

Lista de Exercícios - Aula 8

$$A = B^{-1} \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} \text{ é inversa de } B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} \rightarrow x+y = ?$$

$$A \cdot \bar{A}^1 = I_n \quad \left[\begin{matrix} 3 & -5 \\ y & 2 \end{matrix} \right] \cdot \left[\begin{matrix} x & 1 \\ 5 & 3 \end{matrix} \right] = \left[\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right] \quad \left. \begin{array}{l} 3x - 5 = 1 \\ xy + 10 = 0 \end{array} \right\} \quad \text{⑤}$$

$\text{I} \quad 3x - 5 = 1$ $3x = 1 + 5$ $x = \frac{6}{3} \rightarrow x = 2$ $\{ \begin{array}{l} 3 \\ -5 \\ \hline 1 \end{array}$	$\text{II} \quad xy + 10 = 0$ $2 \cdot y = -10$ $y = \frac{-10}{2} = y = -5$ $\{ \begin{array}{l} 2 \\ -10 \\ \hline -5 \end{array}$	$x + y$ $2 + (-5)$ -3 Alternative C
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2- $A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix}$ + valores de k para que a matriz não Admita inversa ($\det = 0$).

$$\det A = \begin{vmatrix} 1+3k & 0 & 1 & 1 & 0 \\ 1 & 3 & k & 1 & \\ 1 & k & 3 & 1 & k \end{vmatrix} + 3+k^2 - (1+3k)$$

$$\left. \begin{array}{l} a=1 \\ b=-3 \\ c=2 \end{array} \right\} \Delta = (-3)^2 - 4(1)(2) \quad \left| \quad k = \frac{-(-3) \pm \sqrt{3}}{2 \cdot 1} \rightarrow k = \frac{3 \pm 1}{2} \right.$$

$$\Delta = 9 - 8$$

$$\Delta = 1$$

$$k_1 = \frac{3+1}{2} + \frac{4}{2} \rightarrow k_1 = 2 \quad \left| \quad k_{11} = \frac{3-1}{2} + \frac{2}{2} \rightarrow k_{11} = 1 \right.$$

Valores de $k = 1$ e $2 \rightarrow$ Alternativa C

$$3 - A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \quad e \quad B = A^{-1} \quad * \text{Regra das matrizes de ordem 2}$$

$$\det A = 12 - 10$$

$$\det A = 2$$

$$B = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \div 2$$

↓

$$\text{Alternativa C} \rightarrow B = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$$

4. $\det \neq 0 \rightarrow \text{matriz inversível}$

$$\det = \begin{vmatrix} x & 1 & 2 & | & x \\ 3 & | & 2 & 3 & | \\ 50 & | & x & 50 & | \end{vmatrix}$$

$$x^2 + 20 + 6 \rightarrow x^2 + 26$$

$$x^2 + 26 - (20 + 5x)$$

$$x^2 + 26 - 20 - 5x$$

$$x^2 + 6 - 5x$$

$$x^2 - 5x + 6 \neq 0$$

spil

$$x^2 - 5x + 6 \neq 0$$

$$\begin{cases} a=1 \\ b=-5 \\ c=6 \end{cases}$$

$$x \neq \frac{-(-5) \pm \sqrt{25}}{2 \cdot 1}$$

$$x \neq \frac{5 \pm 1}{2}$$

$$x_1 \neq \frac{5+1}{2} \rightarrow \frac{6}{2} \rightarrow x_1 \neq 3 \quad | \quad x_{11} \neq \frac{5-1}{2} \rightarrow \frac{4}{2} \rightarrow x_{11} \neq 2$$

Alternativa A $\rightarrow \{x \neq 3 \text{ e } x \neq 2\}$.

$$5 - A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \rightarrow A + A^{-1} = ?$$

$$\det A = \begin{vmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix} \quad \text{2+2+2=6} \quad \left\{ \begin{array}{l} \det A = 7 - (6) \\ \det A = 1 \end{array} \right.$$

$1+2+4=7$

$$A = \begin{bmatrix} -1_{11} & -1_{12} & 2_{13} \\ 2_{21} & 1_{22} & -2_{23} \\ 1_{31} & 1_{32} & -1_{33} \end{bmatrix} \rightarrow A' = \begin{bmatrix} (-3 - (-2)) & (-2 - (-2)) & (2 - 1) \\ (1 - 2) & (1 - 2) & (-1 - (-1)) \\ (2 - 2) & (2 - 4) & (-3 - (-2)) \end{bmatrix}$$

X+y+impar
bSi ha
muda

$$A' = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \rightarrow \bar{A} = (A')^t = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\bar{A}^{-1} = \frac{\bar{A}}{5} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} \text{A+A}^{-1} \\ \text{próxima} \\ \text{pagina} \end{array}$$

$$A + A^{-1} = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

A

$$\begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix} \rightarrow \text{Alternativa B}$$

B

6- $(X \cdot A)^t = B$

 $((X \cdot A)^t)^t = B^t$
 $X \cdot A = B^t$
 $X \cdot A \cdot A^{-1} = B^t \cdot A^{-1}$
 $\checkmark X = B^t \cdot A^{-1}$

A matriz transposta de uma matriz
transposta é a matriz normal

Alternativa (B)

7- $B = \begin{bmatrix} x \\ y \end{bmatrix}$ e $C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix} \rightarrow A^{-1} = ?$
 $AB = C$

$A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix} \rightarrow A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} \rightarrow \det A = 24 - 25$
 $\det A = -1$

Regras da matriz de ordem 2

$$\left\{ \begin{array}{l} A^{-1} = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \div -1 \\ A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & 4 \end{bmatrix} \end{array} \right.$$

Alternativa (D)

8- $A = \begin{pmatrix} 2 & k \\ 2 & 1 \end{pmatrix} \rightarrow$ soma dos valores de $k = ?$

 $\rightarrow \det A = \det A^{-1}$

$\det A = 2 - (-2k) \quad \left\{ \begin{array}{l} \det A \cdot \det A^{-1} = 1 \\ (2+2k) \cdot (2+k) = 1 \end{array} \right. \rightarrow 4k^2 + 8k + 3 = 0$

 $\det A = 2 + 2k \quad \left\{ \begin{array}{l} 4 + 4k + 4k + 4k^2 = 1 \\ 4k^2 + 8k + 4 - 3 = 0 \end{array} \right.$

(3)

$$4k^2 + 8k + 3 = 0 \rightarrow K = \frac{-8 \pm \sqrt{64}}{2 \cdot 4}$$

$$\begin{array}{l|l} a=4 & \Delta = 8^2 - 4 \cdot 4 \cdot 3 \\ b=8 & \Delta = 64 - 48 \\ c=3 & \Delta = 16 \end{array} \quad K = \frac{-8 \pm 4}{2 \cdot 4}$$

$$K_1 = \frac{-8+4}{8} = \frac{-4}{8} = \frac{1}{2} \quad K_{11} = \frac{-8-4}{8} = \frac{-12}{8} = \frac{-3}{2}$$

$$K_2 = \frac{-8+4}{8} = \frac{-4}{8} = \frac{1}{2} \quad K_{11} = \frac{-8-4}{8} = \frac{-12}{8} = \frac{-3}{2}$$

$$\text{Soma dos valores de } K = \frac{-1}{2} + \frac{-3}{2} + \frac{-1-3}{2} = \frac{-4}{2} \rightarrow \boxed{-2}$$

Alternativa B

q) A e B \rightarrow São matrizes de ordem 2 com determinantes não nulos ($\det(A) \neq 0$ e $\det(B) \neq 0$).

a) $(A+B) \cdot (A+B)$.

$$(A+B) \cdot (A+B) = A^2 - AB + BA - B^2$$

+ Importante: $AB \neq BA$, no geral,
NÃO São a mesma coisa

b) $(A+B)^2 = A^2 + 2AB + B^2$
 \rightarrow Para isso ser possível AB tem que ser igual a BA ($AB = BA$)

c) $\det A$
 $\det(-A) \rightarrow \det(-A) = (-1)^2 \cdot \det A = \det A$

$$\frac{\det A}{\det(-A)} = \frac{1}{1}$$

d) $B = A^{-1}$
 $\rightarrow \det A \cdot \det B = 1$

$$\det B = \frac{1}{\det A}$$