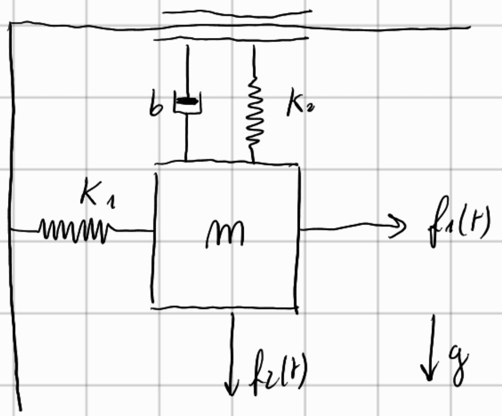
ALCUNE CONSIDERAZIONI:

- È possibile che una piccola parte degli esercizi sia scorretta
- Alcuni esercizi possono essere risolti in modi diversi, ma ugualmente corretti
- Gli esercizi sono svolti in ordine cronologico, quindi più si va in fondo nel file più sarà probabile che con più esperienza alle spalle saranno corretti.

Buon LAVORO!



$$\frac{d}{dt}\left(\frac{\delta L(\dot{q},q)}{\delta \dot{q}_k}\right) - \frac{\delta L(\dot{q},q)}{\delta q_k} + \frac{\delta D(\dot{q},q)}{\delta \dot{q}_k} = Q_k \quad k=1,\dots,n$$

$q_1 = x \quad q_2 = y$, 2 gradi di libertà.

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2$$

$$U = \frac{1}{2}k_1x^2 + \frac{1}{2}k_2y^2 - mgy$$

$$D = \frac{1}{2}b\dot{y}^2 \qquad Q_1 = f_1$$

$$Q_2 = f_2$$

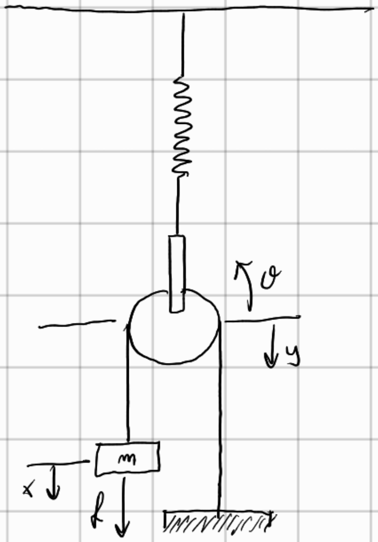
$$\mathcal{L}(x,\dot{x},y,\dot{y}) = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 - \frac{1}{2}k_1x^2 - \frac{1}{2}k_2y^2 + mgy$$

1ª equazione:

$$m\ddot{x} + k_1x = u_1$$

$$m\ddot{y} + k_2y - mg + b\dot{y} = u_2$$

Esercizio 2



$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} M \dot{y}^2 + \frac{1}{2} I \dot{\theta}^2$$

NOTA: $x = 2y$ $y = R\theta$ Perché cilindro non può traslare senza ruotare.

$\theta = \frac{y}{R}$ $\theta = \frac{x}{2R}$

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} M \cdot \frac{\dot{x}^2}{4} + \frac{1}{2} \frac{M\cancel{R^2}}{2} \cdot \frac{\dot{x}^2}{4\cancel{R^2}} = \dot{x}^2 \left(\frac{1}{2} m + \frac{1}{8} M + \frac{1}{16} M \right)$$

$$T = \dot{x}^2 \left(\frac{1}{2} m + \frac{3}{16} M \right)$$

$$U = -mgx - Mgy + \frac{1}{2} K y^2 = -mgx - \frac{Mgx}{2} + \frac{1}{2} K \cdot \frac{x^2}{4} = \frac{1}{8} K x^2 - \frac{Mgx}{2} - mgx$$

$$\mathcal{L} = T - U = \dot{x}^2 \left(\frac{1}{2} m + \frac{3}{16} M \right) - \frac{1}{8} K x^2 + \frac{Mgx}{2} + mgx$$

$$\ddot{x} \left(m + \frac{3}{8} M \right) + \frac{Kx}{4} - \frac{Mg}{2} - mg = u$$

$x_1(t) = x(t)$

$x_2(t) = \dot{x}(t)$ $\Rightarrow$

$u_1 = u$

$u_2 = \frac{Mg}{2} + mg$

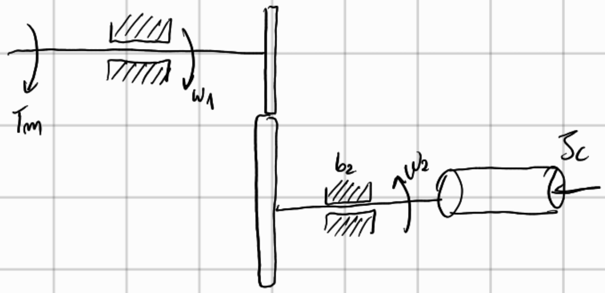
$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = \frac{u_1}{m + \frac{3}{8} M} + \frac{u_2}{m + \frac{3}{8} M} - \frac{K x_1(t)}{4\left(m + \frac{3}{8} M \right)}$$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ \frac{-K}{4\left(m + \frac{3}{8} M\right)} & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 & 0 \\ \frac{1}{m + \frac{3}{8} M} & \frac{1}{m + \frac{3}{8} M} \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 & 0 \end{pmatrix} u(t)$$

3)



1 grado di libertà. Ci arrestiamo.

$$\begin{cases} T_m - b_1\omega_1 - T_1 = J_1\dot{\omega} \\ T_2 - b_2\omega_2 = (J_2 + J_c)\dot{\omega} \end{cases}$$

$T_1\omega_1 = T_2\omega_2 \qquad \frac{\omega_2}{\omega_1} = n$

$\frac{T_1}{T_2} = n \Rightarrow T_1 = nT_2 \qquad \omega_2 = n\omega_1$

$$\begin{cases} T_m - b_1\omega_1 - T_1 = J_1\dot{\omega}_1 \\ T_2 - b_2\omega_2 = (J_2 + J_c)\dot{\omega}_2 \end{cases}$$

$T_2 = b_2\omega_2 + (J_2 + J_c)\dot{\omega}_2$

$\hookrightarrow T_m - b_1\omega_1 - nb_2\omega_2 - n(J_2 + J_c)\dot{\omega}_2 = J_1\dot{\omega}_1$

$T_m - b_1\omega_1 - n^2 b_2\omega_1 - n^2(J_2 + J_c)\dot{\omega}_1 = J_1\dot{\omega}_1$

$\dot{\omega}_1\left(J_1 + n^2 J_2 + n^2 J_c\right) = T_m - b_1\omega_1 - n^2 b_2\omega_1$

$x_1(t) = \theta(t) \qquad \alpha = J_1 + n^2 J_2 + n^2 J_c$

$x_2(t) = \omega_1(t)$

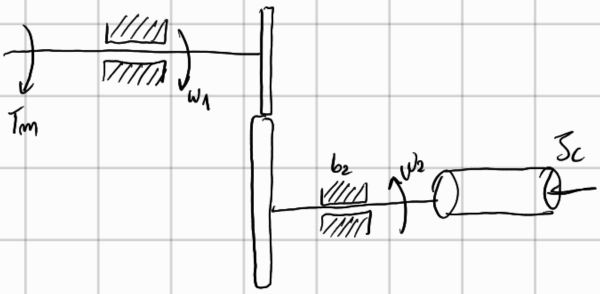
$\dot{x}_1(t) = x_2(t)$

$\dot{x}_2(t) = \frac{u_1}{\alpha} - \frac{b_1 x_2(t)}{\alpha} - \frac{n^2 b_2 x_2(t)}{\alpha}$

$y(t) = n x_2(t)$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 0 & \frac{-b_1 - n^2 b_2}{\alpha} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{\alpha} \end{pmatrix} u(t)$$

$y(t) = (0 \quad 1)x(t) + (0)\,u(t)$



Con Lagrange:

$$T = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2}(J_2 + J_m)\dot{\theta}_2^2 = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} n^2 (J_2 + J_m)\dot{\theta}_1^2$$

$$U = 0$$

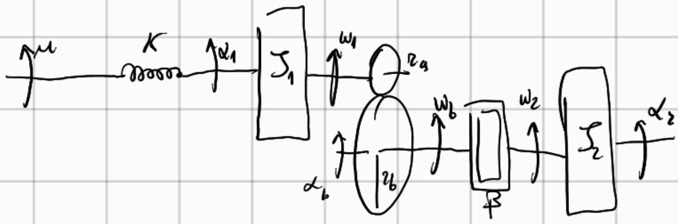
$$D = \frac{1}{2} b_1 \dot{\theta}_1^2 + \frac{1}{2} b_2 \dot{\theta}_2^2 = \frac{1}{2} b_1 \dot{\theta}_1^2 + \frac{1}{2} n^2 b_2 \dot{\theta}_1^2$$

$$Q = T_m$$

$$J_1 \ddot{\theta}_1 + n^2 (J_2 + J_m)\ddot{\theta}_1 + b_1 \dot{\theta}_1 + b_2 n^2 \dot{\theta}_2 = T_m$$

Stesso risultato.

2)



$\omega_b = \omega_2$?

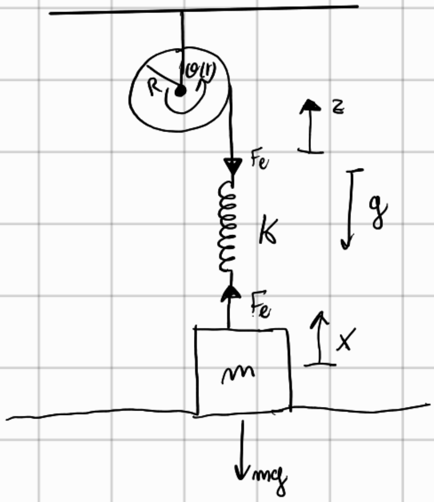
$\frac{\omega_2}{\omega_1} = n$

$T = \frac{1}{2} J_1 \omega_1^2 + \frac{1}{2} J_2 \omega_2^2$

$U = \frac{1}{2} K \theta_1^2$

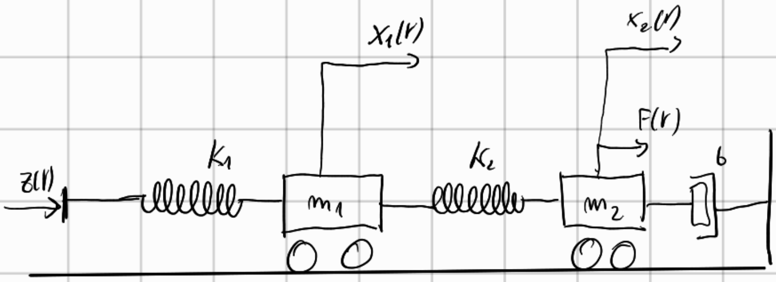
$D = \frac{1}{2} \beta \omega_2^2$

2)



Ho un grado di libertà?

$$m\ddot{x} = -mg + K($$

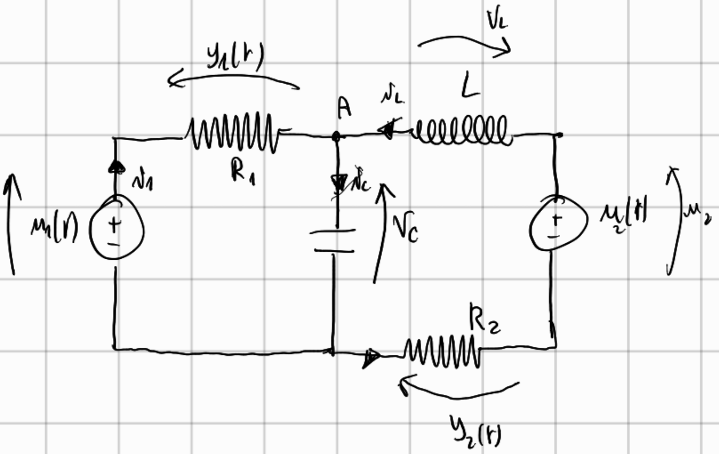


2 gradi di libertà.

Corpo 2:

$$m_2 \ddot{x}_2 = -b\dot{x}_2 + K_2(x_2 - x_1)$$

1)



VAR. STATO:

i_L
V_C

$$u_1 - R_1 i_1 - V_C = 0$$

$$V_C + L\frac{di_L}{dt} - u_2 + R_2 i_L = 0$$

$$i_1 + i_L - i_C = 0 \Rightarrow i_1 = i_C - i_L$$

$$\begin{cases} u_1 - R_1 C\frac{dv_C}{dt} + R_1 i_L - V_C = 0 \\ V_C + L\frac{di_L}{dt} - u_2 + R_2 i_L = 0 \end{cases}$$

$$y_1 = i_1 R_1 = i_C R_1 - i_L R_1 =$$

$$= R_1 C\frac{dV_C}{dt} - i_L R_1 =$$

$$=$$

$$\dot{x}_1(t) = \frac{u_2}{L} - \frac{R_2 x_1}{L} - \frac{x_2}{L}$$

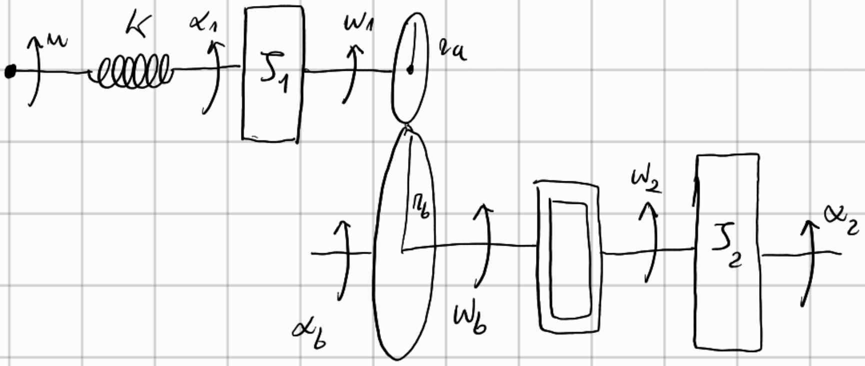
$$\dot{x}_2(t) = \frac{u_1}{R_1 C} + \frac{x_1}{C} - \frac{x_2}{R_1 C}$$

$$y_1(t) = u_1 - x_2$$

$$y_2(t) = R_2 x_1$$

$$\dot{x}(t) = \begin{pmatrix} -\frac{R_2}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_1 C} \end{pmatrix} x(t) + \begin{pmatrix} 0 & \frac{1}{L} \\ \frac{1}{R_1 C} & 0 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 0 & -1 \\ R_2 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} u(t)$$



NON CAMBIA
ENERGIA POTENZIALE
GRAVITAZIONALE

Gradi di libertà: 2
Coordinate lagrangiane: θ_1, θ_2.

$$T = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2$$

$$\frac{w_a}{w_b} = \frac{1}{n} \Rightarrow \frac{w_b}{n} = w_a \Rightarrow$$
$$w_b = n w_a$$

$$U = \frac{1}{2} k \theta_1^2$$

$$D = \frac{1}{2} b (\dot{\theta}_2 - n \dot{\theta}_1)^2$$

$$L(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2) = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 - \frac{1}{2} k \theta_1^2$$

1^A EQUAZIONE

$$J_1 \ddot{\theta}_1 + k\theta_1 + b(\dot{\theta}_2 - n\dot{\theta}_1) \cdot (-n) = u$$

2^A EQUAZIONE

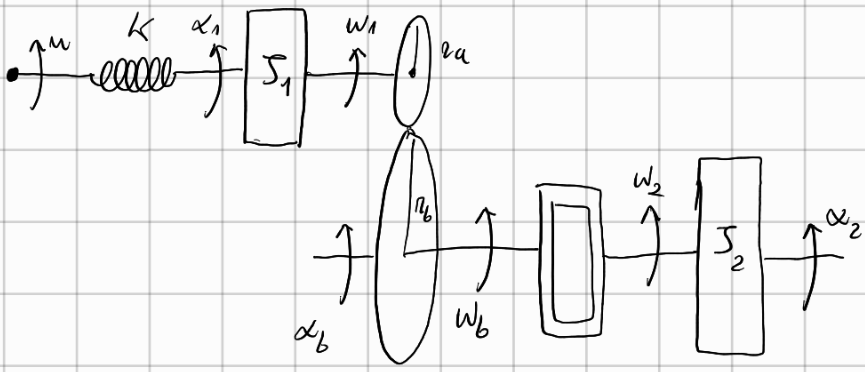
$$J_2 \ddot{\theta}_2 + b(\dot{\theta}_2 - n\dot{\theta}_1) = 0$$

$$\begin{cases} J_1 \ddot{\theta}_1 = u + nb\dot{\theta}_2 - n^2 b \dot{\theta}_1 - k\theta_1 \\ J_2 \ddot{\theta}_2 = nb\dot{\theta}_1 - b\dot{\theta}_2 \end{cases}$$

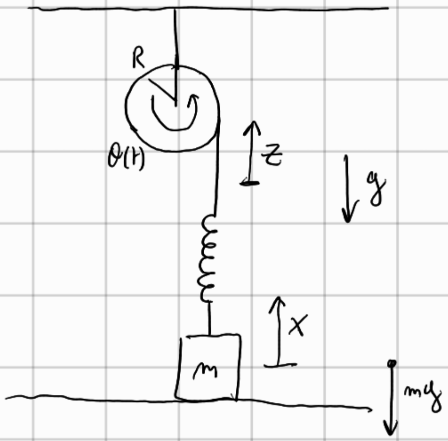
$x_1(t) = \theta_1(t)$ $\quad$ $x_3(t) = \omega_1(t)$

$x_2(t) = \theta_2(t)$ $\quad$ $x_4(t) = \omega_2(t)$

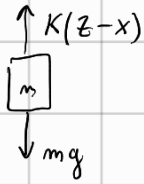
$$
\begin{cases}
\dot{x}_1(t) = x_3(t) \\
\dot{x}_2(t) = x_4(t) \\
\dot{x}_3(t) = \dfrac{u}{J_1} + \dfrac{nb\,x_4}{J_1} - \dfrac{n^2 b\,x_3}{J_1} - \dfrac{k x_1}{J_1} \\
\dot{x}_4(t) = \dfrac{nb\,x_3}{J_2} - \dfrac{b}{J_2} x_4 \\
y = \dfrac{u}{J_1} + \dfrac{nb\,x_4}{J_1} - \dfrac{n^2 b\,x_3}{J_1} - \dfrac{k x_1}{J_1}
\end{cases}
$$



1)



$z = R\theta$



1 grado di libertà:

$$m\ddot{x} = K(z-x) - mg \quad \Rightarrow \quad \begin{aligned} x_1(t) &= x(t) \\ x_2(t) &= \dot{x}(t) \\ u_1(t) &= \theta(t) \\ u_2(t) &= g \end{aligned}$$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \frac{K}{m} R u_1 - \frac{K}{m} x_1 - u_2 \\ y(t) = x_1(t) \end{cases}$$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ -\frac{K}{m} & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 & 0 \\ \frac{KR}{m} & -1 \end{pmatrix} u(t)$$

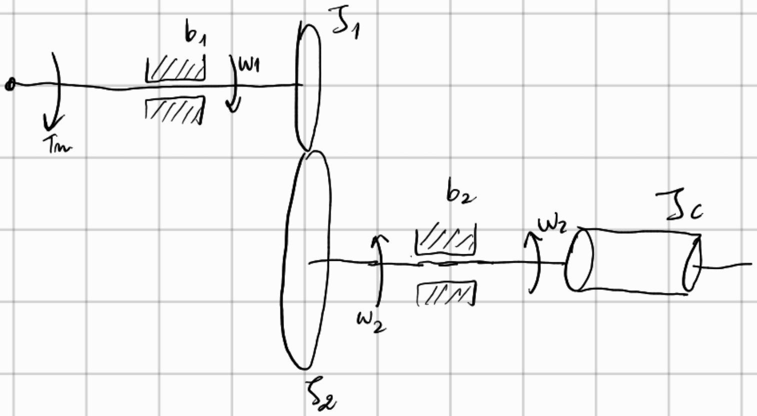
$$y(t) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(t)$$

LAGRANGE:

$T = \frac{1}{2} m\dot{x}^2$ $\qquad$ $L = \frac{1}{2} m\dot{x}^2 - mgx - \frac{1}{2} K(R\theta - x)^2$

$U = mgx + \frac{1}{2} K(R\theta - x)^2$

$m\ddot{x} + mg - K(R\theta - x) = 0$ $\quad$ EQUAZIONE



1 grado.

$\omega_2 = n\omega_1$

$$T = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2}(J_2 + J_c)(n\,\dot{\theta}_1)^2$$

$$U = 0$$

$$D = \frac{1}{2} b_1 \dot{\theta}_1^2 + \frac{1}{2} b_2 (n\dot{\theta}_1)^2$$

EQUAZIONE:

$$J_1 \ddot{\theta}_1 + n^2 (J_2 + J_c) \ddot{\theta}_1 + b_1 \dot{\theta}_1 + b_2 n^2 \dot{\theta}_1 = T_m$$

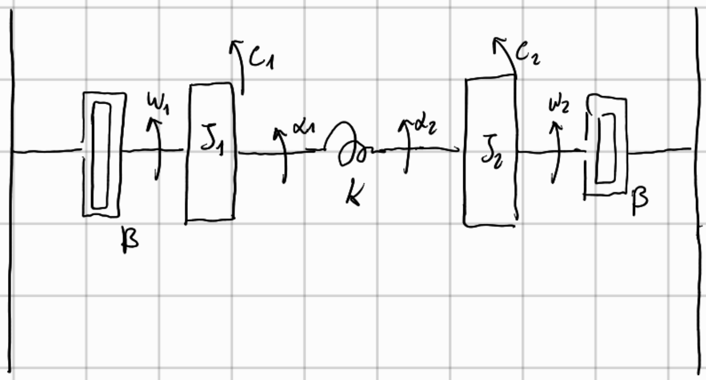
$$\ddot{\theta}_1 = \frac{T_m}{J_1 + n^2(J_2 + J_c)} - \frac{b_1 + b_2 n^2}{J_1 + n^2(J_2 + J_c)} \dot{\theta}_1$$

$\omega_2 = n\omega_1$

$$\frac{\ddot{\theta}_2}{n} = \frac{T_m}{J_1 + n^2(J_2 + J_c)} - \frac{b_1 + b_2 n^2}{J_1 + n^2(J_2 + J_c)} \cdot \frac{\dot{\theta}_2}{n}$$

$$\ddot{\theta}_2 = \frac{n\,T_m}{J_1 + n^2(J_2 + J_c)} - \frac{b_1 + b_2 n^2}{J_1 + n^2(J_2 + J_c)}$$

2)



C_1 e C_2 ingressi.

$$\begin{cases} J_1 \ddot{\alpha}_1 = C_1 - \beta\omega_1 - K(\alpha_1 - \alpha_2) \\ J_2 \ddot{\alpha}_2 = C_2 - \beta\omega_2 - K(\alpha_2 - \alpha_1) \end{cases}$$

2 equazioni per 2 gradi di libertà.

$x_1(t) = \alpha_1(t)$ $\quad x_3(t) = \omega_1(t)$ $\quad u_1(t) = C_1$

$x_2(t) = \alpha_2(t)$ $\quad x_4(t) = \omega_2(t)$ $\quad u_2(t) = C_2$

$$\begin{cases} \dot{x}_1(t) = x_3(t) \\ \dot{x}_2(t) = x_4(t) \\ \dot{x}_3(t) = \frac{u_1}{J_1} - \frac{\beta}{J_1} x_3(t) - \frac{K}{J_1} x_1(t) + \frac{K}{J_1} x_2(t) \\ \dot{x}_4(t) = \frac{u_2}{J_2} - \frac{\beta}{J_2} x_4(t) - \frac{K}{J_2} x_2(t) + \frac{K}{J_2} x_1(t) \end{cases}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K}{J_1} & \frac{K}{J_1} & -\frac{\beta}{J_1} & 0 \\ \frac{K}{J_2} & -\frac{K}{J_2} & 0 & -\frac{\beta}{J_2} \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{J_1} & 0 \\ 0 & \frac{1}{J_2} \end{bmatrix}$$

LAVORO CON LAGRANGE

$T = \frac{1}{2} J_1 \dot{\alpha}_1^2 + \frac{1}{2} J_2 \dot{\alpha}_2^2$

$U = \frac{1}{2} K (\alpha_2 - \alpha_1)^2$

$D_1 = \frac{1}{2} \beta \dot{\alpha}_1^2 \qquad D_2 = \frac{1}{2} \beta \dot{\alpha}_2^2$

$L = \frac{1}{2} J_1 \dot{q}_1^2 + \frac{1}{2} J_2 \dot{q}_2^2 - \frac{1}{2} K (q_2 - q_1)^2$

$D = \frac{1}{2} \beta \dot{q}_1^2 + \frac{1}{2} \beta \dot{q}_2^2$

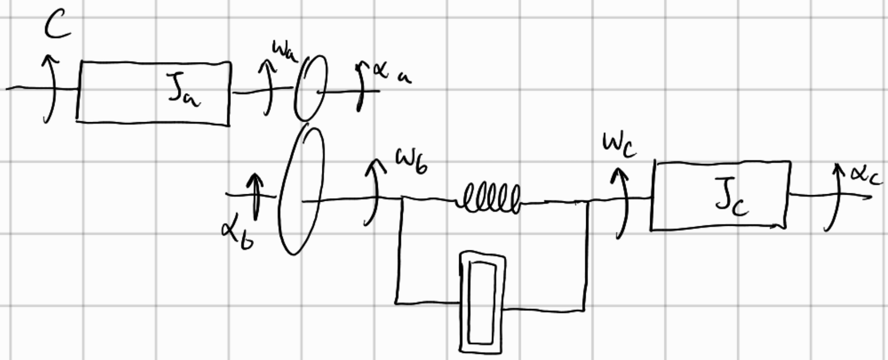
$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_1}\right) = J_1 \ddot{q}_1 \qquad \frac{\delta L}{\delta q_1} = +K(q_2 - q_1) \qquad \frac{\delta D}{\delta \dot{q}_1} = \beta \dot{q}_1 \qquad Q_1 = C_1$

$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_2}\right) = J_2 \ddot{q}_2 \qquad \frac{\delta L}{\delta q_2} = -K(q_2 - q_1) \qquad \frac{\delta D}{\delta \dot{q}_2} = \beta \dot{q}_2 \qquad Q_2 = C_2$

1) $J_1 \ddot{q}_1 - K(q_2 - q_1) + \beta \dot{q}_1 = C_1$

2) $J_2 \ddot{q}_2 + K(q_2 - q_1) + \beta \dot{q}_2 = C_2$

4)



$$T = \frac{1}{2} J_a w_a^2 + \frac{1}{2} J_c w_c^2$$

$w_a R_a = w_b R_b$

$$U = \frac{1}{2} K(\alpha_c - \alpha_b)^2 = \frac{1}{2} K(\alpha_c - m\alpha_a)^2$$

$w_b = w_a \frac{R_a}{R_b} = w_a m$

$$L = \frac{1}{2} J_a \dot{q}_1^2 + \frac{1}{2} J_c \dot{q}_2^2 - \frac{1}{2} K(q_2 - m q_1)^2$$

$q_1 = \alpha_a$

$q_2 = \alpha_c$

$$D = \frac{1}{2} \beta (\dot{q}_2 - m\dot{q}_1)^2$$

2 equations:

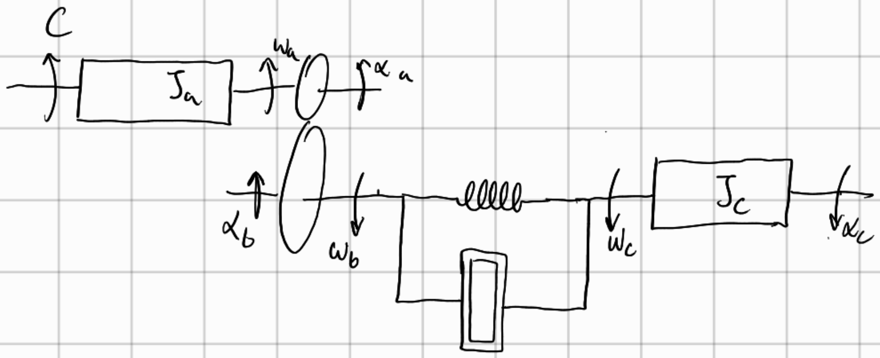
$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_1}\right) = J_a \ddot{q}_1$ $\quad\quad$ $\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_2}\right) = J_c \ddot{q}_2$

$\frac{\delta L}{\delta q_1} = +K(q_2 - m q_1) \cdot m$ $\quad\quad$ $\frac{\delta L}{\delta q_2} = -K(q_2 - m q_1)$

$\frac{\delta D}{\delta \dot{q}_1} = -\beta(\dot{q}_2 - m\dot{q}_1) m$ $\quad\quad$ $\frac{\delta D}{\delta \dot{q}_2} = \beta(\dot{q}_2 - m\dot{q}_1)$

$Q_1 = C$ $\quad\quad$ $Q_2 = 0$

$$\begin{cases} J_a \ddot{q}_1 - mK(q_2 - m q_1) - m\beta(\dot{q}_2 - m\dot{q}_1) = C \\ J_c \ddot{q}_2 + K(q_2 - m q_1) + \beta(\dot{q}_2 - m\dot{q}_1) = 0 \end{cases}$$



$$\begin{cases} J\ddot{\theta}_a = C - C_0 \\ J\ddot{\theta}_b = +K(\theta_b - \theta_c) + b(\omega_b - \omega_c) \end{cases}$$

Segno + perché T va verso alto a destra

$$\begin{cases} J\ddot{\theta}_a = C - nK(n\theta_a - \theta_c) - nb(n\omega_a - \omega_c) \\ J\ddot{\theta}_b = +K(n\theta_a - \theta_c) + b(n\omega_a - \omega_c) \end{cases}$$

$\overset{\nearrow T_0}{\omega_a T_a} = \overset{\nearrow T_1}{\omega_c T_c}$

$r_a \omega_a = r_b \omega_b$

$\omega_b = \frac{r_a}{r_b}\omega_a = \omega_a n$

$\omega_a T_a = \omega_b T_b$

$T_b = \frac{\omega_a}{\omega_b} T_a$

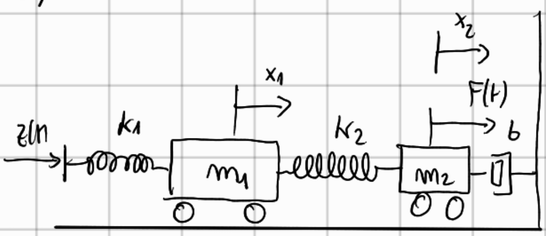
$T_b = \frac{1}{n} T_a$

$T_a = nK(\theta_b - \theta_c) + nb(\omega_b - \omega_c)$

$T_a = nK(n\theta_a - \theta_c) + nb(n\omega_a - \omega_c)$

$$\begin{cases} J_a\ddot{q}_1 - nK(q_2 - nq_1) - n\beta(\dot{q}_2 - n\dot{q}_1) = C \\ J_c\ddot{q}_2 + K(q_2 - nq_1) + \beta(\dot{q}_2 - n\dot{q}_1) = 0 \end{cases}$$

2)



Lagrange: Ho 2 gradi di libertà.

Coordinate Lagrangiane $q_1 = x_1$, $q_2 = x_2$

$$T = \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2} m_2 \dot{q}_2$$

$$U = \frac{1}{2} K_1 (q_1 - z)^2 + \frac{1}{2} K_2 (q_2 - q_1)^2$$

$$D = \frac{1}{2} \beta \dot{q}_2^2$$

$$L = \frac{1}{2} m_1 \dot{q}_1^2 + \frac{1}{2} m_2 \dot{q}_2^2 - \frac{1}{2} K_1 (q_1 - z)^2 - \frac{1}{2} K_2 (q_2 - q_1)^2$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_1}\right) = m_1 \ddot{q}_1 \qquad \frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_2}\right) = m_2 \ddot{q}_2$$

$$\frac{\delta L}{\delta q_1} = -K_1 (q_1 - z) + K_2 (q_2 - q_1) \qquad \frac{\delta L}{\delta q_2} = -K_2 (q_2 - q_1)$$

$$\frac{\delta D}{\delta \dot{q}_1} = 0 \qquad \frac{\delta D}{\delta \dot{q}_2} = \beta \dot{q}_2$$

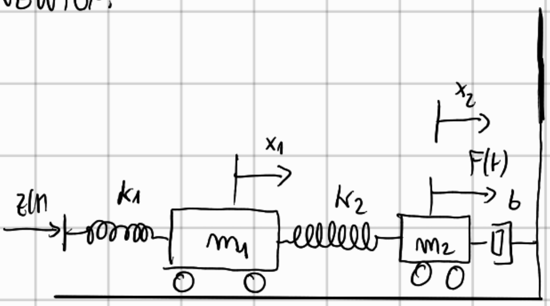
1ª EQ: $m_1 \ddot{q}_1 + K_1 (q_1 - z) - K_2 (q_2 - q_1) = 0$

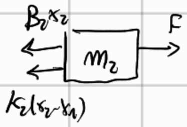
2ª EQ: $m_2 \ddot{q}_2 + K_2 (q_2 - q_1) + \beta \dot{q}_2 = F(t)$

$$m_2 \ddot{x}_2 = F - K_2 (x_2 - x_1) - \beta_2 \dot{x}_2$$

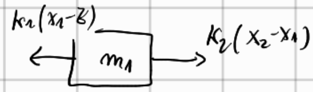
$$m_1 \ddot{x}_1 = K_2 (x_2 - x_1) - K_1 (x_1 - z)$$

NEWTON:



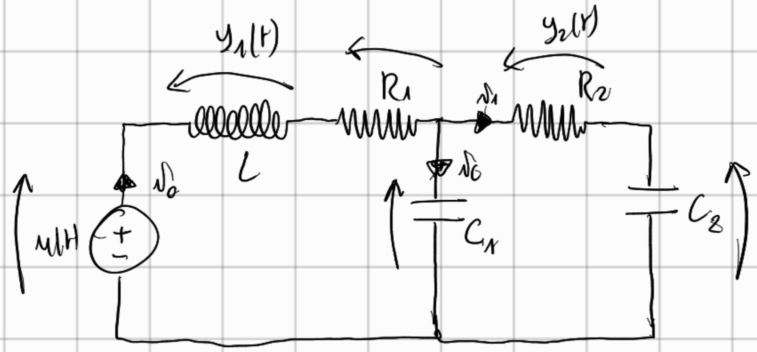


$$m_2 \ddot{x}_2 = F - k_2(x_2 - x_1) - \beta_2 \dot{x}_2$$



$$m_1 \ddot{x}_1 = k_2(x_2 - x_1) - k_1(x_1 - z)$$

4)



$$\begin{cases} u - L\dfrac{di_L}{dt} - R_1 i_L - V_{C_1} = 0 \\ V_{C_1} - R_2 C_2 \dfrac{dV_{C_2}}{dt} - V_{C_2} = 0 \\ i_L - C_1\dfrac{dV_{C_1}}{dt} - C_2\dfrac{dV_{C_2}}{dt} = 0 \end{cases}$$

$x_1(t) = i_L(t)$

$x_2(t) = V_{C_1}(t)$

$x_3(t) = V_{C_2}(t)$

$$\begin{cases} \dot{x}_1(t) = \dfrac{u}{L} - \dfrac{R_1}{L}x_1(t) - \dfrac{x_2(t)}{L} \\ \dot{x}_3(t) = \dfrac{x_2(t)}{R_2C_2} - \dfrac{x_3(t)}{R_2C_2} \\ \dot{x}_2(t) = \dfrac{x_1}{C_1} - \dfrac{C_2}{C_1}\left(\dfrac{x_2(t)}{R_2C_2} - \dfrac{x_3(t)}{R_2C_2}\right) \end{cases}$$

$$\begin{cases} \dot{x}_1(t) = -\dfrac{R_1}{L}x_1 - \dfrac{1}{L}x_2 + \dfrac{1}{L}u \\ \dot{x}_2(t) = \dfrac{1}{R_2C_2}x_2 - \dfrac{1}{R_2C_2}x_3 \\ \dot{x}_3(t) = \dfrac{1}{C_1}x_1 - \dfrac{1}{C_1R_2}x_2 + \dfrac{1}{C_1R_2}x_3 \end{cases}$$

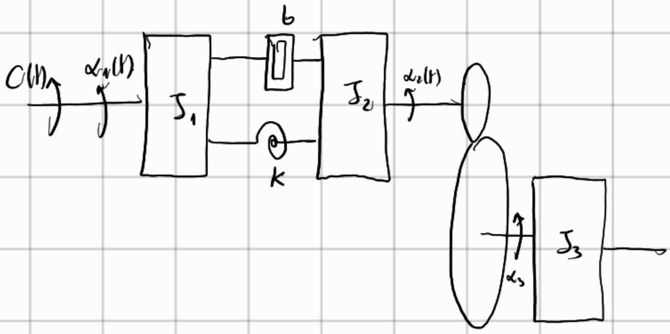
$y_1(t) = -R_1x_1 - x_2 + u$

$y_2(t) = x_2 - x_3$

$$\dot{x}(t) = \begin{bmatrix} -\frac{R_1}{L} & -\frac{1}{L} & 0 \\ 0 & \frac{1}{R_2C_2} & -\frac{1}{R_2C_2} \\ \frac{1}{C_2} & -\frac{C_1}{R_2C_2^2} & \frac{C_1}{R_2C_2^2} \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} -R_1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

1)



$$J_1\ddot{\theta}_1 = C(t) - b(\omega_1-\omega_2) - K(\theta_1-\theta_2)$$
$$J_2\ddot{\theta}_2 = b(\omega_1-\omega_2) + K(\theta_1-\theta_2) - C_0$$
$$J_3\ddot{\theta}_3 = C_1$$

$C_0\omega_2 = C_1\omega_3$

$C_0 = C_1\frac{\omega_3}{\omega_2} = n\,C_1$

$\Downarrow$

$$J_1\ddot{\theta}_1 = C(t) - b(\omega_1-\omega_2) - K(\theta_1-\theta_2)$$
$$J_2\ddot{\theta}_2 = b(\omega_1-\omega_2) + K(\theta_1-\theta_2) - nJ_3\ddot{\theta}_3$$

$r_a\omega_2 = r_b\omega_3$

$\omega_3 = \frac{r_a}{r_b}\omega_2 = n\omega_2$

$\Downarrow$

$$J_1\ddot{\theta}_1 = C(t) - b(\omega_1-\omega_2) - K(\theta_1-\theta_2)$$
$$J_2\ddot{\theta}_2 = b(\omega_1-\omega_2) + K(\theta_1-\theta_2) - n^2J_3\ddot{\theta}_2$$

$\frac{r_a}{r_b} = \frac{\omega_3}{\omega_2} = n$

$\omega_3 = n\omega_2$

LAGRANGE:

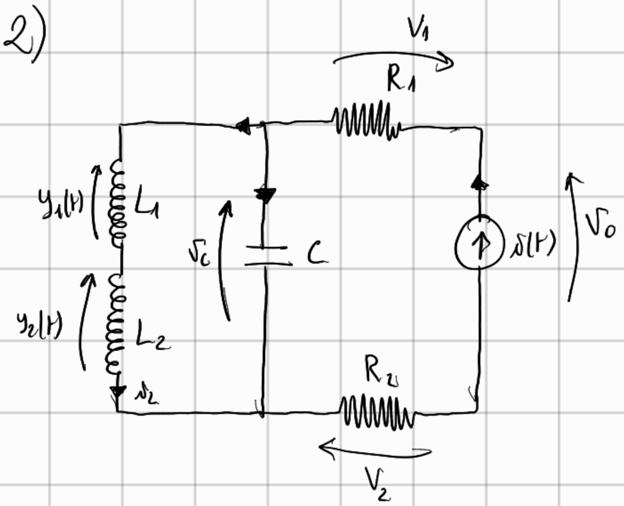
$$T = \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}J_2\dot{\theta}_2^2 + \frac{1}{2}J_3\dot{\theta}_3^2$$
$$U = \frac{1}{2}K(\theta_2-\theta_1)^2$$

$$L = \frac{1}{2}J_1\dot{q}_1^2 + \frac{1}{2}J_2\dot{q}_2^2 + \frac{1}{2}J_3n^2\dot{q}_2^2 - \frac{1}{2}K(q_2-q_1)^2$$
$$D_1 = \frac{1}{2}b(\dot{q}_2-\dot{q}_1)^2$$
$$Q_1 = C$$

EQ. 1:

$$J_1\ddot{q}_1 - K(q_2-q_1) - b(\dot{q}_2-\dot{q}_1) = C$$
$$J_2\ddot{q}_2 + J_3n^2\ddot{q}_2 + K(q_2-q_1) + b(\dot{q}_2-\dot{q}_1) = 0$$

2)



$$\begin{cases} v_0 - v_1 - v_{L_1} - v_{L_2} - v_2 = 0 \\ v_0 - v_1 - v_C - v_2 = 0 \\ i_0 - i_L - i_C = 0 \end{cases}$$

$$\begin{cases} v_0 - R_1 \cancel{i_0} - L_1 \frac{di_L}{dt} - L_2 \frac{di_L}{dt} - R_2 \cancel{i_0} = 0 \\ v_0 - R_1 \cancel{i_0} - v_C - R_2 \cancel{i_0} = 0 \\ i_0 - i_L - C\frac{dv_C}{dt} = 0 \end{cases}$$

$$\begin{cases} v_C - (L_1 + L_2)\frac{di_L}{dt} = 0 \\ i_0 - i_L - C\frac{dv_C}{dt} = 0 \end{cases}$$

$x_1(t) = v_C(t)$

$x_2(t) = i_L(t)$

$i_0 = u_1$

$\dot{x}_1(t) = \frac{u_1}{C} - \frac{x_2}{C}$

$\dot{x}_2(t) = \frac{x_1}{L_1 + L_2}$

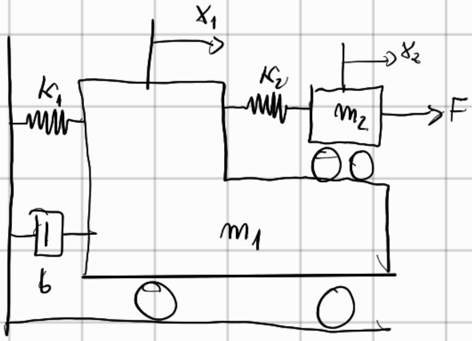
$y_1(t) = \frac{L_1 \cdot x_1}{L_1 + L_2}$

$y_2(t) = \frac{L_2 \cdot x_1}{L_1 + L_2}$

$$\dot{x}(t) = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L_1 + L_2} & 0 \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} \frac{L_1}{L_1 + L_2} & 0 \\ \frac{L_2}{L_1 + L_2} & 0 \end{bmatrix} x(t)$$

3)



$$\begin{cases} m_2\ddot{x}_2 = F - K_2(x_2-x_1) \\ m_1\ddot{x}_1 = K_2(x_2-x_1) - K_1x_1 - b_1\dot{x}_1 \end{cases}$$

$$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2$$

$$U = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(x_2-x_1)^2$$

$$D = \frac{1}{2}b\dot{x}_1^2$$

$$m_1\ddot{x}_1 + K_1x_1 - K_2(x_2-x_1) - b\dot{x}_1 = 0$$

$$m_2\ddot{x}_2 + K_2(x_2-x_1) = F$$

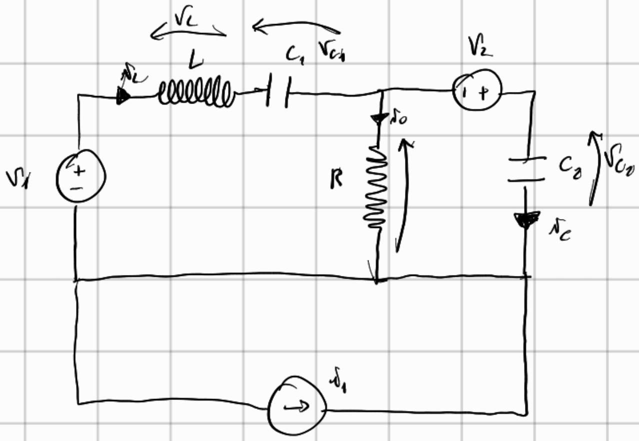
$x_1(t) = x_1 \qquad x_3(t) = \dot{x}_1$

$x_2(t) = x_2 \qquad x_4(t) = \dot{x}_2$

$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \\ \dot{x}_3 = \frac{k_2x_2}{m_1} - \frac{k_2x_1}{m_1} - \frac{k_1x_1}{m_1} + \frac{bx_3}{m_1} \\ \dot{x}_4 = \frac{u}{m_2} - \frac{k_2}{m_2}x_2 + \frac{k_2}{m_2}x_1 \end{cases}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k_1-k_2}{m_1} & \frac{k_2}{m_1} & \frac{b}{m_1} & 0 \\ \frac{k_2}{m_2} & \frac{-k_2}{m_2} & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x(t)$$



$$\begin{cases} v_1 - L\dfrac{di_L}{dt} - v_{C_1} - R i_0 = 0 \\ v_1 - L\dfrac{di_L}{dt} - v_{C_1} + v_2 - v_{C_2} = 0 \\ R i_0 + v_2 - v_{C_2} = 0 \\ i_L - i_0 - i_C = 0 \Rightarrow i_0 = C\dfrac{dv_{C_2}}{dt} - i_L \end{cases}$$

$$\begin{cases} v_1 - L\dfrac{di_L}{dt} - v_{C_1} - RC\dfrac{dv_{C_2}}{dt} + R i_L = 0 \\ v_1 - \cancel{L\dfrac{di_L}{dt}} - \cancel{v_{C_1}} + v_2 - v_{C_2} = 0 \\ RC\dfrac{dv_{C_2}}{dt} - R i_L + v_2 - v_{C_2} = 0 \end{cases}$$

$\Downarrow$

$$v_{C_2} - v_2 - RC\frac{dv_C}{dt} + R i_L = 0$$

$$v_1 - L\frac{di_L}{dt} - v_{C_1} + v_2 - v_{C_2} = 0$$

$$RC\frac{dv_C}{dt} - R i_L + v_2 - v_{C_2} = 0$$

$x_1(t) = i_L(t)$

$x_2(t) = v_{C_1}(t)$

$x_3(t) = v_{C_2}(t)$

$$v_1 - L\frac{di_L}{dt} - v_{C_1} + v_2 - v_{C_2} = 0$$

$$RC\frac{dv_{C_2}}{dt} - Ri_L + v_2 - v_{C_2} = 0$$

$$i_L = C\frac{dv_{C_1}}{dt}$$

$$\begin{cases} \dot{x}_1(t) = \frac{u_1}{L} - \frac{x_2}{L} + \frac{u_2}{L} - \frac{x_3}{L} \\ \dot{x}_2(t) = \frac{x_1}{C} - \frac{u_2}{RC} + \frac{x_2}{RC} \\ \dot{x}_3(t) = \frac{x_1}{C} \\ y(t) = x_3(t) - u_2 \end{cases}$$

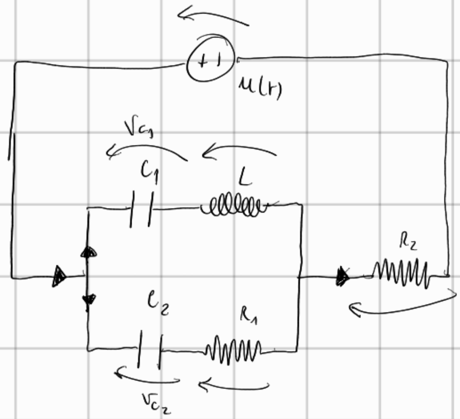
$$\dot{x}(t) = \begin{bmatrix} 0 & -\frac{1}{L} & -\frac{1}{L} \\ \frac{1}{C} & \frac{1}{RC} & 0 \\ \frac{1}{C} & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{L} & \frac{1}{L} \\ 0 & -\frac{1}{RC} \\ 0 & 0 \end{bmatrix} u(t)$$

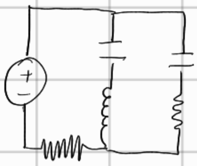
$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & -1 \end{bmatrix} u(t)$$

3) Un motore in corrente continua è suddivisibile in 3 componenti principali: un rotore, uno statore e delle spazzole.

1) STATORE: è la componente fissa composta da struttura esterna e magnete permanente. Imprime un campo magnetico che permette la rotazione del rotore per effetto della legge di Lenz. Le spire percorse da corrente sono interpretabili come fili, soggetti a forza $= i\,\vec{l} \times \vec{B}$. La coppia di forze agente sulle spire imprime un momento che mette in rotazione il sistema di spire, fornendo il momento necessario all'albero motore.

2) ROTORE: formato da un agglomerato di spire percorse da corrente. Il rotore ruota per effetto della coppia di forze appena descritte. La presenza di tanti avvolgimenti di spire, dà una relazione lineare tra corrente e coppia sull'albero e tra forza controelettromotrice e velocità angolare. I parametri di proporzionalità rendono il sistema più consono all'utilizzo come generatore oppure come motore.





$u - v_{C_1} - L\frac{di_L}{dt} - R_2 i_0 = 0$ $\qquad$ $i_L = C_1\frac{dv_{C_1}}{dt}$

$v_{C_2} + R_1 C_2\frac{dv_{C_2}}{dt} - L\frac{di_L}{dt} - v_{C_1} = 0$

$i_0 = i_L + C_2\frac{dv_{C_2}}{dt}$ $\qquad$ $C_2\frac{dv_{C_2}}{dt} = \frac{L}{R_1}\frac{di_L}{dt} + \frac{v_{C_1}}{R_1} - \frac{v_{C_2}}{R_1}$

$u - v_{C_1} - L\frac{di_L}{dt} - R_2 i_L - R_2 C_2\frac{dv_{C_2}}{dt} = 0$

$\Downarrow$

1) $u - \cancel{v_{C_1}} - v_{C_2} - R_1 C_2\frac{dv_{C_2}}{dt} + \cancel{v_{C_1}} - R_2 i_L - R_2 C_2\frac{dv_{C_2}}{dt} = 0$

2) $u - v_{C_1} - L\frac{di_L}{dt} - R_2 i_L - \frac{R_2}{R_1}L\frac{di_L}{dt} - \frac{R_2}{R_1}v_{C_1} + \frac{R_2 v_{C_2}}{R_1} = 0$

3) $i_L = C\frac{dv_{C_1}}{dt}$

1) $\frac{dv_{C_2}}{dt}(R_1+R_2)C_2 = u - v_{C_2} - R_2 i_L$

2) $\frac{di_L}{dt}\left(L + \frac{R_2}{R_1}L\right) = u - \frac{R_1+R_2}{R_1}v_{C_1} - R_2 i_L + \frac{R_2}{R_1}v_{C_2}$

3) $i_L = C\frac{dv_{C_1}}{dt}$

$x_1(t) = i_L$

$x_2(t) = v_{c_1}$

$x_3(t) = v_{c_2}$

$$\dot{x}_1(t) = \frac{u}{L\left(\frac{R_1+R_2}{R_1}\right)} - \frac{x_2}{L} - \frac{R_2 x_1}{L\left(\frac{R_1+R_2}{R_1}\right)} + \frac{\frac{R_2}{R_1} \cdot x_3}{L\left(\frac{R_1+R_2}{R_1}\right)}$$

$$\dot{x}_2(t) = \frac{x_1}{C}$$

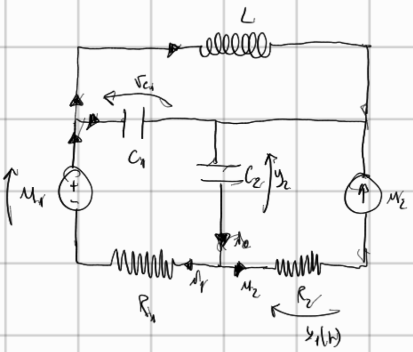
$$\dot{x}_3(t) = \frac{u}{(R_1+R_2)C} - \frac{x_3}{(R_1+R_2)C} - \frac{R_2 x_1}{(R_1+R_2)C}$$

$$u - v_{c_1} - L\frac{di_L}{dt} - R_2 i_0 = 0$$

$$v_{c_2} + R_1 C_2 \frac{dv_{c_2}}{dt} - L\frac{di_L}{dt} - v_{c_1} = 0$$

$$i_0 = i_L + C_2\frac{dv_{c_2}}{dt}$$

4)



1) $v_{C_1} = L\dfrac{di_2}{dt}$

2) $u_1 - v_{C_1} - v_{C_2} - R_1 i_1 = 0$

3) $i_1 + u_2 - i_0 = 0 \Rightarrow i_1 = C_2\dfrac{dv_{C_2}}{dt} - u_2$

$\Rightarrow$ 2) $u_1 - v_{C_1} - v_{C_2} = R_1 C_2 \dfrac{dv_{C_2}}{dt} + R_1 u_2 = 0$

4) $i_1 - i_2 - i_{C_1} = 0$

$$C_1\frac{dv_{C_1}}{dt} = i_1 - i_2 = C_2\frac{dv_{C_2}}{dt} - u_2 - i_2$$

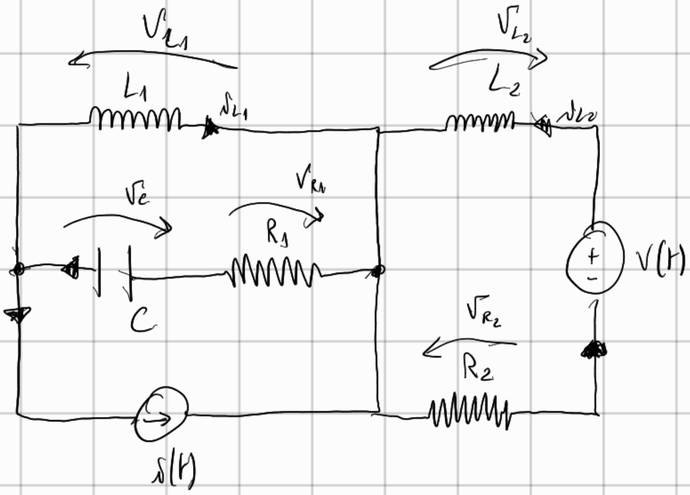
$$R_1 C_1 \frac{dv_{C_1}}{dt} = R_1 C_2 \frac{dv_{C_2}}{dt} - R_1 u_2 - R_1 i_2$$

$$R_1 C_1 \frac{dv_{C_1}}{dt} = \cancel{R_1 u_2} + u_1 - v_{C_1} - v_2 - \cancel{R_1 u_2} - R_1 i_2$$

$$R_1 C_1 \frac{dv_{C_1}}{dt} = u_1 - v_{C_1} - v_{C_2} - R_1 i_2$$

$$\begin{cases} \dfrac{di_2}{dt} = \dfrac{v_{C_1}}{L} \\[2ex] \dfrac{dv_{C_1}}{dt} = \dfrac{u_1}{R_1 C_1} - \dfrac{v_{C_1}}{R_1 C_1} - \dfrac{v_{C_2}}{R_1 C_1} - \dfrac{i_2}{C_1} \\[2ex] \dfrac{dv_{C_2}}{dt} = \dfrac{u_1}{R_1 C_2} - \dfrac{v_{C_1}}{R_1 C_2} - \dfrac{v_{C_2}}{R_1 C_2} + \dfrac{u_2}{C_2} \end{cases}$$

2)



$$i_C = i_{L_1} + i$$

1) $$C\frac{dv_C}{dt} = i_{L_1} + i$$

2) $$v_C + R_1\left(i_{L_1} + i\right) - L_1\frac{di_{L_1}}{dt} = 0$$

$$v_{L_2} - v + v_{R_2} = 0$$

3) $$L_2\frac{di_{L_2}}{dt} - v + R_2 i_{R_2} = 0$$

$i_{L_1} = x_1(t)$
$i_{L_2} = x_2(t)$
$v_C = x_3(t)$

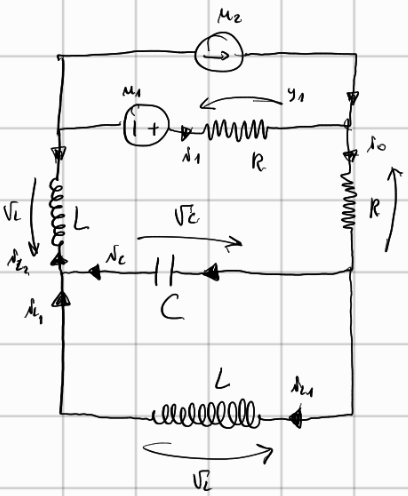
$$\begin{cases} \dot{x}_1 = \frac{x_3}{L_1} + \frac{R_1 x_1}{L_1} + \frac{u_2}{L_1} \\ \dot{x}_2 = \frac{u_1}{L_2} - \frac{R_2}{L_2}x_2 \\ \dot{x}_3 = \frac{x_1}{C} + \frac{u_2}{C} \end{cases}$$

$y_1 = \frac{1}{2}L_1 x_1^2$

$y_2 = \frac{1}{2}L_2 x_2^2$

$y_3 = \frac{1}{2}C x_3^2$

1)



1) $V_C = L\dfrac{di_{L1}}{dt}$

2) $i_C + i_{L1} = i_0 \Rightarrow C\dfrac{dV_C}{dt} = i_{L2} - i_{L1}$

NOTA: $u_2 + i_1 = i_0$
$u_2 + i_1 = i_{L2}$ $\Rightarrow i_0 = i_{L2}$
$i_1 = u_2 - i_{L2}$

3) $V_{L2} + V_C + R\, i_0 + R\, i_1 - u_1 = 0$

$V_{L2} + V_C + R\cancel{i_{L2}} + R u_2 - R\cancel{i_{L2}} - u_1 = 0$

(3) $L\dfrac{di_{L2}}{dt} = u_1 - V_C - R u_2$

$$\begin{cases} \dot{x}_1 = \dfrac{x_3}{L} \\ \dot{x}_2 = -\dfrac{x_3}{L} + \dfrac{u_1}{L} - \dfrac{R}{L}u_2 \\ \dot{x}_3 = \dfrac{-x_1}{C} + \dfrac{x_2}{C} \end{cases}$$

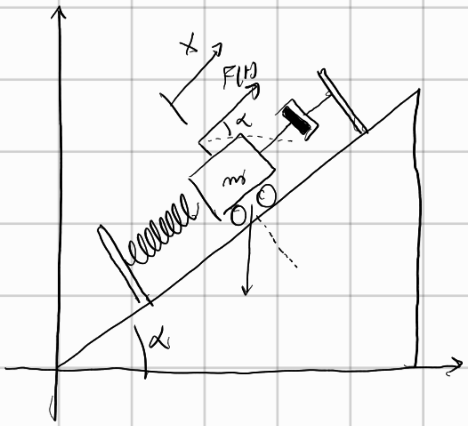
$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & \frac{1}{L} \\ 0 & 0 & -\frac{1}{L} \\ -\frac{1}{C} & \frac{1}{C} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ \frac{1}{L} & -\frac{R}{L} \\ 0 & 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & R \\ 0 & -R \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix} u(t)$$

$y_2 = R\, x_2$

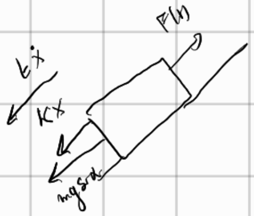
$y_1 = -R\, x_2 + R u_2$

2)



Ho 1 grado di libertà.

Scrivo equazioni



$$m\ddot{x} = F - mg\sin\alpha - Kx - b\dot{x}$$

$x_1(t) = x(t) \qquad x_2(t) = \dot{x}(t)$

$u_1 = F$

$u_2 = g$

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\frac{K}{m}x_1 - b x_2 + \frac{u_1}{m} - u_2 \sin\alpha \end{cases}$$

$T = \frac{1}{2}m\dot{x}^2 \qquad U = mgx\sin\alpha + \frac{1}{2}kx^2$

$\mathcal{L} = \frac{1}{2}m\dot{x}^2 - mgx\sin\alpha - \frac{1}{2}kx^2 \qquad D = \frac{1}{2}B\dot{x}^2$

3) I metodi di integraz. numerica sono usati per calcolare una soluz. approssimata di un sistema di equaz. differenziali, e si basano sul calcolo del valore della soluzione in determinati istanti di tempo t_i.

Partendo da un problema del tipo:

$\dot{x}(t) = f(x,t) \quad t \in [T_0, T_N] \quad x(t_0) = x_0$, che se ben posto ammette 1 e 1 sola soluzione, si decompone l'intervallo $[T_0, T_N]$ definendo n valori $T_0 = t_0 < t_1 < \dots < t_N = T_N$ e si esprime il valore di $x(t_i)$ in funzione dei valori precedenti di x calcolati (single step se dipende da $x(t_{i-1})$, multistep se da più parametri).

L'obiettivo è trovare un valore x_i che approssimi al meglio possibile il valore $x(t_i)$. Si parte quindi da un'uguaglianza:

$$x(t_{i+1}) = x(t_i) + h\Delta(f, x_i, t, h)$$

, dove Δ è una funzione che mi indica il comportamento dell'eq. differenziale, e da essa si approssima il valore di $x(t_{i+1})$.

METODO DI EULERO:

Il metodo di Eulero usa l'approssimazione di derivata come rapp. incrementale:

$$\frac{x_{i+1} - x_i}{h} \approx f(x_i, t_i)$$

Quindi:

$$x_{i+1} = x_i + h f(x_i, t_i).$$

Risulta essere un algoritmo single step e a passo fisso per come definito.

Il calcolo di x_{i+1}, da un punto di vista algoritmico si riduce a un sistema di 2 equaz. non accoppiate:

$K_1 = f(x_i, t_i)$
$x_{i+1} = x_i + hK_1$

L'approccio di Runge-Kutta usa invece l'approssimaz. di area sotto la curva come base del rettangolo per valore intermedio della funzione:

$$x(t_{i+1}) = x(t_i) + \int_{t_i}^{t_i+h} f(x(\tau), \tau)\, d\tau$$

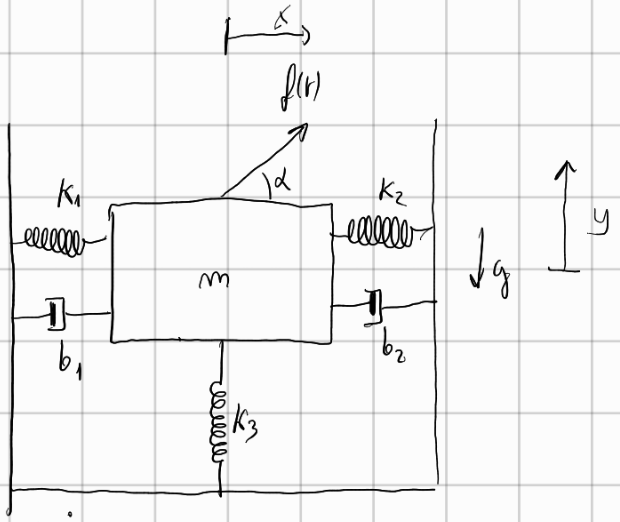
Dove $x(t_{i+1}) \approx x(t_i) + h f\left(x\left(t_i + \frac{h}{2}\right), t_i + \frac{h}{2}\right)$

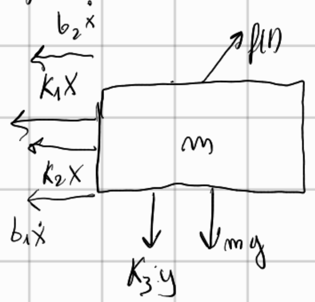
Per il calcolo di $x\left(t_i + \frac{h}{2}\right)$ si può procedere con il metodo di Eulero oppure approcciare il problema ricorsivamente riapplicando Runge-Kutta n volte.

$x\left(t_i + \frac{h}{2}\right) \approx x(t_i) + \frac{h}{2} f(x(t_i), t_i)$
Il tutto si riduce a un sistema di eq. non accopp.

$K_1 = f(x_i, t_i)$
$K_2 = f\left(x_i + \frac{h}{2}K_1, t_i + \frac{h}{2}\right)$
$x_{i+1} = x(t_i) + hK_2$

1)





$$\begin{cases} m\ddot{x} = f\cos\alpha - b_1\dot{x} - b_2\dot{x} - k_1 x - k_2 x \\ m\ddot{y} = f\sin\alpha - mg - K_3 y \end{cases}$$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2$$

$$U = \frac{1}{2}k_1 x^2 + \frac{1}{2}k_2 x^2 + \frac{1}{2}k_3 y^2 + mgy$$

$$D = \frac{1}{2}b_1\dot{x}^2 + \frac{1}{2}b_2\dot{x}^2$$

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 - \frac{1}{2}k_1 x^2 - \frac{1}{2}k_2 x^2 - \frac{1}{2}k_3 y^2 - mgy$$

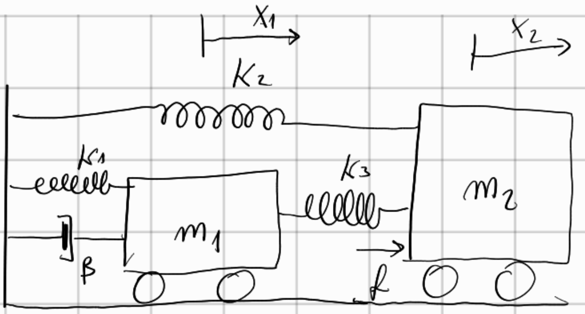
1° eq:)

$$m\ddot{x} + K_1 x + K_2 x + b_1\dot{x} + b_2\dot{x} = f\cos\alpha$$

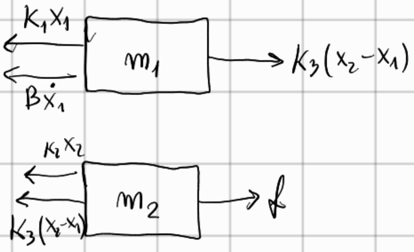
$$m\ddot{y} + mg + K_3 y = f\sin\alpha$$

$X_1(t) = x$ $\quad X_2(t) = y$ $\quad X_3(t) = \dot{x}$ $\quad X_4(t) = \dot{y}$ $\quad f = u_1$ $\quad g = u_2$

$$
\begin{cases}
\dot{x}_1 = x_3 \\
\dot{x}_2 = x_4 \\
\dot{x}_3 = -\dfrac{(k_1+k_2)}{m}x_1 - \dfrac{(b_1+b_2)}{m}x_3 + \dfrac{u_1\cos\alpha}{m} \\
\dot{x}_4 = -\dfrac{k_3}{m}x_2 + \dfrac{u_1\sin\alpha}{m} - u_2
\end{cases}
$$



2 gradi di libertà del sistema.



$$\begin{cases} m_2 \ddot{x}_2 = f - K_2 x_2 - K_3(x_2 - x_1) \\ m_1 \ddot{x}_1 = K_3(x_2 - x_1) - K_1 x_1 - \beta \dot{x}_1 \end{cases}$$

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$$U = \frac{1}{2} K_3 (x_2 - x_1)^2 + \frac{1}{2} K_1 x_1^2 + \frac{1}{2} K_2 x_2^2$$

$$D = \frac{1}{2} \beta \dot{x}_1^2$$

1) $m_1 \ddot{x}_1 - \left[+K_3(x_2 - x_1) - K_1 x_1 \right] + \beta \dot{x}_1 = 0$

$$m_2 \ddot{x}_2 - \left[-K_3(x_2 - x_1) - K_2 x_2 \right] = f$$

$x_1(t) = x_1 \qquad x_2(t) = x_2$

$x_3(t) = \dot{x}_1 \qquad x_4(t) = \dot{x}_2$

$$\begin{cases} \dot{x}_1(t) = x_3 \\ \dot{x}_2(t) = x_4 \\ \dot{x}_3(t) = \frac{K_3}{m_1}(x_2 - x_1) - \frac{K_1}{m_1} x_1 - \frac{\beta}{m_1} x_3 \end{cases}$$

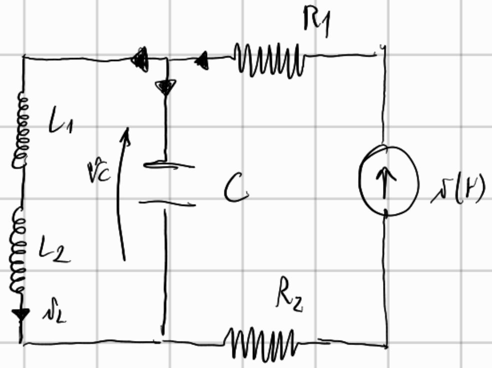
$$\dot{x}_4(t) = \frac{u}{m_2} - \frac{K_2}{m_2}x_2 - \frac{K_3}{m_2}(x_2 - x_1)$$

$$y_1 = x_2(t)$$

$$y_2(t) = x_4(t)$$

$$y_3(t) = \frac{u}{m_2} - \frac{K_2}{m_2}x_2 - \frac{K_3}{m_2}(x_2 - x_1)$$

2)



(2) $$v_C = (L_1 + L_2)\frac{d\,i_L}{dt}$$

$$i_C + i_L - i = 0$$

$$i_C = i - i_L$$

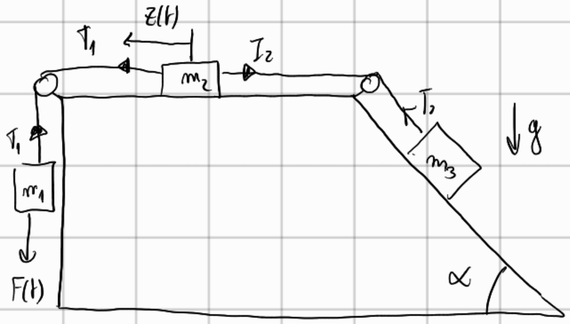
(1) $$C\frac{dv_C}{dt} = i - i_L$$

$$U = \frac{1}{2}L_1 i_L^2 + \frac{1}{2}L_2 i_L^2 + \frac{1}{2}C v_C^2$$

$$x_1(t) = i_L(t)$$
$$x_2(t) = v_C(t)$$

$$\begin{cases} \dot{x}_1(t) = \dfrac{x_2(t)}{L_1 + L_2} \\ \dot{x}_2(t) = \dfrac{u_1}{C} - \dfrac{x_1}{C} \\ y_1 = \dfrac{1}{2}(L_1 + L_2)x_1^2 + \dfrac{1}{2}C x_2^2 \end{cases}$$

1)



$$m_1\ddot{z} = F + m_1 g - T_1$$

$$m_2\ddot{z} = T_1 - T_2$$

$$m_3\ddot{z} = T_2 - m_3 g \sin\alpha$$

$$(m_1 + m_2 + m_3)\ddot{z} = F + m_1 g - m_3 g \sin\alpha$$

$$\ddot{z} = \frac{u_1}{m_1+m_2+m_3} + \frac{u_2(m_1 - m_3 \sin\alpha)}{m_1+m_2+m_3}$$

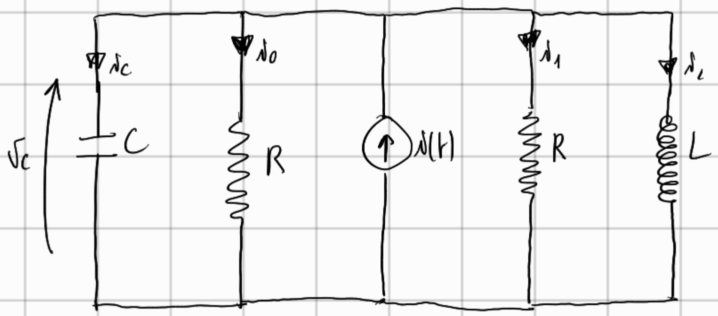
$$T = \frac{1}{2}m_1\dot{z}^2 + \frac{1}{2}m_2\dot{z}^2 + \frac{1}{2}m_3\dot{z}^2$$

$$U = -m_1 g z + m_3 g z \sin\alpha$$

$$\mathcal{L} = \frac{1}{2}M\dot{z}^2 + m_1 g z - m_3 g z \sin\alpha$$

$$M\ddot{z} - m_1 g + m_3 g \sin\alpha = F$$

2)



$$v_c = L \frac{d i_L}{dt}$$

$$J - i_0 - i_1 - i_c - i_L = 0$$

$$J - \frac{v_c}{R} - \frac{v_c}{R} - i_L - C\frac{dv_c}{dt} = 0$$

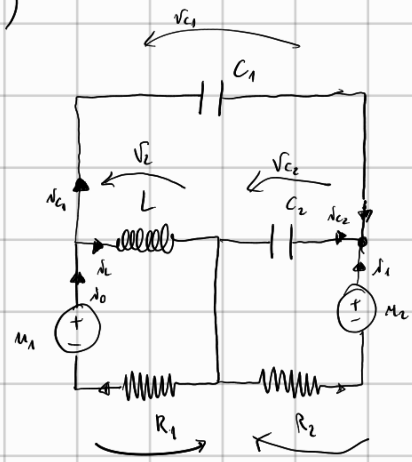
$$\begin{cases} ① \quad \dfrac{d i_L}{dt} = \dfrac{v_c}{L} \\ \\ ② \quad \dfrac{dv_c}{dt} = \dfrac{J}{C} - \dfrac{2 v_c}{CR} - \dfrac{i_L}{C} \end{cases}$$

$$\begin{cases} \dot{x}_1(t) = \dfrac{x_2}{L} \\ \\ \dot{x}_2(t) = \dfrac{u_1}{C} - \dfrac{2 x_2}{CR} - \dfrac{x_1}{C} \end{cases}$$

$$y_1 = \frac{1}{2} L x_1^2$$

$$y_2 = \frac{1}{2} C x_2^2$$

1)



$y_1 = R_2 i_1 = -R_2 i_{C_1} - R_2 i_{C_2}$

$y_2 = i_0 R_1 = R_1 i_{C_1} + R_1 i_L$

$v_{C_1} - v_L - v_{C_2} = 0$

(1) $v_{C_1} - v_{C_2} = L\dfrac{di_L}{dt}$

$i_0 - i_{C_1} - i_L = 0$ $\qquad i_0 = i_{C_1} + i_L$

$i_{C_1} + i_{C_2} + i_1 = 0$ $\qquad i_1 = -i_{C_1} - i_{C_2}$

$i_L - i_0 - i_1 - i_{C_2} = 0$

$u_1 - v_L - R_1 i_0 = 0 \quad \Rightarrow \quad u_1 - v_L - R_1 i_{C_1} - R_1 i_L = 0$

$u_2 + v_{C_2} - R_2 i_1 = 0 \qquad u_2 + v_{C_2} + R_2 i_{C_1} + R_2 i_{C_2} = 0$

(1) $R_1 C \dfrac{dv_{C_1}}{dt} = u_1 - v_L - R_1 i_L = u_1 - (v_{C_1} - v_{C_2}) - R_1 i_L$

(2) $R_2 C_2 \dfrac{dv_{C_2}}{dt} = -u_2 - v_{C_2} - R_2 i_{C_1}$

(1) $\dfrac{dv_{C_1}}{dt} = \dfrac{u_1}{R_1 C} - \dfrac{v_{C_1}}{R_1 C} + \dfrac{v_{C_2}}{R_1 C} - \dfrac{i_L}{C}$

$$\frac{dv_{C_2}}{dt} = \frac{-u_2}{R_2 C_2} - \frac{v_{C_2}}{R_2 C_2} - \frac{1}{C_2}\cdot\left(\frac{u_1}{R_1} - \frac{v_{C_1}}{R_1} + \frac{v_{C_2}}{R_1} - i_L\right)$$

$$\frac{dv_{C_2}}{dt} = \frac{-u_2}{R_2 C_2} - v_{C_2}\left(\frac{1}{R_2 C_2} + \frac{1}{R_1 C_2}\right) + \frac{v_{C_1}}{R_1 C_2} - \frac{u_1}{R_1 C_2} + \frac{i_L}{C_2}$$

$$\begin{cases} v_{C_1} - v_{C_2} = L\dfrac{di_L}{dt} \\[2ex] \dfrac{dv_{C_1}}{dt} = \dfrac{u_1}{R_1C} - \dfrac{v_{C_1}}{R_1C} + \dfrac{v_{C_2}}{R_1C} - \dfrac{i_L}{C} \\[2ex] \dfrac{dv_{C_2}}{dt} = -\dfrac{u_1}{R_1C_2} - \dfrac{u_2}{R_2C_2} + \dfrac{v_{C_1}}{R_1C_2} - v_{C_2}\left(\dfrac{1}{R_2C_2} + \dfrac{1}{R_1C_2}\right) + \dfrac{i_L}{C_2} \end{cases}$$

$x_1(t) = i_L(t) \quad x_2(t) = v_{C_1}(t) \quad x_3(t) = v_{C_2}(t)$

$$\begin{cases} \dot{x}_1(t) = \dfrac{x_2(t)}{L} - \dfrac{x_3(t)}{L} \\[2ex] \dot{x}_2(t) = -\dfrac{x_1(t)}{C} - \dfrac{x_2(t)}{R_1C} + \dfrac{x_3(t)}{R_1C} + \dfrac{u_1}{R_1C} \\[2ex] \dot{x}_3(t) = \dfrac{x_1(t)}{C_2} + \dfrac{x_2(t)}{R_1C_2} - \left(\dfrac{1}{R_2C_2} + \dfrac{1}{R_1C_2}\right)x_3(t) - \dfrac{u_1}{R_1C_2} - \dfrac{u_2}{R_2C_2} \end{cases}$$

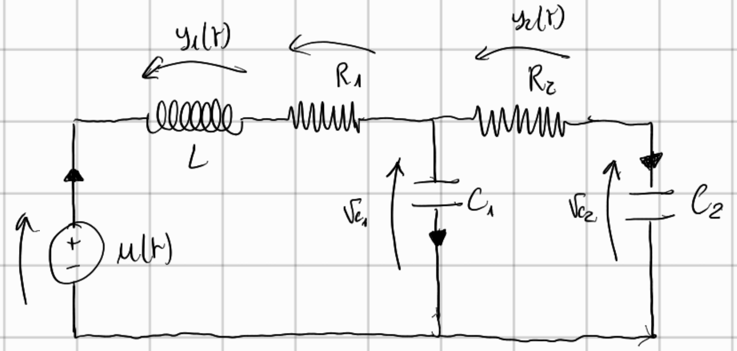
$$y_1(t) = x_3(t)\left(1 + \frac{R_1}{R_2} - \frac{R_2}{R_1}\right) + u_2$$

$$y_2(t) = -x_2(t) + x_3(t) + u_1$$

$$\dot{x}(t) = \begin{bmatrix} 0 & \frac{1}{L} & -\frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{R_1C} & \frac{1}{R_1C} \\ \frac{1}{C_2} & \frac{1}{R_1C_2} & -\left(\frac{1}{R_2C_2} + \frac{1}{R_1C_2}\right) \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ \frac{1}{R_1C} & 0 \\ -\frac{1}{R_1C_2} & -\frac{1}{R_2C_2} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 + \frac{R_1}{R_2} - \frac{R_2}{R_1} \\ 0 & -1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} u(t)$$

1)



(0) $u(t) - L\dfrac{di_L}{dt} - R_1 i_L - v_{C_1} = 0$

$v_{C_1} - R_2 i_{C_2} - v_{C_2} = 0$

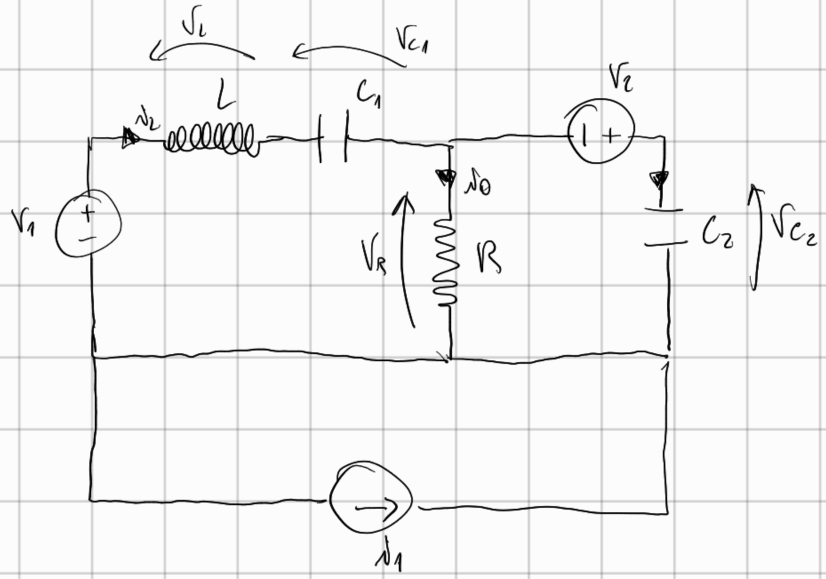
(1) $v_{C_1} - R_2 C_2 \dfrac{dv_{C_2}}{dt} - v_{C_2} = 0 \Rightarrow R_2 C_2 \dfrac{dv_{C_2}}{dt} = v_{C_1} - v_{C_2}$

$C_2 \dfrac{dv_{C_2}}{dt} = \dfrac{v_{C_1}}{R_2} - \dfrac{v_{C_2}}{R_2}$

(2) $i_L = i_{C_1} + i_{C_2}$

(3) $i_L = C_1 \dfrac{dv_{C_1}}{dt} + \dfrac{v_{C_1}}{R_2} - \dfrac{v_{C_2}}{R_2}$

1)



$i_L = i_0 + i_{C_2}$

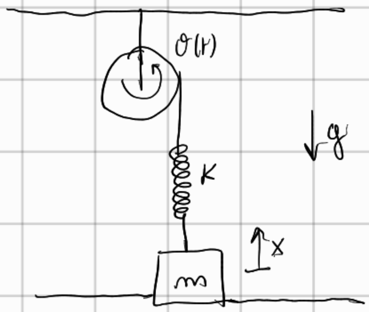
(1) $v_1 - L\dfrac{di_L}{dt} - v_{C_1} + v_2 - v_{C_2} = 0$

(3) $i_L = \dfrac{v_{C_2} - v_2}{R} + C_2\dfrac{dv_{C_2}}{dt}$

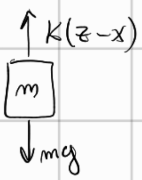
(2) $C_1\dfrac{dv_{C_1}}{dt} = i_L$

(3) $R i_0 + v_2 - v_{C_2} = 0 \Rightarrow i_0 = \dfrac{v_{C_2} - v_2}{R}$

2)



$Z = R\theta$



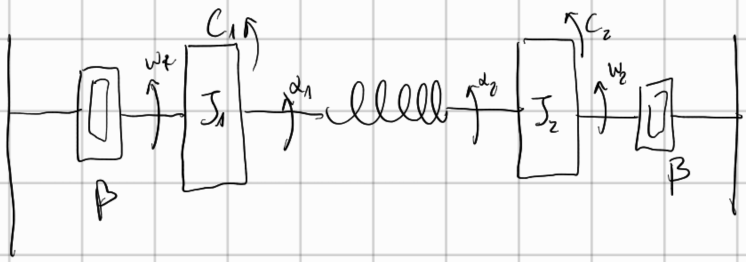
$$m\ddot{x} = K(z-x) - mg$$

$$m\ddot{x} = K(R\theta - x) - mg$$

$$U = mgx + \frac{1}{2}K(R\theta - x)^2 \qquad T = \frac{1}{2}m\dot{x}^2$$

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 - mgx - \frac{1}{2}K(R\theta - x)^2$$

$$m\ddot{x} + mg - K(R\theta - x) = 0$$



$$J_1\dot{\omega}_1 = C_1 - \beta\omega_1 - K(\alpha_1 - \alpha_2)$$

$$J_2\dot{\omega}_2 = C_2 - \beta\omega_2 - K(\alpha_2 - \alpha_1)$$

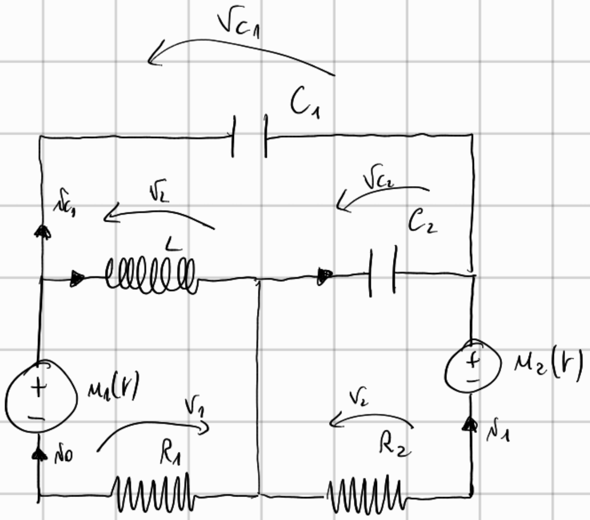
$$U = \frac{1}{2}K(\alpha_1 - \alpha_2)^2 \qquad \mathcal{L} = \frac{1}{2}J_1\dot{q}_1^2 + \frac{1}{2}J_2\dot{q}_2^2 - \frac{1}{2}K(q_1 - q_2)^2$$

$$T = \frac{1}{2}J_1\dot{\alpha}_1^2 + \frac{1}{2}J_2\dot{\alpha}_2^2 \qquad D = \frac{1}{2}\beta\dot{q}_1^2 + \frac{1}{2}\beta\dot{q}_2^2$$

$$J_1\ddot{q}_1 + K(q_1 - q_2) + \beta\dot{q}_1 = C_1$$

$$J_2\ddot{q}_2 - K(q_1 - q_2) + \beta\dot{q}_2 = C_2$$

1)



(1) $v_{C_1} = L\dfrac{di_L}{dt} + v_{C_2}$

$u_1 - v_L - R_1 i_0 = 0$ $\qquad i_0 = \dfrac{u_1}{R_1} - \dfrac{v_L}{R_1}$

$u_2 + v_{C_2} - R_2 i_1 = 0$ $\qquad i_1 = \dfrac{u_2}{R_2} + \dfrac{v_{C_2}}{R_2}$

$i_0 = i_{C_1} + i_L$ $\qquad y_1 = R_1 i_0$

$\dfrac{u_1}{R_1} - \dfrac{v_L}{R_1} = C_1\dfrac{dv_{C_1}}{dt} + i_L$ $\qquad = u_1 - v_L = u_1 - v_{C_1} + v_{C_2}$

$R_2 i_1 = u_2 + v_{C_2}$

$\dfrac{u_1}{R_1} - \dfrac{v_{C_1}}{R_1} + \dfrac{v_{C_2}}{R_1} = \underbrace{C_1\dfrac{dv_{C_1}}{dt}}_{i_{C_1}} + i_L$

(2) $C_1\dfrac{dv_{C_1}}{dt} = \dfrac{u_1}{R_1} - \dfrac{v_{C_1}}{R_1} + \dfrac{v_{C_2}}{R_1} - i_L$

$i_1 + i_{C_2} + i_{C_1} = 0$

$\dfrac{u_2}{R_2} + \dfrac{v_{C_2}}{R_2} + C_2\dfrac{dv_{C_2}}{dt} + \dfrac{u_1}{R_1} - \dfrac{v_{C_1}}{R_1} + \dfrac{v_{C_2}}{R_1} - i_L = 0$

(3) $C_2\dfrac{dv_{C_2}}{dt} = \dfrac{v_{C_1}}{R_1} - v_{C_2}\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) - \dfrac{u_1}{R_1} - \dfrac{u_2}{R_2} + i_L$

$i_L(t) = x_1(t)$

$v_{C_1}(t) = x_2(t)$

$v_{C_2}(t) = x_3(t)$

$$\begin{cases} \dot{x}_1(t) = \dfrac{x_2(t)}{L} - \dfrac{x_3(t)}{L} \\[2ex] \dot{x}_2(t) = -\dfrac{x_1(t)}{C_1} - \dfrac{x_2(t)}{C_1R_1} + \dfrac{x_3(t)}{C_1R_1} + \dfrac{u_1}{C_1R_1} \\[2ex] \dot{x}_3(t) = \dfrac{x_1(t)}{C_2} + \dfrac{x_2(t)}{C_2R_1} - \dfrac{x_3(t)}{C_2} \cdot \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) - \dfrac{u_1}{C_2R_1} - \dfrac{u_2}{C_2R_2} \end{cases}$$

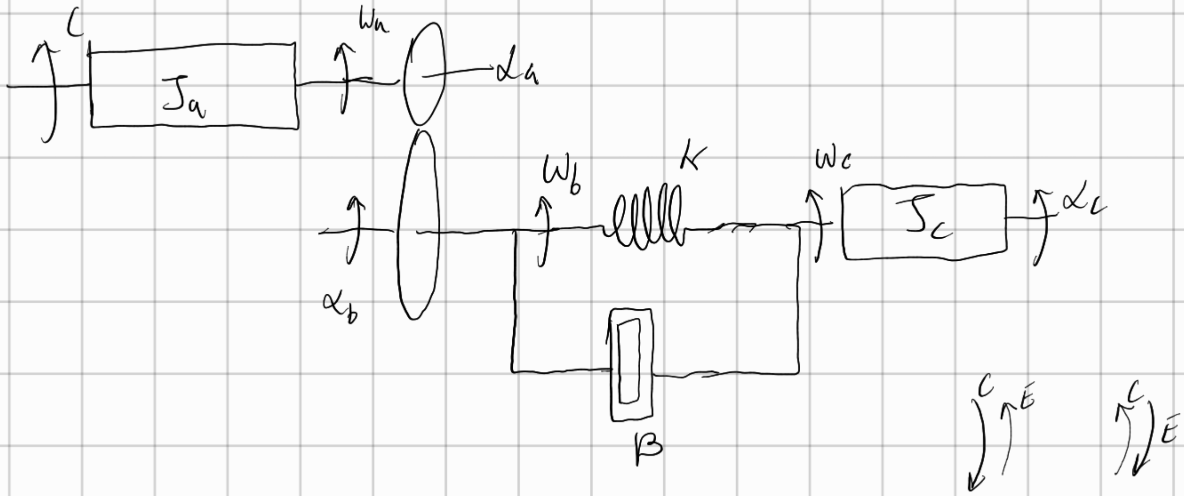
$$y_1(t) = -x_2(t) + x_3(t) + u_1(t)$$

$$y_2(t) = x_3(t) + u_2(t)$$

$$\dot{x}(t) = \begin{bmatrix} 0 & \frac{1}{L} & -\frac{1}{L} \\ -\frac{1}{C_1} & -\frac{1}{C_1R_1} & +\frac{1}{C_1R_1} \\ \frac{1}{C_2} & \frac{1}{C_2R_1} & -\frac{1}{C_2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ \frac{1}{C_1R_1} & 0 \\ -\frac{1}{C_2R_1} & -\frac{1}{C_2R_2} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u(t)$$

2)



$$J_a \dot{W}_a = C - C_r \qquad C_r = K(W_c - W_b) + K($$

$$J_c \dot{W}_c = K(\alpha_c - \alpha_b) + \beta(W_c - W_b)$$

$$K(\alpha_c - m\alpha_a) + \beta(W_c - mW_a)$$

NOTA: $W_a r_a = W_b r_b$

$$\frac{W_a}{W_b} = \frac{1}{m} \Rightarrow W_b = mW_a \qquad \theta_b = m\theta_a$$

$$J_a \dot{W}_a = C - K(\alpha_c - m\alpha_a) - b(W_c - mW_a)$$

$$J_c \dot{W}_c = K(\alpha_c - \alpha_b) + \beta(W_c - mW_a)$$

$$U = \frac{1}{2}K(\alpha_c - m\alpha_a)^2 \qquad \mathcal{L} = f(\alpha_1, \alpha_2) - \frac{1}{2}K(\alpha_c - m\alpha_a)^2$$

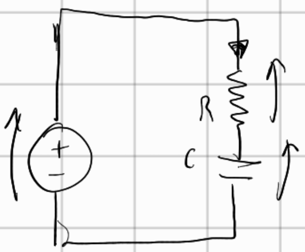
$$T = \frac{1}{2}J_a \dot{\alpha}_a^2 + \frac{1}{2}J_c \dot{\alpha}_c^2$$

$$D = \frac{1}{2}b(W_c - mW_a)^2$$

1) $J_a \ddot{\alpha}_a + (-1)(-1) \cdot K(\alpha_c - m\alpha_a)(-1)m + b(W_c - mW_a)(-1)m = C$

$$J_a \ddot{\alpha}_a = C + mK(\alpha_c - m\alpha_a) + bm(W_c - mW_a)$$

$$J_c \dot{\alpha}_c + K(\alpha_c - m\alpha_a) + b(W_c - mW_a) = 0$$



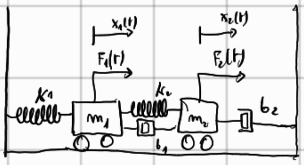
$e - Ri - v_c = 0$

$v_c + RC\dfrac{dv_c}{dt} = e$

$\dfrac{dv_c}{dt} + \dfrac{v_c}{RC} = \dfrac{e}{RC}$

$\dot{x}_1(t) = -\dfrac{x_1(t)}{RC} + \dfrac{e}{RC}$

1)



$$m_1\ddot{x}_1 = F_1(t) - k_1x_1 - K_2(x_1-x_2) - b_1(\dot{x}_1-\dot{x}_2)$$

$$m_2\ddot{x}_2 = F_2(t) - b_2\dot{x}_2 - K_2(x_2-x_1) - b_1(x_2-x_1)$$

$$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2$$

$$U = \frac{1}{2}K_1x_1^2 + \frac{1}{2}K_2(x_2-x_1)^2$$

$$D = \frac{1}{2}b_1(\dot{x}_2-\dot{x}_1)^2 + \frac{1}{2}b_2\dot{x}_2^2$$

$$\mathcal{L} = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 - \frac{1}{2}K_1x_1^2 - \frac{1}{2}K_2(x_2-x_1)^2$$

1) $m_1\ddot{x}_1 - \left(-K_1x_1 - K_2(x_2-x_1)(-1)\right) - b_1(\dot{x}_2-\dot{x}_1) = F_1$

$$m_2\ddot{x}_2 - \left(-K_2(x_2-x_1)\right) + b_1(\dot{x}_2-\dot{x}_1) + b_2\dot{x}_2 = F_2$$

$x_1(t) = x_1 \qquad x_3(t) = \dot{x}_1$

$x_2(t) = x_2 \qquad x_4(t) = \dot{x}_2$

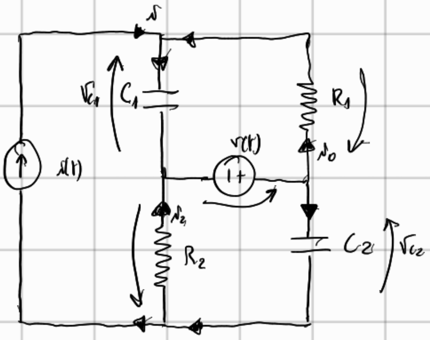
$$\begin{cases} \dot{x}_1(t) = x_3(t) \\ \dot{x}_2(t) = x_4(t) \\ \dot{x}_3(t) = -\frac{K_1}{m_1}x_1(t) - \frac{K_2}{m_1}(x_1-x_2) - \frac{b_1}{m_1}(x_3-x_4) + \frac{u_1}{m_1} \\ \dot{x}_4(t) = -\frac{b_2}{m_2}x_4 - \frac{K_2}{m_2}(x_2-x_1) - \frac{b_1}{m_2}(x_4-x_3) \end{cases}$$

$$\begin{cases} \dot{x}_1(t) = x_3(t) \\ \dot{x}_2(t) = x_4(t) \\ \dot{x}_3(t) = -\frac{1}{m_1}(K_1+K_2)\,x_1(t) + \frac{K_2}{m_1}x_2(t) - \frac{b_1}{m_1}x_3(t) + \frac{b_1}{m_1}x_4(t) + \frac{u_1}{m_1} \\ \dot{x}_4(t) = \frac{K_2}{m_2}x_1(t) - \frac{K_2}{m_2}x_2(t) + \frac{b_1}{m_2}x_3(t) - \frac{1}{m_2}(b_1+b_2)\,x_4(t) + \frac{u_2}{m_2} \end{cases}$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(K_1+K_2)}{m_1} & \frac{K_2}{m_1} & -\frac{b_1}{m_1} & \frac{b_1}{m_1} \\ \frac{K_2}{m_2} & -\frac{K_2}{m_2} & \frac{b_1}{m_2} & -\frac{(b_1+b_2)}{m_2} \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x(t)$$

2)



$v_{C_1} + R_1 i_0 - v = 0$ $\qquad$ $i_0 = \frac{v}{R_1} - \frac{v_{C_1}}{R_1}$

$v - v_{C_2} - R_2 i_2 = 0$ $\qquad$ $i_2 = \frac{v}{R_2} - \frac{v_{C_2}}{R_2}$

$i_{C_1} = i + i_0$

(1) $i_{C_1} = i + \frac{v}{R_1} - \frac{v_{C_1}}{R_1}$

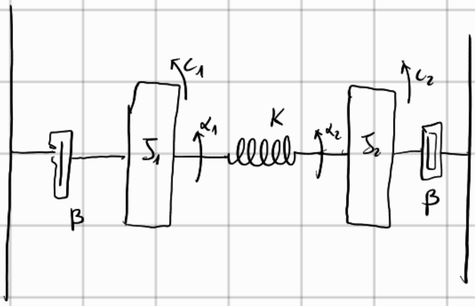
$i_{C_2} = i_2 + i$

(2) $i_{C_2} = \frac{v}{R_2} - \frac{v_{C_2}}{R_2} + i$

$x_1(t) = v_{C_1}(t)$ $\qquad$ $x_2(t) = v_{C_2}(t)$ $\qquad$ $u_1 = i$ $\quad$ $u_2 = v$

$$\begin{cases} \dot{x}_1(t) = \frac{-x_1(t)}{R_1 C_1} + \frac{u_1}{C_1} + \frac{u_2}{R_1 C_1} \\ \dot{x}_2(t) = -\frac{x_2(t)}{R_2 C_2} + \frac{u_1}{C_2} + \frac{u_2}{R_2 C_2} \\ y(t) = \frac{1}{2} C_1 x_1(t)^2 + \frac{1}{2} C_2 x_2(t)^2 \end{cases}$$

2)



$$J_1\dot{\omega}_1 = C_1 - K(\alpha_1 - \alpha_2) - \beta\omega_1$$

$$J_2\dot{\omega}_2 = C_2 - K(\alpha_2 - \alpha_1) - \beta\omega_2$$

$$\mathcal{L} = \frac{1}{2}J_1\omega_1^2 + \frac{1}{2}J_2\omega_2^2 - \frac{1}{2}K(\alpha_1-\alpha_2)^2$$

$$D = \frac{1}{2}\beta\omega_1^2 + \frac{1}{2}\beta\omega_2^2$$

$$\mathcal{L}(q_1, q_2, \dot{q}_1, \dot{q}_2) = \frac{1}{2}J_1\dot{q}_1^2 + \frac{1}{2}J_2\dot{q}_2^2 - \frac{1}{2}K(q_1-q_2)^2$$

$$D = \frac{1}{2}\beta\dot{q}_1^2 + \frac{1}{2}\beta\dot{q}_2^2$$

$$J_1\ddot{q}_1 = -K(q_1-q_2) - \beta\dot{q}_1 + C_1$$

$$J_2\dot{q}_2 - K(q_1-q_2) + \beta\dot{q}_2 = C_2$$

$x_1(t) = q_1 \quad x_2(t) = q_2$

$x_3(t) = \dot{q}_1 \quad x_4(t) = \dot{q}_2$

$$\begin{cases} \dot{x}_1(t) = x_3(t) \\ \dot{x}_2(t) = x_4(t) \\ \dot{x}_3(t) = \frac{-K}{J_1}x_1 + \frac{K}{J_1}x_2 - \frac{\beta}{J_1}x_3 + \frac{u_1}{J_1} \end{cases}$$

$$\dot{x}_4(t) = \frac{K}{J_2}x_1 - \frac{K}{J_2}x_2 - \frac{\beta}{J_2}x_4 + \frac{u_2}{J_2}$$

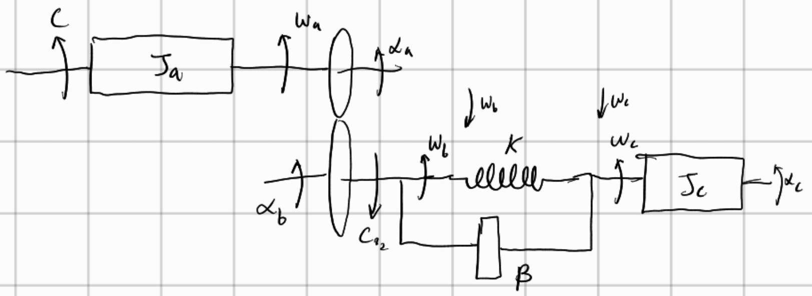
$y_1(t) = x_1(t)$

$y_2(t) = x_2(t)$

$$\dot{X}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K}{J_1} & \frac{K}{J_1} & -\frac{\beta}{J_1} & 0 \\ \frac{K}{J_2} & -\frac{K}{J_2} & 0 & -\frac{\beta}{J_2} \end{bmatrix} X(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{J_1} & 0 \\ 0 & \frac{1}{J_2} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} X(t)$$

2)



$J_a\ddot{\alpha}_a = C - C_r$

$K(\omega_b-\omega_c)$

$C_{r_2}=K(\alpha_c-\alpha_b)+\beta(\omega_c-\omega_b)$

$J_b\ddot{\alpha}_b = K(\dot{\alpha}_b-\dot{\alpha}_c)+\beta(\dot{\alpha}_b-\dot{\alpha}_c)$

$C_r\omega_a = C_{r_2}\omega_b$

$C_r = C_{r_2}\frac{\omega_b}{\omega_a} = C_{r_2}n$

$\frac{\omega_b}{\omega_a}=n$

$\omega_b = n\omega_a$

$J_a\ddot{\alpha}_a = C - nK(\alpha_c - n\alpha_a) - n\beta(\dot{\alpha}_c - n\dot{\alpha}_a)$

$J_b\ddot{\alpha}_b = K(n\dot{\alpha}_a-\dot{\alpha}_c)+\beta(n\dot{\alpha}_a-\dot{\alpha}_c)$

$\mathcal{L} = \frac{1}{2}J_a\dot{\alpha}_a^2 + \frac{1}{2}J_c\dot{\alpha}_c^2 - \frac{1}{2}K(\alpha_c-\alpha_b)^2$

$D = \frac{1}{2}\beta(\dot{\alpha}_c-\dot{\alpha}_b)^2$

$\mathcal{L}(q_1,q_2,\dot{q}_1,\dot{q}_2) = \frac{1}{2}J_a\dot{q}_1^2+\frac{1}{2}J_c\dot{q}_2^2-\frac{1}{2}K(q_2-nq_1)^2$

$D(\dot{q}_1,\dot{q}_2)=\frac{1}{2}\beta(\dot{q}_2-n\dot{q}_1)^2$

1ª EQ:

$-nKq_2+n^2Kq_1-n\beta\dot{q}_2+n^2\beta\dot{q}_1$

$J_a\ddot{q}_1 - nK(q_2-nq_1) - n\beta(\dot{q}_2-n\dot{q}_1) = C_1$

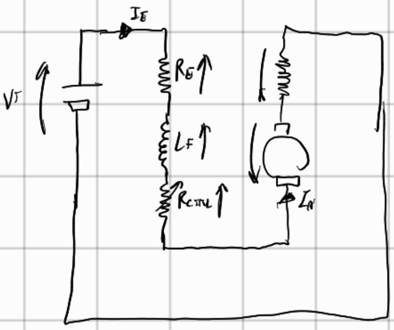
$J_b\ddot{q}_2 + K(q_2-nq_1)+\beta(\dot{q}_2-n\dot{q}_1)=0$

$Kq_2-nKq_1+\beta\dot{q}_2-n\beta\dot{q}_1$

$q_1=x_1$
$q_2=x_2$
$\dot{q}_1=x_3$
$\dot{q}_2=x_4$

$$\begin{cases}\dot{x}_1(t)=x_3(t)\\ \dot{x}_2(t)=x_4(t)\\ \dot{x}_3(t)=\frac{u_1}{J_a}-\frac{n^2K}{J_a}x_1+\frac{nK}{J_a}x_2-\frac{n^2\beta}{J_a}x_3+\frac{n\beta}{J_a}x_4\\ \dot{x}_4(t)=\frac{nK}{J_b}x_1(t)-\frac{K}{J_b}x_2(t)+\frac{n\beta}{J_b}x_3-\frac{\beta}{J_b}x_4\end{cases}$$

$$\dot{x}(t)=\begin{bmatrix}0&0&1&0\\0&0&0&1\\-\frac{n^2K}{J_a}&\frac{nK}{J_a}&-\frac{n^2\beta}{J_a}&\frac{n\beta}{J_a}\\ \frac{nK}{J_b}&-\frac{K}{J_b}&\frac{n\beta}{J_b}&-\frac{\beta}{J_b}\end{bmatrix}x(t)+\begin{bmatrix}0\\0\\\frac{1}{J_a}\\0\end{bmatrix}u(t)$$



$$\Phi = K_\phi I_F$$

$$\Phi = K_\phi I_A$$

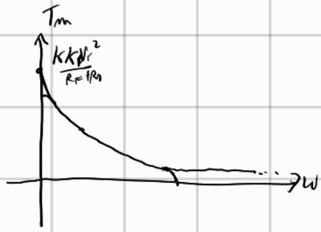
$$E_A = K\Phi\omega_m = K K_\phi \omega_m I_A$$

$$T_m = K\Phi I_A = K K_\phi I_A^2$$

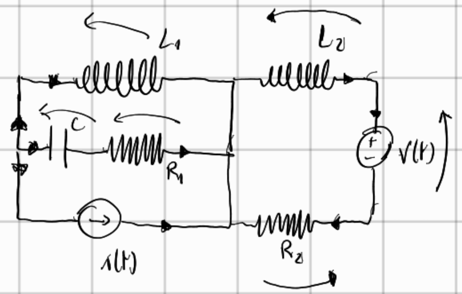
$$V_T = R_F I_A + R_A I_A + E_A$$

$$I_A = \frac{V_T}{R_F + R_A + K K_\phi \omega_m}$$

$$T_m = \frac{K K_\phi V_T^2}{(R_F + R_A + K K_\phi \omega_m)^2}$$



3)



$X_1 = i_{L1}$ $\quad X_2 = i_{L2}$ $\quad X_3 = v_C$ $\quad u_1 = v$

$u_2 = i$

$$\begin{cases} \dot{x}_1(t) = -\frac{R_1}{L_1} x_1(t) + \frac{x_3(t)}{L_1} - \frac{R_1}{L_1} u_2 \\ \dot{x}_2(t) = -\frac{R_2}{L_2} x_2(t) - \frac{u_1}{L_2} \\ \dot{x}_3(t) = -\frac{x_1(t)}{C_1} - \frac{u_2}{C_1} \end{cases}$$

$$y(t) = \frac{1}{2} L_1 x_1^2 + \frac{1}{2} L_2 x_2^2 + \frac{1}{2} C x_3^2$$

1) $R_2 i_{L_2} + v + L_2 \frac{di_{L_2}}{dt} = 0$

$$\frac{di_{L_2}}{dt} = -\frac{v}{L_2} - \frac{R_2}{L_2} i_{L_2}$$

$$L_1 \frac{di_{L_1}}{dt} - v_C - R_1 C_1 \frac{dv_C}{dt} = 0$$

2) $i_{L_1} + i + C_1 \frac{dv_C}{dt} = 0$

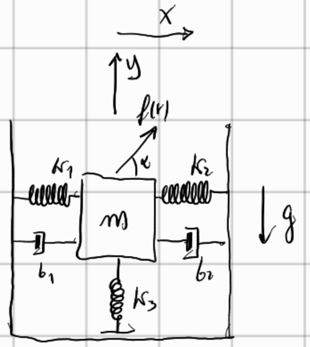
$$\frac{dv_C}{dt} = -\frac{i_{L_1}}{C_1} - \frac{i}{C_1}$$

$$L_1 \frac{di_{L_1}}{dt} - v_C - R_1(-i - i_{L_1}) = 0$$

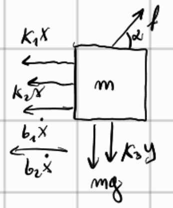
3) $L_1 \frac{di_{L_1}}{dt} - v_C + R_1 i + R_1 i_{L_1} = 0$

$$\frac{di_{L_1}}{dt} = \frac{v_C}{L_1} - \frac{R_1}{L_1} i - \frac{R_1}{L_1} i_{L_1}$$

1)



$u_1(t) = f(t)$
$u_2(t) = g$



$$m\ddot{x} = f\cos\alpha - k_1 x - k_2 x - b_1 \dot{x} - b_2 \dot{x}$$

$$m\ddot{y} = f\sin\alpha - k_3 y - mg$$

$$T = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m\dot{y}^2$$

$$U = mgy + \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 x^2 + \frac{1}{2} k_3 y^2$$

$$D = \frac{1}{2} b_1 \dot{x}^2 + \frac{1}{2} b_2 \dot{x}^2$$

$$\mathcal{L} = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m\dot{y}^2 - mgy - \frac{1}{2} k_1 x^2 - \frac{1}{2} k_2 x^2 - \frac{1}{2} k_3 y^2$$

1) $m\ddot{x} + k_1 x + k_2 x + b_1 \dot{x} + b_2 \dot{x} = f\cos\alpha$

$m\ddot{y} + mg + k_3 y = f\sin\alpha$

3) L'equazione di continuità si applica a fluidi in moto stazionario considerando una linea di flusso e due punti su di essa, P_1 e P_2. A partire da P_1 si sceglie una superficie S_1 piccola tale da poter accettare l'approssimaz. di p, ρ e v costanti. Si considera un tubo di flusso a partire dal bordo di S_1, e si considera la superficie chiusa che posso delimitare tra S_1 e S_2 data dal tubo di flusso. Allora, se al suo interno non ci sono né sorgenti né pozzi di massa, allora la portata di massa è costante.

$$\rho_1 S_1 v_1 = \rho_2 S_2 v_2.$$

Se ho un liquido, approssimabile come incomprimibile, allora $\rho_1 = \rho_2 \Rightarrow S_1 v_1 = S_2 v_2$. PORTATA VOLUMICA.