Giovanna Santana Pennisi - CTII 350

Lista de Exercícios 3 - Determinantes de Matrizes de ordem 1, 2 e 3

Lista de exercícios

1-a) $\begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix}$ = Diagonal principal = $2 \cdot 5 = 10$

Diagonal secundária = $1 \cdot 3 = 3$

det = $10 - 3$ → det = 7

b) $\begin{vmatrix} -2 & -4 \\ 3 & 6 \end{vmatrix}$ − Diagonal principal = $-2 \cdot 6 = -12$

− Diagonal secundária = $3 \cdot -4 = -12$

det = $-12 - (-12)$ → det = $-12 + 12$ → det = 0

c) $\begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 4 & -2 \end{vmatrix} \begin{matrix} 3 & -1 \\ 2 & 1 \\ 1 & 4 \end{matrix}$

$\left.\begin{array}{l} \text{Diagonal principal} = 3 \cdot 1 \cdot (-2) = -6 \\ \text{Paralela } 1 = (-1) \cdot (-1) \cdot 1 = 1 \\ \text{Paralela } 2 = 1 \cdot 2 \cdot 4 = 8 \end{array}\right\} 3$

$\left.\begin{array}{l} \text{Diagonal secundária: } 1 \cdot 1 \cdot 1 = 1 \\ \text{Paralela } 2\text{: } 4 \cdot (-1) \cdot 3 = -12 \\ \text{Paralela } 3\text{: } (-2) \cdot 2 \cdot (-1) = 4 \end{array}\right\} -7$

det = principal − paralela

det $3 - (-7)$

det = $3 + 7$

det = 10

d) $\left|\begin{matrix} 3 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{matrix}\right|\begin{matrix} 3 & 2 \\ 2 & 3 \\ 1 & 1 \end{matrix}$

D. principal $= 3 \cdot 3 \cdot 4 = 36$
Paralela 1 $= 2 \cdot 1 \cdot 1 = 2$
Paralela 2 $= (-1) \cdot 2 \cdot 1 = -2$
} 36

D. secundária $= 1 \cdot 3 \cdot (-1) = -3$
Paralela 1 $= 1 \cdot 1 \cdot 3 = 3$
Paralela 2 $= 4 \cdot 2 \cdot 2 = 16$
} 16

$\det = 36 - (+16)$
$\det = 36 - 16$
$\det = 20$

2 - $a_{ij} \rightarrow$ Se $i = j \rightarrow -3$ / Se $i \neq j = 0$. / $\det A = ?$

$\underset{3 \times 3}{A} = \left[\begin{matrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{matrix}\right]\begin{matrix} -3 & 0 \\ 0 & -3 \\ 0 & 0 \end{matrix}$ } $\det A = ?$

D. Principal $= (-3) \cdot (-3) \cdot (-3) = -27$
Paralela 1 $= 0 \cdot 0 \cdot 0 = 0$
Paralela 2 $= 0 \cdot 0 \cdot 0 = 0$
} -27

D. secundária $= 0 \cdot (-3) \cdot 0 = 0$
P. 1 $= 0 \cdot 0 \cdot (-3) = 0$
P. 2 $= (-3) \cdot 0 \cdot 0 = 0$
} 0

$\det A = -27 - 0 > \det A = -27$ Alternativa A

$\det A = -27 \Rightarrow \det A = (-27)$ Alternativa A

3- $\left|\begin{array}{ccc|cc} x & 1 & x & x & 1 \\ 3 & x & 4 & 3 & x \\ -1 & 3 & 3 & -1 & 3 \end{array}\right| = -3$

D. Principal: $x \cdot x \cdot 3 = 3x^2$	D. Secundária: $1 \cdot x \cdot x = x^2$
Paralela 1: $1 \cdot 4 \cdot 1 = 4$	Paralela 1: $3 \cdot 4 \cdot x = 12x$
Paralela 2: $x \cdot 3 \cdot 3 = 9x$	Paralela 2: $3 \cdot 3 \cdot 1 = 9$
$3x^2 + 9x + 4$	$x^2 + 12x + 9$

$3x^2 + 9x + 4 - (x^2 + 12x + 9) = -3$

$3x^2 + 9x + 4 - x^2 - 12x - 9 + 3 = 0$

$2x^2 - 3x - 2 = 0$

$a = 2$ | $\Delta = (-3)^2 - 4 \cdot 2 (-2)$

$b = -3$ | $\Delta = 9 + 16$

$c = -2$ | $\Delta = 25$

$x = \frac{3 \pm 5}{4}$

$x' = \frac{3-5}{4} = \frac{-2^{:2}}{4_{:2}} = -\frac{1}{2}$

$x'' = \frac{3+5}{4} = \frac{8}{4} = 2$

Alternativa E $\{-\frac{1}{2}, 2\}$

4- $\left|\begin{matrix} x-1 & -1 & 0 \\ 0 & x+1 & -1 \\ 2 & +1 & x+1 \end{matrix}\right| \begin{matrix} x-1 & -1 \\ 0 & x+1 \\ 2 & -1 \end{matrix} = 2$

D. Principal: $(x-1)\cdot(x+1)\cdot(x+1)$ *

Paralela 1: $(-1)\cdot(-1)\cdot 2 = 2$

Paralela 2: $0\cdot 0\cdot(-1) = 0$

$(x-1)(x+1)\cdot(x+1) + 2 + 0$

D. Secundária: $2\cdot(x+1)\cdot 0 = 0$

Paralela 1: $(-1)\cdot(-1)\cdot(x-1) = (x-1)$

Paralela 2: $(x+1)\cdot 0\cdot(-1) = 0$

$0 + (x-1) + 0$

$(x-1)(x+1)(x+1) + 2 - (x-1) = 2$

$(x^2 + x - \cancel{x} - 1)(x+1) + \cancel{2} - x + 1 - \cancel{2} = 0$

$x^3 + x^2 - x - 1 - x + 1 = 0$

$x^3 + x^2 - 2x = 0$ → Usar relações de Girard

$a = 1$
$b = 1$
$c = -2$
$d = 0$

$x_1 + x_2 + x_3 = -b/a$

$x_1 + x_2 + x_3 = \frac{-1}{1}$ → -1 → Alternativa C

5- $\underset{3\times 2}{A} = \begin{bmatrix} -1 & -4 \\ 1 & -2 \\ 3 & 0 \end{bmatrix}$ → $a_{ij} = 2i - 3j$

$a_{11} = (2\cdot 1) - (3\cdot 1) = -1$

$a_{12} = (2\cdot 1) - (3\cdot 2) = -4$

$a_{21} = (2\cdot 2) - (3\cdot 1) = 1$

$a_{22} = (2\cdot 2) - (3\cdot 2) = -2$

$a_{31} = (2\cdot 3) - (3\cdot 1) = 3$

$a_{32} = (2\cdot 3) - (3\cdot 2) = 0$

$\underset{2\times 3}{B} = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ → $b_{jk} = k - j$

$b_{11} = 1 - 1 = 0$	$b_{12} = 2 - 1 = 1$	$b_{13} = 3 - 1 = 2$
$b_{21} = 1 - 2 = -1$	$b_{22} = 2 - 2 = 0$	$b_{23} = 3 - 2 = 1$

5- continuação $\left\{ \underset{3\times 2}{A} = \begin{bmatrix} -1 & -4 \\ 1 & -2 \\ 3 & 0 \end{bmatrix} \cdot \underset{2\times 3}{B} = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \right.$

$$A.B = \begin{bmatrix} 0+4 & -1+0 & -2-4 \\ 0+2 & 1-0 & 2-2 \\ 0+0 & 3+0 & 6+0 \end{bmatrix} \Rightarrow A.B = \begin{bmatrix} 4 & -1 & -6 \\ 2 & 1 & 0 \\ 0 & 3 & 6 \end{bmatrix}$$

$$A.B = \begin{vmatrix} 4 & -1 & -6 & 4 & -1 \\ 2 & 1 & 0 & 2 & 1 \\ 0 & 3 & 6 & 0 & 3 \end{vmatrix}$$

D. Principal: $4 \cdot 1 \cdot 6 = 24$
Paralela 1ª: $(-1) \cdot 0 \cdot 0 = 0$
Paralela 2ª: $(-6) \cdot 2 \cdot 3 = -36$
} -12

D. Secundária: $0 \cdot 1 \cdot (-6) = 0$
Paralela 1ª: $3 \cdot 0 \cdot 4 = 0$
Paralela 2ª: $6 \cdot 2 \cdot (-1) = -12$
} -12

$\det AB = -12 - (-12)$
$\det A.B = -12 + 12$
$\det A.B = 0$
↳ Alternativa (C)

6 = $\underset{2\times 3}{A} = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$ $\quad \underset{3\times 2}{B} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}$ } $A \cdot B = ?$

$$A.B = \begin{bmatrix} 2-0-0 & -2+0-2 \\ -1-1+0 & +1+1+0 \end{bmatrix} = AB = \begin{bmatrix} 2 & -4 \\ -2 & +2 \end{bmatrix}$$

$$A.B = \begin{vmatrix} 2 & -4 \\ -2 & +2 \end{vmatrix}$$

- D. Principal $= 2 \cdot 2 = 4$
- D. Secundária $= (-2) \cdot (-4) = 8$

Det $AB = 4 - (+8)$ → Det $AB = 4 - 8$ → Det $A.B = -4$
↳ Alternativa D