Error estimates and Convergence order

$E_h = |I - I_h|$: error = true integral I minus approximate integral I_h with interval width h.

$$E_h = Ch^\alpha$$

α is the **convergence order**.
The theory says:

- Composite rectangle rule: $\alpha = 2$
- Composite trapezoid rule: $\alpha = 2$
- Composite Simpson rule: $\alpha = 4$

Absolute

How do I evaluate α?

$$E_h = Ch^\alpha$$

$$E_{h/2} = C\left(\frac{h}{2}\right)^\alpha$$

Do the ratio of the two equations

$$2^\alpha = \frac{E_h}{E_{h/2}}$$

$$\alpha = \log_2 \frac{E_h}{E_{h/2}} = \log_2 10 \log_{10} \frac{E_h}{E_{h/2}} = \frac{\log_{10} \frac{E_h}{E_{h/2}}}{\log_{10} 2}$$

Remember: $\log_a b = \log_a c \log_c b$

Relative

What if I don't know the true analytical value of the integral I?
There is another way to compute α given by

$$\alpha = \frac{\log_{10} \left| \frac{I_h - I_{h/2}}{I_{h/2} - I_{h/4}} \right|}{\log_{10} 2}$$