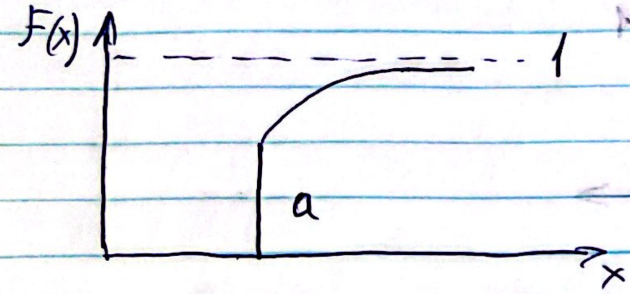
1

1. $F(\infty)=1$ so $A=1$

2.



3.

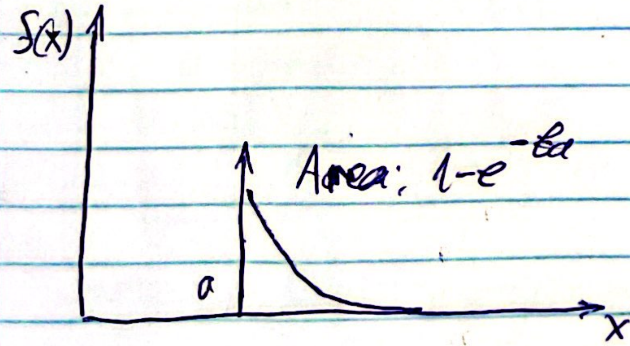
$$h(x)\delta(x-\tau)=h(\tau)\delta(x-\tau)$$

$$\Rightarrow f(x)=\frac{d}{dx}F(x)=\frac{d}{dx}(1-e^{-bx})u(x-a)$$

$$=\left(\frac{d}{dx}(1-e^{-bx})\right)u(x-a)+(1-e^{-bx})\frac{d}{dx}u(x-a)$$

$$=be^{-bx}u(x-a)+(1-e^{-bx})\delta(\cancel{x}-a)$$

$$=be^{-bx}u(x-a)+(1-e^{-ba})\delta(x-a)$$



2.

$P(x_2=0) = P(x_2=0|x_1=0)P(x_1=0) + P(x_2=0|x_1=1)$
$P(x_1=1)$ $P(x_2=1) = P(x_2=1|x_1=0)P(x_1=0) + P(x_2=1|x_1=1)P(x_1=1)$

$\{P(x_2=0), P(x_2=1)\} = (0.2, 0.8)\begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix} = (0.36, 0.64)$

where $P(x_2=0|x_1=0)=0.6$, $P(x_2=1|x_1=0)=0.4$
$P(x_2=0|x_1=1)=0.3$, $P(x_2=1|x_1=1)=0.7$

a) $P(x_2=1)=0.64$

b) $\{P(x_3=0), P(x_3=1)\} = (0.36, 0.64)\begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix} = (0.408, 0.592)$

or $\{P(x_3=0), P(x_3=1)\} = (0.2, 0.8)\begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix}^2 = (0.408, 0.592)$

So, $P(x_3=1)=0.592$

c) $\{P(x_3=0), P(x_3=1)\} = (0.2, 0.8)\begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix}^2 = (0.408, 0.592)$

$\begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix}^2 = \begin{pmatrix} 0.48 & 0.52 \\ 0.39 & 0.61 \end{pmatrix}$

where

$P(x_3=0|x_1=0)=0.48$, $P(x_3=1|x_1=0)=0.52$
$P(x_3=0|x_1=1)=0.39$, $P(x_3=1|x_1=1)=0.61$

$P(x_1=0|x_3=1) = \dfrac{P(x_1=0, x_3=1)}{P(x_3=1)} = \dfrac{P(x_3=1|x_1=0)P(x_1=0)}{P(x_3=1)}$

$= \dfrac{(0.52)(0.2)}{0.592} = \dfrac{13}{74} \approx 0.176$

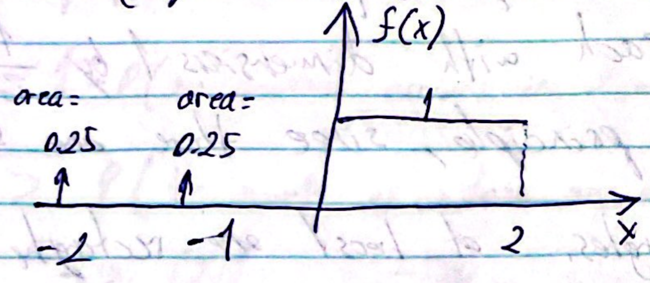
d) $\{P(X_\infty=0), P(X_\infty=1)\} = \lim_{n\to\infty} (0.2, 0.8) \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix}^n$

$= (0.2, 0.8) \begin{pmatrix} \frac{3}{7} & \frac{4}{7} \\ \frac{3}{7} & \frac{4}{7} \end{pmatrix} = \left(\frac{3}{7}, \frac{4}{7}\right)$

so, $\lim_{n\to\infty} P(X_n=0) = \frac{3}{7}$

3.

1) $f(x) = 0.25\delta(x+2) + 0.25\delta(x+1) + 0.25(u(x) - u(x-2))$



2)

$E\{x\} = \int_{-\infty}^{\infty} x f(x) dx = \int_{-\infty}^{\infty} x \{0.25\delta(x+2) + 0.25\delta(x+1) + 0.25(u(x) - u(x-2))\} dx = 0.25 \{\int_{-\infty}^{\infty} x\delta(x+2)dx + \int_{-\infty}^{\infty} x\delta(x+1)dx + \int_0^2 x dx\} = 0.25\{-2-1+(\frac{1}{2})4\} = -0.25$

4.

We can divide the 1 by 1 square into 2 equal rectangles, each with dimensions 1 by $\frac{1}{2}$ by the pigeonhole principle, since there are 5 points and only 2 rectangles. at least one rectangle must contain 3 or more of these points. Area of each rectangle is $\frac{1}{2}$. and any triangle formed within a rectangle cannot have an area larger than the rectangle itself. So, a triangle formed by 3 points in the same rectangle will have a maximum possible area. that is half area of ~~the~~ rectangle which is $\frac{1}{4}$ since one rectangle must contain at least 3 points, there must be at least one triangle with an area less than or equal to $\frac{1}{4}$

So Probability that there is at least one triangle with an area less than or equal to $\frac{1}{4}$ when 5 points are dropped into a 1 by 1 square is 1. means it is certain to occur.

Scanned with CamScanner

5.

1) P(at least one customer arrives in 2 mins)

$= 1 - P$(no customer arrives in 2 mins) $= 0.3$

$\Rightarrow P$(the number of customer arrivals in $(0,2) = 0$)

$$= e^{-2\lambda}\frac{(2\lambda)^0}{0!} = e^{-2\lambda} = 0.7 \Rightarrow \lambda = \frac{-(\ln 0.7)}{2} \approx 0.178$$

2. P(no customer in 3 mins)

$$= e^{-3\lambda}\frac{(3\lambda)^0}{0!} = e^{-3\lambda} \approx e^{-3 \cdot 0.178} = 0.586$$

3. P($\overset{\text{at least}}{2}$ customers in 10 mins) $= 1 - P$(at most one in 10 mins)

$= 1 - P$(number of customer arrivals in $(0,10) = 0$ or 1)

$$= 1 - \left(e^{-\lambda t}\frac{(\lambda t)^0}{0!} + e^{-\lambda t}\frac{(\lambda t)^1}{1!}\right) = 1 - e^{-\lambda t}(1 + \lambda t)$$

$$\approx 1 - e^{-1.78}(1 + 1.78) = 0.531$$

4. P(exactly one customer in 1 min) $= P$(number of customer in $(0,1) = 1$)

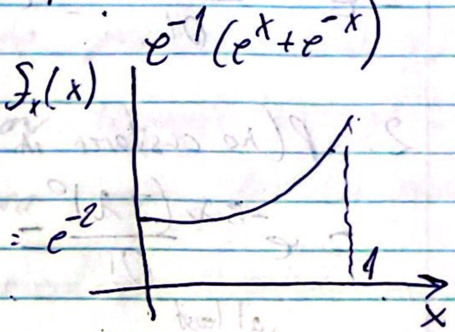
$$= e^{-\lambda}\frac{(\lambda)^1}{1!} = \lambda e^{-\lambda} \approx 0.178 e^{-0.178} = 0.149$$

6.

1) $\int_{-\infty}^{\infty} f(x)\,dx = A + e^{-1}\int_0^1 (e^x+e^{-x})\,dx = A+1-e^{-2} = 1 \Rightarrow A = e^{-2}$

2) $f(x) = e^{-2}\delta(x) + e^{-1}(e^x+e^{-x})(u(x)-u(x-1))$

3) $f_X(x) = \begin{cases} e^{-2}\delta(x, & x=0 \\ e^{-1}(e^x+e^{-x}), & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$



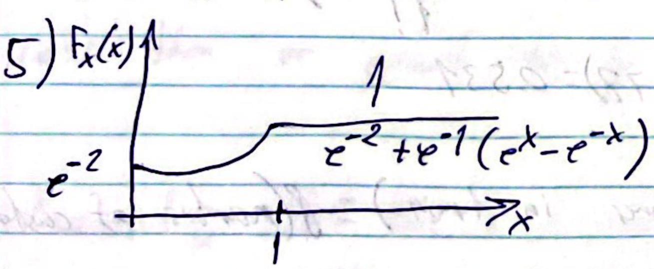
4) $x<0,\ F_X(x)=0$

in $(0,1)$, $F_X(x) = \int_{-\infty}^{x} f_X(a)\,da = \int_0^x (e^{-2}\delta(a) + e^{-1}(e^a+e^{-a}))\,da$

$= e^{-2} + e^{-1}(e^x - e^0 - e^{-x} + e^{-0}) = e^{-2} + e^{-1}(e^x - e^{-x})$

for $x \geq 1$, $F_X(x) = 1$

5)



7.

$$\min(x,y) = x\,u(y-x) + y\,u(x-y)$$

$$\frac{d}{dx}\min(x,y) = \frac{d}{dx}\{x\,u(y-x)\} + \frac{d}{dx}\{y\,u(x-y)\}$$

$$= u(y-x) - x\delta(y-x) + y\delta(x-y)$$

$$= u(y-x) - x\delta(y-x) + x\delta(x-y)$$

$$= u(y-x) - x\delta(y-x) + x\delta(y-x)$$

$$= u(y-x)$$

$$\frac{d}{dy}\frac{d}{dx}\min(x,y) = \frac{d}{dy}u(y-x) = \delta(y-x)$$

8.

If k is number of heads

$P(\text{even}) = P(k=0) + P(k=2) + \dots$

$= q^n + \binom{n}{2} p^2 q^{n-2} + \binom{n}{4} p^4 q^{n-4} + \dots$

$1 = (q+p)^n = \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} = \binom{n}{0} p^0 q^n + \binom{n}{1} p^1 q^{n-1} + \binom{n}{2} p^2 q^{n-2} + \dots$

$(q-p)^n = \sum_{k=0}^{n} \binom{n}{k} (-p)^k q^{n-k} = \sum_{k=0}^{n} \binom{n}{k} (-1)^k p^k q^{n-k}$

$= \binom{n}{0} p^0 q^n - \binom{n}{1} p^1 q^{n-1} + \binom{n}{2} p^2 q^{n-2} - \dots$

$1 + (q-p)^n = 2P(\text{even})$ so, $P(\text{even}) = \dfrac{\{1+(q-p)^n\}}{2}$

Scanned with CamScanner

9.

number of accident the driver has is a random variable that follows Poisson distribution, which is appropriate for modeling the number of times an event occurs within a fixed interval of time or space

$$P(X=k) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

$$P(X=3) = \frac{e^{-\lambda t}(\lambda t)^3}{3!}$$

$$P(X=3) = \frac{e^{-0.03 \cdot 100}(0.03 \cdot 100)^3}{3!}$$

$$P(X=3) = \frac{e^{-3}(3)^3}{3!}$$

$$P(X=3) = \frac{e^{-3} \cdot 27}{6}$$

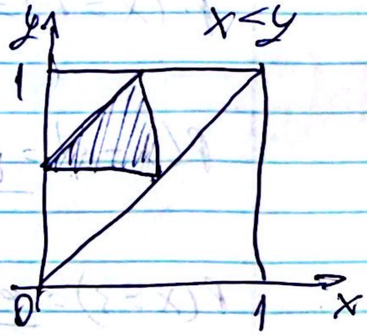
$$P(X=3) = \frac{27}{e^3 \cdot 6}$$

$$P(X=3) = \frac{27}{6e^3} \approx 0.224$$

10.

for three segment to form a triangle the sum of the lengths of any two segments must be greater then the length of the remaining segment.

for two segment x and y $(x<y)$ the three segments are $x, y-x$, and $1-y$.



$$\begin{cases} x+(y-x)>1-y \\ x+(1-y)>y-x \\ y-x+(1-y)>x \end{cases} \Rightarrow \begin{cases} y>\frac{1}{2} \\ y<x+\frac{1}{2} \\ x<\frac{1}{2} \end{cases}$$

case 2 $y<x$ mirrors the conditions with x and y interchanged

$$\begin{cases} x>\frac{1}{2} \\ x<y+\frac{1}{2} \\ y<\frac{1}{2} \end{cases}$$

so, ratio of the shaded area that satisfies the triangle inequality to the total area which is $\frac{1}{4}$

even though there are many points that satisfy the right triangle condition, the probability based on area is zero since the region that forms a right triangle has no area in a continuous space.

so, probability that those three pieces can form a right triangle is zero.