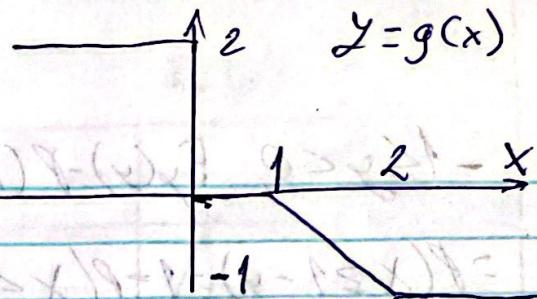


①



$$y = g(x) = \begin{cases} 2, & x < 0 \\ 0, & 0 \leq x < 1 \\ -x+1, & 1 \leq x \leq 2 \\ -1, & x > 2 \end{cases}$$

$$-1 \leq y \leq 1, \text{ so, } F_X(y) = \begin{cases} 0, & y < -1 \\ 1, & y \geq 1 \end{cases}$$

~~For~~  $-1 \leq y \leq 1$

$$y = -1, P(Y = -1) = P(X > 2) = 1 - F_X(2)$$

$$y = 0 \quad P(Y = 0) = P(0 < X < 1) = F_X(1) - F_X(0)$$

$$y = 2 \quad P(Y = 2) = P(X < 2) = F_X(0)$$

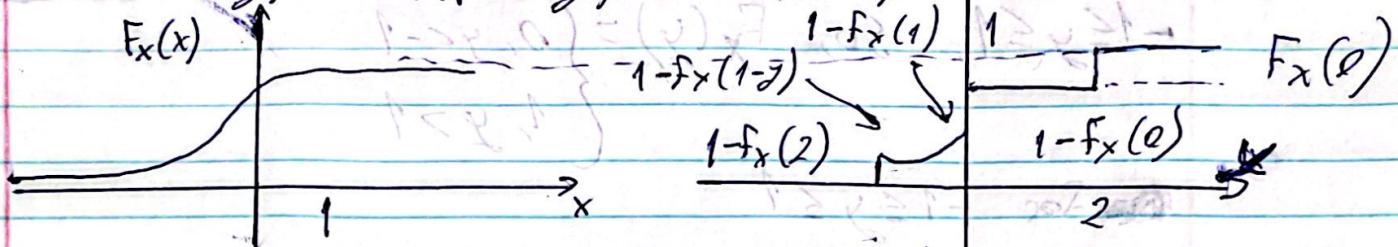
$$\text{In } (-1, 0), y = -x+1, f_Y(y) = \frac{f_X(x)}{|g'(x)|} = \frac{f_X(1-y)}{|-1|} = f_X(1-y)$$

$$f_Y(y) = \begin{cases} (1 - F_X(2))\delta(y + 1), & y = -1 \\ f_X(1-y), & -1 < y < 0 \\ (F_X(1) - F_X(0))\delta(y), & y = 0 \\ F_X(0)\delta(y - 2), & y = 2 \\ 0, & \text{otherwise} \end{cases}$$

$$F_Y(y) = \begin{cases} 0 & y < -1 \\ 1 - F_X(1-y) & -1 \leq y < 0 \\ 1 - F_X(0), & 0 \leq y < 2 \\ 1, & 2 \leq y \end{cases}$$

$$-1 \leq y < 0, \quad F_y(y) = P(Y \leq y) = P(1-X \leq y) = P(-X \leq y-1)$$

$$= P(X \geq 1-y) = 1 - P(X \leq 1-y) = 1 - F_x(1-y) \quad \uparrow F_x(y)$$



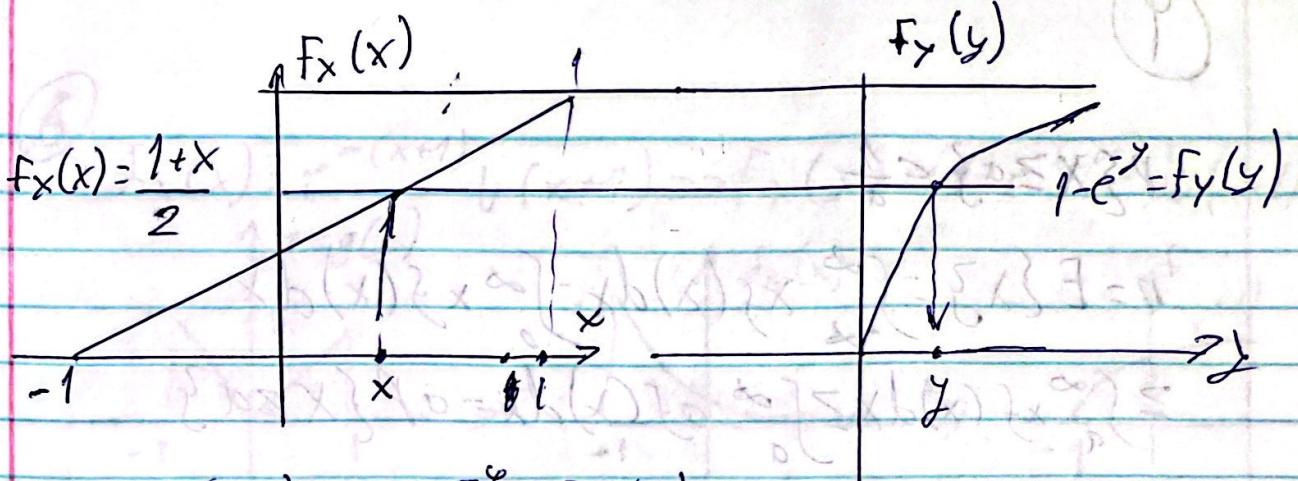
The figure shows a graph of a function  $f_x(x)$  on a coordinate plane. The horizontal axis is labeled  $x$  and has tick marks at 0, 1, and 2. The vertical axis is labeled  $f_x(x)$ . The function starts at  $(0, 0)$ , rises to a peak between  $x=0$  and  $x=1$ , and then decreases towards  $x=2$ . A horizontal line segment connects the point  $(2, 0)$  on the  $x$ -axis to the curve at  $x=2$ . The region under the curve from  $x=0$  to  $x=2$  is shaded light blue and labeled "area = f\_x(0)". The region under the curve from  $x=2$  to  $x=1$  is shaded light red and labeled "area = 1-f\_x(2)". To the right of the graph, the total area is given as "area = f\_x(1) - f\_x(0)".

$$f = g \circ h \quad \text{where } h(x) = ((x+1)^2 - 1)^{\frac{1}{2}} = (x+1)^2$$

$$S = \{x_1, (x_2)z((x_3)x_4 - (x_5)x_6)\}$$

$$S = \{x \mid (x-a)B(1) > 0\}$$

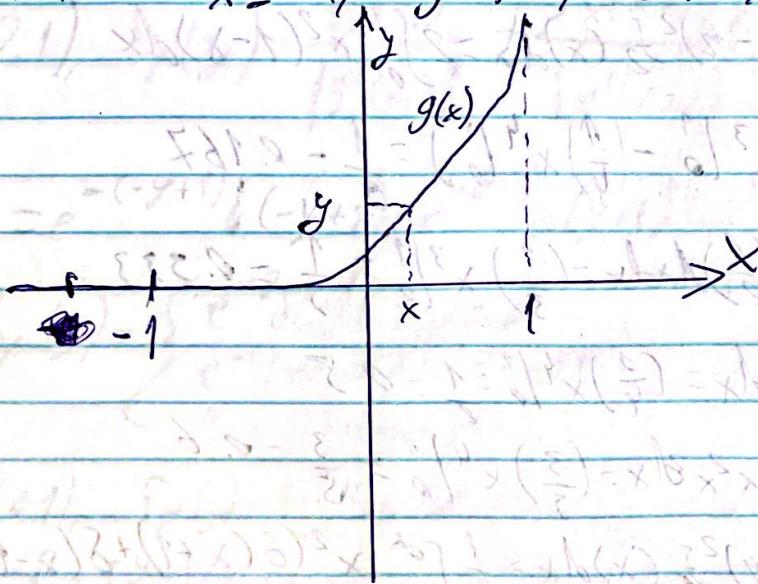
$$\left. \begin{array}{l} 1 \geq x \\ 0 \leq 2x+1 - (x-1) < 1 \\ 3 \geq x-1 - (x-1) \end{array} \right\} \Rightarrow \{x \in \mathbb{R} \mid 1 \geq x \}$$



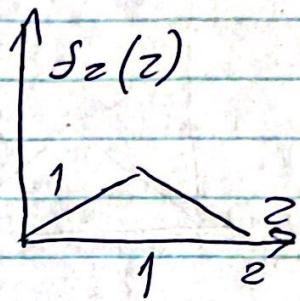
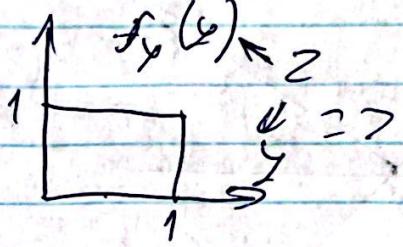
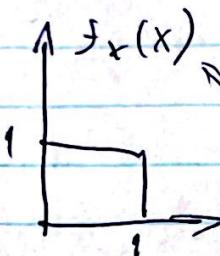
$$f_x(x) = \frac{1+x}{2} = 1 - e^{-y} = f_y(y)$$

$$y = -\ln \frac{1-x}{2} = g(x)$$

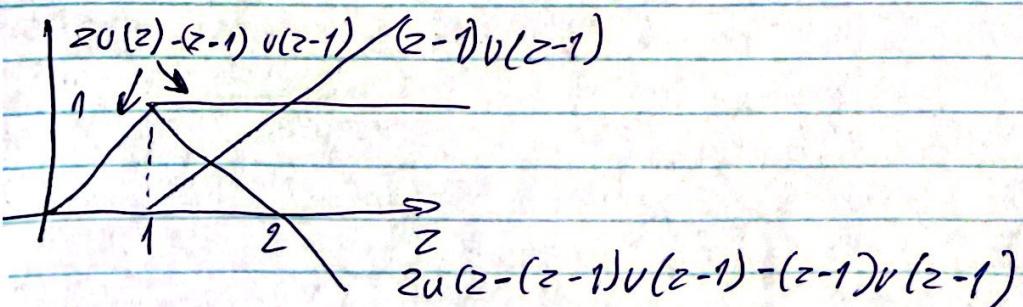
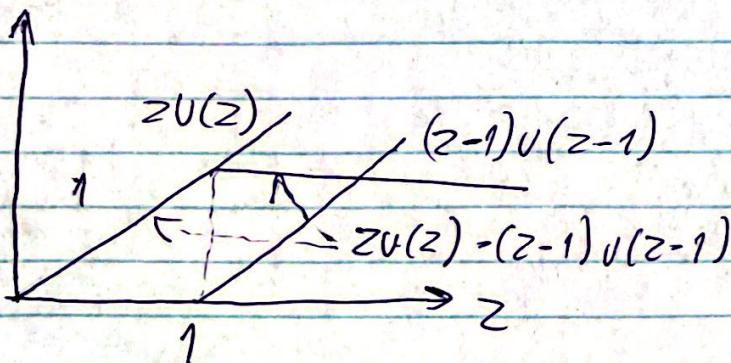
$-1 < x < 1$  :  $x \leq -1, g(x) = 0$ , for  $x \geq 1, g(x) = \infty$

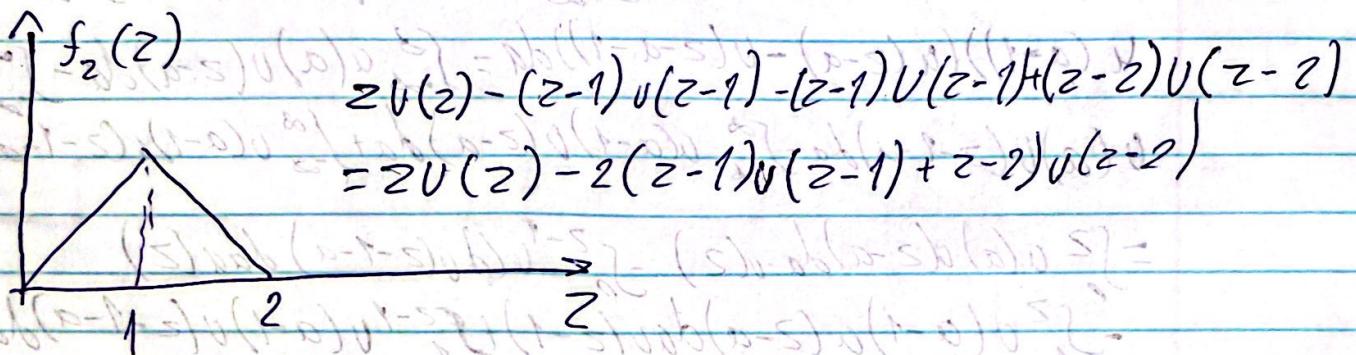
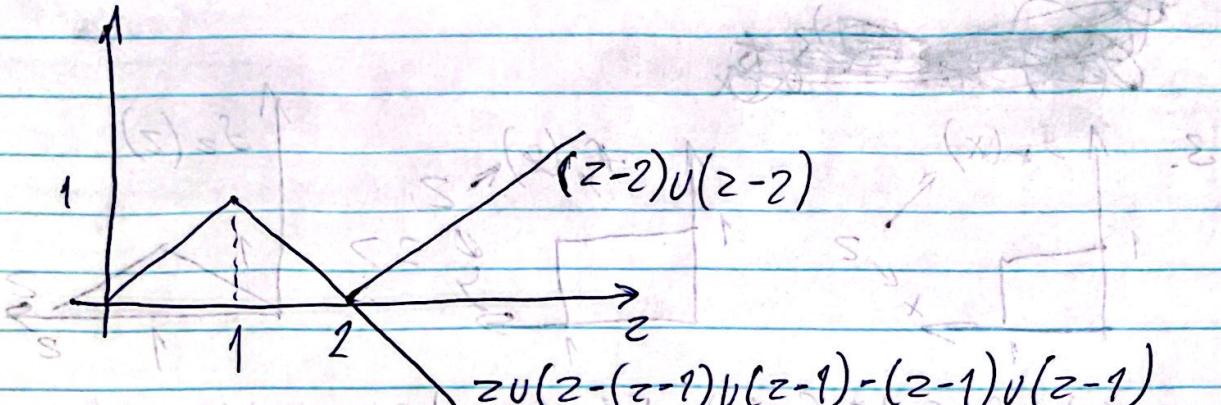


~~scribbles~~



$$\begin{aligned}
 f_z(z) &= f_x(z) * f_y(z) = \int_{-\infty}^{\infty} f_x(a) f_y(z-a) da = \int_{-\infty}^{\infty} (u(a) - \\
 &u(a-1))(u(z-a) - u(z-a-1)) da = \int_{-\infty}^{\infty} u(a) u(z-a) da - \int_{-\infty}^{\infty} \\
 &u(a) u(z-1-a) da - \int_{-\infty}^{\infty} u(a-1) u(z-a) da + \int_{-\infty}^{\infty} u(a-1) u(z-1-a) da \\
 &= \int_0^z u(a) u(z-a) da \cdot u(z) - \int_0^{z-1} u(a) u(z-1-a) da \cdot u(z) \\
 &\quad - \int_1^z u(a-1) u(z-a) da u(z-1) + \int_1^{z-1} u(a-1) u(z-1-a) da \cdot u(z-2) \\
 &= \int_0^z 1 da \cdot u(z) - \int_0^{z-1} 1 da u(z) = \int_0^z 1 da \cdot u(z-1) + \int_1^{z-1} 1 da u(z-2) \\
 &= z u(z) - (z-1) u(z-1) - (z-1) u(z-1) + (z-2) u(z-2)
 \end{aligned}$$





$$(s-s)v(s-s)+(r-s)v(r-s)-(r-s)v(r-s)-(s)v(s) =$$

$$(r-s)v(r-s)-(s)v(s)$$

$$(r-s)v(r-s)-(r-s)v(r-s)(s)v(s)$$

$$(r-s)(r-s)-(r-s)v(r-s)-s)v(s)$$

(9)

$$P\{X \geq a\} \leq \frac{n}{a}$$

$$\eta = E\{\lambda\} = \int_{-\infty}^{\infty} x f(x) dx - \int_0^{\infty} x f(x) dx$$

$$\geq \int_a^{\infty} x f(x) dx \geq \int_a^{\infty} a f(x) dx = a P\{X \geq a\}$$

$$\text{let } a = \sqrt{\eta}, \quad P\{X \geq \sqrt{\eta}\} \leq \frac{n}{\sqrt{\eta}} = \sqrt{\eta}$$

(5)

$$1. \sigma_1^2 = \int_{-\infty}^{\infty} (x-n)^2 f_1(x) dx = \int_{-\infty}^{\infty} x^2 \delta(x) dx = \int_{-\infty}^{\infty} n^2 \delta(x) dx = 0$$

$$2. \sigma_2^2 = \int_{-\infty}^{\infty} (x-n)^2 f_2(x) dx = 2 \int_0^1 x^2 (1-x) dx \\ = 2 \left( \frac{1}{3} x^3 \Big|_0^1 - \left( \frac{1}{6} x^4 \Big|_0^1 \right) \right) = \frac{1}{6} = 0.167$$

$$3. \sigma_3^2 = 2 \int_0^1 x^2 \left( \frac{1}{2} \right) dx = \left( \frac{1}{3} \right) x^3 \Big|_0^1 = \frac{1}{3} = 0.333$$

$$4. \sigma_4^2 = 2 \int_0^1 x^2 x dx = \left( \frac{1}{4} \right) x^4 \Big|_0^1 = \frac{1}{4} = 0.5$$

$$5. \sigma_5^2 = 2 \int_0^1 \left( \frac{3}{2} \right) x^2 x^3 dx = \left( \frac{3}{5} \right) x^4 \Big|_0^1 = \frac{3}{5} = 0.6$$

$$6. \sigma_6^2 = \int_{-\infty}^{\infty} (x-n)^2 f_6(x) dx = \frac{1}{2} \int_{-\infty}^{\infty} x^2 (\delta(x+1) + \delta(x-1)) dx \\ = \left( \frac{1}{2} \right) \left( \int_{-\infty}^{\infty} x^2 \delta(x+1) dx + \int_{-\infty}^{\infty} x^2 \delta(x-1) dx \right) \\ = \left( \frac{1}{2} \right) \left( \int_{-\infty}^{\infty} (-1)^2 \delta(x+1) dx + \int_{-\infty}^{\infty} 1^2 \delta(x-1) dx \right) \\ = \left( \frac{1}{2} \right) \left( \int_{-\infty}^{\infty} \delta(x+1) dx + \int_{-\infty}^{\infty} \delta(x-1) dx \right) = \left( \frac{1}{2} \right) (1+1) = 1$$

$$7. \sigma_7^2 = 2 \int_0^{0.5} 0.75 x^2 dx + 2 \int_{0.5}^1 0.25 x^2 dx = 2 \left( 0.75 \frac{1}{3} \frac{1}{8} + 0.25 \frac{1}{3} \frac{1}{8} \right) = \frac{5}{24} \approx 0.208$$

$$8. \sigma_8^2 = 2 \int_{0.5}^1 x^2 dx = \frac{7}{12} \approx 0.583$$

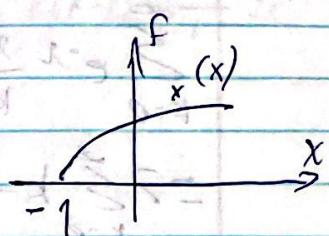
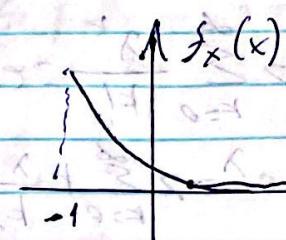
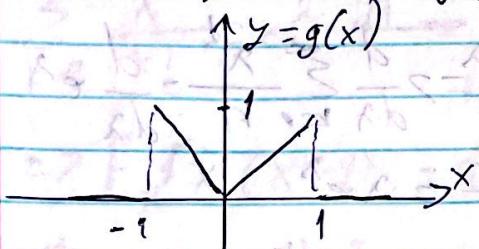
$$9. \int_0^1 a(1-x^2) dx = 1 \Rightarrow a = \frac{3}{4} \quad f_g(x) = \left( \frac{3}{4} \right) (1-x^2)$$

$$\sigma_9^2 = 2 \left( \frac{3}{4} \right) \int_0^1 x^2 (1-x^2) dx = \left( \frac{3}{2} \right) \left( \frac{1}{3} - \frac{1}{5} \right) = \frac{1}{5} = 0.2$$

$$\underline{\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2, \sigma_6^2, \sigma_7^2, \sigma_8^2}$$

(6)

$$f_X(x) = e^{-(x+1)} \nu(x+1) \Rightarrow F_X(x) = (1 - e^{-(x+1)}) \nu(x+1)$$



$$P(Y=0) = P(X < -1) + P(X > 1) = P(X < -1) + P(X > 1)$$

$$= f_X(-1) + (1 - f_X(1)) = 0 + 1 - (1 - e^{-(1+1)}) \nu(1+1) = e^{-2}$$

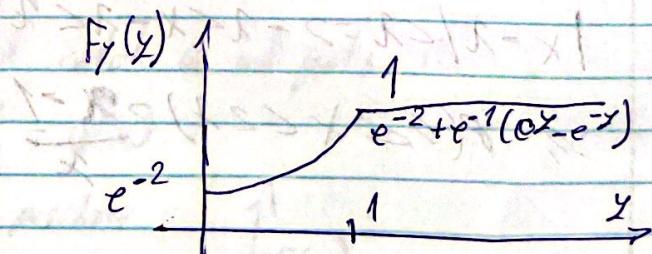
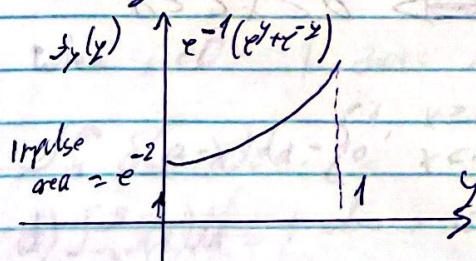
$$\text{In } (0, 1), f_Y(y) = \frac{f_X(x_1)}{|g'(x_1)|} + \frac{f_X(x_2)}{|g'(x_2)|} = f_X(-y) + f_X(y)$$

$$= e^{-(-y+1)} \nu(-y+1) + e^{-y+1} \nu(y+1) = e^{-1}(e^y + e^{-y})$$

$$f_Y(y) = \begin{cases} e^{-2}\delta(y), & y=0 \\ e^{-1}(e^y + e^{-y}), & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

$$\begin{aligned} \text{In } (0, 1), F_Y(y) &= \int_{-\infty}^y f_Y(a) da = \int_{-2}^0 f_Y(a) da + \int_0^y f_Y(a) da \\ &= e^{-2} + \int_0^y f_Y(a) da = e^{-2} + e^{-1} \int_0^y (e^a + e^{-a}) da \\ &= e^{-2} + e^{-1} (e^a \Big|_0^y - e^{-a} \Big|_0^y) = e^{-2} + e^{-1} (e^y - 1 - e^{-y} + 1) = e^{-2} + e^{-1} (e^y - e^{-y}) \end{aligned}$$

$$\text{for } y \geq 1, f_Y(y) = 1$$



(7)

$$P(X=t) = e^{-\lambda} \frac{\lambda^t}{t!}$$

$$\sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} = 1 \Rightarrow \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^{\lambda} \Rightarrow \frac{d}{d\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = \frac{d}{d\lambda} e^{\lambda}$$

$$\Rightarrow \sum_{k=0}^{\infty} k \frac{\lambda^{k-1}}{k!} = e^{\lambda} \Rightarrow \sum_{k=0}^{\infty} k \frac{\lambda^k}{k!} = \lambda e^{\lambda}$$

$$\Rightarrow \sum_{k=0}^{\infty} k e^{-\lambda} \frac{\lambda^k}{k!} = \lambda \Rightarrow E\{X\} = \sum_{k=0}^{\infty} k e^{-\lambda} \frac{\lambda^k}{k!} = \lambda$$

$$\sigma^2 = E\{(X-\mu)^2\} = E\{X^2\} - \mu^2 = E\{X^2\} - \lambda^2$$

$$E\{X^2\} = \sum_{k=0}^{\infty} k^2 e^{-\lambda} \frac{\lambda^k}{k!} = ?$$

$$\sum_{k=0}^{\infty} k \frac{\lambda^k}{k!} = \lambda e^{\lambda} \Rightarrow \frac{d}{d\lambda} \sum_{k=0}^{\infty} k \frac{\lambda^k}{k!} = \frac{d}{d\lambda} \lambda e^{\lambda} = e^{\lambda} + \lambda e^{\lambda}$$

$$\Rightarrow \sum_{k=0}^{\infty} k^2 \frac{\lambda^{k-1}}{k!} = (1+\lambda) e^{\lambda} \Rightarrow \sum_{k=0}^{\infty} k^2 e^{-\lambda} \frac{\lambda^k}{k!} = (1+\lambda) \lambda$$

$$\sigma^2 = E\{X^2\} - \lambda^2 = (1+\lambda)\lambda - \lambda^2 = \lambda$$

$$P\{|X-\mu| \geq \epsilon\} \leq \frac{\sigma^2}{\epsilon^2}$$

$$= P\{|X-\mu| \geq \epsilon\} \geq 1 - \frac{\sigma^2}{\epsilon^2}$$

$$\text{Let } \epsilon = \lambda, \quad P\{|X-\lambda| \geq \lambda\} \geq 1 - \frac{\lambda^2}{\lambda^2} = \frac{\lambda-1}{\lambda}$$

$$|X-\lambda| \geq \lambda \Rightarrow -\lambda \leq X-\lambda \leq \lambda \Leftrightarrow 0 \leq X \leq 2\lambda$$

$$\text{So, } P(0 \leq X \leq 2\lambda) \geq \frac{\lambda-1}{\lambda}$$

(7b)

$$\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x \xrightarrow{\frac{d}{dx}} \sum_{k=0}^{\infty} k \frac{x^{k-1}}{k!} = x \xrightarrow{\frac{d}{dx}} \sum_{k=0}^{\infty} k(k-1) \frac{x^{k-2}}{k!} = x^2$$

$$\Rightarrow E[x(x-1)] = x^2$$

$$\sum_{k=0}^{\infty} k(k-1) \frac{x^{k-2}}{k!} = e^x \xrightarrow{\frac{d}{dx}} \sum_{k=0}^{\infty} k(k-1)(k-2) \frac{x^{k-3}}{k!} = x^3 \Rightarrow$$

$$E[x(x-1)(x-2)] = x^3$$

8.

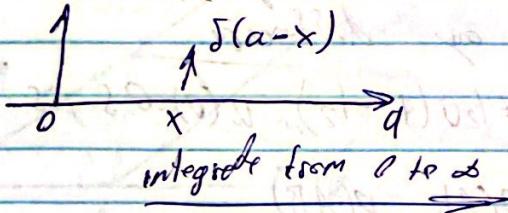
$$P(|X-\mu| \geq \epsilon) \leq \frac{\sigma^2}{\epsilon^2} \Rightarrow P(|X-\mu| \geq k\sigma) \leq \frac{\sigma^2}{k^2\epsilon^2} = \frac{1}{k^2}$$

$$\Rightarrow P(|X-\mu| \leq k\sigma) \geq 1 - \frac{1}{k^2} = \frac{k^2-1}{k^2} \quad \text{In(d), } k=4 \quad (d)$$

9. a)  $\int_0^\infty \delta(x-a) da = - \int_x^\infty \delta(\beta dB = \int_{-\infty}^x \delta(\beta dB = u(x))$

b)  $a$  and  $B$  are some. input function is  $u(x)$

c)

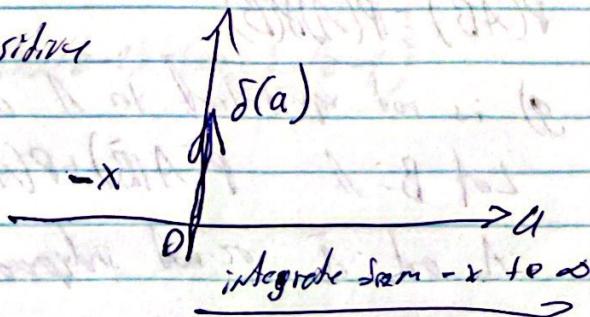


~~When~~ when  $x \geq 0$  integral catches the impulse and gets 1

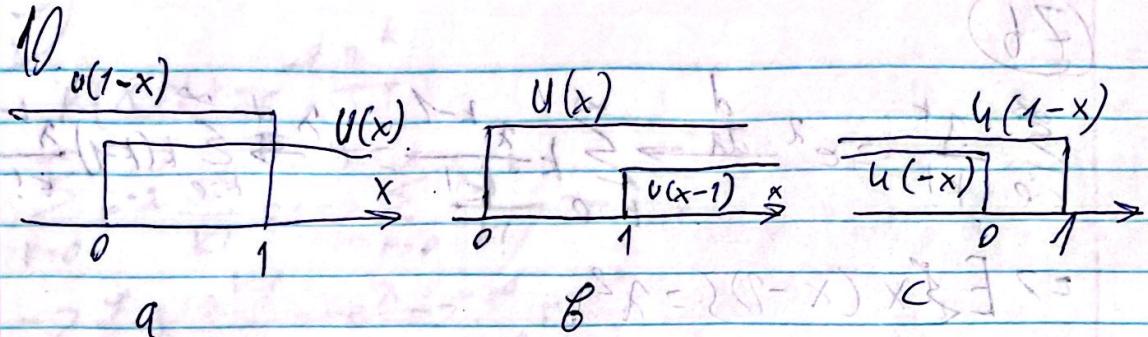
when  $x < 0$  it does not catch up and gets 0

$$\int_0^\infty \delta(a-x) da = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases} = u(x)$$

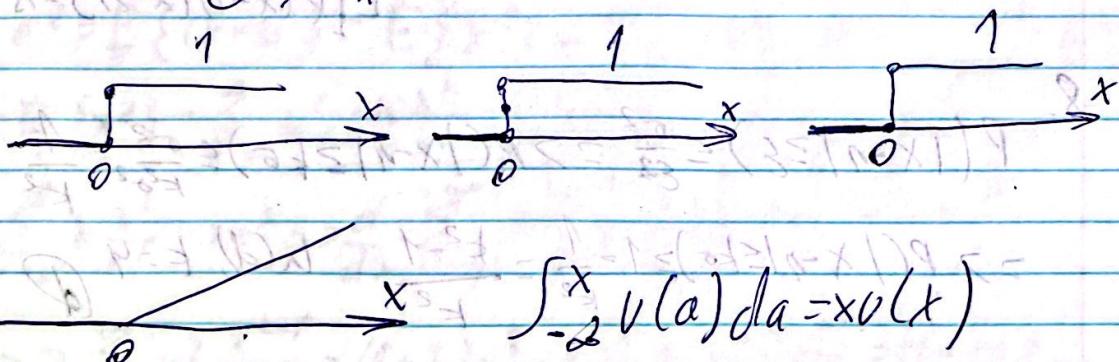
d)  $\int_{-x}^\infty \delta(a) da = 1$  when  $x$  is positive



10



$$u(x) = \begin{cases} 1, & x > 0 \\ A, & x = 0 \\ 0, & x < 0 \end{cases}$$



$$\int_{-\infty}^x v(a) da = xv(x)$$

A can be any finite value

When  $x$  is continuous variable, single point with finite value does not make any difference

$\min(t_1, t_2) = t_1 v(t_2 - t_1) + t_2 v(t_1 - t_2)$ ,  $v(0) = 0.5$  is reasonable

$$11. P(A|B) = P(A|\bar{B}) \Rightarrow \frac{P(AB)}{P(B)} = \frac{P(\bar{A}\bar{B})}{P(\bar{B})}$$

$$P(AB(1-P(B))) = P(B)P(\bar{A}\bar{B}) = P(B)P(A) - P(AB)$$

$$P(\bar{A}\bar{B}) - P(AB)P(\bar{B}) = P(A)P(\bar{B}) - P(\bar{B})P(AB)$$

$$P(AB) = P(A)P(B)$$

It is not equivalent to  $A$  and  $B$  are independent

$$\text{Let } B = A \quad P(A|\bar{B}) + P(\bar{A}|\bar{B}) = P(A|\bar{A}) + P(\bar{A}|A) = 0 + 1 = 1$$

$A$  and  $\bar{A}$  are not independent