1) $A \subset B \Rightarrow P(A) \leq P(B)$

C is a subset of $A \cup B \cup C \Rightarrow P(A \cup B \cup C) \geq P(C)$

$\Rightarrow P((A \cup B)C) + P(\overline{A \cup B \cup C}) \geq P(C)$

$= P((A \cup B)C) \geq P(C) - P(\overline{A \cup B \cup C})$

$$\Rightarrow \frac{P((A \cup B)C)}{P(C)} \geq 1 - \frac{P(\overline{A \cup B \cup C})}{P(C)}$$

$$\Rightarrow P((A \cup B)|C) \geq 1 - \frac{P(\overline{A \cup B \cup C})}{P(C)}$$

2)

$P(A|B) + P(\bar{A}|\bar{B}) = 1.$

$$\frac{P(AB)}{P(B)} + \frac{P(\bar{A}\bar{B})}{P(\bar{B})} = 1$$

$$\frac{P(AB)}{P(B)} + \frac{P(\bar{A}) - P(\bar{A}B)}{1 - P(B)} = 1$$

$$\frac{P(AB)}{P(B)} + \frac{1 - P(A) - P(B) + P(AB)}{1 - P(B)} = 1$$

$$P(AB) - P(AB)P(B) + P(B) - P(A)P(B) - P^2(B) + P(AB)P(B) = P(B) - P^2(B)$$

$P(AB) = P(A)P(B)$

Since A and B are independent, if we write all the above steps reversed the proof is done.

Scanned with CamScanner

3)
T_0 — "0" is transmitted, T_1 — "1" is trasnitted

R_0 — "0" is received R_1 — "1" is received

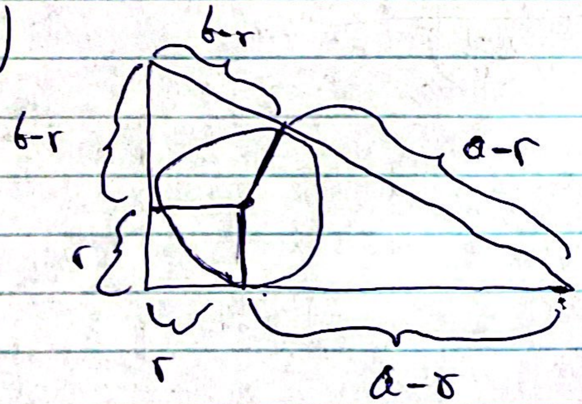
$P(R_0) = P(R_0|T_0)P(T_0) + P(R_0|T_1)P(T_1) = (0.7)(0.6) + (0.2)(0.4) = 0.5$

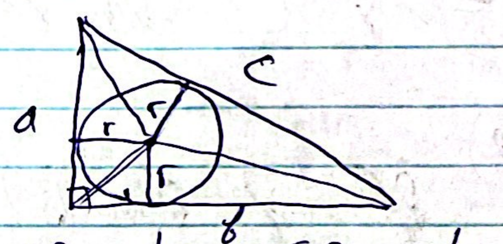
$P(R_1) = P(R_1|T_0)P(T_0) + P(R_1|T_1)P(T_1) = (0.3)(0.6) + (0.8)(0.4) = 0.5$

Using the total probability:

$P(\text{error}) = P(R_1|T_0)P(T_0) + P(R_0|T_1)P(T_1) = 0.3 \cdot 0.6 + 0.2 \cdot 0.4 = 0.26$

4)





$$\frac{ar}{2} + \frac{br}{2} + \frac{cr}{2} = \frac{ab}{2}$$

$$r = \frac{ab}{a+b+c}$$

The radius of the inscribed circle of a right triangle: $r = \frac{a+b-c}{2} = \frac{3+4-5}{2} = 1$

R in problem can be as large as 1. So, probability that circle stays inside of the triangle $= \frac{1}{3}$.

$r = \frac{a+b-c}{2}$ and $r = \frac{ab}{a+b+c}$ together we get $a^2+b^2=c^2$

5)

in second method let k = number of tests for each 10-people group. then k ~~takes~~ takes only 1 or 11.

for each group the average number of tests equals:

$1 \cdot P(k=1) + 11 \cdot P(k=11)$

$= P(\text{All 10 people are negative}) + 11P(\text{not all 10 people are negative})$

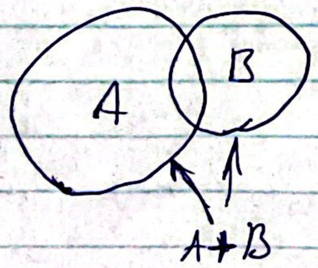
$= 0.9^{10} + 11(1-0.9^{10}) \approx 7.513$

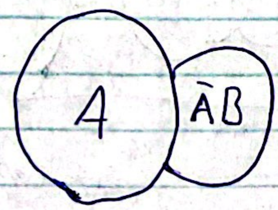
Then, for 100 people, the average number of tests equals approximately $10 \cdot 7.513 \approx 75$. that is by second method we can save about 25 tests.

6.

A - the event that the first slot; the probability of success for his second shot is $\frac{1}{8}$

That is $P(B|\bar{A}) = \frac{1}{8}$ Then the probability we are looking for: $P(A+B)$





$A+B = A+\bar{A}B$. we see that A and $\bar{A}B$ are mutually exclusive. then

$P(A+B) = P(A+\bar{A}B) = P(A) + P(\bar{A}B) = P(A) + P(B|\bar{A})P(\bar{A})$

$= \left(\frac{1}{2}\right) + \left(\frac{1}{8}\right)\left(\frac{1}{2}\right) = \frac{9}{16} = 0.5625$

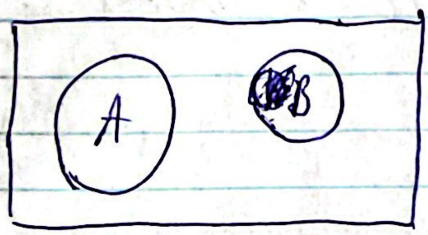
A and B are mutually exclusive. thus $P(A+B) = P(A) + P(B)$

but $P(B)$ was not given.

7)

If A and B are independent $P(AB)=P(A)P(B)$ if mutually exclusive $P(AB)=0$

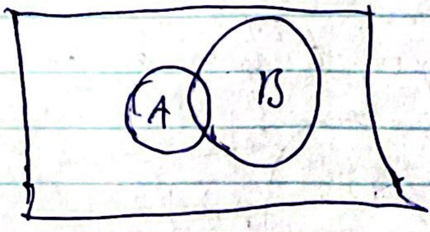
thus A and B are mutually exclusive and independent only if $P(A)P(B)=0 \Rightarrow P(A)=0,\ P(B)=0$ or both are equal to 0



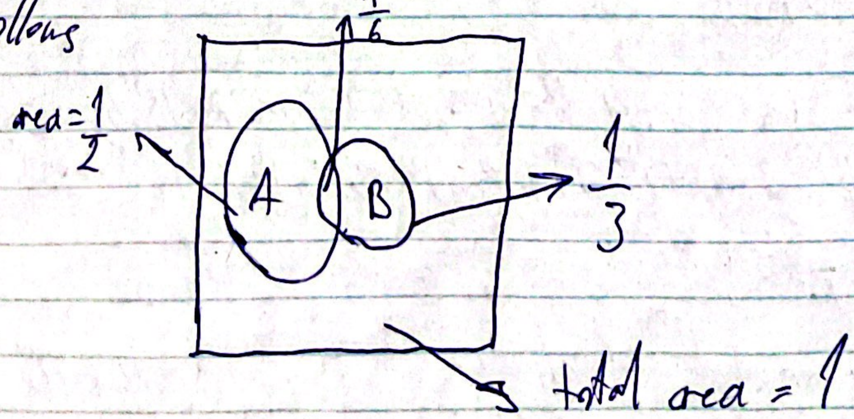
A and B are mutually exclusive

suppose probability = area

If A is x % of the total and portion of A in B is x % of B then A and B are independent.



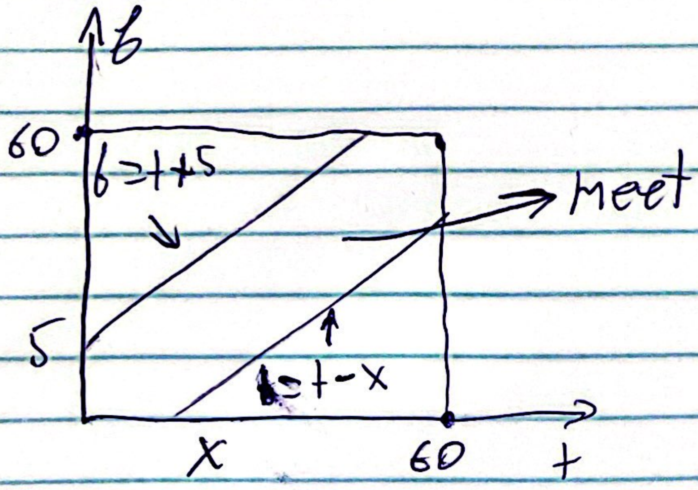
more specific example of independent events is given as follows



8.

t – ~~arrival~~ time of train

B – arrival time of bus



$$P(\text{meet}) = \frac{60^2 - \triangledown - \triangle}{60^2}$$

$$= \frac{\left\{ \frac{60^2 - 55^2}{2} - \frac{60 - x^2}{2} \right\}}{60^2} = 0.2$$

$x = 7.70277$