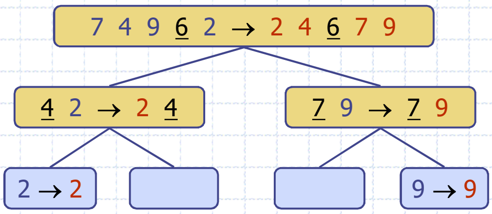
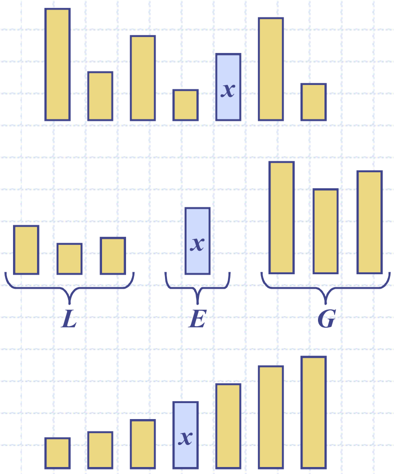
Quick-Sort



Quick-Sort

- Quick-sort is a randomized sorting algorithm based on the divide-and-conquer paradigm:
  - Divide: pick a random element x (called pivot) and partition S into
    - L elements less than x
    - E elements equal x
    - G elements greater than x
  - Recur: sort L and G
  - Conquer: join L, E and G



Separazione avviene con più sforzo: dobbiamo usare 3 liste.
Fase di discesa ha sforzo maggiore.
Ripeto fino a che non arrivo a liste tutte da 1 elemento.

A furia di scendere, quando ricompongo li ho ordinati. Fusione ci fa risparmiare.

Impegnativa la divisione sul pivot.

Partition

- We partition an input sequence as follows:
  - We remove, in turn, each element y from S and
  - We insert y into L, E or G, depending on the result of the comparison with the pivot x
- Each insertion and removal is at the beginning or at the end of a sequence, and hence takes $O(1)$ time
- Thus, the partition step of quick-sort takes $O(n)$ time

Algorithm *partition*(S, p)
    Input sequence S, position p of pivot
    Output subsequences L, E, G of the elements of S less than, equal to, or greater than the pivot, resp.
    $L, E, G \leftarrow$ empty sequences
    $x \leftarrow S.remove(p)$
    while $\neg S.isEmpty()$
        $y \leftarrow S.remove(S.first())$
        if $y < x$
            $L.addLast(y)$
        else if $y = x$
            $E.addLast(y)$
        else $\{ y > x \}$
            $G.addLast(y)$
    return L, E, G

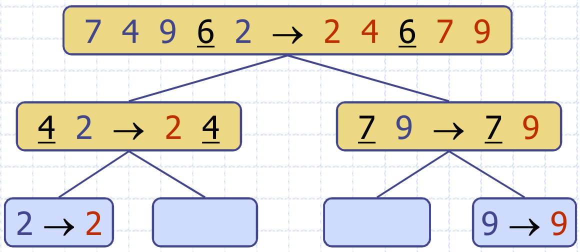
Java Implementation

```java
/** Quick-sort contents of a queue. */
public static <K> void quickSort(Queue<K> S, Comparator<K> comp) {
  int n = S.size();
  if (n < 2) return;                          // queue is trivially sorted
  // divide
  K pivot = S.first();                        // using first as arbitrary pivot
  Queue<K> L = new LinkedQueue<>();
  Queue<K> E = new LinkedQueue<>();
  Queue<K> G = new LinkedQueue<>();
  while (!S.isEmpty()) {                      // divide original into L, E, and G
    K element = S.dequeue();
    int c = comp.compare(element, pivot);
    if (c < 0)                                // element is less than pivot
      L.enqueue(element);
    else if (c == 0)                          // element is equal to pivot
      E.enqueue(element);
    else                                      // element is greater than pivot
      G.enqueue(element);
  }
  // conquer
  quickSort(L, comp);                         // sort elements less than pivot
  quickSort(G, comp);                         // sort elements greater than pivot
  // concatenate results
  while (!L.isEmpty())
    S.enqueue(L.dequeue());
  while (!E.isEmpty())
    S.enqueue(E.dequeue());
  while (!G.isEmpty())
    S.enqueue(G.dequeue());
}
```

E è già ordinata

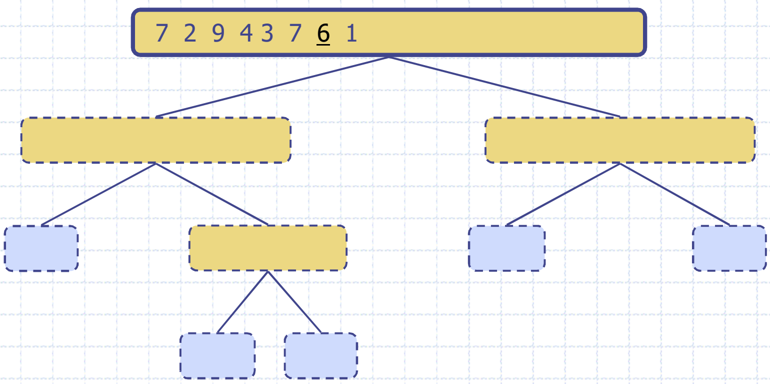
Quick-Sort Tree

- An execution of quick-sort is depicted by a binary tree
  - Each node represents a recursive call of quick-sort and stores
    - Unsorted sequence before the execution and its pivot
    - Sorted sequence at the end of the execution
  - The root is the initial call
  - The leaves are calls on subsequences of size 0 or 1



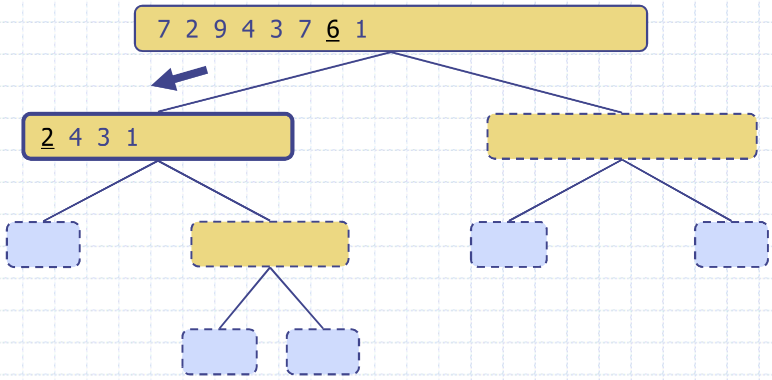
Execution Example

- Pivot selection



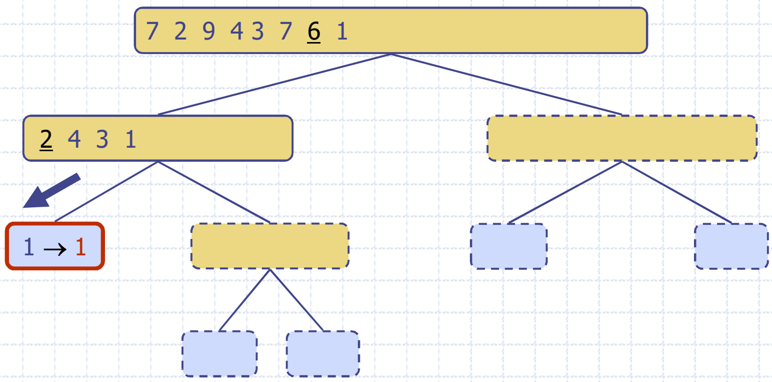
Execution Example (cont.)

- Partition, recursive call, pivot selection



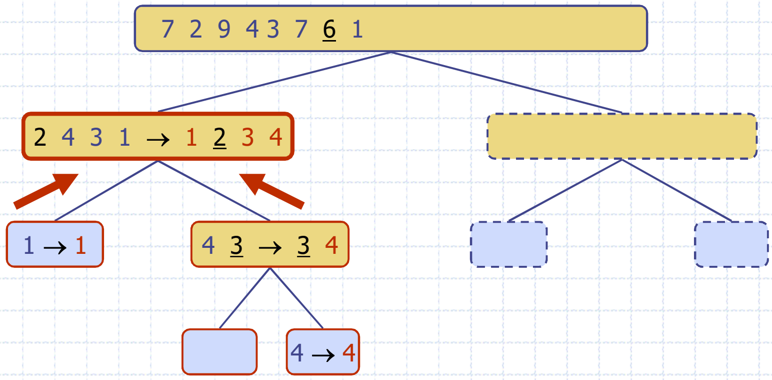
Execution Example (cont.)

- Partition, recursive call, base case



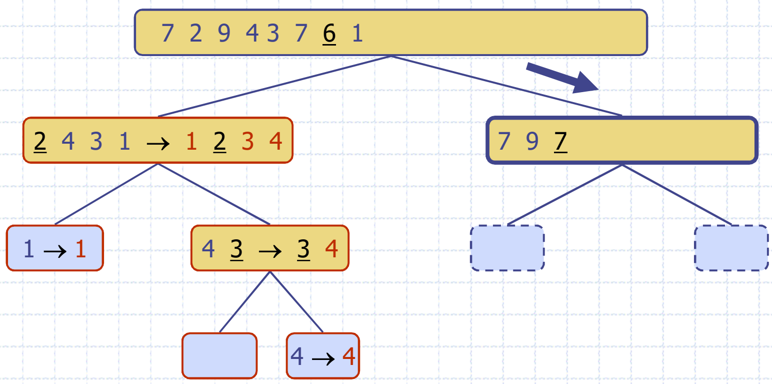
Execution Example (cont.)

- Recursive call, …, base case, join



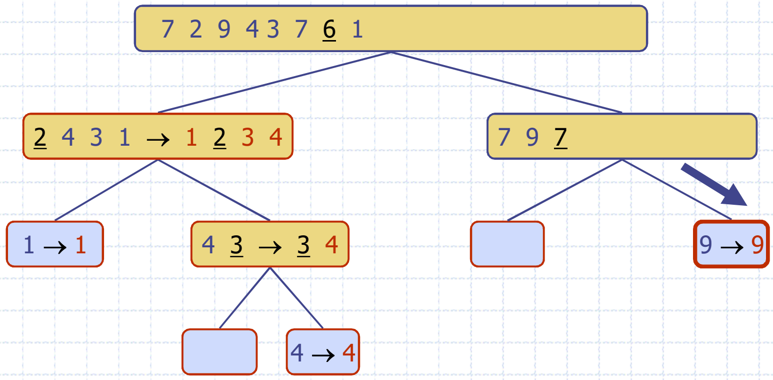
Execution Example (cont.)

- Recursive call, pivot selection



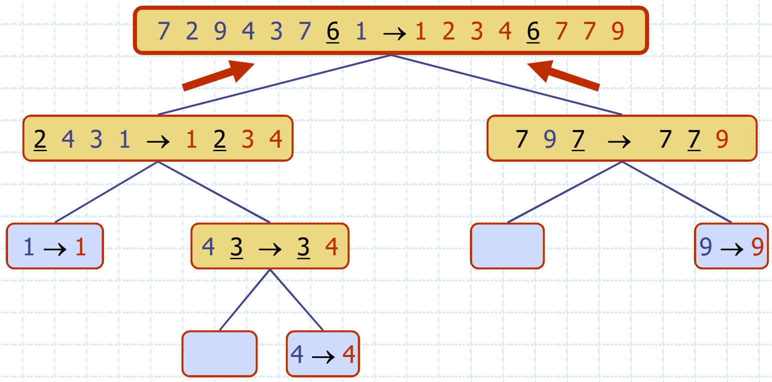
Execution Example (cont.)

- Partition, …, recursive call, base case



Execution Example (cont.)

- Join, join



Grosso del lavoro si fa nella suddivisione

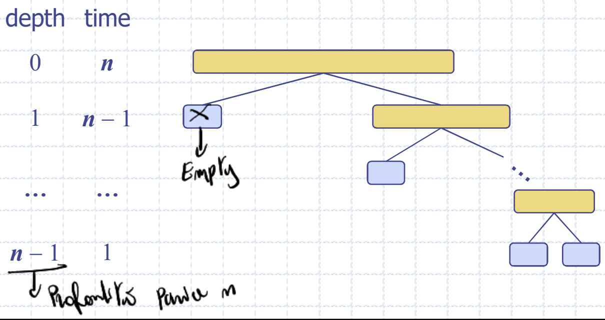
Prima suddivisione richiede n confronti

Worst-case Running Time

- The worst case for quick-sort occurs when the pivot is the unique minimum or maximum element
- One of L and G has size $n-1$ and the other has size 0
- The running time is proportional to the sum

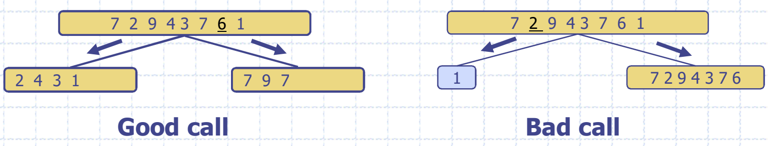
$$n + (n-1) + \ldots + 2 + 1$$

- Thus, the worst-case running time of quick-sort is $O(n^2)$

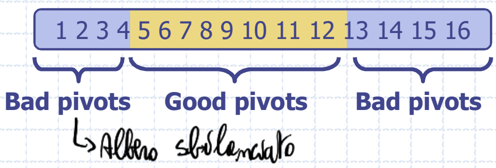


Expected Running Time

- Consider a recursive call of quick-sort on a sequence of size s
  - **Good call:** the sizes of L and G are each less than $3s/4$
  - **Bad call:** one of L and G has size greater than $3s/4$



- A call is good with probability $1/2$
  - $1/2$ of the possible pivots cause good calls:



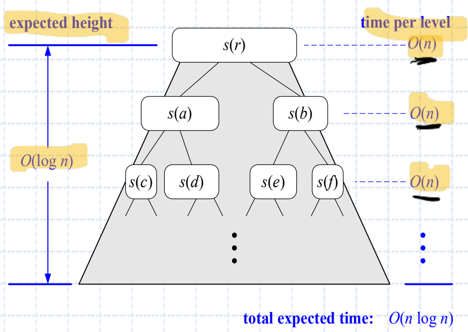
50% di avere pivot che divide in 2 pezzi uguali.

Expected Running Time, Part 2

- The amount or work done at the nodes of the same depth is $O(n)$
- Thus, the expected (probabilistic) running time of quick-sort is $O(n \log n)$

Su ogni livello n confronti

2^i problemi di dimensione $\frac{n}{2^i}$



Nel caso medio tende a dividere in 2 pezzi uguali ⇒ altezza albero medio è $\log n$

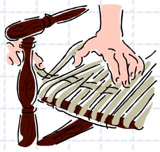
→ Più leggero a livello di memoria

In-Place Quick-Sort

- Quick-sort can be implemented to run in-place
- In the partition step, we use replace operations to rearrange the elements of the input sequence such that
  - the elements less than the pivot have rank less than h
  - the elements equal to the pivot have rank between h and k
  - the elements greater than the pivot have rank greater than k
- The recursive calls consider
  - elements with rank less than h
  - elements with rank greater than k

Algorithm *inPlaceQuickSort*(S, l, r)
  Input sequence S, ranks l and r
  Output sequence S with the elements of rank between l and r rearranged in increasing order
  if $l \geq r$
    return
  $i \leftarrow$ a random integer between l and r
  $x \leftarrow S.elemAtRank(i)$
  $(h, k) \leftarrow inPlacePartition(x)$
  $inPlaceQuickSort(S, l, h-1)$
  $inPlaceQuickSort(S, k+1, r)$

In-Place Partitioning



- Perform the partition using two indices to split S into L and E U G (a similar method can split E U G into E and G).

left right

3 2 5 1 0 7 3 5 9 2 7 9 8 9 7 9 **6** (pivot = 6)

- Repeat until left and right cross:
  - Scan left to the right until finding an element ≥ x.
  - Scan right to the left until finding an element < x.
  - Swap elements at indices left and right

left right

3 2 5 1 0 7 3 5 9 2 7 9 8 9 7 9 **6**

left

3 2 5 1 0 2 3 5 **6** 7 7 9 8 9 7 9 **9**

Indice sinistro si incrementa fino a che trovo un elemento più grande del pivot. Destro uguale. → Scambio 2 con 7 e ricomincio. Finché left e right si incontrano. ⇒ Tutto ordinato! Scambio pivot con elemento puntato da left.

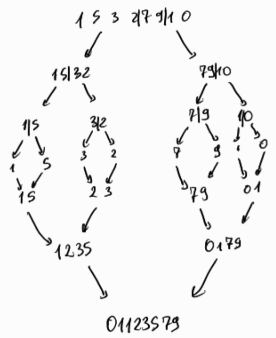
Java Implementation

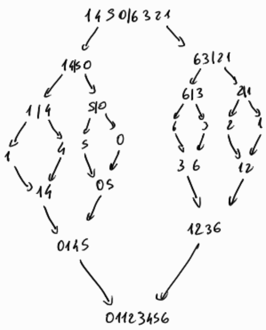
```
/** Sort the subarray S[a..b] inclusive. */
private static <K> void quickSortInPlace(K[ ] S, Comparator<K> comp,
                                                            int a, int b) {
  if (a >= b) return;           // subarray is trivially sorted
  int left = a;
  int right = b−1;
  K pivot = S[b];
  K temp;                       // temp object used for swapping
  while (left <= right) {
    // scan until reaching value equal or larger than pivot (or right marker)
    while (left <= right && comp.compare(S[left], pivot) < 0) left++;
    // scan until reaching value equal or smaller than pivot (or left marker)
    while (left <= right && comp.compare(S[right], pivot) > 0) right−−;
    if (left <= right) {        // indices did not strictly cross
      // so swap values and shrink range
      temp = S[left]; S[left] = S[right]; S[right] = temp;
      left++; right−−;
    }
  }
  // put pivot into its final place (currently marked by left index)
  temp = S[left]; S[left] = S[b]; S[b] = temp;
  // make recursive calls
  quickSortInPlace(S, comp, a, left − 1);
  quickSortInPlace(S, comp, left + 1, b);
}
```

valore maggiore

Summary of Sorting Algorithms

Algorithm	Time	Notes
selection-sort	$O(n^2)$	▪ in-place ▪ slow (good for small inputs)
insertion-sort	$O(n^2)$	▪ in-place ▪ slow (good for small inputs)
quick-sort	$O(n \log n)$ expected	▪ in-place, randomized ▪ fastest (good for large inputs)
merge-sort	$O(n \log n)$	▪ sequential data access ▪ fast (good for huge inputs)





1 5 3 2 7 9 4 5

12|3456|78