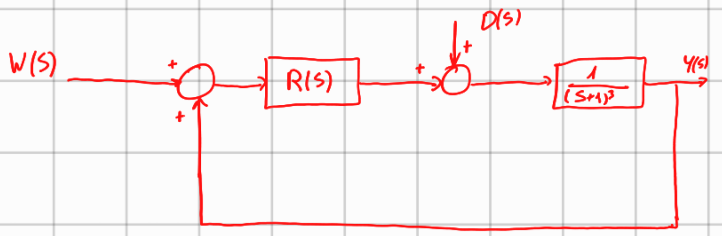
ALCUNE CONSIDERAZIONI:

- É possibile che una piccola parte degli esercizi sia scorretta

- Alcuni esercizi possono essere risolti in modi diversi, ma ugualmente corretti (specialmente nella scelta del posizionamento di poli e zeri per i tempi di risposta)

- Gli esercizi sono svolti in ordine cronologico, quindi più si va in fondo nel file più sará probabile che con più esperienza alle spalle saranno corretti.

Buon LAVORO!



1) Astatico a disturbo costante d(t), margine di guadagno di 6dB

$R(s) = \frac{M_R}{s}$. $L(s) = \frac{M_R}{s} \cdot \frac{1}{(s+1)^3}$ Trovo ω_π:

$\angle L(j\omega_\pi) = -\pi \Rightarrow -\frac{\pi}{2} - 3\,\mathrm{atan}(\omega_\pi) = -\pi$

$\mathrm{atan}(\omega_\pi) = \frac{\pi}{6} \Rightarrow \omega_\pi = \frac{\sqrt{3}}{3}$

$K_{m\,dB} = 6 \Rightarrow 20\log_{10}(K_m) = 6$

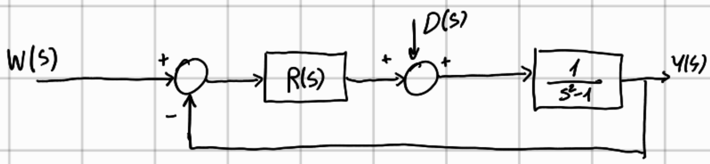
$\log_{10} K_m = \frac{6}{20}$

$K_m \cong 2$

$|L(j\omega_\pi)| = \frac{1}{K_m} \Rightarrow \frac{M_R}{|j\omega_\pi|} \cdot \frac{1}{\sqrt{(1+\omega_\pi^2)^3}} = \frac{1}{2}$

$\Rightarrow M_R\sqrt{3} \cdot \frac{1}{\sqrt{(1+\frac{1}{3})^3}} = \frac{1}{2} \Rightarrow \frac{M_R\sqrt{3}}{\sqrt{\frac{64}{27}}} = \frac{1}{2}$

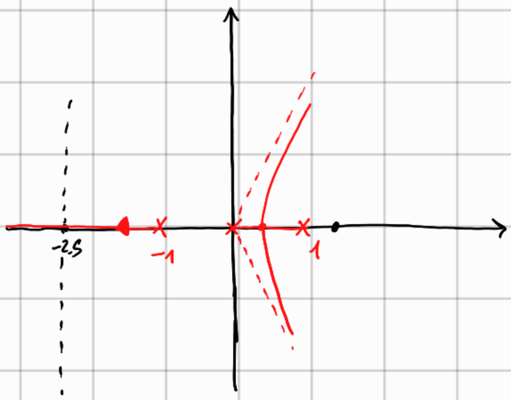
$\frac{M_R\sqrt{3} \cdot 3\sqrt{3}}{8} = \frac{1}{2} \Rightarrow M_R = \frac{4}{9}$



- Astatismo a $d(t)$ costante
- $T_{a1} \simeq 2s \Rightarrow \bar{\sigma} = -\frac{5}{2} = -2.5$
- Modi aperiodici: poli reali

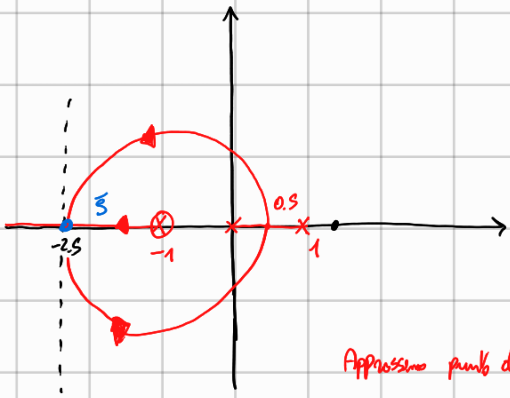
$L(s) = \frac{M}{s} \cdot \frac{1}{s^2-1}$, Provo con $R(s) = \frac{M}{s}$

$x_a = \frac{1}{3}(0+1-1) = 0$



Inizio cancellando polo in -1. Inserisco poi uno zero per ottenere punto doppio in -2.5.

$$R(s) = \frac{M}{s}(1+s)(1+s\tau)$$



Approssimo punto doppio in $\frac{1}{2}$.

$d = 3 \Rightarrow$ Centro circonferenza in -1!

Troviamo il valore di ρ:

$\bar{s} = -2.5$

$|\rho| = \frac{2.5 \cdot 3.5}{1.5} = 5.83$ $\quad R(s) = \frac{5.83}{s}(s+1)^2$ $\quad$ Non fisicamente realizzabile

$$R_{PID}(s) = \frac{5.83}{s} \cdot (s^2 + 2s + 1) = \frac{5.83 s^2 + 11.66 s + 5.83}{s}$$

$$K_P + \frac{K_I}{s} + K_D s = \frac{K_P s + K_I + K_D s^2}{s}$$

$K_D = 5.83$ $\quad K_P = 11.66$ $\quad K_I = 5.83$ $\quad T_D = \frac{K_P}{K_I} = 2$

$$R_D(s) = \frac{K_P T_D s}{1 + \frac{s T_D}{N}} = \frac{23.32 s}{1 + \frac{2}{N} s}$$

$$\overbrace{\frac{K_P T_D s^2 + K_P s + K_P / K_I}{s}}^{\text{Regolatore PID in questa forma}}$$

$K_P T_D = 5.83$

$K_P = 11.66$

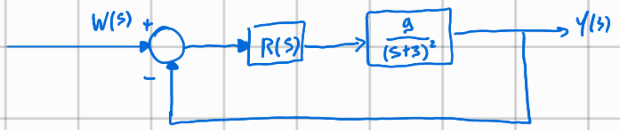
$\frac{K_P}{T_I} = 5.83$

$\Rightarrow T_I = 2$

$K_P = 11.66$

$T_D = \frac{1}{2}$

$$R_{PID}(s) = 11.66 \left(1 + \frac{1}{2s} + \frac{\frac{1}{2} s}{1 + \frac{1}{2N} s} \right)$$



• Errore nullo a regime per grad. unitario

• $\varphi_m = 45^\circ$ a $\omega_c = 3$ rad/s.

$R_1(s) = \frac{M}{s}$ $\qquad L^*(s) = \frac{9M}{s(s+3)^2}$ $\qquad L^*(j\omega_c) = \frac{9M}{3j(3+3j)^2} = \frac{3M}{9j(1+j)^2} = -\frac{1}{6}M = -\frac{M}{6}$

• La fase é -180° se $M>0$. Devo avere anticipo di 45°.

$\Delta\varphi = 45^\circ \Rightarrow \qquad R_2(s) = 1+sT \;/\; \angle R_2(j\omega_c) = 45^\circ$

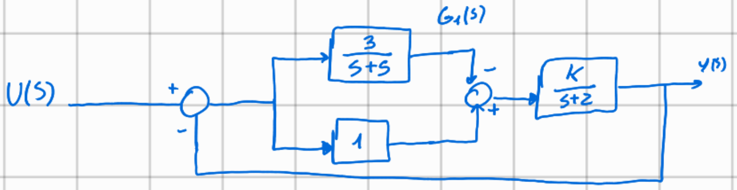
$\arctan(\omega_c T) = 45^\circ$

$\omega_c T = \tan 45^\circ$

$T = \frac{1}{\omega_c} = \frac{1}{3}$

$L^*(s) = \frac{9(1+\frac{1}{3}s)M}{s(s+3)^2}$ $\qquad |L^*(j\omega_c)| = M\sqrt{\frac{1}{36}+\frac{1}{36}} = \frac{1}{6}\sqrt{2}M \Rightarrow M = \frac{6}{\sqrt{2}} = 3\sqrt{2}$

$L(s) = \frac{9\sqrt{2}(3+s)}{s(s+3)^2}$



$$G_1(s) = \frac{-3}{s+5} + 1 = \frac{s+2}{s+5}$$

$$G_{TOT}(s) = \frac{s+2}{s+5} \cdot \frac{K}{s+2} = \frac{K}{s+5}$$. Non compl. raggiungibile.

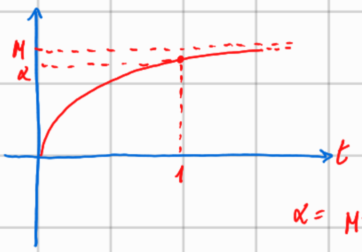
$$F(s) = \frac{\frac{K}{s+5}}{1 + \frac{K}{s+5}} = \frac{K}{s+5+K}$$

$p_1 = -5-K$. Per stabilità, $K > -5$

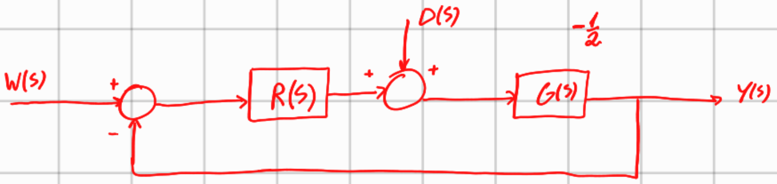
$\downarrow$ $\tau = \dfrac{1}{5+K} \Rightarrow T_{a1} = \dfrac{\ln(100)}{5+K} = 1$

$\Rightarrow K = \ln(100) - 5$

$$F(0) = \frac{K}{5+K} = \frac{\ln(100)-5}{\ln(100)} = M$$



$\alpha = M - \dfrac{M}{100}$



- $G(s) = -\frac{1}{2}$

$$\frac{-\frac{1}{2}}{1-\frac{1}{2}R(s)}$$

- Astatico a disturbo costante $d(t)$
- $K_{dB} = 6dB \iff K = 2$

$R_{PID}(s) = K_P\left(1 + \frac{1}{T_i s} + T_D s\right)$ $\qquad K_P\, G(s) = -\frac{1}{K_m} \Rightarrow K_P = 1$

$$\frac{1}{T_i s} + T_D s = 0$$

$$\frac{1}{4T_D} - T_D = 0 \Rightarrow \quad T_D^2 = \frac{1}{4} \qquad T_D = \frac{1}{2}$$

$$T_i = 2$$

$$R_{PID}(s) = 1 + \frac{1}{2s} + \frac{s}{2}$$

2)

$R_0(s) = \frac{M}{s^2}$ $\quad \omega_c = 1\,rad/s \quad \varphi_m = 40°$

$L_0(s) = \frac{M}{s^2} \cdot \frac{1}{s+2} \quad L_0(j\omega_c) = \frac{-M}{2+j} \Rightarrow \angle L_0(j\omega_c) = -\pi - atan(\frac{1}{2})$ e deve essere $-\pi + \frac{40° \cdot \pi}{180°}$

$-atan(\frac{1}{2}) = -26.6°$

$\Rightarrow$ Anticipo $\Delta\varphi = 66.6°$

$R_2(s) = 1 + sT \;, \quad \angle(1+Tj) = \Delta\varphi$

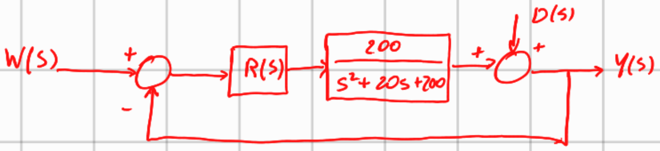
$T = \tan(\Delta\varphi) = 2.31$

$R(s) = \frac{M(1+2.31s)}{s^2}$

Ora, $|L(j)| = 1 \Rightarrow \frac{M|1+2.31j|}{1} \cdot \frac{1}{|2+j|} = 1 \Rightarrow M = \frac{|2+j|}{|1+2.31j|} = 0.89$

$L(s) = \frac{0.89(1+2.31s)}{s^2} \cdot \frac{1}{s+2}$

2)



$G(s) = \dfrac{1}{1+\frac{1}{10}s+\frac{s^2}{200}}$

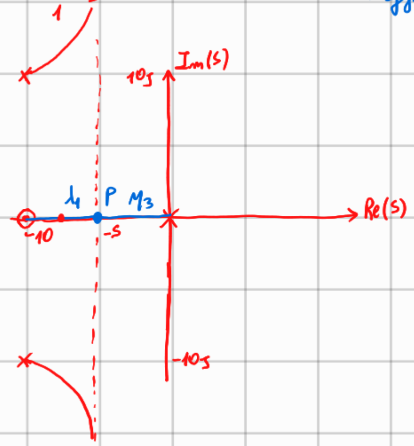
$s_{1,2} = -10 \pm 10j$

• $T_{ass} = 1s \Rightarrow \bar{\sigma} = \frac{5}{1} = 5$

Aggiungo polo in origine e zero in -10.

Impongo che polo a Re(·) maggiore si trovi in -5, per avere $T_{ass} = 1s$.

$x_a = \frac{1}{2}(-20+10) = -5$



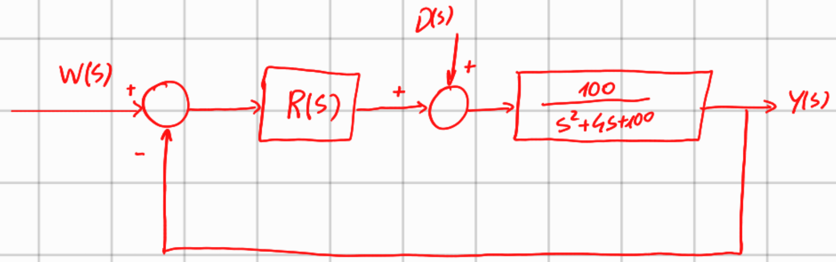
Impongo che ci sia un polo in -5: calcolo $\bar{\rho}$: $\bar{\rho} = \dfrac{\eta_1\eta_2\cancel{\eta_3}}{\cancel{\lambda_1}} = \eta^2 = (10^2+5^2) = 125 = \bar{\rho} = 200\mu \Rightarrow \mu = 0.625$

$L(s) = \dfrac{125\cdot(s+10)}{s(s^2+20s+200)}$

$$e_\infty = \lim_{s\to0} sE(s) = \lim_{s\to0} s\frac{S(s)}{s^\nu} = \lim_{s\to0} s^{1-\nu}\frac{s^g}{s^g+\mu} = \lim_{s\to0}\frac{s^{g-(\nu-1)}}{s^g+\mu} = \begin{cases} \frac{1}{\mu} & g=\nu+1 \\ 0 & g>\nu+1 \\ \infty & g<\nu+1 \end{cases}$$

$\nu=3$, $g=1$ $\quad g<4$, errore ∞.

2)



- Astatismo a disturbo $d(t)$ a gradino $\Rightarrow$ polo in $R(S)$ nell'origine.
- $K_m = 20dB = 10$

$$L^*(S) = \frac{M}{S} \cdot \frac{100}{S^2+4s+100}$$

$100 - \omega_\pi^2 + 4j\omega_\pi$

- Trovo ω_π: $\angle L^*(j\omega_\pi) = -\pi \Rightarrow s^2+4s+100$ deve essere puramente immaginario.

$$\omega_\pi = 10 \, rad/s$$

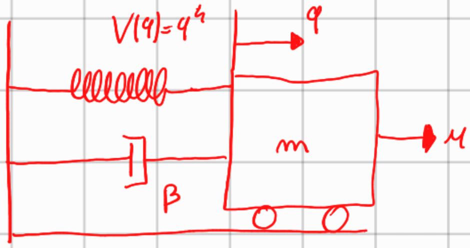
$$|L^*(j\omega_\pi)| = \frac{100M}{10 \cdot 40} = \frac{1}{K_m} \Rightarrow 100M = 40$$

$$M = \frac{4}{10}$$

$$R(S) = \frac{4}{10S} \qquad L(S) = \frac{40}{S(S^2+4s+100)}$$

Se $W(s) = \frac{1}{S^2}$, $y_\infty = \lim_{s\to 0} \frac{\cancel{s} S(s)}{s^{\cancel{2}}} = \lim_{s\to 0} \frac{1}{S} \cdot \frac{1}{1+L(S)} = \frac{\cancel{s}(s^2+4s+100)}{S(s^2+4s+100)+40} \cdot \frac{1}{\cancel{s}} = \frac{10}{4}$

1)



$m = 4\,kg$

$$T(\dot q) = \frac{1}{2} m \dot q^2 \qquad \mathcal{L}(q,\dot q) = T - U = \frac{1}{2} m \dot q^2 - q^4$$

$$U(q) = q^4$$

$$D(\dot q) = \frac{1}{2}\beta \dot q^2$$

$$\Rightarrow \quad m\ddot q + 4q^3 + \beta \dot q = u$$

Equazione:

$$q = x_1, \quad \dot q = x_2$$

$$\begin{cases} \dot x_1(t) = x_2(t) \\ \dot x_2(t) = -\dfrac{4x_1(t)^3}{4} - \dfrac{\beta}{4} x_2(t) + \dfrac{u(t)}{4} \\ y = x_1(t) \end{cases}$$

$$f(x(t),u(t)) = \begin{pmatrix} x_2(t) \\ -x_1(t)^3 - \frac{\beta}{4} x_2(t) + \frac{u(t)}{4} \end{pmatrix}$$

Trovo punti di eq. con $u = 4N$.

$$\begin{aligned} 0 &= x_2 \\ 0 &= -x_1^3 - \frac{\beta}{4} x_2 + 1 \end{aligned} \Rightarrow \begin{aligned} x_2 &= 0 \\ x_1^3 &= 1 \Rightarrow \end{aligned} \quad \bar x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \bar u = 4$$

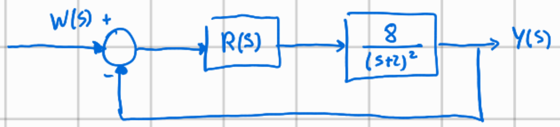
$$A = \frac{\delta f}{\delta x}\Big|_{\substack{x=\bar x \\ u=\bar u}} = \begin{pmatrix} 0 & 1 \\ -3x_1^2 & -\frac{\beta}{4} \end{pmatrix}\Big|_{\substack{x=\bar x \\ u=\bar u}} = \begin{pmatrix} 0 & 1 \\ -3 & -\frac{\beta}{4} \end{pmatrix}$$

$$p(s) = s^2 + \frac{\beta}{4} s + 3 = 0$$

$$\Delta = 0 = \frac{\beta^2}{16} - 12 \Rightarrow \beta^2 - 192 = 0$$

$$\beta = 13.86$$

2)



• Errore a regime con rampa unitaria: 5%

• $\varphi_m = 45°$ e $\omega_c = 2$ rad/s

NOTA: Per avere quell'errore, ho bisogno di polo nell'origine. $g=1$

$$e_\infty = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \cdot \frac{S(s)}{s^2} = \lim_{s\to 0} \frac{S(s)}{s} = \lim_{s\to 0} \frac{1}{s} \cdot \frac{s^g}{s^g + M} = \lim_{s\to 0} \frac{s^{g-1}}{s^g+M} = \frac{1}{M}$$

$$M = \frac{1}{e_\infty} = \frac{100}{5} = 20 \Rightarrow \alpha = 10 \qquad R_1(s) = \frac{\alpha}{s} = \frac{10}{s}$$

$$G(s) = \frac{8}{4\left(1+\frac{s}{2}\right)\left(1+\frac{s}{2}\right)} = \frac{2}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{2}\right)}$$

Calcolo margine di fase:

$$\angle R_1(j\omega_c)G(j\omega_c) = \angle\left(\frac{2}{\underset{\uparrow\ 1-1+2j}{(1+j)(1+j)}}\right)\left(\frac{5}{j}\right) = -\frac{\pi}{2} - \frac{\pi}{2} = -\pi$$

Devo avere un anticipo di $45°$, ma devo imporre che $|L^*(j\omega_c)| = 1$

$$|L^*(j\omega_c)| = \left|\frac{2}{2j}\right|\left|\frac{5}{j}\right| = 5 \Rightarrow \text{ho bisogno di attenuazione di } \frac{1}{5}.$$

$\Delta\varphi_a = \Delta\varphi + \bar{\varphi}$, con $\Delta\varphi = 45°$, $\bar{\varphi} = 5°$

$\Delta g_a = \bar{g}$, con $\bar{g} = 8$ dB

$\Delta\varphi_r = -\bar{\varphi}$

$\Delta g_r = \Delta g - \bar{g}$, con $\Delta g = \frac{1}{5} = -14$ dB

$R_{2R}(s) = \dfrac{1+Ts}{1+\alpha Ts}$ $\quad M = -22\text{dB} = 0.08$ $\quad \alpha = -5°$

$R_{2A}(s) = \dfrac{1+Ts}{1+\alpha Ts}$ $\quad M = 2.5$ $\quad \varphi = 50°$

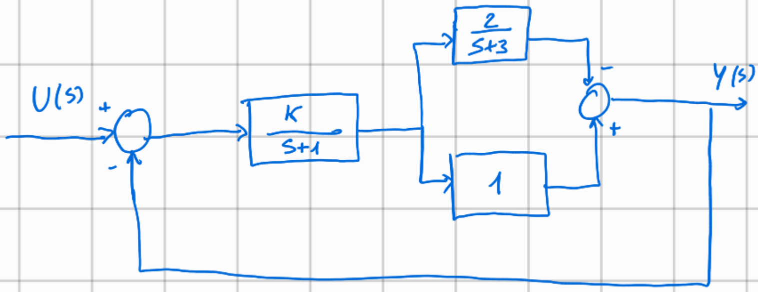
$\alpha = \dfrac{M\cos\varphi - 1}{M(M-\cos\varphi)}$ $\quad T = \dfrac{M - \cos\varphi}{\omega_c \sin\varphi}$

$\alpha = 12.56$ $\quad T = 5.26$

$\alpha = 0.13$ $\quad T = 1.21$

$$L(s)=\frac{10}{s}\,\frac{2}{(1+\frac{s}{2})(1+\frac{s}{2})}\cdot\frac{1+5.26s}{1+66s}\cdot\frac{1+1.21s}{1+0.16s}$$

1)



$$L(s)=\frac{K}{s+1}\cdot\left(\frac{-2}{s+3}+1\right)=\frac{K}{s+1}\cdot\frac{s+1}{s+3}$$

Cancellazione con polo a monte ⇒ non oss.

$$L(s)=\frac{K}{s+3}$$

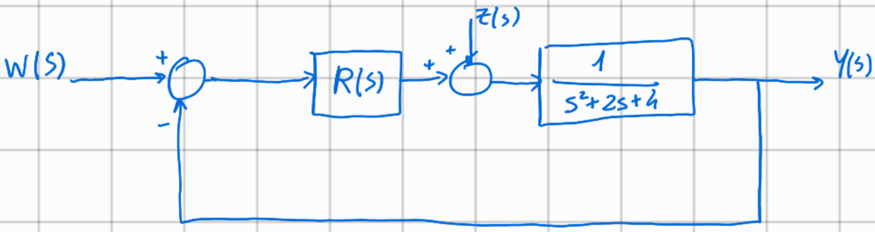
$$F(s)=\frac{\frac{K}{s+3}}{1+\frac{K}{s+3}}=\frac{K}{s+3+K}$$

Modo di evoluzione in uscita:

$e^{(-3-K)t}$ $\quad \tau=\frac{1}{3+K}$ $\quad 4.6\,\tau\cong\frac{1}{2}\Rightarrow$

$$\frac{1}{3+K}=\frac{1}{9.2}\Rightarrow K=6.2$$

2)



- $\varphi_m = 40°$
- $\omega_c = 2$ rad/s
- Astatico a disturbi costanti

$R_1(s) = \frac{M}{s}$, per astatismo.

$$L^*(j\omega_c) = R_1(j\omega_c)G(j\omega_c) = \frac{M}{2j} \cdot \frac{1}{4j} = \frac{-M}{8}$$

$\angle L^*(j\omega_c) = -\pi$

$|L^*(j\omega_c)| = \frac{M}{8}$

- Ho bisogno di anticipo di 40°.

$\Delta\varphi = 40°$

$R_2(s) = 1 + s\tau \; / \; \angle R_2(j\omega_c) = 40°$

$\operatorname{atan}(\omega_c \tau) = 40° \Rightarrow \omega_c \tau = \tan 40°$

$\tau = \frac{\tan 40°}{\omega_c} = 0.42$

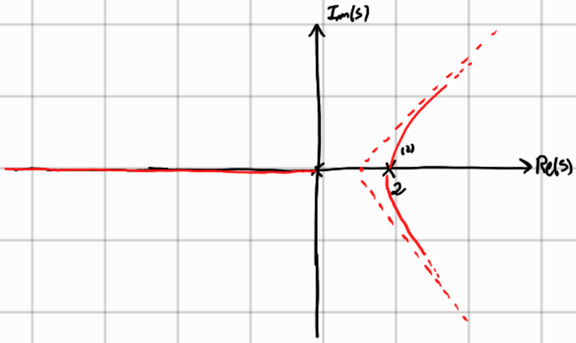
$$L(s) = \frac{M(1+0.42s)}{s} \cdot \frac{1}{s^2+2s+4}$$

$$|L(j\omega_c)| = 1 \Rightarrow \frac{M|1+0.84j|}{2} \cdot \frac{1}{4} = 1 \Rightarrow M = \frac{8}{1.31} = 6.1$$

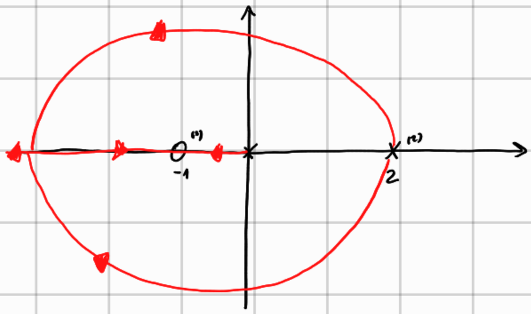
$$L(s) = \frac{1.525(1+0.42s)}{s} \cdot \frac{1}{1+\frac{s}{2}+\frac{s^2}{4}}$$

2)

• R(s) ha almeno un polo nell'origine



Inserisco 2 zeri coincidenti in −1, per scelta casuale.



Trovo punto doppio:

$$L(s) = \frac{\mu(s+1)^2}{s}\cdot\frac{4}{(s-2)^2}$$

$$\frac{d\gamma(x)}{dx} = \frac{d}{dx}\left(\frac{s(s-2)^2}{(s+1)^2}\right) = \frac{(s-2)^2(s+1)^2 + s(s+1)^2(s-2)\cdot 2 - 2s(s-2)^2(s+1)}{(s+1)^2}$$

$$(s-2)^{\cancel{2}}(s+1)^{\cancel{2}} + s(s+1)^{\cancel{2}}(\cancel{s-2})\cdot 2 - 2s(s-2)^{\cancel{2}}(\cancel{s+1}) = 0$$

$$(s-2)(s+1) + s(s+1)\cdot 2 - 2s(s-2) = 0$$

$$s^2 - s - 2 + \cancel{2}s^2 + 2s - \cancel{2}s^2 + 4s = 0$$

$$s^2 + 5s - 2 = 0 \qquad s_{1,2} = \frac{-5 \pm \sqrt{25+8}}{2} = \begin{cases} 0.37 \\ -5.37 \end{cases}$$

Trovo $\bar{p}$: $\dfrac{5.37 \cdot 7.37^2}{4.37^2} = 15.27 \Rightarrow M = \dfrac{15.27}{4} = 3.82$

$$L(s) = \frac{15.17(s+1)^2}{s} \cdot \frac{1}{(s-2)^2} \qquad M_0 = \frac{15.17}{4} = 3.82 \Rightarrow e_\infty = \frac{1}{3.82}$$

$$F(s) = \frac{15.17(s+1)^2}{s(s-2)^2 + 15.17(s+1)^2} = \frac{15.17(s^2+2s+1)}{s(s^2-4s+4) + 15.17(s^2+2s+1)}$$

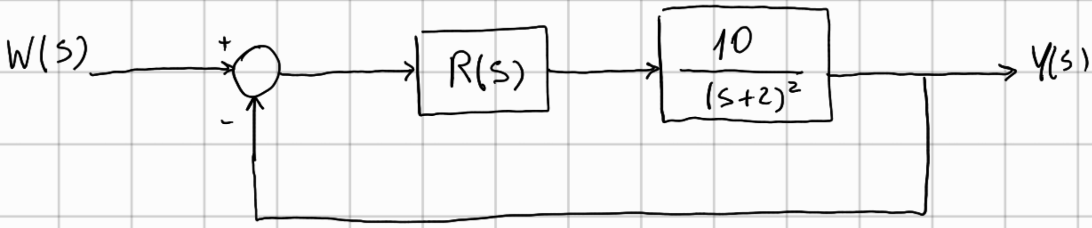
Il polo dominante è tra 0 e -1, dopo gli zeri. Non si presenta sovraelongazione

NOTA: $R(s) = \dfrac{3.82}{s}(s^2+2s+1) = \dfrac{3.82s^2 + 7.64s + 3.82}{s}$

Regolatore PID in questa forma

$$\frac{\overbrace{K_p T_D s^2 + K_p s + K_p/T_I}}{s}$$

2)



$e_\infty = 0.2$ con $w(t) = t$, $t \geq 0$. $\varphi_m = 40°$ con $\omega_c = 0.7$ rad/s

$e_\infty = \frac{1}{M}$ e un polo nell'origine $\Rightarrow M = 5$

$R_1(s) = \frac{a}{s}$ $L_0(s) = \frac{a}{s} \cdot \frac{10}{4(1+\frac{s}{2})^2}$, con $\frac{10}{4}a = 5 \Rightarrow a = 2$

$L_0(s) = \frac{5}{s(1+\frac{s}{2})^2}$

Calcolo $L_0(j\omega_c) = \frac{5}{0.7j(1+\frac{0.7}{2}j)^2} =$ $|L_0(j\omega_c)| = 6.36 \approx 16.1\,dB$

$\angle L_0(j\omega_c) = -128.6°$

, $\varphi_m = 180° - 128.6° = 51.4°$

Mi serve un ritardo e un'attenuazione.

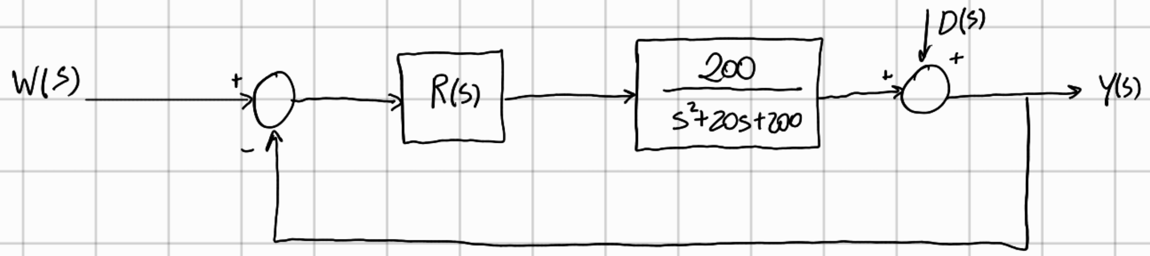
$M = \frac{1}{6.36} = 0.158$ $\alpha = -11.4°$

$R_2(s) = \frac{1+Ts}{1+\alpha Ts}$ $\alpha = \frac{M\cos\varphi - 1}{M(M-\cos\varphi)}$ $T = \frac{M - \cos\varphi}{\hat{\omega}\sin\varphi}$

$\alpha = 6.5$ $T = 5.94$

$R_2(s) = \frac{1+5.94s}{1+38.61s}$ $\Rightarrow L(s) = \frac{5}{s(1+\frac{s}{2})^2}\frac{1+5.94s}{1+38.61s}$

2)

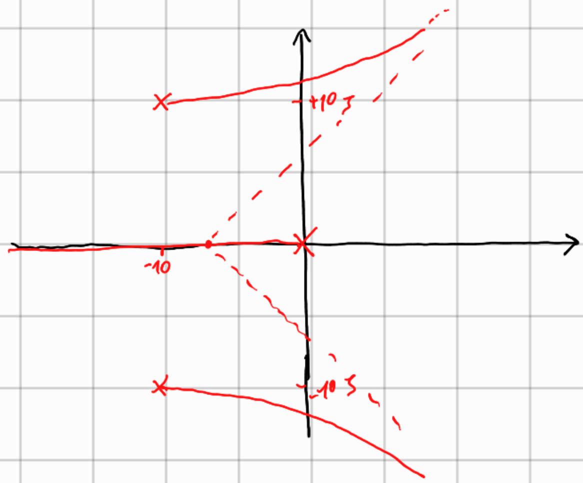


$T_{a5\%} = 1s \Rightarrow \bar{\sigma} = -\frac{5}{1} = -5$. Impongo che poli si trovino su asse $Re(s) = -5$.

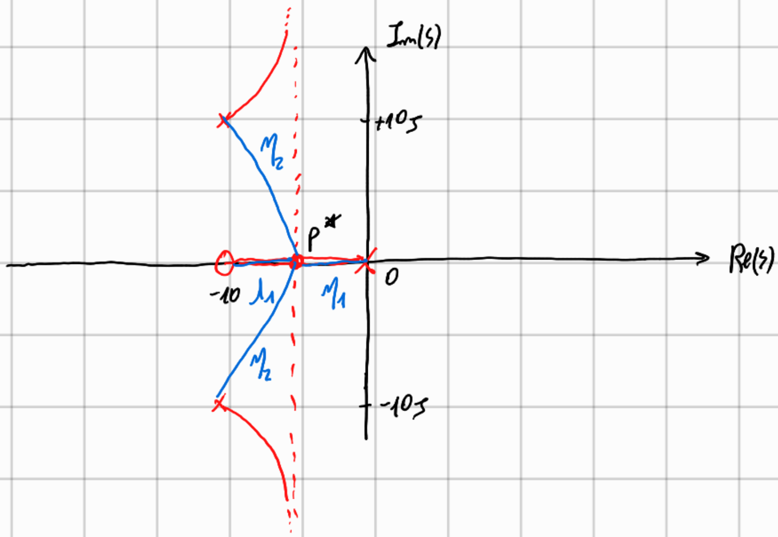
$S_{1,2} = \frac{-10 \pm \sqrt{100-200}}{1} = -10 \pm 10j$

Con regolatore PI ho 1 polo in origine e 1 zero.

$X_1 = \frac{1}{3}(-20) = -6.66$



Piazzo uno zero in −10 e impongo che alloco poli in −5.

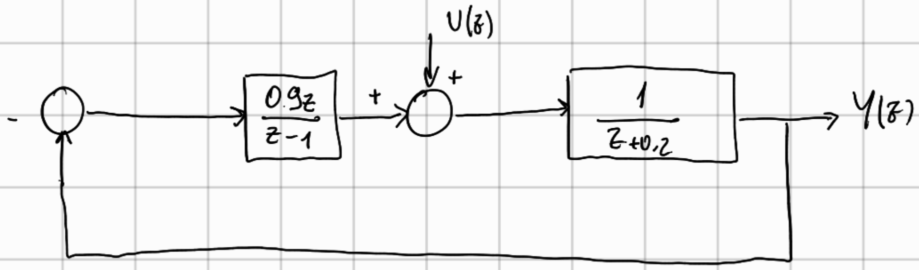


Calcolo $\bar{\rho}$ associato a p^*: $\frac{\cancel{M_1} M_2^2}{\cancel{X_1}} = 10^2+5^2 = 125 = \mu \cdot 200 \Rightarrow \mu = 0.625$

$L(s) = \frac{125}{s} \cdot \frac{(s+10)}{s^2+20s+200}$

$e_\infty = \frac{2}{M} = \frac{2}{125 \cdot 10/200} = \frac{400}{10 \cdot 125} = \frac{40}{125} = \frac{8}{25} = 0.32$

1)



$$G(z) = \frac{\frac{1}{z+0.2}}{1 + \frac{0.9z}{z-1} \cdot \frac{1}{z+0.2}} = \frac{z-1}{(z-1)(z+0.2)+0.9z} = \frac{z-1}{z^2-z+0.2z-0.2+0.9z} = \frac{z-1}{z^2+0.1z-0.2}$$

$$G(z) = \frac{z-1}{(z+0.5)(z-0.4)}$$

$$U(z) = \frac{z}{(z-1)^2}$$

$$Y(z) = Z\left[\frac{\cancel{z-1}}{(z+0.5)(z-0.4)(z-1)^{\cancel{2}}}\right] = Z\left[\frac{r_1}{z+0.5} + \frac{r_2}{z-0.4} + \frac{r_3}{z-1}\right]$$

$$r_1 = \frac{z-1}{(z-0.4)(z-1)^2}\Bigg|_{z=-0.5} = \frac{-1.5}{-0.9\cdot(-1.5)^2} = 0.74$$

$$r_2 = \frac{z-1}{(z+0.5)(z-1)^2}\Bigg|_{z=+0.4} = -1.85$$

$$r_{32} = \frac{1}{(z+0.5)(z-0.4)}\Bigg|_{z=1} = 1.1$$

$$y(k) = 0.74\cdot(-0.5)^k - 1.85\cdot(0.4)^k + 1.1\cdot 1^k$$

1)

$$\begin{cases} \dot{x}_1(t) = -x_1^2(t) + x_2(t)u(t) \\ \dot{x}_2(t) = x_1(t) - x_2(t)u(t) \\ y(t) = x_1(t) + x_2(t) \end{cases}$$

$u(t) = 1.1$

$\bar{u} = 1 \Rightarrow \begin{matrix} 0 = -x_1^2 + x_2 \\ 0 = x_1 - x_2 \end{matrix} \Rightarrow x_1 = x_2$, $\quad x_1^2 - x_1 = 0 \Rightarrow x_1 = 1, 0$, $\quad x_2 = 1, 0.$

$$\bar{x}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \bar{x}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$$f(x(t), u(t)) = \begin{pmatrix} -x_1^2(t) + x_2(t)u(t) \\ x_1(t) - x_2(t)u(t) \end{pmatrix}$$

$$A_1 = \left.\frac{\delta f}{\delta x}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} = \begin{pmatrix} -2x_1 & u \\ 1 & -u \end{pmatrix}_{\substack{x=\bar{x} \\ u=\bar{u}}} = \begin{pmatrix} -2 & 1 \\ 1 & -1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$$

Forma compagna, vedo che ho autovalore a parte reale positiva ⇒ instabile.

$$b_1 = \left.\frac{\delta f}{\delta u}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} = \begin{pmatrix} x_2 \\ -x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$c^T = \begin{pmatrix} 1 & 1 \end{pmatrix} \qquad d = 0$$

Sistema linearizzato:

$$\delta\dot{x}(t) = \begin{pmatrix} -2 & 1 \\ 1 & -1 \end{pmatrix} \delta x(t) + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \delta u(t) \qquad \delta u(t) = 0.1$$

$$p(s) = \begin{vmatrix} s+2 & -1 \\ -1 & s+1 \end{vmatrix} = (s+2)(s+1) - 1 = s^2 + 3s + 1$$

$$\delta y(t) = \begin{pmatrix} 1 & 1 \end{pmatrix} \delta x(t)$$

Trovo $G(0)$: $\quad c^T(-A)^{-1}b + d = \begin{pmatrix} 1 & 1 \end{pmatrix} (-A)^{-1} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} =$

$$= \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = -1$$

$\delta y_\infty = -0.1 \quad \Rightarrow \quad y(t) = \bar{y} + \delta y_\infty = 2 - 0.1 = 1.9$

1)

$$g_y(t) = te^{-3t} + 2e^{-t}\cos(2t) \qquad t \geq 0 \qquad u(t) = 45$$

$$G(s) = \frac{1}{(s+3)^2} + 2\frac{s+1}{s^2+2s+1+4}$$

$$s^2+2s+5=0 \qquad s_{1,2} = \frac{-1 \pm \sqrt{1-5}}{1} = -1 \pm 2j$$

$$Y_1(s) = \frac{G(s)}{s} = \underbrace{\frac{1}{s(s+3)^2}}_{Y_a(s)} + \underbrace{2 \cdot \frac{s+1}{s(s+1-2j)(s+1+2j)}}_{Y_b(s)}$$

$$Y_a(s) = \frac{M}{s} + \frac{r_{11}}{s+3} + \frac{r_{12}}{(s+3)^2} = \frac{1}{9s} - \frac{1}{9(s+3)} - \frac{1}{3(s+3)^2}$$

$$r_{12} = \frac{1}{s}\Big|_{s=-3} = -\frac{1}{3} \qquad y_a(t) = \frac{1}{9} - \frac{1}{9}e^{-3t} - \frac{1}{3}te^{-3t}$$

$$r_{11} = \frac{d}{ds}\frac{1}{s}\Big|_{s=-3} = -\frac{1}{s^2}\Big|_{s=-3} = -\frac{1}{9}$$

$$Y_b(s) = 2 \cdot \frac{s+1}{s(s+1-2j)(s+1+2j)} = \frac{M}{s} + \frac{Q}{(s+1-2j)} + \frac{Q^*}{(s+1+2j)}$$

$$Q = \frac{2(s+1)}{s(s+1+2j)}\Big|_{s=-1+2j} = \ldots$$

$$M = 0.4$$

$$y(t) = 45y_a(t) + 45y_b(t)$$

$$Y_b(s) = 2 \cdot \frac{s+1}{s((s+1)^2+4)}$$

Definisco $p = s+1 \Rightarrow s = p-1$. Antitrasformo e poi riaggiungo ritardo.

$$Y_b(p) = 2 \cdot \frac{p}{(p-1)(p^2+4)} = \frac{r_1}{p-1} + \frac{R}{(p-2j)} + \frac{R^*}{(p+2j)}$$

$$r_1 = \left.\frac{2p}{p^2+4}\right|_{p=1} = \frac{2}{5}$$

$$R = \left.\frac{2p}{(p-1)(p+2j)}\right|_{p=2j} = \frac{4j}{4j(2j-1)} = \frac{1}{2j-1} \left\langle \begin{array}{l} |R| = 0.45 \\ \angle R = -2 \end{array} \right.$$

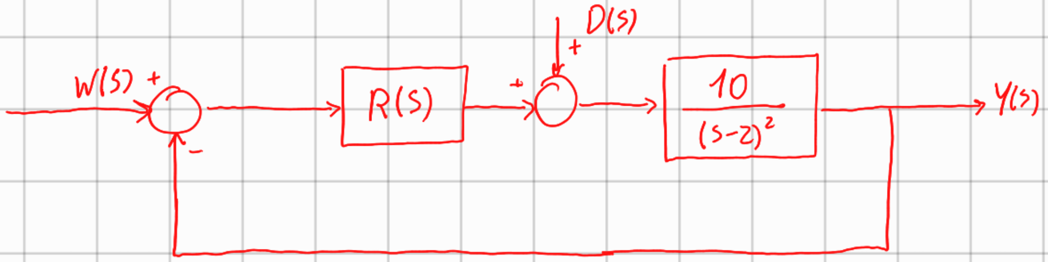
$y_b^*(t) = \frac{2}{5} e^{t} + 0.9 \cos(2t-2)$, ma devo ricordare che $s = p-1$, quindi $p = s+1$.

In p ho anticipo di 1s. Devo moltiplicare per e^{-t}.

$$y_b(t) = \frac{2}{5} + 0.9 e^{-t} \cos(2t-2)$$

$$\cos(2t)\cos(2) + \sin(2t)\sin(2)$$

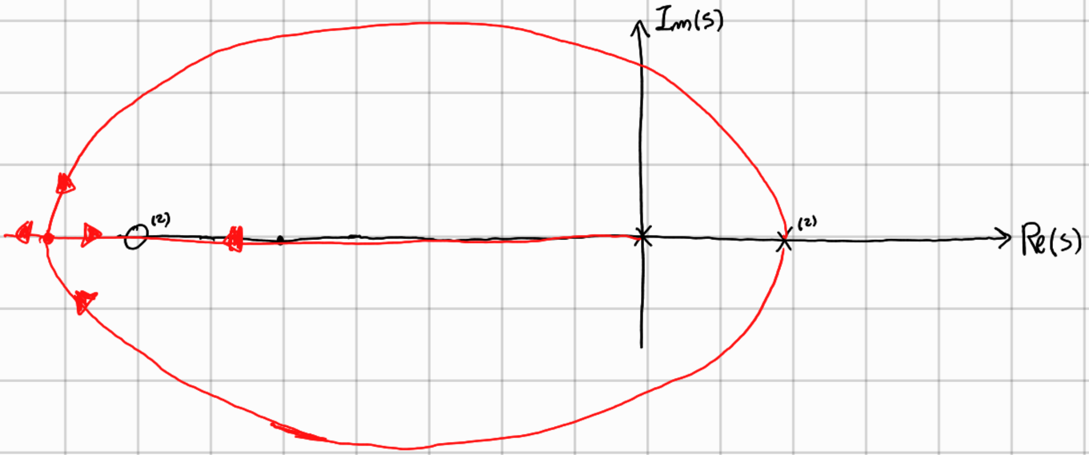
2)



• Astatismo a disturbi a gradino: Polo in origine.

• Modo aperiodico con costante di tempo pari a 0.2s su polo doppio.

Se $\tau=0.2$, allora, polo in -5, con punto doppio.



Aggiungo 2 zeri, imponendo p. doppio in -5.

$$R(s)=\frac{(s+z)^2}{s}M \qquad L(s)=\frac{M\cdot 10\,(s+z)^2}{s(s-2)^2}$$

$$\gamma(x)=\frac{x(x-2)^2}{(x+z)^2} \Rightarrow \gamma'(x)=0 \Rightarrow \frac{(x-2)^{\cancel{2}}(x+z)^{\cancel{2}}+2x\cancel{(x-2)}(x+z)^{\cancel{2}}-2x(x-2)^{\cancel{2}}\cancel{(x+z)}}{(x+z)^4}=0$$

$$(x-2)(x+z)+2x(x+z)-2x(x-2)=0$$

$$x^2-2x+zx-2z+\cancel{2}x^2+2zx-\cancel{2}x^2+4x=0$$

$$x^2+2x+3zx-2z=0$$

$x^2+2x+3zx-2z=0$

$$x_{1_2}=\frac{-2-3z\pm\sqrt{4+9z^2+12z+8z}}{2}=-5$$

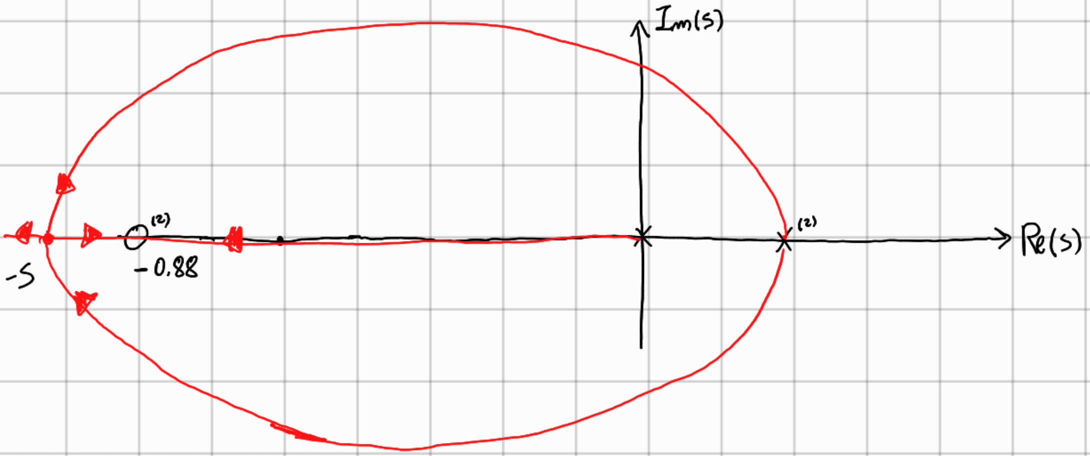
$$\Longrightarrow \quad -2-3z\pm\sqrt{9z^2+20z+4}=-10$$

$$\pm\sqrt{9z^2+20z+4}=-8+3z$$

$$\cancel{9z^2}+20z+4=64+\cancel{9z^2}-48z$$

$$z=\frac{15}{17}$$

Trovo ρ:

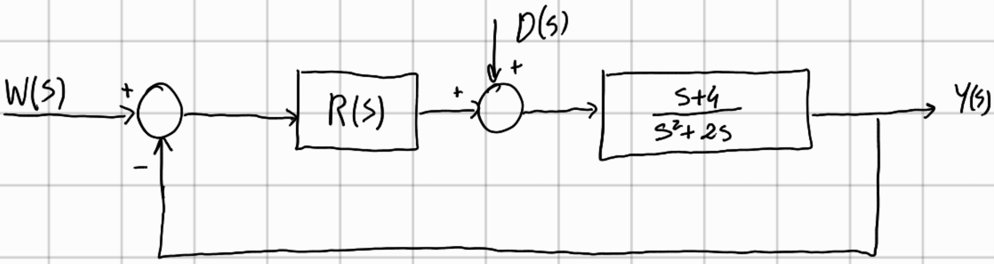


$$\bar{\rho}=\frac{7\cdot 7\cdot 5}{4{,}12^2}=14.4=10\mu \Rightarrow \quad \mu=1.44$$

$$L(s)=\frac{14.4\,(s+0.88)^2}{s\,(s-2)^2}$$

$$F(s)=\frac{39.2\,(s-0.88)^2}{s\,(s-2)^2+14.4\,(s+0.88)^2}$$

2)

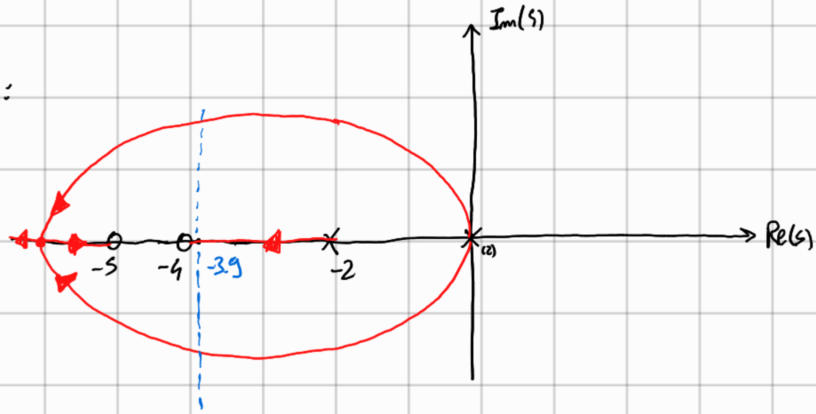


• Errore nullo con W(s) a rampa (1 solo polo nell'origine perché ne ho 1 in S)

• $T_{ass\,2\%} = 1s$

$$\bar{\sigma} = \frac{\ln(0.02)}{1} = -3.9$$

Uso il Root Locus:



Inserisco uno zero non troppo vicino per sostenibilità su guadagno. Lo metto in -5.

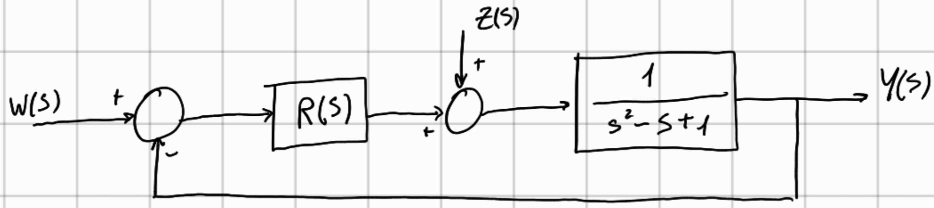
Impongo polo con Re(·) più alta di -3.9.

$$\bar{P} = \frac{3.9 \cdot 3.9 \cdot 1.9}{0.1 \cdot 1.1} = 262.7$$

$$L(s) = \frac{262.7}{s} \cdot \frac{(s+4)(s+5)}{s^2+2s} \qquad F(s) = \frac{262.7(s+4)(s+5)}{262.7(s+4)(s+5) + s(s^2+2s)}$$

• Errore nullo per poli nell'origine.

1/12/17



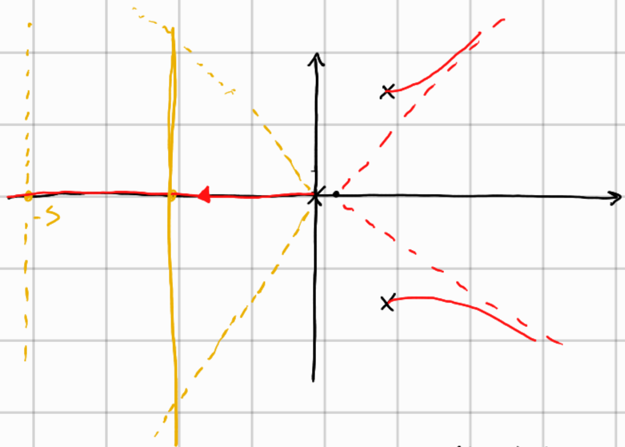
• Aggiungiamo anche polo in origine su R(S)

$$R_0(S) = \frac{M}{S}$$

Disegno luogo delle radici: voglio modo pseudoperiodico con $\xi = 0.5$

e modo aperiodico con $T_a \leq 1s \Rightarrow \bar{\sigma} \leq -5s$

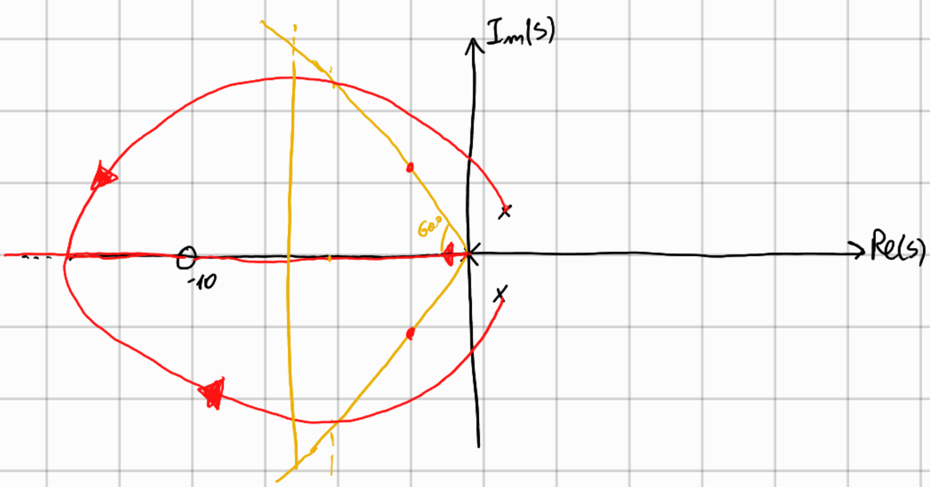
$$S_{1,2} = \frac{1 \pm \sqrt{1-4}}{2} = 0.5 \pm 0.866 j$$



$$X_A = \frac{1}{3}\left(0.5 + 0.5\right) = \frac{1}{3}$$

Inserisco uno zero doppio in modo da attirare i rami sulla destra.

Lo provo a inserire in -2.5.



Impongo poli con parte reale in -5. Impongo poli $(-5 \pm 8.66 j)$ con una $\bar{P} = 20$.

Calcolo posizione degli zeri:

$$20 = \frac{\sqrt{5^2+8.66^2} \cdot \sqrt{4.5^2+7.794^2}}{(8.66^2+b^2)}$$

$$2\cancel{0}(8.66^2+b^2) = 9\cancel{0}$$

$$150+2b^2 = 9$$ *

Non ho soluzioni. Abbassiamo la $\bar{p}$: la scelgo 1.

$$8.66^2+b^2 = 90 \Rightarrow b^2 = 15 \quad b = 3.87$$

$$b = |a+5| \Rightarrow a^2+25+10a = b^2$$

$$a^2+10a+10=0$$

$$a_{1,2} = -5 \pm \sqrt{25-10} = \begin{cases} -1.13 \\ -8.87 \end{cases}$$

Scelgo -8.87 come soluzione.

Calcolo il polo reale se $\bar{p}=1$

$x = |Re(p)|$

$$1 = \frac{x \cdot ((x+0.5)^2 + 0.866^2)}{(x-8.87)^2}$$

$$(x-8.87)^2 = x((x+0.5)^2+0.866^2)$$

Unica soluzione reale (e quindi accettabile) é $x_0 = 2.9$.

Quindi polo non va bene.

$$G(s) = \frac{1}{s^2+s+1}$$

$$s^2+s+1=0$$

$$s_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} j$$

- Assumiamo a dinamica costante $\Rightarrow R(s) = \frac{M}{s}$
- $K_{m_{dB}} = 6\,dB \Leftrightarrow K_m = 2$

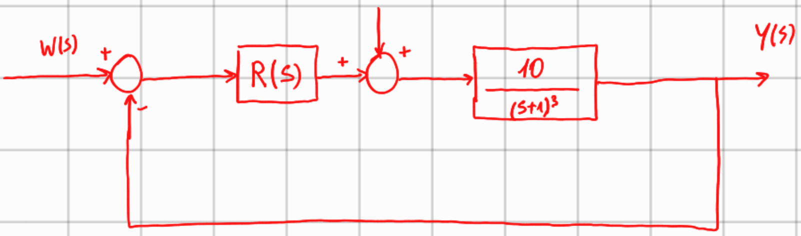
NOW: Trovo ω_c: $\angle L(j\omega_c) = -\pi \Rightarrow -\frac{\pi}{2} - \angle(1+j\omega_c - \omega_c^2) = -\pi$

$\Rightarrow \angle(1+j\omega_c - \omega_c^2) = \frac{\pi}{2} \Rightarrow \omega_c \,/\, 1+j\omega_c-\omega_c^2$ sia puram. imm.

$\omega_c = 1\,rad/s$.

$$|L(j\omega_c)| = \frac{M}{1} \cdot \frac{1}{|j|} = \frac{1}{2} \Rightarrow M = \frac{1}{2} \qquad R(s) = \frac{0.5}{s(s^2+s+1)}$$

2)



• Astatico a disturbi a gradino.

• $K_m = 2$.

$$R_{PID}(s) = K_p\left(1+\frac{1}{T_I s}+T_D s\right) = K_p\left(\frac{T_I T_D s^2 + T_I s + 1}{T_I s}\right) \quad , \; T_I = 4T_D.$$

Calcolo ω_π, ma per semplicità di calcoli impongo che $\angle R_{PID}(j\omega_\pi) = 0$.

Trovo ω_π per la $G(s)$:

$$\angle G(j\omega_\pi) = -\pi \Rightarrow \quad -3\arctan(\omega_\pi) = -\pi$$

$$\omega_\pi = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\angle R_{PID}(j\omega_\pi) = \angle\left(\frac{4T_D^2(-3) + \sqrt{3}\,j\,4T_D + 1}{4T_D\, j\sqrt{3}}\right)$$

$$\Downarrow$$

$$\angle\left(\frac{12T_D^2 j - j + 4\sqrt{3}\,T_D}{4\sqrt{3}\,T_D}\right) = 0$$

, deve essere un numero reale: $12T_D^2 = 1$

$$T_D = \frac{1}{\sqrt{12}} = 0.289$$

$$T_I = 1.15$$

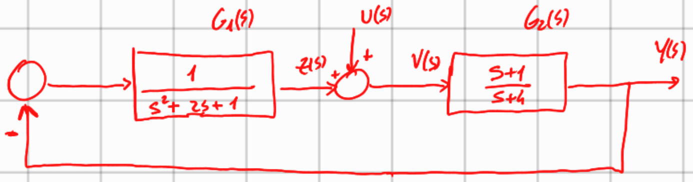
Ora impongo richiesta di guadagno:

$$|L(j\omega_\pi)| = |R(j\omega_\pi)|\,|G(j\omega_\pi)| = \left|K_p\left(1+\frac{1}{1.15\,j\sqrt{3}} + 0.289\,j\sqrt{3}\right)\right|\left|\frac{10}{(1+j\sqrt{3})^3}\right| = 1.25K_p$$

$$1.25K_p = \frac{1}{2} \Rightarrow \quad K_p = 0.4$$

$$R_{PID}(s) = 0.4\left(1+\frac{1}{1.15s} + 0.289s\right)$$

1)



$G_1(s) = \frac{1}{(s+1)^2}$. Uno zero cancella un polo sulla catena di ritorno. Non ragg. né osservabile.

Trovo ISU con forma di raggiungibilità:

$$G(s) = \frac{G_2(s)}{1+G_1(s)G_2(s)} = \frac{\frac{s+1}{s+4}}{1+\frac{\cancel{s+1}}{s+4}\cdot\frac{1}{(s+1)^{\cancel{2}}}} = \frac{(s+1)^2}{(s+4)(s+1)+1} = \frac{s^2+2s+1}{s^2+5s+5}$$

Dimostrazione che tutti poli a parte reale neg. e la dinamica cancellata era a. stabile.
Sistema è asintoticamente stabile.

$$A = \begin{pmatrix} 0 & 1 \\ -a_0 & -a_1 \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad c^T = \begin{pmatrix} b_0 - b_2 a_0 & b_1 - b_2 a_1 \end{pmatrix} \qquad d = b_2$$

$$A = \begin{pmatrix} 0 & 1 \\ -5 & -5 \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad c^T = \begin{pmatrix} 1-1\cdot 5 & 2-1\cdot 5 \end{pmatrix} \qquad d = 1$$

$$c^T = \begin{pmatrix} -4 & -3 \end{pmatrix}$$

1)

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\operatorname{atan}(x_1(t)) - 2x_2(t) + u(t) \\ y(t) = x_1(t) \end{cases} \qquad u(t) = 0.1\cos(t)$$

Linearizzo intorno a $\bar{u} = 0$

$x_2 = 0$ $\qquad \bar{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\operatorname{atan} x_1 = 0 \Rightarrow x_1 = 0 \qquad \bar{y} = 0$

$$A = \left.\frac{\delta f}{\delta x}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{1+x_1^2} & -2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$$

Punto di linearizz. è as. stabile.

$$b = \left.\frac{\delta f}{\delta u}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad c^T = \begin{pmatrix} 0 & 1 \end{pmatrix} \qquad d = 0.$$

$$G(s) = C(sI-A)^{-1}B + D = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} s & -1 \\ 1 & s+2 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{s^2+2s+1} \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} s+2 & 1 \\ -1 & s \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$= \frac{s}{s^2+2s+1} \qquad \delta u(t) = 0.1\cos(t) \Rightarrow \delta U = 0.1 \frac{s}{s^2+1}$$

$$Y(s) = 0.1 \cdot \frac{s^2}{(s^2+1)(s^2+2s-1)} = \frac{r_1}{(s+1+\sqrt{2})} + \frac{r_2}{(s+1-\sqrt{2})} + \frac{Q}{(s-j)} + \frac{Q^*}{(s+j)}$$

$$r_1 = \left.\frac{0.1\ s^2}{(s^2+1)(s+1-\sqrt{2})}\right|_{s=-1-\sqrt{2}} = -0.03 \qquad r_2 = 0.05 \qquad Q = \left.\frac{0.1\ s^2}{(s+j)(s^2+2s-1)}\right|_{s=j} = 0.025$$

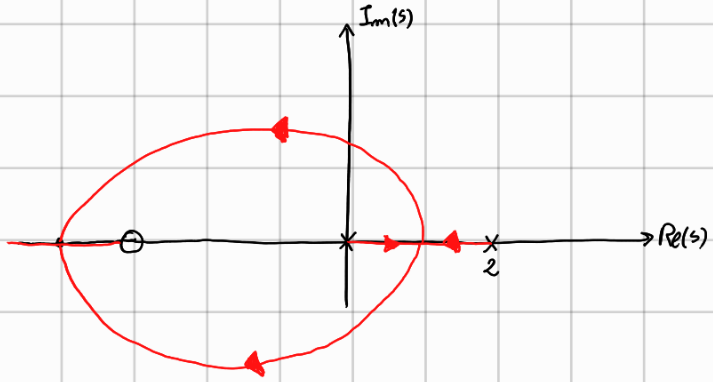
$$\delta y(t) = 0.05 e^{(-1+\sqrt{2})t} - 0.03 e^{(-1-\sqrt{2})t} + 0.05\cos(t) + 0.$$

2)

$$G(s) = \frac{1}{s-2}$$

• Astatismo a disturbi costanti e modo aperiodico con $T_{a_2} = 1s$

$\bar{\sigma} = \frac{\ln(0.02)}{1} = -3.9$, Devo imporre poli in -3.9 e polo in origine.



Impongo punto doppio in -3.9.

$$\gamma(x) = \frac{x(x-2)}{x-z} \qquad \gamma'(x) = \frac{(x-2)(x-z)+x(x-z)-x(x-2)}{(x-z)^2} = 0$$

$$\cancel{x^2} - \cancel{2x} - zx + 2z + x^2 - zx - \cancel{x^2} + \cancel{2x} = 0$$

$$x^2 - 2zx + 2z = 0$$

Impongo soluzione in -3.9.

$$15.21 + 7.8z + 2z = 0$$

$$15.21 + 9.8z = 0$$

$$z = -1.55$$

Trovo $\bar{\rho}$ per avere poli in -3.9:

$$\bar{\rho} = \frac{\eta_1 \eta_2}{\lambda_1} = \frac{3.9 \cdot 5.9}{2.35} = 9.79$$

$$L(s) = \frac{9.79\,(s+1.55)}{s(s-2)}$$

1)

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -\arctan(x_1(t)) - 2x_2(t) + u(t) \qquad u(t) = 0.1\cos(t)$$

$$y(t) = x_1(t)$$

Linearizzo con $\bar{u} = 0$:

$x_2 = 0$

$\arctan x_1 = 0 \Rightarrow \bar{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

forma di raggiung. $\Rightarrow$ Per Cartesio p.eq. è A.S.

$$A = \left.\frac{\delta f}{\delta x}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{1+x_1^2} & -2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$$

$$b = \left.\frac{\delta f}{\delta u}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad c^T = (1 \quad 0) \qquad d = 0$$

$$G(s) = C(sI-A)^{-1}B = (1 \quad 0)\begin{pmatrix} s & -1 \\ 1 & s+2 \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{s^2+2s+1}(1 \quad 0)\begin{pmatrix} s+2 & 1 \\ -1 & s \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$= \frac{1}{\underbrace{s^2+2s+1}_{(s+1)^2}}$$

$$\delta u(t) = 0.1\cos t \qquad \delta U(s) = \frac{1}{10} \cdot \frac{s}{s^2+1}$$

$$\delta Y(s) = \left[\frac{r_{11}}{s+1} + \frac{r_{12}}{(s+1)^2} + \frac{Q}{s-j} + \frac{Q^*}{s+j} \right]$$

$$r_{12} = \frac{1}{10} \cdot \left.\frac{s}{s^2+1}\right|_{s=-1} = \frac{-1}{20} = -\frac{1}{20}$$

$$r_{11} = \frac{1}{10} \cdot \left.\frac{(s^2+1) - 2s^2}{(s^2+1)^2}\right|_{s=-1} = \frac{1}{10}\,\frac{2-2}{2} = 0$$

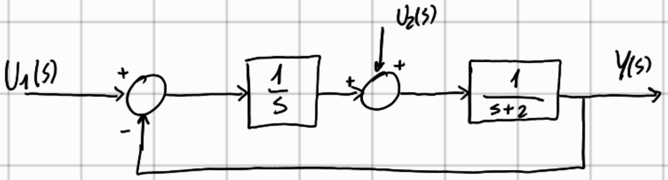
$$Q = \frac{1}{10}\,\frac{s}{s+j\omega} \cdot \left.\frac{1}{(s+1)^2}\right|_{s=j} =$$

$$= \frac{1}{10}\,\frac{\cancel{j\omega}}{2\cancel{j\omega}} \cdot \frac{1}{(1+j)^2} =$$

$$= \frac{1}{20} \cdot \frac{1}{2j} = \begin{cases} |Q| = \frac{1}{40} \\ \angle Q = -\frac{\pi}{2} \end{cases}$$

$$\delta y(t) = -\frac{1}{20}\,t e^{-t} + \frac{1}{20}\cos\left(t - \frac{\pi}{2}\right)$$

1)



$u_1(t) = e^{-3t}\delta_{-1}(t) \qquad u_2(t) = t\,\delta_{-1}(t)$

$$F(s) = \frac{\frac{1}{s}\cdot\frac{1}{s+2}}{1+\frac{1}{s(s+2)}} = \frac{1}{s^2+2s+1} = \frac{1}{(s+1)^2}$$

$$U_1(s) = \frac{1}{s+3} \qquad Y(s) = \frac{1}{s+3}\cdot\frac{1}{(s+1)^2} = \frac{r_1}{s+3} + \frac{r_{21}}{s+1} + \frac{r_{22}}{(s+1)^2}$$

$$r_1 = \left.\frac{1}{(s+1)^2}\right|_{s=-3} = \frac{1}{4} \qquad r_{22} = \left.\frac{1}{s+3}\right|_{s=-1} = \frac{1}{2}$$

$$r_{21} = \left.\frac{-1}{(s+3)^2}\right|_{s=-1} = -\frac{1}{4}$$

$$y_1(t) = -\frac{1}{4}e^{-t} + \frac{1}{2}te^{-t} + \frac{1}{4}e^{-3t}, \quad t \geq 0.$$

$$F_2(s) = \frac{\frac{1}{s+2}}{1+\frac{1}{s}\cdot\frac{1}{s+2}} = \frac{s}{s^2+2s+1} \qquad U_2(s) = \frac{1}{s^2} \qquad Y(s) = \frac{1}{s(s^2+2s+1)}$$

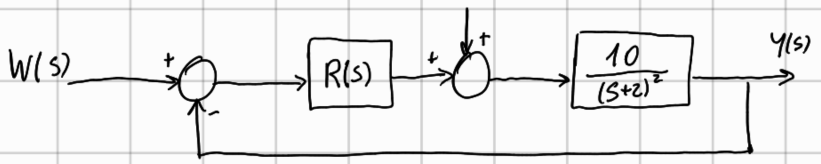
$$Y(s) = \frac{r_1}{s} + \frac{r_{21}}{s+1} + \frac{r_{22}}{(s+1)^2}$$

$$r_1 = \left.\frac{1}{s^2+2s+1}\right|_{s=0} = 1$$

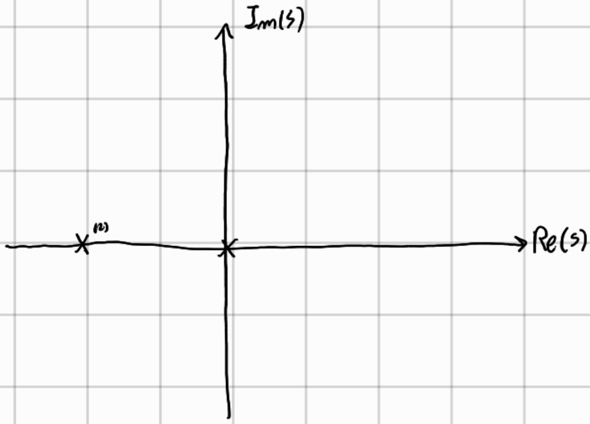
$$r_{22} = \left.\frac{1}{s}\right|_{s=-1} = -1 \qquad r_{21} = \left.-\frac{1}{s^2}\right|_{s=-1} = -1$$

$$y_2(t) = 1 - te^{-t} - e^{-t}, \quad t \geq 0$$

2)



$\bar{\sigma} = -4.6$



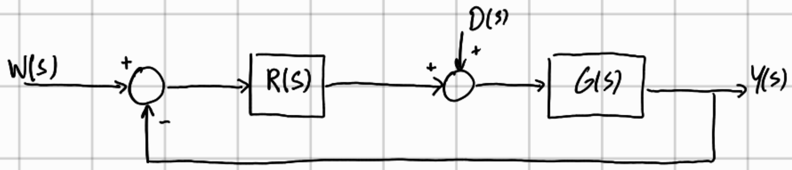
Provo ad inserire due zeri per bilanciare poli e imporre punto doppio in -5, con polo in -6.64, perché con polo doppio $T_{a1} = 6.64T \Rightarrow T = \frac{1}{6.64} \Rightarrow \bar{\sigma} = -6.64$

$$R(s) = \frac{M(s+2)^2}{s(s+13.28)} \qquad \bar{P} = 6.64^2 = 44.1 = 10M \Rightarrow M = 4.41$$

$$L(s) = \frac{44.1}{s(s+13.28)}$$

NOTA: Qui ho usato il $T_{a1} \cong 6.64T$ per trovare il polo doppio per il tempo di assestamento. Va bene?

2)



- $G(s) = -\frac{1}{2}$
- Voglio $K_m = 6\,dB \iff K_m = 2$

- Scelgo $T_I = 4T_D$ per avere zeri coincidenti.

$\omega_c = 1\,rad/s$. Impongo che $|R_{PID}(j\omega_c)| = 1$, $\angle R_{PID}(j\omega_c) = 0$

$$\left|K_P\left(1 + \frac{1}{4T_D j} + jT_D\right)\right| = 1 \quad \Rightarrow \quad |K_P|\left|1 + j\left(T_D - \frac{1}{4T_D}\right)\right| = 1$$

$$\Rightarrow K_P\sqrt{1 + \left(T_D - \frac{1}{4T_D}\right)^2} = 1$$

Nota: $\angle K_P\left(1 + j\left(T_D - \frac{1}{4T_D}\right)\right) = 0 \Rightarrow \quad T_D - \frac{1}{4T_D} = 0$

$$4T_D^2 = 1 \Rightarrow T_D = \frac{1}{2} \qquad T_I = 2$$

$$\Rightarrow K_P\sqrt{1 + 0} = 1 \Rightarrow K_P = 1$$

$$R_{PID}(s) = \left(1 + \frac{1}{2s} + \frac{1}{2}s\right)$$

1)

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -64x_1(t) - a x_2(t) + u(t)$$
$$y(t) = 64 x_1(t)$$

$$A = \begin{pmatrix} 0 & 1 \\ -64 & -7.81 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$C^T = \begin{pmatrix} 64 & 0 \end{pmatrix} \quad d = 0$$

a/ sistema ha modi pseudo periodici convergenti.

$$(sI-A)^{-1} = \begin{pmatrix} s+7.81 & -1 \\ 64 & s \end{pmatrix} \cdot \frac{1}{s(s+7.81)+64}$$

$$A = \begin{pmatrix} 0 & 1 \\ -64 & -a \end{pmatrix}, \text{ forma compagna.}$$

$$\begin{pmatrix} 64(s+7.81) & -64 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -64$$

$$p(s) = \underbrace{s^2}_{s^2} + \underbrace{as}_{2\cdot 8\cdot s} + \underbrace{64}_{8^2} = 0.$$

$\Rightarrow a = 16.$

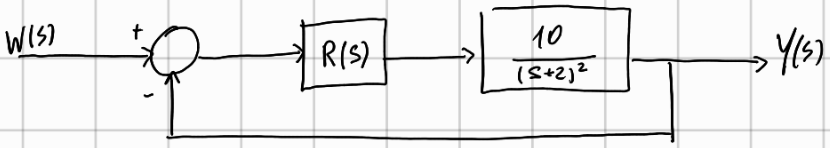
Se voglio $T_{a2} = 1s$, allora $\frac{\ln(0.02)}{-\xi\omega_n} = 1 \Rightarrow -\xi\omega_n = \ln(0.02) = -3.906$. Parte reale dei poli é $-\frac{a}{2}$.

$a = 2 \cdot 3.906 = 7.81$ Il discriminante é $<0 \Rightarrow$ ho poli complessi e convergenti.

$S\% = 100\, e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$, con $-\xi\omega_n = -\frac{a}{2}$ $\omega_n = \sqrt{64} = 8. \Rightarrow \xi = \frac{a}{2\omega_n} = 0.49$

$S\% = 17,1\,\%$

2)



$R_1(s) = \frac{M}{s}$, voglio astatismo per disturbi costanti

Errore a regime $= 20\%$ su $W(t)$. $\quad e_\infty = \frac{A}{10M} = 0.2 \Rightarrow \frac{1}{10M} = 0.2 \Rightarrow 10M = 5 \Rightarrow M = 0.5$

$R_1(s) = \frac{0.5}{s}$

$\varphi_m = 40°$, $\omega_c = 0.7\ rad/s$.

$L^*(s) = \frac{5}{s(s+2)^2} \qquad L^*(0.75) = \frac{5}{0.75(2+0.75)^2} = 1.591 e^{-2.245}$

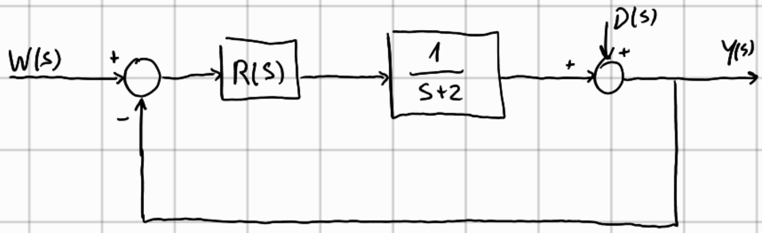
Mi serve attenuazione di 1.591 e ritardo di $11.42°$ $(\theta = -128.58°)$

$M = \frac{1}{1.591} = 0.629 \quad \varphi = -11.42°$

$\alpha = \frac{M\cos\varphi - 1}{M(M - \cos\varphi)} \qquad T = \frac{M - \cos\varphi}{\hat{\omega}\sin\varphi} \qquad R_2(s) = \frac{1 + 2.54s}{1 + 4.42s}$

$\alpha = 1.74 \qquad T = 2.54$

2)



- Astatico con disturbo a rampa: 2 poli nell'origine.
- $\omega_c = 1$ rad/s con $\varphi_m = 40°$.

$$R_1(s) = \frac{M}{s^2} \qquad L^*(s) = \frac{M}{s^2} \cdot \frac{1}{s+2} \Rightarrow L^*(s) = -M \cdot \frac{1}{2+s}$$

0.45M

153.44°

Ho bisogno di ritardo di 13.44°.

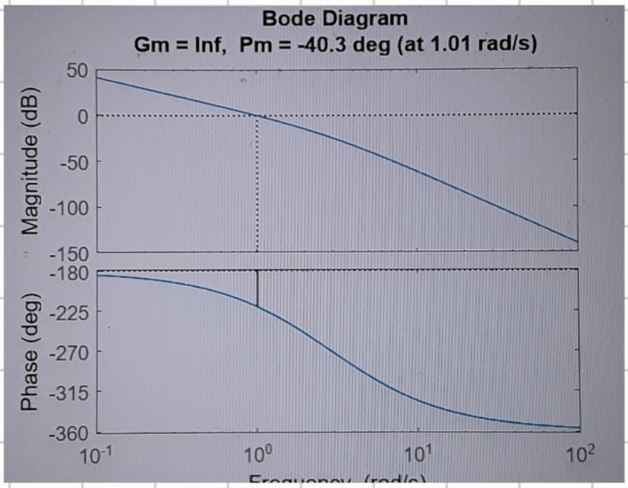
$$R_2(s) = \frac{1}{1+sT} \qquad \angle R_2(s) = \angle(1+Ts) = +13.44°$$

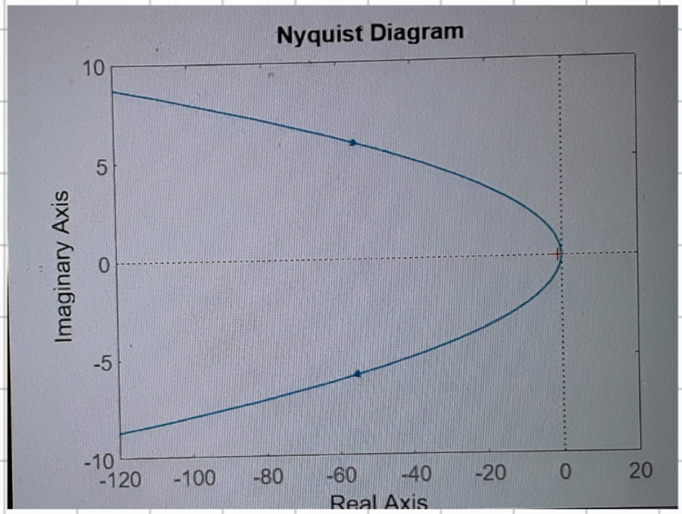
$$T = \tan(+13.44°) = +0.24$$

$$L(s) = \frac{M}{s^2(1+0.24s)(s+2)}$$ Trovo $M/|L(s)| = 1$

$$\Rightarrow \frac{M}{|1+0.24s||2+s|} = M \cdot 0.43 = 1 \Rightarrow M = 2.33$$

$$L(s) = \frac{2.33}{s^2(1+0.24s)(s+2)} = \frac{1.66}{s^2(1+0.24s)(1+0.5s)}$$





Matlab vede 140° come -220°. Come funziona?

• Alternativa:

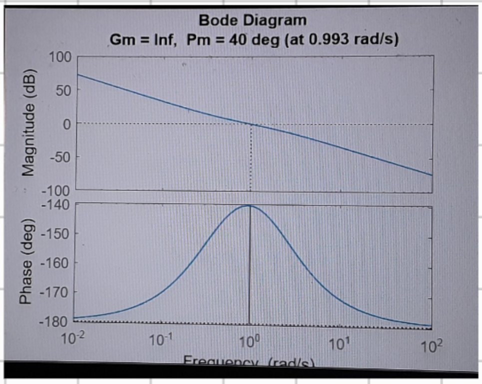
$$L^*(s) = -M \cdot \frac{1}{2+s} \begin{cases} 0.45M \\ 153.44^\circ = -206.56^\circ \Rightarrow \text{Mi serve anticipo di } 66.56^\circ. \end{cases}$$

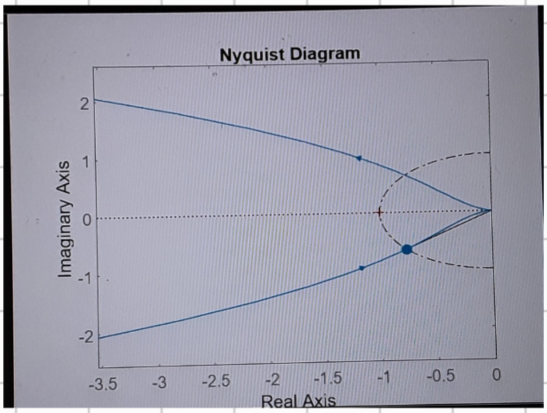
$R_2(s) = 1+sT \qquad \angle R_2(s) = \angle(1+sT) = 66.56^\circ \Rightarrow T = \tan(66.56^\circ) = 2.31$

$R_2(s) = 1+2.31s$

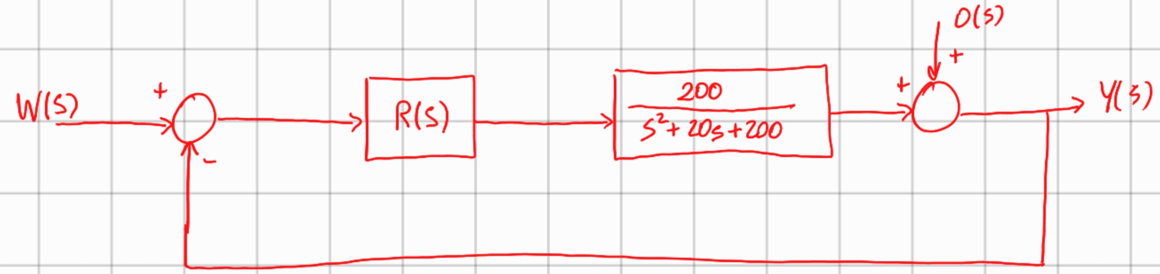
$$L(s) = \frac{M(1+2.31s)}{s^2(s+2)} \qquad |L(s)| = 1 \Rightarrow \frac{M\,|1+2.31s|}{|2+s|} = M \cdot 1.13 = 1 \Rightarrow M = 0.88$$

$$L(s) = \frac{0.88(1+2.31s)}{s^2(s+2)} = \frac{0.44(1+2.31s)}{s^2(1+0.5s)}$$



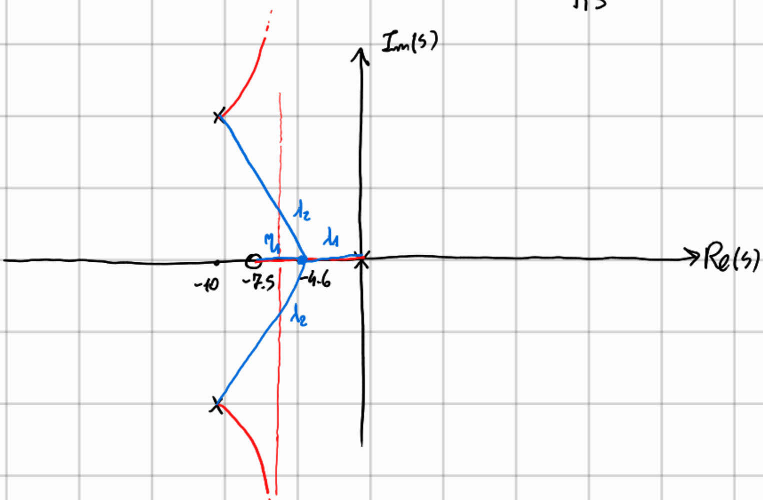


2)



• $T_{a1} = 1s$

$s^2+20s+200=0$ $\quad s=-10\pm 10j$ $\quad R_{PI}(s)= K_p \dfrac{1+sT_I}{T_I s}$



Voglio imporre polo in -4.6 per avere $T_{a1}=1s$. Provo ad inserire zero in -7.5.

$$x_A = \frac{1}{2}(-10-10+7.5) = -6.25.$$

Trovo $\bar{p}$ / polo si trova in -4.6: $\bar{p} = \dfrac{l_1 l_2^2}{m_1} = \dfrac{4.6\cdot(5.4^2+10^2)}{2.9} \cong 204.87 = 200\mu = 1.02$

$$R(s) = \frac{1.02}{s}(s+7.5) = \frac{7.65}{s}(1+0.13s) \qquad T_I = 0.13 \qquad K_p = T_I \cdot 7.65 = 0.995$$

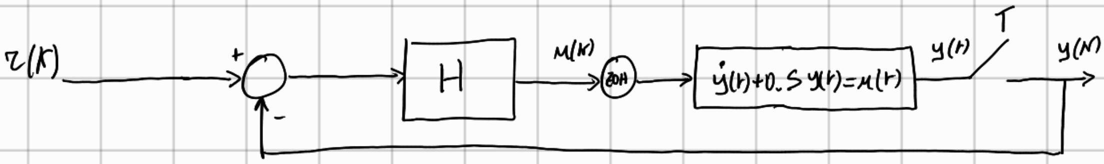
$$R_1(s) = 0.995\,\frac{1+0.13s}{0.13s}$$

$$L(s) = R_1(s)G(s)$$

Calcolo $M_1 = \lim_{s\to 0} sL(s) = \dfrac{0.995(1+0.13s)}{0.13}\cdot\dfrac{200}{s^2+20s+200} = 7.65$

$$e_\infty = \frac{2}{M_1} = 0.26$$

1)



$$y(t)=x(t) \Rightarrow \begin{cases} \dot x(t) = -0.5x(t)+u(t) \\ y(t)=x(t) \end{cases}$$

$$a=-0.5 \qquad b=1 \qquad c=1 \qquad d=0$$

$$a^* = e^{AT_s} = e^{-0.5\ln 2} = e^{\ln 2^{-0.5}} = 2^{-0.5} = \frac{\sqrt2}{2}$$

$$b = \int_0^{\ln 2} e^{a\sigma} b\, d\sigma = \int_0^{\ln 2} e^{-0.5\sigma}\, d\sigma =$$

$$= -2\left[e^{-0.5\sigma}\right]_0^{\ln 2} = -2\left[e^{\ln 2^{-0.5}} - 1\right] = -2\left[\frac{\sqrt2}{2}-1\right] = 2-\sqrt2 = 0.586$$

Sistema:

$$x(k+1) = \frac{\sqrt2}{2}x(k) + 0.586\,u(k)$$

$$y(k) = x(k)$$

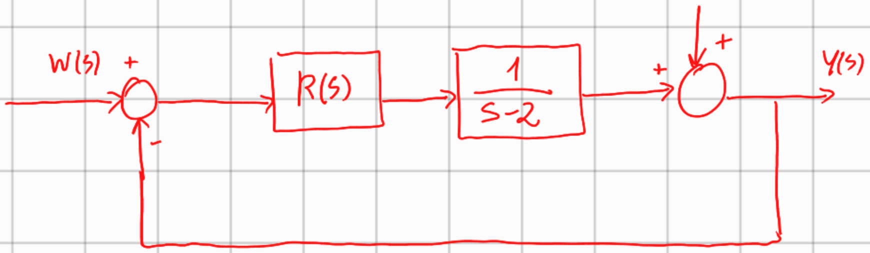
$$G(z) = C(zI-A)^{-1}B+D = 1\cdot\frac{1}{z-\frac{\sqrt2}{2}}\cdot 0.586 = \frac{0.586}{z-\frac{\sqrt2}{2}}$$

$$L(z) = \frac{0.586H}{z-\frac{\sqrt2}{2}} \qquad F(z) = \frac{0.586H}{z-\frac{\sqrt2}{2}+0.586H} \Rightarrow 0.586H - \frac{\sqrt2}{2} > 0$$

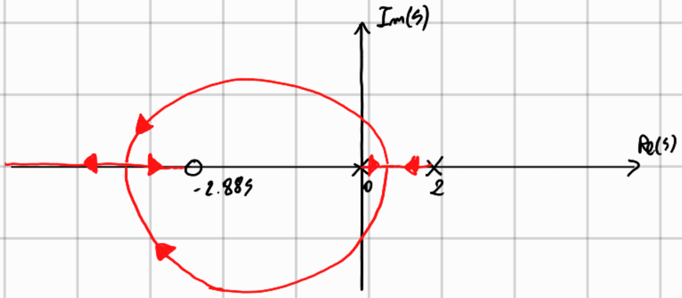
$$(2-\sqrt2)H > \frac{\sqrt2}{2}$$

$$H > \frac{\sqrt2}{2(2-\sqrt2)} \qquad H > 1.207$$

2)



$R(s) = \frac{M}{s}$ almeno per astatismo. Aggiungo zero per avere polo doppio in -6.64.



$$\gamma(x) = \frac{(x-2)x}{x+z} \qquad 0 = \gamma'(x) \Rightarrow (x-2)(x+z) + x(x+z) - x(x-2) = 0$$

Impongo -6.64 come soluzione.

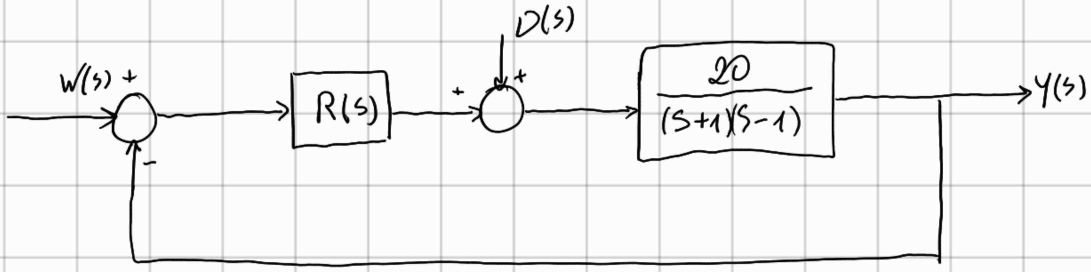
$$-8.64(6.64+z) - 6.64(6.64+z) - 6.64 \cdot 8.64 = 0$$

$$z = 2.885$$

Trovo $\bar{\rho}$ che porta poli in -6.64:

$$\bar{\rho} = \frac{6.64 \cdot 8.64}{3.755} = 15.28 \qquad R(s) = \frac{15.28\,(s+2.885)}{s}$$

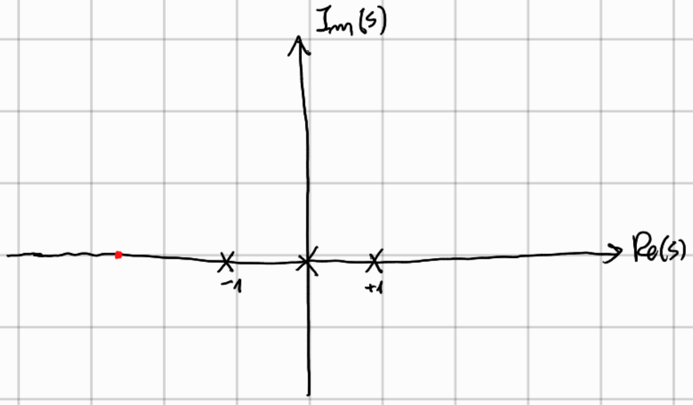
2)



• Associamo a $D(s) = \frac{1}{s}$ e $T_{a1} = 2s$. $\Rightarrow$ Devo avere poli in $\frac{-5}{T_{a1}} = -2.5$.

• Mi serve un polo nell'origine per gradino.

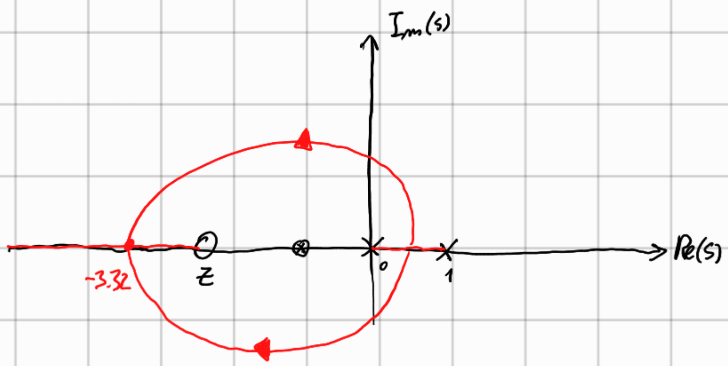
• Disegno Root Locus



• Inserisco zero in -1.

Caso polo doppio $\Rightarrow T_a \simeq 6.64\, T$, quindi, polo è in $-\frac{1}{T}$, $\frac{1}{T} = \frac{6.64}{2} = 3.32$.

Punto doppio si troverà in -3.32.



$\gamma(x) = \frac{(x-1)x}{(x-z)}$ $\quad 0 = \gamma'(x) \Rightarrow (x-1)(x-z) + x(x-z) - x(x-1) = 0$

Impongo che -3.32 sia soluzione.

$z = -1.44$

Trovo $\bar{\rho}$ che impone quel polo: $\bar{\rho} = \frac{3.32 \cdot 4.32}{1.88} = 7.63 = \mu \cdot 20 \Rightarrow \mu = 0.38$

$R(s) = \frac{0.38}{s}(s+1.44)(s+1) = \frac{0.38}{s}(s^2 + 2.44s + 1.44) = 0.38\left(s + 2.44 + \frac{1.44}{s}\right) = R_{PID}(s) = 0.93\left(1 + \frac{1}{1.69s} + 0.41s\right)$

1)

$$x(k+1) = \begin{pmatrix} 0 & 1 \\ 0.2h & h-0.2 \end{pmatrix} x_k + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = (0.2 \quad 1)\, x(k)$$

• Matrice A in forma compagna. $p(s) = s^2 - (h-0.2)s - 0.2h$

Stabilità asintotica:

$$s = \frac{z+1}{1-z} \qquad p(z) = \frac{z^2+2z+1}{(1-z)^2} - \frac{(h-0.2)(z+1)}{(1-z)} - 0.2h$$

Studio gli zeri:

$$z^2+2z+1-(h-0.2)(1-z^2)-0.2h(1+z^2-2z)=0$$

$$z^2+2z+1-h+hz^2+0.2-0.2z^2-0.2h-0.2hz^2+0.4hz=0$$

$$z^2(0.8+0.8h)+z(2+0.4h)+1.2-1.2h=0$$

$$\begin{cases} 0.8+0.8h>0 \\ 2+0.4h>0 \\ 1.2-1.2h>0 \end{cases} \Rightarrow \begin{array}{l} h>-1 \\ h>-\frac{2}{0.4} \\ h<1 \end{array} \Rightarrow -1<h<1$$

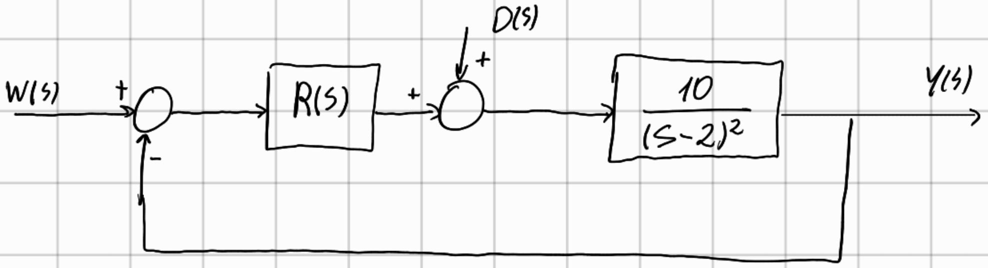
$h=\pm1$, sistema stabile.

• Nota: sistema è in forma di oss.

$$\mathcal{O} = \begin{pmatrix} C^T \\ C^T A \end{pmatrix} \qquad C^T A = \begin{pmatrix} 0.2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0.2h & h-0.2 \end{pmatrix} = \begin{pmatrix} 0.2h & h \end{pmatrix}$$

$$\mathcal{O} = \begin{pmatrix} 0.2 & 1 \\ 0.2h & h \end{pmatrix}$$ ha rango 1 $\forall h$. Non osservabile.

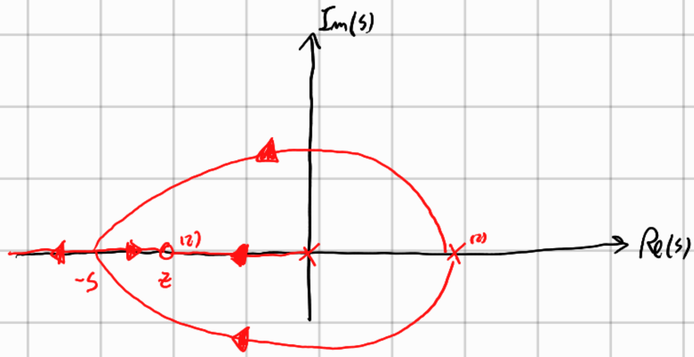
2)



- Astatismo a d(t) a gradino. $\Rightarrow$ $R(s)$ ha polo nell'origine.
- Modo di evoluzione aperiodico con $\tau = 0.2s$ doppio

$\Rightarrow$ Voglio un polo $(1+0.2s)^2 \Rightarrow s_0 = -\frac{1}{0.2} = -5$

Disegno il luogo:



Nota: metteremo due zeri per avere punto doppio in -5.

$$\gamma(x) = \frac{(x-2)^2 x}{(x-z)^2}$$

$$\gamma'(x) = 0 \Rightarrow (x-2)^{\cancel{2}}(x-z)^{\cancel{2}} + 2x\cancel{(x-2)}(x-z)^{\cancel{2}} - 2x(x-2)^{\cancel{2}}\cancel{(x-z)} = 0$$

$$(x-2)(x-z) + 2x(x-z) - 2x(x-2) = 0$$

$$x^2 - 2x - zx + 2z + \cancel{2x^2} - 2zx - \cancel{2x^2} + 4x = 0$$

Impongo soluzione in -5:

$$25 + 10 + 5z + 2z + 10z - 20 = 0$$

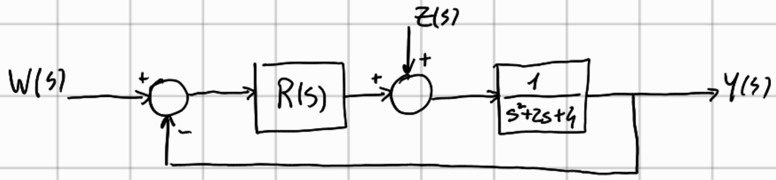
$$17z + 15 = 0 \qquad z = \frac{-15}{17} = -0.882$$

Trovo la $\bar{\rho}$: $\bar{\rho} = \frac{5 \cdot 7^2}{4.118^2} = 14.45 = 10\mu \Rightarrow \mu = 1.445$

$$R_{PID}(s) = \frac{1.445}{s}(s+0.882)^2 = \frac{1.445}{s}\left(s^2 + 0.78 + 1.76s\right) = 1.445\left(s + \frac{0.78}{s} + 1.76\right) =$$

$$= 2.54\left(1 + \frac{1}{2.26s} + 0.57s\right)$$

2)



• Astatico a disturbo $z(t)$ costante $\Rightarrow$ Polo in origine.

• $\varphi_m = 40^\circ$ in $\omega_c = 2$ rad/s.

$$R_1(s) = \frac{M}{s} \qquad L^*(s) = \frac{M}{s(s^2+2s+4)}$$

Calcolo fase in $\omega = 2$ rad/s: $L^*(2j) = \dfrac{M}{2j(4j)} = \dfrac{-M}{8}$, se $M>0$, fase è -180°.

Mi serve un anticipo di 40°.

$$R_2(s) = 1+sT \;/\; \angle R_2(j\cdot 2) = 40^\circ$$

$$\Rightarrow \operatorname{atan}(2T) = 40^\circ$$

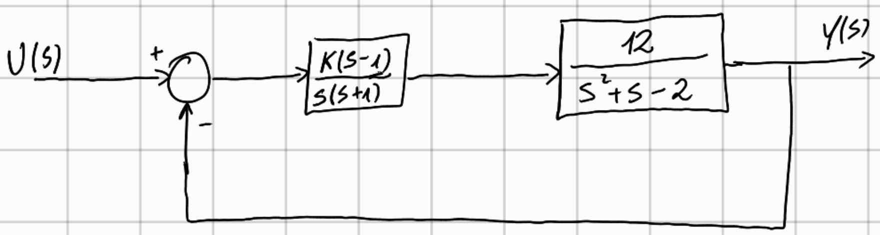
$$T = \frac{\tan(40^\circ)}{2} = 0.42$$

Impongo che $|L(2j)| = 1$

$$\frac{|1+0.84j|M}{|2j|} \cdot \frac{1}{|4j|} = 1 \Rightarrow M = \frac{8}{\sqrt{1^2+0.84^2}} = 6.13$$

$$R(s) = \frac{6.13}{s}(1+0.42s)$$

1)



Nota: Ci sarà sempre cancellazione sulla catena di andata di polo instabile. Non posso avere A.S.

$$F(s) = \frac{\dfrac{12}{(\cancel{s-1})(s+2)} \cdot \dfrac{K(\cancel{s-1})}{s(s+1)}}{1 + \dfrac{12K}{(s+2)(s+1)s}} =$$

$$= \frac{12K}{s(s+1)(s+2) + 12K}$$

Denominatore: $(s^2+s)(s+2) + 12K = s^3 + 2s^2 + s^2 + 2s + 12K = 0$

$$s^3 + 3s^2 + 2s + 12K = 0$$

Routh:

3	1	2
2	3	12K
1	6-12K	
0	12K	

$\Rightarrow$ Per stabilità bibo, $\begin{cases} 6-12K > 0 \\ 12K > 0 \end{cases} \Rightarrow 0 < K < \frac{1}{2}$

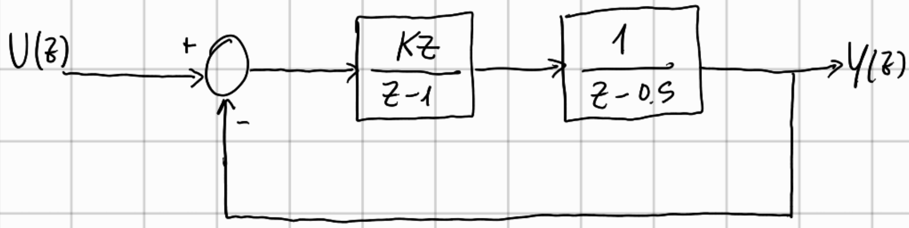
Scelgo $K = \frac{1}{12}$

$$F(s) = \frac{1}{s^3 + 3s^2 + 2s + 1}$$

$F(0) = 1$ $\quad F(j) = \frac{1}{-j - 3 + 2j + 1} = \frac{1}{j - 2} = -0.4 - 0.2j = \begin{cases} |\,| & 0.447 \\ \angle & 3.6 \end{cases}$

$$y_r(t) = 5 \cdot 1 + 0.894 \cos(t + 3.6)$$

1)



$$F(z) = \frac{\frac{Kz}{z-1} \cdot \frac{1}{z-0.5}}{1 + \frac{Kz}{z-1} \cdot \frac{1}{z-0.5}} =$$

$$= \frac{Kz}{(z-1)(z-0.5) + Kz}$$

Valuto denominatore: $z^2 - 1.5z + 0.5 + Kz = 0$

$$z^2 + (K-1.5)z + 0.5 = 0$$

$$z = \frac{1+s}{1-s}$$

$$(1+s)^2 + (K-1.5)(1+s)(1-s) + 0.5(1-s)^2 = 0$$

$$s^2 + 2s + 1 + (K-1.5)(1-s^2) + 0.5(s^2 - 2s + 1) = 0$$

$$2s^2 + 4s + 2 + (2K-3)(1-s^2) + s^2 - 2s + 1 = 0$$

$$3s^2 + 2s + \cancel{3} + 2K - \cancel{3} - 2Ks^2 + 3s^2 = 0$$

$$(6-2K)s^2 + 2s + 2K = 0$$

$$\begin{cases} 6 - 2K > 0 \\ K > 0 \end{cases} \Rightarrow 0 < K < 3$$

Scelgo $K = 1$

$$F(z) = \frac{z}{(z-1)(z-0.5) + z}$$

$$u(k) = 5\delta_{-1}(k)$$

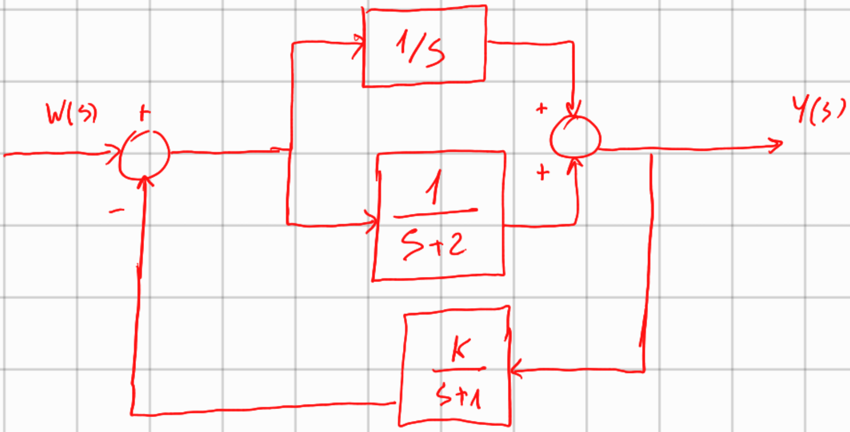
$$M = F(1) = 1 \Rightarrow \quad y(k) = 5\delta_{-1}(k)$$

2)

$$G(s) = \frac{10}{(s+1)(s-1)}$$

- Errore nullo a gradino e $T_{a1} = 2s$ con modi aperiodici.

1)



$$L(s) = \frac{1}{s} + \frac{1}{s+2} = \frac{2s+2}{s(s+2)} \qquad F(s) = \frac{\frac{2(\cancel{s+1})}{s(s+2)} \cdot \frac{K}{\cancel{s+1}}}{1 + \frac{2(\cancel{s+1})}{s(s+2)} \cdot \frac{K}{\cancel{s+1}}} =$$

$$= \frac{2K}{s^2+2s+2K}$$

Se voglio modo pseudoperiodico convergente, $K>0$.

Se $\xi = 0.5$,

$$2 = 2\xi\omega_n \Rightarrow \omega_n = 2 \qquad 2K = \omega_n^2 \Rightarrow K = 2$$

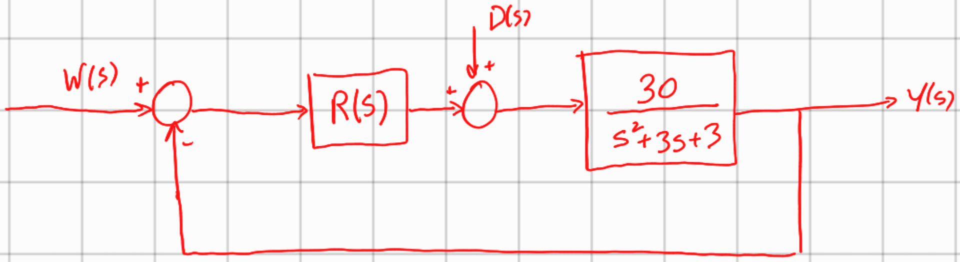
$$\hookrightarrow s_{1,2} = -1 \pm \sqrt{-3} = -1 \pm j\sqrt{3}$$

Se $F(s) = \dfrac{4}{s^2+2s+4} = \dfrac{Q}{(s+1-j\sqrt{3})} + \dfrac{Q^*}{(s+1+j\sqrt{3})}$

$$Q = \left.\frac{4}{(s+1+j\sqrt{3})}\right|_{s=-1+j\sqrt{3}} = \frac{4}{2\sqrt{3}\,j} = \begin{cases} |\cdot| \ \frac{2}{3}\sqrt{3} \\ \angle \ -\frac{\pi}{2} \end{cases}$$

$$g_y(t) = \frac{4}{3}\sqrt{3}\, e^{-t} \cos\left(\sqrt{3}t - \frac{\pi}{2}\right) = \frac{4}{3}\sqrt{3}\, e^{-t} \sin(\sqrt{3}t)$$

2)



• Astatismo a disturbo costante ⇒ Polo in R(s).

• Margine di fase a 40° a $\omega_c = 1.5$ rad/s.

$$L^*(s) = \frac{M}{s}\cdot\frac{30}{s^2+3s+3}$$

Calcolo $L(j\omega_c) = \dfrac{30M}{1.5j\,(-1.5^2+4.5j+3)} = 4.38\,e^{-2.98j}$

$\varphi_c = -170.5°$, ma serve margine di 40° ⇒ $\varphi_m = 180° - |\varphi_c|$ ⇒ $\varphi_c = -140°$

Ho bisogno di anticipo di 30.5°.

$R_2(s) = 1+sT$ / $\angle R_2(j\omega_c) = 30.5°$

$$\arctan(\omega_c T) = 30.5°$$

$$T = \frac{\tan(30.5°)}{\omega_c} = 0.39$$

$$L(s) = \frac{M}{s}(1+0.39s)\cdot\frac{30}{s^2+3s+3}$$

Impongo $|L(j\omega_c)|=1$ $|L(j\omega_c)| = 5.08M = 1 \Rightarrow M = 0.197$

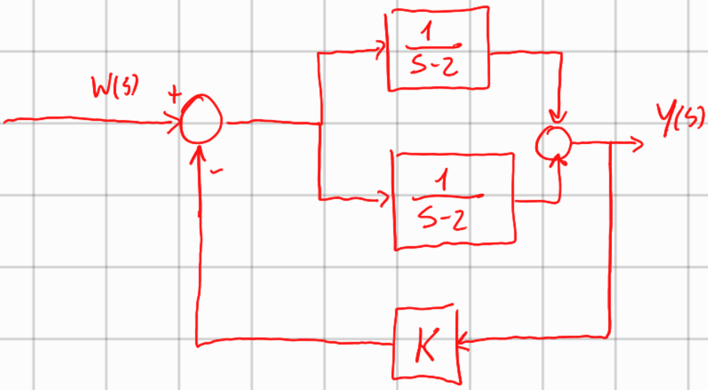
$$L(s) = \frac{0.197(1+0.39s)}{s}\cdot\frac{30}{s^2+3s+3}$$

$$R(s) = \frac{0.197(1+0.39s)}{s}$$

$$= \frac{0.077s+0.197}{s} = \overset{K_P}{0.077} + \frac{\overset{K_I}{0.197}}{s}$$

$T_I = \dfrac{K_P}{K_I} = 0.015$

1)



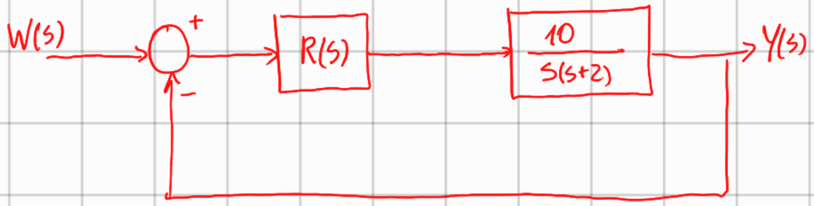
$$L(s) = \frac{2}{s-2}$$

$$F(s) = \frac{\frac{2K}{s-2}}{1+\frac{2K}{s-2}} = \frac{2K}{s+2K-2}$$

In questo caso, non é possibile avere la stabilitá asintotica perché avviene una cancellazione di un polo a parte reale positiva sul parallelo. Il sistema non é né compl. raggiungibile né osservabile.

Per la stabilitá bibo, $2K-2>0 \Rightarrow K>1$

2)



• Errore al 10% in ingresso $W(s) = \frac{1}{s^3}$, con $\varphi_m = 40°$ con $\omega_c = 5 rad/s$

$\nu = 3$, se $g = \nu - 1$, errore a regime è $\frac{A}{M}$.

$$L^*(s) = \frac{M_0}{s} \cdot \frac{10}{s(s+20)} \Rightarrow s^2 L(s)\Big|_{s=0} = \frac{M_0}{2} \Rightarrow \frac{A}{M} = \frac{1}{10}$$

$$\frac{2}{M_0} = \frac{1}{10} \Rightarrow M_0 = 20$$

$$L^*(s) = \frac{20}{s} \cdot \frac{10}{s(s+20)}$$

Calcolo $L^*(j5) \Rightarrow |\ | = 0.39$

$\angle = 166° = -194°$.

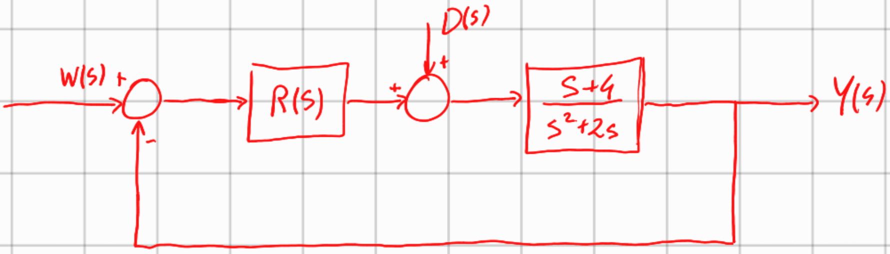
Per questo margine, mi serve anticipo di $\underset{\varphi}{54°}$ e un amplificazione di $\underset{M}{2.56}$.

$$\alpha = \frac{M\cos\varphi - 1}{M(M - \cos\varphi)} = 0.1 \qquad T = \frac{M - \cos\varphi}{\hat{\omega}\sin\varphi} = 0.49$$

$$R(s) = \frac{20}{s} \cdot \frac{1 + 0.49s}{1 + 0.049s} \qquad L(s) = \frac{20}{s} \cdot \frac{1 + 0.49s}{1 + 0.049s} \cdot \frac{10}{s(s+20)}$$

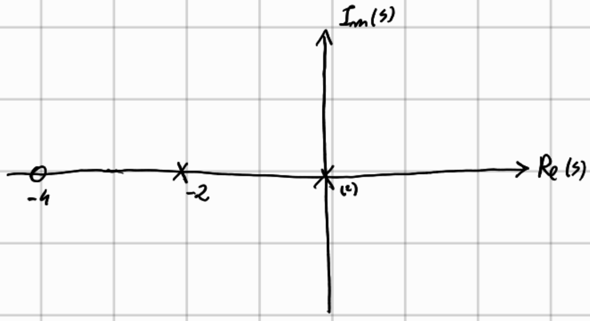
$$S(s) = \frac{s^2(s+20)(1+0{,}049s)}{s^2(s+20)(1+0{,}049s) + 200(1+0{,}49s)}$$

2)

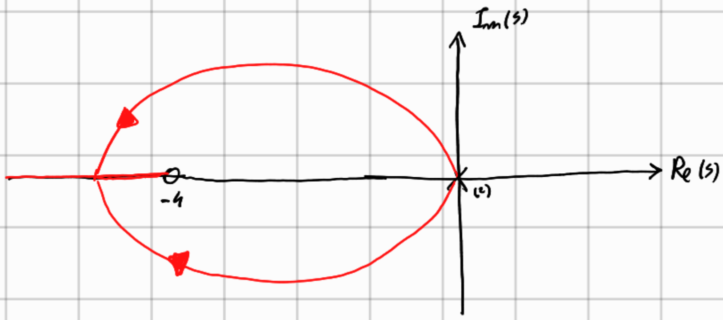


• Errore nullo a regime su $w(t)$ a gradino: voglio un polo nell'origine.

• $T_{a2} = 1s$



• Cancello polo in -2 con zero:



Trovo $\bar{p}$ che porta coppia di poli con parte reale in -3.9.

$s = z - 3.9$

Scrivo $1+L(s)=0 \Rightarrow$ $1+\frac{p(s+4)}{s^2}=0$

$\Rightarrow s^2+p(s+4)=0\Big|_{s=z-3.9}$

$z^2 - 7.8x + 15.21 + pz + 0.1p = 0$

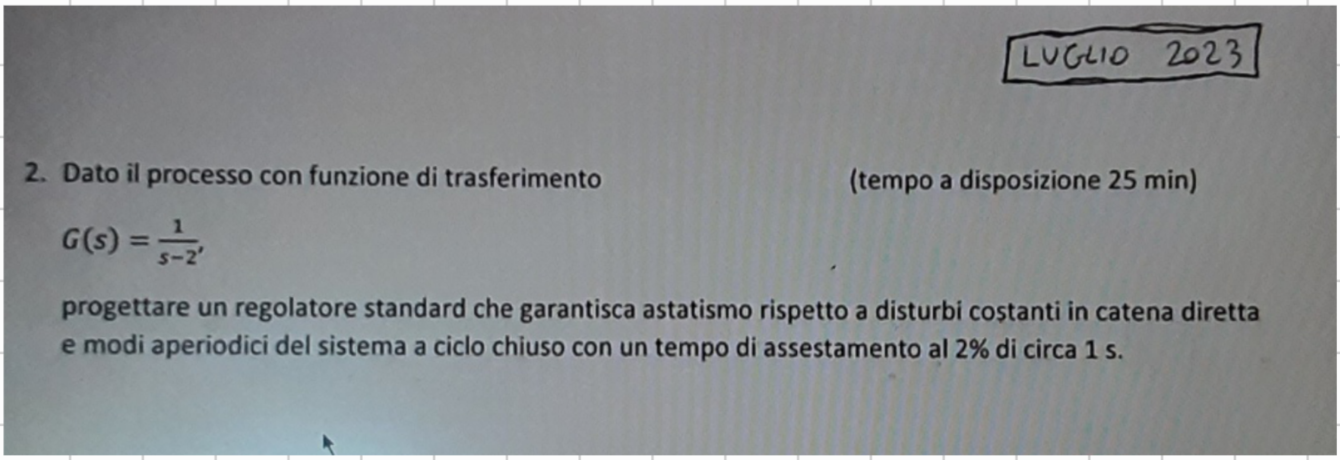
Devo imporre parte reale a 0:

$b=0 \Rightarrow p = 7.8$

Si vede che $\Delta<0$. Va bene

Quindi, se $p=7,8$ $R(s)=\frac{7.8}{s}(s+2)$

2)

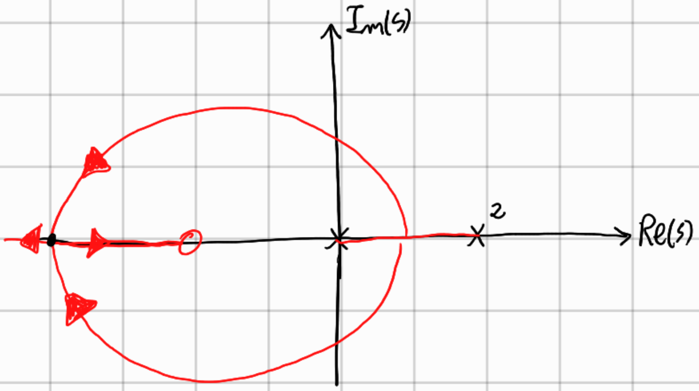


LUGLIO 2023

2. Dato il processo con funzione di trasferimento (tempo a disposizione 25 min)

$G(s) = \frac{1}{s-2}$,

progettare un regolatore standard che garantisca astatismo rispetto a disturbi costanti in catena diretta e modi aperiodici del sistema a ciclo chiuso con un tempo di assestamento al 2% di circa 1 s.



$T_{a2} \cong 5.83\,T \Rightarrow \sigma = -\frac{5.83}{T_{a2}} = -5.83$. Devo imporre punto doppio in 5.83.

$$\gamma(x) = \frac{x(x-2)}{x-z} \Rightarrow \gamma'(x) = 0 \Rightarrow (x-2)(x-z) + x(x-z) - x^2 + 2x = 0$$

$$x^2 - 2x - zx + 2z + x(x-z) - x^2 + 2x = 0$$

Impongo che $x = -5.83$ sia soluzione

$z = -2.49$

Quindi, zero é in -4.195.

Trovo $\bar{p}$: $\bar{p} = \frac{5.83 \cdot 7.83}{(5.83 - 2.49)} \cong 13.67$

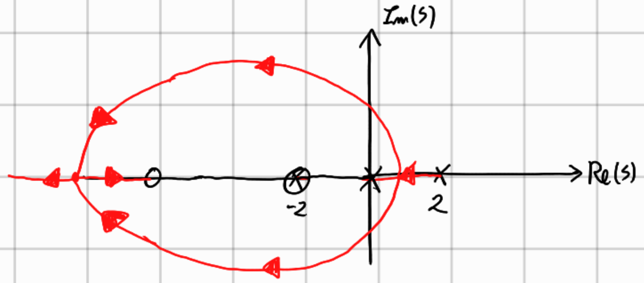
$$R(s) = \frac{13.67}{s}(s + 2.49) = \frac{13.67\,(1 + 0.4\,s)}{0.4}$$

SETTLING TIME: 1.0001 s

2)

$$G(s) = \frac{10}{(s+1)(s-1)}$$

R(s) ha almeno un polo in origine per astatismo.



Inserisco zero per avere punto doppio in $\frac{-6.64}{2} = -3.32$, perché $T_{a1} \cong 6.64T$

Cancello polo in -2.

$$\gamma(x) = \frac{x(x-2)}{x-z}$$

$$\gamma'(x) = (x-2)(x-z) - x(x-z) - x(x-2) = 0$$

Impongo $x = -3.32$ come soluzione.

$z = -1.28$

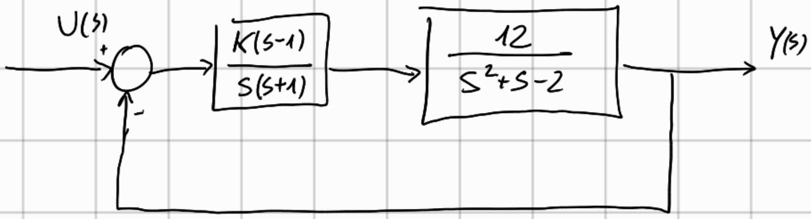
$$\bar{\rho} = \frac{3.32 \cdot 5.32}{(3.32-1.28)} = 8.67 = 10\mu \Rightarrow \mu = 0.867$$

$$R(s) = \frac{0.867}{s}(s+2)(s+1.28) = \frac{0.867}{s}(s^2+3.28s+2.56) =$$

$$= 2.84 + \frac{2.22}{s} + 0.867s$$

$$L(s) = \frac{8.67(s+1.28)}{s(s-2)}$$

1)



$s^2+s-2=(s-1)(s+2) \Rightarrow$ C'é cancellaz. di polo a $Re(\cdot)>0$.

Sistema non può essere A.S.

$$L(s)=\frac{12K}{s(s+1)(s+2)} \qquad F(s)=\frac{12K}{s(s+1)(s+2)+12K}$$

$$s^3+3s^2+2s+12K=0$$

3	1	2
2	3	12K
1	6-12K	
0	12K	

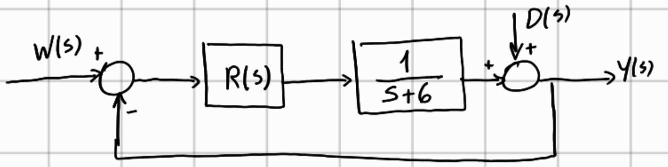
$K<\frac{1}{2}$

$K>0$

$K=\frac{1}{12}$ $\qquad F(s)=\frac{1}{s^3+3s^2+2s+1}$

$F(0)=1 \qquad F(j)=\frac{1}{-j-3+2j+1}=\frac{1}{j-2}=\frac{j+2}{1-4}$

2)



• Astatico con $D(s)$ a gradino.

• $\omega_c = 8$ rad/s

$$R(s) = \frac{\mu}{s} \Rightarrow L(s) = \frac{\mu}{s} \cdot \frac{1}{s+6} \qquad L(8j) = \frac{\mu}{8j} \cdot \frac{1}{8j+6}$$

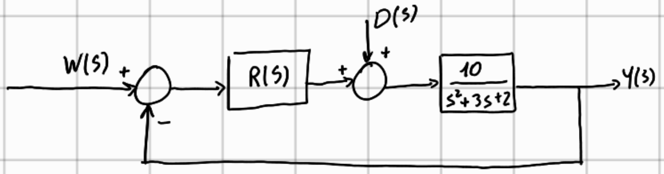
Impongo che $|L(8j)| = 1 = 0.0125\mu \Rightarrow \mu = 80$

• Calcolo $\angle L(j\omega_c)$: $\angle L(8j) = \frac{80}{8j} \cdot \frac{1}{8j+6} = -143^\circ \Rightarrow \varphi_m = 37^\circ \Rightarrow \xi \cong \frac{\varphi_m}{100} \approx 0.37 = 37\%$

• Calcolo ω_π: $\angle L(j\omega_\pi) = -\pi \Rightarrow -\frac{\pi}{2} - \arctan\left(\frac{\omega_\pi}{6}\right) = -\pi \Rightarrow$ deve avere numero puramente immaginario.

La $\omega \to \infty$. Sistema è stabile intrinsecamente.

2)



• Astatico con $d(t)$ a gradino

• $\varphi_m = 45°$ con $\omega_c = 1$ rad/s

$$R_1(s) = \frac{M}{s} \qquad L^*(s) = \frac{10M}{s(s^2+3s+2)} = \frac{10M}{s(s+2)(s+1)}$$

$$\text{Calcolo } L^*(j) = \frac{10M}{j(1+j)(2+j)} \Rightarrow \angle L^*(j) = -161.57°$$

Per avere $\varphi_m = 45°$, ho bisogno di anticipo di 26.57°.

$$\angle R_2(j) = \angle(1+jT) = 26.57° \Rightarrow T = \tan(26.57°) = 0.5$$

$$L(s) = \frac{10M(1+0.5s)}{s(s+1)(s+2)} = \frac{5M(\cancel{s+2})}{s(s+1)(\cancel{s+2})} = \frac{5M}{s(s+1)}$$

Impongo $|L(j)| = 1 \Rightarrow \left|\frac{5M}{j(1+j)}\right| = 1 \qquad \frac{5\sqrt{2}}{2}M = 1 \qquad \frac{\sqrt{2}}{5} = M \cong 0.28$

$$R(s) = \frac{0.28(1+0.5s)}{s} = \frac{0.14}{0.5s}(1+0.5s)$$

K_p T_I

1)

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -\arctan(x_1(t)) - x_2(t) + u(t)$$
$$y(t) = x_1(t)$$

$u(t) = \frac{\pi}{4}$. $\qquad u(t) = \frac{\pi}{4} + 0.1\cos(2t)$

$\bar{u} = \frac{\pi}{4} \Rightarrow \qquad x_2 = 0 \qquad\qquad \bar{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\arctan(x_1) = \frac{\pi}{4} \Rightarrow x_1 = 1$

$$\left.\frac{\delta f}{\delta x}\right|_{\substack{x=\bar{x}\\ u=\bar{u}}} = \left.\begin{pmatrix} 0 & 1 \\ -\frac{1}{1+x_1^2} & -1 \end{pmatrix}\right|_{\substack{x=\bar{x}\\ u=\bar{u}}} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{2} & -1 \end{pmatrix}$$

Matrice in forma compagna.
Polin. caratteristico ha coeff. dello stesso segno => Punto di eq. localmente stabile.

$$\left.\frac{\delta f}{\delta u}\right|_{\substack{x=\bar{x}\\ u=\bar{u}}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \left.\frac{\delta g}{\delta x}\right|_{\substack{x=\bar{x}\\ u=\bar{u}}} = (1 \quad 0) \qquad d = 0$$

$$G(s) = c^T(sI-A)^{-1}b = \frac{1}{s^2+s+0.5}(1 \quad 0)\begin{pmatrix} s+1 & 1 \\ -0.5 & s \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$= \frac{1}{s^2+s+0.5}$$

$\delta u(t) = 0.1\cos(2t)$. $\quad \omega = 2\,rad/s$

Calcolo $G(2j) = \frac{1}{-4+2j+0.5} = \begin{cases} |\cdot| \; 0.25 \\ -150^\circ \end{cases}$

$\delta y_r(t) = 0.05\cos(2t - 2.62) \qquad y_r(t) = 1 + 0.05\cos(2t - 2.62)$

1)

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -64x_1(t) - \alpha x_2(t) + u(t)$$

$$y(t) = 64x_1(t)$$

$$b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 64 & 0 \end{pmatrix}$$

Voglio modi di evoluz. pseudoper. convergenti.

$$A = \begin{pmatrix} 0 & 1 \\ -64 & -\alpha \end{pmatrix} \quad \text{forma compagna:} \quad P(s) = s^2 + \alpha s + 64$$

$$s_{1,2} = \frac{-\alpha \pm \sqrt{\alpha^2 - 256}}{2}$$

$$\begin{cases} \bullet \ \alpha > 0 \\ \bullet \ \alpha^2 - 256 < 0 \end{cases} \Rightarrow \begin{cases} \alpha > 0 \\ -16 < \alpha < 16 \end{cases} \Rightarrow \alpha \in (0, 16)$$

Voglio $T_{a2} = 1s$.

$$G(s) = \frac{1}{s^2 + \alpha s + 64} \begin{pmatrix} 64 & 0 \end{pmatrix} \begin{pmatrix} s+\alpha & 1 \\ -64 & s \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{64}{s^2 + \alpha s + 64}$$

Per modi pseudoperiodici: $T_{a2} \approx \dfrac{\ln\left(\frac{100}{2}\right)}{\xi\omega_n} \Rightarrow \xi\omega_n = 3.91$

$\Rightarrow$ Parte reale $= -3.91 = -\dfrac{\alpha}{2} \Rightarrow \alpha = 7.82 \qquad \omega_n = 8 \Rightarrow \xi = \dfrac{3.91}{\omega_n} = 0.49$

$$S\% = 100\, e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = 17.1\%$$

$$G(s) = \frac{64}{s^2 + 7.82s + 64} = \frac{64}{(s + 3.91 - 6.97j)(s + 3.91 + 6.97j)}$$

$$Y(s) = \frac{M}{s} + \frac{Q}{s+3.91-6.97j} + \frac{Q^*}{s+3.91+6.97j}$$

$$M = G(0) = 1$$

$$Q = \left.\frac{64}{s(s+3.91+6.97j)}\right|_{s=-3.91+6.97j} = \begin{cases} 0.57 \\ 150.7^\circ \end{cases}$$

$$y(t) = 1 + 1.14\, e^{-3.91t} \cos(6.97t + 2.63)$$

1)

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -\sin(x_1(t)) - x_2(t) + u(t)$$
$$y(t) = x_1(t)$$

$u(t) = \frac{1}{2}$ $\qquad$ $u(t) = \frac{1}{2} + 0.1\cos(3t)$

• Se $\bar{u} = \frac{1}{2}$:
$$0 = x_2$$
$$0 = -\sin x_1 + \frac{1}{2} \Rightarrow x_1 = \frac{\pi}{6}$$
$$\bar{x} = \begin{pmatrix} \frac{\pi}{6} \\ 0 \end{pmatrix}$$

$$A = \left.\frac{\delta f}{\delta x}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} = \left.\begin{pmatrix} 0 & 1 \\ -\cos x_1 & -1 \end{pmatrix}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} = \begin{pmatrix} 0 & 1 \\ -\frac{\sqrt{3}}{2} & -1 \end{pmatrix}$$

Punto di eq é A.S. localmente.
Matrice in forma compagna

$$b = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad c^T = \begin{pmatrix} 1 & 0 \end{pmatrix} \qquad d = 0$$

Calcolo la $G(s) = c^T (sI - A)^{-1} b = \frac{1}{s^2 + s + \frac{\sqrt{3}}{2}} \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} s+1 & 1 \\ -\frac{\sqrt{3}}{2} & s \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$

$$= \frac{1}{s^2 + s + \frac{\sqrt{3}}{2}}$$

$$s_{1,2} = \frac{-1 \pm \sqrt{1 + 2\sqrt{3}}}{2} = \begin{cases} -0.5 - 0.78j \\ -0.5 + 0.78j \end{cases}$$

$$u(t) = 0.1\cos(3t) \Rightarrow U(s) = 0.1 \cdot \frac{s}{s^2 + 9}$$

$$Y(s) = \frac{r}{(s + 0.5 - 0.78j)} + \frac{r^*}{(s + 0.5 + 0.78j)} + \frac{Q}{s - 3j} + \frac{Q^*}{s + 3j}$$

$$r = \left.\frac{0.1 s}{(s^2 + 9)(s + 0.5 + 0.78j)}\right|_{s = -0.5 + 0.78j} = \begin{cases} 6.84 \cdot 10^{-3} \\ 37.8° \end{cases}$$

$$Q = \left. \frac{0.1s}{\left(s^2+s+\frac{\sqrt{3}}{2}\right)(s+3j)} \right|_{s=3j} = \begin{cases} 5.77 \cdot 10^{-3} \\ -159.8^\circ \end{cases}$$

$$\delta y(t) = 1.37 \cdot 10^{-2} e^{-0.5t} \cos(0.78t + 0.66) + 1.15 \cdot 10^{-2} \cos(3t - 2.79)$$

$$y(t) = \frac{\pi}{6} + \delta y(t)$$

2)

$$\dot{x}_1(t) = -x_2(t) + e^{-2x_1(t)} - u(t)$$
$$\dot{x}_2(t) = -x_2(t) + e^{-x_1(t)}$$
$$y(t) = x_2(t)$$

$\bar{u} = 2$:

$$0 = -x_2 + e^{-2x_1} - 2 \qquad\qquad 0 = x_2^2 - x_2 - 2$$
$$0 = -x_2 + e^{-x_1} \quad \Rightarrow \quad x_{2_1} = 2 \qquad x_{1_1} = -\ln 2 \qquad \bar{x} = \begin{pmatrix} -\ln 2 \\ 2 \end{pmatrix}$$
$$x_{2_2} = -1 \quad \Rightarrow$$

Soluzione non accett. perché $x_2 = e^{-x_1}$

$$A = \left.\frac{\delta f}{\delta x}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} = \left.\begin{pmatrix} -2e^{-2x_1} & -1 \\ -e^{-x_1} & -1 \end{pmatrix}\right|_{x=\bar{x}} = \begin{pmatrix} -8 & -1 \\ -2 & -1 \end{pmatrix}$$

$$b = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \qquad c^T = \begin{pmatrix} 0 & 1 \end{pmatrix} \qquad d = 0$$

$$(sI-A)^{-1} = \begin{pmatrix} s+8 & 1 \\ 2 & s+1 \end{pmatrix}^{-1} = \frac{1}{s^2+9s+6} \begin{pmatrix} s+1 & -1 \\ -2 & s+8 \end{pmatrix}$$

P. eq. localmente A.S. per Cartesio

$$G(s) = \frac{1}{s^2+9s+6} \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} s+1 & -1 \\ -2 & s+8 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \frac{2}{s^2+9s+6}$$

$$G(5j) = \frac{2}{-25+45j+6} = 0.041e^{-112° j}$$

$$\delta y_1(t) = 0.082\cos(5t - 1.95)$$
$$y(t) = 2 + 0.082\cos(5t - 1.95)$$

1)

$$x(k+1) = \begin{pmatrix} 0 & 1 \\ -h & 0.1h \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} 0 & 1 \end{pmatrix} x(k)$$

$$P(s) = s^2 - 0.1hs + h$$

$$s = \frac{1+z}{1-z} \Rightarrow (1+z)^2 - 0.1h(1-z)(1+z) + h(1-z)^2 = 0$$

$$1+z^2+2z - 0.1h(1-z^2) + h(1+z^2-2z) = 0$$

$$z^2 + 2z + 1 - 0.1h + 0.1hz^2 + h + hz^2 - 2hz = 0$$

$$z^2(1+1.1h) + z(2-2h) + 1 + 0.9h = 0$$

$$\begin{cases} 1+1.1h > 0 & h > -\frac{10}{11} \\ 2-2h > 0 & h < 1 \\ 1+0.9h > 0 & h > -\frac{10}{9} \end{cases} \Rightarrow -\frac{10}{11} < h < 1$$

$\Rightarrow$ Scelgo $h = 0.5$

$$G(z) = \frac{1}{z^2 - 0.05z + 0.5} \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} z-0.05 & 1 \\ -0.5 & z \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{z^2 - 0.05z + 0.5}$$

$G(1) = 1.45 \qquad y_r(k) = 5 \cdot 1.45 = 7.25$

2)

$R(S)=\frac{K_I}{S}$ $\qquad \varphi_m = 45°$

$$L(S) = \frac{K_I}{S}\cdot\frac{1}{(S+1)^2}$$

Trovo w_c: $\qquad \varphi_m = 180° - |\angle L(jw_c)| \Rightarrow |\angle L(jw_c)| = 135°$

$$\angle L(jw_c) = -135° \Rightarrow -\frac{\pi}{2} - 2\,\text{atan}(w_c) = -\frac{3}{4}\pi$$

$$\text{atan}(w_c) = +\frac{\pi}{8} \Rightarrow w_c = \tan\left(+\frac{\pi}{8}\right) = 0.414$$

Impongo $|L(0.414j)| = 1 \Rightarrow K_I = 0.48$

$L(S) = \frac{0.48}{S}\cdot\frac{1}{(S+1)^2}$ $\qquad$ Risposta a regime a quel disturbo è $\frac{A}{M}$ perché

ho un unico polo: $d_\infty = \frac{10}{0.48} = 20.83$

2)

$G(s) = \frac{10}{(s+2)^2}$

• e_∞ a rampa = 20%

• $\varphi_m = 40°$ con $\omega_c = 0.7$ rad/s

• Per errore a rampa $\neq 0$ e $< +\infty$ devo avere un polo nell'origine.

$e_\infty = \frac{A}{M}$, con $A=1$. $M = M_0 \cdot 10 \Rightarrow$ $0{,}2 = \frac{1}{M_0 10} \Rightarrow M_0 = 0.5$

SBAGLIATO: $M = \frac{10}{4}$

$L^*(s) = \frac{5}{s(s+2)^2}$. Calcolo modulo e fase in 0.7 rad/s.

$L(0.7j) = 1.59 e^{-128.6° j}$. Ho bisogno di compressione di $\frac{1}{1.59}$ e ritardo di 11.4°.

$M \cong 0.629$ $\quad \alpha = -11.4°$

$\alpha = \frac{M\cos\varphi - 1}{M(M - \cos\varphi)}$ $\quad T = \frac{M - \cos\varphi}{\hat{\omega}\sin\varphi}$

$\alpha = 1.735$ $\quad T = 2.539$

$R_2(s) = \frac{1 + 2.539s}{1 + 4.405s}$

$R(s) = \frac{0.5}{s} \cdot \left(\frac{1+2.539s}{1+4.405s} \right)$

1)

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} K & 0 & 1 \end{pmatrix} x(t)$$

- Trovo K affinché $|G(10j)| = 0$.

NOTA: $b_n = 0 \quad a_0 = 1 \quad a_1 = 2 \quad a_2 = 2$, forma di raggiungibilità.

$$K = b_0 - a_0 b_n \Rightarrow b_0 = K \quad b_1 = 0 \quad b_2 = 1$$

$$G(s) = \frac{b_n s^n + \dots + b_0}{a_n s^n + \dots + a_0} = \frac{s^2 + K}{s^3 + 2s^2 + 2s + 1}$$

$$\Rightarrow s^2 + K = 0 \Big|_{s = 10j} \Rightarrow K = 100$$

$$G(s) = \frac{s^2 + 100}{s^3 + 2s^2 + 2s + 1}$$

	1	2	2	1
		-1	-1	-1
-1	1	1	1	0

$$s^3 + 2s^2 + 2s + 1 = (s^2 + s + 1)(s + 1)$$

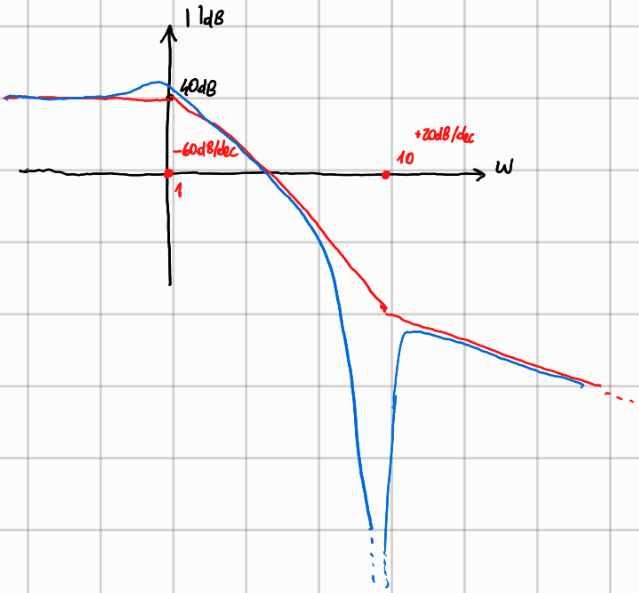
$$s_{1,2} = \frac{-1 \pm \sqrt{1 - 4}}{2} \Rightarrow \omega_n = 1$$

Punti rottura: 10 $(s^2 + 100)$ (con $\xi = 0$, ho antirisonanza come previsto) $\Rightarrow +40$ dB/dec

: 1, con $\xi = \frac{1}{2}$, ho piccola risonanza. $\Rightarrow -40$ dB/dec

: 1 per polo reale $\Rightarrow -20$ dB/dec.

$\mu = 100 \Rightarrow |\mu|_{dB} = 40$



| |dB
40dB
-60dB/dec
+20dB/dec
10
1
ω

1)

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} x(t)$$

Sistema è raggiungibile per forma di raggiungibilità.

$$\mathcal{O} = \begin{pmatrix} c^T \\ c^T A \\ c^T A^2 \end{pmatrix} \Rightarrow c^T A = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix}$$

$$c^T A \cdot A = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{pmatrix} = \begin{pmatrix} -1 & -2 & -1 \end{pmatrix}$$

$$\mathcal{O} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & -2 & -1 \end{pmatrix}$$, ha rango 2 perché $-S_1^T - S_2^T = S_3^T$.

Non osservabile.

Calcolo $G(s)$: $\quad b_n = 0 \quad a_0 = 1 \quad a_1 = 2 \quad a_2 = 2 \quad a_3 = 1$

$1 = b_0 - a_0 b_n \Rightarrow b_0 = 1$

$1 = b_1 - a_1 b_n \Rightarrow b_1 = 1$

$0 = b_2 - a_2 b_n \Rightarrow b_2 = 0$

$$G(s) = \frac{s+1}{s^3 + 2s + 2s + 1} = \frac{s+1}{(s^2 + s + 1)(s+1)} = \frac{1}{s^2 + s + 1}$$

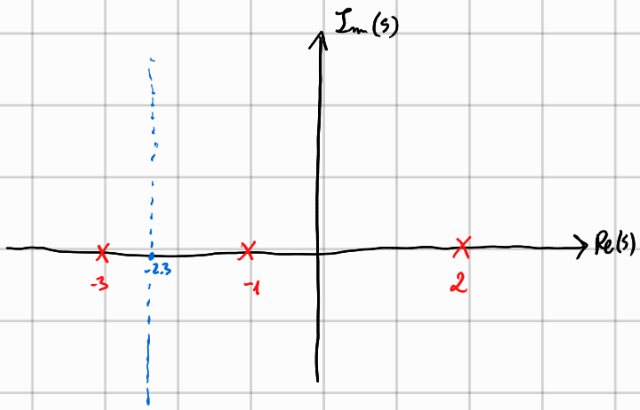
$$S_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} j \Rightarrow \xi\omega_n = 0.5$$

$$T_{a_1} \cong \frac{\ln(100)}{\xi\omega_n} = 9.21\,s$$

2)

$$G(s) = \frac{1}{(s+1)(s-2)(s+3)}$$

$$T_a \leq 2s \Rightarrow -\bar{\sigma} \simeq -\frac{4.6}{2} = -2.3$$



- Piazzo zero tra -1 e -3 e un polo in modo che punto doppio cada in -3.32 (6.64T ⇒ Ta1 se ho polo doppio).

Punto doppio si troverà a metà tra 2 e il polo ⇒ lo piazzo in -8.64.

$$R(s) = \frac{M(s+1)(s+3)}{(s+8.64)}$$ Trovo M per avere punto doppio:

$$\bar{P} = 5.32^2 = 28.3$$

$$R(s) = \frac{28.3(s+1)(s+3)}{(s+8.64)}$$

1)

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} x(t)$$

Sistema in forma di raggiungibilità. È compl. raggiungibile.
Verifico osservabilità:

$$\mathcal{O} = \begin{pmatrix} C^T \\ C^T A \\ C^T A^2 \end{pmatrix} \qquad C^T A = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix}$$

$$C^T A \cdot A = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -2 \end{pmatrix} = \begin{pmatrix} -1 & -2 & -1 \end{pmatrix}$$

$$\mathcal{O} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & -2 & -1 \end{pmatrix}$$ $rk = 2 \Rightarrow$ Non osservabile completamente.

$$P(s) = s^3 + 2s^2 + 2s + 1 = 0$$

$$\begin{array}{c|ccc|c} & 1 & 2 & 2 & 1 \\ & & -1 & -1 & -1 \\ \hline -1 & 1 & 1 & 1 & 0 \end{array}$$

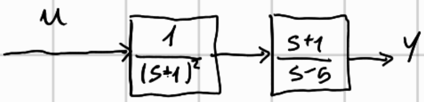
$$P(s) = (s+1)(s^2+s+1)$$

$$\lambda_1 = -1 \qquad \lambda_{2,3} = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$$

$$T_{a1} \approx \frac{\ln(100)}{\xi \omega_n} = 2 \ln(100) \cong 9{,}2\ s$$

STA FATTO SOPRA BENE

1)

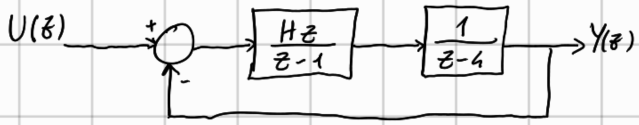


$$G(s) = \frac{1}{(s+1)(s-5)} = \frac{r_1}{s+1} + \frac{r_2}{s-5} = \frac{1}{6}\cdot\frac{1}{s+1} - \frac{1}{6}\cdot\frac{1}{s-5}$$

$$g_y(t) = \frac{1}{6}e^{-t} - \frac{1}{6}e^{5t}$$

- Sistema non é A.S, non é stabile Bibo, é ragg. ma non completamente osservabile.

1)



H/ sistema é stabile BIBO

$$F(z) = \frac{\frac{Hz}{z-1} \cdot \frac{1}{z-4}}{1 + \frac{Hz}{z-1} \cdot \frac{1}{z-4}} = \frac{Hz}{z^2 - 5z + 4 + Hz}$$

$$\Delta = (H-5)^2 - 16$$

$$p(z) = z^2 + (H-5)z + 4 \qquad z = \frac{1+s}{1-s}$$

$$h^2 + 25 - 10h - 16 = h^2 - 10h + 9$$

$$h_{1,2} = 5 \pm \sqrt{25-9} = 5 \pm 4 \begin{cases} 1 \\ 10 \end{cases}$$

$$p'(s) = (1+s)^2 + (H-5)(1+s)(1-s) + 4(1-s)^2 = 0$$

$$1 + 2s + s^2 + (H-5)(1-s^2) + 4(1+s^2-2s) = 0$$

$$\cancel{1} + 2s + s^2 - Hs^2 + 5s^2 + H - \cancel{5} + \cancel{4} + 4s^2 - 8s = 0$$

$$(10-H)s^2 - 6s + H = 0$$

$$\begin{cases} 10 - H < 0 \\ H < 0 \end{cases} \qquad \nexists H \in \mathbb{R}$$

- $H = 9 \Rightarrow F(z) = \frac{9z}{z^2 - 4z + 4} = \frac{9z}{(z-2)^2} \qquad y(k) = \frac{9}{2} k 2^k$

$$\Downarrow$$

$$(z-2)^2 \Rightarrow z_{1,2} = 2$$

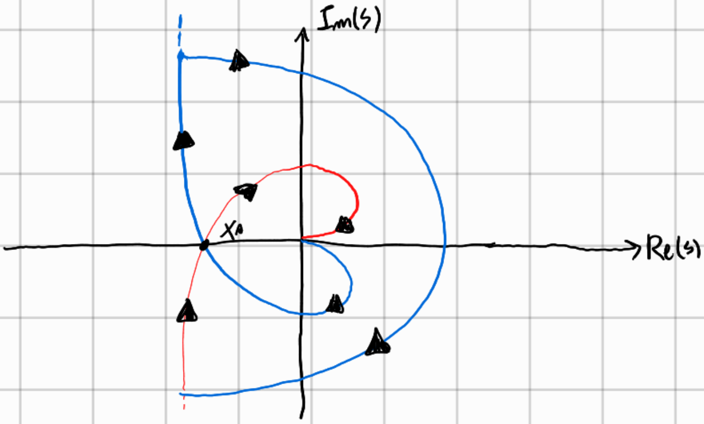
1)

$$G(s) = \frac{8}{s(s+2)^3}$$

$M_0 = \lim_{\omega \to 0} |G(j\omega)| = +\infty$ $\varphi_0 = -90°$

$\varphi_F = -360°$

$M_\infty = 0$

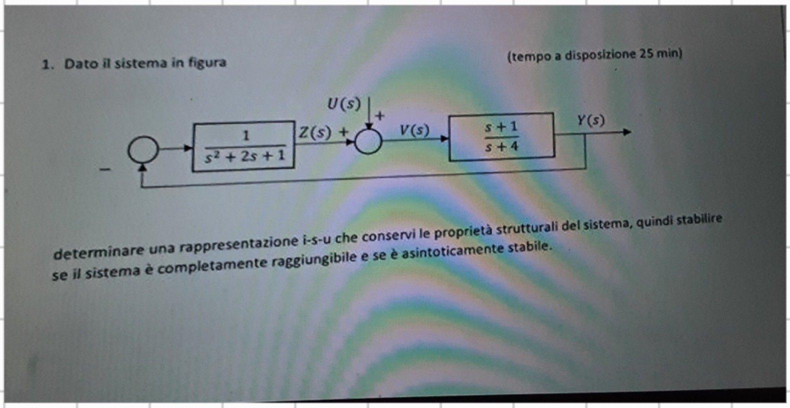


$\angle G(j\omega_\pi) = -\pi$

$$-\frac{\pi}{2} - 3\arctan\left(\frac{\omega_\pi}{2}\right) = -\pi \Rightarrow \arctan\left(\frac{\omega_\pi}{2}\right) = \frac{\pi}{6}$$

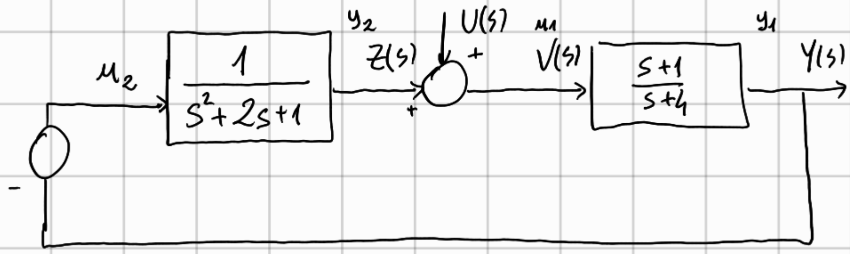
$$\Rightarrow \frac{\omega_\pi}{2} = \frac{\sqrt{3}}{3} \Rightarrow \omega_\pi = \frac{2}{3}\sqrt{3}$$

$$|G(j\omega_\pi)| = \frac{8}{\frac{2}{3}\sqrt{3}\,\left|2+\frac{2}{3}\sqrt{3}j\right|^3} = \frac{9}{16} \Rightarrow K_m = \frac{16}{9}.$$ Sistema è stabile.



1. Dato il sistema in figura (tempo a disposizione 25 min)

determinare una rappresentazione i-s-u che conservi le proprietà strutturali del sistema, quindi stabilire se il sistema è completamente raggiungibile e se è asintoticamente stabile.



$G_1(s) = \dfrac{s+1}{s+4}$ Ordine 1. $a_1 = 1$ $a_0 = 4$ $\Rightarrow A = -4$

$b_1 = 1$ $b_0 = 1$ $b = 1$

$C = (b_0 - a_0 b_1) = -3$ $d = b_1 = 1$

$$\begin{cases} \dot{x}_1(t) = -4x_1(t) + u_1(t) \\ y_1(t) = -3x_1(t) + u_1(t) \end{cases}$$

$G_2(s) = \dfrac{1}{s^2+2s+1}$ $a_2 = 1$ $a_1 = 2$ $a_0 = 1$

$b_2 = 0$ $b_1 = 0$ $b_0 = 1$

$A = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$ $b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $C^T = (b_0 - a_0 b_2 \quad b_1 - a_1 b_2) = (1 \quad 0)$ $d = 0$

$$\begin{cases} \dot{x}_2(t) = x_3(t) \\ \dot{x}_3(t) = -x_2(t) - 2x_3(t) + u_2(t) \\ y_2(t) = x_2(t) \end{cases}$$

$$\Rightarrow \begin{cases} \dot{x}_1(t) = -4x_1(t) + u_1(t) \\ y_1(t) = -3x_1(t) + u_1(t) \end{cases}$$

$y_1 = -u_2$

$u_1 = y_2 + u$ $\quad y_2 =$

$y_1 = y$

$$\begin{cases} \dot{x}_2(t) = x_3(t) \\ \dot{x}_3(t) = -x_2(t) - 2x_3(t) + u_2(t) \\ y_2(t) = x_2(t) \end{cases}$$

$$\begin{aligned} \dot{x}_1(t) &= -4x_1(t) + y_2(t) + u(t) \\ y(t) &= -3x_1(t) + y_2(t) + u(t) \\ \dot{x}_2(t) &= x_3(t) \\ \dot{x}_3(t) &= -x_2(t) - 2x_3(t) - y \\ y_2(t) &= x_2(t) \end{aligned}$$

$$\Rightarrow \begin{aligned} \dot{x}_1(t) &= -4x_1(t) + x_2(t) + u(t) \\ \dot{x}_2(t) &= x_3(t) \\ \dot{x}_3(t) &= -x_2(t) - 2x_3(t) - y(t) \\ y(t) &= -3x_1(t) + x_2(t) + u(t) \end{aligned}$$

$$\Downarrow$$

$$\begin{cases} \dot{x}_1(t) = -4x_1(t) + x_2(t) + u(t) \\ \dot{x}_2(t) = x_3(t) \\ \dot{x}_3(t) = 3x_1(t) - 2x_2(t) - 2x_3(t) - u(t) \\ y(t) = -3x_1(t) + x_1(t) + u(t) \end{cases}$$

$$A = \begin{pmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & -2 & -2 \end{pmatrix}$$

$$b = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad c^T = \begin{pmatrix} -3 & 1 & 0 \end{pmatrix}$$

$$\theta = \begin{pmatrix} b & Ab & A^2 b \end{pmatrix} \quad \Rightarrow \quad Ab = \begin{pmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & -2 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \\ 5 \end{pmatrix}$$

$$A \cdot Ab = \begin{pmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & -2 & -2 \end{pmatrix} \begin{pmatrix} -4 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 15 \\ 5 \\ -20 \end{pmatrix}$$

$$\theta = \begin{pmatrix} 1 & -4 & 15 \\ 0 & -1 & 5 \\ -1 & 5 & -20 \end{pmatrix} \Rightarrow |\theta| = 20 + 20 - 15 - 25 = 0$$

$$R=\begin{pmatrix} C^T \\ C^TA \\ C^TA^2 \end{pmatrix} \qquad C^TA=\begin{pmatrix}-3 & 1 & 0\end{pmatrix}\begin{pmatrix}-4 & 1 & 0\\ 0 & 0 & 1\\ 3 & -2 & -2\end{pmatrix}=\begin{pmatrix}12 & -3 & 1\end{pmatrix}$$

$$C^TA\cdot A=\begin{pmatrix}12 & -3 & 1\end{pmatrix}\begin{pmatrix}-4 & 1 & 0\\ 0 & 0 & 1\\ 3 & -2 & -2\end{pmatrix}=\begin{pmatrix}-45 & 10 & -5\end{pmatrix}$$

$$R=\begin{pmatrix}-3 & 1 & 0\\ 12 & -3 & 1\\ -45 & 10 & -5\end{pmatrix} \Rightarrow |R|=-45-45+30+60=0 \ \checkmark.$$

Sistema è asintoticamente stabile perché dinamica cancellata è stabile e il resto dei poli sono:

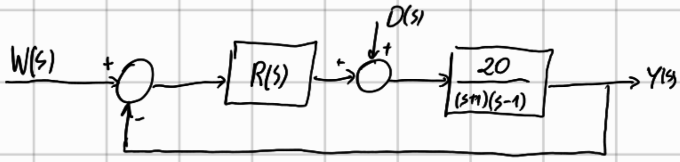
$$F(s)=\frac{\frac{s+1}{s+4}}{1+\frac{s+1}{s+4}\cdot\frac{1}{(s+1)^2}}=\frac{(s+1)^2}{(s+4)(s+1)+1}$$

$$s^2+5s+5=0$$

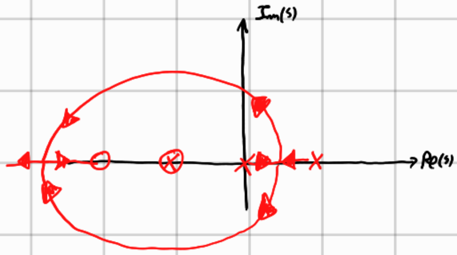
$$s_{1,2}=-2\pm\sqrt{4-5} \Rightarrow \text{Parte reale negativa.}$$

È asint. stabile.

2)



• Astatico a $D(s)=\frac{1}{s}$ e $T_{a1}=2s$
↓
polo in origine



Inserisco 2 zeri: uno in −1 per rimuovere polo, uno in z / punto doppio

Sarà reale e in −3.32 ($\frac{6.64}{2}$, con $6.64T \simeq T_{a1}$ per sistemi con polo doppio coincidente)

$$\gamma(x)=\frac{x(x-1)}{x-z} \Rightarrow \gamma'(x)=0 \Rightarrow (x-1)(x-z)+x(x-z)-x(x-1)=0$$

Impongo $x=-3.32$ come soluzione:

$$-4.32(-3.32-z)-3.32(-3.32-z)+3.32(-4.32)=0$$

$$\cancel{+14.34}+4.32z+11.02+3.32z-\cancel{14.34}=0$$

$$z=-1.44$$

Zero in −1.44.

Calcolo $\bar{p}$: $\bar{p}=\frac{3.32\cdot 4.32}{1.88}=7.63=20\mu \Rightarrow \mu=0.38$

8.776

$$R(s)=0.38\frac{(s+1)(s+1.44)}{s}=0.38\frac{s^2+2.44s+1.44}{s}=\underbrace{0.38}_{K_D}s+\underbrace{0.93}_{K_P}+\underbrace{\frac{0.55}{s}}_{K_I}$$

2)

$$G(s) = \frac{1}{s^2+6s+9}$$

• Voglio una $\xi \approx 0.707$ e astatismo a disturbi cost in catena diretta

$\xi \approx \frac{\varphi_m}{100} \Rightarrow \varphi_m \approx 70.7°$. Per avere astatismo mi serve polo in origine.

$R(s) = \frac{M}{s}$ Verifico w_c e guardo margine di fase considerando $M=1$

Verifico dove ho $\varphi_m = 70.7°$. $2\arctan\left(\frac{w_c}{3}\right) = 19.3°$

$$\varphi_m = 180° - |\angle L(jw_c)| \Rightarrow -90° - 2\arctan\left(\frac{w_c}{3}\right) = -109.3°$$

$$2\arctan\left(\frac{w_c}{3}\right) = 19.3°$$

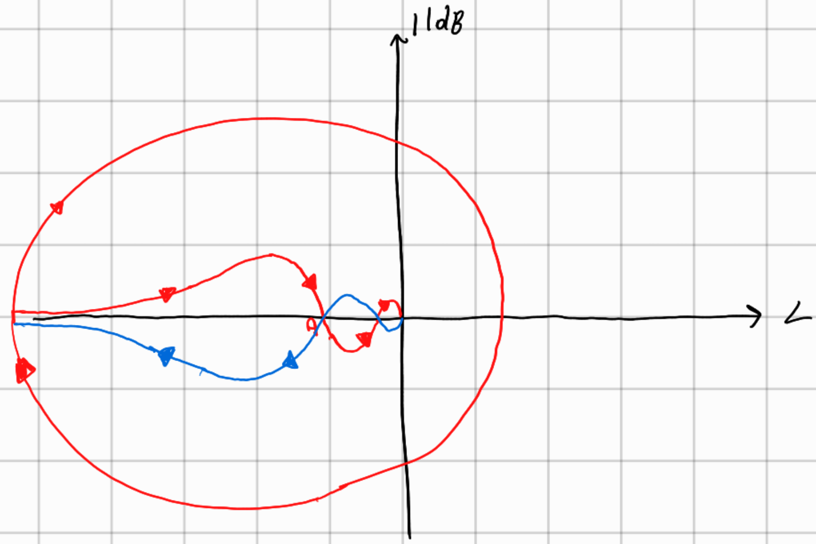
$$w_c = 0.51 \text{ rad/s}$$

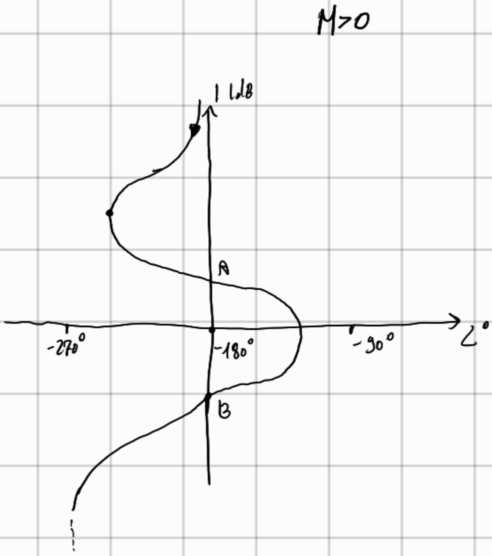
Impongo $0.51 = w_c$

$$\Rightarrow \frac{M}{0.51\,(9+0.51^2)} = 1 \qquad M = 4.72$$

$$S\% = 100\, e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = 4.3\%$$

1)





Se ho $G(s)$ che ha 2 poli, allora se $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, con polo doppio compio un giro orario.

2)

$$G(s) = \frac{1}{s^2+s+1}$$

- Astatismo a d costante $\Rightarrow$ $R(s)$ ha almeno polo in origine.
- $K_m = 6\,dB \Leftrightarrow K_m = 2$

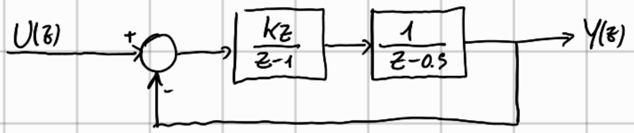
$$L(s) = \frac{M}{s} \cdot \frac{1}{s^2+s+1}$$

Trovo ω_π: $\quad -\frac{\pi}{2} - \angle(s^2+s+1) = -\pi \quad \Rightarrow \quad \angle(s^2+s+1) = \frac{\pi}{2}, \Rightarrow (-\omega_\pi^2 + j\omega_\pi + 1)$ puram. immag.

$\omega_\pi = 1\ rad/s.$ $\qquad |L(j\omega_\pi)| = \frac{M}{1} \cdot \frac{1}{1} = \frac{1}{2} \Rightarrow M = \frac{1}{2}$

$$L(s) = \frac{1}{2s} \cdot \frac{1}{s^2+s+1}$$

1)



$$F(z) = \frac{Kz}{(z-1)(z-0.5) + Kz}$$

$$d(z) = z^2 - 1.5z + Kz + 0.5 \qquad z = \frac{1+s}{1-s}$$

$$d(s) = 1 + 2s + s^2 + (K-1.5)\overbrace{(1+s)(1-s)}^{(1-s^2)} + 0.5(1+s^2-2s) = 0$$

$1 + K - 1.5 + 0.5$

$2s - s$

$s^2 - Ks^2 + 1.5s^2 + 0.5s^2$

$$\Rightarrow (3-K)s^2 + s + K = 0$$

$$\Rightarrow \begin{cases} 3-K > 0 \\ K > 0 \end{cases} \Rightarrow \quad 0 < K < 3$$

Scelgo $K = 1.5$

$$F(z) = \frac{Kz}{(z-1)(z-0.5) + Kz} = \frac{1.5z}{z^2 - 0.5} = \frac{15z}{(z-0.707)(z+0.707)}$$

$$u(k) = 5\,\delta_{-1}(k) \Rightarrow U(z) = \frac{5z}{z-1} \qquad y_\infty = \lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1} G(z)\bar{u} = 5 \cdot \frac{15}{0.5} = 150$$

1)

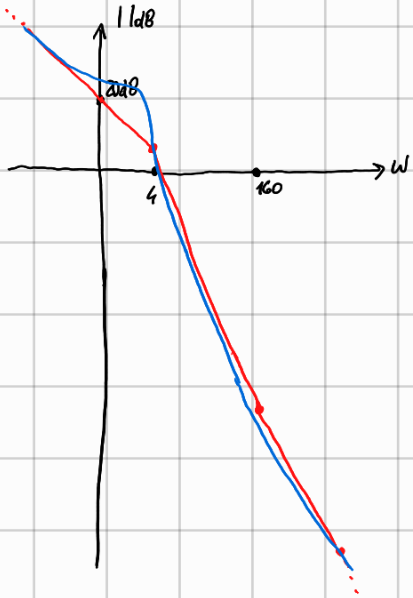
$$G(s) = \frac{s-160}{s^3+2s^2+16s} = \frac{s-160}{s(s^2+2s+16)} = \frac{-160\left(1-\frac{1}{160}s\right)}{16s\left(1+\frac{1}{8}s+\frac{s^2}{16}\right)} =$$

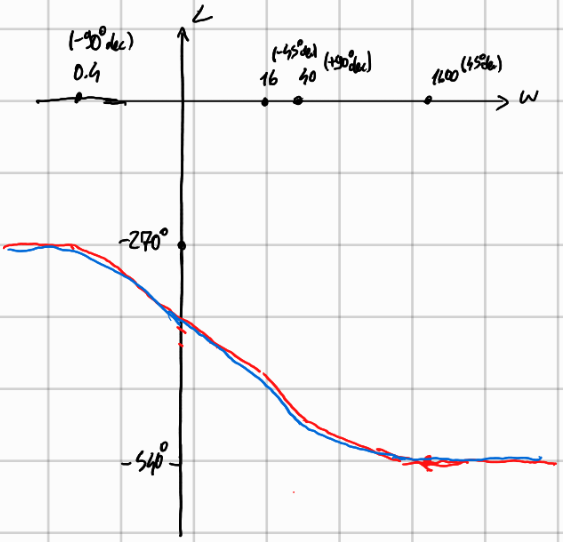
$2\xi\omega_n = 2 \Rightarrow \xi = \frac{1}{\omega_n} = \frac{1}{4}$

$\omega_n = 4$

$$= \frac{-10\left(1-\frac{1}{160}s\right)}{s\left(1+\frac{1}{8}s+\frac{s^2}{16}\right)}$$

- Rottura: 160 rad/s, zero a parte reale positiva. +20 dB/dec e −90° $\qquad |M|_{dB} = 20\,dB$
- 4 rad/s, poli complessi e coniugati, $\xi > 0$. −40 dB/dec e −180°





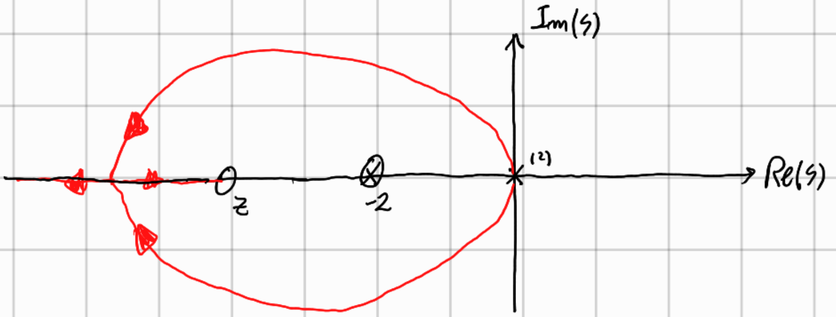
2)

$$G(s) = \frac{1}{s^2+2s}$$

- Errore = 0 a riferimento a rampa
- T_a 1% = 1.6 s

$$G(s) = \frac{1}{s(s+2)}$$

Ho bisogno di un altro polo in origine



Impongo punto doppio in $-\frac{6.64}{1.6} = -4.15$ ($T_{a1} \cong 6.64\,T$ per $G(s)$ con polo doppio).

Inserisco uno zero in -2 e un altro in z.

$$\gamma(x) = \frac{x^2}{(x-z)} \quad \text{impongo } \frac{d\gamma(x)}{dx} = 0 \qquad 2x(x-z) - x^2 = 0$$

Impongo soluzione in -4.15:

$$-8.3(-4.15-z) - 17.22 = 0$$

$$34.45 + 8.3z - 17.22 = 0$$

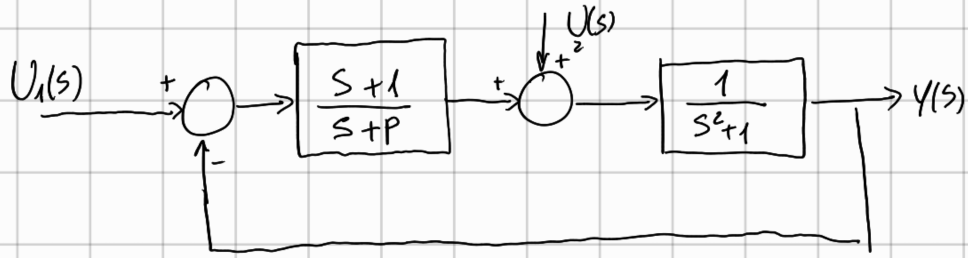
$$z = -2.08$$

Trovo la $\bar{p}$ associata:

$$\bar{p} = \frac{4.15 \cdot 4.15}{2.07} = 8.32$$

$$R(s) = \frac{8.32}{s}(s+2)(s+2.08) = \frac{8.32}{s}(s^2+4.08s+4.16) = 33.95 + \frac{1}{0.029s} + 8.32s$$

1)



• Non ci sono cancellazioni. Sistema compl. ragg. e osservabile.

$G_1(s) = \frac{s+1}{s+p}$ $\quad a_0 = p \quad b_0 = 1 \quad 1^0$ ordine.

$a_1 = 1 \quad b_1 = 1$

$A = -p \qquad b = 1 \qquad C^T = (b_0 - a_0 b_n) = (1-p) \qquad d = b_m = 1$

$$\begin{cases} \dot{x}_1(r) = -p x_1(r) + u_0(r) \\ y_0(r) = (1-p) x_1(r) + u_0(r) \end{cases} \qquad u_0 = u_1 - y$$

$G_2(s) = \frac{1}{s^2+1}$ $\quad a_0 = 1 \quad a_1 = 0 \quad a_2 = 1$

$b_0 = 1 \quad b_1 = 0 \quad b_2 = 0$

$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad c^T = (b_0 - a_0 b_2 \quad b_1 - a_1 b_2) = (1 \quad 0) \qquad d = 0$

$$\begin{cases} \dot{x}_2(r) = x_3(r) \\ \dot{x}_3(r) = -x_2(r) + u_3(r) \\ y(r) = x_2(r) \end{cases} \qquad u_3 = u_2 + y_0$$

$$\begin{cases} \dot{x}_1(r) = -p x_1(r) + u_0(r) \\ y_0(r) = (1-p) x_1(r) + u_0(r) \end{cases} \qquad u_0 = u_1 - y$$

$$\begin{cases} \dot{x}_2(r) = x_3(r) \\ \dot{x}_3(r) = -x_2(r) + u_3(r) \\ y(r) = x_2(r) \end{cases} \qquad u_3 = u_2 + y_0$$

$\dot{x}_1(r) = -p x_1(r) + u_1 - y(r)$

$y_0(r) = (1-p) x_1(r) + u_1 - y(r)$

$\dot{x}_2(r) = x_3(r)$

$\dot{x}_3(r) = -x_2(r) + u_2(r) + y_0(r)$

$y(r) = x_2(r)$

$$\begin{cases} \dot{x}_1(r) = -p x_1(r) + u_1(r) - x_2(r) \\ \dot{x}_2(r) = x_3(r) \\ \dot{x}_3(r) = -x_2(r) + u_2(r) + (1-p)x_1(r) + u_1 - x_2(r) \\ y(r) = x_2(r) \end{cases}$$

$$\begin{cases} \dot{x}_1(r) = -p x_1(r) - x_2(r) + u_1(r) \\ \dot{x}_2(r) = x_3(r) \\ \dot{x}_3(r) = (1-p)x_1(r) - 2x_2(r) + u_1(r) + u_2(r) \\ y(r) = x_2(r) \end{cases}$$

$$A = \begin{pmatrix} -p & -1 & 0 \\ 0 & 0 & 1 \\ 1-p & -2 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{pmatrix} \qquad c^T = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \quad d = 0$$

$$AB = \begin{pmatrix} -p & 0 \\ 1 & 1 \\ 1-p & 0 \end{pmatrix} \qquad A \cdot AB = \begin{pmatrix} p^2-1 & -1 \\ 1-p & 0 \\ p^2-p-2 & -2 \end{pmatrix}$$

$$R = \begin{pmatrix} 1 & 0 & -p & 0 & p^2-1 & -1 \\ 0 & 0 & 1 & 1 & 1-p & 0 \\ 1 & 1 & 1-p & 0 & p^2-p-2 & -2 \end{pmatrix}$$

$$c^T A = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -p & -1 & 0 \\ 0 & 0 & 1 \\ 1-p & -2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

$$c^T A \cdot A = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} A = \begin{pmatrix} 1-p & -2 & 0 \end{pmatrix}$$

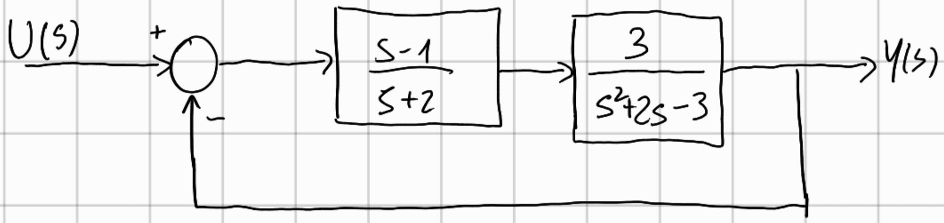
$$\mathcal{O} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1-p & -2 & 0 \end{pmatrix} \qquad rk = 3$$

$$F(s) = \frac{s+1}{(s+p)(s^2+1)+s+1} = \frac{s+1}{s^3+ps^2+s+p+s+1}$$

3	1	2
2	p	p+1
1	p-1	
0	p+1	

$$\begin{cases} p>0 \\ p-1>0 \\ p+1>0 \end{cases} \quad \begin{matrix} p>0 \\ p>1 \\ p>-1 \end{matrix} \Rightarrow p>1$$

1)



• $s^2+2s-3=(s-1)(s+3)$

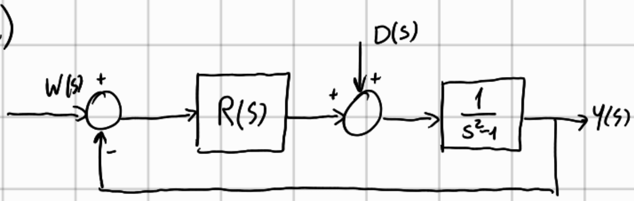
• Nota: ho cancellazione di polo a parte reale positiva. Non ho asintotica stabilità.

• Verifico BIBO:

$$L(s)=\frac{3}{(s+3)(s+2)} \qquad F(s)=\frac{3}{(s+3)(s+2)+3}=\frac{3}{s^2+5s+9}$$

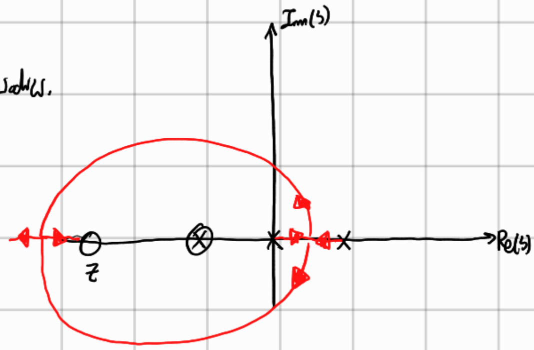
Ho tutti poli a p. reale negativa per cartesio. Lessgoooo.

2)



• Astatismo a d(t) costante.

• $T_{a1\%} = 2s$ e modi aperiodici.



• Cancello polo in −1.

• Inserisco zero / punto doppio su in -3.32 $\left(\frac{6.64}{T_{a1}}, \text{ perché } T_{a1} \approx 6.64T\right)$.

$$\gamma(x) = \frac{x(x-1)}{x-z} \qquad \frac{d\gamma(x)}{dx} = 0 \Rightarrow (x-1)(x-z) + x(x-z) - x(x-1) = 0$$

$$-4.32(-3.32-z) - 3.32(-3.32-z) + 3.32(-4.32) = 0$$

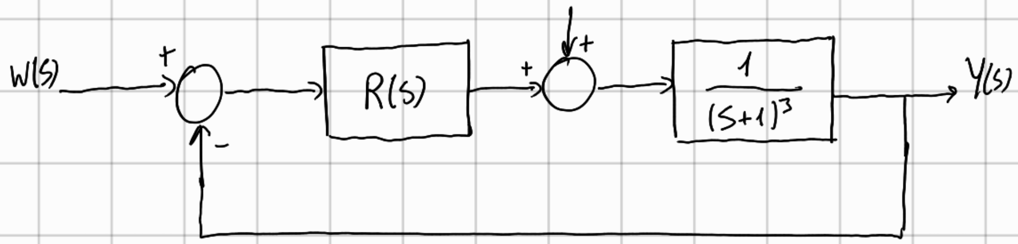
$$4.32z + 11.02 + 3.32z = 0$$

$$z = -1.44$$

• Zero in −1.44. Trovo $\bar{p} = \frac{3.32 \cdot 4.32}{1.88} = 7.63$

$$R(s) = \frac{7.63}{s}(s+1)(s+1.44) = \frac{7.63}{s}(s^2+2.44s+1.44) = 18.62 + \frac{10.99}{s} + 7.63s$$

2)



• Astatico a disturbo costante d(t)

• $K_m = 6dB \iff K_m = 2$

$$R(s) = \frac{M}{s} \qquad L^*(s) = \frac{M}{s(s+1)^3}, \text{ calcolo } \omega_\pi:$$

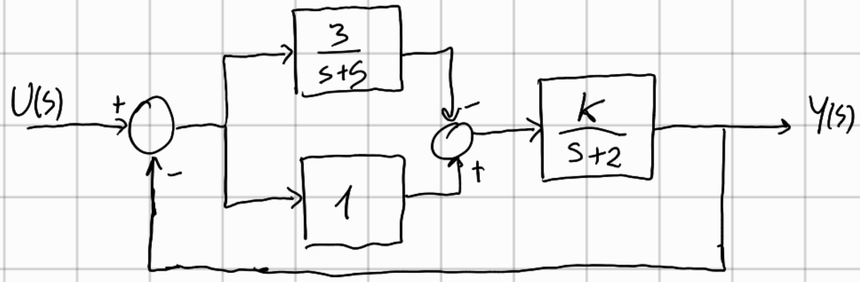
$$\angle L(j\omega_\pi) = -\frac{\pi}{2} - 3\angle(j\omega_\pi+1) = -\pi$$

$$3 \arctan \omega_\pi = \frac{\pi}{2} \Rightarrow \omega_\pi = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\left|L\left(j\tfrac{\sqrt{3}}{3}\right)\right| = \frac{1}{2} \Rightarrow \frac{M}{\frac{\sqrt{3}}{3}\left(\sqrt{\frac{1}{3}+1}\right)^3} = \frac{1}{2} \Rightarrow M = \frac{4}{9}$$

$$R(s) = \frac{4}{9} \cdot \frac{1}{s}$$

1)



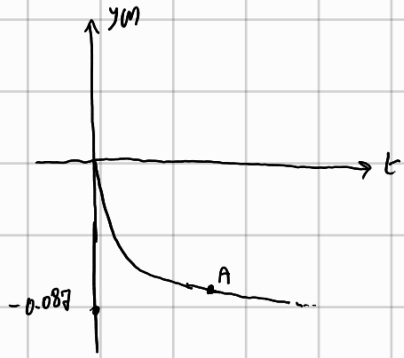
$$L_1(s) = 1 - \frac{3}{s+5} = \frac{s+2}{s+5} \qquad L(s) = \frac{\cancel{s+2}}{s+5} \cdot \frac{K}{\cancel{s+2}}$$

Cancellazione di polo a valle $\Rightarrow$ non raggiungibile.

$$F(s) = \frac{K}{s+5+K} \qquad Y(s) = \frac{1}{s} \cdot \frac{K}{(K+5)(1+\frac{s}{K+5})} \qquad \mu = -0.087$$

$$T_{ass1} = 4.6 \cdot \frac{1}{5+K} = 1s \qquad K = -0.4$$

$$F(s) = \frac{-0.4}{s+4.6} \qquad y(t) = -0.087(1-e^{-4.6t}) \quad t \geq 0$$



$$A = \left(\frac{\ln(100)}{4.6}, -0.087 + 0.0087 \right)$$

$$L_1(s) = \frac{s+2}{s+5} \qquad a_0 = 5 \quad b_0 = 2 \qquad a_1 = 1 \quad b_1 = 1$$

$$A = -5 \qquad b = 1$$

$$c^T = (b_0 - a_0 b_1) = -3 \qquad d = 1$$

$$\dot{x}_1(t) = -5x_1 + u$$

$$y_0(t) = -3x_1 + u$$

$$L_2(s) = \frac{K}{s+2} \qquad a_0 = 2 \quad a_1 = 1 \quad b_0 = K$$

$$A = -2 \qquad b = 1$$

$$c^T = K \qquad d = 0$$

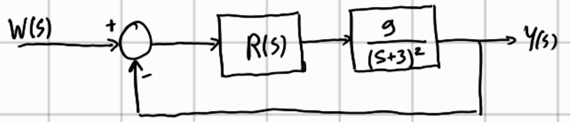
$$\dot{x}_2(t) = -2x_2(t) + u_0$$

$$y(t) = Kx_2(t)$$

$$\dot{x}_1(t) = -5x_1 + u$$
$$\dot{x}_2(t) = -2x_2 - 3x_1 + u$$
$$y(t) = Kx_2(t)$$

$$R = \begin{pmatrix} b & Ab \end{pmatrix} = \begin{pmatrix} 1 & -5 \\ 1 & -5 \end{pmatrix} \checkmark$$

2)



• $R(s)$ ha polo nell'origine.

• $\omega_c = 3\,rad/s$ con $\varphi_m = 45°$.

$$L^*(s) = \frac{M}{s} \cdot \frac{9}{(s+3)^2} \qquad L^*(j3) = \frac{3M}{j(3j+3)^2} = -\frac{1}{6}M$$

Ho fase di $-180°$. Impongo anticipo di $45°$.

$$\angle R_2(j\omega_c) = 45°$$

$$\angle(1 + j \cdot 3T) = 45°$$

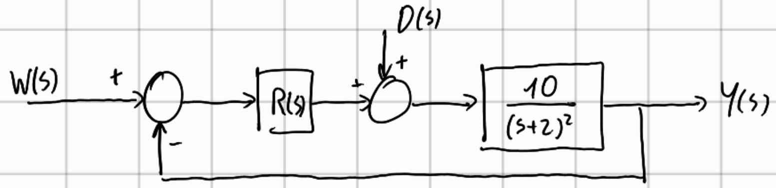
$$\arctan 3T = 45°$$

$$3T = \tan 45° \Rightarrow T = \frac{1}{3}$$

$$|R_1(j\omega_c)||R_2(j\omega_c)||G(j\omega_c)| = 1 \Rightarrow \left|\frac{M}{3j} \cdot \frac{9}{(3j+3)^2}\right| \cdot |1+j| = \frac{\sqrt{2}}{6}M \Rightarrow M = \frac{6}{\sqrt{2}} = 3\sqrt{2}$$

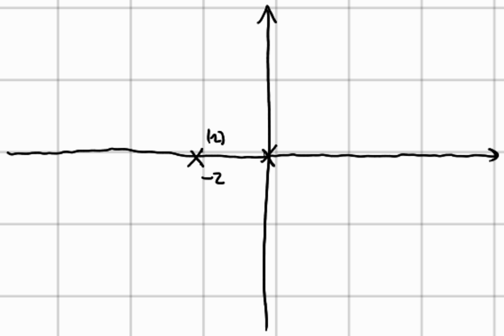
$$R(s) = \frac{3\sqrt{2}}{s}\left(1 + \frac{1}{3}s\right) = \frac{3\sqrt{2}}{s} + \sqrt{2}$$

2)



• Astatico a gradino di W

• $T_{ass,1} = 1s$



$2s+5z+3z=0$

• Zero in -2, zero in z, punto doppio in -6.64.

$$\gamma(x) = \frac{x(x+2)}{x-z} \qquad \gamma'(x)=0 \Rightarrow x(x-z)+(x+2)(x-z) - x(x+2)=0$$

Impongo -6.64 soluz:

$$-6.64(-6.64-z)+(-4.64)(-6.64-z)+6.64(-4.64)=0$$

$$44.1+6.64z+4.64z=0$$

$$z=-3.91$$

$$\frac{8}{s}(s+2)(s+3.125)$$

$$\bar{P} = \frac{6.64 \cdot 4.64}{2.73} = 11.29 = 10\mu \Rightarrow \mu = 1.13$$

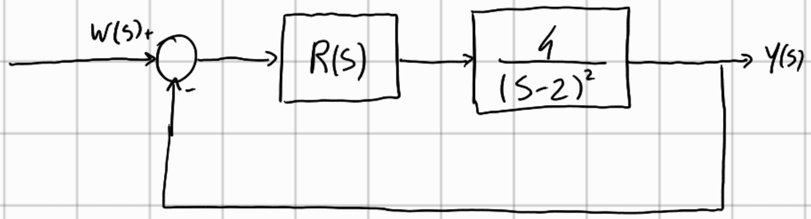
$$R(s) = \frac{1.13}{s}(s+2)(s+3.91) = \frac{1.13}{s}(s^2+5.91s+7.82) = 6.68 + \frac{8.84}{s} + 1.13s$$

$$\parallel$$

$$1.13\left(s+5.91+\frac{7.82}{s}\right) = 6.68\left(1+\frac{1}{0.76s}+0.17s\right)$$

Per la fisica realizzabilità aggiungo polo: $6.68\left(1+\frac{1}{0.76s}+\frac{0.17s}{1+\frac{0.17}{N}s}\right)$

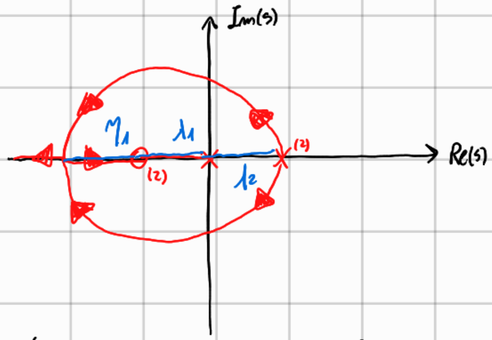
2)



Polo in origine

- Errore a regime nullo a riferimento costante, con solo modi aperiodici.

Inserisco coppia di zeri reali e coincidenti in un punto qualsiasi su semiasse negativo, per esempio in -2.



Calcolo punto doppio e impongo poli in punto doppio.

$$\gamma(x) = \frac{x(x-2)^2}{(x+2)^2} \qquad \gamma'(x)=0 \Rightarrow (x-2)^{\cancel{2}}(x+2)^{\cancel{2}} + 2x(\cancel{x-2})(x+2)^{\cancel{2}} - 2x(x-2)^{\cancel{2}}(\cancel{x+2}) = 0$$

$$(x-2)(x+2) + 2x(x+2) - 2x(x-2) = 0$$

$$x^2 - 4 + \cancel{2x^2} + 4x - \cancel{2x^2} + 4x = 0$$

$$x^2 + 8x - 4 = 0$$

$$x_{1,2} = -4 \pm \sqrt{16+4} = \begin{cases} 0.47 \\ -8.47 \end{cases}$$

Soluzione accettabile é $s = -8.47$.

Calcolo $\bar{\rho} = \frac{l_1 l_2^2}{m_1^2} = \frac{8.47 \cdot 10.47^2}{6.47^2} = 22.18 = 4\mu \Rightarrow \mu = \frac{22.18}{4} = 5.55$

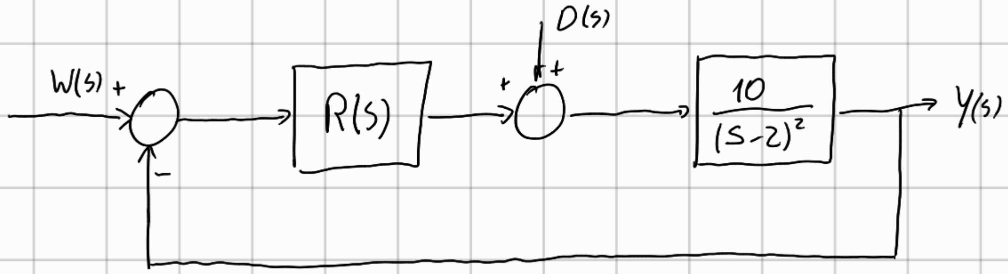
$$R(s) = \frac{M}{s}(s+2)^2 = \frac{5.55}{s}(s^2+4s+4) = \overset{K_D}{5.55s} + \overset{K_P}{22.18} + \overset{K_I}{\frac{22.18}{s}}$$

$$L(s) = \frac{22.18\,(s+2)^2}{(s-2)^2 s} \qquad M = \lim_{s \to 0} sL(s) = 22.18$$

e_∞ con rampa unitaria é $\frac{A}{M} = 0.045$

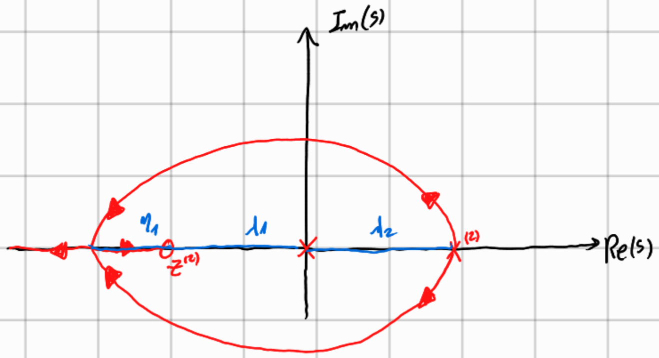
Il sistema presenterá sovraelongazione per la presenza di zeri a bassa frequenza.

2)



- Astatismo a disturbo d(t) costante
- Modo aperiodico con T = 0.2s corrispondente a polo doppio.

⟹ Polo in $(1+sT)^2 \Rightarrow (1+\frac{s}{5})^2 \Rightarrow s=-5$ con polo doppio



Inserisco 2 zeri imponendo p.doppio in −5.

$$\gamma(x) = \frac{x(x-2)^2}{(x-z)^2} \qquad \gamma'(x)=0 \Rightarrow 2x(\cancel{x-2})(x-z)^2 + (x-2)^2(x-z)^2 - 2x(x-2)^2(\cancel{x-z}) = 0$$

Impongo punto doppio in −5:

$$-10(-5-z)+(-7)(-5-z)+10(-7)=0$$

$$50+10z+35+7z-70=0$$

$$15+17z=0$$

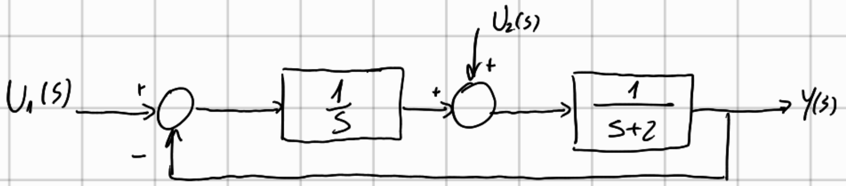
$$z=-0.88$$

Trovo $\bar{p}$ per avere $s=-5$

$$\bar{p} = \frac{\lambda_1\lambda_2^2}{\eta_1^2} = \frac{5\cdot 7^2}{4.12^2} = 14.43 = 10\mu \Rightarrow \mu = 1.44$$

$$R(s) = \frac{\mu}{s}(s+0.88)^2 = \frac{1.44}{s}(s+0.88)^2 = \frac{1.44}{s}(s^2+1.76s+0.77) = \underset{K_P}{2.53} + \underset{K_I}{\frac{1.11}{s}} + \underset{K_D}{1.44s}$$

1)



$u_1(t) = e^{-3t}\delta_{-1}(t)$

$u_2(t) = t\,\delta_{-1}(t)$

$$F(s) = \frac{1}{s(s+2)+1} = \frac{1}{s^2+2s+1}$$

$$S(s) = \frac{s}{s(s+2)+1} = \frac{s}{s^2+2s+1}$$

$$Y_1(s) = \frac{1}{s+3}\cdot\frac{1}{(s+1)^2} = \frac{r}{s+3} + \frac{r_{11}}{s+1} + \frac{r_{12}}{(s+1)^2}$$

$$r = Y(s)(s+3)\big|_{s=-3} = \frac{1}{4}$$

$$r_{12} = \frac{1}{s+3}\Big|_{s=-1} = \frac{1}{2} \qquad r_{11} = \frac{-1}{(s+3)^2}\Big|_{s=-1} = -\frac{1}{4}$$

$$Y_1(s) = \frac{1}{4}\cdot\frac{1}{s+3} - \frac{1}{4}\cdot\frac{1}{s+1} + \frac{1}{2}\cdot\frac{1}{(s+1)^2}$$

$$y_1(t) = \frac{1}{4}e^{-3t} - \frac{1}{4}e^{-t} + \frac{1}{2}te^{-t} \qquad t \geq 0$$

$$Y_2(s) = \frac{1}{s^2}\cdot\frac{s}{(s+1)^2} = \frac{1}{s(s+1)^2} = \frac{r}{s} + \frac{r_{11}}{s+1} + \frac{r_{12}}{s+1}$$

$$r = 1 \qquad r_{12} = \frac{1}{s}\Big|_{s=-1} = -1 \qquad r_{11} = -\frac{1}{s^2}\Big|_{s=-1} = -1$$

$$y_2(t) = 1 - e^{-t} - te^{-t} \qquad t > 0.$$

$$y(t) = \frac{1}{4}e^{-3t} - \frac{1}{4}e^{-t} + \frac{1}{2}te^{-t} + 1 - e^{-t} - te^{-t} = 1 - \frac{5}{4}e^{-t} - \frac{1}{2}te^{-t} \qquad t \geq 0$$

1)

$\dot{x}_1(t) = x_2(t)$

$\dot{x}_2(t) = -\arctan(x_1(t)) - 2x_2(t) + u(t)$

$y(t) = x_1(t)$

$u(t) = 0{,}1\cos(t)$

$\bar{u}=0 \Rightarrow \quad 0 = x_2$

$0 = -\arctan x_1 \quad \Rightarrow \quad \bar{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$A = \frac{\delta f}{\delta x}\bigg|_{\substack{x=\bar{x}\\ u=\bar{u}}} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{1+x_1^2} & -2 \end{pmatrix}\bigg|_{\substack{x=\bar{x}\\ u=\bar{u}}} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} \qquad b = \frac{\delta f}{\delta u}\bigg|_{\substack{x=\bar{x}\\ u=\bar{u}}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$c^T = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad d=0$$

• Per Cartesio, tutti gli autovalori sono a parte reale negativa.

Calcolo $G(s)$: $c^T(sI-A)^{-1}b$

$$(sI-A)^{-1} = \frac{1}{s^2+2s+1}\begin{pmatrix} s+2 & 1 \\ -1 & s \end{pmatrix}$$

$$G(s) = \frac{1}{s^2+2s+1}\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} s+2 & 1 \\ -1 & s \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{(s+1)^2}$$

$u(t) = \cos t \qquad U(s) = \frac{s}{s^2+1} \qquad Y(s) = \frac{s}{(s+j)(s-j)(s+1)^2} = \frac{Q}{s-j} + \frac{Q^*}{s+j} + \frac{r_{11}}{s+1} + \frac{r_{12}}{(s+1)^2}$

$$Q = \frac{s}{(s+j)(s+1)^2}\bigg|_{s=j} = -\frac{1}{4}j \; \begin{cases} \frac{1}{4} \\ -\frac{\pi}{2} \end{cases}$$

$$\frac{s}{s^2+1}$$

$$r_{12} = \frac{s}{(s+j)(s-j)}\Bigg|_{s=-1} = \frac{-1}{(-1+j)(-1-j)} = -\frac{1}{2}$$

$$r_{11} = \frac{s^2+1-2s^2}{(s^2+1)^2}\Bigg|_{s=-1} = \frac{1-s^2}{(s^2+1)^2}\Bigg|_{s=-1} = 0$$

$$\delta y(t) = 0.1\left(\frac{1}{2}\cos(t-\tfrac{\pi}{2}) - \frac{1}{2}t e^{-t}\right) \quad t \geq 0$$

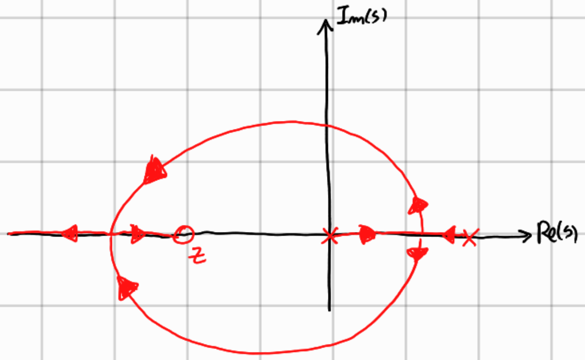
$$y(t) = \overset{0}{\bar{y}} + \delta y(t) = \delta y(t)$$

2)

$$G(s) = \frac{1}{s-2}$$

- Astatismo a disturbi costanti in catena diretta.
- Modi aperiodici con $T_{a2} = 1s$.

- Per poli doppi reali, $T_{a2} \cong 5.83\,T \Rightarrow$ Poli saranno in $-\frac{5.83}{1} = -5.83$.



Impongo z per avere p. doppio in −5.83.

$$\gamma(x) = \frac{x(x-2)}{x-z} \qquad \gamma'(x) = 0 \Rightarrow (x-2)(x-z) + x(x-z) - x(x-2) = 0$$

Impongo −5.83 come soluzione:

$$-7.83(-5.83-z) - 5.83(-5.83-z) + 5.83(-7.83) = 0$$

$$7.83z + 33.99 + 5.83z = 0$$

$$z = -2.49$$

Calcolo $\bar{p} = \frac{\lambda_1 \lambda_2}{\eta_1} = \frac{5.83 \cdot 7.83}{3.34} = 13.67$

$$R(s) = \frac{13.67}{s}(s+2.49) = 13.67 + \frac{34.04}{s}$$

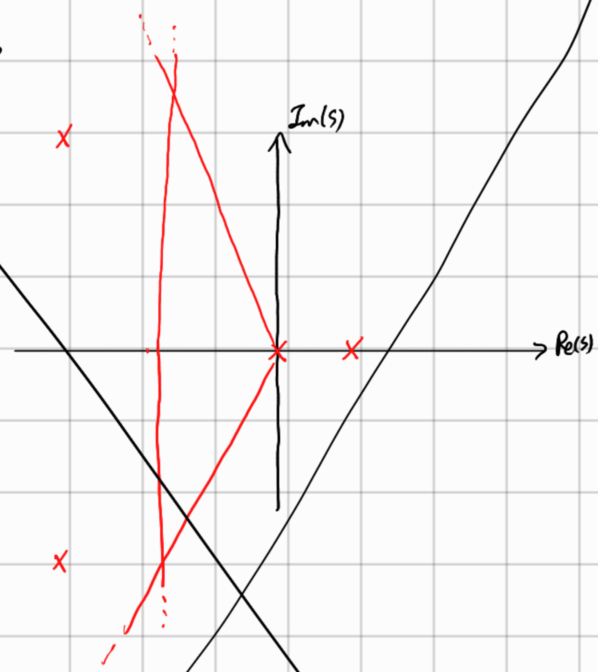
2)

$$G(s) = \frac{1}{(s-1)(s^2+10s+41)}$$

$s^2+10s+41=0 \Rightarrow s_{1,2} = -5 \pm 4j$

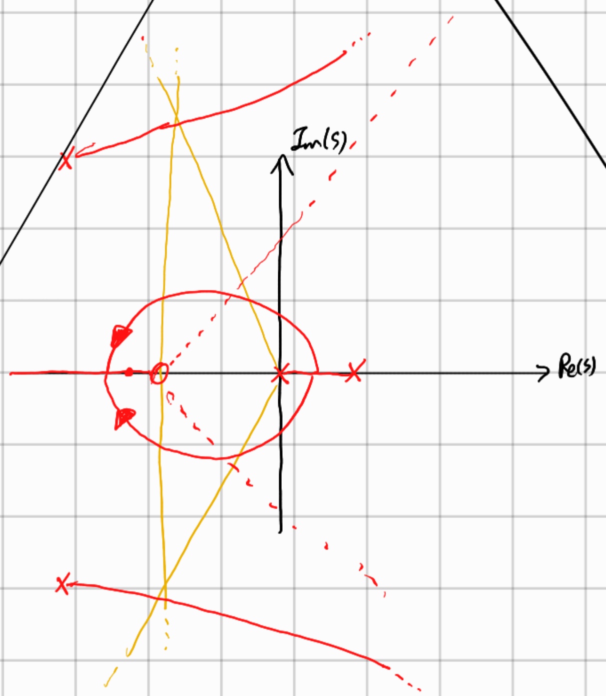
- Errore a regime nullo con gradino
- $\xi > 0{,}2$
- $T_{ass} < 2.5s \Rightarrow \sigma = -2$

$\xi > 0.2 \Rightarrow \theta < a\cos(\xi) = 78.5°$



Inserisco 2 zeri in -2 e vedo come cambia RLocus.

$$x_a = \frac{1}{3}(2-10+1) = -\frac{7}{3}$$



Trovo $\bar{p}$ associato a $Re(s)=-2$.

$s = z+2$

$$s(s+2)(s^2+10s+41)+p(s+2)=0 \Big|_{s=z+2}$$

$$(z+2)z(z^2+4+4z+10z+20+41)+p(z+4)=0$$

$$z(z+2)(z^2+14z+65)+p(z+4)=0$$

$$(z^2+2z)(z^2+14z+65)+p(z+4)=0$$

$$z^4+2z^3+14z^3+28z^2+65z^2+130z+pz+4p=0$$

$$z^4+16z^3+93z^2+(130+p)z+4p=0$$

$$\begin{array}{c|lll} 4 & 1 & 93 & 4p \\ 3 & 16 & 130+p & \\ 2 & 1358-p & 520p+4p^2 & \\ 1 & -65p^2-7092p+176540 & & \\ 0 & 520p+4p^2 & & \end{array}$$

$$-16(520p+4p^2)+(130+p)(1358-p)$$

$$-8320p-64p^2+176540-130p+1358p-p^2$$

$$=-65p^2-7092p+176540$$

Solution: $p=1358$

$p=20.89$

$p=-130$

$p=0$

$p=-130$

1)

$$g_y(t) = te^{-3t} + 2e^{-t}\cos 2t \qquad t \geq 0 \qquad u(t) = 45$$

$$G(s) = \frac{1}{(s+3)^2} + 2\cdot\frac{s+1}{s^2+2s+1+4} = \frac{1}{(s+3)^2} + 2\frac{s+1}{s^2+2s+5}$$

$$G_1(s) = \frac{1}{(s+3)^2} \qquad Y(s) = \frac{1}{s}\cdot\frac{1}{(s+3)^2} = \frac{M}{s} + \frac{r_{11}}{s+3} + \frac{r_{12}}{(s+3)^2}$$

$$r_{12} = \left.\frac{1}{s}\right|_{s=-3} = -\frac{1}{3} \qquad r_{11} = \left.-\frac{1}{s^2}\right|_{s=-3} = \frac{-1}{9} \qquad M = \frac{1}{9}$$

$$y_1(t) = \frac{1}{9} - \frac{1}{9}e^{-3t} - \frac{1}{3}te^{-3t} \qquad t \geq 0$$

$$G_2(s) = 2\frac{s+1}{s^2+2s+5} \qquad Y(s) = \frac{Q}{s+1-2j} + \frac{Q^*}{s+1+2j} + \frac{M}{s}$$

$$s^2 + 2s + 5 = 0$$

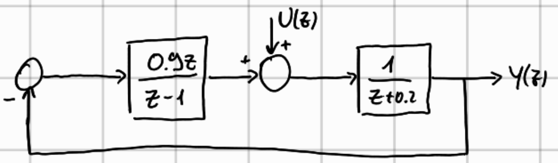
$$s = -1 \pm 2j \qquad M = \frac{2}{5}$$

$$Q = \left.\frac{2(s+1)}{s(s+1+2j)}\right|_{s=-1+2j} = \frac{\cancel{2}\cdot\cancel{2j}}{(-1+2j)(\cancel{4j})} = \frac{1}{2j-1} \begin{cases} 0.447 \\ -116.6^\circ \end{cases}$$

$$y_2(t) = \frac{2}{5} + 0.894\,e^{-t}\cos(2t - 116.6)$$

$$y(t) = 45(y_1(t) + y_2(t))$$

1)



$u(k) = k\,\delta_{-1}(k)$

$$S(z) = \frac{\frac{1}{z+0.2}}{1+\frac{1}{z+0.2}\cdot\frac{0.9z}{z-1}} = \frac{z-1}{(z+0.2)(z-1)+0.9z} = \frac{z-1}{z^2+0.1z-0.2} = \frac{z-1}{(z-0.4)(z+0.5)}$$

$$Y(z) = \frac{z}{(z-1)^2}\cdot\frac{z-1}{(z-0.4)(z+0.5)} = \frac{z}{(z-1)(z-0.4)(z+0.5)} = z\left[\frac{r_1}{z-1}+\frac{r_2}{z-0.4}+\frac{r_3}{z+0.5}\right]$$

$$r_1 = \left.\frac{z}{(z-0.4)(z+0.5)}\right|_{z=1} = \frac{10}{9} \qquad r_2 = \left.\frac{1}{(z-1)(z+0.5)}\right|_{z=0.4} = -1.85$$

$$r_3 = \left.\frac{1}{(z-1)(z-0.4)}\right|_{z=-0.5} = 0.74$$

$$y(k) = \frac{10}{9} - 1.85\cdot 0.4^k + 0.74\cdot(-0.5)^k$$

aperiodico conv. aperiodico alternato convergente

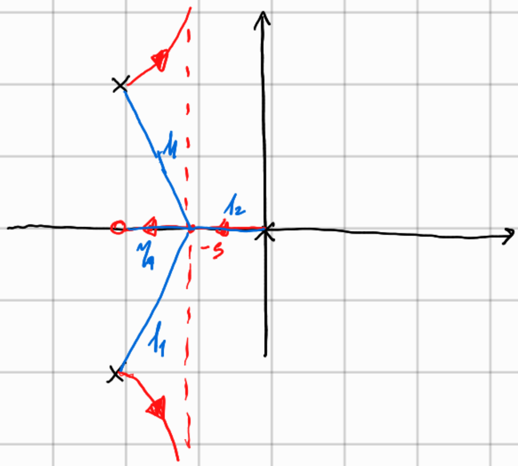
2)

$$G(s) = \frac{200}{s^2+20s+200}$$

$s_{12} = -10 \pm 10j$

$T_{a1} = 1s \Rightarrow$ Voglio polo in -5 s.

Inserisco zero in -10.



Trovo $\bar{p}$ per avere polo in -5:

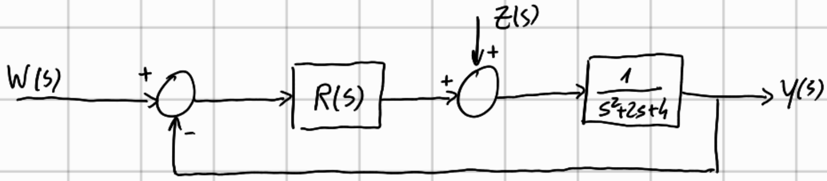
$$\bar{P} = \frac{l_1^2 l_2}{m_1} = \frac{(5^2+10^2)\,5}{5} = 125 = 200\mu \Rightarrow \mu = 0.625$$

$$R_{PI}(s) = \frac{0.625}{s}(s+10) = 0.625 + \frac{62.5}{s}$$

- Disturbo a rampa di $A=2$ ho $e_\infty = \frac{A}{M_0} = \frac{2}{6.25} = 0.32$

$$L(s) = \frac{125\,(s+10)}{s\,(s^2+20s+200)} \Rightarrow M_0 = sL(s)\big|_{s=0} = 6.25$$

2)



• Astatico a disturbo $z(t)$ costante.

• $\varphi_m = 40°$ con $\omega_c = 2$ rad/s.

$$R(s) = \frac{M}{s} \qquad L^*(s) = \frac{M}{s}\cdot\frac{1}{(s^2+2s+4)} \qquad L^*(j\omega_c) = \frac{M}{2j(4j)} = -\frac{M}{8} \begin{cases} |\,| = \frac{M}{8} \\ \angle = -180° \end{cases}$$

Mi serve anticipo di 40°:

$R_2(s) \,/\, \angle R_2(j\omega_c) = 40°$

$$\arctan(2T) = 40°$$

$$T = \frac{\tan 40°}{2} = 0.42$$

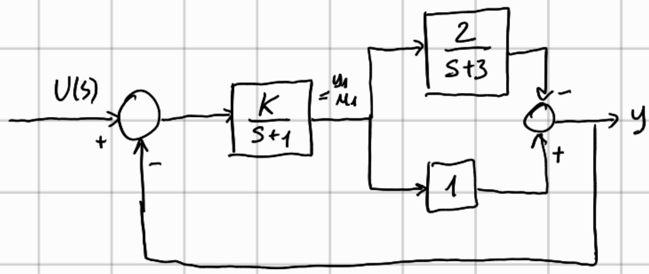
$$R(s) = \frac{M}{s}(1+0.42s)$$

Impongo $|L(j\omega_c)| = 1 \Rightarrow \left|\frac{M}{2j}(1+2j\cdot 0.42)\cdot\frac{1}{4j}\right| = 0.16M = 1$

$\Rightarrow M = 6.25$

$$R(s) = \frac{6.25}{s}(1+0.42s)$$

1)



$$G_2(s) = 1 - \frac{2}{s+3} = \frac{s+1}{s+3}$$ 1° ordine, $a_1 = 1$ $a_0 = 3$ $b_1 = 1$ $b_0 = 1$

$A = -3$ $b = 1$ $\dot{x}_1(t) = -3x_1(t) + u_1(t)$

$c = b_0 - a_0 b_1 = -2$ $d = 1$ $y(t) = -2x_1(t) + u_1(t)$

$$G_1(s) = \frac{K}{s+1}$$ $a_0 = 1$ $a_1 = 1$ $b_0 = K$ $b_1 = 0$ $A = -1$ $b = 1$ $C = b_0 - a_0 b_1 = K$ $d = 0$

$\dot{x}_2(t) = -x_2(t) + u(t)$ $y_1 = u_1$

$y_1(t) = Kx_2(t)$

$\Rightarrow \dot{x}_1(t) = -3x_1(t) + Kx_2(t)$

$\dot{x}_2(t) = -x_2(t) + u(t)$

$y(t) = -2x_1(t) + Kx_2(t)$

$$A = \begin{pmatrix} -3 & K \\ 0 & -1 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad c^T = \begin{pmatrix} -2 & K \end{pmatrix}$$

$$\vartheta = \begin{pmatrix} c^T \\ c^T A \end{pmatrix} \qquad c^T A = \begin{pmatrix} -2 & K \end{pmatrix} \begin{pmatrix} -3 & K \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 6 & -3K \end{pmatrix}$$

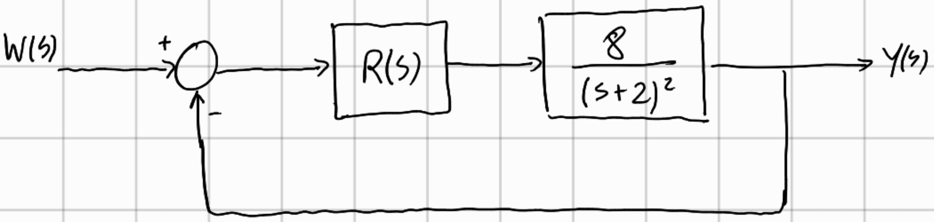
$$\vartheta = \begin{pmatrix} -2 & K \\ 6 & -3K \end{pmatrix} \Rightarrow \text{rk}\,\vartheta = 1$$

$$F(s) = \frac{\frac{K}{s+3}}{1 + \frac{K}{s+3}} = \frac{K}{s+3+K} \qquad T_{a1} = \frac{1}{3+K} \ln(100) = 0.5$$

$\Rightarrow 2\ln(100) = 3 + K$

$K = 6.21$

2)



- e_∞ con rampa unitaria del 5%.
- $\varphi_m = 45°$ a $\omega_c = 2$ rad/s

$e_\infty = \frac{A}{M}$ $\quad R_1(s) = \frac{M_0}{s}$ $\quad L^*(s) = \frac{M_0}{s} \cdot \frac{8}{(s+2)^2}$ $\quad M = sL(s)|_{s=0} = M_0 \cdot 2$

$e_\infty = \frac{A}{2M_0} \Rightarrow \frac{1}{2M_0} = 0.05 \Rightarrow M_0 = 10$

$R_1(s) = \frac{10}{s}$

$L^*(2j) = \frac{10}{s} \cdot \frac{8}{(s+2)^2}\Big|_{s=2j} = -5$. Ho bisogno di anticipo di 30° e attenuazione di $\frac{1}{5}$.

- Rete ritardatrice:
  - Attenua di $\frac{1}{5}$ + compensazione rete anticipatrice (circa -8 dB)
- Rete anticipatrice:
  - Anticipa di 45° + compensazione rete ritardatrice (circa 5°)

- Attenuazione:

$M_{dB} = 20\log_{10}\frac{1}{5} - 8dB = -21.98\ dB \iff M = 0.08$

$\varphi = -5°$.

- Anticipo:

$\varphi = 50°$ $\quad M_{dB} \approx 8\ dB \iff M = 2.51$

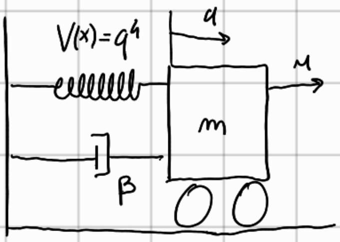
$R_{2A}(s) = \frac{1+sT}{1+s\alpha T}$ $\quad \alpha = \frac{M\cos\varphi - 1}{M(M-\cos\varphi)} = 0.131$ $\quad R_{2R}(s) = \frac{1+sT}{1+s\alpha T}$ $\quad \alpha = 12.56$

$T = 5.26$

$T = \frac{M - \cos\varphi}{\omega_c \sin\varphi} = 1.219$

$$R(s) = \frac{10}{s} \cdot \frac{1+1.219s}{1+0.16s} \cdot \frac{1+5.26s}{1+66.07s}$$

1)



$V(q)=q^4 \quad m=4kg$ $\quad$ β / sistema linearizz. intorno a p. eq. con $u=4N$ ha modo convergente relativo ad autovalore doppio.

$$V(q)=q^4 \quad T(q,\dot q)=\frac{1}{2}m\dot q^2 \quad D=\frac{1}{2}\beta\dot q^2$$

$$\mathcal{L}(q,\dot q)=\frac{1}{2}m\dot q^2-q^4$$

$$\frac{d}{dt}\frac{\delta}{\delta\dot q}\mathcal{L}(q,\dot q)-\underbrace{\frac{\delta}{\delta q}\mathcal{L}(q,\dot q)}_{-4q^3}+\frac{\delta}{\delta\dot q}D(\dot q)=u$$

$$m\ddot q+4q^3+\beta\dot q=u \Rightarrow \ddot q=-q^3-\frac{\beta}{4}\dot q+\frac{u}{4}$$

$$x_1=q \quad x_2=\dot q \Rightarrow \begin{cases}\dot x_1=x_2\\ \dot x_2=-x_1^3-\frac{\beta}{4}x_2+\frac{u}{4}\end{cases}$$

$\bar u=4$

$\hookrightarrow$
$$0=x_2$$
$$0=-x_1^3+1 \Rightarrow x_1=1 \qquad \Rightarrow \bar x=\begin{pmatrix}1\\0\end{pmatrix}$$

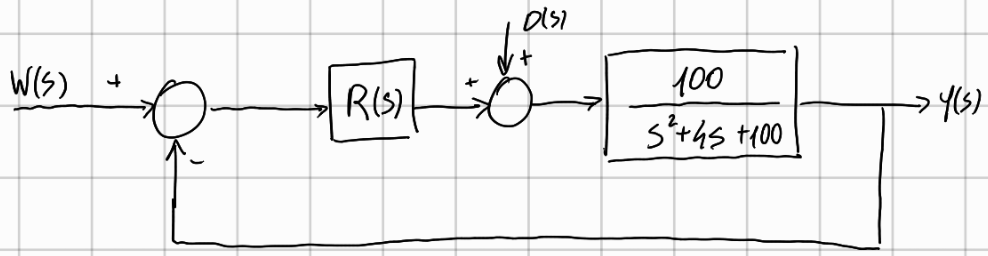
$$A=\frac{\delta f}{\delta x}\Big|_{\substack{x=\bar x\\u=\bar u}}=\begin{pmatrix}0 & 1\\ -3x_1^2 & -\frac{\beta}{4}\end{pmatrix}\Big|_{\substack{x=\bar x\\u=\bar u}}=\begin{pmatrix}0 & 1\\ -3 & -\frac{\beta}{4}\end{pmatrix}$$ Forma compagna.

$$P(s)=s^2+\frac{\beta}{4}s+3=0$$

$$\Delta=0 \Rightarrow \frac{\beta^2}{16}-12=0 \Rightarrow \beta^2=12\cdot 16=192$$
$$\beta=8\sqrt{3}\approx 13.86$$

Solo valore positivo è accettabile per coeff. attrito.

2)



- Astatico con $d(t)$ a gradino.
- $K_m = 20\,dB \iff k_m = 10$

$R(s) = \frac{M}{s}$ per astatismo. $\quad L(s) = \frac{M}{s} \cdot \frac{100}{s^2+4s+100}$

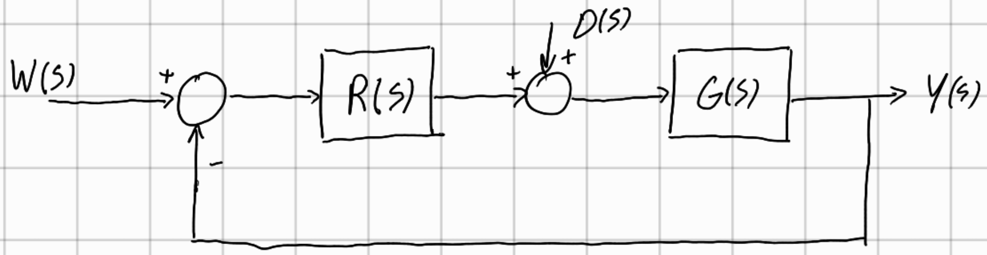
Trovo ω_π: $\quad -\frac{\pi}{2} - \angle\left(4j\omega_\pi + 100 - \omega_\pi^2\right) = -\pi \implies$ Parte reale di $s^2+4s+100 = 0$.

$\implies 100 = \omega_\pi^2 \iff \omega_\pi = 10.$

Trovo $|L(j\omega_\pi)| = \frac{M}{|10j|} \cdot \frac{100}{|40j|} = \frac{M}{4} = \frac{1}{k_m} = \frac{1}{10} \implies M = \frac{4}{10} = \frac{2}{5}$

$R(s) = \frac{0.4}{s} \qquad L(s) = \frac{40}{s(s^2+4s+100)} \qquad M_0 = \lim_{s\to 0} sL(s) = \frac{4}{10}$

$e_\infty = \frac{A}{M_0} = \frac{1}{0.4} = \frac{10}{4} = \frac{5}{2} = 2.5$



$G(s) = -\frac{1}{2}$

• Assumiamo a $d(t)$ costante, con $K_m = 6dB \Leftrightarrow K_m = 2$

$1 \, rad/s = \omega_c$. Nota: Margine è già 2. Quindi devo scegliere parametri / $|R(s)G(s)| = \frac{1}{2}$

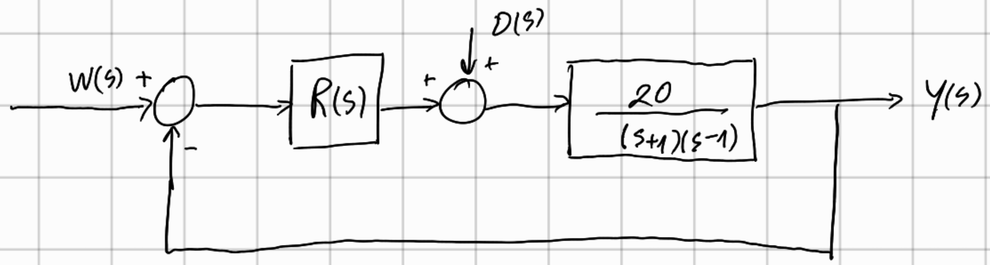
$$R(s) = K_P\left(1 + \frac{1}{T_i s} + T_D s\right)$$

$K_P = 1$, perché non voglio cambiare la G in s.

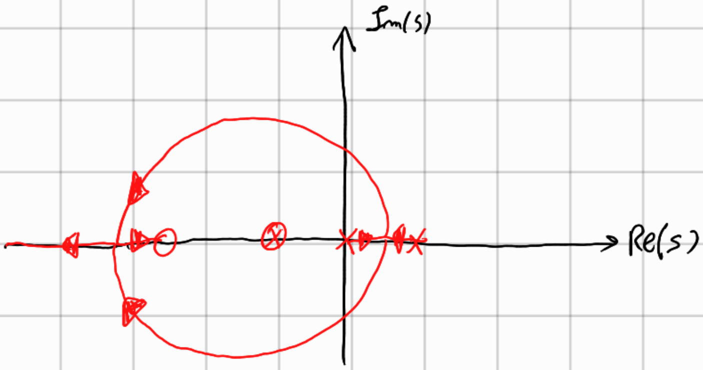
$$\frac{1}{T_i s} + T_D s = 0 \quad \Rightarrow \quad \frac{1}{4T_D} - T_D = 0 \qquad T_D^2 = \frac{1}{4} \qquad T_D = \frac{1}{2}$$

$$T_i = 4T_D = 2$$

2)



• Assumiamo a disturbo $D(s) = \frac{1}{s}$, con $T_{a_1} = 2s$.



Impongo zero in -1 e zero in z / punto doppio è in $-\frac{6.64}{2}$, perché $T_{a1} \cong 6.64T$ per poli reali multipli.

$$\gamma(x) = \frac{x(x-1)}{x-z} \qquad \gamma'(x) = 0 \Rightarrow (x-1)(x-z) + x(x-z) - x(x-1) = 0$$

Impongo -2.5 come soluzione e -3.32 come soluzione.

(1) $$-3.5(-2.5-z) - 2.5(-2.5-z) + 2.5(-3.5) = 0$$

$$3.5z + 6.25 + 2.5z = 0$$

$$z = -1.04$$

(2) $$-4.32(-3.32-z) - 3.32(-3.32-z) + 3.32(-4.32) = 0$$

$$4.32z + 11.02 + 3.32z = 0$$

$$z = -1.44$$

1) Trovo $\bar{p} = \frac{2.5 \cdot 3.5}{(2.5-1.04)} = 5.99 \Downarrow M_0 = \frac{5.99}{20} = 0.3$

2) $\bar{p} = \frac{3.32 \cdot 4.32}{(3.32-1.44)} = 7.63 \Downarrow M_0 = 0.38$

$$R_1(s) = \frac{0.3}{s}(s+1)(s+1.04)$$

$$= \frac{0.3}{s}(s^2+2.04s+1.04) =$$

$$= 0.612 + \frac{0.312}{s} + 0.3s$$

$$R_2(s) = \frac{0.38}{s}(s+1)(s+1.44)$$

$$= \frac{0.38}{s}(s^2+2.44s+1.44) =$$

$$= 0.93 + \frac{0.547}{s} + 0.38s$$

• Risposta a disturbo $u(t) = 10 \sin(10t)$

$$S(s) = \frac{G(s)}{1+R(s)G(s)}$$

$$S(10j) = \frac{G(10j)}{1+R(10j)G(10j)}$$

$$G(10j) = -0.198$$

$$R(10j) = 3.86\, e^{j(76.1^\circ)}$$

$$S(10j) = 0.18\, e^{j(-137.7^\circ)}$$

$$d(t)_r = 1.8 \sin(10t - 137.7^\circ)$$

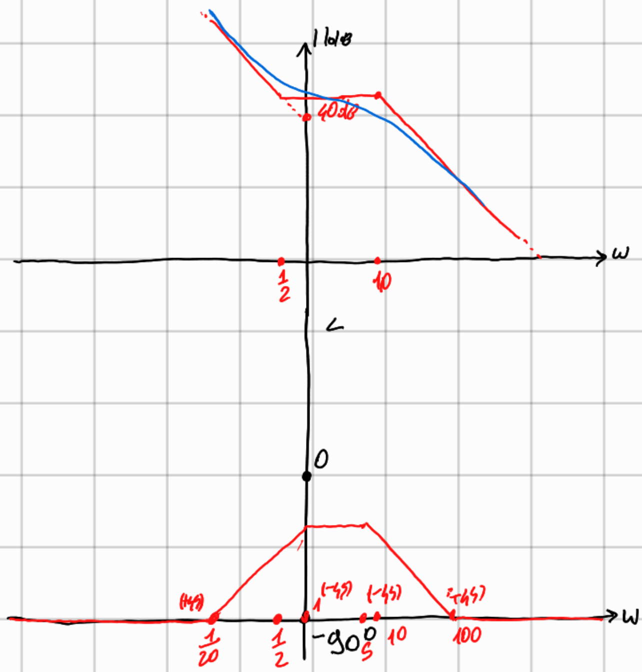
1)

$$G(s) = \frac{2000}{s}\frac{(s+0.5)}{(s+10)} = \frac{2000}{s}\cdot\frac{0.5(1+2s)}{10(1+\frac{1}{10}s)} = \frac{100}{s}\cdot\frac{(1+2s)}{(1+\frac{1}{10}s)}$$

$M = 100 = 40dB$

- Rottura: $\frac{1}{2} = 0.5$, con zero a parte reale negativa: $+20\,dB/dec$ e $+90°$.

  10, con polo a parte reale negativa: $-20\,dB/dec$ e $-90°$.



1)

$$x(k+1) = \begin{pmatrix} 0 & 1 \\ 0.2h & h-0.2 \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} 0.2 & 1 \end{pmatrix} x(k)$$

Sistema é in forma di raggiungibilità, É raggiungibile. $0,2 + h - 0.2$

$$\mathcal{O} = \begin{pmatrix} C^T \\ C^T A \end{pmatrix} \qquad C^T A = \begin{pmatrix} 0,2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0.2h & h-0,2 \end{pmatrix} = \begin{pmatrix} 0.2h & h \end{pmatrix}$$

$$\mathcal{O} = \begin{pmatrix} 0,2 & 1 \\ 0.2h & h \end{pmatrix}$$ non osservabile.

$$P(s) = s^2 - (h-0,2)s - 0.2h = 0$$

$$\begin{cases} 0,2 - h > 0 \\ -0.2h > 0 \end{cases} \Rightarrow \begin{matrix} h < 0,2 \\ h < 0. \end{matrix} \Rightarrow h < 0$$