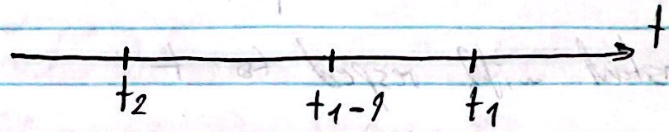
① $E\{Y(t)\} = E\{W(t) - W(t-1)\} = 0$

$R_Y(t_1,t_2) = E\{(W(t_1) - W(t_1-1))(W(t_2) - W(t_2-1))\}$

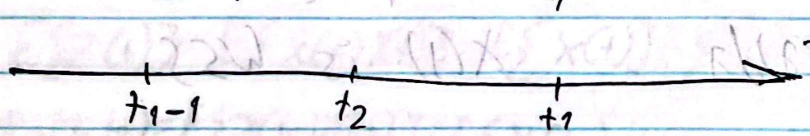
$= \alpha\{\min(t_1,t_2) - \min(t_1,t_2-1) - \min(t_1-1,t_2) + \min(t_1-1,t_2-1)\}$

when $t_2 < t_1$ and $t_2 < t_1 - 1$



$R_Y(t_1,t_2) = \alpha\{\min(t_1,t_2) - \min(t_1,t_2-1) - \min(t_1-1,t_2) + \min(t_1-1,t_2-1)\} = \alpha(t_2 - t_2 + 1 - t_2 + t_2 - 1) = 0$

when $t_2 < t_1$ and $t_2 + 1 > t_1$



$R_Y(t_1,t_2) = \alpha\{\min(t_1,t_2) - \min(t_1,t_2-1) - \min(t_1-1,t_2) + \min(t_1-1,t_2-1)\} = \alpha(t_2 - t_2 + 1 - t_1 + 1 + t_2 - 1) = \alpha(1-(t_1-t_2))$

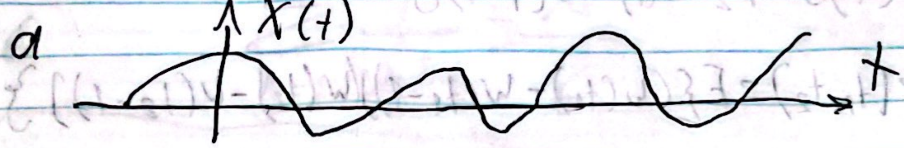
$R_Y(t_1,t_2) = \begin{cases} 0 & |t_1-t_2| > 1 \\ \alpha(1-|t_1-t_2|), & |t_1-t_2| \le 1 \end{cases}$

Therefore, $R_Y(\tau) = \begin{cases} 0, & |\tau| > 1 \\ \alpha(1-|\tau|) & |\tau| \le 1 \end{cases}$

$Y(t)$ is WSS

(2)

a



B. $E\{x(t)\} = \frac{1}{T}\int_0^T h(t+\alpha)\,d\alpha \overset{t+\alpha=\lambda}{=} \frac{1}{T}\int_t^{t+T} h(\lambda)\,d\lambda$

$= \frac{1}{T}\int_0^T h(\lambda)\,d\lambda =$ constant with respect to t

$R_x(t_1,t_2) = E\{X(t_1)X(t_2)\} = E\{h(t_1+\alpha)h(t_2+\alpha)\}$

$= \frac{1}{T}\int_0^T h(t_1+\alpha)h(t_2+\alpha)\,d\alpha \overset{t_2+\alpha=\lambda}{=} \frac{1}{T}\int_{t_2}^{t_2+T} h(t_1+\lambda-t_2)h(\lambda)\,d\lambda$

$= \frac{1}{T}\int_0^T h(\tau+\lambda)h(\lambda)\,d\lambda$ $\quad X(t)$ is WSS

(3)

$\eta_t = E\{X(t)\} = \lambda t, \quad \sigma_x^2 = E\{(X(t)-\eta_t)^2\} = \lambda t$

$\lim_{n\to\infty} P\left\{\left|\frac{X(t)}{t} - \lambda\right| \le \varepsilon\right\} = 1 \Rightarrow \lim_{n\to\infty} P\left\{\left|\frac{X(t)}{t} - \lambda\right| > \varepsilon\right\} = 0$

$\lim_{t\to\infty} P\left\{\left|\frac{X(t)}{t} - \lambda\right| > \varepsilon\right\} \le \lim_{t\to\infty} \frac{E\left\{\left|\frac{X(t)}{t} - \lambda\right|^2\right\}}{\varepsilon^2} = \lim_{t\to\infty} \frac{\lambda t}{t^2\varepsilon^2} = 0$

$\lim_{t\to\infty} \frac{X(t)}{t} = \lambda$

(4)

$E\{y^2(t)\} = E\left\{\int_0^t z(\alpha)\,d\alpha \int_0^t z(\beta)\,d\beta\right\} = \int_0^t\int_0^t E\{z(\alpha)z(\beta)\}\,d\alpha\, d\beta$

$= \int_0^t\int_0^t q(\beta)\delta(\alpha-\beta)\,d\beta\, d\alpha = \int_0^t q(\alpha)\left\{\int_0^t \delta(\alpha-\beta)\,d\beta\right\}d\alpha = \int_0^t q(\alpha)\,d\alpha$

(5)

$$E\{Y(t)\} = E\left\{\int_{-\infty}^{\infty} X(t-\alpha)h(\alpha)\,d\alpha\right\} = \int_{-\infty}^{\infty} E\{X(t-\alpha)\}h(\alpha)\,d\alpha = 0$$

$$E\{Y^2(t)\} = E\left\{\int_{-\infty}^{\infty} X(t-\alpha)h(\alpha)\,d\alpha \int_{-\infty}^{\infty} X(t-\beta)h(\beta)\,d\beta\right\}$$

$$= E\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(t-\alpha)X(t-\beta)h(\alpha)h(\beta)\,d\alpha\,d\beta\right\}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\{X(t-\alpha)X(t-\beta)\}h(\alpha)h(\beta)\,d\alpha\,d\beta$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(\alpha-\beta)h(\alpha)h(\beta)\,d\alpha\,d\beta$$

$$= \int_{-\infty}^{\infty} h^2(\alpha)\left\{\int_{-\infty}^{\infty} \delta(\alpha-\beta)\,d\beta\right\}d\alpha = \int_{-\infty}^{\infty} h^2(\alpha)\,d\alpha$$

(6)

$$E\{Z(t)\} = E\{X(t)Y(t)\} = E\{X(t)\}E\{Y(t)\} = \text{Const}$$

$$R_Z(t_1,t_2) = E\{Z(t_1)Z(t_2)\} = E\{X(t_1)Y(t_1)X(t_2)Y(t_2)\}$$

$$= E\{X(t_1)X(t_2)\}E\{Y(t_1)Y(t_2)\} = R_X(t_1,t_2)R_Y(t_1,t_2) = R_X(\tau)R_Y(\tau) = R_Z(\tau)$$

$Z(t)$ is WSS

(7) $P\{Y(t_n)=y_n, Y(t_{n-1})=y_{n-1} \ldots Y(t_1)=y_1\}$

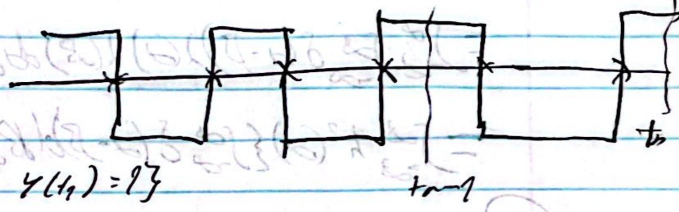
$= P\{Y(t_n+\varepsilon)=y_n, Y(t_{n-1}+\varepsilon)=y_{n-1}, \ldots, Y(t_1+\varepsilon)=y_1\}$

for any n, $t_1, t_2 \ldots t_n$ and any ε

$P\{Y(t_n)=1 \mid Y(t_{n-1})=1, Y(t_{n-2})=1 \ldots Y(t_1)=1\}$

$= P\{Y(t_n)=1 \mid Y(t_{n-1})=1\}$

$= e^{-\lambda(t_n-t_{n-1})}\cosh\lambda(t_n-t_{n-1})$



$P\{Y(t_n)=1, Y(t_{n-1})=1, Y(t_{n-2})=1 \ldots Y(t_1)=1\}$

$= P\{Y(t_n)=1 \mid Y(t_{n-1})=1, \ldots Y(t_1)=1\} \cdot P\{Y(t_{n-1})=1 \mid Y(t_{n-2})=1, \ldots, Y(t_1)=1\}$

$\cdots P\{Y(t_2)=1 \mid Y(t_1)=1\}\, P\{Y(t_1)=1\}$

$= \underbrace{P\{Y(t_n)=1 \mid Y(t_{n-1})=1\}}_{t_n-t_{n-1}} \cdot \underbrace{P\{Y(t_{n-1})=1 \mid Y(t_{n-2})=1\}}_{t_{n-1}-t_{n-2}} \cdots \underbrace{P\{Y(t_2)=1 \mid Y(t_1)=1\}}_{t_2-t_1} \underbrace{P\{Y(t_1)=1}_{\frac{1}{2}}$

$= e^{-\lambda(t_n-t_{n-1})}\cosh\lambda(t_n-t_{n-1}) \ldots e^{-\lambda(t_2-t_1)}\cosh\lambda(t_2-t_1)\cdot\frac{1}{2}$

(8) $E\{X_{n+1} \mid X_n = rs\} = \frac{(r+1)s}{2} + \frac{(r-1)s}{2} = rs$

(9) $P\{Y_{n+1}=y_{n+1} \mid Y_n=y_n, Y_{n-1}=y_{n-1} \ldots Y_1=y_1\}$

$= P\{X_{n+1}+Y_n = y_{n+1} \mid Y_n=y_n, Y_{n-1}=y_{n-1}, \ldots, Y_1=y_1\}$

$= P\{X_{n+1}+y_n = y_{n+1} \mid Y_n=y_n, Y_{n-1}=y_{n-1}, \ldots, Y_1=y_1\}$

$= P\{X_{n+1} = y_{n+1}-y_n \mid Y_n=y_n, Y_{n-1}=y_{n-1} \ldots Y_1=y_1\}$

$= P\{X_{n+1}=y_{n+1}-y_n \mid Y_n=y_n\}$

$= P\{X_{n+1}+y_n = y_{n+1} \mid Y_n=y_n\}$

$= P\{X_{n+1}+Y_n = y_{n+1} \mid Y_n=y_n\}$

$= P\{Y_{n+1} = y_{n+1} \mid Y_n=y_n\}$

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(10) $Y(t) = \{Y(t) - Y(t_{n-1})\} + \{Y(t_{n-1}) - Y(t_{n-2})\} + \dots + \{Y(t_1) - Y(t_0)\}$

$= X_n + X_{n-1} + \dots + X_1$