# Random Walk of Brownian Motion and Its Long Time Behavior

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We use the Einstein random-walk paradigm for diffusion to derive a degenerate nonlinear parabolic equation and study the long-time asymptotics of prototypical non-linear diffusion equations. Specifically, we consider the case of a non-degenerate diffusivity function that is a (non-negative) polynomial of the dependent variable of the problem. We motivate these types of equations using Einstein's random walk paradigm, but instead of Taylor expansion we used Caratheodory theorem unlike Einstein's original work leading to a partial differential equation in non-divergence form. On the other hand, using conservation principles leads to a partial differential equation in divergence form. A transformation is derived to handle both cases. We investigate a qualitative properties of the solution using maximum principle and energy method, in order to obtain bounds above and below for the time-evolution of the solution to the non-linear diffusion problem. Having thus sandwiched, we prove that, unlike the case of degenerate diffusion, the solution converges onto the linear diffusion solution at long times. Select numerical examples support the mathematical theorems and illustrate the convergence process.