Scroll Equalizer

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Scrolls have been scanned at different beam energy and resolution, processed and saved as volumetric images whose voxels have an intensity $\in [0, \dots, 65535]$ that are proportional to the mass attenuation coefficient of the material. First, outliers have been filtered out by clipping at quantiles of order 0.01 and 0.99 (independently for each scroll). Then, the data has been normalized to $\in [0, \dots, 1]$, multiplied by 65535, and saved as uint16. Since this procedure has been performed independently for each different scan, the voxels have intensities that can vary a lot even for the same material scanned at the same resolution and beam energy. We are going to equalize the scrolls to Scroll 1 (PHercParis4) as much as possible.

For simplicity we are going to neglect the fact that the data was clipped.

Let $I_{m,s}$ be the intensity of a voxel for a given material m and a specific scroll s. From the normalization step, we know that:

$$I_{m,s}(\text{energy}) = \frac{I_{m,\text{raw}}(\text{energy}) - \min(I_s)}{\max(I_s) - \min(I_s)} \tag{1}$$

where raw is the raw value before normalization.

Therefore, in order to equalize sN (scroll N) with $s1$ (scroll 1) we need to know not only the raw intensity (or density) of the material scanned with a beam at the given energy, but also what are the min and max values of the intensities before normalization. This data has been lost.

Luckily enough, if we can find two materials with a known mass attenuation coefficient (at that energy beam level) we can write a determined system of four equations and four unknowns.

These materials are air, whose density is 0 by definition, and Nylon-12 which is the material making the casts that covered the scrolls during the scan. In Sookpeng et al. [2016] (Table 1) it is reported the mass attenuation coefficients of Nylon-6 casts when scanned with beams of different energies going from 40 keV to 100 keV. We assume Nylon-6 to be not too different from Nylon-12. We assume that the density of Nylon-12, when scanned at $54keV$, to be 0.219. Let us first rescale the intensities from uint16 to a float $\in [0, \dots, 1]$. Then, we can measure in every scan the median value of air and of the nylon casts. For instance, $I_{\text{air},s1}(54\text{keV}) = 0.39$, $I_{\text{nylon},s1}(54\text{keV}) = 0.66$, $I_{\text{air},s3}(54\text{keV}) = 0.316$, $I_{\text{nylon},s3}(54\text{keV}) = 0.496$.

$$
\begin{aligned}
0.39 &= -\frac{\min(I_{s1})}{\max(I_{s1}) - \min(I_{s1})}, \\
0.316 &= -\frac{\min(I_{s3})}{\max(I_{s3}) - \min(I_{s3})}, \\
0.66 &= \frac{0.219 - \min(I_{s1})}{\max(I_{s1}) - \min(I_{s1})}, \\
0.496 &= \frac{0.219 - \min(I_{s3})}{\max(I_{s3}) - \min(I_{s3})}.
\end{aligned} \tag{2}
$$

We can solve the system and obtain the min and max values used to normalize the intensities in the two scrolls.

Finally, s3 equalized to s1 will be:

$$
I_{m,s3\to s1}(54\text{keV}) = \frac{(\max(I_{s3}) - \min(I_{s3}))I_{m,s3}(54\text{keV}) + \min(I_{s3}) - \min(I_{s1})}{\max(I_{s1}) - \min(I_{s1})}. \tag{3}
$$

A similar reasoning can be applied to equalize different scrolls.

When you insert the value for Nylon-6 mass attenuation coefficient please pay attention to the energy of the beam.

References

S. Sookpeng, P. Cheebsumon, T. Pengpan, and C. Martin. Comparison of computed tomography dose index in polymethyl methacrylate and nylon dosimetry phantoms. *Journal of medical physics*, 41(1):45–51, 2016. doi: 10.4103/0971-6203.177287.