(1) $X(t)$ is WSS, then $E\{X(t)\} = \eta$ constant

a. $E\{Y(t)\} = E\{X'(t)\} = \frac{d}{dt} E\{X'(t)\} = \frac{d}{dt}\eta = 0$

b. $X(t)$ is WSS, then $R(t,t) = R(t-t) = R(0)$ constant

$$\frac{d}{dt} R(t,t) = \frac{d}{dt} E\{X(t)X(t)\} = \frac{d}{dt} E\{X^2(t)\} =$$

$$2E\{X(t)X'(t)\} = 2E\{X(t)Y(t)\} = 0$$

$$E\{X(t)Y(t)\} = 0$$

c. and d. $E\{X(t)Y(t)\} = 0 = E\{X(t)\}E\{Y(t)\}$

so, $X(t)$ and $Y(t)$ are uncorrelated and orthogonal

(2)

a. $e^{\lambda t} = 1 + \lambda t + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^3}{3!} + \frac{(\lambda t)^4}{4!} + \dots$

$$e^{-\lambda t} = 1 - \lambda t + \frac{(\lambda t)^2}{2!} - \frac{(\lambda t)^3}{3!} + \frac{(\lambda t)^4}{4!} - \dots$$

(1)−(2): $e^{\lambda t} - e^{-\lambda t} = 2\left(\lambda t + \frac{(\lambda t)^3}{3!} + \frac{(\lambda t)^5}{5!} + \dots\right)$

$$\frac{(1 - e^{-2\lambda t})}{2} = e^{-\lambda t}\left(\lambda t + \frac{(\lambda t)^3}{3!} + \frac{(\lambda t)^5}{5!} + \dots\right)$$

$$P(X(t)=1) = P(\text{in } (0,t),\ k \text{ is odd}) = \sum_{k=odd} e^{-\lambda t}\frac{(\lambda t)^k}{k!}$$

$$= e^{-\lambda t}\left(\lambda t + \frac{(\lambda t)^3}{3!} + \frac{(\lambda t)^5}{5!} + \dots\right) = \frac{(1 - e^{-2\lambda t})}{2}$$

(1)+(2):

$$e^{\lambda t} + e^{-\lambda t} = 2\left(1 + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^4}{4!} + \dots\right)$$

$$\frac{(1 + e^{-2\lambda t})}{2} = e^{-\lambda t}\left(1 + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^4}{4!} + \dots\right)$$

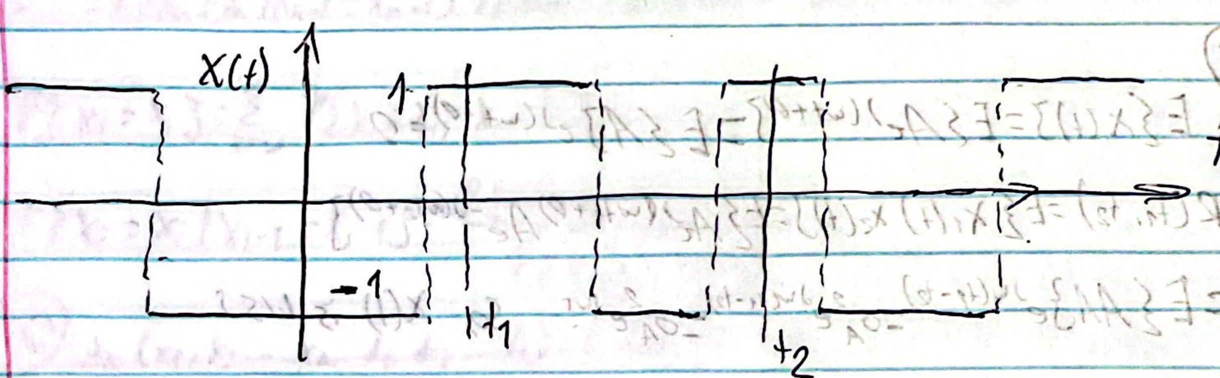
$$P(X(t)=-1) = P(\text{in } (0,t),\ k = \text{even}) = \sum_{k=even} e^{-\lambda t}\frac{(\lambda t)^k}{k!}$$

$$= e^{-\lambda t}\left(1 + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^4}{4!} + \dots\right) = \frac{(1 + e^{-2\lambda t})}{2}$$

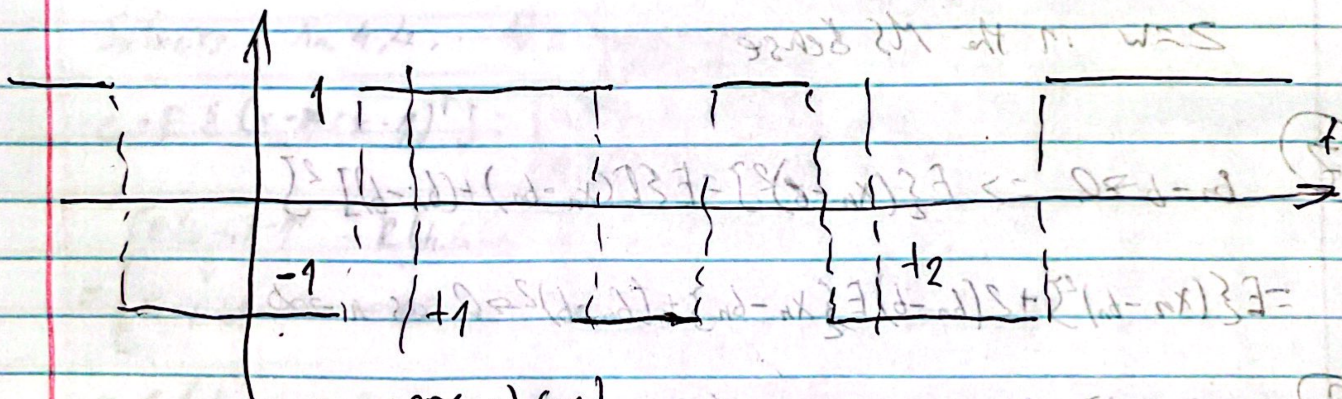
c. $E\{x(t)\} = (+1)P(x(t)=1) + (-1)P(x(t)=-1)$

$= \frac{1}{2}(1-e^{-2\lambda t}) - \frac{1}{2}(1+e^{-2\lambda t}) = -e^{-2\lambda t}$

d. for $t_2 > t_1$,



$+1 = (+1)(+1) \overset{or}{=} (-1)(-1)$



and $-1 = (+1)(-1) \overset{or}{=} (-1)(+1)$

$R(t_1,t_2) = E\{x(t_1)x(t_2)\} = (+1)P(x(t_1)x(t_2)=1) + (-1)P(x(t_1)x(t_2)=-1)$

$= P(x(t_1)x(t_2)=1) - P(x(t_1)x(t_2)=-1)$

$= P(\text{in } (t_1,t_2)\ k=\text{even}) - P(\text{in } (t_1,t_2),\ k=\text{odd})$

$= \frac{(1+e^{-2\lambda(t_2-t_1)})}{2} - \frac{1-e^{-2\lambda(t_2-t_1)}}{2} = e^{-2\lambda(t_2-t_1)}$

$R(t_1,t_2) = e^{-2\lambda|t_1-t_2|}$

3. $\frac{d}{dB}E\{(Y-BX^3)^2\} = 2E\{(Y-BX^3)X^3\} = 0 \Rightarrow B = \frac{E\{YX^3\}}{E\{X^6\}}$

(4)

$E\{(x(t+1)-x(t-1))^2\} = E\{x^2(t+1) - 2x(t+1)x(t-1) + x^2(t-1)\}$

$= E\{x^2(t+1)\} - 2E\{x(t+1)x(t-1)\} + E\{x^2(t-1)\}$

$= 2R(0) - 2R(2) = 4(1-e^{-2}) \approx 3.44$

(5)

$E\{x(t)\} = E\{Ae^{j(\omega t+\theta)}\} = E\{A\}e^{j(\omega t+\theta)} = 0$

$R(t_1, t_2) = E\{x_1(t)\,x_2(t)\} = E\{Ae^{j(\omega t_1+\theta)}Ae^{-j(\omega t_2+\theta)}\}$

$= E\{AA\}e^{j\omega(t_1-t_2)} = \sigma_A^2 e^{j\omega(t_1-t_2)} = \sigma_A^2 e^{j\omega\tau}$. So, $x(t)$ is WSS

(6) If $E\{Z^2\} = E\{W^2\} = E\{ZW\}$, then $E\{(Z-W)^2\} = E\{Z^2\} - 2E\{ZW\} + E\{W^2\} = 0$

$Z = W$ in the MS sense

(7) $b_n - b \to 0 \implies E\{(x_n-b)^2\} = E\{[(x_n-b_n)+(b_n-b)]^2\}$

$= E\{(x_n-b_n)^2\} + 2(b_n-b)E\{x_n-b_n\} + (b_n-b)^2 \to 0$ as $n \to \infty$

(8) $E\{Z(t)\} = E\{A\}\cos\omega_0 t + E\{B\}\sin\omega_0 t = 0$

$R(t_1,t_2) = E\{Z(t_1)Z(t_2)\} = E\{(A\cos\omega_0 t_1 + B\sin\omega_0 t_1)(A\cos\omega_0 t_2 + B\sin\omega_0 t_2)\}$

$= E\{A^2\}\cos\omega_0 t_1 \cos\omega_0 t_2 + \underbrace{E\{AB\}}_{=0}\cdot(\text{cross terms}) + E\{B^2\}\sin\omega_0 t_1 \sin\omega_0 t_2$

$= \cos\omega_0 t_1 \cos\omega_0 t_2 + \sin\omega_0 t_1 \sin\omega_0 t_2 = \cos\omega_0(t_1-t_2)$

where we applied $\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)$

9

$P\{X_1 = x_1, X_2 = x_2, \dots X_n = x_n\}$ 　 $1 \quad 2 \quad 3 \dots n$

$= P\{X_n = x_n \mid X_{n-1} = x_{n-1}, \dots X_1 = x_1\} \cdot P\{X_{n-1} = x_{n-1} \dots X_1 = x_1\}$

$= \dots = P\{X_n = x_n \mid X_{n-1} = x_{n-1}\} \cdot P\{X_{n-1} = x_{n-1} \mid X_{n-2} = x_{n-2}\} \cdot P\{X_2 = x_2 \mid X_1 = x_1\} \cdot P_{X_1 = x}$

$P\{X_i = x_i\} = \sum_{x=-\infty}^{\infty} P\{X_i = x_i, X_j = k\}$

$$P\{X_i = x_i \mid X_{i-1} = x_{i-1}\} = \frac{P\{X_i = x_i, X_{i-1} = x_{i-1}\}}{P\{X_{i-1} = x_{i-1}\}}$$

10 $f_X(x_1, x_2 \dots x_n, t_1, t_2, \dots t_n) = f_X(x_1, x_2, \dots x_n, t_1+\varepsilon, t_2+\varepsilon \dots t_n+\varepsilon)$

for any n, any $t_1\, t_2 \dots t_n$ and any ε

$x(t)$ is normal

$$f_X(x_1, x_2 \dots x_n, t_1, t_2, \dots t_n) = \frac{1}{\sqrt{2\pi}^n \sqrt{\det C}} e^{-\frac{1}{2}(\underline{x} - \underline{\eta})^T C^{-1} (\underline{x} - \underline{\eta})}$$

$$C = E\{(\underline{x} - \underline{\eta})(\underline{x} - \underline{\eta})^T\} = \begin{bmatrix} x(t_1) - E\{x(t_1)\} \\ \vdots \\ x(t_n) - E\{x(t_n)\} \end{bmatrix} \cdot [x(t_1) - E\{x(t_1)\} \dots x(t_n) - E\{x(t_n)\}]$$

$$= \begin{bmatrix} R(t_1 - t_1) - \eta^2 & \dots & R(t_1, t_n) - \eta^2 \\ \vdots & & \vdots \\ R(t_1, t_n) - \eta^2 & \dots & R(t_n, t_n) - \eta^2 \end{bmatrix} = \begin{bmatrix} R(0) - \eta^2 & \dots & R(t_1 - t_2) - \eta^2 \\ \vdots & & \vdots \\ R(t_1 - t_n) - \eta^2 & \dots & R(t_n - t_n) - \eta^2 \end{bmatrix}$$

$= C(t_1 - t_2, t_1 - t_3, \dots t_{n-1} - t_n) = C(t_i - t_j \text{ for } i \neq j \quad i, j = 1, 2, \dots n$

$f_X(x_1, x_2 \dots x_n, t_1, t_2 \dots t_n) = f_X(x_1, x_2 \dots x_n, t_i - t_j, \text{ for } i \neq j, \; i, j = 1, 2 \dots n)$

$f_X(x_1, x_2 \dots x_n, t_1+\varepsilon, t_2+\varepsilon, \dots, t_n+\varepsilon)$

$= f_X(x_1, x_2 \dots x_n, (t_i+\varepsilon) - (t_j+\varepsilon), \text{ for } i \neq j \quad i, j = 1, 2, \dots, n)$

$= f_X(x_1, x_2 \dots x_n \; t_i - t_j \text{ for } i \neq j, \; i, j = 1, 2, \dots, n)$

$= f_X(x_1, x_2 \dots x_n, t_1, t_2, \dots t_n)$