Mathematical Expressions Used in the Document

$$y = \max(x,0) \tag{1}$$

$$\frac{dy}{dx} \cong \frac{1}{2}\left(1 + \frac{x}{\sqrt{x^2 + \varepsilon}}\right) \tag{2}$$

where ε is a small number used in smoothing the derivative of Eq. (1).

$$\max(A,B) = B + \max(A - B,0) \tag{3}$$

$$\min(A,B) = B - \max(B - A,0) \tag{4}$$

Stream-Aquifer Flow Exchange Equations

$$Q = C_*\left[h_s - \max\left(h_g, h_{incip}\right)\right] \tag{5}$$

$$Q = C_*\left[h_s - h_{incip} - \max\left(h_g - h_{incip}, 0\right)\right] \tag{6}$$

$$Q = C_*\left[h_s - h_{incip} - \max(Y,0)\right] \tag{7}$$

where C_* is the SAFE conductance term, $Y = h_g - h_{incip}$.

$$h_{incip} = h_s - C_*^{'} W_p\left(h_s - h_b + e_{cl} + h_{ce}\right) \tag{8}$$

where h_b is the stream bottom elevation and $C_*^{'} = \frac{K_{cl}}{2K_H e_{cl}\Gamma}$.

$$
\begin{aligned}
Q^* &= \min\left(Q, Q_{avl}\right) \\
&= Q_{avl} - \max\left(Q_{avl} - Q, 0\right) \\
&= Q_{avl} - \max\left(X, 0\right)
\end{aligned}
\tag{9}
$$

where Q_{avl} is the available flow in the stream and $X = Q_{avl} - Q$.

Derivatives

$$
\frac{\partial h_{incip}}{\partial h_s} = 1 - C_*^{'}\left[\frac{\partial W_p}{\partial h_s}\left(h_s - h_b + e_{cl} + h_{ce}\right) + W_p\right] \tag{10}
$$

$$
\frac{\partial h_{incip}}{\partial h_g} = 0 \tag{11}
$$

$$
\frac{\partial Y}{\partial h_s} = -\frac{\partial h_{incip}}{\partial h_s} \tag{12}
$$

$$
\frac{\partial Y}{\partial h_s} = 1 \tag{13}
$$

$$
\begin{aligned}
\frac{\partial Q}{\partial h_s} &= C_*\left[1 - \frac{\partial h_{incip}}{\partial h_s} - \frac{1}{2}\left(1 + \frac{Y}{\sqrt{Y^2 + \varepsilon}}\right)\frac{\partial Y}{\partial h_s}\right] \\
&= C_*\left[1 - \frac{\partial h_{incip}}{\partial h_s} + \frac{1}{2}\left(1 + \frac{Y}{\sqrt{Y^2 + \varepsilon}}\right)\frac{\partial h_{incip}}{\partial h_s}\right] \\
&= C_*\left\{1 - \frac{\partial h_{incip}}{\partial h_s}\left[1 - \frac{1}{2}\left(1 + \frac{Y}{\sqrt{Y^2 + \varepsilon}}\right)\right]\right\}
\end{aligned}
\tag{14}
$$

$$\frac{\partial Q}{\partial h_g} = -C_* \frac{1}{2}\left(1 + \frac{Y}{\sqrt{Y^2 + \varepsilon}}\right)\frac{\partial Y}{\partial h_g}$$

$$= -C_* \frac{1}{2}\left(1 + \frac{Y}{\sqrt{Y^2 + \varepsilon}}\right) \tag{15}$$

$$\frac{\partial X}{\partial h_s} = -\frac{\partial Q}{\partial h_s} \tag{16}$$

$$\frac{\partial X}{\partial h_g} = -\frac{\partial Q}{\partial h_g} \tag{17}$$

Gaining Stream (stream-aquifer flow not limited by available stream flow):

$$\frac{\partial Q^*}{\partial h_s} = \frac{\partial Q}{\partial h_s} \tag{18}$$

$$\frac{\partial Q^*}{\partial h_g} = \frac{\partial Q}{\partial h_g} \tag{19}$$

Losing Stream (stream-aquifer flow limited by available stream flow):

$$\frac{\partial Q^*}{\partial h_s} = -\frac{1}{2}\left(1 + \frac{X}{\sqrt{X^2 + \varepsilon}}\right)\frac{\partial X}{\partial h_s}$$

$$= \frac{1}{2}\left(1 + \frac{X}{\sqrt{X^2 + \varepsilon}}\right)\frac{\partial Q}{\partial h_s} \tag{20}$$

$$
\begin{aligned}
\frac{\partial Q^*}{\partial h_g} &= -\frac{1}{2}\left(1+\frac{X}{\sqrt{X^2+\varepsilon}}\right)\frac{\partial X}{\partial h_g} \\
&= \frac{1}{2}\left(1+\frac{X}{\sqrt{X^2+\varepsilon}}\right)\frac{\partial Q}{\partial h_g}
\end{aligned}
\tag{21}
$$