Degree of precision

Degree of precision d = the highest polynomial degree for which the quadrature rule gives the same result as the analytical integral (the error $E_h = |I - I_h|$ is equal to zero)

- Midpoint rectangle: $d = 1$
- Trapezoid: $d = 1$
- Simpson: $d = 2$

It can be shown that for a quadrature rule the maximum degree of precision we can have is $d_{max} = 2n - 1$, where n is the number of nodes.

If n is fixed, clearly it is a big advantage!

$n = 1$, $d_{max} = 1$
$n = 2$, $d_{max} = 3$
$n = 3$, $d_{max} = 5$

Is it possible to construct a quadrature rule that achieves this maximum degree of precision?

Gauss integration rule

- The theory says what **nodes** and what **weights** are needed to achieve this.
- It turns out that the nodes are the zeros of certain polynomials, called *orthogonal polynomials*.
- It is common practice to provide these nodes on the canonical interval $[-1, 1]$
- If you have a general interval $[a, b]$, you can always remap the nodes between the two intervals

n	x_k	w_k
1	0	2
2	$\pm\frac{\sqrt{3}}{3} = \pm 0.5773502692$	1
3	0 $\pm\sqrt{3/5} = \pm 0.7745966692$	$\frac{8}{9} = 0.8888888889$ $\frac{5}{9} = 0.5555555556$

Table 18.1: Zeros of Legendre polynomials and weights of Gauss integration