Additional Ratings' Changes

In our model, the public signal realizes once, before agents offer contracts and before the investor employs an agent. In reality, public signals, such as credit ratings, are upgraded and downgraded frequently and investors and agents have long-term relationships. In the model,

[13] The **?** demonstrates that bond's funds expense ratio is countercyclical.

if the public signal changes after the investor has employed an agent, the optimal contract above still induces the agent to invest efficiently. The new public signal influences the portfolio allocation only insofar as it improves the agent's information (cf. the preceding discussion of imperfect signals). Changes in public information after the investor and agent commit to a relationship never decrease efficiency and can be beneficial if they improve information. Our model therefore suggests that public information matters in delegated asset management largely because funds are looking to attract new investors or because their current investors may withdraw their funds.

5 Conclusion

In this paper, we theoretically investigate the effect of public information on investor welfare in the context of delegated asset management. Specifically, we ask whether more precise public information can increase investor welfare by decreasing an asset manager's informational advantage and thereby mitigating the agency problem between him and his client. We demonstrate a new cost that public information imposes on the economy, even when they are unbiased. In our model, public information does not mitigate the incentive problem between asset managers and their clients, but only prevent asset managers from providing insurance to their clients.

We characterize the optimal contract (asset manager's fee) between a risk-averse principal (the client) and a risk-averse agent (the asset manager) and show that the presence of the public signal does not mitigate the incentive problem; in fact, a contract (i.e. management fee) written on returns alone can solve the incentive problem. Further, in contrast to previous literature, we find that decreasing the precision of the public signal is Pareto improving. The reason is that precise credit ratings prevent the asset manager from providing insurance to his client.

Given the interpretation of the public information in our model as credit ratings, our results suggest that the information that credit ratings provide to the clients of delegated asset managers does harm and not good. As a result, our main policy prescription is that regulators should make ratings categories wider, so that credit ratings are less informative. This helps asset managers to provide insurance to their clients in addition to expertise. Whereas our results stand in contrast to some established results in the contract theory literature, they are consistent with some empirical facts about delegated asset management fees.

A Appendices

A.1 General Case

A.1.1 Proof of Lemma 3.1

Suppose, in anticipation of a contradiction, an equilibrium in which the employed agent offers contract $\hat{\Phi}$ given signal $\hat{\rho}$ and receives strictly in excess of his reservation utility,

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}\big(\tilde{w}\big)\Big)\,\Big|\,\tilde{\rho}=\hat{\rho}\right]>\bar{u}. \tag{26}$$

We now show that another agent $\hat{\mathrm{A}}$ has a profitable deviation. In order for a contract $\hat{\Phi}_\varepsilon$ to be a profitable deviation for $\hat{\mathrm{A}}$ it must (i) make the principal employ him given $\hat{\rho}$ and (ii) give him expected utility greater than his reservation utility $\bar{u}$ given $\hat{\rho}$. The subtlety in this proof is that $\hat{\mathrm{A}}$'s contract determines not only the allocation of surplus, but also his action x. To circumvent the effect of changing actions on payoffs, we construct $\hat{\Phi}_\varepsilon$ to induce the agent to choose the same action that he would have chosen under $\hat{\Phi}$, but still to change the division of surplus. To summarize, $\hat{\Phi}_\varepsilon$ is a profitable deviation if given $\hat{\rho}$ (i) it gives the principal higher utility than does $\hat{\Phi}$,

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}-\hat{\Phi}_\varepsilon\big(\tilde{w}\big)\Big)\mid\tilde{\rho}=\hat{\rho}\right]>\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}-\hat{\Phi}\big(\tilde{w}\big)\Big)\mid\tilde{\rho}=\hat{\rho}\right],$$

(ii) it gives the agent utility in excess of $\bar{u}$,

$$\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}_\varepsilon\big(\tilde{w}\big)\Big)\,\Big|\,\tilde{\rho}=\hat{\rho}\right]>\bar{u},$$

and (iii) the set of incentive compatible actions under $\hat{\Phi}$ and $\hat{\Phi}_\varepsilon$ coincide,

$$\arg\max_x\left\{\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}_\varepsilon\big(\tilde{w}\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]\right\}=\arg\max_x\left\{\mathbb{E}\left[u_{\mathrm{A}}\Big(\hat{\Phi}\big(\tilde{w}\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]\right\}.$$

One example of a contract that satisfies these three conditions is

$$\hat{\Phi}_\varepsilon(\tilde{w}):=u_{\mathrm{A}}^{-1}\Big(u_{\mathrm{A}}\Big(\hat{\Phi}\big(\tilde{w}\big)\Big)-\varepsilon\Big) \tag{27}$$

given $\hat{\rho}$, so that

$$u_{\mathrm{A}}\big(\hat{\Phi}_{\varepsilon}\big) = u_{\mathrm{A}}\big(\hat{\Phi}\big) - \varepsilon. \tag{28}$$

Since $u_{\mathrm{P}}' > 0$, a sufficient condition for $\hat{\Phi}_{\varepsilon}$ to satisfy condition (i) is that

$$\tilde{w} - \hat{\Phi}_{\varepsilon}(\tilde{w}) > \tilde{w} - \hat{\Phi}(\tilde{w}),$$

or, substituting from equation (27),

$$\hat{\Phi}(\tilde{w}) > u_{\mathrm{A}}^{-1}\Big(u_{\mathrm{A}}\Big(\hat{\Phi}(\tilde{w})\Big) - \varepsilon\Big),$$

which is satisfied for $\varepsilon > 0$ by the inverse function theorem since $u_{\mathrm{A}}' > 0$.

Condition (ii) holds for $\varepsilon > 0$ and sufficiently small. This follows from equation (28) and inequality (26) with the continuity of u_{A}.

Finally, condition (iii) is immediate from equation (28) since affine transformations of utility do not affect choices.

Thus the investor will employ agent $\hat{\mathrm{A}}$ who will receive, given $\hat{\rho}$, utility greater than the utility that he would have received in the supposed equilibrium (in the supposed equilibrium he was unemployed and he was obtaining $\bar{u}$). Thus $\hat{\Phi}_{\varepsilon}$ is a profitable deviation for $\hat{\mathrm{A}}$ and Φ cannot be the contract of an agent employed at equilibrium given $\hat{\rho}$.

We have shown that the agent's expected utility given any ρ cannot exceed $\bar{u}$. To conclude the proof, note that his utility can never be strictly less than $\bar{u}$ because then his expected utility would be less than his reservation utility.

A.1.2 Proof of Proposition 3.2

The proof relies on the following lemma.

LEMMA A.1. *The efficient sharing rule φ is affine in wealth w.*

Proof. Assumptions (1) and (2) imply that

$$u_{\mathrm{P}}(w) = \frac{1}{b-1}\left(\frac{w}{b} + \frac{a_{\mathrm{P}}}{b^2}\right)^{\frac{b-1}{b}}$$

and

$$u_{\mathrm{A}}(w) = \frac{1}{b-1}\left(\frac{w}{b} + \frac{a_{\mathrm{A}}}{b^2}\right)^{\frac{b-1}{b}}.$$

The contract that implements efficient risk sharing is that which maximizes social surplus (for appropriate welfare weight μ) for every realization of wealth. Now compute the efficient sharing rule directly via the first order approach:

$$\frac{\partial}{\partial \varphi}\Big(u_{\mathrm{P}}(w-\varphi)+\mu_\rho u_{\mathrm{A}}(\varphi)\Big)=0$$

so

$$\Big(\frac{w-\varphi}{b}+\frac{a_{\mathrm{P}}}{b^2}\Big)^{-\frac{1}{b}}=\mu_\rho\Big(\frac{\varphi}{b}+\frac{a_{\mathrm{A}}}{b^2}\Big)^{-\frac{1}{b}}.$$

This implies

$$\varphi(w)=\frac{a_{\mathrm{P}}-\mu_\rho^{-b}a_{\mathrm{A}}+bw}{b\left(1+\mu_\rho^{-b}\right)},$$

which is affine in w. □

We begin the proof of Proposition 3.2 with the agent's incentive problem given the contract φ and show through a series of manipulations that his incentives are aligned with those of the social planner. We rely on the fact that $u_{\mathrm{P}}'(w-\varphi)=\mu_\rho u_{\mathrm{A}}'(\varphi)$, from the definition of efficient risk sharing.

Incentive compatibility implies the first-order condition

$$\frac{\partial}{\partial x}\mathbb{E}\left[u_{\mathrm{A}}\Big(\varphi\big(\tilde{w}(x)\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]=0$$

or

$$\mathbb{E}\left[u_{\mathrm{A}}'\Big(\varphi\big(\tilde{w}(x)\big)\Big)\varphi'\big(\tilde{w}(x)\big)\tilde{w}'(x)\,\Big|\,\tilde{\sigma}=\sigma\right]=0.$$

By lemma A.1 φ' is a constant, thus we can pass it under the expectation operator. Further, since the right-hand side above is zero, we can remove φ' from the equation entirely to get

$$\mathbb{E}\left[u_{\mathrm{A}}'\Big(\varphi\big(\tilde{w}(x)\big)\Big)\tilde{w}'(x)\,\Big|\,\tilde{\sigma}=\sigma\right]=0.$$

And, since $u_{\mathrm{P}}'(w-\varphi)-\mu_\rho u_{\mathrm{A}}'(\varphi)=0$, the equation above re-writes as

$$\mathbb{E}\left[u_{\mathrm{P}}'\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)\tilde{w}'(x)\,\Big|\,\tilde{\sigma}=\sigma\right]=0. \tag{29}$$

Now, in order to back out the social planner's objective from (29) observe that it suffices to subtract

$$\mathbb{E}\left[\varphi'\big(\tilde{w}(x)\big)\tilde{w}'(x)\left[u'_{\mathrm{P}}\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)-\mu_\rho u'_{\mathrm{A}}\Big(\varphi\big(\tilde{w}(x)\big)\Big)\right]\;\Big|\;\tilde{\sigma}=\sigma\right], \tag{30}$$

which is zero since $u'_{\mathrm{P}}(w-\varphi)-\mu_\rho u'_{\mathrm{A}}(\varphi)=0$. Now subtracting expression (30) from equation (29) gives

$$\mathbb{E}\left[\Big(\tilde{w}'(x)-\varphi'\big(\tilde{w}(x)\big)\tilde{w}'(x)\Big)u'_{\mathrm{P}}\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)\;\Big|\;\tilde{\sigma}=\sigma\right]+$$
$$+\mu_\rho\mathbb{E}\left[\varphi'\big(\tilde{w}(x)\big)\tilde{w}'(x)u'_{\mathrm{A}}\Big(\varphi\big(\tilde{w}(x)\big)\Big)\;\Big|\;\tilde{\sigma}=\sigma\right]=0$$

or

$$\frac{\partial}{\partial x}\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}(x)-\varphi\big(\tilde{w}(x)\big)\Big)+\mu_\rho u_{\mathrm{A}}\Big(\varphi\big(\tilde{w}(x)\big)\Big)\;\Big|\;\tilde{\sigma}=\sigma\right]=0.$$

This is the first-order condition of the social planner's problem if he knows σ. Since u_{P} and u_{A} are concave and $\tilde{w}$ is concave in x, the first order condition implies a global maximum, viz. the incentive compatible x is a social optimum.

A.2 Application: Portfolio Choice

A.2.1 Computation of Optimal Investment

The problem stated in line (10) is to find the optimal investment x^* given the optimal sharing rule stated in equation (9), namely

$$\varphi_\mu(w)=A+Bw,$$

where

$$A=\frac{\mu a_{\mathrm{A}}-a_{\mathrm{P}}}{1+\mu}\quad\text{and}\quad B=\frac{1}{1+\mu}. \tag{31}$$

That is, x^* must maximize the expectation

$$-\frac{1}{2}\,\mathbb{E}\Bigg[\Big(R_f+x(\tilde{R}-R_f)-A-B\left(R_f+x(\tilde{R}-R_f)\right)-a_{\mathrm{P}}\Big)^2$$
$$+\mu\left(\left(A+B\left(R_f+x(\tilde{R}-R_f)\right)-a_{\mathrm{A}}\right)^2\right)\;\Big|\;\tilde{\sigma}=\sigma\Bigg]$$

over all x. Thus the first-order condition says that for optimum x^*

$$\mathbb{E}\Bigg[(1-B)(\tilde{R}-R_f)\Big(R_f+x^*(\tilde{R}-R_f)-A-B\Big(R_f+x^*(\tilde{R}-R_f)\Big)-a_{\mathrm{P}}\Big)$$
$$+\mu\,B(\tilde{R}-R_f)\Big(A+B\Big(R_f+x^*(\tilde{R}-R_f)\Big)-a_{\mathrm{A}}\Big)\;\Big|\;\tilde{\sigma}=\sigma\Bigg]=0,$$

thus

$$x^*=\frac{\big(\bar{R}-R_f\big)}{\mathbb{E}\big[(\tilde{R}-R_f)^2\,\big|\,\tilde{\sigma}=\sigma\big]}\left(\frac{(1-B)(A+a_{\mathrm{P}})-\mu\,B(A-a_{\mathrm{A}})}{(1-B)^2+B^2\mu}-R_f\right).$$

Substituting in for A and B from equation (31) in the numerator gives

$$(1-B)(A+a_{\mathrm{P}})-\mu B(A-a_{\mathrm{A}})=\frac{\mu\,(a_{\mathrm{A}}+a_{\mathrm{P}})}{1+\mu}$$

and substituting in for A and B from equation (31) in the denominator gives

$$(1-B)^2+B^2\mu=\frac{\mu}{1+\mu}.$$

Therefore

$$\begin{aligned} x &= \frac{\big(\bar{R}-R_f\big)\big(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\big)}{\mathbb{E}\big[\big(\tilde{R}-R_f\big)^2\,\big|\,\tilde{\sigma}=\sigma\big]} \\ &= \frac{\big(\bar{R}-R_f\big)\big(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\big)}{\sigma^2+\big(\bar{R}-R_f\big)^2}. \end{aligned}$$

A.2.2 Computation of the Social Planner's Weight

Immediately from plugging in the expressions for u_{A}, φ_{μ_ρ}, and x^* into equation (12), observe that

$$\begin{aligned} 2|\bar{u}|\left(1+\mu_\rho\right)^2 &= \mathbb{E}\left[\left(R_f + \frac{\left(\bar{R}-R_f\right)\left(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\right)}{\tilde{\sigma}^2+\left(\bar{R}-R_f\right)^2}\left(\tilde{R}-R_f\right) - a_{\mathrm{P}} - a_{\mathrm{A}}\right)^2 \middle| \tilde{\rho}=\rho\right] \\ &= \left(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\right)^2 \mathbb{E}\left[\left(\frac{\left(\bar{R}-R_f\right)\left(\tilde{R}-R_f\right)}{\tilde{\sigma}^2+\left(\bar{R}-R_f\right)^2} - 1\right)^2 \middle| \tilde{\rho}=\rho\right] \\ &= \left(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\right)^2 \left\{1 - 2\mathbb{E}\left[\frac{\left(\bar{R}-R_f\right)\left(\tilde{R}-R_f\right)}{\tilde{\sigma}^2+\left(\bar{R}-R_f\right)^2} \middle| \tilde{\rho}=\rho\right] + \right. \\ &\qquad \left. + \mathbb{E}\left[\left(\frac{\left(\bar{R}-R_f\right)\left(\tilde{R}-R_f\right)}{\tilde{\sigma}^2+\left(\bar{R}-R_f\right)^2}\right)^2 \middle| \tilde{\rho}=\rho\right]\right\}. \end{aligned} \tag{32}$$

Applying the law of iterated expectations gives

$$\begin{aligned} &1 - \frac{2|\bar{\mu}|\left(1+\mu_\rho\right)^2}{\left(a_{\mathrm{P}}+a_{\mathrm{A}}-R_f\right)^2} \\ &= 2\mathbb{E}\left[\mathbb{E}\left[\frac{\left(\bar{R}-R_f\right)\left(\tilde{R}-R_f\right)}{\tilde{\sigma}^2+\left(\bar{R}-R_f\right)^2} \middle| \tilde{\sigma}\right] \middle| \tilde{\rho}=\rho\right] - \mathbb{E}\left[\mathbb{E}\left[\left(\frac{\left(\bar{R}-R_f\right)\left(\tilde{R}-R_f\right)}{\tilde{\sigma}^2+\left(\bar{R}-R_f\right)^2}\right)^2 \middle| \tilde{\sigma}\right] \middle| \tilde{\rho}=\rho\right] \\ &= 2\mathbb{E}\left[\frac{\left(\bar{R}-R_f\right)\mathbb{E}\left[\left(\tilde{R}-R_f\right) \middle| \tilde{\sigma}\right]}{\tilde{\sigma}^2+\left(\bar{R}-R_f\right)^2} \middle| \tilde{\rho}=\rho\right] + \mathbb{E}\left[\frac{\left(\bar{R}-R_f\right)^2\mathbb{E}\left[\left(\tilde{R}-R_f\right)^2 \middle| \tilde{\sigma}\right]}{\left(\tilde{\sigma}^2+\left(\bar{R}-R_f\right)^2\right)^2} \middle| \tilde{\rho}=\rho\right] \end{aligned}$$

and since

$$\mathbb{E}\left[\left(\tilde{R}-R_f\right)^2 \middle| \tilde{\sigma}\right] = \tilde{\sigma}^2 + \left(\bar{R}-R_f\right)^2$$

we have

$$
\begin{aligned}
&1 - \frac{2|\bar{\mu}|\big(1+\mu_\rho\big)^2}{\big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\big)^2} \\
&= \big(\bar{R} - R_f\big)^2 \left\{ \mathbb{E}\left[\frac{2}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2} \,\middle|\, \tilde{\rho} = \rho \right] - \mathbb{E}\left[\frac{1}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2} \,\middle|\, \tilde{\rho} = \rho \right] \right\} \\
&= \mathbb{E}\left[\frac{\big(\bar{R} - R_f\big)^2}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2} \,\middle|\, \tilde{\rho} = \rho \right].
\end{aligned}
$$

Finally, solve for $(1+\mu_\rho)^2$ and cross multiply to recover equation (13).

A.2.3 Computation of Expected Utility Given ρ

Plug in to equation (14) and compute, maintaining at first the shorthand

$$
\tilde{w} := \tilde{w}\big(x^*(\tilde{\sigma})\big) = R_f + x^*(\tilde{\sigma})\big(R - R_f\big),
$$

that is:

$$
\begin{aligned}
&\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}\big(x^*(\tilde{\sigma})\big) - \varphi_{\mu_\rho}\big(\tilde{w}\big(x^*(\tilde{\sigma})\big)\big)\Big) \,\middle|\, \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2}\mathbb{E}\left[\Big(a_{\mathrm{P}} - \tilde{w} + \varphi_{\mu_\rho}\big(\tilde{w}\big)\Big)^2 \,\middle|\, \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2}\mathbb{E}\left[a_{\mathrm{P}} - \tilde{w} + a_{\mathrm{A}} + \frac{\tilde{w} - a_{\mathrm{P}} - a_{\mathrm{A}}}{1+\mu_\rho} \,\middle|\, \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2}\mathbb{E}\left[a_{\mathrm{P}} - \tilde{w} + a_{\mathrm{A}} + \frac{\tilde{w} - a_{\mathrm{P}} - a_{\mathrm{A}}}{1+\mu_\rho} \,\middle|\, \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2 \mathbb{E}\left[\big(a_{\mathrm{P}} + a_{\mathrm{A}} - \tilde{w}\big)^2 \,\middle|\, \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2 \mathbb{E}\left[\Big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f - x^*(\tilde{\sigma})\big(\tilde{R} - R_f\big)\Big)^2 \,\middle|\, \tilde{\rho} = \rho \right] \\
&= -\frac{1}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2 \mathbb{E}\left[\left(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f - \big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\big)\frac{\big(\bar{R} - R_f\big)\big(\tilde{R} - R_f\big)}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2}\right)^2 \,\middle|\, \tilde{\rho} = \rho \right] \\
&= -\frac{\big(a_{\mathrm{P}} + a_{\mathrm{A}} - R_f\big)^2}{2}\left(\frac{\mu_\rho}{1+\mu_\rho}\right)^2 \mathbb{E}\left[\left(1 - \frac{\big(\bar{R} - R_f\big)\big(\tilde{R} - R_f\big)}{\tilde{\sigma}^2 + \big(\bar{R} - R_f\big)^2}\right)^2 \,\middle|\, \tilde{\rho} = \rho \right].
\end{aligned}
$$

Now, from equation (32) above,

$$\mathbb{E}\left[\left(1-\frac{(\bar{R}-R_f)(\tilde{R}-R_f)}{\tilde{\sigma}^2+(\bar{R}-R_f)^2}\right)^2\middle|\tilde{\rho}=\rho\right]=2|\bar{u}|\left(\frac{1+\mu_\rho}{a_{\mathrm{P}}+a_{\mathrm{A}}-R_f}\right)^2,$$

so, finally,

$$\mathbb{E}\left[u_{\mathrm{P}}\Big(\tilde{w}\big(x^*(\tilde{\sigma})\big)-\varphi\big(\tilde{w}\big(x^*(\tilde{\sigma})\big),\tilde{\sigma},\rho\big)\Big)\;\Big|\;\tilde{\rho}=\rho\right]=\bar{u}\,\mu_\rho^2.$$

A.2.4 Proof of Proposition 4.1

Since

$$\frac{\sigma^2}{\sigma^2+(\bar{R}-R_f)^2}$$

is increasing in σ^2,

$$\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+(\bar{R}-R_f)^2}\middle|\rho_i\right]<\frac{\sigma_{i+1}^2}{\sigma_{i+1}^2+(\bar{R}-R_f)^2}<\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+(\bar{R}-R_f)^2}\middle|\rho_{i+1}\right].$$

This follows from definition (19), which implies that σ_{i+1} is greater than the expectation of $\tilde{\sigma}$ conditional on ρ_i but less than the expectation of $\tilde{\sigma}$ conditional on ρ_{i+1}. Now, immediately from equation (13), $\mu_{\rho_i}<\mu_{\rho_{i+1}}$ and by induction $\mu_{\rho_i}<\mu_{\rho_j}$ whenever $i<j$. Combined with equation (16), the result implies that lower expected variances correspond to steeper wealth compensation for agents.

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