

```
In[1]:= sys = {y1'[t] == y1[t-1],
              y2'[t] == y1[t-1] + y2[t-1/5],
              y3'[t] == y2[t],
              y1[t /; t < 0] == 1, y2[t /; t < 0] == 1, y3[t /; t < 0] == 1}
```

```
Out[1]= {y1'[t] == y1[-1+t], y2'[t] == y1[-1+t] + y2[-1/5+t],
          y3'[t] == y2[t], y1[t /; t < 0] == 1, y2[t /; t < 0] == 1, y3[t /; t < 0] == 1}
```

```
In[2]:= solx = DSolve[sys, {y1[t], y2[t], y3[t]}, {t, 0, 5}];
sol = Simplify[solx]
```

```
Out[3]= { {y1[t] -> {
  1                                     t <= 0
  Indeterminate                       t > 5
  1+t                                 0 < t <= 1
  1/2 (3+t^2)                         1 < t <= 2
  1/6 (1+12 t-3 t^2+t^3)              2 < t <= 3
  1/24 (85-60 t+42 t^2-8 t^3+t^4)     3 < t <= 4
  1/120 (-599+980 t-430 t^2+120 t^3-15 t^4+t^5) True
```

```
1
Indeterminate
1+2 t
26/25 + 8 t/5 + t^2
1/375 (382+660 t+225 t^2+125 t^3)
7721+12660 t+5850 t^2+1000 t^3+625 t^4
7500
1/187500 (192001+322900 t+130250 t^2+45000 t^3+3125 t^4+3125 t^5)
1/1125000 (1717631+793650 t+1390875 t^2+207500 t^3+65625 t^4+3125 t^6)
1/196875000 (243605489+282271710 t+121195725 t^2+
74795000 t^3+6759375 t^4+2362500 t^5-109375 t^6+78125 t^7)
1/7875000000 (11010509361+7656426680 t+8788933300 t^2+1036651000 t^3+
703543750 t^4+34475000 t^5+17062500 t^6-1250000 t^7+390625 t^8)
1 (464372943517+442062175320 t+272654561700 t^2+
```

$$\begin{aligned}
& 354\,375\,000\,000 \setminus \\
& 125\,001\,807\,000 \, t^3 + 5\,453\,988\,750 \, t^4 + 6\,117\,300\,000 \, t^5 + \\
& 95\,812\,500 \, t^6 + 123\,750\,000 \, t^7 - 10\,546\,875 \, t^8 + 1\,953\,125 \, t^9) \\
& \frac{1}{177\,187\,500\,000\,000} (24\,059\,153\,535\,251 + \\
& 19\,293\,672\,491\,550 \, t + 17\,556\,246\,271\,125 \, t^2 + 3\,307\,908\,315\,000 \, t^3 + \\
& 1\,538\,466\,693\,750 \, t^4 - 6\,417\,337\,500 \, t^5 + 50\,928\,281\,250 \, t^6 - \\
& 646\,875\,000 \, t^7 + 896\,484\,375 \, t^8 - 78\,125\,000 \, t^9 + 9\,765\,625 \, t^{10}) \\
& \frac{1}{194\,906\,250\,000\,000} (-194\,311\,112\,239 + 619\,477\,897\,407\,050 \, t - \\
& 28\,050\,041\,017\,625 \, t^2 + 90\,940\,116\,465\,000 \, t^3 + 5\,682\,508\,631\,250 \, t^4 + \\
& 3\,466\,596\,787\,500 \, t^5 - 125\,570\,156\,250 \, t^6 + 83\,118\,750\,000 \, t^7 - \\
& 3\,029\,296\,875 \, t^8 + 1\,289\,062\,500 \, t^9 - 107\,421\,875 \, t^{10} + 9\,765\,625 \, t^{11}) \\
& \frac{1}{58\,471\,875\,000\,000\,000} (57\,813\,100\,565\,417\,521 + 79\,164\,318\,128\,378\,340 \, t + 66\,982\,597\,344\,054\,150 \, t^2 + \\
& 1\,592\,191\,015\,497\,500 \, t^3 + 6\,585\,416\,665\,059\,375 \, t^4 + 127\,014\,225\,525\,000 \, t^5 + \\
& 184\,436\,333\,812\,500 \, t^6 - 10\,552\,843\,125\,000 \, t^7 + 3\,372\,380\,859\,375 \, t^8 - \\
& 185\,195\,312\,500 \, t^9 + 45\,761\,718\,750 \, t^{10} - 3\,515\,625\,000 \, t^{11} + 244\,140\,625 \, t^{12}) \\
& \frac{1}{3\,800\,671\,875\,000\,000\,000} (1\,208\,131\,797\,409\,159\,793 + \\
& 10\,504\,068\,982\,889\,728\,740 \, t - 198\,887\,302\,319\,321\,850 \, t^2 + \\
& 2\,098\,158\,023\,378\,345\,500 \, t^3 - 59\,401\,857\,115\,940\,625 \, t^4 + \\
& 85\,441\,262\,917\,725\,000 \, t^5 - 1\,134\,528\,654\,187\,500 \, t^6 + 1\,886\,186\,176\,875\,000 \, t^7 - \\
& 132\,122\,112\,890\,625 \, t^8 + 27\,393\,437\,500\,000 \, t^9 - 1\,851\,738\,281\,250 \, t^{10} + \\
& 319\,921\,875\,000 \, t^{11} - 22\,216\,796\,875 \, t^{12} + 1\,220\,703\,125 \, t^{13}) \\
& \frac{1}{266\,047\,031\,250\,000\,000\,000} (199\,755\,531\,810\,208\,252\,299 + \\
& 467\,865\,895\,928\,929\,679\,090 \, t + 246\,306\,144\,652\,072\,052\,275 \, t^2 + \\
& 9\,833\,178\,155\,393\,001\,500 \, t^3 + 38\,152\,549\,454\,706\,561\,875 \, t^4 - \\
& 2\,044\,950\,569\,242\,831\,250 \, t^5 + 1\,031\,939\,602\,490\,671\,875 \, t^6 - \\
& 39\,422\,452\,801\,875\,000 \, t^7 + 18\,716\,079\,022\,265\,625 \, t^8 - \\
& 1\,440\,474\,191\,406\,250 \, t^9 + 222\,888\,681\,640\,625 \, t^{10} - 16\,653\,710\,937\,500 \, t^{11} + \\
& 2\,199\,462\,890\,625 \, t^{12} - 136\,718\,750\,000 \, t^{13} + 6\,103\,515\,625 \, t^{14}) \\
& \frac{1}{19\,953\,527\,343\,750\,000\,000\,000} (9\,598\,156\,695\,605\,625\,910\,201 + 48\,608\,382\,040\,949\,066\,782\,950 \, t + \\
& 3\,871\,392\,977\,755\,211\,792\,625 \, t^2 + 9\,569\,375\,674\,924\,820\,072\,500 \, t^3 - \\
& 406\,396\,933\,417\,377\,459\,375 \, t^4 + 612\,447\,891\,327\,078\,056\,250 \, t^5 - \\
& 41\,563\,583\,056\,229\,609\,375 \, t^6 + 11\,883\,627\,100\,434\,375\,000 \, t^7 - \\
& 671\,866\,885\,142\,578\,125 \, t^8 + 181\,115\,954\,550\,781\,250 \, t^9 - \\
& 14\,451\,908\,173\,828\,125 \, t^{10} + 1\,811\,291\,015\,625\,000 \, t^{11} - 139\,854\,736\,328\,125 \, t^{12} + \\
& 14\,868\,164\,062\,500 \, t^{13} - 823\,974\,609\,375 \, t^{14} + 30\,517\,578\,125 \, t^{15}) \\
& \frac{1}{319\,256\,437\,500\,000\,000\,000\,000} (1\,283\,366\,938\,567\,708\,813\,391\,341 - 798\,878\,182\,250\,576\,650\,222\,800 \, t + \\
& 944\,507\,880\,943\,643\,935\,557\,000 \, t^2 - 117\,180\,637\,543\,488\,035\,090\,000 \, t^3 + \\
& 53\,889\,555\,846\,938\,171\,587\,500 \, t^4 - 3\,351\,018\,060\,544\,094\,850\,000 \, t^5 + \\
& 1\,793\,720\,446\,735\,091\,875\,000 \, t^6 - 135\,039\,986\,412\,581\,250\,000 \, t^7 + \\
& 26\,468\,576\,798\,167\,968\,750 \, t^8 - 1\,824\,705\,762\,343\,750\,000 \, t^9 + \\
& 342\,829\,283\,671\,875\,000 \, t^{10} - 27\,405\,574\,218\,750\,000 \, t^{11} + \\
& 2\,927\,729\,492\,187\,500 \, t^{12} - 223\,535\,156\,250\,000 \, t^{13} + \\
& 19\,775\,390\,625\,000 \, t^{14} - 976\,562\,500\,000 \, t^{15} + 30\,517\,578\,125 \, t^{16})
\end{aligned}$$

$y_2[t] \rightarrow$



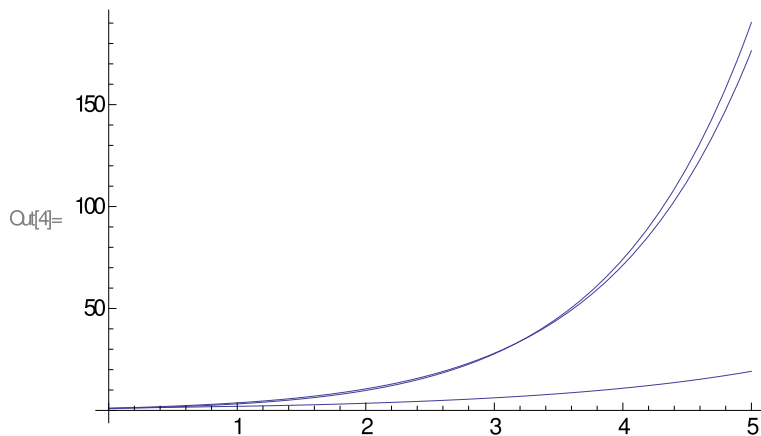
$$\begin{aligned}
 & 2\,122\,776\,091\,844\,133\,613\,232\,393\,404\,843\,265\,625\,t^6 + \\
 & 362\,413\,806\,731\,160\,865\,694\,886\,457\,571\,250\,000\,t^7 - \\
 & 40\,472\,108\,309\,077\,303\,925\,840\,692\,539\,843\,750\,t^8 + \\
 & 5\,150\,873\,947\,890\,563\,876\,957\,519\,648\,437\,500\,t^9 - \\
 & 445\,042\,011\,966\,532\,699\,795\,583\,417\,968\,750\,t^{10} + \\
 & 44\,129\,503\,423\,644\,931\,036\,453\,515\,625\,000\,t^{11} - \\
 & 3\,568\,989\,996\,988\,554\,027\,049\,316\,406\,250\,t^{12} + \\
 & 299\,144\,538\,086\,669\,449\,077\,148\,437\,500\,t^{13} - \\
 & 20\,203\,269\,205\,836\,931\,945\,800\,781\,250\,t^{14} + \\
 & 1\,484\,302\,246\,931\,254\,394\,531\,250\,000\,t^{15} - \\
 & 102\,239\,024\,319\,423\,065\,185\,546\,875\,t^{16} + 6\,956\,270\,000\,212\,097\,167\,968\,750\,t^{17} - \\
 & 410\,318\,866\,512\,298\,583\,984\,375\,t^{18} + 22\,722\,554\,779\,052\,734\,375\,000\,t^{19} - \\
 & 1\,061\,171\,817\,779\,541\,015\,625\,t^{20} + 39\,087\,295\,532\,226\,562\,500\,t^{21} - \\
 & 932\,216\,644\,287\,109\,375\,t^{22} + 11\,920\,928\,955\,078\,125\,t^{23}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{18\,490\,803\,293\,384\,296\,875\,000\,000\,000\,000\,000\,000\,000} \\
 & (3\,614\,261\,787\,040\,018\,614\,132\,785\,333\,078\,394\,472\,081 + \\
 & 17\,998\,689\,860\,392\,705\,275\,653\,730\,366\,556\,642\,613\,160\,t + \\
 & 47\,043\,491\,356\,643\,885\,885\,547\,977\,643\,253\,488\,467\,100\,t^2 - \\
 & 19\,132\,422\,283\,145\,635\,357\,201\,497\,938\,996\,019\,179\,000\,t^3 + \\
 & 10\,013\,654\,544\,796\,278\,119\,004\,306\,540\,667\,448\,141\,250\,t^4 - \\
 & 1\,940\,042\,942\,650\,501\,429\,210\,403\,137\,113\,588\,525\,000\,t^5 + \\
 & 385\,505\,812\,725\,049\,208\,301\,679\,048\,485\,463\,187\,500\,t^6 - \\
 & 45\,927\,460\,214\,989\,074\,827\,157\,632\,994\,005\,625\,000\,t^7 + \\
 & 6\,525\,568\,762\,659\,051\,178\,510\,682\,186\,480\,859\,375\,t^8 - \\
 & 674\,746\,157\,729\,681\,852\,314\,366\,265\,093\,750\,000\,t^9 + \\
 & 73\,736\,159\,483\,884\,923\,815\,047\,373\,671\,875\,000\,t^{10} - \\
 & 6\,024\,734\,551\,442\,529\,152\,963\,036\,718\,750\,000\,t^{11} + \\
 & 536\,906\,325\,983\,437\,465\,558\,788\,085\,937\,500\,t^{12} - \\
 & 40\,855\,574\,336\,954\,173\,187\,753\,906\,250\,000\,t^{13} + \\
 & 3\,106\,572\,403\,981\,551\,706\,201\,171\,875\,000\,t^{14} - \\
 & 201\,494\,892\,275\,065\,828\,613\,281\,250\,000\,t^{15} + \\
 & 13\,891\,005\,854\,080\,937\,347\,412\,109\,375\,t^{16} - \\
 & 907\,666\,759\,374\,334\,716\,796\,875\,000\,t^{17} + 57\,326\,002\,093\,063\,354\,492\,187\,500\,t^{18} - \\
 & 3\,190\,461\,526\,336\,669\,921\,875\,000\,t^{19} + 163\,843\,374\,824\,523\,925\,781\,250\,t^{20} - \\
 & 7\,052\,120\,208\,740\,234\,375\,000\,t^{21} + 236\,234\,664\,916\,992\,187\,500\,t^{22} - \\
 & 5\,149\,841\,308\,593\,750\,000\,t^{23} + 59\,604\,644\,775\,390\,625\,t^{24})
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2\,311\,350\,411\,673\,037\,109\,375\,000\,000\,000\,000\,000\,000\,000} \\
 & (-8\,432\,587\,480\,804\,165\,525\,393\,593\,543\,984\,229\,191\,240\,499 + \\
 & 19\,134\,277\,210\,222\,448\,189\,731\,402\,315\,175\,459\,472\,117\,000\,t - \\
 & 8\,498\,046\,229\,913\,495\,119\,337\,462\,643\,627\,722\,205\,292\,500\,t^2 + \\
 & 4\,856\,808\,663\,262\,306\,659\,470\,860\,761\,661\,997\,468\,225\,000\,t^3 - \\
 & 1\,163\,410\,571\,626\,122\,217\,241\,459\,089\,224\,125\,078\,343\,750\,t^4 + \\
 & 324\,226\,747\,194\,773\,991\,868\,307\,045\,454\,213\,018\,375\,000\,t^5 - \\
 & 50\,922\,804\,123\,672\,284\,270\,368\,614\,935\,630\,701\,562\,500\,t^6 + \\
 & 8\,171\,286\,110\,300\,649\,685\,804\,751\,165\,093\,296\,875\,000\,t^7 - \\
 & 897\,095\,518\,369\,638\,772\,811\,021\,393\,569\,892\,578\,125\,t^8 + \\
 & 109\,461\,559\,071\,466\,849\,923\,299\,849\,663\,281\,250\,000\,t^9 - \\
 & 10\,455\,833\,043\,135\,241\,286\,403\,455\,891\,015\,625\,000\,t^{10} +
 \end{aligned}$$

$$\begin{aligned}
& 1\,005\,757\,525\,209\,818\,437\,594\,500\,410\,156\,250\,000\, t^{11} - \\
& 77\,607\,142\,589\,843\,357\,735\,851\,489\,257\,812\,500\, t^{12} + \\
& 6\,300\,139\,445\,892\,286\,618\,343\,261\,718\,750\,000\, t^{13} - \\
& 450\,590\,237\,476\,989\,676\,724\,853\,515\,625\,000\, t^{14} + \\
& 31\,524\,704\,774\,096\,216\,423\,339\,843\,750\,000\, t^{15} - \\
& 1\,963\,114\,579\,479\,068\,378\,448\,486\,328\,125\, t^{16} + \\
& 127\,602\,629\,454\,815\,826\,416\,015\,625\,000\, t^{17} - \\
& 7\,898\,798\,129\,773\,330\,688\,476\,562\,500\, t^{18} + \\
& 465\,610\,664\,188\,385\,009\,765\,625\,000\, t^{19} - \\
& 24\,425\,356\,375\,694\,274\,902\,343\,750\, t^{20} + 1\,164\,994\,430\,541\,992\,187\,500\,000\, t^{21} - \\
& 46\,276\,330\,947\,875\,976\,562\,500\, t^{22} + 1\,416\,206\,359\,863\,281\,250\,000\, t^{23} - \\
& 28\,312\,206\,268\,310\,546\,875\, t^{24} + 298\,023\,223\,876\,953\,125\, t^{25}
\end{aligned}$$

In[4]:= `Plot[{y1[t], y2[t], y3[t]} /. sol, {t, 0, 5}, PlotRange -> All]`



In[5]:= `Ny = {y1[t], y2[t], y3[t]} /. sol /. t -> 5`

Out[5]= 
$$\left\{ \left\{ \frac{767}{40}, \frac{1\,372\,977\,775\,497\,546\,065\,372\,181\,595\,185\,280\,327\,502\,633}{7\,782\,324\,618\,427\,734\,375\,000\,000\,000\,000\,000\,000\,000}, \frac{2\,118\,288\,127\,243\,946\,981\,292\,253\,783\,821\,715\,691\,529\,048\,793}{11\,128\,724\,204\,351\,660\,156\,250\,000\,000\,000\,000\,000\,000\,000} \right\} \right\}$$

In[11]:= `N[Ny, 20]`

Out[11]=  $\{\{19.175000000000000000, 176.42257844738031770, 190.34420193607041693\}\}$

In[7]:= y1[t] /. sol // InputForm

Out[7]/InputForm= {Piecewise[{{1, t <= 0}, {Indeterminate, t > 5}, {1 + t, Inequality[0, Less, t, {(3 + t^2)/2, Inequality[1, Less, t, LessEqual, 2]}], {(1 + 12\*t - 3\*t^2 + t^3) (85 - 60\*t + 42\*t^2 - 8\*t^3 + t^4)/24, Inequality[3, Less, t, LessEqual, 4]}]}

In[8]:= tab = {Table[First[y1[t] /. sol], {t, 5}],  
Table[First[y2[t] /. sol], {t, 5}],  
Table[First[y3[t] /. sol], {t, 5}]}

Out[8]=  $\left\{ \left\{ 2, \frac{7}{2}, \frac{37}{6}, \frac{87}{8}, \frac{767}{40} \right\}, \left\{ \frac{696401}{187500}, \frac{187120972422851}{17718750000000}, \frac{11404193203167808022899}{162019172005425000223647627894251}, \frac{40721484375000000000}{2274702117187500000000000000}, \frac{1372977775497546065372181595185280327502633}{7782324618427734375000000000000000000} \right\}, \left\{ \frac{17896711}{5625000}, \frac{9523954835239571}{97453125000000}, \frac{4025234611713839832224131}{145116562500000000000000}, \frac{82125609204103677328910337492038771}{110736634886718750000000000000000}, \frac{2118288127243946981292253783821715691529048793}{11128724204351660156250000000000000000} \right\} \right\}$

In[9]:= N[tab] // TraditionalForm

Out[9]/TraditionalForm=  $\begin{pmatrix} 2 & 3.5 & 6.16667 & 10.875 & 19.175 \\ 3.71414 & 10.5606 & 28.0053 & 71.2265 & 176.423 \\ 3.18164 & 9.77286 & 27.7379 & 74.163 & 190.344 \end{pmatrix}$

In[10]:= tab // TraditionalForm

Out[10]/TraditionalForm=  $\begin{pmatrix} 2 & \frac{7}{2} & \frac{37}{6} & \frac{87}{8} & \frac{767}{40} \\ \frac{696401}{187500} & \frac{187120972422851}{1771875000000} & \frac{11404193203167808022899}{40721484375000000000} & \frac{162019172005425000223647627894251}{2274702117187500000000000000} & \frac{1372977775497546065372181595185280327502633}{7782324618427734375000000000000000000} \\ \frac{17896711}{5625000} & \frac{9523954835239571}{97453125000000} & \frac{4025234611713839832224131}{145116562500000000000000} & \frac{82125609204103677328910337492038771}{110736634886718750000000000000000} & \frac{2118288127243946981292253783821715691529048793}{11128724204351660156250000000000000000} \end{pmatrix}$