SAE III

Part I

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A)

i) Since the variable gain function does not exist, we have a closed loop system with G(s)= $\frac{4}{s(s+1)}$so: G $_c$ (s)= $\frac{H(s)}{1+H(s)} = \frac{4}{s^2+s+4}$and since s ^{2} + s + 4 = s ^{2} + 2ζo $_n$ + ω $_n$ ^{2} , it follows that ω $_n$ =2 rad / s and ζ=0.25. It holds that e= r - y $\overset{y=Hr}{\Longleftrightarrow}$e= r -$\frac{4r}{s^2+s+4} \Leftrightarrow e = r\left(\frac{s^2+s}{s^2+s+4}\right) \Leftrightarrow r(s^2+s) = e(s^2+s+4)$

And with inverse Laplace transform we have the differential equation:
r''(t) + r'(t) = e''(t) + e'(t) + e(t)

To find the state equations we consider as the state the phase variables of the error.

X $_1$ = e

X $_1$ '= X $_2$

X $_2$ = e ' X $_2$ '= e '' = r '' + r ' - e ' -4 e = r '' + r ' - X $_2$ -4 X $_1$

So the system is:

X $_1$ ' = X $_2$

X $_2$ '= -4 X $_1$ - X $_2$ + r '' + r '

II) For entering the step function r (t)=1 , t>0, with r'=0, r''=0, the equilibrium point is obtained:

$$\begin{cases} X'_1 = 0 \Leftrightarrow X_2 = 0 \\ X'_2 = 0 \Leftrightarrow -4X_1 - X_2 = 0 \overset{X_2=0}{\Longleftrightarrow} X_1 = 0 \end{cases}$$

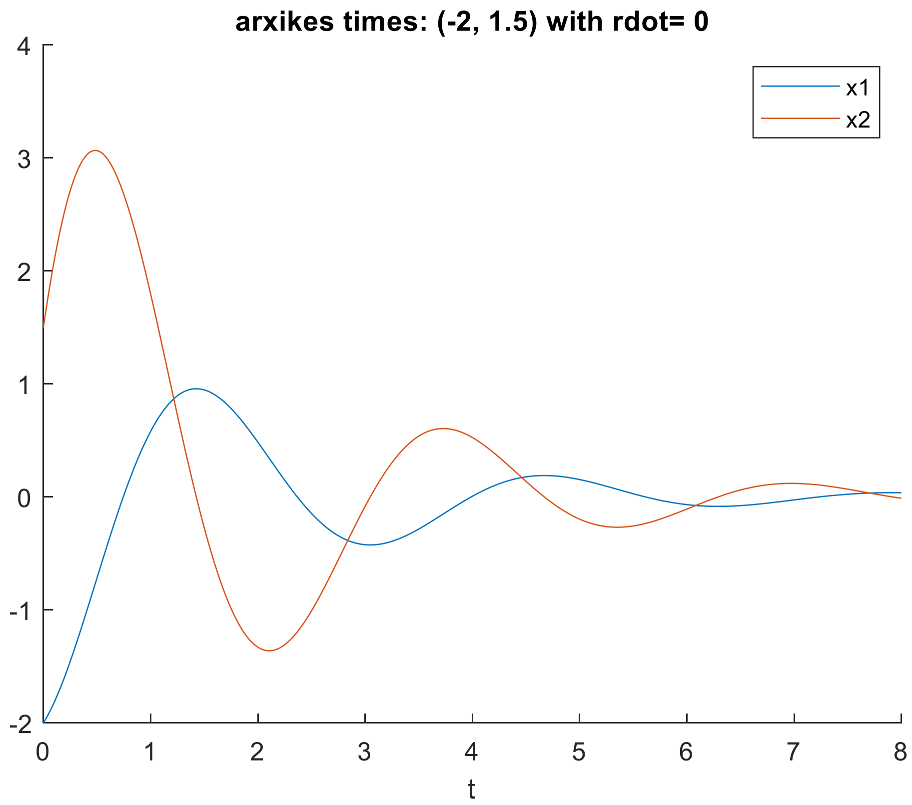
So SI (0 , 0)

For input ramp function with slope V =1.2, r (t)=1.2 t , r '=1.2, r ''=0, the equilibrium point is obtained:

$$\begin{cases} X'_1 = 0 \\ X'_2 = 0 \end{cases} \Leftrightarrow \begin{cases} X_2 = 0 \\ -4X_1 - X_2 + 1.2 = 0 \end{cases} \overset{X_2=0}{\Longleftrightarrow} \begin{cases} X_2 = 0 \\ X_1 = 0.3 \end{cases}$$

So SI (0.3 , 0)

III) The time response of the state variables for entering the step function is shown in the figures below:



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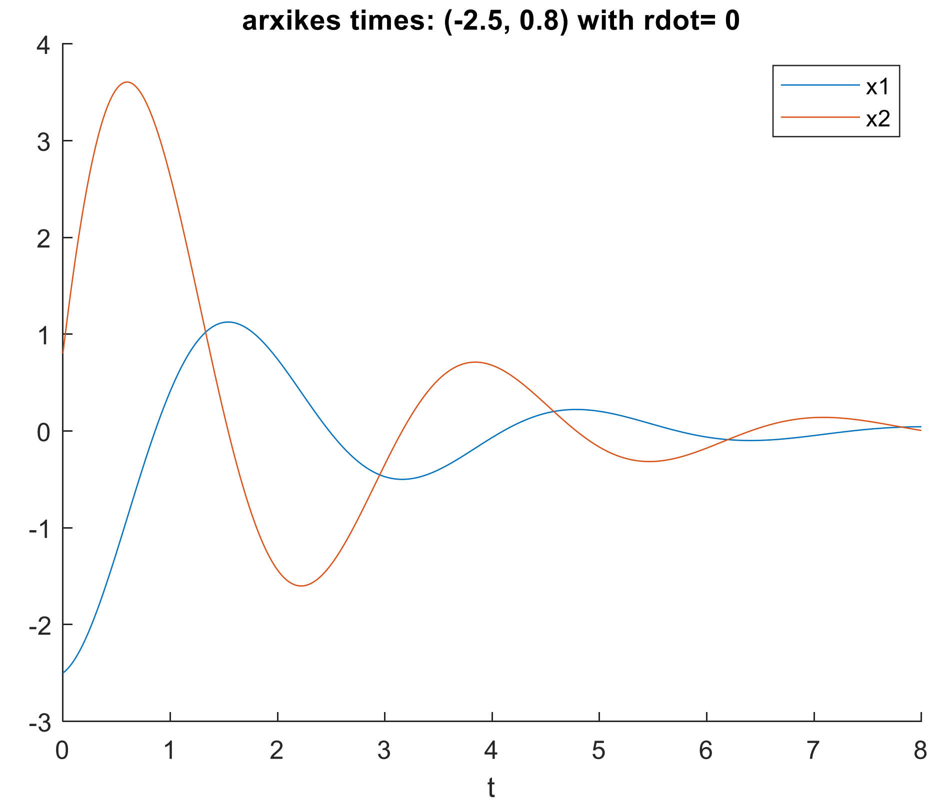

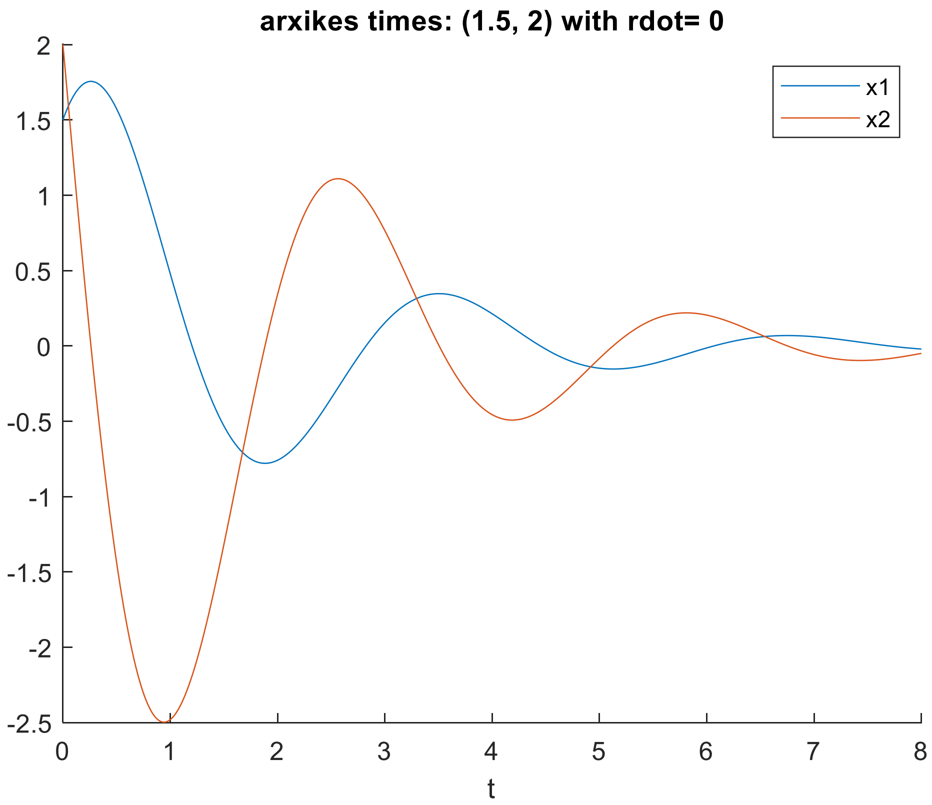

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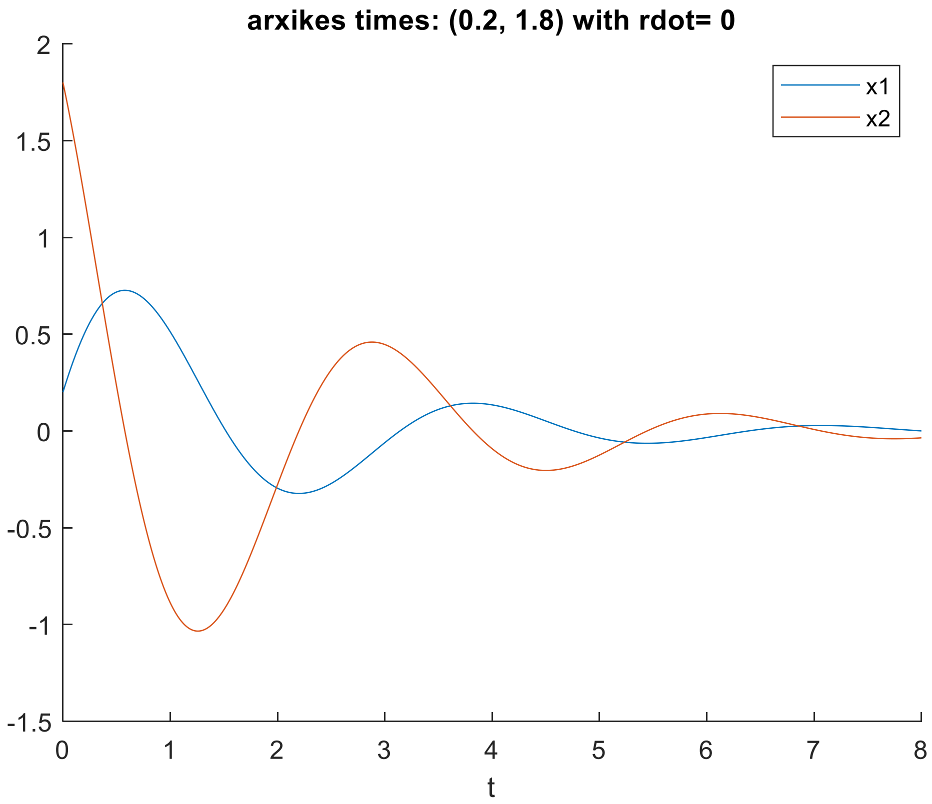

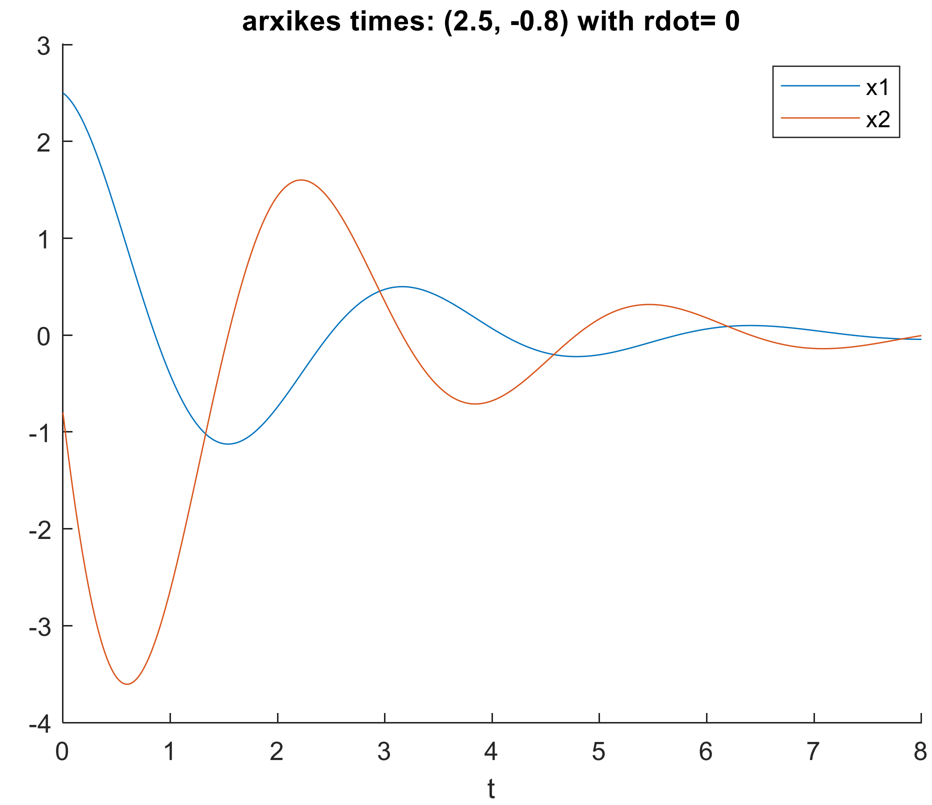

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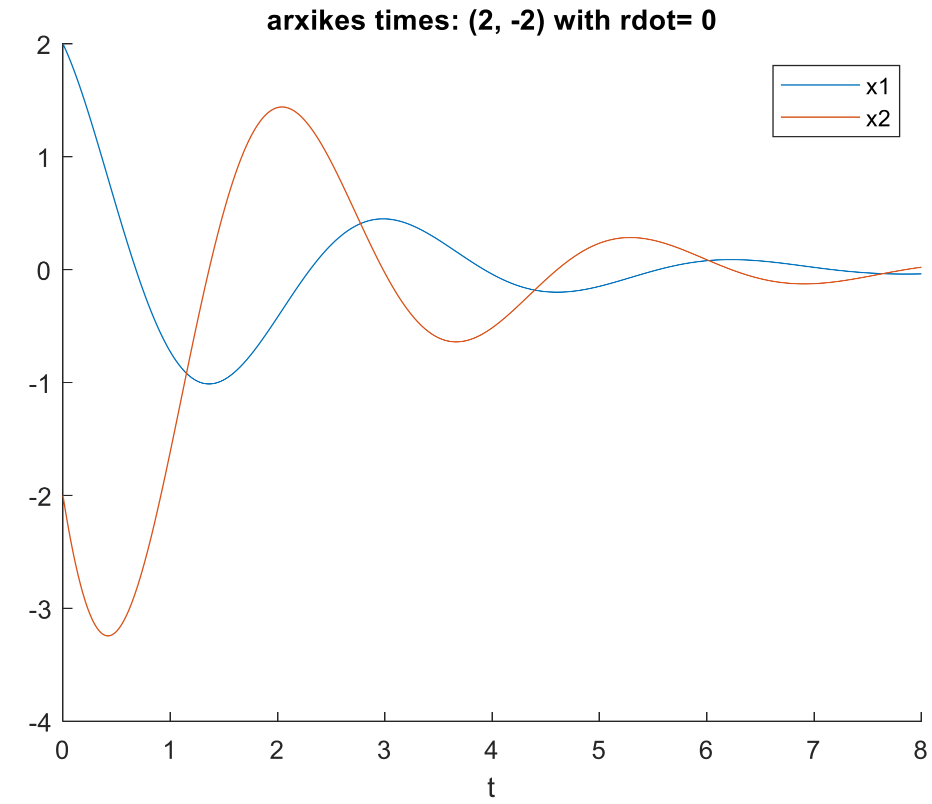
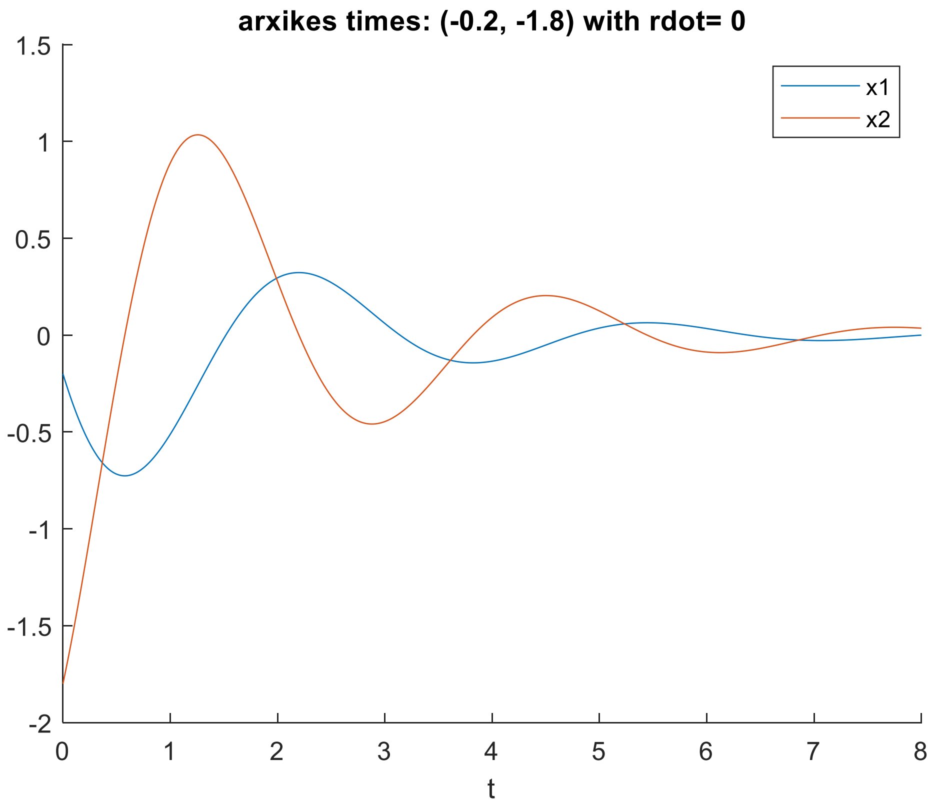
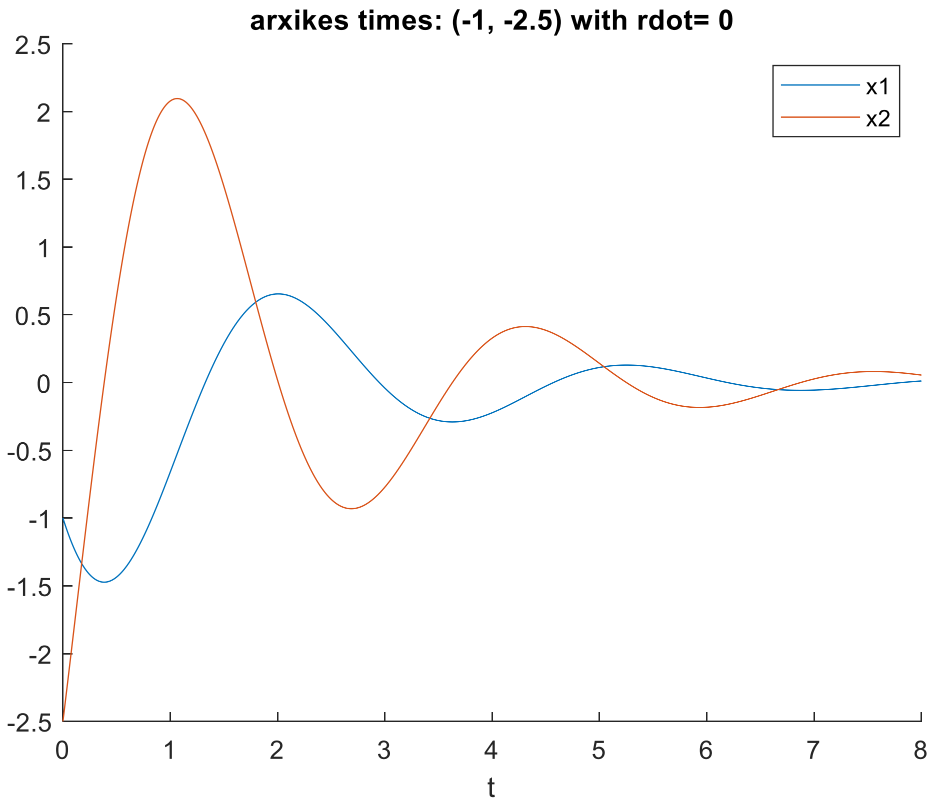
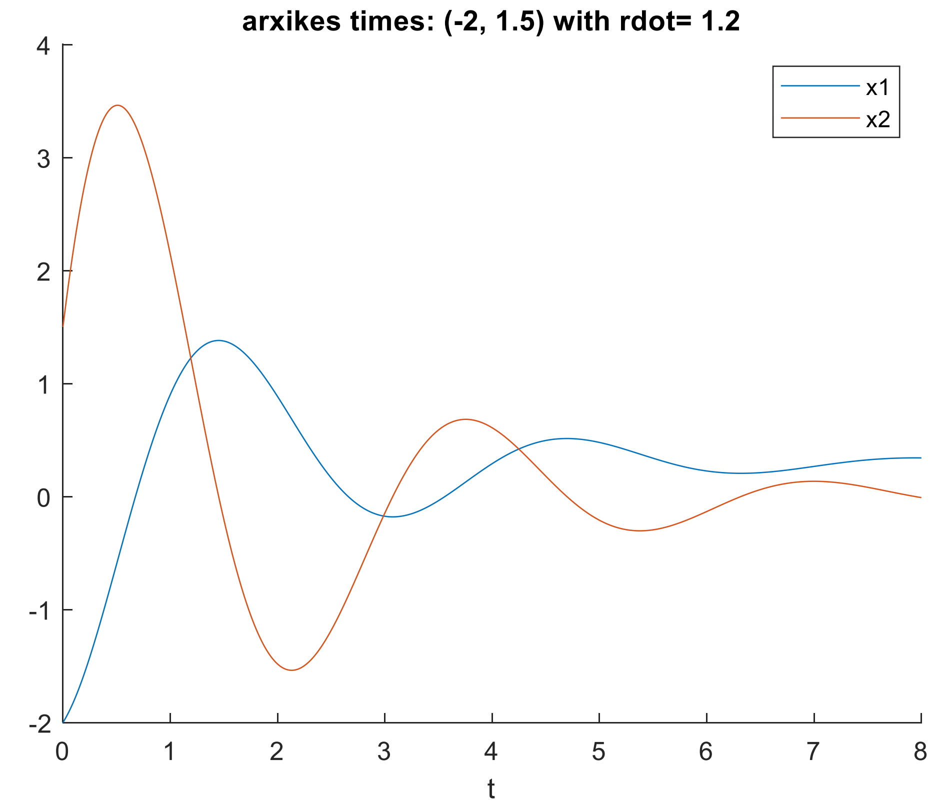






The time response of the state variables for input to the ramp function is shown in the figures below:



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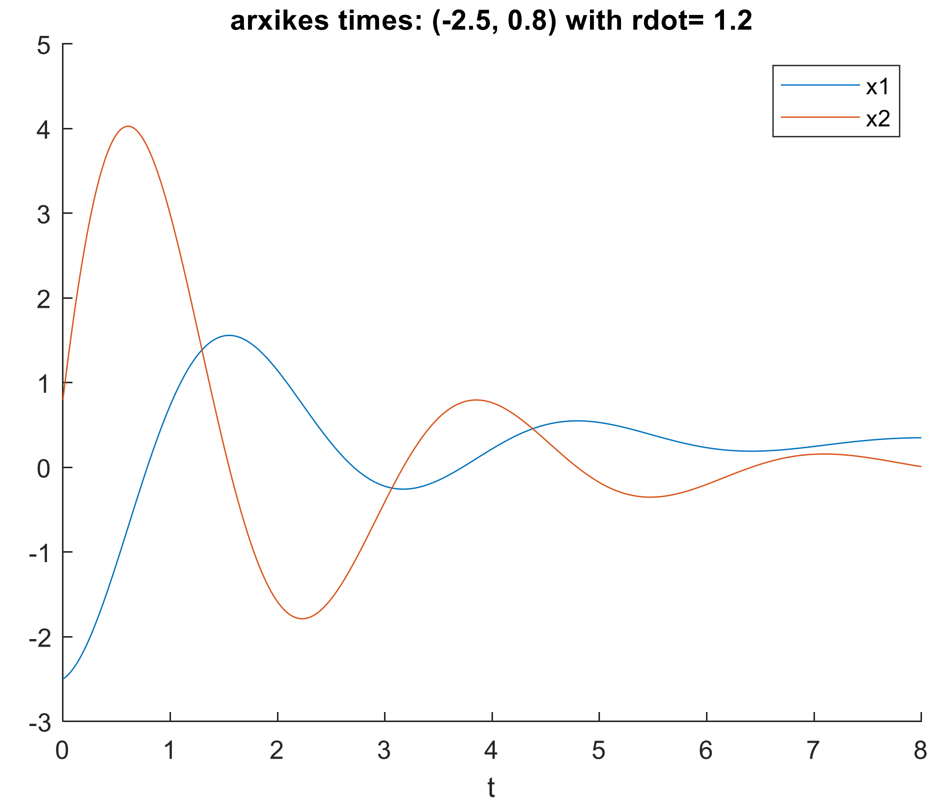

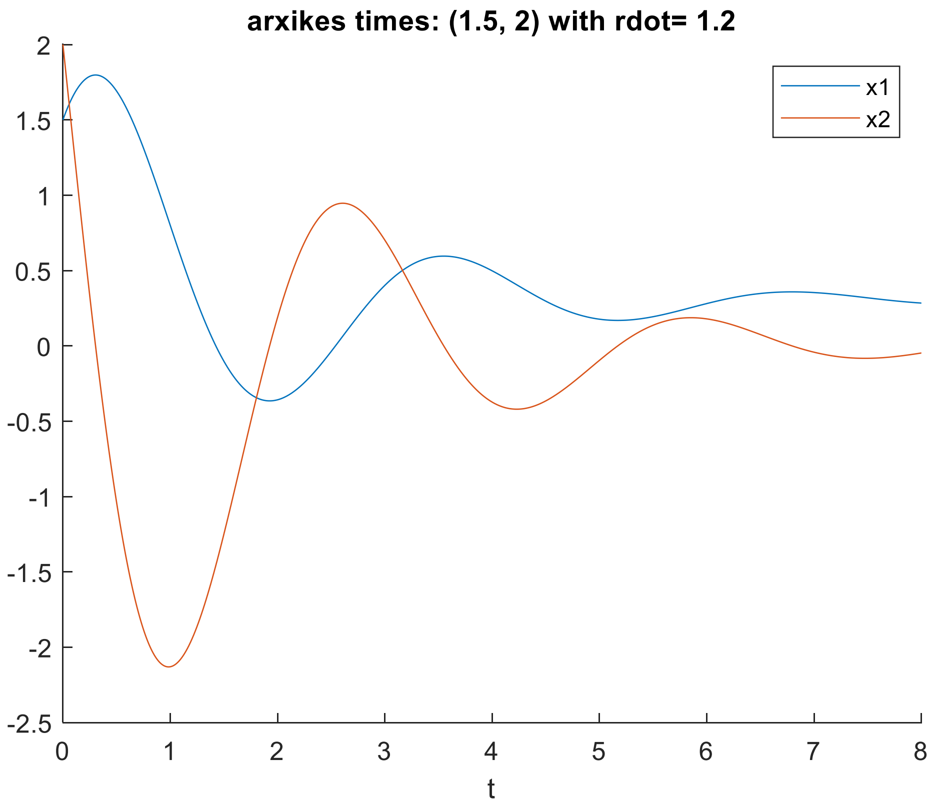

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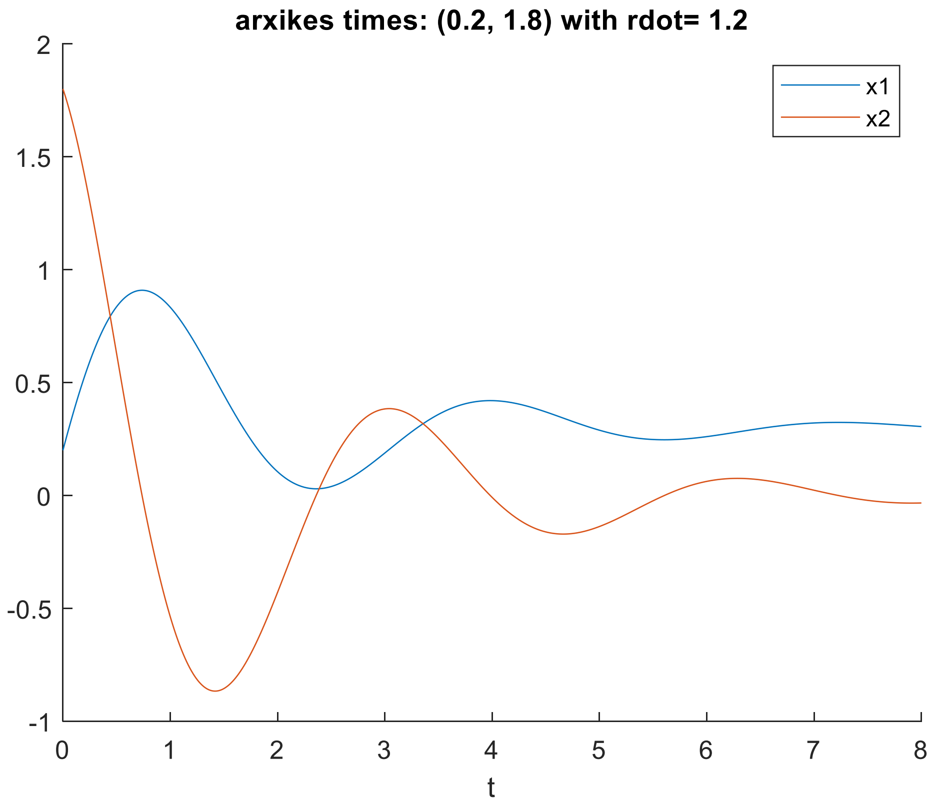

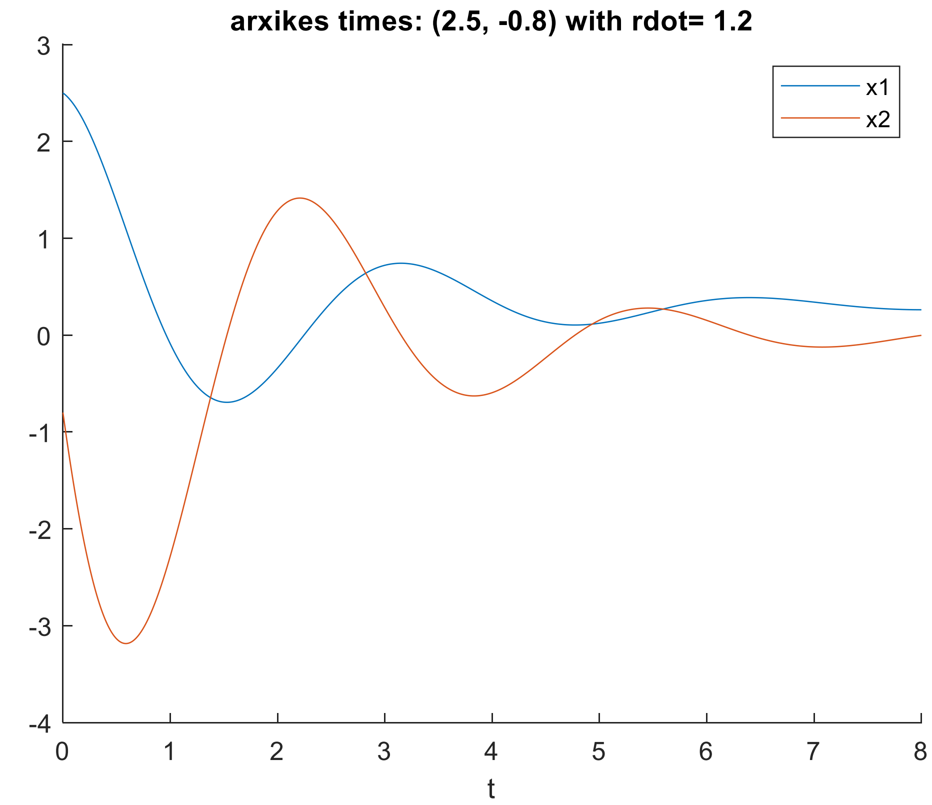

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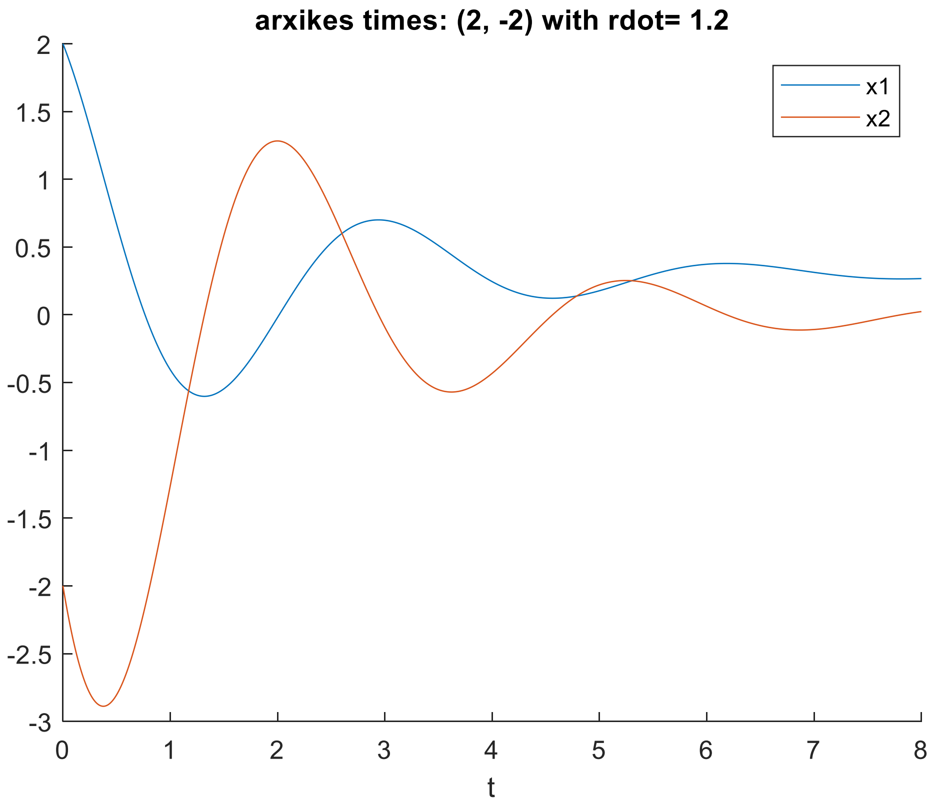

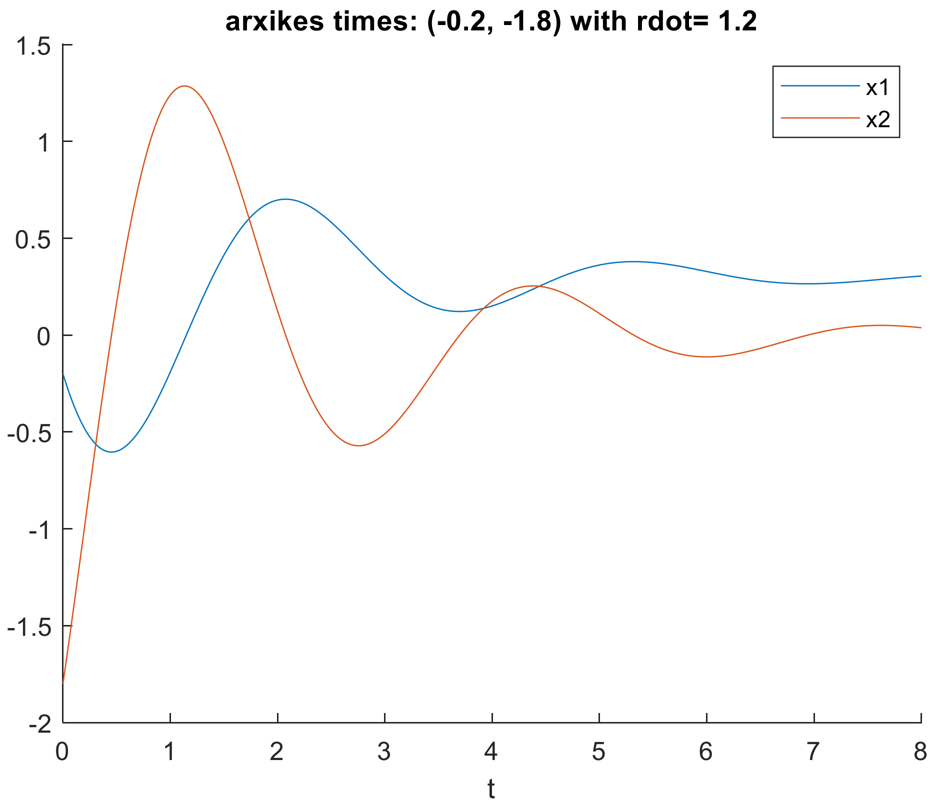

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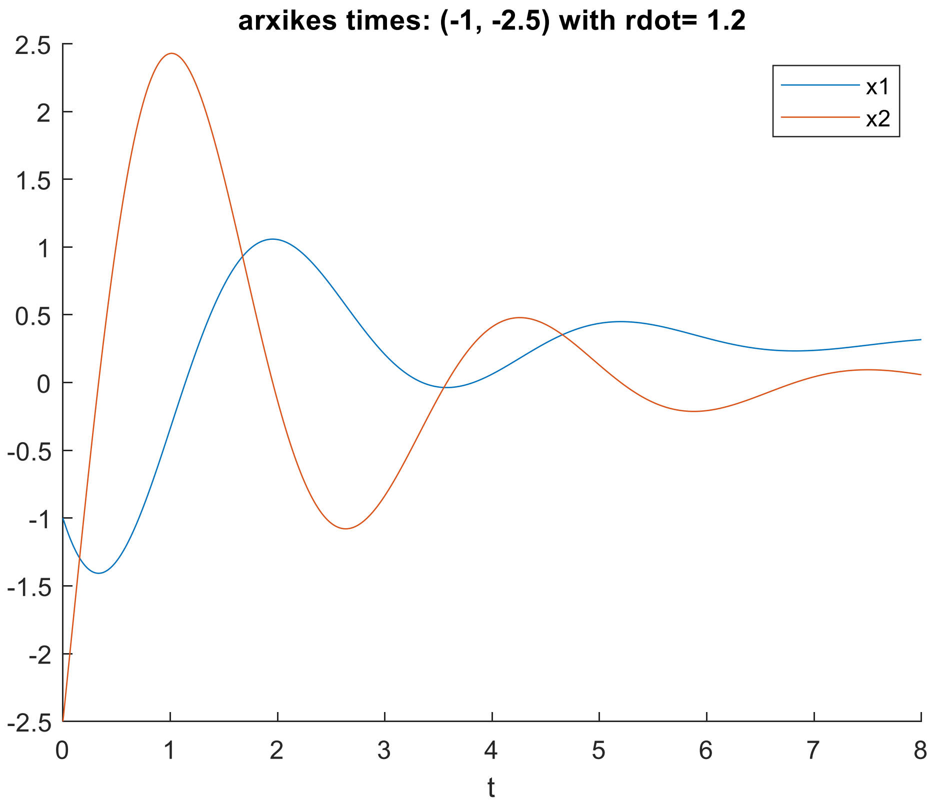
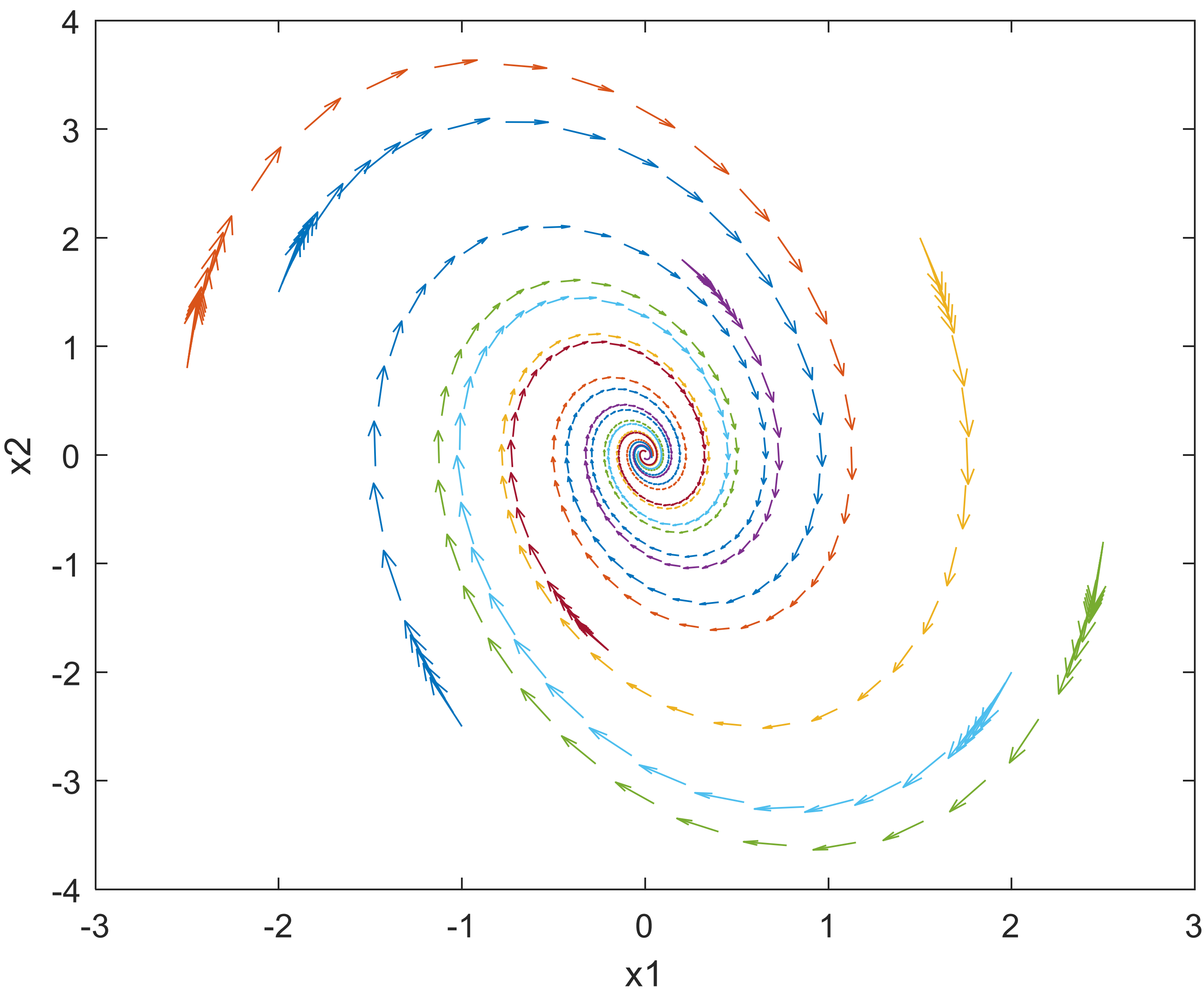
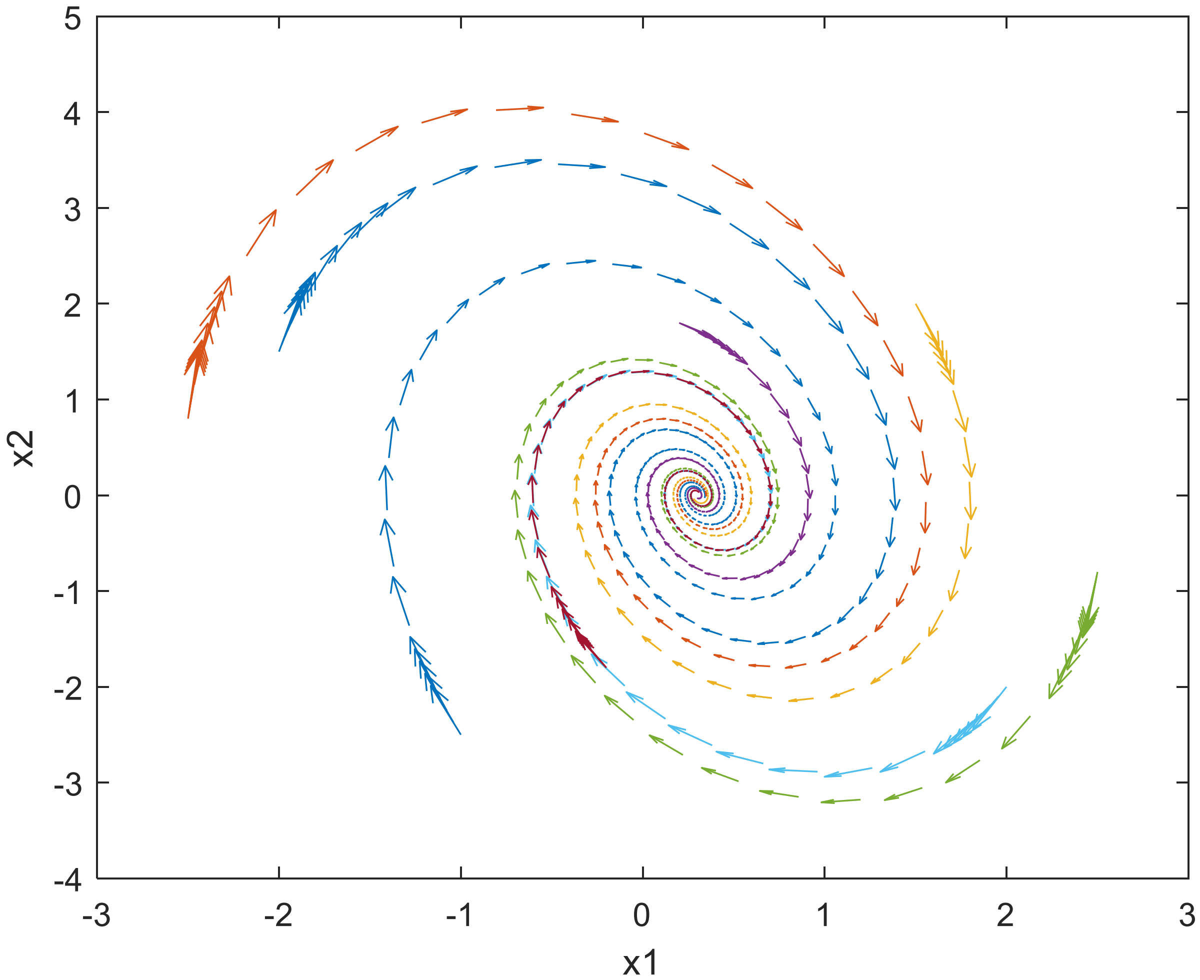


The phase portrait of the system for entering the step function is as follows:



The phase portrait of the system for entering the ramp function is as follows:



B) I) We assume that the variable profit function N(s) exists. For $\varepsilon > \varepsilon_o$ and $\varepsilon < -\varepsilon_0$, u(t) =e(t) holds, as it did for A, so for this case we have the same equations of state as for A, that is:

$$X_1' = X_2$$

$$X_2' = -4X_1 - X_2 + r'' + r'$$

In the case $-\varepsilon_0 < \varepsilon < \varepsilon_0$, u(t)=a*e(t) holds. To find the equations of state we work as follows:

$$\frac{y(s)}{u(s)} = \frac{4}{s^2 + s} \Leftrightarrow y(s^2 + s) = 4u \overset{ILT}{\Leftrightarrow} y''(t) + y'(t) = 4u(t) \ \ (\mathbf{1})$$

Valid: $e(t) = r(t) - y(t)$

We will distinguish between the cases where the input is the step function and where the input is the ramp function of gradient V.

Step Entry

$e' = -y'$ and $e'' = -y''$

So (1): $-e'' - e' = 4u \Leftrightarrow e'' + e' = -4u \overset{u=a*e(t)}{\Longleftrightarrow} e'' + e' = -4\, a * e$

$\mathbf{X_1 = e \quad X_1' = X_2}$

$\mathbf{X_2 = e' \quad X_2' = -4aX_1 - X_2}$

Input Ramp gradient V (r '= V)

$e' = V - y'$ and $e'' = -y''$

So (1): $-e'' + V - e' = 4u \Leftrightarrow e'' + e' = V - 4u \overset{u=a*e(t)}{\Longleftrightarrow} e'' + e' = V - 4\, a * e$

$\mathbf{X_1 = e \quad X_1' = X_2}$

$\mathbf{X_2 = e' \quad X_2' = -4aX_1 - X_2 + V}$

So in both input cases we can consider that the state equations are those of the ramp input, since the slope of the step function is equal to its derivative which is equal to 0. Finally we have:

$X_1 = e$, for each e

$X_2 = e'$, for each e

$X_1' = X_2$, for each e

$X_2' = -4aX_1 - X_2 + r'$, for $-e_0 < e < e_0$

$X_2' = -4X_1 - X_2 + r'$, for $e > e_0$ or $e < -e_0$

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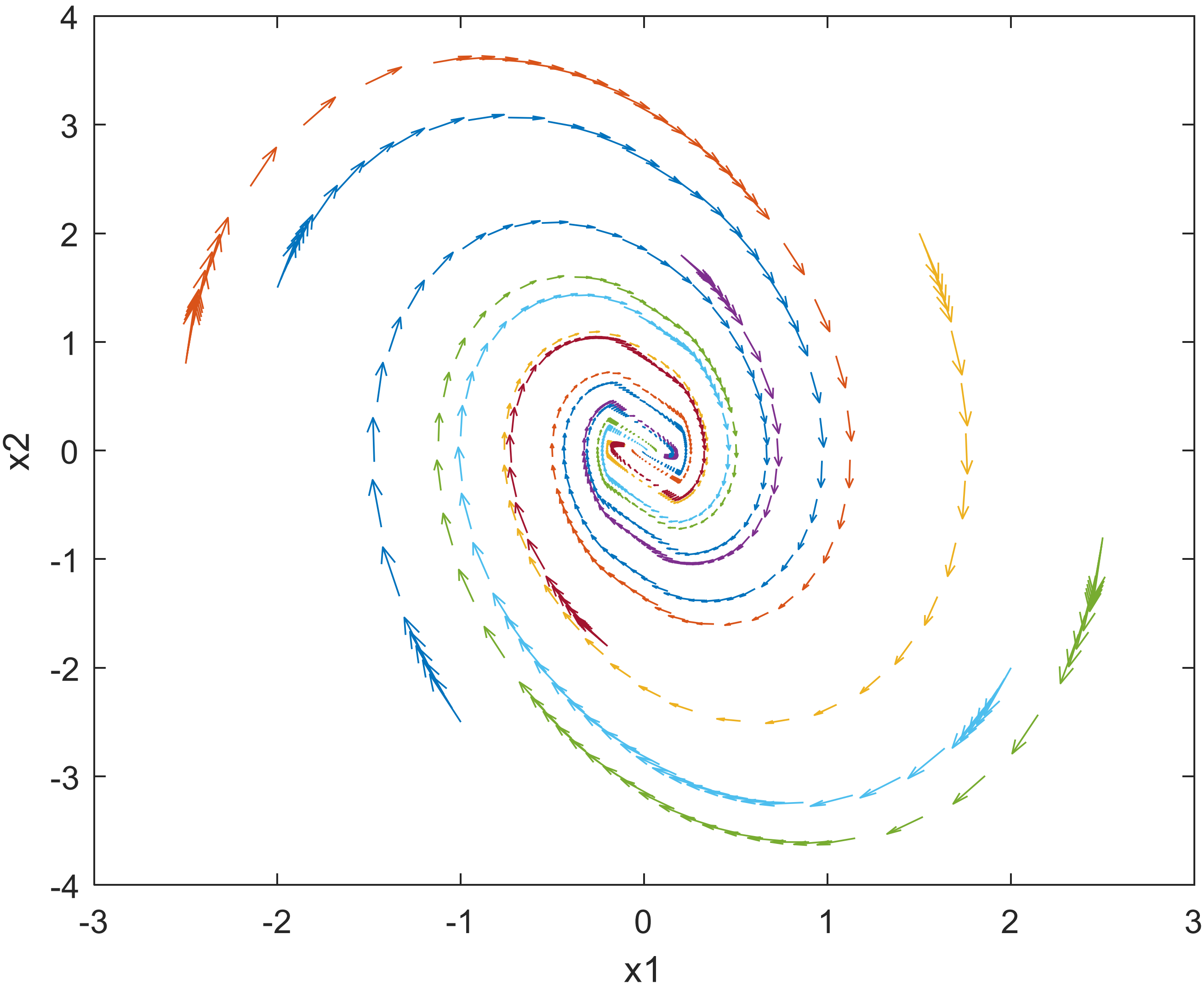
II)phase portrait for step input



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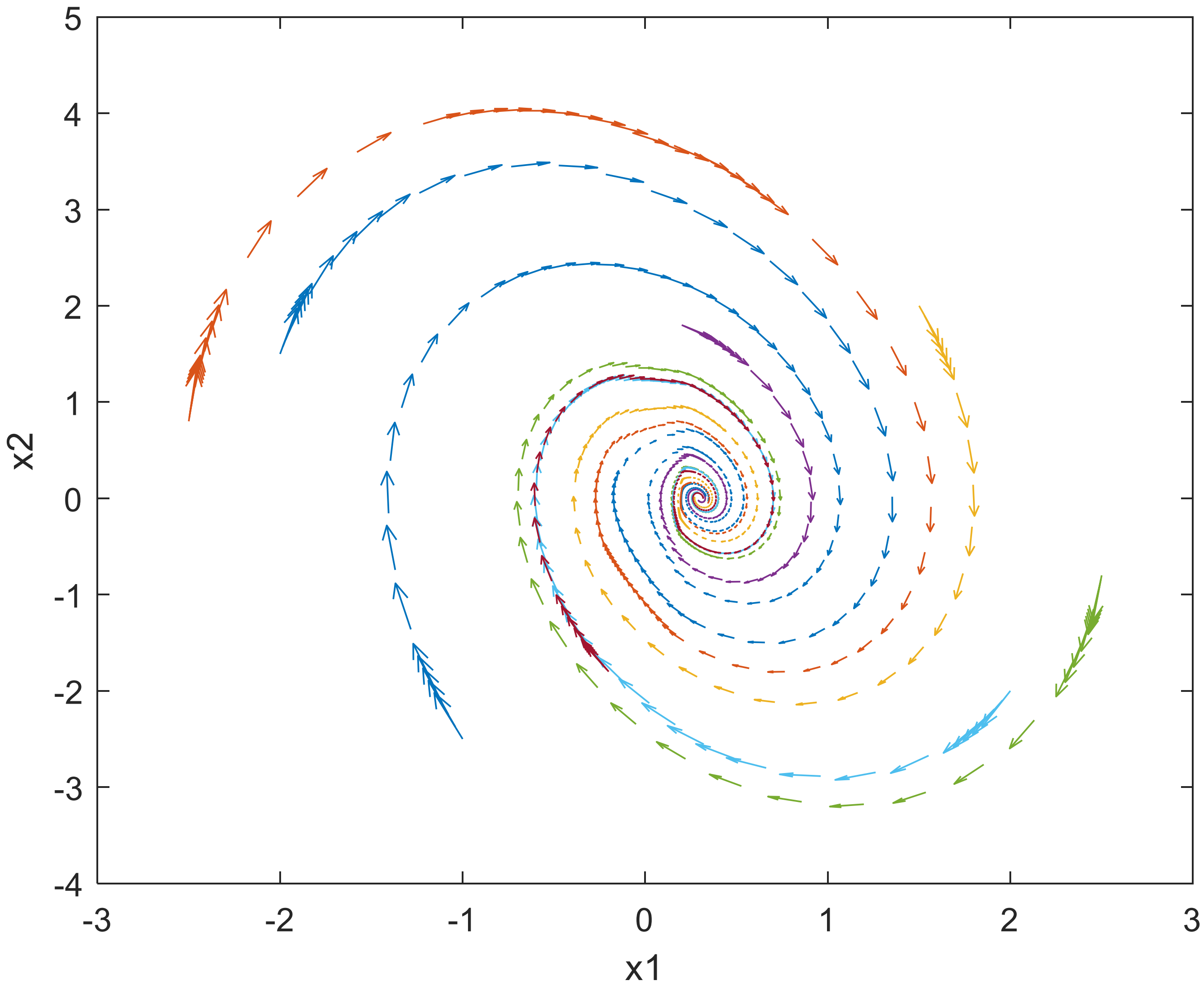
phase portrait for gradient ramp input 1.2



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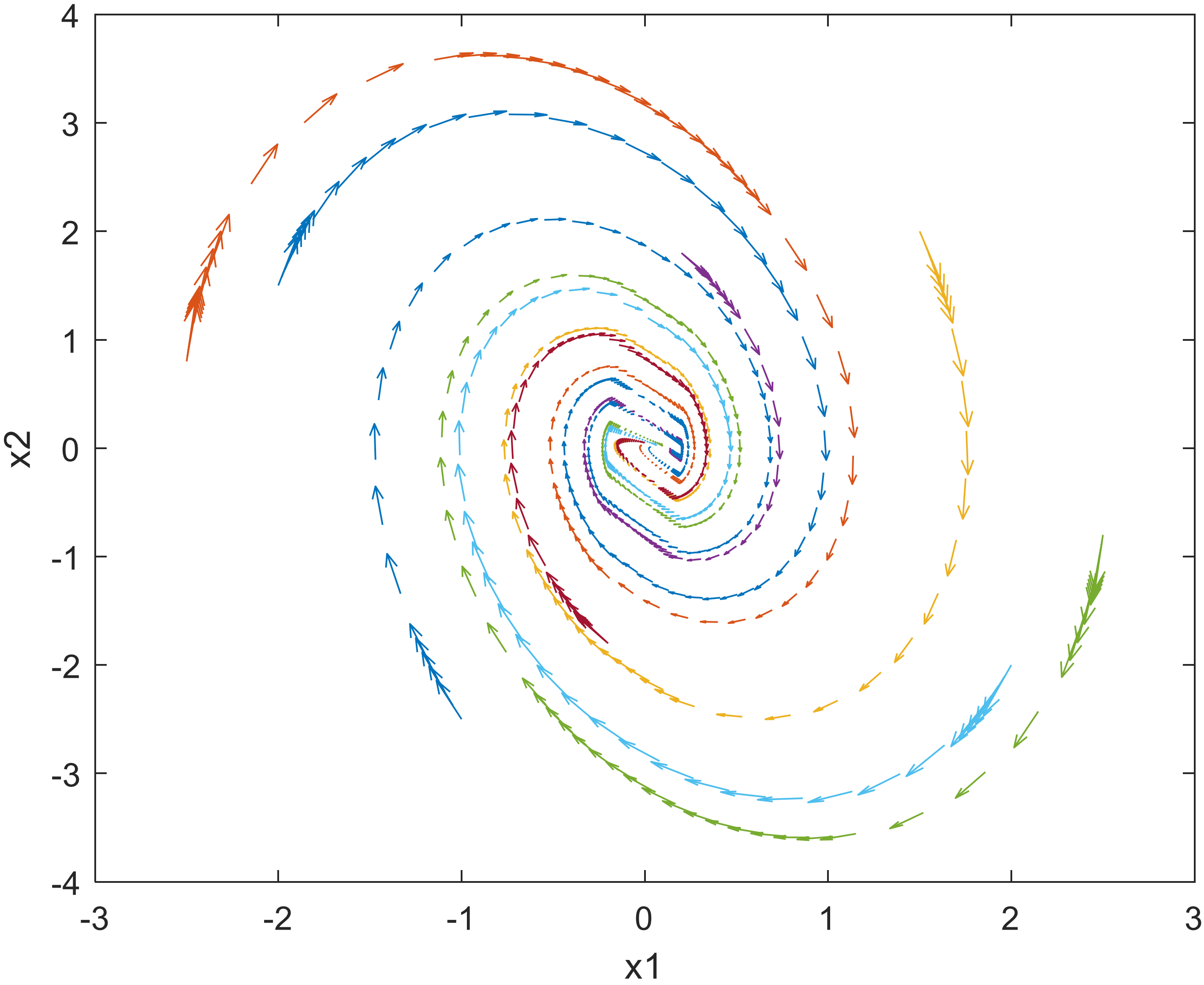
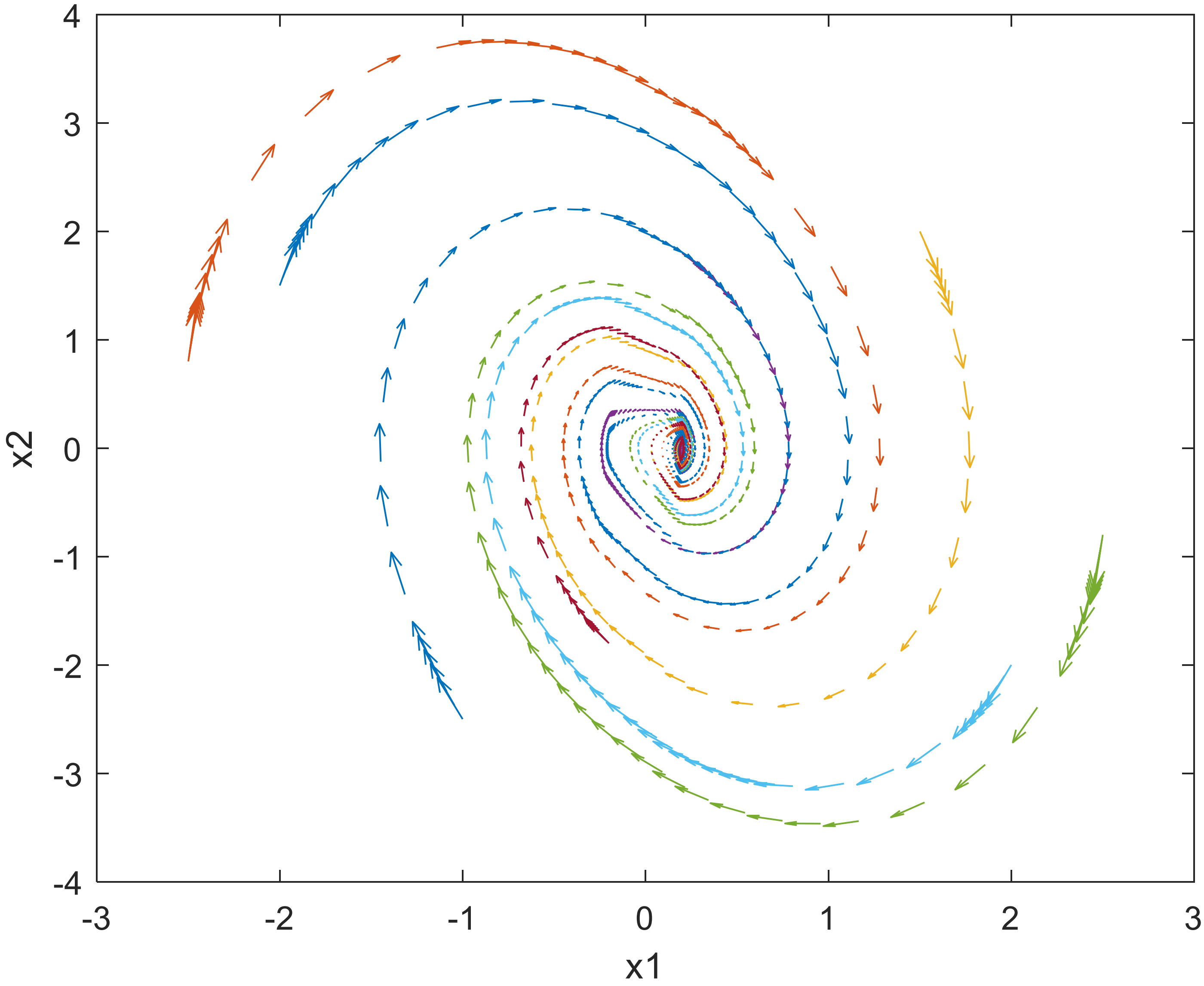
phase portrait for 0.04 gradient ramp input



phase portrait for 0.4 gradient ramp input

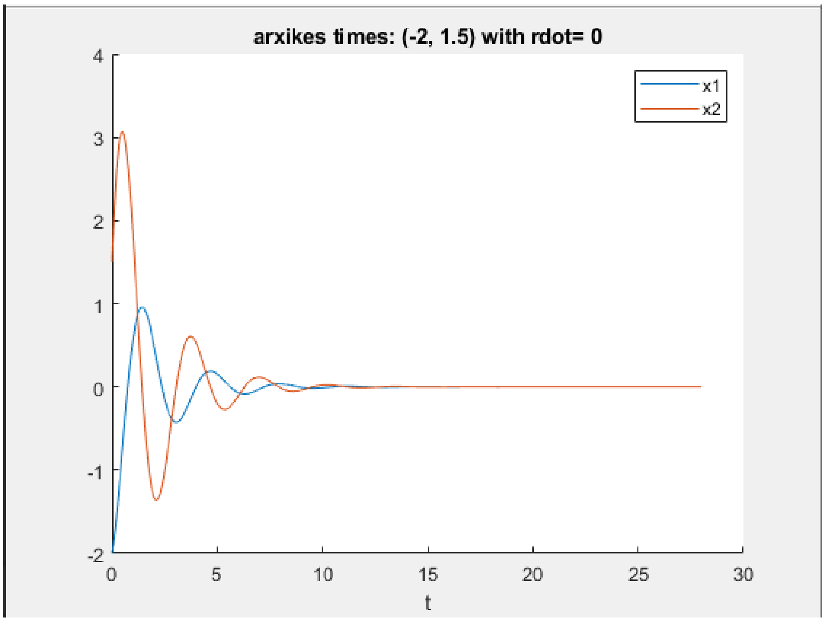


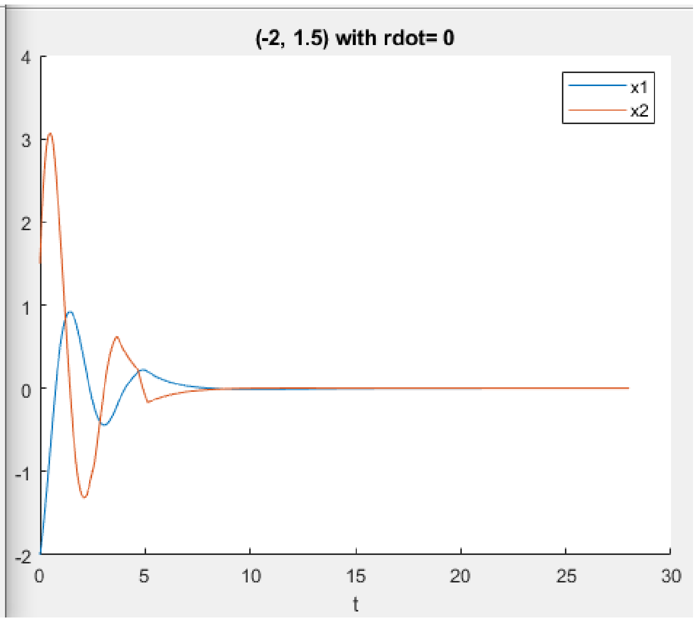
III)

(The following conclusions were drawn by observing the diagrams I made for 28 s . However, in the code provided the time is 8 s when the above diagrams were made. For confirmation you can change the time from 8 to 28)

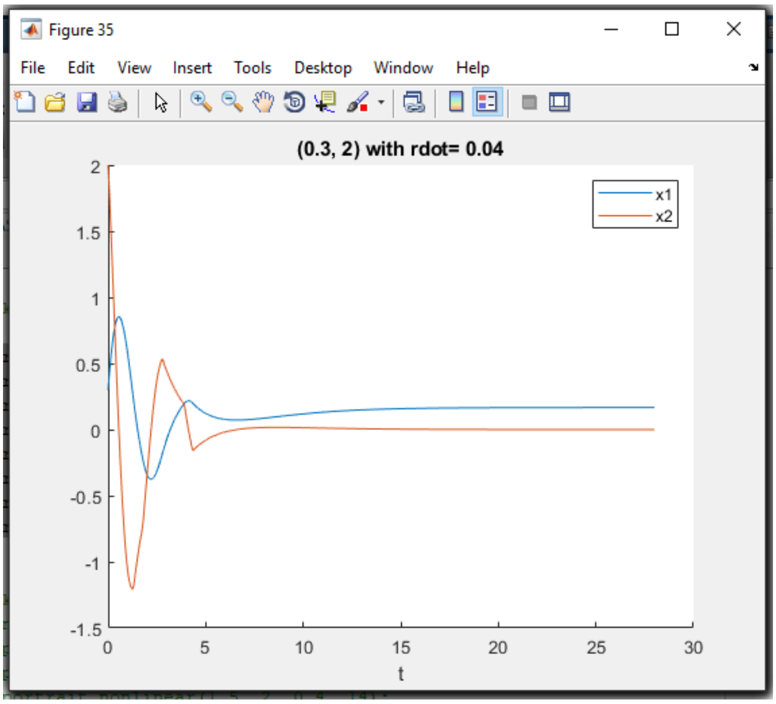
For **step input** the nonlinear system is asymptotically stable for S.I. the (0,0) and the response speed decreases relative to the linear.

Linear Non-Linear

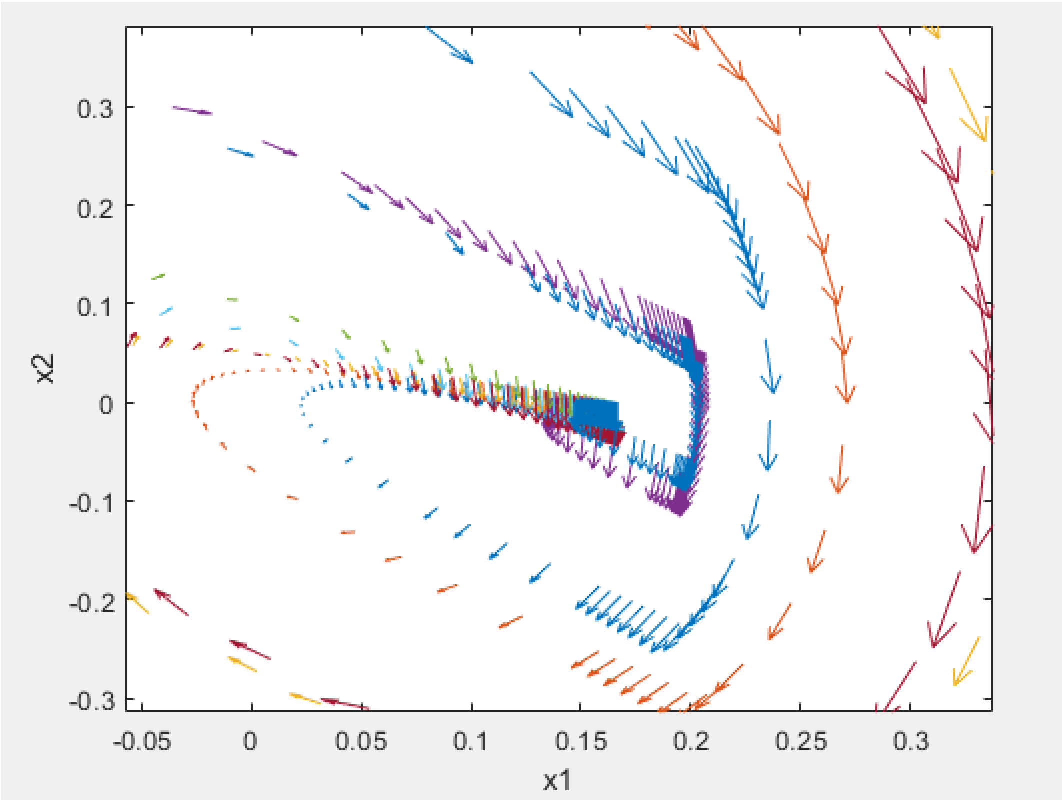




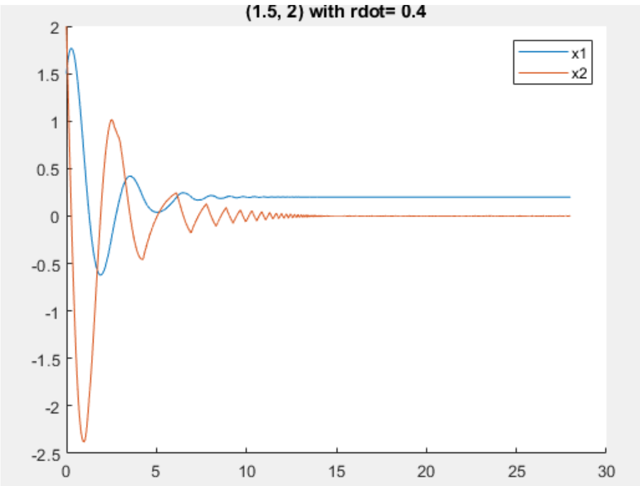
For **a ramp input with a slope of 0.04** the nonlinear system is asymptotically stable at S.I. (0.166,0) (if -0.2< x 1<0.2, valid) resulting from the above formulas for small error and thus the response speed is reduced compared to linear.

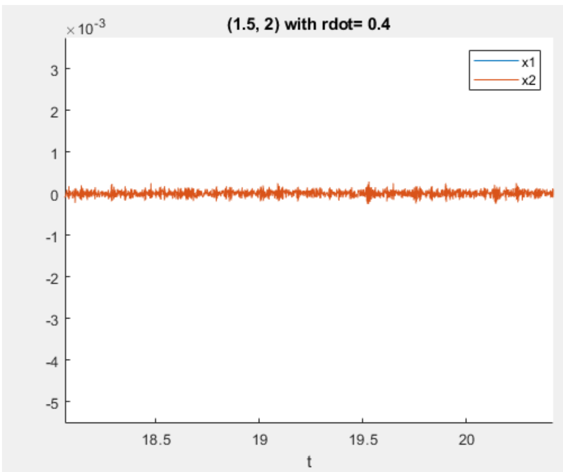


Zoom in phase portrait for 0.04 ramp nonlinearity showing asymptotic stability



For **a ramp entry with a slope of 0.4** the nonlinear system does not reach the equilibrium point ((1.67,0) if -0.2<x1<0.2, not possible or (1,0) if ε>0.2, accepted) due to the discontinuity and x1 tends at 0.2 (at the discontinuity) while x2 oscillates around 0. . Zoomed in on the left





Finally, for **a ramp entry with a slope of 1.2** the nonlinear system is asymptotically stable for S.I. the (0.3,0) (x1>0.2) and thus the response speed is slower. If we zoom in a lot on the phase portrait we notice that around the equilibrium point some circles appear which may indicate the existence of limit cycles. However, due to their very small dimensions, we cannot be sure of their existence.