Giovanna Santana Pennisi - CTII 350

Lista de Exercícios - Aula 10

Lista de Exercícios

1- $\binom{8}{3} = \frac{8!}{3!\,(8-3)!} = \frac{8\cdot7\cdot6\cdot\cancel{5!}}{3\cdot2\cdot1\cdot\cancel{5!}} = \frac{336}{6} = 56$

Alternativa B

2- $\binom{200}{198} = \frac{200!}{198!\,(200-198)!} = \frac{200\cdot199\cdot\cancel{198!}}{\cancel{198!}\cdot2\cdot1} = \frac{39{,}800}{2} = 19{,}900$

Alternativa A

3- $\binom{n-1}{2} = \binom{n+1}{4} \to$ 1º Caso: $\frac{(n-1)!}{2!\,(n-1-2)!} = \frac{(n+1)!}{4!\,(n+1-4)!}$

$\to \frac{(n-1)!\,4!\,\cancel{(n-3)!}}{2!\,\cancel{(n-3)!}} = (n+1)\,n!$

$\frac{(n-1)!\,4\cdot3\cdot\cancel{2!}}{\cancel{2!}} = (n+1)\,n\cdot(n-1)!$

$12 = \frac{(n^2+n)\cdot\cancel{(n-1)!}}{\cancel{(n-1)!}}$

$\to n^2+n-12=0$

$-4 + 3 = -1$

$-4 \cdot 3 = -12$

não convém

convém

$n=3$

2º Caso: $\binom{n-1}{2} = \binom{n+1}{4} = 0 \to n < k$

$n-1<2$	$n+1<4$
$n<2+1$	$n<4-1$
$\vee$	$\vee$
$n<3$	$n<3$

$\vee$

$n<3$

$V = \{1, 2, 3\}$

4- $\binom{20}{13}+\binom{20}{14} = ?$ | Soma de 2 consecutivos $\rightarrow \binom{n}{k}+\binom{n}{k+1}=\binom{n+1}{k+1}$

$\hookrightarrow \binom{20}{13}+\binom{20}{14} = \binom{21}{14}$

★ Complementares : $\binom{n}{k}=\binom{n}{n-k}$

$\binom{21}{14} = \binom{21}{21-14} = \binom{21}{7}$ → Alternativa C

5- $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\dots+\binom{n}{n} = 2^n$ (soma na linha n)

6- a) $\sum_{p=0}^{10}\binom{10}{p} = \binom{10}{0}+\binom{10}{1}+\binom{10}{2}+\dots+\binom{10}{10} = 2^{10} = 1024$

b) $\sum_{p=0}^{9}\binom{10}{p} = \binom{10}{0}+\binom{10}{1}+\binom{10}{2}+\dots+\binom{10}{9} = 2^{10}-\binom{10}{10} = 1024-1$

1023

c) $\sum_{p=2}^{9}\binom{9}{p} = \binom{9}{2}+\binom{9}{3}+\dots+\binom{9}{9} = 2^9-\binom{9}{1}-\binom{9}{0} = 512-9-1$

502

d) $\sum_{p=4}^{10}\binom{p}{4} = \binom{4}{4}+\binom{5}{4}+\dots\binom{10}{4} = \binom{11}{5}$ → soma na coluna 4

$\rightarrow \binom{11}{5} = \frac{11!}{5!\,(11-5)!} = \frac{11\cdot10\cdot9\cdot8\cdot7\cdot6!}{5!\cdot6!} = \frac{11\cdot10\cdot9\cdot8\cdot7}{5\cdot4\cdot3\cdot2\cdot1}$

$\rightarrow \frac{55,440}{120} = 462$

e) $\sum_{p=5}^{10} \binom{p}{5} = \binom{5}{5} + \binom{6}{5} + \dots + \binom{10}{5} = \binom{11}{6}$ → Soma na coluna 5

→ $\binom{11}{6} = \frac{11!}{6!(11-5)!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot 5!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

→ $\frac{55.440}{120} = 462$

7 – $\sum_{k=0}^{m} \binom{m}{k} = 512$

$\sum_{k=0}^{m} \binom{m}{k} = \binom{m}{0} + \dots + \binom{m}{m} = 512$ (soma na linha m)

$2^m = 512$

$\cancel{2}^m = \cancel{2}^9$

$m = 9$ → Alternativa e