Scroll Equalizer

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Scrolls have been scanned at different beam energies and resolutions, processed, and saved as volumetric images with voxel intensities in the range $[0, \ldots, 65535]$, which are proportional to the mass attenuation coefficient of the material. Initially, outliers were filtered by clipping at the 0.01 and 0.99 quantiles (independently for each scroll). The data was then normalized to the range $[0, \ldots, 1]$, multiplied by 65535, and saved as uint16. Since this procedure was performed independently for each scan, the voxel intensities can vary significantly, even for the same material scanned at the same resolution and beam energy. Our goal is to equalize the scrolls to Scroll 1 (PHercParis4) as closely as possible.

For simplicity, we will neglect the fact that the data was clipped.

Let $I_{m,s}$ represent the intensity of a voxel for a given material m and a specific scroll s. From the normalization step, we have:

$$I_{m,s}(\text{energy}) = \frac{I_{m,\text{raw}}(\text{energy}) - \min(I_s)}{\max(I_s) - \min(I_s)} \tag{1}$$

where raw denotes the raw value before normalization.

To equalize scroll *sN* (scroll N) with *s1* (scroll 1), we need to know not only the raw intensity (or density) of the material scanned with a beam at the given energy but also the minimum and maximum values of the intensities before normalization. Unfortunately, this data has been lost.

However, if we can identify two materials with known mass attenuation coefficients at the specified energy beam level, we can establish a system of four equations with four unknowns.

These materials are air, whose density is 0 by definition, and Nylon-12, which is the material used to make the casts covering the scrolls during the scan. According to Sookpeng et al. [2016] (Table 1), the mass attenuation coefficients of Nylon-6 casts scanned with beams of different energies (ranging from 40 keV to 100 keV) are reported. We assume that Nylon-6 is similar to Nylon-12. We further assume the density of Nylon-12, when scanned at 54 keV, to be 0.219. First, we rescale the intensities from uint16 to a float in the range $[0, \ldots, 1]$. Then, we can measure the median value of air and the nylon casts in each scan. For example, $I_{\text{air},s1}(54\text{keV}) = 0.39$, $I_{\text{nylon},s1}(54\text{keV}) = 0.66$, $I_{\text{air},s3}(54\text{keV}) = 0.316$, $I_{\text{nylon},s3}(54\text{keV}) = 0.496$.

$$
\begin{aligned}
0.39 &= -\frac{\min(I_{s1})}{\max(I_{s1}) - \min(I_{s1})}, \\
0.316 &= -\frac{\min(I_{s3})}{\max(I_{s3}) - \min(I_{s3})}, \\
0.66 &= \frac{0.219 - \min(I_{s1})}{\max(I_{s1}) - \min(I_{s1})}, \\
0.496 &= \frac{0.219 - \min(I_{s3})}{\max(I_{s3}) - \min(I_{s3})}.
\end{aligned} \tag{2}
$$

We can solve the system and obtain the min and max values used to normalize the intensities in the two scrolls.

Finally, s3 equalized to s1 will be:

$$
I_{m,s3\to s1}(54\text{keV}) = \frac{(\max(I_{s3}) - \min(I_{s3}))I_{m,s3}(54\text{keV}) + \min(I_{s3}) - \min(I_{s1})}{\max(I_{s1}) - \min(I_{s1})}. \tag{3}
$$

A similar reasoning can be applied to equalize different scrolls.

When you insert the value for Nylon-6 mass attenuation coefficient please pay attention to the energy of the beam.

References

S. Sookpeng, P. Cheebsumon, T. Pengpan, and C. Martin. Comparison of computed tomography dose index in polymethyl methacrylate and nylon dosimetry phantoms. *Journal of medical physics*, 41(1):45–51, 2016. doi: 10.4103/0971-6203.177287.