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DISTRIBUTION SHIFT

A Study on Their Effects on Statistical Models and
Strategies for Mitigation

Andrea Spinelli, Giacomo Amerio,
Giovanni Lucarelli, Tommaso Piscitelli

University of Trieste

Table of contents

1. Introduction
2. Data Generation
3. Performance Degradation
4. Performance Enhancement

Introduction

Dataset shift

- **Dataset shift** is a common problem in machine learning.
- It occurs when the distribution of the training data differs from the distribution of the test data.
- This can lead to a decrease in the performance of the model.

The two most common and well-studied causes of dataset shift are:

- Sample selection bias
- non stationary environments

Aims of project

This project aims to evaluate the impact of simple **Covariate shift** in the input distribution on the performance of robust models within the context of a synthetic binary classification task.

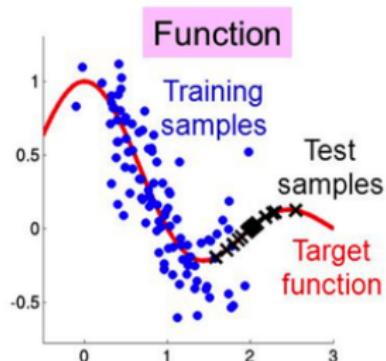
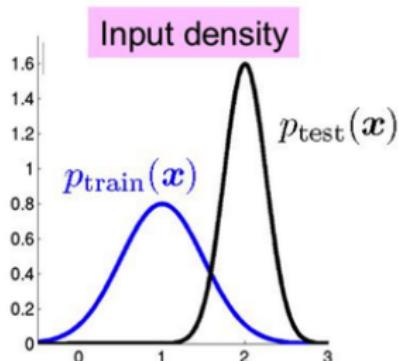
Key questions addressed in this study include:

- How do different types of covariate shifts affect the performance of robust models?
- Are certain models inherently more robust to simple covariate shifts?
- What strategies can be employed to improve model performance following such shifts?

Covariate shift

Can be formally defined as follows. Consider an input variable X and a response variable Y , where $X \rightarrow Y$ represents the relationship between the two. Let P_{tra} denote the probability distribution of the training data and P_{tst} denote the probability distribution of the test data. A covariate shift occurs when:

$$P_{\text{tra}}(Y | X) = P_{\text{tst}}(Y | X) \quad \text{but} \quad P_{\text{tra}}(X) \neq P_{\text{tst}}(X).$$



Example

Consider a model designed to distinguish between cats and dogs:

Training set:



Test set:



- Model will not accurately distinguish between cats and dogs because the feature distribution will differ.
- Changes in the input distribution can significantly impact the model's accuracy.

Inaccurate model

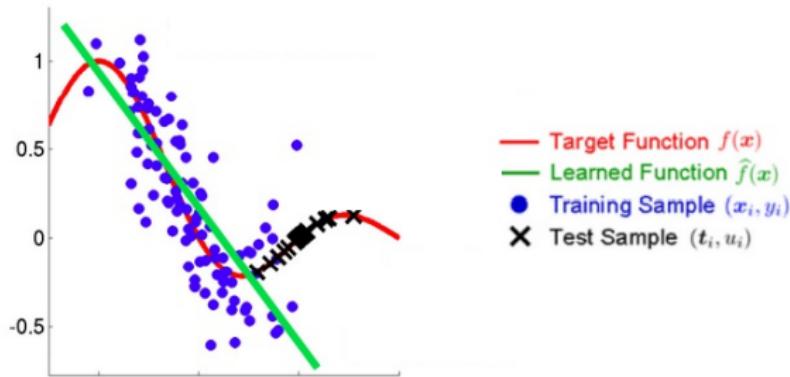


Figure 1: Example of inaccurate model.

In this study, we analyze the effects of distribution shift on different statistical models and propose strategies for its mitigation.

Data Generation

Training Dataset: Features

The dataset consists of $n = 10^4$ observations with 3 features and 1 binary target variable.

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Features:

- $X_{\text{train}} = (X_{\text{train}1}, X_{\text{train}2}, X_{\text{train}3}) \sim \mathcal{N}(\boldsymbol{\mu}_{\text{train}}, \boldsymbol{\Sigma}_{\text{train}})$
- $\mu_{\text{train}i} \sim \mathcal{U}_{[0,1]}$ for $i = 1, 2, 3$
- $[\boldsymbol{\Sigma}_{\text{train}}]_{i,j} \sim \mathcal{U}_{[-1,1]}$ for $i, j = 1, 2, 3$

Note: The $\boldsymbol{\Sigma}$ randomly generated has been transformed to a symmetric and positive semidefinite matrix by computing $\boldsymbol{\Sigma}\boldsymbol{\Sigma}^T$.

Training Dataset: Target Variable

Building the **target variable** $Y \in \{0, 1\}$:

1.

$$z = \beta_0 + \sum_{i=1}^3 \beta_i x_i + \sum_{i=1}^3 \beta_{ii} x_i^2 + \sum_{i=1}^2 \sum_{j=i+1}^3 \beta_{ij} x_i x_j, \quad \beta. \sim \mathcal{U}_{[-1,1]}$$

2.

$$p = \frac{1}{1 + e^{-z}}$$

3.

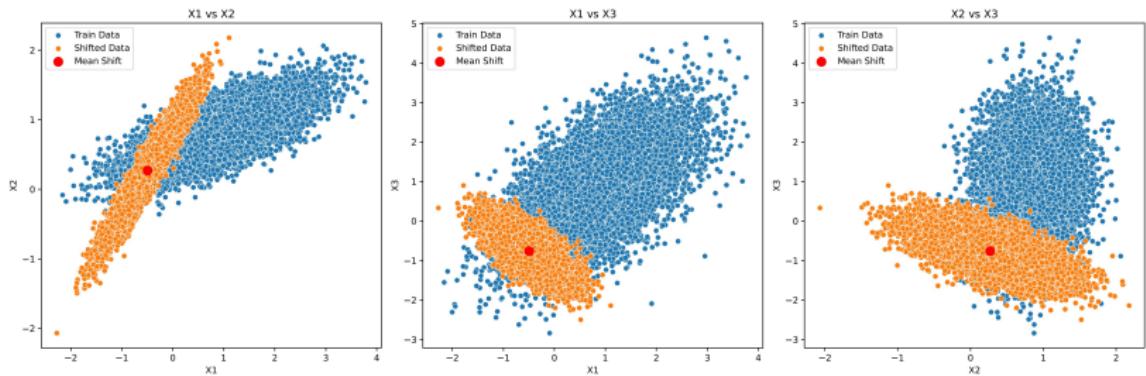
$$Y \sim \text{Be}(p)$$

Testing Dataset

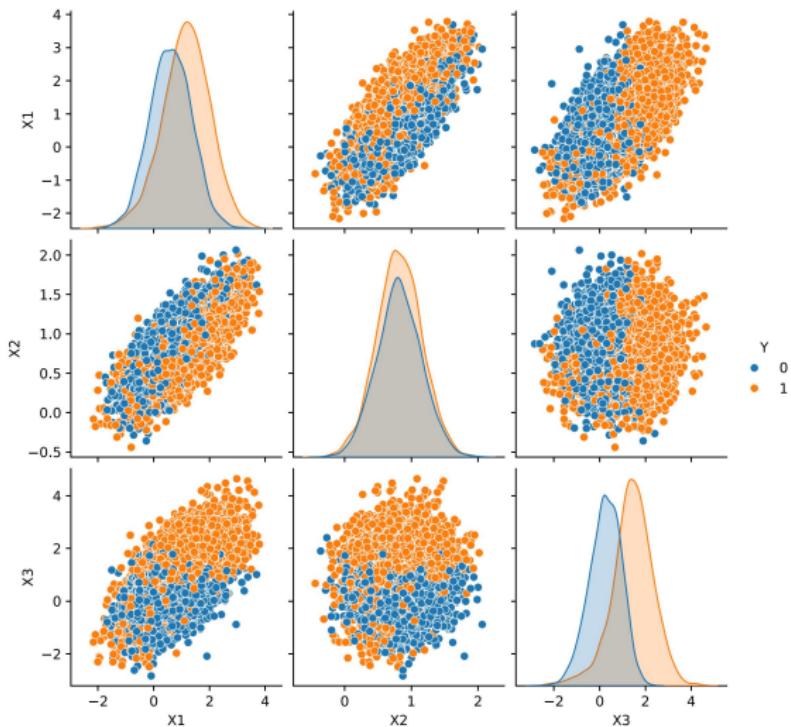
Same dataset structure as the train set, but:

- $X_{\text{shift}} = (X_{\text{shift}1}, X_{\text{shift}2}, X_{\text{shift}3}) \sim \mathcal{N}(\boldsymbol{\mu}_{\text{shift}}, \boldsymbol{\Sigma}_{\text{shift}})$
- $\boldsymbol{\mu}_{\text{shift}} = Q_{0.05}(X_{\text{train}})$
- $[\boldsymbol{\Sigma}_{\text{shift}}]_{i,j} \sim \mathcal{U}_{[-0.5, 0.5]}$ for $i, j = 1, 2, 3$

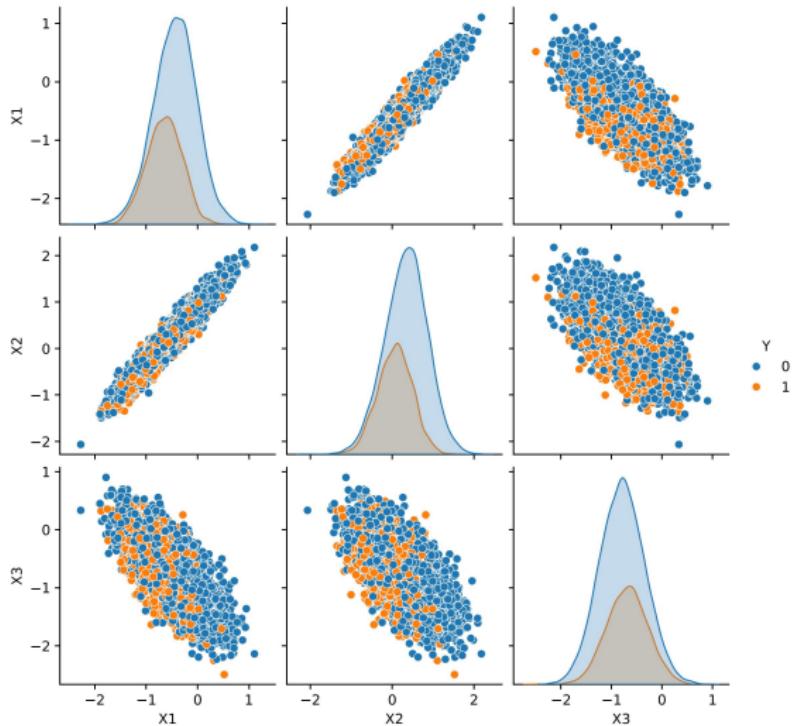
Original and Shifted Features



Label Distribution in Train Set



Label Distribution in Shifted Test Set



Note: IR from 1.19 to 2.36

Testing Mixture

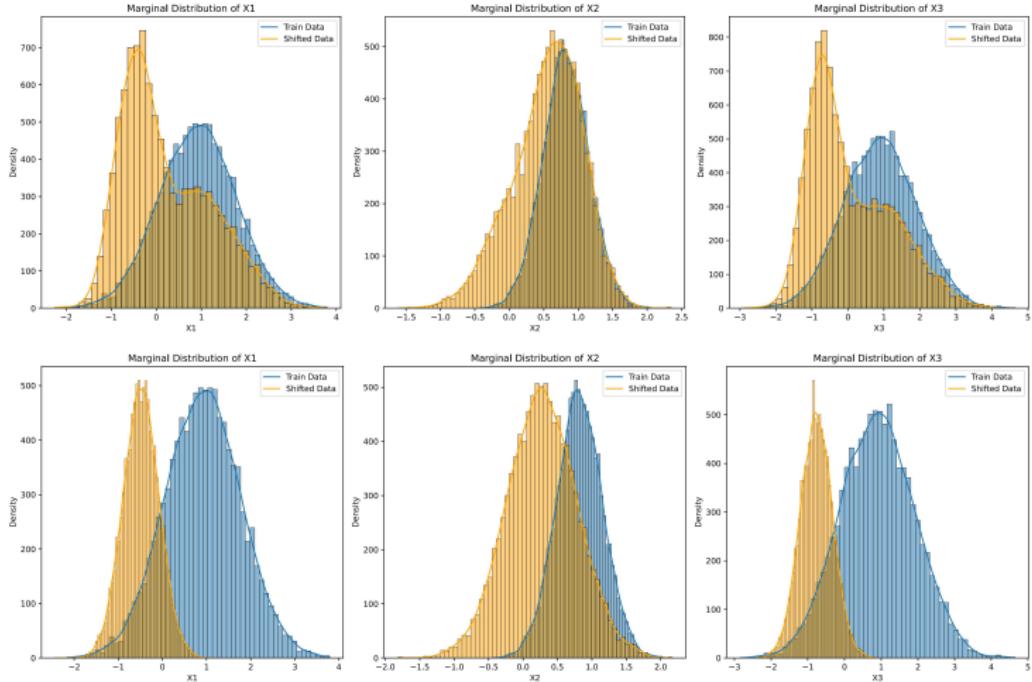
Series of datasets using **statistical mixtures** of the training features distribution and the fully shifted distribution.

$$X_\alpha \sim \alpha \cdot \mathcal{N}(\boldsymbol{\mu}_{\text{shift}}, \boldsymbol{\Sigma}_{\text{shift}}) + (1 - \alpha) \cdot \mathcal{N}(\boldsymbol{\mu}_{\text{train}}, \boldsymbol{\Sigma}_{\text{train}})$$

$$\alpha \in \{0.0, 0.1, \dots, 1.0\}$$

Y_α generated as before

Note: $X_{0.0}$ and X_{train} come from the same distribution, but the former are used as fresh new data.



Top: $\alpha = 0.5$. Bottom: $\alpha = 1.0$.

Performance Degradation

Performance Enhancement

1. No Prior Shift Knowledge Needed

- Simplifies implementation by eliminating the need for shift estimation.
- Adaptable to various datasets without additional shift information.

2. Built-in Regularization

- Prevents overfitting by introducing controlled noise.
- Enhances model generalization on unseen data.

- Random
- Augmentation
- Walk

Input: $Data_{train}$, $Size$, N , ε .

$Data\% \leftarrow$ random subset of $N\%$ of Data

For x_i in $Data\%$

$x'_i \leftarrow \begin{cases} X_i + \varepsilon & \text{with probability 0.5} \\ X_i - \varepsilon & \text{with probability 0.5} \end{cases}$

$y'_i \leftarrow y_i$

End For

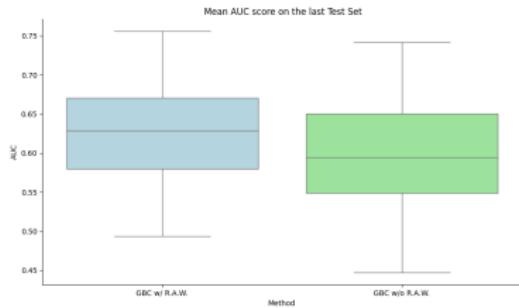
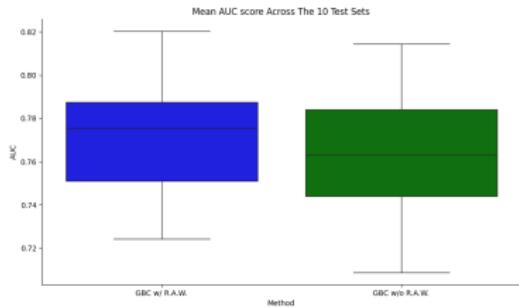
$Data_{aug} \leftarrow Data_{new} \cup Data\%$

$Data_{final} \leftarrow$ Downsample($Data_{aug}$, $Size$)

Return $Data_{final}$

Classify With Gradient Boosting Using R.A.W.

1. Apply the R.A.W. pre-processing method to the training data to address covariate shift.
2. Train a Gradient Boosting Classifier on the augmented dataset.
3. Evaluate the model's performance on shifted test sets.



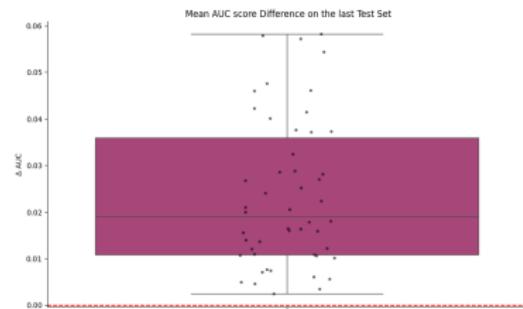
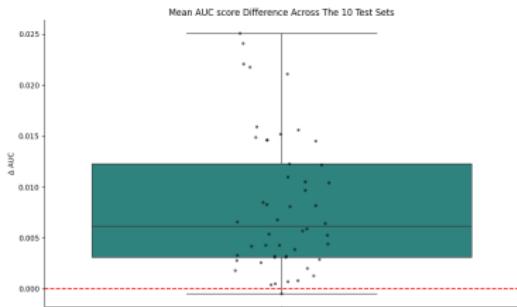
A Statistical Analysis Of The Results

- Null Hypothesis (H_0): $\Delta\bar{\mu} = \overline{\text{AUC}}_{\text{R.A.W.}} - \overline{\text{AUC}}_{\text{base}} = 0$
- Alternative Hypothesis (H_1): $\Delta\bar{\mu} = \overline{\text{AUC}}_{\text{R.A.W.}} - \overline{\text{AUC}}_{\text{base}} \neq 0$
The R.A.W. method improves the AUC score of the Gradient Boosting Classifier.
- Test Used: Paired t-test on 50 independent $\overline{\text{AUC}}$ differences.

	$\Delta\bar{\mu}$	t-stat	p-value	95% CI
$\Delta\overline{\text{AUC}}^*$	0.0083	8.75	1.39×10^{-11}	[0.006, 0.010]
$\Delta\overline{\text{AUC}}_{\text{last}}^{**}$	0.0235	10.59	2.86×10^{-14}	[0.019, 0.028]

* Mean AUC score difference across all 10 shifted test sets.

** Mean AUC score difference on the most shifted test set.



Questions?

References i