

# A New Block Preconditioner for Implicit Runge-Kutta Methods for Parabolic PDE

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**Abstract:** Explicit time integrators for parabolic PDE are subject to a restrictive time-step limit, so A-stable integrators are essential. It is well known that although there are no A-stable explicit linear multistep methods and implicit multistep methods cannot be A-stable beyond order two, there exist A-stable and L-stable implicit Runge-Kutta (IRK) methods at all orders. IRK methods offer an appealing combination of stability and high order; however, these methods are not widely used for PDE because they lead to large, strongly coupled linear systems. An  $s$ -stage IRK system has  $s$ -times as many degrees of freedom as the systems resulting from backward Euler or implicit trapezoidal rule discretization applied to the same equation set. In this talk, I will introduce a new block preconditioner for IRK methods, based on a block LDU factorization with algebraic multigrid subsolves for scalability. I will demonstrate the effectiveness of this preconditioner on the heat equation as a simple test problem, and compare in condition number and eigenvalue distribution, and in numerical experiments with other preconditioners currently in the literature. Experiments are run with IRK stages upto  $s = 7$ , and it is found that the new preconditioner outperforms the others, with the improvement becoming more pronounced as spatial discretization is refined and as temporal order is increased.

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