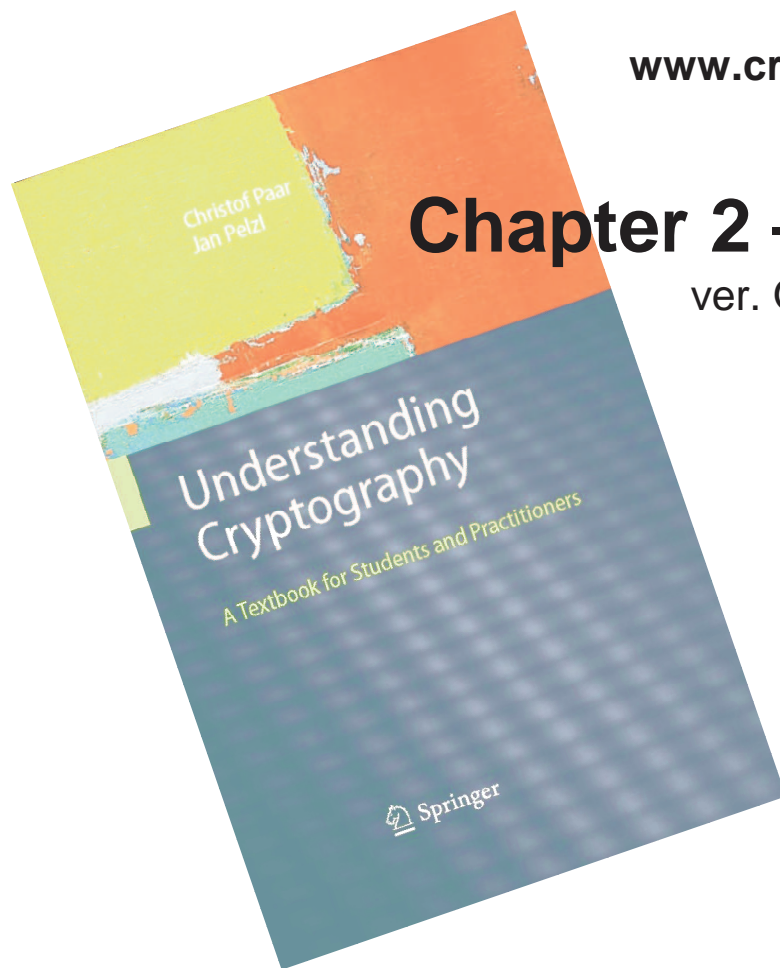


# Understanding Cryptography – A Textbook for Students and Practitioners

by Christof Paar and Jan Pelzl

[www.crypto-textbook.com](http://www.crypto-textbook.com)



## Chapter 2 – Stream Ciphers

ver. October 29, 2009

These slides were prepared by Thomas Eisenbarth, Christof Paar and Jan Pelzl

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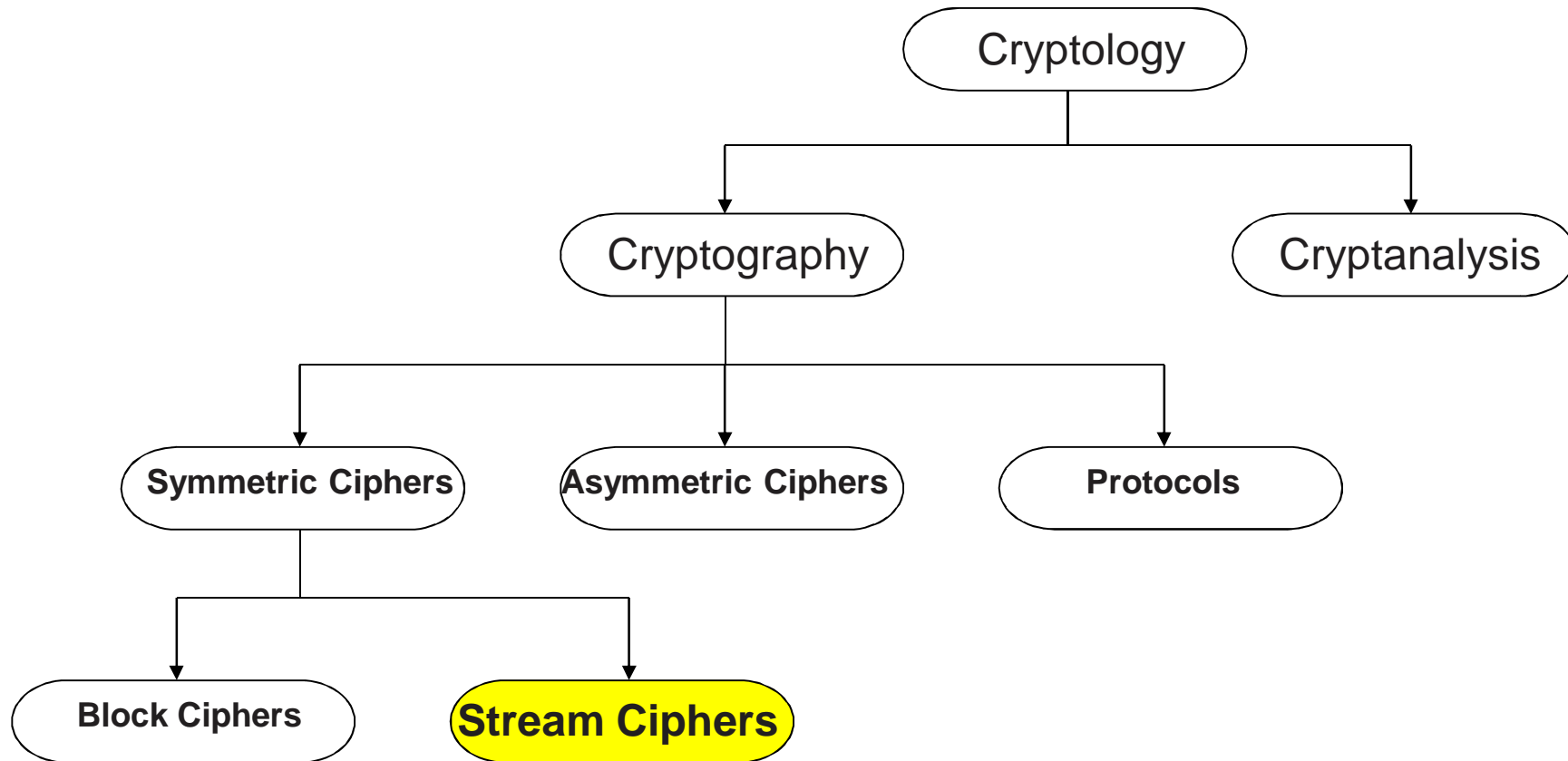
# Content of this Chapter

- Intro to stream ciphers
- Random number generators (RNGs)
- One-Time Pad (OTP)
- Linear feedback shift registers (LFSRs)
- Trivium: a modern stream cipher

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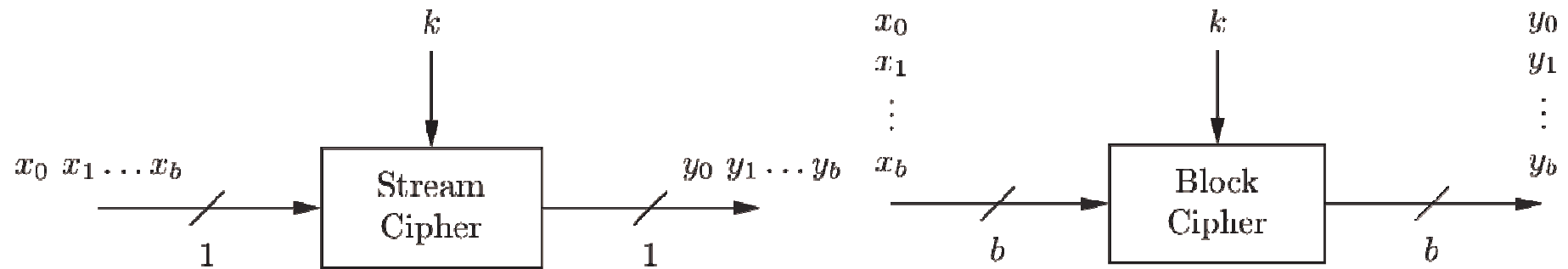
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## ■ Stream Ciphers in the Field of Cryptology



Stream Ciphers were invented in 1917 by Gilbert Vernam

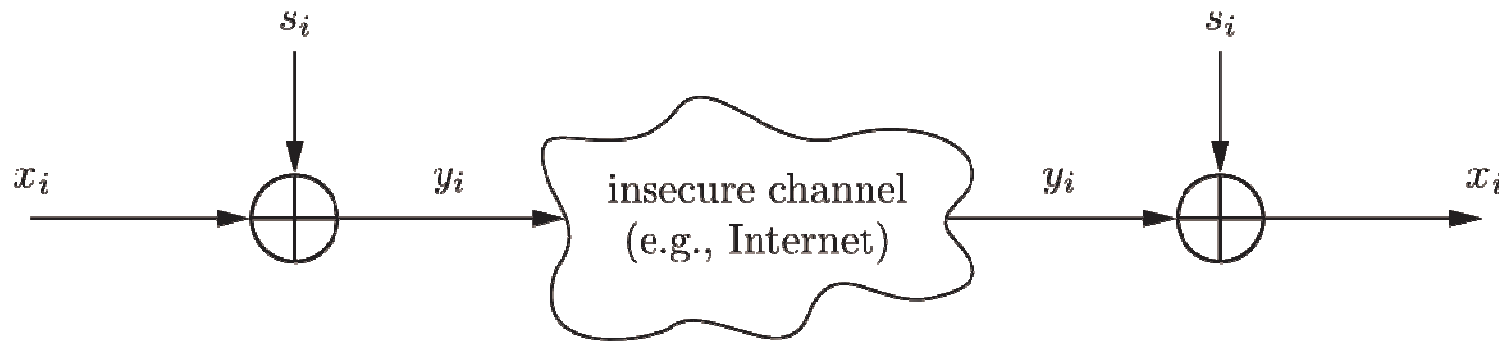
## ■ Stream Cipher vs. Block Cipher



- **Stream Ciphers**
  - Encrypt bits individually
  - Usually small and fast → common in embedded devices (e.g., A5/1 for GSM phones)
- **Block Ciphers:**
  - Always encrypt a full block (several bits)
  - Are common for Internet applications

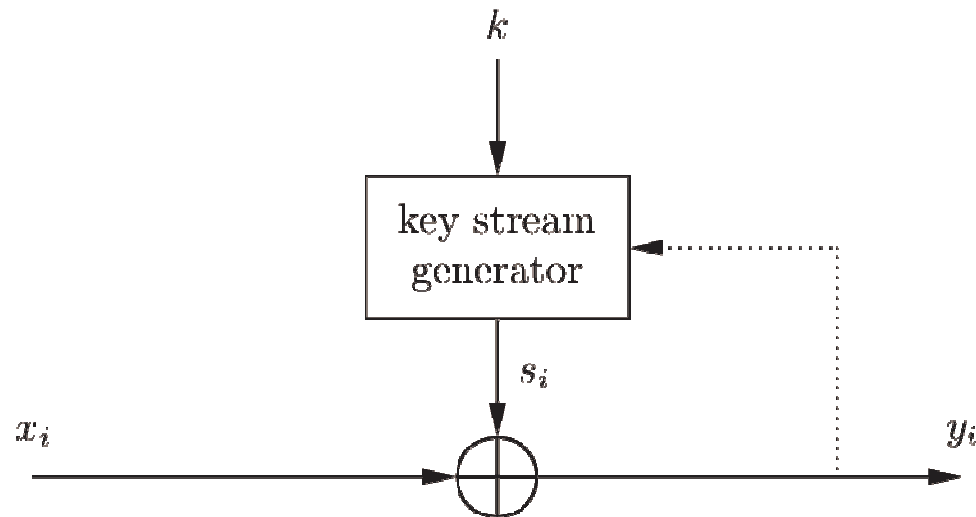
## ■ Encryption and Decryption with Stream Ciphers

Plaintext  $x_i$ , ciphertext  $y_i$  and key stream  $s_i$  consist of individual bits



- Encryption and decryption are simple additions modulo 2 (aka XOR)
- Encryption and decryption are the same functions
- **Encryption:**  $y_i = e_{s_i}(x_i) = x_i + s_i \bmod 2$   $x_i, y_i, s_i \in \{0,1\}$
- **Decryption:**  $x_i = e_{s_i}(y_i) = y_i + s_i \bmod 2$

## ■ Synchronous vs. Asynchronous Stream Cipher



- Security of stream cipher depends entirely on the key stream  $s_i$  :
  - Should be **random** , i.e.,  $\Pr(s_i = 0) = \Pr(s_i = 1) = 0.5$
  - Must be **reproducible** by sender and receiver
- **Synchronous Stream Cipher**
  - Key stream depend only on the key (and possibly an initialization vector IV)
- **Asynchronous Stream Ciphers**
  - Key stream depends also on the ciphertext (dotted feedback enabled)



## ■ Why is Modulo 2 Addition a Good Encryption Function?

- Modulo 2 addition is equivalent to XOR operation
- For perfectly random key stream  $s_i$ , each ciphertext output bit has a 50% chance to be 0 or 1  
→ Good statistic property for ciphertext
- Inverting XOR is simple, since it is the same XOR operation

$x_i$	$s_i$	$y_i$
0	0	0
0	1	1
1	0	1
1	1	0

## ■ Stream Cipher: Throughput

Performance comparison of symmetric ciphers (Pentium4):

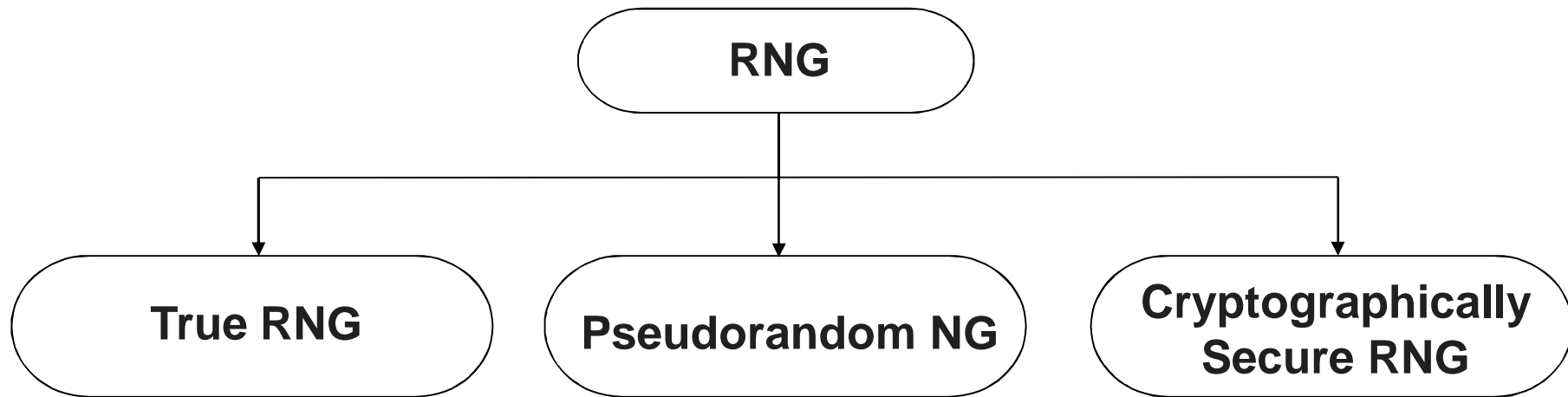
Cipher	Key length	Mbit/s
DES	56	36.95
3DES	112	13.32
AES	128	51.19
RC4 (stream cipher)	(choosable)	211.34

Source: Zhao et al., Anatomy and Performance of SSL Processing, ISPASS 2005

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## ■ Random number generators (RNGs)



## ■ True Random Number Generators (TRNGs)

- Based on physical random processes: coin flipping, dice rolling, semiconductor noise, radioactive decay, mouse movement, clock jitter of digital circuits
- Output stream  $s_i$  should have good statistical properties:  
 $\Pr(s_i = 0) = \Pr(s_i = 1) = 50\%$  (often achieved by post-processing)
- Output can neither be predicted nor be reproduced

Typically used for generation of keys, nonces (used only-once values) and for many other purposes



## ■ Pseudorandom Number Generator (PRNG)

- Generate sequences from initial seed value
- Typically, output stream has good statistical properties
- Output can be reproduced and can be predicted

Often computed in a recursive way:

$$s_0 = \text{seed}$$

$$s_{i+1} = f(s_i, s_{i-1}, \dots, s_{i-t})$$

Example: *rand()* function in ANSI C:

$$s_0 = 12345$$

$$s_{i+1} = 1103515245s_i + 12345 \bmod 2^{31}$$

**Most PRNGs have bad cryptographic properties!**

## ■ Cryptanalyzing a Simple PRNG

Simple PRNG: **Linear Congruential Generator**

$$S_0 = \textit{seed}$$

$$S_{i+1} = AS_i + B \bmod m$$

### Assume

- unknown  $A$ ,  $B$  and  $S_0$  as key
- Size of  $A$ ,  $B$  and  $S_i$  to be 100 bit
- 300 bit of output are known, i.e.  $S_1$ ,  $S_2$  and  $S_3$

### Solving

$$S_2 = AS_1 + B \bmod m$$

$$S_3 = AS_2 + B \bmod m$$

...directly reveals  $A$  and  $B$ . All  $S_i$  can be computed easily!

**Bad cryptographic properties due to the linearity of most PRNGs**

## ■ Cryptographically Secure Pseudorandom Number Generator (CSPRNG)

- Special PRNG with additional property:
  - Output must be **unpredictable**

**More precisely:** Given  $n$  consecutive bits of output  $s_i$ , the following output bits  $s_{n+1}$  cannot be predicted (in polynomial time).

- Needed in cryptography, in particular for stream ciphers
- Remark: There are almost no other applications that need unpredictability, whereas many, many (technical) systems need PRNGs.



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## ■ One-Time Pad (OTP)

### Unconditionally secure cryptosystem:

- A cryptosystem is unconditionally secure if it cannot be broken even with *infinite* computational resources

### One-Time Pad

- A cryptosystem developed by Mauborgne that is based on Vernam's stream cipher:
- Properties:

Let the plaintext, ciphertext and key consist of individual bits

$$x_i, y_i, k_i \in \{0,1\}.$$

$$\text{Encryption:} \quad e_{k_i}(x_i) = x_i \oplus k_i.$$

$$\text{Decryption:} \quad d_{k_i}(y_i) = y_i \oplus k_i$$

**OTP is unconditionally secure if and only if the key  $k_i$  is used once!**

## ■ One-Time Pad (OTP)

Unconditionally secure cryptosystem:

$$y_0 = x_0 \oplus k_0$$

$$y_1 = x_1 \oplus k_1$$

:

Every equation is a linear equation with two unknowns

⇒ for every  $y_i$  are  $x_i = 0$  and  $x_i = 1$  equiprobable!

⇒ This is true iff  $k_0, k_1, \dots$  are independent, i.e., all  $k_i$  have to be generated truly random

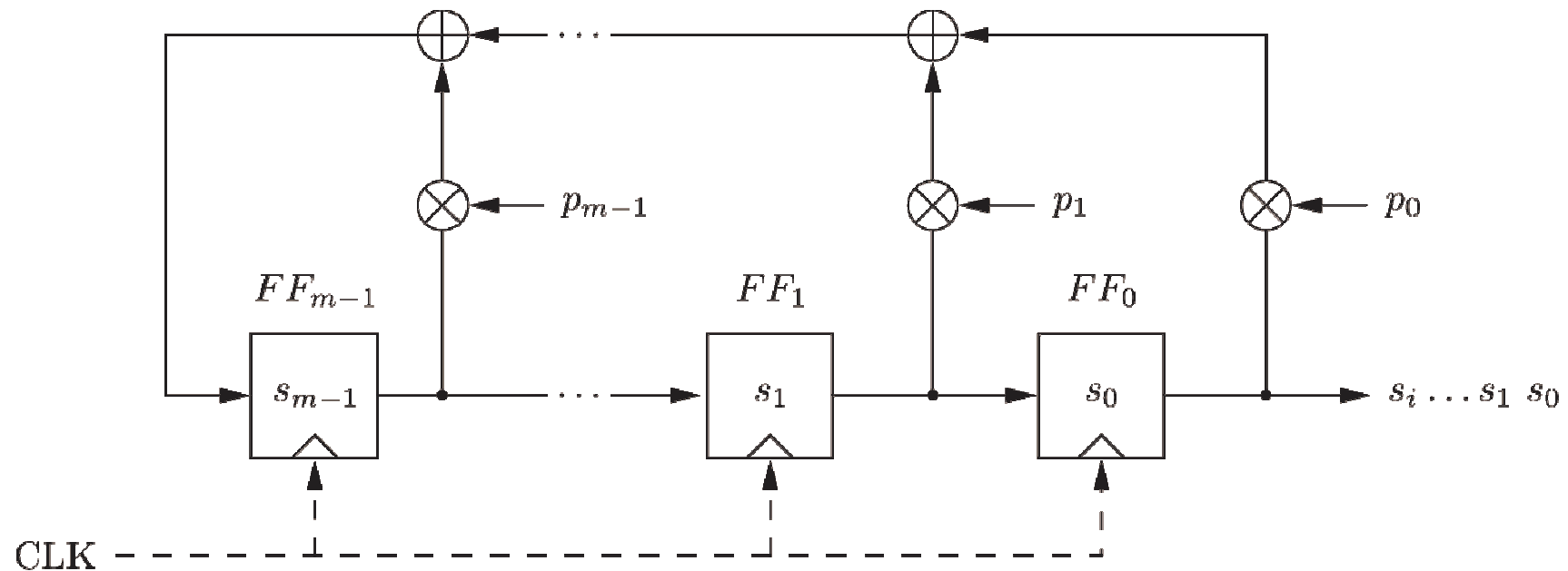
⇒ It can be shown that this systems can *provably* not be solved.

**Disadvantage:** For almost all applications the OTP is **impractical** since the key must be as long as the message! (Imagine you have to encrypt a 1GByte email attachment.)

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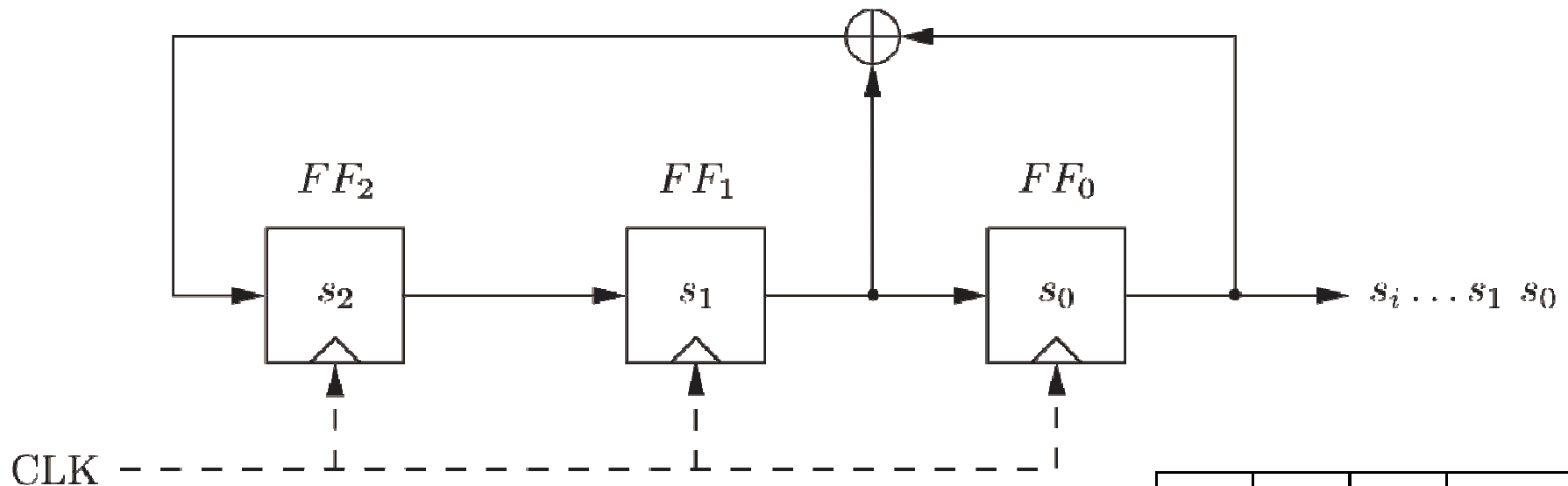
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## ■ Linear Feedback Shift Registers (LFSRs)



- Concatenated *flip-flops* ( $FF$ ), i.e., a shift register together with a feedback path
- Feedback computes fresh input by XOR of certain state bits
- *Degree*  $m$  given by number of storage elements
- If  $p_i = 1$ , the feedback connection is present (“closed switch”), otherwise there is not feedback from this flip-flop (“open switch”)
- Output sequence repeats periodically
- Maximum output length:  $2^m - 1$

## ■ Linear Feedback Shift Registers (LFSRs): Example with m=3



- LFSR output described by recursive equation:

$$s_{i+3} = s_{i+1} + s_i \text{ mod } 2$$

- Maximum output length (of  $2^3-1=7$ ) achieved only for certain feedback configurations, .e.g., the one shown here.

<i>clk</i>	$FF_2$	$FF_1$	$FF_0=s_i$
0	1	0	0
1	0	1	0
2	1	0	1
3	1	1	0
4	1	1	1
5	0	1	1
6	0	0	1
7	1	0	0
8	0	1	0

## ■ Security of LFSRs

LFSRs typically described by polynomials:

$$P(x) = x^m + p_{l-1}x^{m-1} + \dots + p_1x + p_0$$

- Single LFSRs generate highly predictable output
- If  $2m$  output bits of an LFSR of degree  $m$  are known, the feedback coefficients  $p_i$  of the LFSR can be found by solving a system of linear equations\*
- Because of this many stream ciphers use **combinations** of LFSRs

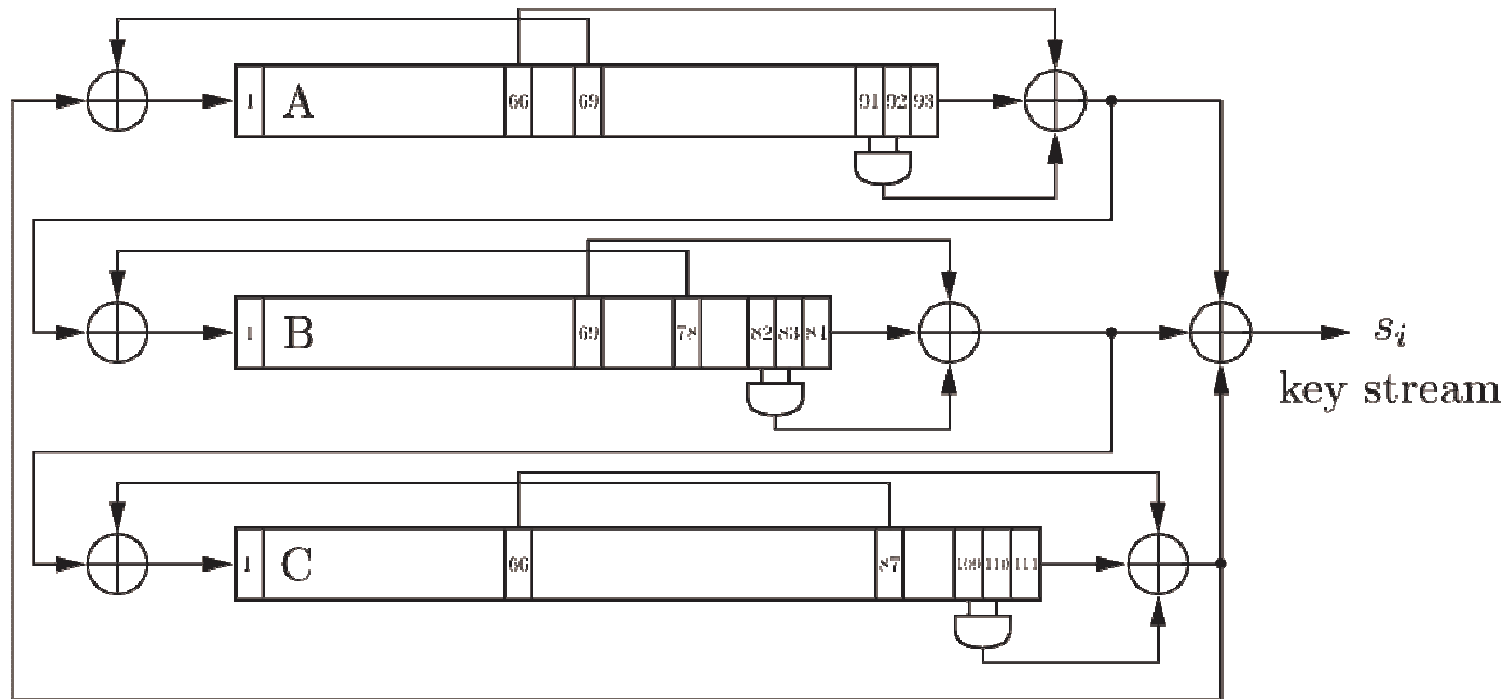
\*See Chapter 2 of *Understanding Cryptography* for further details.

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## ■ A Modern Stream Cipher - Trivium



- Three *nonlinear* LFSRs (NLFSR) of length 93, 84, 111
- XOR-Sum of all three NLFSR outputs generates key stream  $s_i$
- Small in Hardware:
  - Total register count: 288
  - Non-linearity: 3 AND-Gates
  - 7 XOR-Gates (4 with three inputs)

## ■ Trivium

### Initialization:

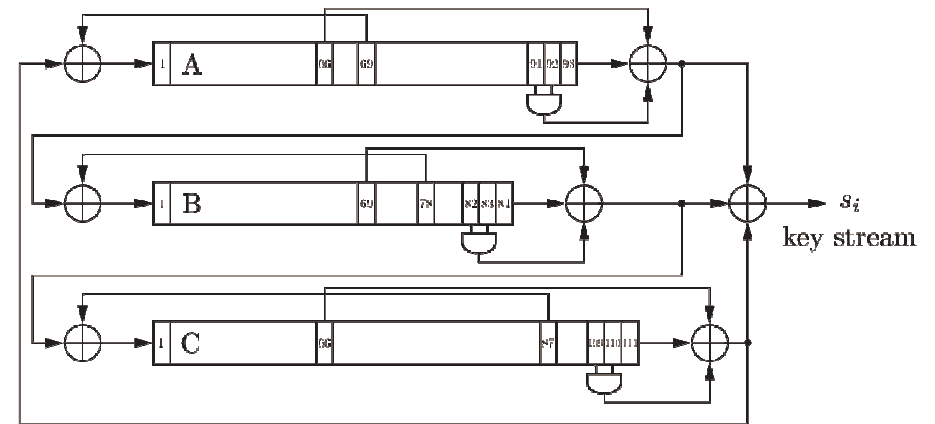
- Load 80-bit IV into A 
- Load 80-bit key into B
- Set  $c_{109}, c_{110}, c_{111} = 1$ , all other bits 0

### Warm-Up:

- Clock cipher  $4 \times 288 = 1152$  times without generating output

### Encryption:

- XOR-Sum of all three NLFSR outputs generates key stream  $s_i$



Design can be parallelized to produce up to 64 bits of output per clock cycle

	Register length	Feedback bit	Feedforward bit	AND inputs
A	93	69	66	91, 92
B	84	78	69	82, 83
C	111	87	66	109, 110

## ■ Lessons Learned

- Stream ciphers are less popular than block ciphers in most domains such as Internet security. There are exceptions, for instance, the popular stream cipher RC4.
- Stream ciphers sometimes require fewer resources, e.g., code size or chip area, for implementation than block ciphers, and they are attractive for use in constrained environments such as cell phones.
- The requirements for a *cryptographically secure pseudorandom number generator* are far more demanding than the requirements for pseudorandom number generators used in other applications such as testing or simulation
- The One-Time Pad is a provable secure symmetric cipher. However, it is highly impractical for most applications because the key length has to equal the message length.
- Single LFSRs make poor stream ciphers despite their good statistical properties. However, careful combinations of several LFSR can yield strong ciphers.