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Lista de Exercícios - Matrizes

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Lista de Exercícios - Matrizes

1-

$$A = \begin{bmatrix} 5 & 8 \\ 7 & 10 \\ 9 & 12 \end{bmatrix} \rightarrow a_{ij} = 2i + 3j$$

$a_{11} = 2 \cdot 1 + 3 \cdot 1$	$a_{12} = 2 \cdot 1 + 3 \cdot 2$	$a_{21} = 2 \cdot 2 + 3 \cdot 1$	$a_{22} = 2 \cdot 2 + 3 \cdot 2$	$a_{31} = 2 \cdot 3 + 3 \cdot 1$	$a_{32} = 2 \cdot 3 + 3 \cdot 2$
$a_{11} = 5$	$a_{12} = 8$	$a_{21} = 7$	$a_{22} = 10$	$a_{31} = 9$	$a_{32} = 12$

2- $$A = \begin{bmatrix} 5 & 17 \\ 8 & 20 \end{bmatrix}_{2 \times 2} \rightarrow a_{ij} = i + 4j^2$$ Alternativa A

$a_{11} = 1^2 + 4 \cdot 1^2$	$a_{12} = 1^2 + 4 \cdot 2^2$	$a_{21} = 2^2 + 4 \cdot 1^2$	$a_{22} = 2^2 + 4 \cdot 2^2$
$a_{11} = 1 + 4 \cdot 1$	$a_{12} = 1 + 4 \cdot 4$	$a_{21} = 4 + 4 \cdot 1$	$a_{22} = 4 + 4 \cdot 4$
$a_{11} = 5$	$a_{12} = 17$	$a_{21} = 8$	$a_{22} = 20$

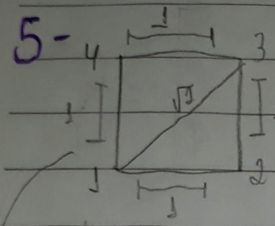
3- $\begin{bmatrix} 1 & x+2 \\ y-1 & z+1 \end{bmatrix} = \begin{bmatrix} 1 & -x \\ 2y & -2z \end{bmatrix}$

$x + 2 = -x$	$y-1 = 2y$	$z + 1 = -2z$
$x + x = -2$	$2y - y = -1$	$z + 2z = -1$
$x = \frac{-2}{2}$		$z = \frac{-1}{3}$
$x = -1$	$y = -1$	

4- $\begin{bmatrix} 3 & -x \\ 3x & x \end{bmatrix} = \begin{bmatrix} 3 & y \\ 2x+1 & z-1 \end{bmatrix}$

$3x = 2x + 1$	$-x = y$	$x = z - 1$
$3x - 2x = 1$	$y = -(1)$	$1 = z - 1$
		$z = 1 + 1$
$x = 1$	$y = -1$	$z = 2$

5-



$1 \rightarrow l = 1$

diagonal: $d^2 = l^2 + l^2$

$d^2 = 2l^2$

$\sqrt{d^2} = \sqrt{2 \cdot 1^2}$

$d = \sqrt{2}$

A = Matriz de ordem 4

Distâncias entre os vértices de número i e j:

$a_{11} = 1$ a 1	$a_{12} = 1$ a 2	$a_{13} = 1$ a 3	$a_{14} = 1$ a 4	$a_{21} = 2$ a 1	$a_{22} = 2$ a 2
$a_{11} = 0$	$a_{12} = 1$	$a_{13} = \sqrt{2}$	$a_{14} = 1$	$a_{21} = 1$	$a_{22} = 0$

$a_{23} = 2$ a 3	$a_{24} = 2$ a 4	$a_{31} = 3$ a 1	$a_{32} = 3$ a 2	$a_{33} = 3$ a 3
$a_{23} = 1$	$a_{24} = \sqrt{2}$	$a_{31} = \sqrt{2}$	$a_{32} = 1$	$a_{33} = 0$

$a_{34} = 3$ a 4	$a_{41} = 4$ a 1	$a_{42} = 4$ a 2	$a_{43} = 4$ a 3	$a_{44} = 4$ a 4
$a_{34} = 1$	$a_{41} = 1$	$a_{42} = \sqrt{2}$	$a_{43} = 1$	$a_{44} = 0$

Ficando = $A = \begin{bmatrix} 0 & 1 & \sqrt{2} & 1 \\ 1 & 0 & 1 & \sqrt{2} \\ \sqrt{2} & 1 & 0 & 1 \\ 1 & \sqrt{2} & 1 & 0 \end{bmatrix}$ Alternativa B

6 - $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ e $B = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \rightarrow 2A - B = ?$

$1^{\underline{o}} = 2 \cdot \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 6 \end{bmatrix}$

$2^{\underline{o}} = \begin{bmatrix} -2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} -2 \\ +6 \\ 5 \end{bmatrix} \rightarrow$ Alternativa D

7 - $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ e $B = \begin{bmatrix} -1 & 3 & 2 \\ 2 & 0 & 1 \end{bmatrix} \rightarrow A - B^t = ?$

$1^{\underline{o}} = B^t = B = \begin{bmatrix} -1 & 3 & 2 \\ 2 & 0 & 1 \end{bmatrix} \Rightarrow B = \begin{bmatrix} -1 & 2 \\ 3 & 0 \\ 2 & 1 \end{bmatrix}$

$2^{\underline{o}} = A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} - B = \begin{bmatrix} -1 & 2 \\ 3 & 0 \\ 2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}$

$\rightarrow$ Alternativa B

8- $A = \begin{bmatrix} 2 & -1 & 2y \\ x & 0 & -z \\ 4 & 3 & 2 \end{bmatrix} \rightarrow A = A^t$

1º $A^t = \begin{bmatrix} 2 & x & 4 \\ -1 & 0 & 3 \\ 2y & 3 & 2 \end{bmatrix} = A = \begin{bmatrix} 2 & -1 & 2y \\ x & 0 & -z \\ 4 & 3 & 2 \end{bmatrix}$

$x = -1$ | $2y = 4$, $y = 4/2$, $y = 2$ | $-z = 3 \; (-1)$, $z = -3$ | $x + y + z$, $-1 + 2 + (-3)$, $+1 - 3$

Alternativa A $\rightarrow -2$

9- $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \\ 4 & 5 \end{bmatrix}$ | $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$

$a_{ij} = i + j \; (i \neq j) \,/\, a_{ij} = 1 \; (i = j)$ | $b_{ij} = 0 \; (i \neq j) \,/\, b_{ij} = 2i - j \; (i = j)$

$a_{11} = 1$ | $a_{22} = 1$ | $b_{11} = (2 \cdot 1) - 1 = 1$ | $b_{22} = (2 \cdot 2) - 2 = 2$

$a_{12} = 1 + 2 = 3$ | $a_{31} = 3 + 1 = 4$ | $b_{12} = 0$ | $b_{31} = 0$

$a_{21} = 2 + 1 = 3$ | $a_{32} = 3 + 2 = 5$ | $b_{21} = 0$ | $b_{32} = 0$

$A + B = A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \\ 4 & 5 \end{bmatrix} + B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 3 \\ 4 & 5 \end{bmatrix}$

Alternativa C

10- $M = \begin{bmatrix} x & 8 \\ 10 & y \end{bmatrix}$, $N = \begin{bmatrix} y & 6 \\ 12 & x+4 \end{bmatrix}$ e $P = \begin{bmatrix} 7 & 16 \\ 23 & 13 \end{bmatrix}$

$\frac{3}{2}M + \frac{2}{3}N = P$

$\frac{3}{2} \cdot \begin{bmatrix} x & 8 \\ 10 & y \end{bmatrix} + \frac{2}{3} \cdot \begin{bmatrix} y & 6 \\ 12 & x+4 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 23 & 13 \end{bmatrix}$

$\begin{bmatrix} \frac{3}{2}x & \frac{24}{2} \\ \frac{30}{2} & \frac{3}{2}y \end{bmatrix} + \begin{bmatrix} \frac{2}{3}y & \frac{12}{3} \\ \frac{24}{3} & \frac{2}{3}x + \frac{8}{3} \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 23 & 13 \end{bmatrix}$

$\frac{30}{2} \quad \frac{3}{2}y \quad \frac{24}{3} \quad \frac{2}{3}x + \frac{8}{3} = 23 \quad 13$

$\frac{3}{2}x + \frac{2}{3}y = 7$

$\frac{9x}{\cancel{6}} + \frac{4y}{\cancel{6}} = \frac{42}{\cancel{6}}$

I $9x + 4y = 7$

$\frac{3}{2}y + \frac{2}{3}x + \frac{8}{3} = 13$

$\frac{9y}{\cancel{6}} + \frac{4x}{\cancel{6}} + \frac{16}{\cancel{6}} = \frac{78}{\cancel{6}}$

$9y + 4x = 78 - 16$

II $9y + 4x = 62$

$\begin{cases} 9x + 4y = 42 \\ 9y + 4x = 62 \end{cases}$ Subtração

II $9y + 4x - \overset{\text{I}}{(9x + 4y)} = 62 - 42$

$9y + 4x - 9x - 4y = 20$

$5y - 5x = 20$

$5(y - x) = 20$

$y - x = 20/5$

$y - x \rightarrow$

$y - x = 4$ → Alternativa B