

Giovanna Santana Pennisi - CTII 350

Lista de Exercícios - Aula 8

Lista de Exercícios

1- $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$ é inversa de $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} \rightarrow x+y=?$

$A \cdot B^{-1} = I_n$ $\rightarrow \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} \cdot \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\left\{ \begin{array}{l} 3x - 5 = 1 \text{ (I)} \\ xy + 10 = 0 \text{ (II)} \end{array} \right.$

$B \cdot A = I_n$

<p>(I) $3x - 5 = 1$ $3x = 1 + 5$ $x = \frac{6}{3} \rightarrow x = 2$</p>	<p>(II) $xy + 10 = 0$ $2 \cdot y = -10$ $y = \frac{-10}{2} = y = -5$</p>	<p>$x + y$ $2 + (-5)$ -3 Alternativa C</p>
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2- $A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix} \rightarrow$ valores de k para que a matriz não admita inversa ($\det = 0$)

$\det A = \begin{vmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{vmatrix} = 1(3 - k) - 0(3 - k) + 1(3k - 1) = 3 - k + 3k - 1 = 2 + 2k$

$2 + 2k = 0 \rightarrow k = -1$

$\Delta = (-3)^2 - 4(1)(2) = 9 - 8 = 1$

$k = \frac{-(-3) \pm \sqrt{1}}{2 \cdot 1} = \frac{3 \pm 1}{2}$

$k_1 = \frac{3+1}{2} = 2$ $k_2 = \frac{3-1}{2} = 1$

Valores de $k = 1$ e $2 \rightarrow$ Alternativa C

$$3-A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \quad \text{e} \quad B = A^{-1} \quad \begin{array}{l} \downarrow \\ \text{* Regra das matrizes} \\ \text{de ordem 2} \end{array}$$

$$\det A = 12 - 10$$

$$\det A = 2_{\neq 0}$$

$$B = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \div 2_{\neq}$$

↓

$$\text{Alternativa C}_{\neq} \rightarrow B = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}_{\neq}$$

4. $\det \neq 0 \rightarrow$ matriz inversível

$$90 + 2x + 3x \rightarrow 20 + 5x$$

$$\det = \begin{vmatrix} x & 1 & 2 & x & 1 \\ 3 & 1 & 2 & 3 & 1 \\ 10 & 1 & x & 10 & 1 \end{vmatrix}$$

$$x^2 + 20 + 6 \rightarrow x^2 + 26$$

$$\left. \begin{array}{l} x^2 + 26 - (20 + 5x) \\ x^2 + 26 - 20 - 5x \\ x^2 + 6 - 5x \end{array} \right\} \begin{array}{l} x^2 + 26 - 20 - 5x \\ x^2 + 6 - 5x \\ \checkmark \\ x^2 - 5x + 6 \neq 0_{\neq} \end{array}$$

spi

$$x^2 - 5x + 6 \neq 0$$

$$\begin{array}{l|l} a=1 & \Delta = (-5)^2 - 4(1)(6) \\ b=-5 & \Delta = 25 - 24 \\ c=6 & \Delta = 1 \end{array}$$

$$x \neq \frac{-(-5) \pm \sqrt{1}}{2 \cdot 1}$$

$$x \neq \frac{5 \pm 1}{2}$$

$$x_1 \neq \frac{5+1}{2} \rightarrow \frac{6}{2} \rightarrow x_1 \neq 3_{\neq} \quad \bigg| \quad x_{11} \neq \frac{5-1}{2} \rightarrow \frac{4}{2} \rightarrow x_{11} \neq 2_{\neq}$$

$$\text{Alternativa A} \rightarrow \{x \neq 3 \text{ e } x \neq 2\}.$$

$$5-A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \rightarrow A + A^{-1} = ?$$

$$\det A = \begin{vmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix} \xrightarrow{2+2+2=6} \det A = 7 - (6) = 1$$

$$A = \begin{bmatrix} -1_{11} & -1_{12} & 2_{13} \\ 2_{21} & 1_{22} & -2_{23} \\ 1_{31} & 1_{32} & -1_{33} \end{bmatrix} \rightarrow A' = \begin{bmatrix} (-1-(-2)) & (-2-(-2)) & (2-1) \\ (1-2) & (1-2) & (-1-(-1)) \\ (2-2) & (2-1) & (-1-(-2)) \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \rightarrow \bar{A} = (A')^t = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\bar{A}}{1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$A + A^{-1}$
→ próxima
página

$$A + A^{-1} = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix} \rightarrow \text{Alternativa B}$$

$$6 - (X \cdot A)^t = B$$

$$((X \cdot A)^t)^t = B^t$$

$$X \cdot A = B^t$$

$$X \cdot A \cdot A^{-1} = B^t \cdot A^{-1}$$

$$X = B^t \cdot A^{-1}$$

Alternativa

(B)

A matriz transposta de uma matriz transposta é a matriz normal

$$7 - B = \begin{bmatrix} x \\ y \end{bmatrix} \text{ e } C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix} \rightarrow A^{-1} = ?$$

$$AB = C$$

$$A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix} \rightarrow A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} \rightarrow \det A = 24 - 25$$

$$\det A = -1$$

Regra de
matriz
de ordem 2

$$A^{-1} = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \div -1$$

$$A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & 4 \end{bmatrix} \rightarrow \text{Alternativa (D)}$$

$$8 - A = \begin{bmatrix} 2 & k \\ -2 & 1 \end{bmatrix}$$

→ soma dos valores de $k = ?$

$$\rightarrow \det A = \det A^{-1}$$

$$\det A = 2 - (-2k)$$

$$\det A = 2 + 2k$$

$$\det A \cdot \det A^{-1} = 1$$

$$(2 + 2k) \cdot (2 + 2k) = 1$$

$$4 + 4k + 4k + 4k^2 = 1$$

$$4k^2 + 8k + 4 - 1 = 0$$

$$4k^2 + 8k + 3 = 0$$

$$4k^2 + 8k + 3 = 0$$

$$a=4 \quad \Delta = 8^2 - 4 \cdot 4 \cdot 3$$

$$b=8 \quad \Delta = 64 - 48$$

$$c=3 \quad \Delta = 16$$

$$K = \frac{-(8) \pm \sqrt{16}}{2 \cdot 4}$$

$$K = \frac{-8 \pm 4}{8}$$

$$K_1 = \frac{-8+4}{8} = \frac{-4}{8} = \frac{-1}{2} \quad K_{II} = \frac{-8-4}{8} = \frac{-12}{8} = \frac{-3}{2}$$

$$\text{Soma dos valores de } K = \frac{-1}{2} + \frac{-3}{2} = \frac{-4}{2} = -2 \rightarrow \text{Alternative B}$$

9- A e B \rightarrow são matrizes de ordem 2, com determinantes não nulos ($\det(A) \neq 0$ e $\det(B) \neq 0$).

$$a) (A+B) \cdot (A-B)$$

$$\rightarrow A^2 - AB + BA - B^2$$

+ Importante: AB e BA, no geral, NÃO são a mesma coisa

$$b) (A+B)^2 = A^2 + 2AB + B^2$$

\rightarrow Para isso ser possível AB tem que ser igual a BA ($AB = BA$)

$$c) \frac{\det A}{\det(-A)}$$

$$\rightarrow \det(-A) = (-1)^2 \cdot \det A = \det A$$

$$\frac{\det A}{\det(-A)} = 1$$

$$d) B = A^{-1}$$

$$\rightarrow \det A \cdot \det B = 1$$

$$\det B = \frac{1}{\det A}$$