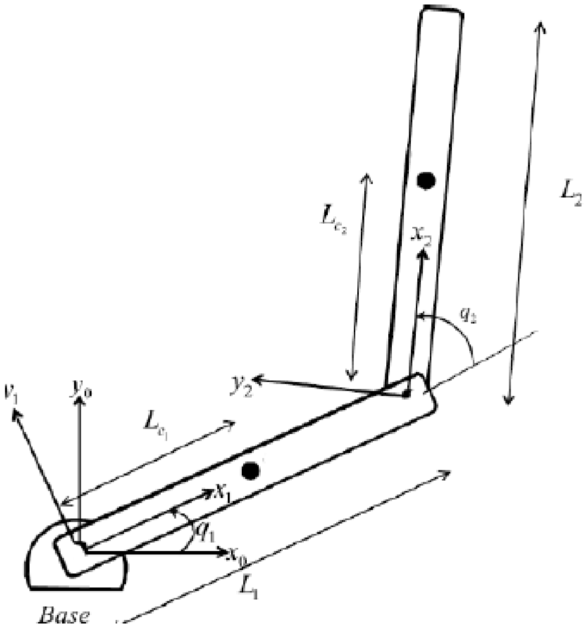
Adaptive Control of a Robotic Arm

Given the following system:

$$\boldsymbol{H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u}$$



Where

$$\mathbf{q} = [q_1 \ \ q_2]^T \, , \dot{\mathbf{q}} = [\dot{q}_1 \ \ \dot{q}_2]^T$$

$$\mathbf{H(q)} = \begin{bmatrix} m_2(L_{c2}^2 + 2L_1L_{c2}\cos(q_2) + L_1^2\ ) \ + L_{c1}^2 m_1 + I_{z2} + I_{z1} & m_2L_{c2}^2 + L_1L_{c2}m_2\cos(q_2) + I_{z2} \\ L_{c2}^2 m_2 + L_1L_{c2}m_2\cos(q_2) + I_{z2} & L_{c2}^2 m_2 + I_{z2} \end{bmatrix}$$

$$\mathbf{C(q,\dot{q})} = \begin{bmatrix} -m_2L_1L_{c2}\sin(q_2)\dot{q}_2 & -m_2L_1L_{c2}\ \sin(q_2)\,(\dot{q}_2 + \dot{q}_1) \\ m_2L_1L_{c2}\ \sin(q_2)\,\dot{q}_1 & \mathbf{0} \end{bmatrix}$$

$$\mathbf{G(q)} = \begin{bmatrix} m_2L_{c2}g\ \cos(q_1 + q_2) + (m_2L_1 + m_1L_{c1})g\cos(q_1) \\ m_2L_{c2}\ g\ \cos(q_1 + q_2) \end{bmatrix}$$

The purpose of the control is to monitor the trajectory:

$$q_{1d}(t) \ = \begin{cases} (-90^o + 50^o(1 - \cos(0.63t))\,, t \le 5 \\ 10^o\,, t > 5 \end{cases}$$

$$q_{2d}(t) \ = \begin{cases} 170^o - 60^o(1 - \cos(0.63t)\,, t \le 5 \\ 50^o\,, t > 5 \end{cases}$$

If the values for $\mathbf{I_{z1}\ I_{z2}\ L_{c1}\ L_{c2}\ m_2}$ are unknown, they can be estimated as

$$\widehat{I_{z1}} = 0.05,\ \widehat{I_{z2}} = 0.02,\ \widehat{L_{c1}} = 0.35,\ \widehat{L_{c2}} = 0.1\ \widehat{m_2} = 2$$

It is given that:

$$0.02 < I_{z1} < 0.5$$

$$0.01 < I_{z2} < 0.15$$

$$0.1 < L_{c1} < 0.4$$

$$0.05 < L_{c2} < 0.45$$

$$0.5 < m_2 < 5$$

$$\lambda_{min}(H) \geq 0.2$$

$$\lambda_{max}(H) \leq 4$$

a] Simulate the system if you know every value of the system, select the best input function to linearize the system to reach the goal (set the poles of the Closed Loop Monitoring Error System to -10). Given

$$m_1 = 6\,kg, m_2 = 4\,kg\,, L_1 = 0.5\,m\,, L_2 = 0.4\,m\,, L_{c1} = 0.2\,m\,, L_{c2} = 0.4\,m$$
$$I_{z1} = 0.43\,kg\,m^2,\ I_{z2} = 0.05\,kg\,m^2,\ g = 9.81\,m/s^2$$

$$q_1(0) = -87^o\ \ q_2(0) = 167^o\,, \dot{q}_1(0) = \dot{q}_2(0) = 0$$

b] Create a robust control law with the error slip method with Λ = 10 I 2x2 so that tracking is achieved despite the existing uncertainties. For the smooth form of your controller sign function use the saturation function given below.

$$g(x) = \begin{cases} \dfrac{x}{|x|}, |x| \geq \varepsilon \\ \dfrac{x}{\varepsilon}, |x| < \varepsilon \end{cases}$$

, ε a small positive constant