HPC Exam Project

Scaling Study of the Stencil Method

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October 19, 2025

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DI TRIESTE

Introduction

Goals

1. **Optimize** the stencil method for the 2d heat equation
2. **Parallelize** using hybrid approach
3. Perform **scalability** study:
   - 3.1 Thread scaling
   - 3.2 Strong scaling
   - 3.3 Weak scaling

Algorithm

Heat equation (2d)

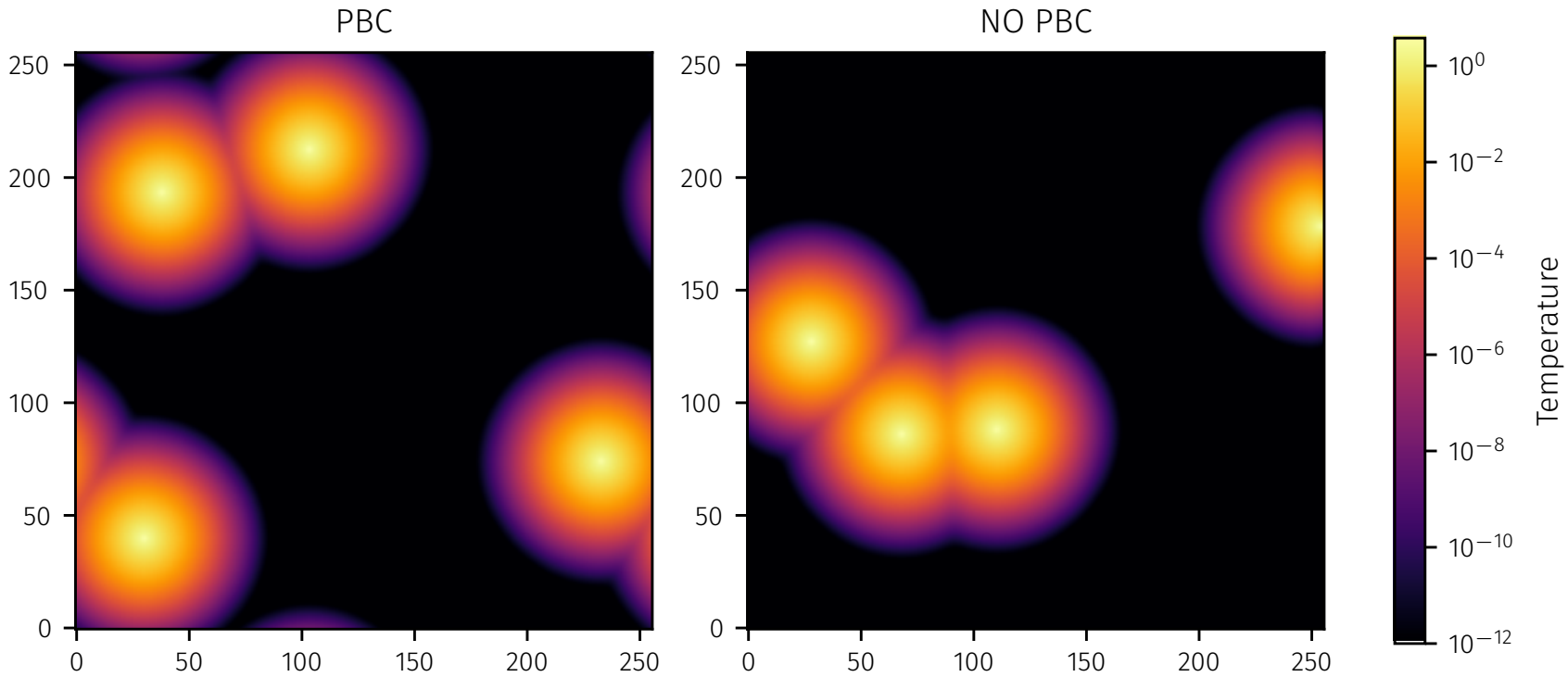
$$\partial_t u = \alpha(\partial_x^2 u + \partial_y^2 u)$$

Finite difference integration

$$u_{i,j}^{(t+1)} = (1 - 4\alpha)u_{i,j}^{(t)} + \alpha \sum_{\langle i,j \rangle} u_{i,j}^{(t)}$$

$$x \in [0, L_x] \to i \in \{1, \ldots, N_x - 1\}$$
$$y \in [0, L_y] \to j \in \{1, \ldots, N_y - 1\}$$

Code Correctness



Optimization

- Compiler flags:
  `-O3 -Wall -march=native`
- Preprocessor directive:
  `#pragma GCC unroll`

Parallelization: shared memory

Implementation

```
1      #pragma omp parallel for schedule(static)
2      for (uint j = 1; j <= ysize; j++){
3          for ( uint i = 1; i <= xsize; i++){
4
5              // update rule
6
7          }
8      }
```

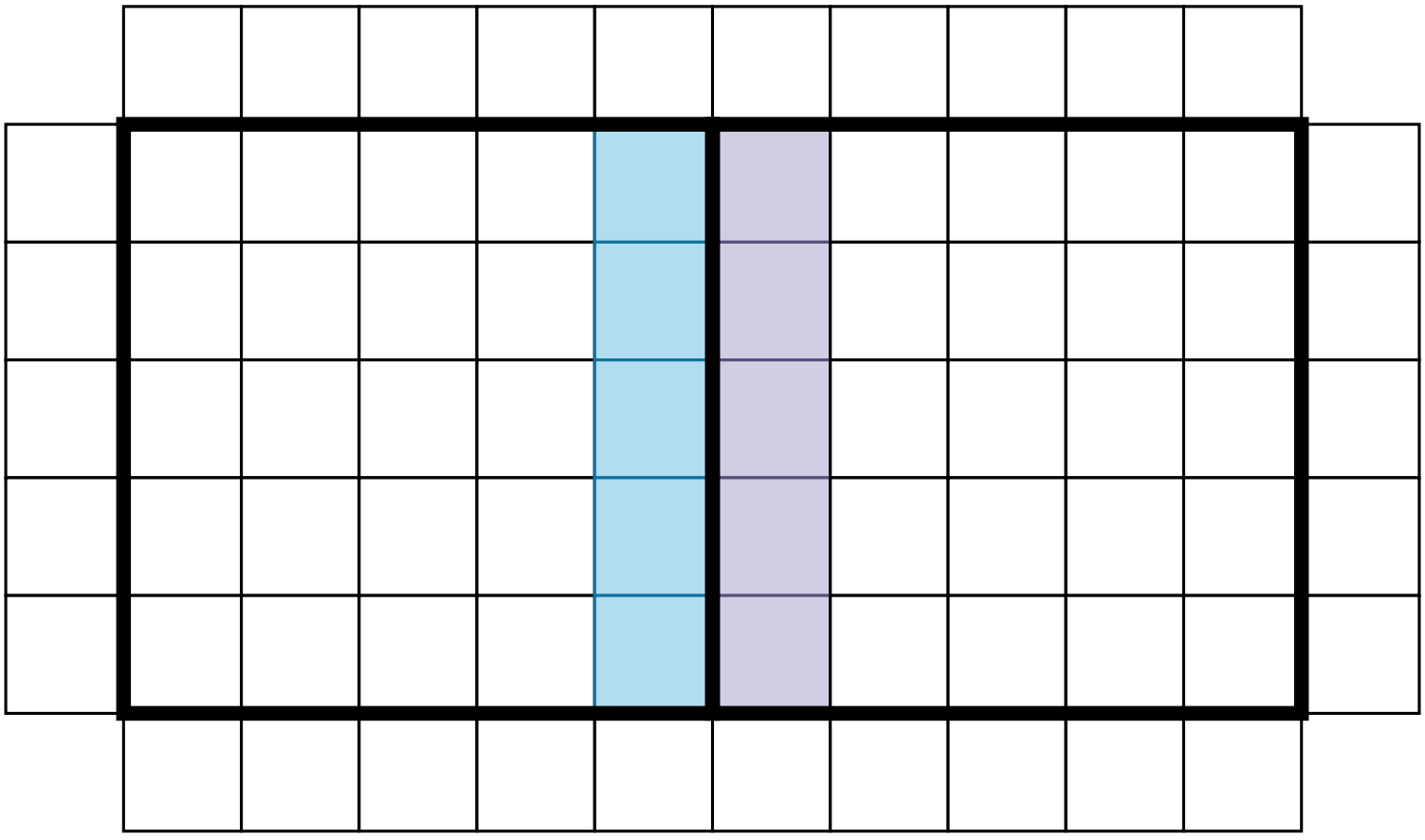
add omp parallel ove compute energy stats

Thread placement and affinity

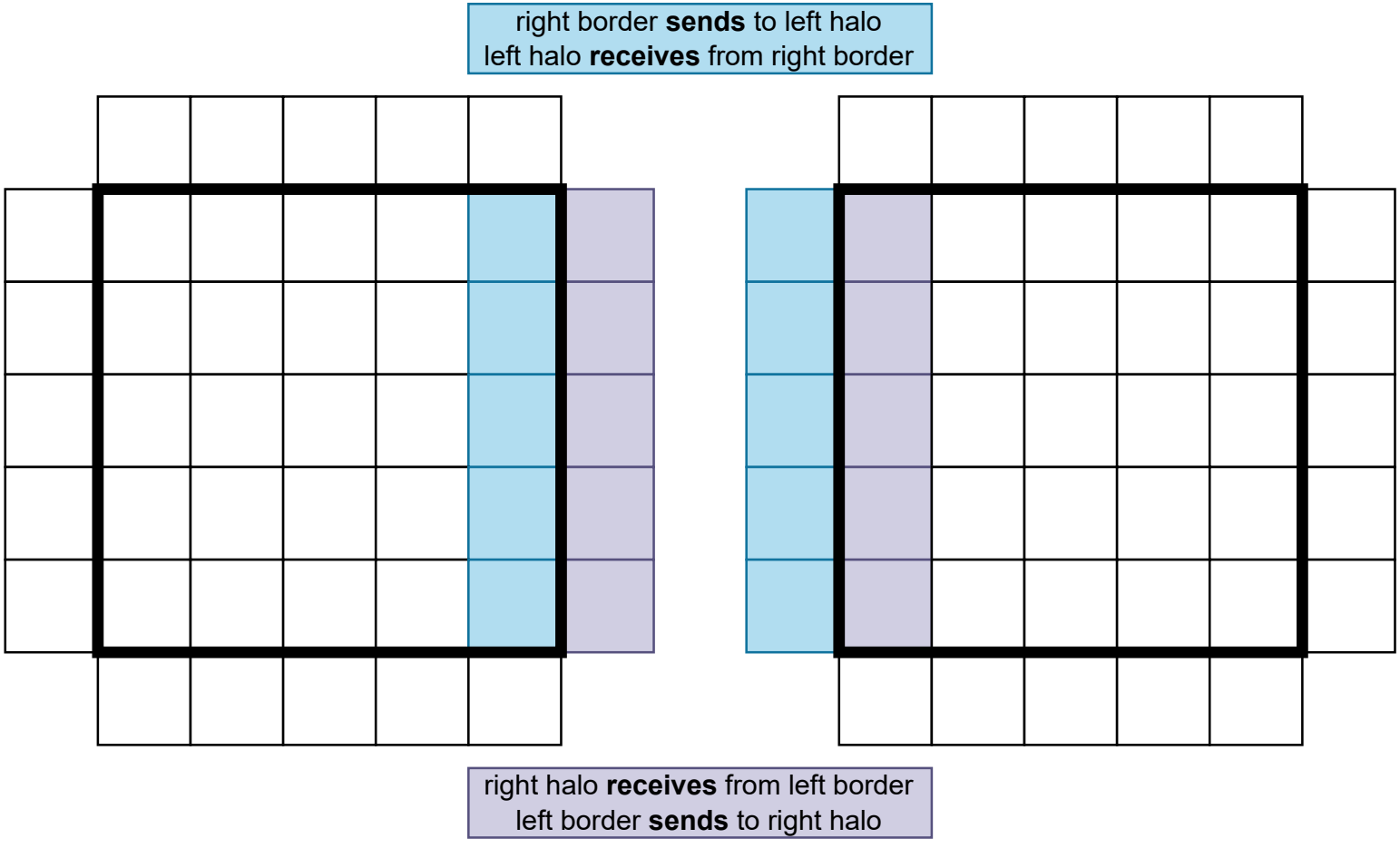
```
1      export OMP_PLACES=cores
2      export OMP_PROC_BIND=close
```

first touch policy

Parallelization: distributed memory



Parallelization: distributed memory



Parallelization: distributed memory

For each task:

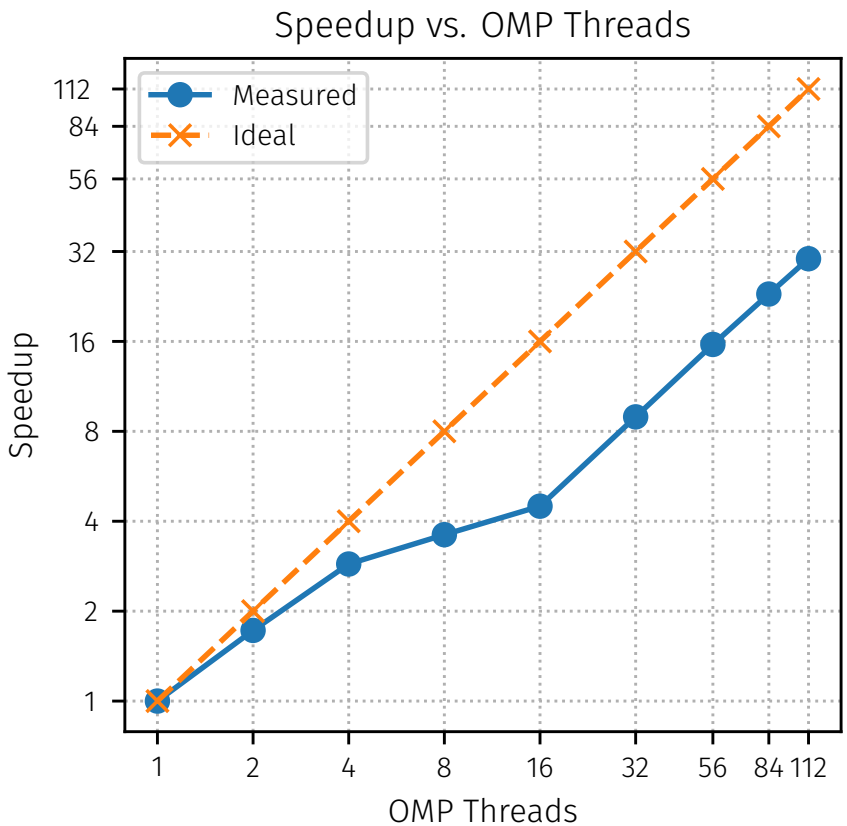
```
1     // pack buffers
2
3     MPI_Irecv(...);
4
5     MPI_Isend(...);
6
7     update_internal();
8
9     MPI_Waitall();
10
11    // unpack buffers
12
13    update_border();
```

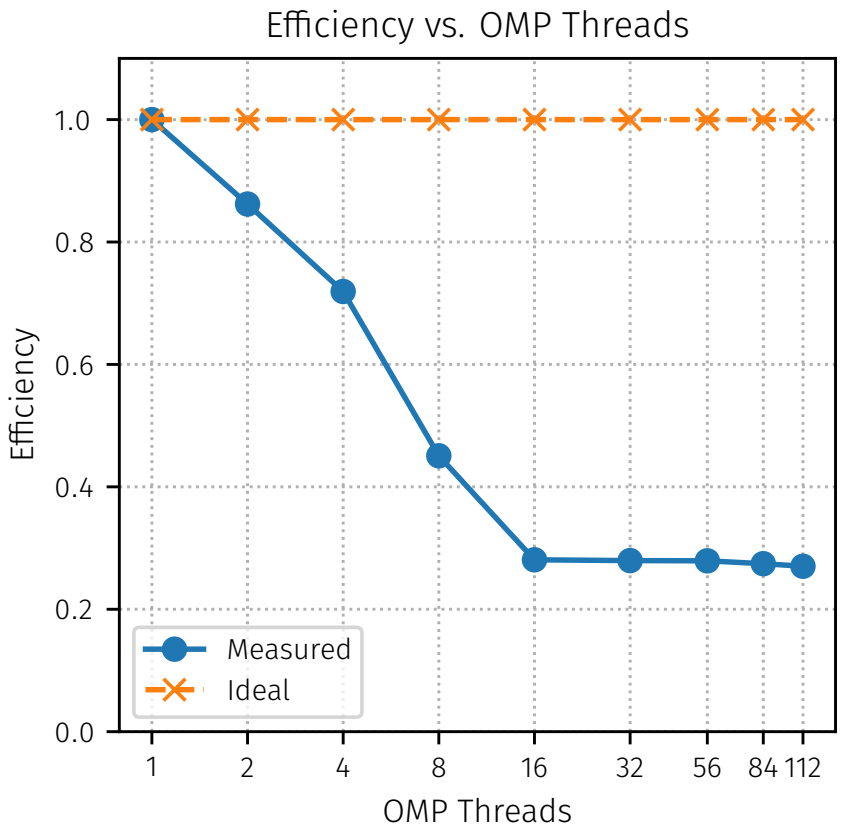
Results

Thread Scaling

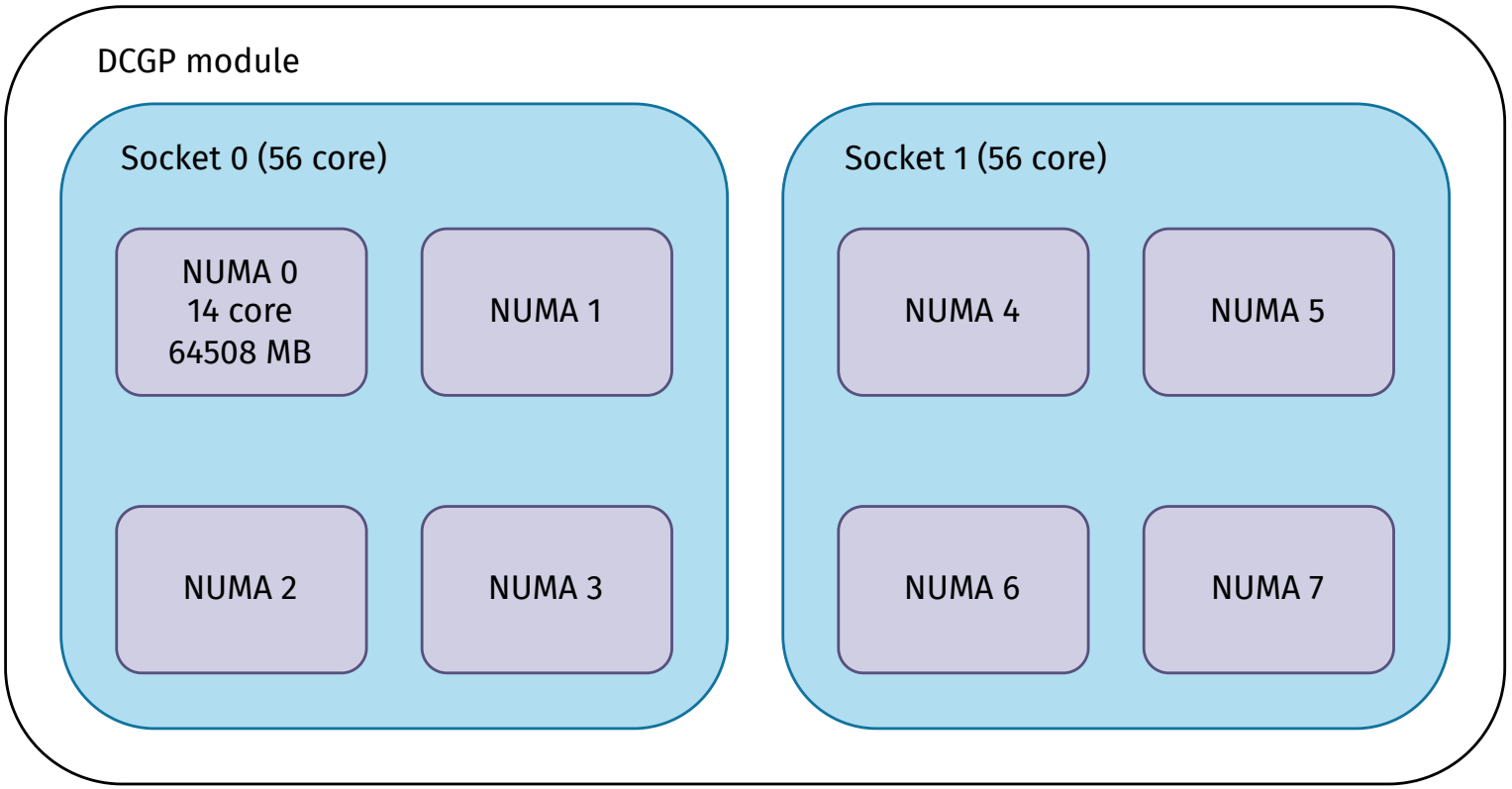
```
GRID_SIZE_X=16384
GRID_SIZE_Y=16384
N_STEPS=500

NODES=1
N_TASKS_PER_NODE=1

THREAD_LIST="1 2 4 8 16 32 56 84 112"
```

Thread Scaling





Node Architecture



Node Architecture: distance matrix

```
[glucarel@lrdn4293 HPC-leonardo]$ numactl --hardware
    available: 8 nodes (0-7)
      node   0   1   2   3   4   5   6   7
        0:  10  12  12  12  21  21  21  21
        1:  12  10  12  12  21  21  21  21
        2:  12  12  10  12  21  21  21  21
        3:  12  12  12  10  21  21  21  21
        4:  21  21  21  21  10  12  12  12
        5:  21  21  21  21  12  10  12  12
        6:  21  21  21  21  12  12  10  12
        7:  21  21  21  21  12  12  12  10
```

Node Architecture: memory

```
[glucarel@lrdn4293 HPC-leonardo]$ lscpu | egrep 'L1d|L1i|L2|L3'
    L1d cache:                   48K
    L1i cache:                   32K
    L2 cache:                    2048K
    L3 cache:                    107520K
```

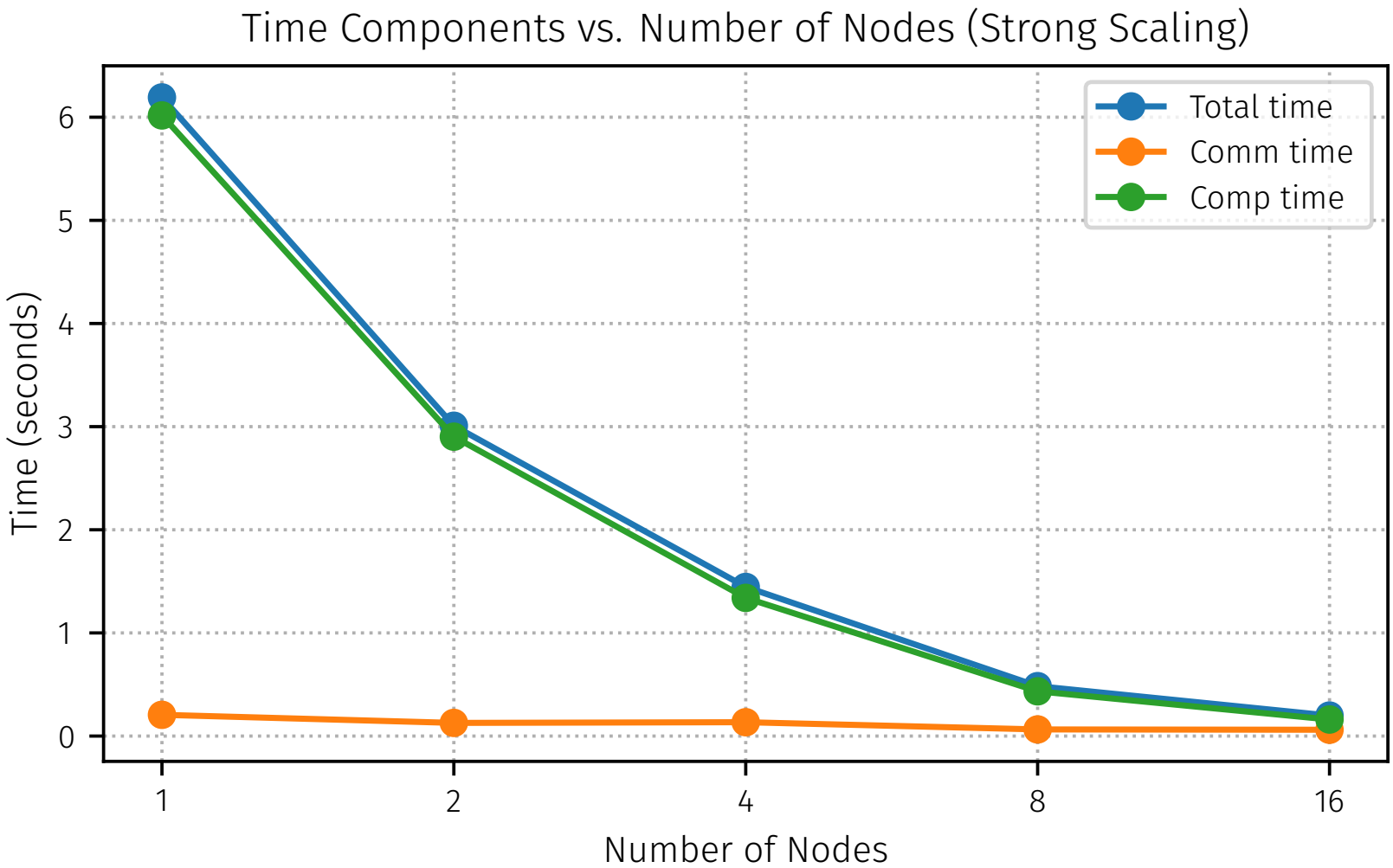
$$N = N_x = N_y = 2^{14}$$

$$\text{memory}_{grid} = 2 \times N^2 \times 8\text{B} \approx 4\text{GB}$$

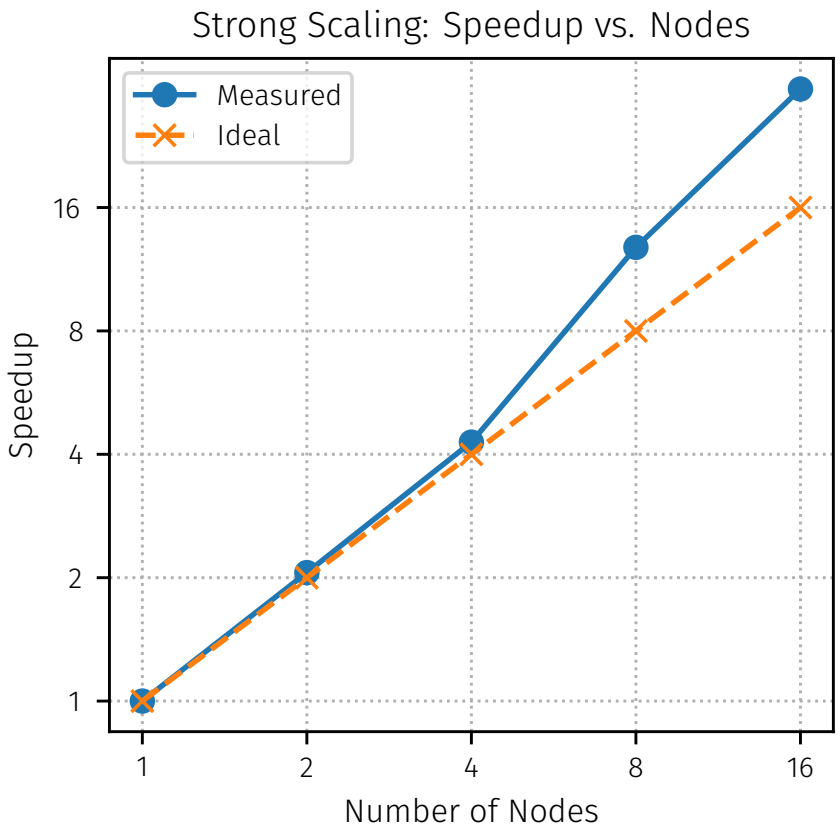
Strong Scaling

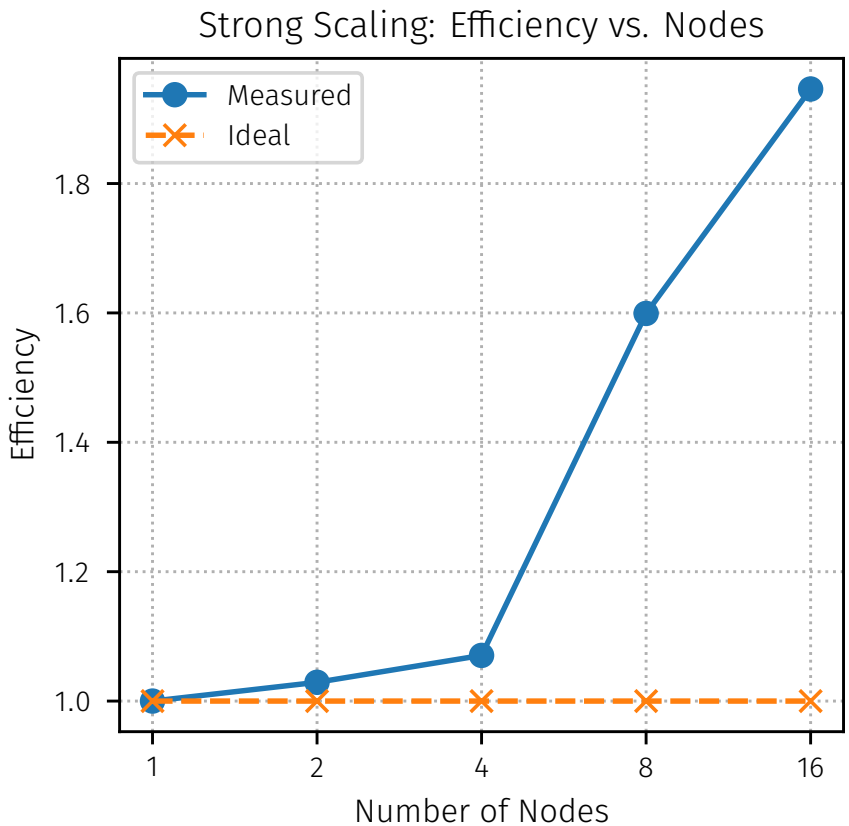
```
GRID_SIZE_X=16384
GRID_SIZE_Y=16384
N_STEPS=500

OMP_THREADS=14
N_TASKS_PER_NODE=8

NODE_LIST="1 2 4 8 16"
```

Strong Scaling



Strong Scaling

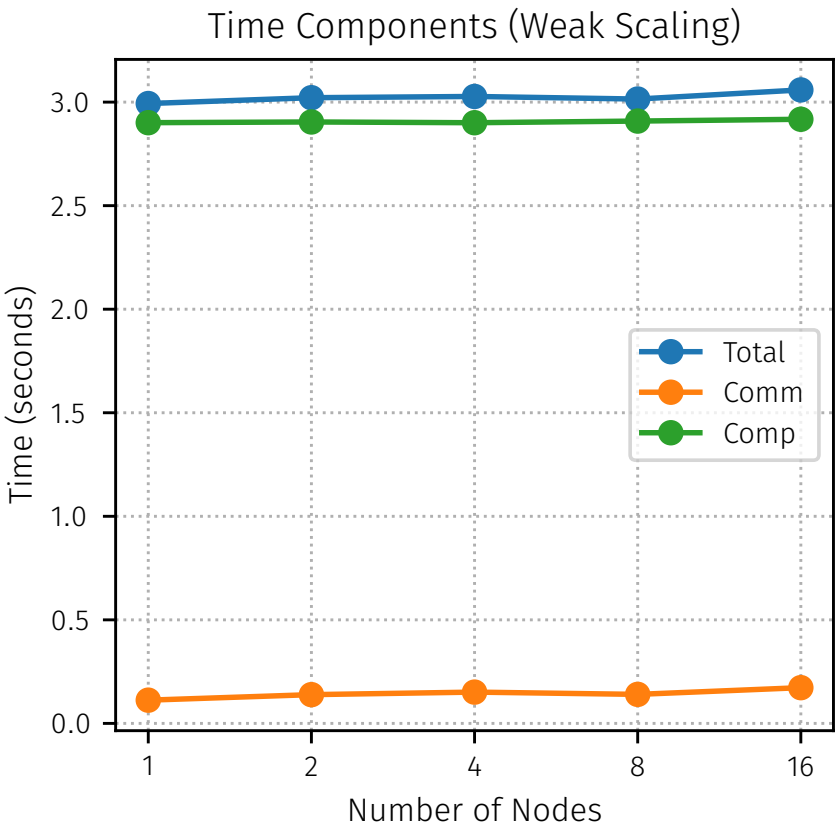


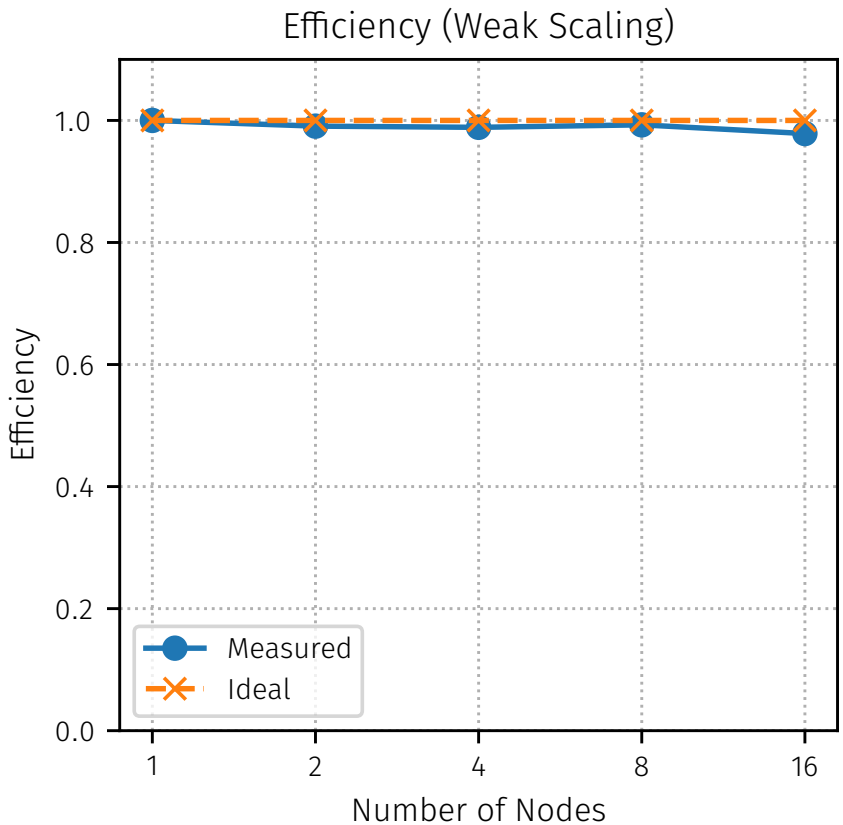


Weak Scaling

```
1   LOCAL_X=4096
2   LOCAL_Y=4096
3
4   OMP_THREADS=14
5   TASKS_PER_NODE=8
6
7   for NODES in "1 2 4 8 16"; do
8       TOTAL_TASKS=$(( NODES * TASKS_PER_NODE ))
9
10      case "${TOTAL_TASKS}" in
11      8)    PX=4;  PY=2  ;;   # 1 node  (8 ranks)
12      ...
13      esac
14
15      GRID_SIZE_X=$(( LOCAL_X * PX ))
16      GRID_SIZE_Y=$(( LOCAL_Y * PY ))
17      ...
18  done
```

Weak Scaling





Conclusion

- Stencil method is memory bound!

Thank You!