

HPC Exam Project

Scaling Study of the Stencil Method

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Introduction

Goals

1. **Optimize** the stencil method for the 2d heat equation
2. **Parallelize** using hybrid approach
3. Perform **scalability** study:
 - 3.1 Thread scaling
 - 3.2 Strong scaling
 - 3.3 Weak scaling

Heat equation (2d)

$$\partial_t u = \alpha(\partial_x^2 u + \partial_y^2 u)$$

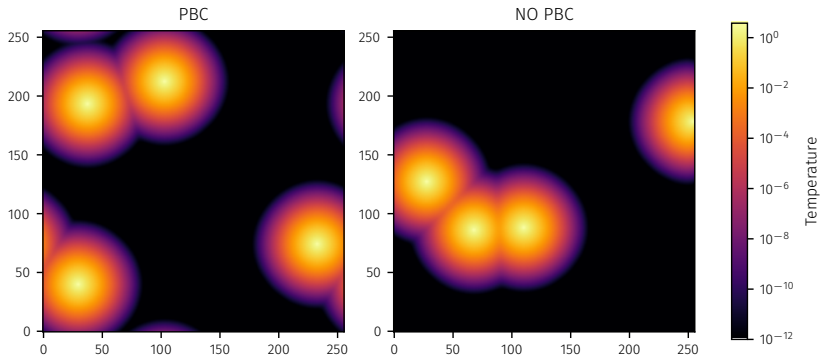
Finite difference integration

$$u_{i,j}^{(t+1)} = (1 - 4\alpha)u_{i,j}^{(t)} + \alpha \sum_{\langle i,j \rangle} u_{i,j}^{(t)}$$

$$x \in [0, L_x] \rightarrow i \in \{1, \dots, N_x - 1\}$$

$$y \in [0, L_y] \rightarrow j \in \{1, \dots, N_y - 1\}$$

Code Correctness



Optimization

- Compiler flags:
-O3 -Wall -march=native
- Preprocessor directive:
`#pragma GCC unroll`

Parallelization: shared memory

Implementation

```
1  #pragma omp parallel for schedule(static)
2  for (uint j = 1; j <= ysize; j++){
3      for ( uint i = 1; i <= xsize; i++){
4
5          // update rule
6
7      }
8  }
```

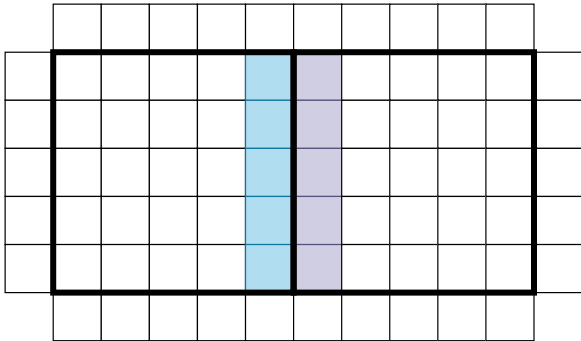
add omp parallel ove compute energy stats

Thread placement and affinity

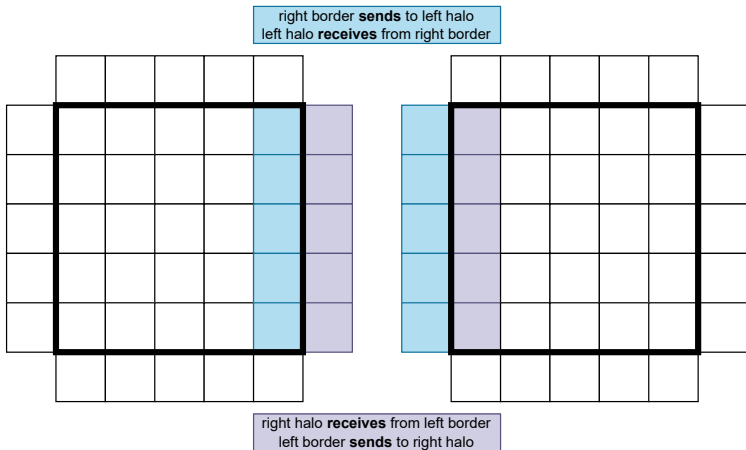
```
1  export OMP_PLACES=cores
2  export OMP_PROC_BIND=close
```

first touch policy

Parallelization: distributed memory



Parallelization: distributed memory



Parallelization: distributed memory

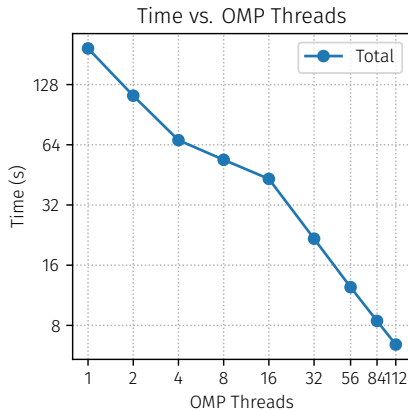
For each task:

```
1  // pack buffers
2
3  MPI_Irecv(...);
4
5  MPI_Isend(...);
6
7  update_internal();
8
9  MPI_Waitall();
10
11 // unpack buffers
12
13 update_border();
```

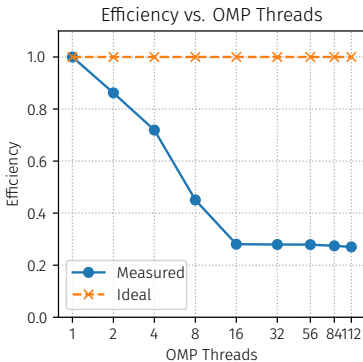
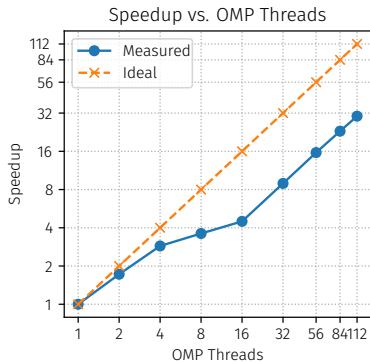
Results

Thread Scaling

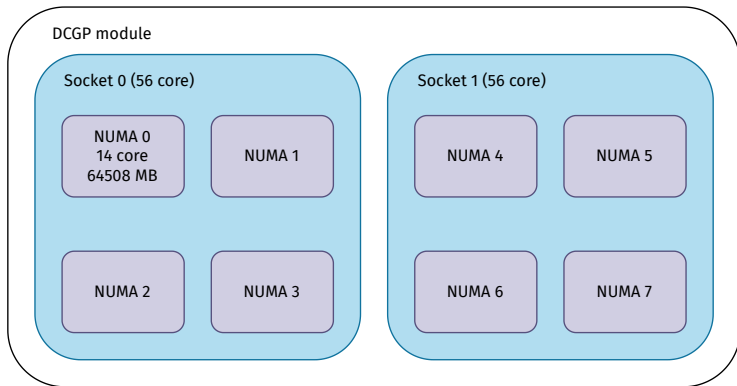
```
1  GRID_SIZE_X=16384
2  GRID_SIZE_Y=16384
3  N_STEPS=500
4
5  NODES=1
6  N_TASKS_PER_NODE=1
7  THREADS="1 2 4 8
           16 32 56 84
           112"
```



Thread Scaling



Node Architecture



Node Architecture: distance matrix

```
1 [glucarel@lrndn4293 HPC-leonardo]$ numactl --hardware
2 available: 8 nodes (0-7)
3 node    0    1    2    3    4    5    6    7
4 0:    10   12   12   12   21   21   21   21
5 1:    12   10   12   12   21   21   21   21
6 2:    12   12   10   12   21   21   21   21
7 3:    12   12   12   10   21   21   21   21
8 4:    21   21   21   21   10   12   12   12
9 5:    21   21   21   21   12   10   12   12
10 6:    21   21   21   21   12   12   10   12
11 7:    21   21   21   21   12   12   12   10
```


Node Architecture: memory

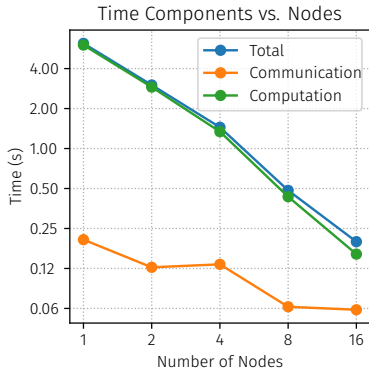
```
1 [glucarel@lrdsn4293 HPC-leonardo]$ lscpu | egrep 'L1d|L1i|L2|L3'
2     L1d cache:           48K
3     L1i cache:           32K
4     L2 cache:            2048K
5     L3 cache:            107520K
```

$$N = N_x = N_y = 2^{14}$$

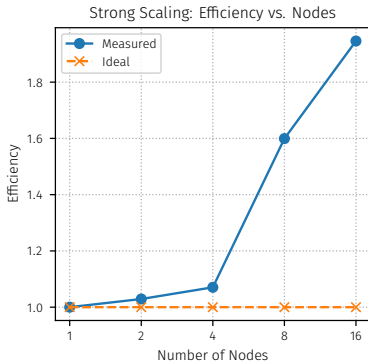
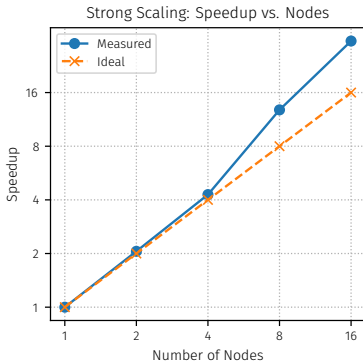
$$\text{memory}_{\text{grid}} = 2 \times N^2 \times 8\text{B} \approx 4\text{GB}$$

Strong Scaling (1/2)

```
1  GRID_SIZE_X=16384
2  GRID_SIZE_Y=16384
3  N_STEPS=500
4
5  OMP_THREADS=14
6  N_TASKS_PER_NODE=8
7
8  NODES="1 2 4 8 16"
```

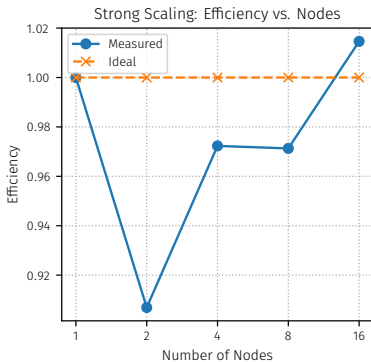
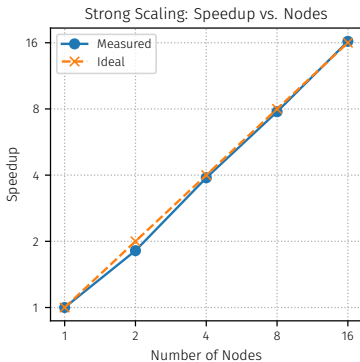


Strong Scaling (1/2)



Strong Scaling (2/2)

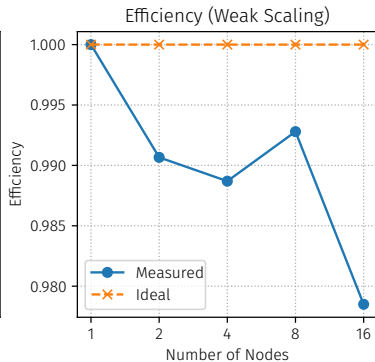
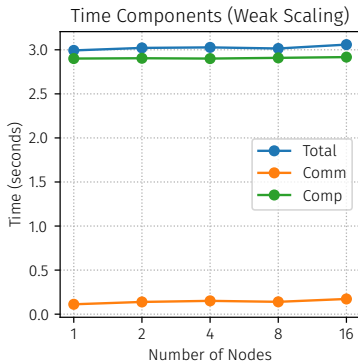
- 1 OMP_THREADS=112
- 2 N_TASKS_PER_NODE=1



Weak Scaling

```
1  LOCAL_X=4096
2  LOCAL_Y=4096
3  OMP_THREADS=14
4  TASKS_PER_NODE=8
5
6  for NODES in "1 2 4 8 16"; do
7      TOTAL_TASKS=$(( NODES * TASKS_PER_NODE ))
8
9      case "${TOTAL_TASKS}" in
10         8)    PX=4; PY=2 ;;    # 1 node (8 ranks)
11         ...
12     esac
13
14     GRID_SIZE_X=$(( LOCAL_X * PX ))
15     GRID_SIZE_Y=$(( LOCAL_Y * PY ))
16     ...
17 done
```

Weak Scaling



Conclusion

- Stencil method is memory bound!

Thank You!