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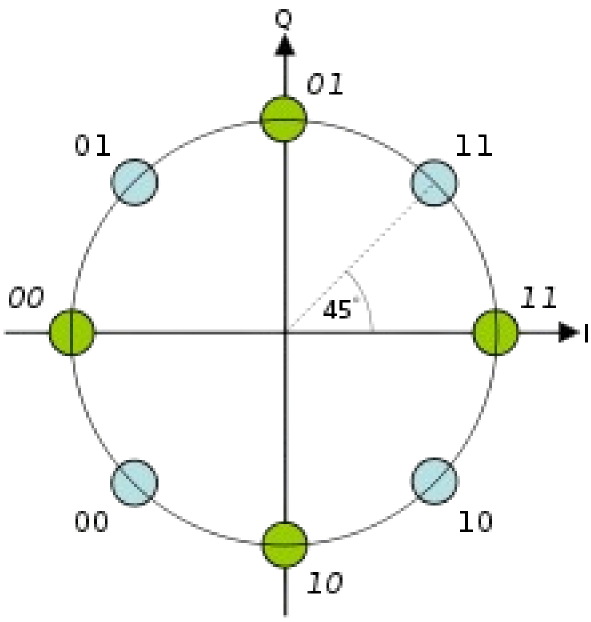
Communication Systems II

Research Project



Contents

The basic constellation schemes are pictured below:

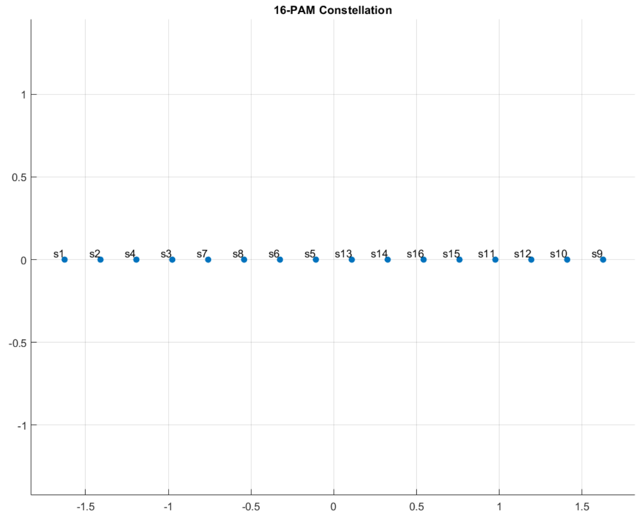


Figure 1: 16-PAM

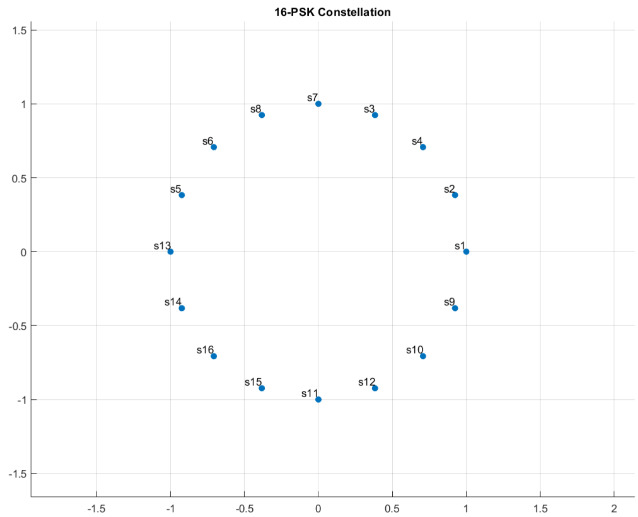


Figure 2: 16-PSK

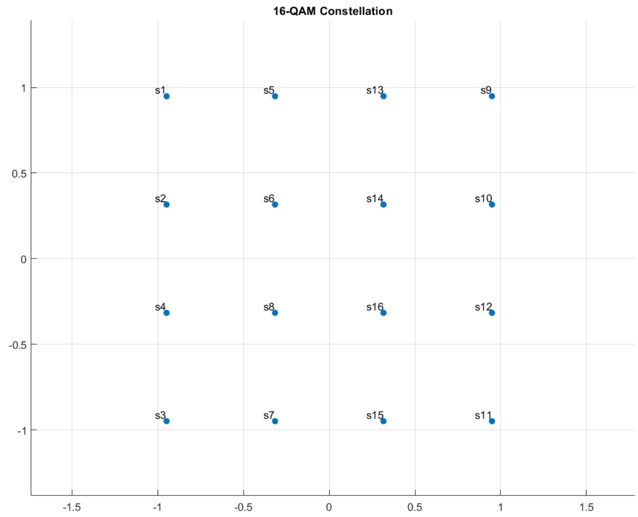


Figure 3: 16-QAM

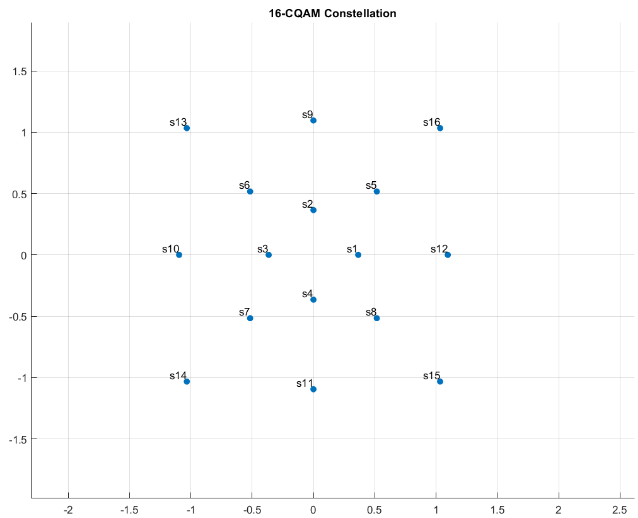


Figure 4: 16-CQAM

1 Problem 1

1.1 Theoretical Analysis

Euclidean Distance (d_{min})

Euclidean Distance is the geometric distance between adjacent constellation points in a modulation scheme's constellation diagram. A larger d_{min} indicates that the constellation points are more widely spaced, which can improve the robustness of the modulation scheme against noise and interference. A smaller d_{min} means that the constellation points are closer together, which can increase the likelihood of symbol errors, especially in the presence of noise or channel impairments. [5]

M-PAM

The Euclidean distance for a PAM constellation between symbols is:

$$d_{s_i,s_j} = \sqrt{||s_i - s_j||^2} = 2 \cdot ||i-j|| \cdot \sqrt{E_g}$$

The minimum Euclidean distance between points is given for $j = i \pm 1$ with $1 \leq j \leq M$ as:

$$d_{min} = \sqrt{||s_i - s_{i\pm1}||^2} = 2 \cdot \sqrt{E_g}$$

where E_g is the average energy per pulse:

$$\begin{aligned} E_g &= \frac{3 \cdot E_s}{M^2 - 1} \\ \Rightarrow E_s &= \frac{E_g \cdot (M^2-1)}{3} \\ \Rightarrow d_{min} &= 2 \cdot \sqrt{\frac{3 \cdot E_s}{M^2-1}} \\ \Rightarrow d_{min} &= 2 \cdot \sqrt{E_s} \cdot \sqrt{\frac{3}{M^2-1}} \end{aligned}$$

and E_s is the average energy per symbol.

M-PSK

Each symbol s_i is:

$$s_i = [\sqrt{E_s} \cdot \cos\theta_i, \sqrt{E_s} \cdot \sin\theta_i]$$

and the Euclidean distance between two symbols s_i and s_j:

$$d_{s_i,s_j} = \sqrt{||s_i - s_j||^2} = \sqrt{2 \cdot E_s \cdot (1 - \cos\frac{2\cdot\pi}{M} \cdot (i-j))}$$

When the symbols are on the constellation, meaning $j = i \pm 1$, then the minimum Euclidean distance is:

$$d_{s_i,s_j} = \sqrt{2 \cdot E_s \cdot (1 - \cos\frac{2\cdot\pi}{M})} = 2 \cdot \sqrt{E_s} \cdot \sin\frac{\pi}{M}$$

$$\Rightarrow d_{min} = 2 \cdot \sqrt{E_s} \cdot \sin\frac{\pi}{M}$$

where E_s is the average energy per symbol.

M-QAM

For Quadrature Amplitude Modulation each symbol is:

$$s_i = [\sqrt{E_i} \cdot \cos\theta_i, \sqrt{E_i} \cdot \sin\theta_i]$$

The minimum Euclidean distance between symbols in an orthogonal QAM constellation is:

$$d_{min} = 2 \cdot \sqrt{E_g} = \sqrt{E_s}\sqrt{\frac{6}{M-1}}$$

where E_s is the average energy per symbol.

M-CQAM

For Circular QAM, the minimum Euclidean distance between points of a M-CQAM constellation is:

$$d_{min} = \sqrt{R_2^2 + R_1^2 - 2R_2R_1\cos(\theta_2 - \theta_1)}$$

Because we use constellations with equal distances and 45° shift between each circle layer we have the following equation :

$$d_{min} = \sqrt{5R^2 - 4R^2 \cdot \cos 45°} = R \cdot \sqrt{5 - 2\sqrt{2}}$$

We need to find R, through E_s :

$$E_s = \frac{4}{M}\sum_{i=1}^{M/4} R_i^2 = \frac{4}{M}\sum_{i=1}^{M/4}(i \cdot R)^2 = \frac{4}{M} \cdot R^2 \cdot \sum_{i=1}^{M/4} i^2 \Rightarrow R = \sqrt{\frac{E_s \cdot M}{4 \cdot \sum_{i=1}^{M/4} i^2}}$$

$$d_{min} = \sqrt{\frac{E_s \cdot M}{4 \cdot \sum_{i=1}^{M/4} i^2} \cdot (5 - 2\sqrt{2})}$$

So, when $M = 16$, $E_s = 1$:

- $d_{min,16-PAM} = 2 \cdot \sqrt{1} \cdot \sqrt{\frac{3}{255}}$

  $\Rightarrow d_{min,16-PAM} = 2 \cdot \sqrt{1} \cdot \sqrt{\frac{1}{85}} = 0.21$
- $d_{min,16-PSK} = 2 \cdot \sqrt{1} \cdot \sin\frac{\pi}{16} = 0.39$
- $d_{min,16-QAM} = \sqrt{1}\sqrt{\frac{6}{16-1}} = 0.63$
- $d_{min,16-QAM} = \sqrt{\frac{1 \cdot 16}{4 \cdot (1+4+9+16)} \cdot (5 - 2\sqrt{2})} = 0.5381$

The minimum Euclidean distance seems to be higher in 16-QAM, then 16-PAM, and then 16-PSK.

Peak-to-Average Power Ratio (PAPR)

Peak-to-Average Power Ratio quantifies the maximum power level compared to the average power level of the transmitted signal [6]:

$$PAPR = \frac{Peak\ Power}{Average\ Power}$$

High PAPR values indicate that the signal has large peaks relative to its average power, which can lead to issues like distortion and reduced efficiency.

M-PAM

Since M-PAM signals can have varying amplitude levels, the peak power corresponds to the highest amplitude level present in the signal, while the average energy is calculated over a period of time or over the entire pulse duration. The peak energy of the symbol is:

$$E_{max} = E_g \cdot (M-1)^2$$

$$\Rightarrow E_{max} = \frac{3 \cdot E_s \cdot (M-1)^2}{M^2-1}$$

while the average power of the symbol is: E_s

$$\Rightarrow PAPR_{M-PAM} = 3 \cdot \frac{(M-1)^2}{M^2-1}$$

M-PSK

In PSK modulation, the symbols are represented by different phase shifts of the carrier signal. The amplitudes of the symbols are constant, and only the phase changes. As a result, the power variations are primarily determined by the phase transitions between symbols. The peak energy of the symbol is:

$$E_{max} = E_s$$

while the average energy of the symbol is: E_s

$$\Rightarrow PAPR_{M-PSK} = 1$$

M-QAM

We are calculating the PAPR for an orthogonal QAM [7]: The peak energy of the symbol is:

$$E_{max} = (\sqrt{M}-1)^2 + (\sqrt{M}-1)^2$$

while the average energy of the symbols is:

$$E_s = \frac{1}{M} \cdot \sum_{i=1}^{M} E_i$$

M-CQAM

As mentioned in 1.1 in a M-CQAM constellation, E_{max} can be written as :

$$E_{\max} = R_{M/4}^2$$

$$\Rightarrow PAPR_{M-CQAM} = \frac{E_{max,CQAM}}{E_{s,CQAM}} = \frac{R_{M/4}^2}{\frac{4}{M}\sum_{i=1}^{M/4} R_i^2} = \frac{(\frac{M}{4}R)^2}{\frac{4}{M} \cdot R^2 \cdot \sum_{i=1}^{M/4} i^2} = \frac{M^3}{4^3 \cdot \sum_{i=1}^{M/4} i^2}$$

So, when $M = 16$:

- $PAPR_{M-PAM} = 3 \cdot \frac{(16-1)^2}{16^2-1} = 2.64$
- $PAPR_{M-PSK} = 1$
- $PAPR_{M-QAM} = \frac{E_{max}}{E_s} = \frac{2 \cdot (4-1)^2}{\frac{1}{16}\sum_{n=1}^{16} E_i}$
  $= \frac{18A^2}{\frac{1}{16}[4*(A^2+A^2)+4*((3A)^2+(3A)^2)+8*((3A)^2+A^2)]} = \frac{18A^2}{10A^2} = 1.8$
- $PAPR_{M-CQAM} = \frac{16^3}{4^3 \cdot (1+4+9+16)} = 2.13$

1.2 Simulation

The code, written in Matlab for the simulation is in Section 3.1. The results of the simulation are the following:

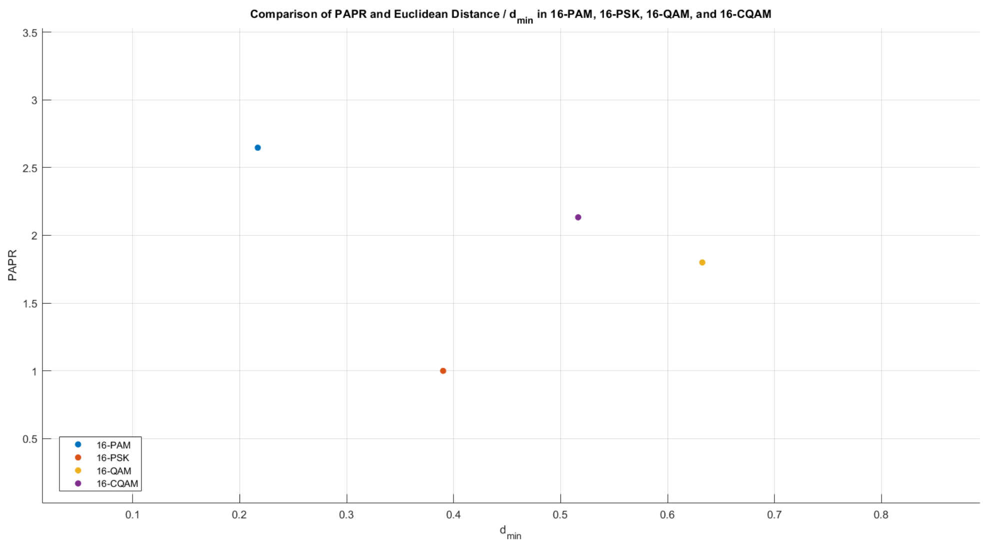


Figure 5: PAPR vs d_{min}

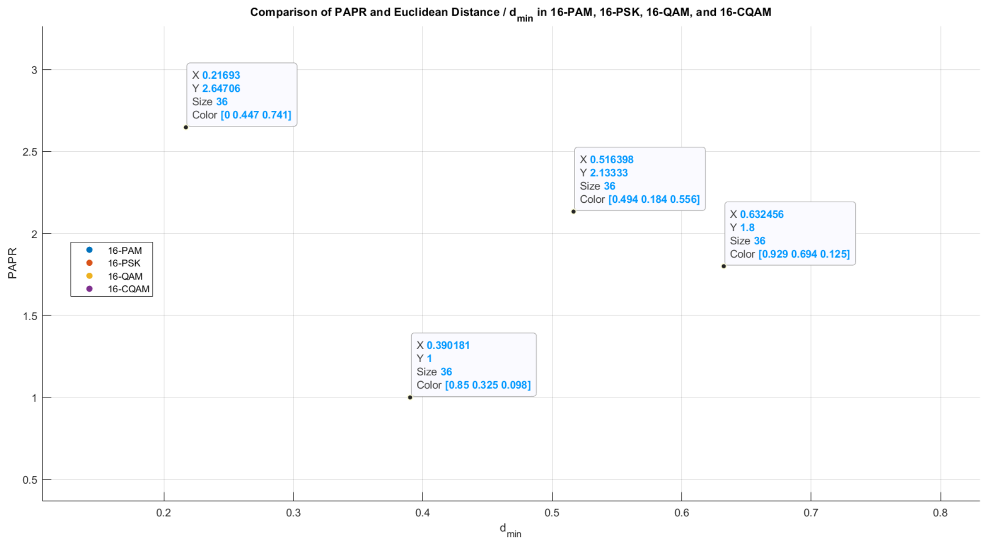


Figure 6: PAPR vs d_{min} with the respected values

The simulation shows exactly what the theoretical analysis predicted.

2 Problem 2

2.1 Theoretical Analysis

Symbol Error Probability (SEP)

The average symbol error probability or symbol error probability is:

$$P_s = \sum_{i=1}^{M} P_{s|i} Pr_(s_i)$$

or for equiprobable symbols:

$$P_s = \frac{1}{M} \sum_{i=1}^{M} P_{s|i}$$

M-PAM

For the M-PAM the SEP is:

$$P_{s(M-PAM)} = \frac{2(M-1)}{M} \cdot Q(\sqrt{\frac{2E_g}{No}})$$
$$P_{s(M-PAM)} = \frac{2(M-1)}{M} \cdot Q(\frac{d_{min}}{\sqrt{2No}})$$
$$P_{s(M-PAM)} = \frac{2(M-1)}{M} \cdot Q(\sqrt{\frac{6E_s}{(M^2-1) \cdot No}})$$

M-PSK

When $M > 4$ there is no closed-form expression for the symbol error probability of M-PSK. However, a very good approximation for large values of $\frac{E_s}{No}$ and M is:

$$P_{s(M-PSK)} \approx 2Q(\sqrt{\frac{2E_s}{No}} \cdot \sin\frac{\pi}{M})$$

M-QAM

The symbol error probability for an orthogonal constellation M-QAM can be found using the maximum likelihood criterion. For the case of square M-QAM, the symbols error probability is:

$$P_{s(M-QAM)} = 1 - (1 - P_{\sqrt{M}})^2$$

where $P_{\sqrt{M}}$ is the symbol error probability of the pass band $\sqrt{M}$PAM, and it is given by the following equation:

$$P_{s,\sqrt{M}} = 2(1 - \frac{1}{\sqrt{M}})Q(\sqrt{\frac{3}{M-1}\frac{E_s}{No}})$$

M-CQAM

Article [8] analyses different methods to calculate the symbol error probability of a CQAM:

$$d_n = \sqrt{(5 + 2n \cdot (n+1)) - (2 + n \cdot (n-1)) \cdot \sqrt{2}}$$

where d_i is the distance between neighboring symbols of circle layers i and $i + 1$.

Signal to Noise ratio (SNR)

The signal-to-noise ratio is defined as the ratio of the received signal power to the noise power introduced by the receiver.

$$SNR = \frac{P_r}{P_N}$$

SNR can also be expressed in terms of E_s and N_0:

$$SNR = \frac{E_s}{N_0}$$

M-PAM
The average power for the M-PAM is:

$$E_s = \frac{1}{M}\sum_{i=1}^{M} E_{s_i}$$

M-PSK
Because of the scheme of the modulation, the average energy per symbol is:

$$E_s = \frac{1}{M}\sum_{i=1}^{M} E_{s_i} = \frac{1}{M}\sum_{i=1}^{M} R_i^2$$

M-QAM
The average power for the M-QAM is:

$$E_s = \frac{1}{M}\cdot\sum_{i=1}^{M} E_{s_i}$$

M-CQAM
Accordingly the average power of the Circular QAM is:

$$E_s = \frac{4}{M}\sum_{i=1}^{M/4} R_i^2 = \frac{4}{M}\sum_{i=1}^{M/4}(i\cdot R)^2 = \frac{4}{M}\cdot R^2\cdot\sum_{i=1}^{M/4} i^2$$

So, when $M = 16$, $E_s = 1$, $N_0 = 0,1$:

- $P_{s(16-PAM)} = \dfrac{2(16-1)}{16}\cdot Q(\sqrt{\dfrac{6\cdot 1}{(16^2-1)\cdot 0,1}})$

  $\Rightarrow P_{s(M-PAM)} = \dfrac{15}{8}\cdot Q(\sqrt{\dfrac{6}{255\cdot 0,1}}) \approx 0,6$
- $P_{s(16-PSK)} \approx 2Q(\sqrt{\dfrac{2}{0,1}}\cdot\sin\dfrac{\pi}{16}) \approx 2Q(0.87) \approx 0.38$
- $P_{s,\sqrt{16}} = 2(1-\dfrac{1}{\sqrt{16}})Q(\sqrt{\dfrac{3}{16-1}\dfrac{E_s}{0,1}}) = (2-\dfrac{2}{4})Q(\sqrt{\dfrac{3}{15}\dfrac{1}{0,1}}) = \dfrac{3}{2}Q(\sqrt{\dfrac{1}{3\cdot 0,1}})$

  $\Rightarrow P_{s(M-QAM)} = 1-(1-\dfrac{3}{2}\cdot Q(\sqrt{\dfrac{1}{3\cdot 0,1}}))^2 \approx 1-(1-\dfrac{3}{2}\cdot 0,034)^2 \approx 0,022$

These results are for:

$$SNR = \frac{1}{0.1} = 10$$

2.2 Simulation

The code, written in Matlab for the simulation is in Section 3.2. The results of the simulation are the following:

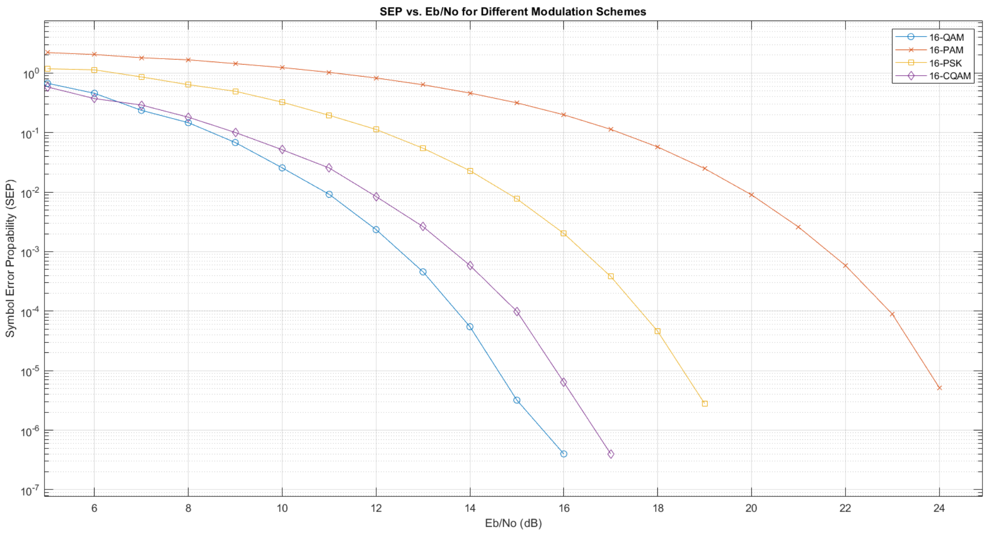


Figure 7: SEP vs SNR

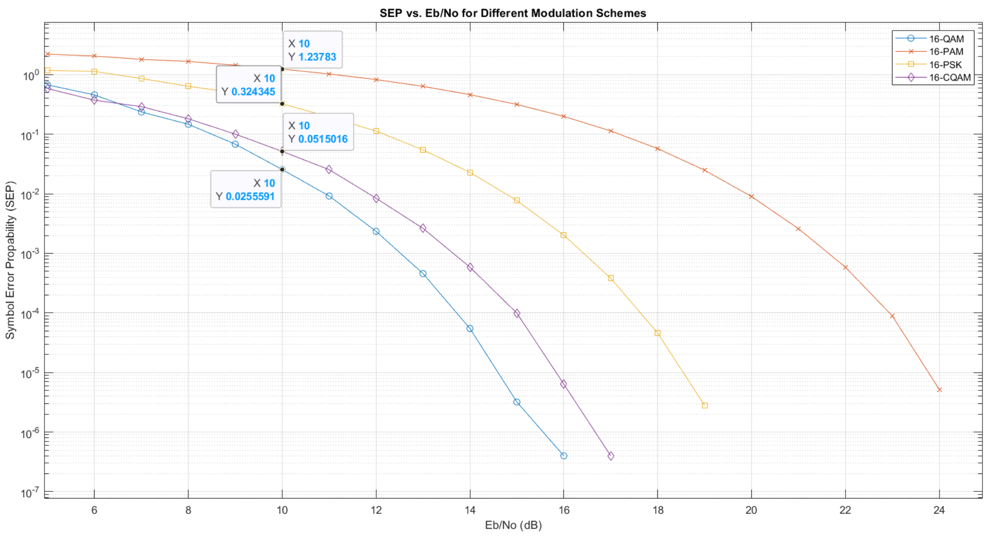


Figure 8: SEP vs SNR

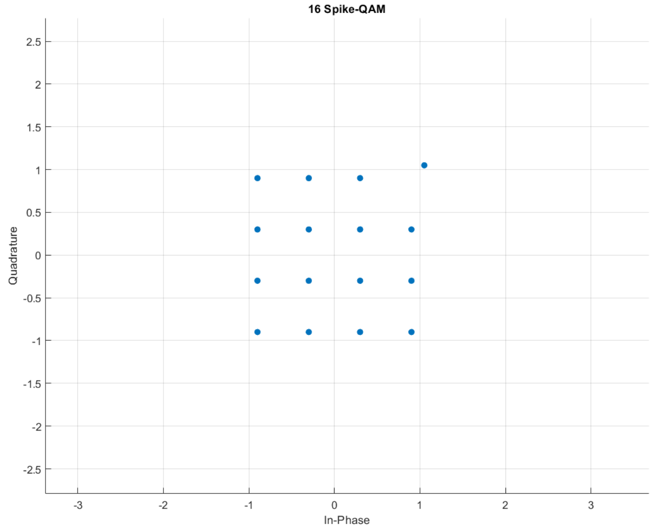


Figure 9: 16-Spike QAM with $PAPR = 3.14$, $d_{min} = 0.6$ ([9])

3 Appendix

Here is the code written in Matlab for each simulation:

3.1 Problem 1 ([1] [2] [3])

```
% Define the constellation points for 16-PAM, 16-PSK, and 16-QAM
M = 16;
constellation_pam = pammod(0:(M-1), M, 0, 'gray');
constellation_psk = pskmod(0:(M-1), M, 0, 'gray');
constellation_qam = qammod(0:(M-1), M, 'gray');

% Normalize constellations so that average symbol energy Es = 1
constellation_pam = normalize_energy(constellation_pam);
constellation_psk = normalize_energy(constellation_psk);
constellation_qam = normalize_energy(constellation_qam);

% Generate 16-CQAM constellation
N = 4;
constellation_cqam = generate_16CQAM(M, N);
constellation_cqam = normalize_energy(constellation_cqam);

% Calculate Euclidean distances for each modulation
dmin_pam = calculate_dmin(constellation_pam);
dmin_psk = calculate_dmin(constellation_psk);
dmin_qam = calculate_dmin(constellation_qam);
dmin_cqam = calculate_dmin(constellation_cqam);

% Calculate PAPR for each modulation
calculatePAPR = @(signal) max(abs(signal).^2) / mean(abs(signal).^2);
papr_pam = calculatePAPR(constellation_pam);
papr_psk = calculatePAPR(constellation_psk);
papr_qam = calculatePAPR(constellation_qam);
papr_cqam = calculatePAPR(constellation_cqam);

% Plot PAPR vs dmin
figure;
hold on;
scatter(dmin_pam, papr_pam, 'filled', 'DisplayName', '16-PAM');
scatter(dmin_psk, papr_psk, 'filled', 'DisplayName', '16-PSK');
scatter(dmin_qam, papr_qam, 'filled', 'DisplayName', '16-QAM');
scatter(dmin_cqam, papr_cqam, 'filled', 'DisplayName', '16-CQAM');
xlabel('d_{min}');
ylabel('PAPR');
title('Comparison of PAPR and Euclidean Distance / d_{min} in 16-PAM, 16-PSK, 16-QAM,
    and 16-CQAM');
legend('Location', 'best');
grid on;
hold off;

% Plot constellations
figure;
subplot(2,2,1);
plot(real(constellation_pam), imag(constellation_pam), 'o');
title('16-PAM Constellation');
grid on;
axis equal;
```

```
subplot(2,2,2);
plot(real(constellation_psk), imag(constellation_psk), 'o');
title('16-PSK Constellation');
grid on;
axis equal;

subplot(2,2,3);
plot(real(constellation_qam), imag(constellation_qam), 'o');
title('16-QAM Constellation');
grid on;
axis equal;

subplot(2,2,4);
plot(real(constellation_cqam), imag(constellation_cqam), 'o');
title('16-CQAM Constellation');
grid on;
axis equal;

% Function to generate 16-CQAM constellation
function constellation = generate_16CQAM(M, N)
    n = M / N;
    R1 = 1;
    constellation = [];
    for i = 1:N
        Ri = R1 * i;
        base_angle = pi/4 * (i-1);
        angles = (0:n-1) * (2 * pi / n) + base_angle;
        constellation = [constellation, Ri * exp(1i * angles)];
    end
end

% Function to calculate the minimum distance (dmin) in a constellation
function dmin = calculate_dmin(constellation)
    num_points = length(constellation);
    distances = inf(num_points*(num_points-1)/2, 1);
    k = 1;
    for i = 1:num_points
        for j = i+1:num_points
            distances(k) = abs(constellation(i) - constellation(j));
            k = k + 1;
        end
    end
    dmin = min(distances);
end

% Function to normalize the average symbol energy to 1
function normalized_constellation = normalize_energy(constellation)
    avg_energy = mean(abs(constellation).^2);
    normalized_constellation = constellation / sqrt(avg_energy);
end
```

3.2 Problem 2 [4]

```
% Parameters
M = 16;
k = log2(M);
EbNoVec = (5:30);
numSymPerFrame = 50;

% Generate and normalize constellations
constellation_pam = normalize_energy(pammod(0:(M-1), M, 0, 'gray'));
constellation_psk = normalize_energy(pskmod(0:(M-1), M, 0, 'gray'));
constellation_qam = normalize_energy(qammod(0:(M-1), M, 'gray'));
constellation_cqam = normalize_energy(generate_16CQAM(M, 4));

% Convert Eb/No to SNR
snrdB = convertSNR(EbNoVec, "ebno", "snr", BitsPerSymbol=k);

% Pre-allocate SER arrays
serEstQAM = zeros(size(EbNoVec));
serEstPAM = zeros(size(EbNoVec));
serEstPSK = zeros(size(EbNoVec));
serEstCQAM = zeros(size(EbNoVec));

for n = 1:length(snrdB)
    % Reset the error and symbol counters
    numErrsQAM = 0;
    numErrsPAM = 0;
    numErrsPSK = 0;
    numErrsCQAM = 0;
    numSymbols = 0;

    while numErrsQAM < 200 && numSymbols < 1e7
        dataIn = randi([0 1], numSymPerFrame * k, 1);
        dataSym = bit2int(dataIn, k);

        txSigQAM = qammod(dataSym, M, 'gray');
        txSigPAM = pammod(dataSym, M, 0, 'gray');
        txSigPSK = pskmod(dataSym, M, 0, 'gray');
        txSigCQAM = constellation_cqam(dataSym + 1);

        rxSigQAM = awgn(txSigQAM, snrdB(n), 'measured');
        rxSigPAM = awgn(txSigPAM, snrdB(n), 'measured');
        rxSigPSK = awgn(txSigPSK, snrdB(n), 'measured');
        rxSigCQAM = awgn(txSigCQAM, snrdB(n), 'measured');

        rxSymQAM = qamdemod(rxSigQAM, M, 'gray');
        rxSymPAM = pamdemod(rxSigPAM, M, 0, 'gray');
        rxSymPSK = pskdemod(rxSigPSK, M, 0, 'gray');
        rxSymCQAM = dsearchn(constellation_cqam.', rxSigCQAM.');

        numErrsQAM = numErrsQAM + symerr(dataSym, rxSymQAM);
        numErrsPAM = numErrsPAM + symerr(dataSym, rxSymPAM);
        numErrsPSK = numErrsPSK + symerr(dataSym, rxSymPSK);
        numErrsCQAM = numErrsCQAM + symerr(dataSym, rxSymCQAM - 1);

        numSymbols = numSymbols + numSymPerFrame;
    end
```

```
    % Estimate the SER
    serEstQAM(n) = numErrsQAM / numSymbols;
    serEstPAM(n) = numErrsPAM / numSymbols;
    serEstPSK(n) = numErrsPSK / numSymbols;
    serEstCQAM(n) = numErrsCQAM / numSymbols;

    sepEstQAM(n) = serEstQAM(n) * 4;
    sepEstPAM(n) = serEstPAM(n) * 4;
    sepEstPSK(n) = serEstPSK(n) * 4;
    sepEstCQAM(n) = serEstCQAM(n) * 4;
end

% Plot the SEP
figure;
semilogy(EbNoVec, sepEstQAM, '-o');
hold on;
semilogy(EbNoVec, sepEstPAM, '-x');
semilogy(EbNoVec, sepEstPSK, '-s');
semilogy(EbNoVec, sepEstCQAM, '-d');
grid on;
legend('16-QAM', '16-PAM', '16-PSK', '16-CQAM');
xlabel('Eb/No (dB)');
ylabel('Symbol Error Probability (SEP)');
title('SEP vs. Eb/No for Different Modulation Schemes');

function constellation = generate_16CQAM(M, N)
    n = M / N;
    R1 = 1
    constellation = [];
    for i = 1:N
        Ri = R1 * i;
        base_angle = pi/4 * (i-1);
        angles = (0:n-1) * (2 * pi / n) + base_angle;
        constellation = [constellation, Ri * exp(1i * angles)];
    end
end

% Function to normalize energy
function norm_constellation = normalize_energy(constellation)
    Es = mean(abs(constellation).^2);
    norm_constellation = constellation / sqrt(Es);
end
```

References

[1] Https://www.mathworks.com/help/comm/ref/pammod.html.

[2] Https://www.mathworks.com/help/comm/ref/pskmod.html.

[3] Https://www.mathworks.com/help/comm/ref/qammod.html.

[4] Https://www.mathworks.com/help/comm/ref/biterr.html.

[5] G. N. Karagiannidis and D. K. Pappi, *Telecommunication Systems.* Tziola Publications, 2018.

[6] G. Sikri and Rajni, *Advances in Computing and Information Technology.* Springer Berlin Heidelberg, July 2012, pp. 685–690.

[7] E. A. Fatih Cogen, "Hexagonal quadrature amplitude modulation aided spatial modulation," *Conference: 11th International Conference on Electrical and Electronics Engineering*, October 2019.

[8] M. I. Hisham Alasady and Q. Rahmanz, "Symbol error rate calculation and data predistortion for 16-qam transmission over nonlinear memoryless satellite channels," *Wiley InterScience*, 2006.

[9] K. C.-H. Manuel Jose Lopez Morales and A. G. Armada, "Optimum constellation for symbol-error-rate to papr ratio minimization in swipt," *IEEE 95th Vehicular Technology Conference*, january 2022.