ALCUNE CONSIDERAZIONI:

- É possibile che una piccola parte degli esercizi sia scorretta
- Alcuni esercizi possono essere risolti in modi diversi, ma ugualmente corretti
- Gli esercizi sono svolti in ordine cronologico, quindi più si va in fondo nel file più sarà probabile che con più esperienza alle spalle saranno corretti.

Buon LAVORO!

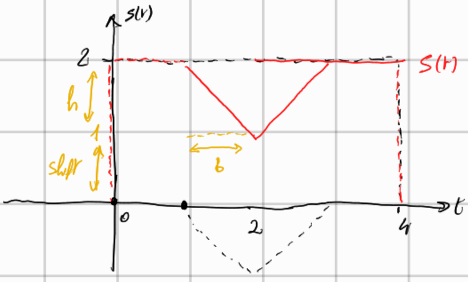
Prova 2K18

1) Potenza, energia, Trasformata

a)

$$s(t) = 2\Pi\left(\frac{t}{4}-\frac{1}{2}\right) - \Delta(t-2)$$

$$= 2\pi\left(\frac{t-2}{4}\right) - \Delta(t-2)$$



s(t) é segnale di energia: $P_s = 0$

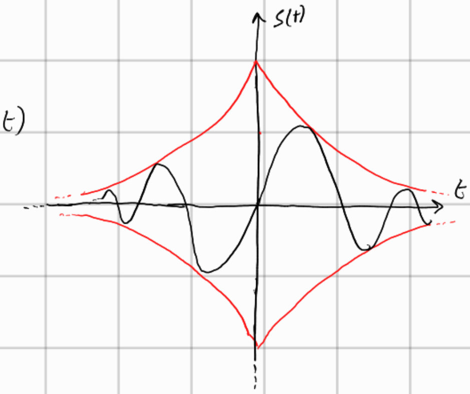
$$E_s = \int_{-\infty}^{+\infty} s(t)^2 dt = 2\left(\frac{h^2 \cdot b}{3} + h \cdot b \cdot \text{shift} + b \cdot \text{shift}^2\right) + 4 \cdot 1 + 4 \cdot 1$$

$$= 2\left(\frac{1}{3} + 1 + 1\right) + 8 = 2\left(\frac{7}{3}\right) + 8 = \frac{14}{3} + 8 = \frac{38}{3}$$

$$F[s(t)] = 2 \cdot 4 e^{-4\pi j f} \operatorname{sinc}(4f) - e^{-4\pi j f} \operatorname{sinc}^2(f)$$

b)

$$s(t) = e^{-|t|} \sin(2\pi f_0 t)$$



NOTA: $s(t)$ è dispari.

$|s(t)|^2$ è pari.

$$E_s = 2\int_0^{+\infty} |s(t)|^2 dt = 2\int_0^{+\infty} e^{-t} \frac{1}{2j}\left[e^{2\pi f_0 j t} - e^{-2\pi f_0 j t}\right] dt = \frac{1}{j}\int_0^{+\infty} \left(e^{-(1-2\pi f_0 j)t} - e^{-(1+2\pi f_0 j)t}\right) dt =$$

$$= \frac{1}{j}\int_0^{+\infty} e^{-(1-2\pi j f_0)t}\, dt - \frac{1}{j}\int_0^{+\infty} e^{-(1+2\pi j f_0)t}\, dt = \frac{e^{-(1-2\pi j f_0)t}}{-j(1-2\pi j f_0)}\Bigg|_0^{+\infty} - \frac{e^{-(1+2\pi f_0 j)t}}{-j(1+2\pi f_0 j)}\Bigg|_0^{+\infty} =$$

$$= \frac{1}{j+2\pi f_0} - \frac{1}{j-2\pi f_0} = \frac{j-2\pi f_0 - j - 2\pi f_0}{(j+2\pi f_0)(j-2\pi f_0)} = \frac{-4\pi f_0}{-1-4\pi^2 f_0^2} = \frac{4\pi f_0}{1+4\pi^2 f_0^2}$$

$P_s = 0$

Calcolo trasf. di Fourier:

$$S(f) = (S_1 * S_2)(f)$$

$$s_2(t) = \sin(2\pi f_0 t) = \cos\left(2\pi f_0 t - \frac{\pi}{2}\right) = \cos\left[2\pi f_0\left(t - \frac{1}{4f_0}\right)\right]$$

$$S_2(f) = e^{-2\pi j f \cdot \frac{1}{4f_0}} \left[\delta(f-f_0) + \delta(f+f_0)\right] =$$

$$e^{-j\frac{\pi}{2}\frac{f}{f_0}}$$

$$= \frac{e^{-j\frac{\pi}{2}}}{2}\delta(f-f_0) + \frac{e^{j\frac{\pi}{2}}}{2}\delta(f+f_0) = \frac{-j}{2}\delta(f-f_0) + \frac{j}{2}\delta(f+f_0)$$

$$s_1(t) = e^{-t}u(t) + e^{-(-t)}u(-t)$$

$$S_1(f) = \frac{1}{1+2\pi j f} + \frac{1}{1-2\pi j f} = \frac{2}{1+4\pi^2 f^2}$$

$$(S_1 * S_2)(f) = \frac{-j}{1+4\pi^2(f-f_0)^2} + \frac{j}{1+4\pi^2(f+f_0)^2}$$

c)

$$s(t) = 1+\cos(2\pi t) + \sin^2(4\pi t+2) = 1+\cos(2\pi t) + 1-\cos^2(4\pi t+2) =$$

$$= 2+\cos(2\pi t) - \frac{1}{2} - \frac{1}{2}\cos(8\pi t+4) = \frac{3}{2} + \cos(2\pi t) - \frac{1}{2}\cos(8\pi t+4)$$

$\mathcal{E} = +\infty$ perché segnale di potenza.

Periodo minimo è 1, perché prendo il più grande tra le f sommate.

$$P = \frac{1}{T_0}\int_0^{T_0} |s(t)|^2 dt = \int_0^1 \left(\frac{3}{2} + \cos(2\pi t) - \frac{1}{2}\cos(8\pi t+4)\right)^2 dt = \frac{9}{4} + \int_0^1 \cos^2(2\pi t)\,dt + \frac{1}{4}\int_0^1 \cos^2(8\pi t+4)\,dt +$$

$$+ 3\cancel{\int_0^1 \cos(2\pi t)\,dt}^{\,0} - \frac{3}{2}\cancel{\int_0^1 \cos(8\pi t+4)\,dt}^{\,0} - \cancel{\int_0^1 \cos(2\pi t)\cos(8\pi t+4)\,dt}^{\,0}$$

integrale del cos su un suo periodo

↳ $\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$

↳ che è una funzione che ha periodo almeno 1, quindi integrale va a 0.

$$= \frac{9}{4} + \frac{1}{2} + \frac{1}{2}\cancel{\int_0^1 \cos(4\pi t)\,dt}^{\,0} + \frac{1}{8} + \frac{1}{8}\cancel{\int_0^1 \cos(16\pi t+8)\,dt}^{\,0} = \frac{9}{4} + \frac{1}{2} + \frac{1}{8} =$$

$$= \frac{18}{8} + \frac{4}{8} + \frac{1}{8} = \frac{23}{8} \quad \square$$

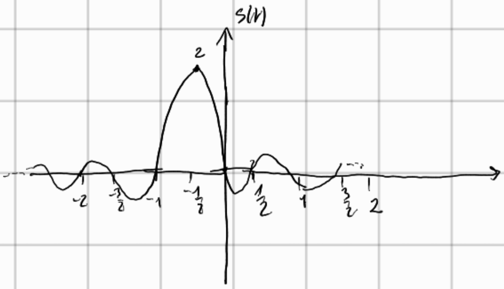
Per calcolare la trasformata:

$$s(t) = \frac{3}{2} + \cos(2\pi t) - \frac{1}{2}\cos(8\pi t+4)$$

Per formula da es. precedente

$$S(f) = \frac{3}{2}\delta(f) + \frac{1}{2}\delta(f-1) + \frac{1}{2}\delta(f+1) - \frac{1}{4}\left[\delta(f-4)e^{j\cdot 4} + \delta(f+4)e^{-j\cdot 4}\right]$$

d) $s(t) = 2\,\mathrm{sinc}\,(2t+1)$



Zeri: $2t+1=K \Rightarrow t=\frac{K-1}{2}$

$P=0$ perché segnale di energia

$$s(t) = 2\,\frac{\sin((2t+1)\pi)}{(2t+1)\pi}$$

$$\mathcal{E}_s = \int_{-\infty}^{+\infty} \frac{4\sin^2((2t+1)\pi)}{[(2t+1)\pi]^2}\,dt = \qquad (2t+1)\pi = y \quad dy = 2\pi\,dt \qquad = \int_{-\infty}^{+\infty} 2\pi\,\frac{\sin^2(y)}{y^2}\,dy$$

RISOLVO DOPO

NOTA:

Io so che considerando $s_0(t) = \Pi(t)$, allora $S_0(f) = \mathrm{sinc}(f)$

E che $F[S_0(t)] = s_0(-f)$. Quindi, $F[S_0(t)] = \Pi(-f) = \Pi(f)$

Sia $\mathrm{sinc}\,t = f(t)$. Allora, $s(t) = 2f(2(t+\frac{1}{2}))$

Quindi, $F[2f(2(t+\frac{1}{2}))] = 2\cdot\frac{1}{2}\,e^{\pi j f}\cdot F(\frac{f}{2}) = S(f)$

Che conclude: $S(f) = e^{\pi j f}\,\Pi\left(\frac{f}{2}\right)$.

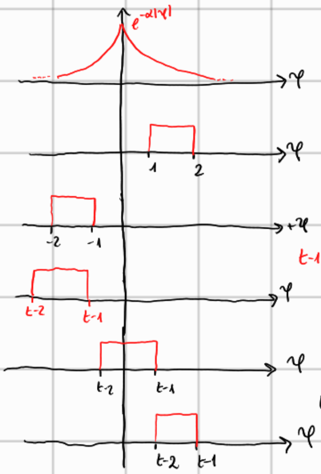
Per parseval $\mathcal{E}_s = \int_{-\infty}^{+\infty} |S(f)|^2 df = \int_{-\infty}^{+\infty} [\Pi(\frac{f}{2})]^2 df = \int_{-1}^{1} df = 2$

2)

$h(t) = e^{-\alpha|t|}$ $\qquad$ $s(t) = u(t-1) - u(t-2)$

$y(t) = h * s$

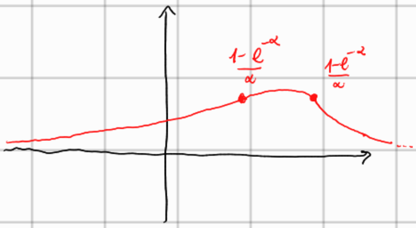
Nota: $s(t-\varphi) = s(-(\varphi - t))$



$t-1<0 \Rightarrow t<1$

$\begin{matrix} t-1>0 \\ t-2<0 \end{matrix} \Rightarrow 1<t<2$

$t>2$



$\frac{1-e^{-\alpha}}{\alpha}$

① $t<1$ $\qquad y(t) = \int_{t-2}^{t-1} e^{+\alpha\varphi} \cdot 1 \, d\varphi = \frac{e^{\alpha\varphi}}{\alpha}\Big|_{t-2}^{t-1} = \frac{e^{\alpha(t-1)} - e^{\alpha(t-2)}}{\alpha}$

② $1<t<2$ $\qquad y(t) = \int_{t-2}^{0} e^{\alpha\varphi} d\varphi + \int_{0}^{t-1} e^{-\alpha\varphi} d\varphi = \frac{e^{\alpha\varphi}}{\alpha}\Big|_{t-2}^{0} + \frac{e^{-\alpha\varphi}}{-\alpha}\Big|_{0}^{t-1} = \frac{1-e^{\alpha(t-2)}}{\alpha} + \frac{1-e^{-\alpha(t-1)}}{\alpha} = \frac{2-e^{\alpha(t-2)}-e^{-\alpha(t-1)}}{\alpha} =$

③ $t>2$ $\qquad y(t) = \int_{t-2}^{t-1} e^{-\alpha\varphi} d\varphi = \frac{e^{-\alpha\varphi}}{-\alpha}\Big|_{t-2}^{t-1} = \frac{e^{-\alpha(t-2)} - e^{-\alpha(t-1)}}{\alpha}$

3)

$h(t) = a(t)\cos(2\pi f_0 t)$ $\qquad$ $a(t) = 2b\,\mathrm{sinc}^2(bt)\cos(\pi b t)$

Disegna risposta armonica.

$$H(f) = A * F[\cos(2\pi f_0 t)] = A(f) * \left[\tfrac{1}{2}\delta(f-f_0) + \tfrac{1}{2}\delta(f+f_0)\right] = \tfrac{1}{2}A(f-f_0) + \tfrac{1}{2}A(f+f_0)$$

$$A(f) = F[2b\,\mathrm{sinc}^2(bt)\cos(\pi b t)]$$

$$A(f) = F[2b\,\mathrm{sinc}^2(bt)] * F[\cos(\pi b t)] = F[2b\,\mathrm{sinc}^2(bt)] * \left[\tfrac{1}{2}\delta\left(f-\tfrac{b}{2}\right) + \tfrac{1}{2}\delta\left(f+\tfrac{b}{2}\right)\right]$$

$$F[2b\,\mathrm{sinc}^2(bt)] = 2\,\Delta\left(\frac{f}{b}\right)$$

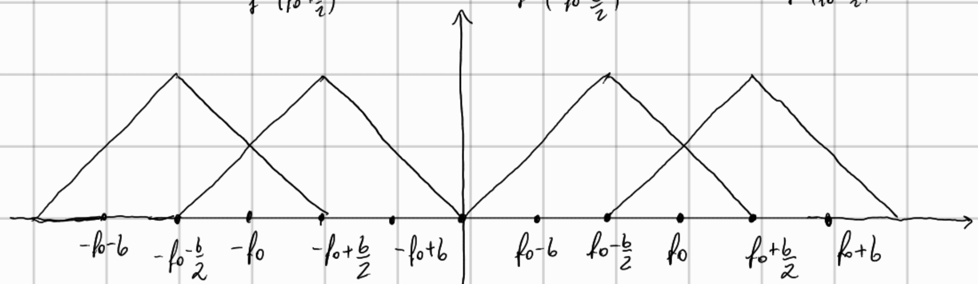
Trasf. notevole con prop. di simmetria $\left(F[S(t)] = S(-f)\right)$ e parità:

$$A(f) = \Delta\left(\frac{f-\frac{b}{2}}{b}\right) + \Delta\left(\frac{f+\frac{b}{2}}{b}\right) = \Delta\left(\frac{f}{b}-\frac{1}{2}\right) + \Delta\left(\frac{f}{b}+\frac{1}{2}\right)$$

$$H(f) = \frac{1}{2}\Delta\left(\frac{f-f_0}{b}-\frac{1}{2}\right) + \frac{1}{2}\Delta\left(\frac{f+f_0}{b}-\frac{1}{2}\right) + \frac{1}{2}\Delta\left(\frac{f-f_0}{b}+\frac{1}{2}\right) + \frac{1}{2}\Delta\left(\frac{f+f_0}{b}+\frac{1}{2}\right)$$

$\frac{1}{b}\left(f-f_0-\frac{b}{2}\right)$ $\qquad$ $\frac{1}{b}\left(f+f_0-\frac{b}{2}\right)$ $\qquad$ $\frac{1}{b}\left(f-f_0+\frac{b}{2}\right)$ $\qquad$ $\frac{1}{b}\left(f+f_0+\frac{b}{2}\right)$

$f-\left(f_0+\frac{b}{2}\right)$ $\qquad$ $f-\left(-f_0+\frac{b}{2}\right)$ $\qquad$ $f-\left(f_0-\frac{b}{2}\right)$ $\qquad$ $f-\left(-f_0-\frac{b}{2}\right)$



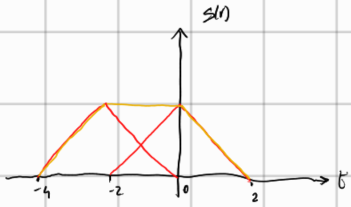
Sia $f_0 = 100\,\mathrm{kHz}$, $b = 4\,\mathrm{kHz}$ $\qquad$ $f_0-\frac{b}{2} = 98\,\mathrm{kHz}$ $\qquad$ $f_0+\frac{b}{2} = 102\,\mathrm{kHz}$ $\qquad$ $s(t) = \sin(200000\pi t + 0.5) +$
$+ 7\cos(201000\pi t)$

$f_1 = 100000$ Hz. Rientra nella banda. $f_2 = 100500$ Hz. Rientra nella banda. Non distorcente.

PROVA 2K19

1)

a) $s(t) = \Delta\left(\frac{t}{2}\right) + \Delta\left(\frac{t}{2}+1\right) = \Delta\left(\frac{t}{2}\right) + \Delta\left(\frac{t+2}{2}\right)$



$P_s = 0$, segnale di energia.

$\mathcal{E}_s = 2\cdot\left(\frac{b\cdot h}{3}\right) + 2\cdot 1 = 2\cdot\left(\frac{2\cdot 1}{3}\right) + 2 = \frac{4}{3} + 2 = \frac{10}{3}$

$S(f) = 2\,\mathrm{sinc}^2(2f) + 2\,e^{4\pi j f}\,\mathrm{sinc}^2(2f)$

b)

$s(t) = 3e^{-3t}\cos(2\pi f_0 t)\,u(t)$

$P_s = 0$, segnale di energia.

$$\mathcal{E}_s = \int_0^{+\infty} 9e^{-6t}\cos^2(2\pi f_0 t)\,dt = 9\int_0^{+\infty} e^{-6t}\left(\frac{1}{2} + \frac{1}{2}\cos(4\pi f_0 t)\right)dt = \frac{9}{2}\int_0^{+\infty} e^{-6t}\,dt + \frac{9}{2}\int_0^{+\infty} e^{-6t}\cos(4\pi f_0 t)\,dt =$$

$$= \frac{9}{2}\left.\frac{e^{-6t}}{-6}\right|_0^{+\infty} + \frac{9}{4}\int_0^{+\infty} e^{-6t}\left(e^{4\pi j f_0 t} + e^{-4\pi j f_0 t}\right)dt = \frac{9}{12} + \frac{9}{4}\int_0^{+\infty} e^{-(6-4\pi j f_0)t}\,dt + \frac{9}{4}\int_0^{+\infty} e^{-(6+4\pi j f_0)t}\,dt =$$

$$= \frac{3}{4} + \frac{9}{4}\left.\frac{e^{-(6-4\pi j f_0)t}}{-(6-4\pi j f_0)}\right|_0^{+\infty} + \frac{9}{4}\left.\frac{e^{-(6+4\pi j f_0)t}}{-(6+4\pi j f_0)}\right|_0^{+\infty} =$$

$$= \frac{3}{4} + \frac{9}{4(6-4\pi j f_0)} + \frac{9}{4(6+4\pi j f_0)} = \frac{3}{4} + \frac{54+54}{4(36+16\pi^2 f_0^2)} = \frac{108+48\pi^2 f_0^2+108}{36+16\pi^2 f_0^2}$$

$$S(f) = F\left[3e^{-3t}u(t)\right] * F\left[\cos(2\pi f_0 t)\right] =$$

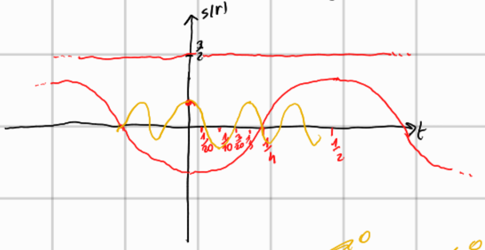
$$= \left(\frac{3}{3+2\pi j f}\right) * \left(\frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)\right) =$$

$$= \frac{3}{2(3+2\pi j(f-f_0))} + \frac{3}{2(3+2\pi j(f+f_0))} = \frac{9+6\pi j f + 9 + 6\pi j f}{2(3+2\pi j(f-f_0))(3+2\pi j(f+f_0))} = \frac{18+12\pi j f}{2(9-4\pi^2(f^2-f_0^2)+12\pi j f)}$$

c)

$$s(t) = 1 - \cos(2\pi t) + \cos^2(5\pi t) = 1 - \cos(2\pi t) + \frac{1}{2} + \frac{1}{2}\cos(10\pi t) = \frac{3}{2} - \cos(2\pi t) + \frac{1}{2}\cos(10\pi t)$$

$\mathcal{E}_s = \infty$, segnale di potenza.



Periodo minimo $= 1$

$\frac{1}{2} + \frac{1}{2}\cos(4\pi t) \to 0$ $\quad$ $\frac{1}{8} + \frac{1}{8}\cos(20\pi t) \to 0$

$$P_s = \frac{1}{T}\int_0^T |s(t)|^2 dt = \int_0^1 \left(\frac{3}{2} - \cos(2\pi t) + \frac{1}{2}\cos(10\pi t)\right)^2 dt = \int_0^1 \frac{9}{4}\,dt + \int_0^1 \cos^2(2\pi t)\,dt + \frac{1}{4}\int_0^1 \cos^2(10\pi t)\,dt +$$

$$- 3\int_0^1 \cancel{\cos(2\pi t)}\,dt + \frac{3}{2}\int_0^1 \cancel{\cos(10\pi t)}\,dt - \int_0^1 \cancel{\cos(2\pi t)\cos(10\pi t)}\,dt \quad \to 0$$

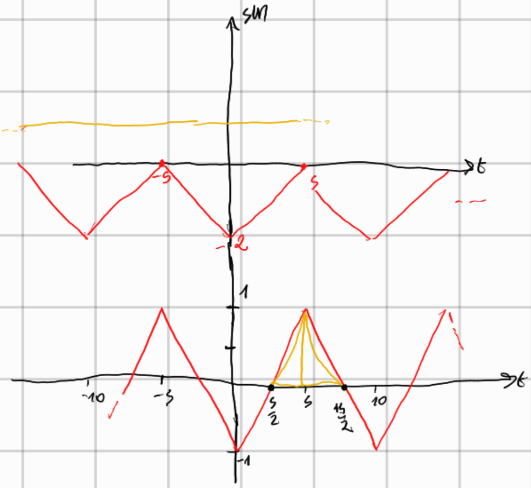
Prostaferesi, 1 è multiplo dei periodi $\to \int_0^1 f(t)\,dt = 0$

$$= \frac{9}{4} + \frac{1}{2} + \frac{1}{8} = \frac{18}{8} + \frac{4}{8} + \frac{1}{8} = \frac{23}{8}.$$

$$S(f) = \frac{3}{2}\delta(f) - \frac{1}{2}\delta(f-1) - \frac{1}{2}\delta(f+1) + \frac{1}{4}\delta(f-5) + \frac{1}{4}\delta(f+5)$$

d)

$$s(t) = 1 - 2\sum_{k=-\infty}^{+\infty} \Delta\left(\frac{t-10k}{5}\right)$$

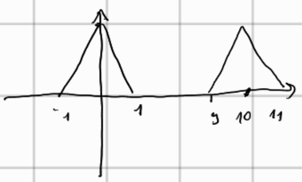


$\mathcal{E}_s = \infty$

$$P_s = \frac{1}{T_0}\int_0^{T_0} |s(t)|^2 dt = \frac{1}{10}\cdot 4\left(\frac{5}{2}\cdot {}^{1}\!/_{3}\right) =$$

$$= \frac{1}{3}$$

Sia $f(t) = 2\sum_{k=-\infty}^{+\infty} \Delta(t - 10k)$



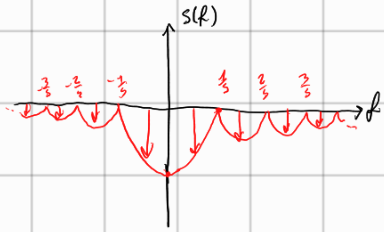
$$F(f) = \frac{2}{10}\cdot \text{sinc}^2(f)\sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{10}\right)$$

allora, se $f_2(t) = 2\sum_{k=-\infty}^{+\infty} \Delta\left(\frac{t-10k}{5}\right) = f\left(\frac{t}{5}\right)$

$$F_2(f) = \frac{2\cdot 5}{10}\,\text{sinc}^2(5f)\sum_{k=-\infty}^{+\infty} \delta\left(5\left(f - \frac{k}{10}\right)\right)$$

Quindi, $S(f) = \delta(f) - \text{sinc}^2(5f) \sum_{k=-\infty}^{+\infty} \delta\left(5\left(f - \frac{k}{10}\right)\right)$

$$S(f) = -\,\text{sinc}^2(5f) \sum_{\substack{k=-\infty \\ k\neq 0}}^{+\infty} \delta\left(5\left(f - \frac{k}{10}\right)\right)$$



MISTAKE:

$$s(t) = -2\,\Lambda\left(\frac{t}{3}\right) + \Pi\left(\frac{t}{10}\right)$$

$$S(t) = \sum_{k=-\infty}^{+\infty} s(t-10k) = s(t) * \sum_{k=-\infty}^{+\infty} \delta(t-10k)$$

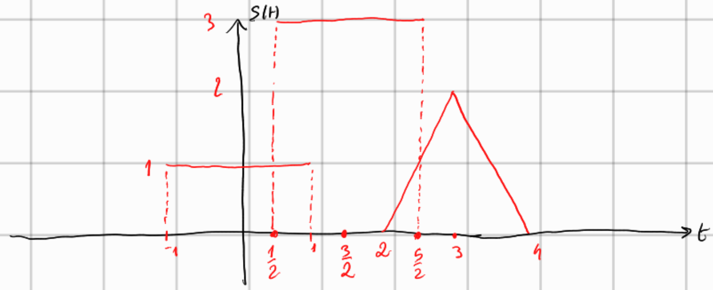
Prova 2K22

1)

a) $s(t) = \Pi\left(\frac{t}{2}\right) + 3\Pi\left(\frac{2t-3}{2}\right) + 2\Delta(t-3)$

$\downarrow$

$3\Pi\left(t-\frac{3}{2}\right)$



$$\mathcal{E}_s = \frac{3}{2}\cdot 1 + \frac{1}{2}\cdot 16 + 1\cdot 9 + \left(\frac{1^2\cdot\frac{1}{2}}{3} + (3+1)\cdot\frac{1}{2}\cdot 3\right) +$$

$$+\left(\frac{\frac{1}{2}\cdot 1^2}{3} + (1+1)\cdot\frac{1}{2}\cdot 1\right) + \frac{1\cdot 4}{3} =$$

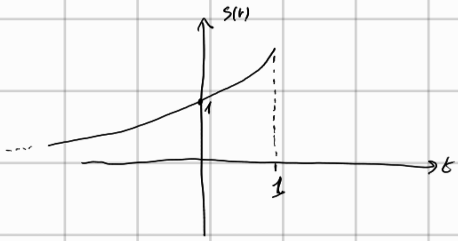
$$= \frac{3}{2} + 8 + 9 + \frac{37}{6} + \frac{7}{6} + \frac{4}{3} = \frac{9+48+54+37+7+8}{6} = \frac{163}{6}$$

$P_s = 0$, segnale di energia.

$$F[s(t)] = 2\,\mathrm{sinc}(2f) + 3e^{-3\pi jf}\,\mathrm{sinc}(f) + 2e^{-6\pi jf}\,\mathrm{sinc}^2(f)$$

b)

$$s(t) = e^{2t} u(-t+1)$$



$P_s = 0$, segnale di energia

$$\mathcal{E}_s = \int_{-\infty}^{1} e^{4t}\,dt = \left.\frac{e^{4t}}{4}\right|_{-\infty}^{1} = \frac{e^4}{4}$$

$$s(t) = e^{2t} u(-t+1)$$

$$S(f) = \int_{-\infty}^{1} e^{2t} e^{-2\pi j f t}\,dt = \int_{-\infty}^{1} e^{(2-2\pi j f)t}\,dt = \left.\frac{e^{(2-2\pi j f)t}}{(2-2\pi j f)}\right|_{-\infty}^{1} = \frac{e^{2-2\pi j f}}{(2-2\pi j f)} = e^2 \cdot \frac{e^{-2\pi j f}}{(2-2\pi j f)}$$

c) $s(t) = 2 - \cos\left(\frac{\pi}{2}t + \frac{1}{10}\right) + \cos^2(3\pi t) =$

$= 2 - \cos\left(\frac{\pi}{2}t + \frac{1}{10}\right) + \frac{1}{2} + \frac{1}{2}\cos(6\pi t) =$

$= \frac{5}{2} - \cos\left(\frac{\pi}{2}t + \frac{1}{10}\right) + \frac{1}{2}\cos(6\pi t)$

Segnale di potenza ⇒ $\mathcal{E} = \infty$. Periodo minimo è 4.

$$P_s = \frac{1}{T}\int_0^T |s(t)|^2 dt = \frac{1}{4}\int_0^4 \left(\frac{5}{2} - \cos\left(\frac{\pi}{2}t + \frac{1}{10}\right) + \frac{1}{2}\cos(6\pi t)\right)^2 dt =$$

$$= \frac{1}{4}\int_0^4 \frac{25}{4}\,dt + \frac{1}{4}\int_0^4 \left(\frac{1}{2} + \frac{1}{2}\cos\left(\pi t + \frac{1}{10}\right)\right)dt + \frac{1}{4}\int_0^4 \left(\frac{1}{8} + \frac{1}{8}\cos(12\pi t)\right)dt$$

Tutti i doppi prodotti fanno 0.

$$\frac{1}{4}\left[\frac{25}{4}\cdot 4\right] + \frac{1}{4}\int_0^4 \frac{1}{2}\,dt + \frac{1}{32}\cdot 4 = \frac{25}{4} + \frac{1}{2} + \frac{4}{32} = \frac{25}{4} + \frac{2}{4} + \frac{1}{8} = \frac{50+4+1}{4} = \frac{55}{4}$$

$$F[s(t)] = F\left[\frac{5}{2} - \cos\left(\frac{\pi}{2}t + \frac{1}{10}\right) + \frac{1}{2}\cos(6\pi t)\right] = \frac{5}{2}\delta(f) - \frac{1}{2}\left[\delta\left(f - \frac{1}{4}\right)e^{j\cdot\frac{1}{10}} + \delta\left(f + \frac{1}{4}\right)e^{-j\cdot\frac{1}{10}}\right] + \frac{1}{4}\left[\delta(f-3) + \delta(f+3)\right]$$

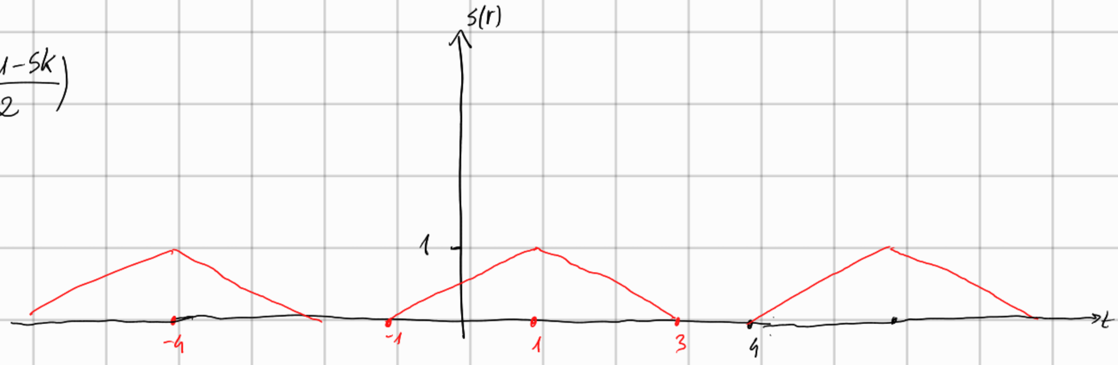
$f = \frac{1}{4}$

d)

$$s(t) = \sum_{k=-\infty}^{+\infty} \Delta\left(\frac{t-1-5k}{2}\right)$$

$k=0$: $\Delta\left(\frac{t-1}{2}\right)$

$k=1$: $\Delta\left(\frac{t-6}{2}\right)$



$$P = \frac{1}{5}\int_0^5 |s(t)|^2 dt = \frac{1}{5}\left(\frac{2}{3}+\frac{2}{3}\right) = \frac{4}{15}$$

$\delta(t) = \Delta\left(\frac{t-1}{2}\right)$ con periodo 5.

$$I(f) = 2e^{-2\pi j f}\,\mathrm{sinc}^2(2f)$$

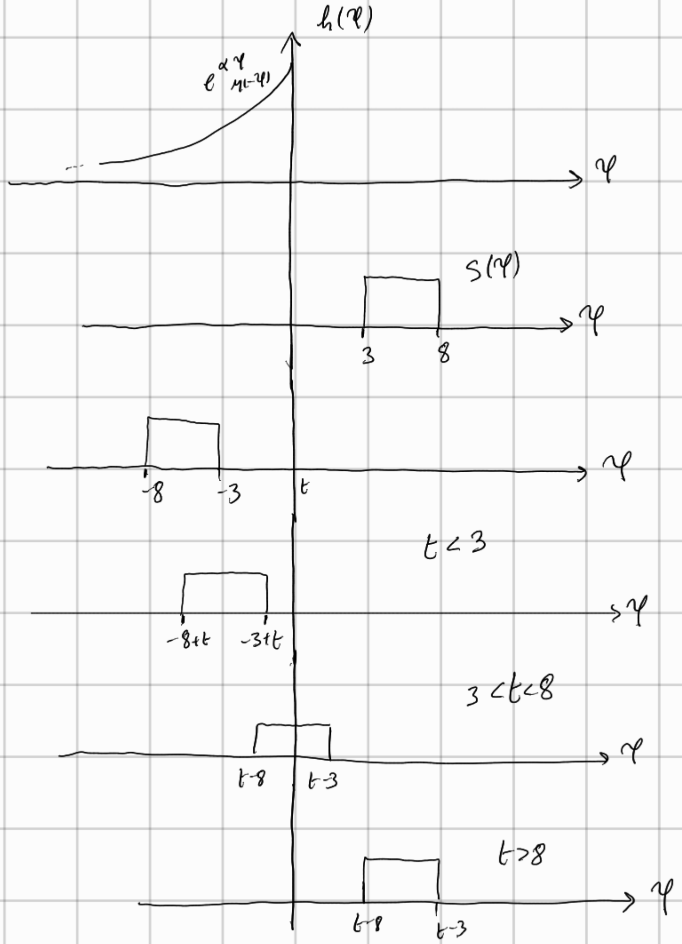
$$s(t) = \delta(t) * \sum_{k=-\infty}^{+\infty} \delta(t-5k)$$

$$S(f) = 2e^{-2\pi j f}\,\mathrm{sinc}^2(2f)\cdot\frac{1}{5}\sum_{k=-\infty}^{+\infty}\delta\left(f-\frac{k}{5}\right)$$

$$S(f) = \frac{2}{5}\sum_{k=-\infty}^{+\infty} e^{-2\pi j\frac{k}{5}}\,\mathrm{sinc}^2\left(\frac{2k}{5}\right)\delta\left(f-\frac{k}{5}\right)$$

2)

$h(t) = e^{\alpha t} u(-t)$ $\qquad$ $s(t) = u(t-3) - u(t-8)$



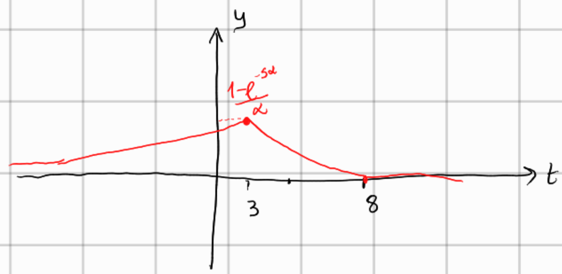
$y(t) = (h * s)(t)$

$s(-(\tau - t))$

$$y(t) = \int_{t-8}^{t-3} e^{\alpha\tau}\, d\tau = \frac{e^{\alpha(t-3)} - e^{\alpha(t-8)}}{\alpha} = \frac{e^{\alpha t}\left(e^{-3} - e^{-8}\right)}{\alpha}$$

$$y(t) = \int_{t-8}^{0} e^{\alpha\tau}\, d\tau = \frac{1 - e^{\alpha(t-8)}}{\alpha}$$

$y(t) = 0$

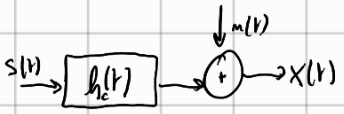


3)

$$h(t) = 2e^{-\alpha(t-1)} u(t-1) \qquad x(t) = \frac{1}{2} - \cos\pi t + 5\cos^2(5t-4)$$

$$x(t) = \frac{1}{2} - \cos(\pi t) + \frac{5}{2} + \frac{5}{2}\cos(10t-8)$$

$$H(f) = \frac{e^{-2\pi j f}}{\alpha + 2\pi j f}$$



$$x(t) = (h_c * s)(t) + m(t)$$

$$R_x(t,\varphi) = E\left[\left(\int_{-\infty}^{+\infty} h_c(\xi)s(t-\xi)d\xi + m(t)\right)\left(\int_{-\infty}^{+\infty} h_c(\alpha-\varphi)s(t-\alpha-\varphi)d\alpha + m(t-\varphi)\right)\right] =$$

$$= E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_c(\xi)h_c(\alpha-\varphi)s(t-\xi)s(t-\alpha-\varphi)\,d\alpha\, d\xi\right] +$$

$$+ E\left[\int_{-\infty}^{+\infty} h_c(\xi)s(t-\xi)d\xi \cdot m(t-\varphi)\right] + E\left[\int_{-\infty}^{+\infty} h_c(\alpha-\varphi)s(t-\alpha-\varphi)\,d\alpha\, m(t)\right] +$$

incorrelati   incorrelati

$$E\left[m(t)m(t-\varphi)\right] =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} R_s(\varphi-(\xi-\alpha))h_c(\xi)h(\alpha-\varphi)\,d\alpha\, d\xi + R_m(\varphi) =$$

$$\xi - \alpha = \beta$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} R_s(\varphi-\beta)h_c(\xi)h(\xi-\beta-\varphi)\,d\beta\, d\xi + R_m(\varphi) =$$

$$= \int_{-\infty}^{+\infty} R_s(\varphi-\beta)\,\tau_{h_c}(\beta+\varphi)\,d\beta + R_m(\varphi)$$

$$\beta + \varphi = \alpha \qquad \varphi - (\alpha - \varphi)$$

$$= \int_{-\infty}^{+\infty} R_s(2\varphi-\alpha)\,\tau_{h_c}(\alpha)\,d\alpha$$

$y(t) = x(t) + n(t)$

$$P_y = E[Y^2(t)] = E[(x(t)+n(t))^2] = E[x(t)^2] + E[n^2(t)] + 2E[x(t)n(t)]$$

$$R_y(t,\tau) \overset{\Delta}{=} E[(x(t)+n(t))(x(t-\tau)+n(t-\tau))] =$$

$$R_x(\tau) + R_n(\tau) + E[x(t)n(t-\tau)] + E[n(t)x(t-\tau)]$$

$$t-\tau=\alpha$$

$$R_{xy}(t,\tau) = E[x(t)y^*(t-\tau)] = (E[x^*(t)\,y(t-\tau)])^* = (E[y(\alpha)\,x(\tau+\alpha)])^* = R^*_{yx}(-\tau)$$

$$R_x(\tau) = E[x(t)x^*(t-\tau)] = (E[x^*(t)x(t-\tau)])^* = R^*_x(-\alpha)$$

$$E[Y(t)] = E\left[\int_{-\infty}^{+\infty} h(\xi)x(t-\xi)d\xi\right] =$$

$$R_y(t,\tau) = E\left[\int_{-\infty}^{+\infty} h(\xi)x(t-\xi)d\xi \int_{-\infty}^{+\infty} h^*(\eta)x^*(t-\tau-\eta)d\eta\right] =$$

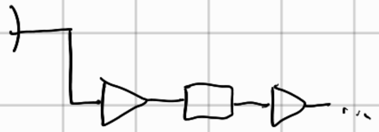
$$= E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(\xi)h^*(\eta)\,x(t-\xi)x^*(t-\tau-\eta)\,d\xi\,d\eta\right] =$$

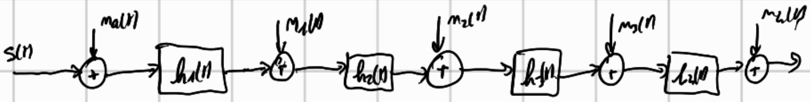
$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(\xi)h^*(\eta)\,R_x(\tau+\eta-\xi)\,d\xi\,d\eta$$

$$\xi-\eta = y$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(\xi)\underbrace{h^*(\xi-y)}\,R_x(\tau-y)\,d\xi\,dy$$

$$\int_{-\infty}^{+\infty} r_h(y)R_x(\tau-y)dy = (r_h * R_x)(\tau)$$



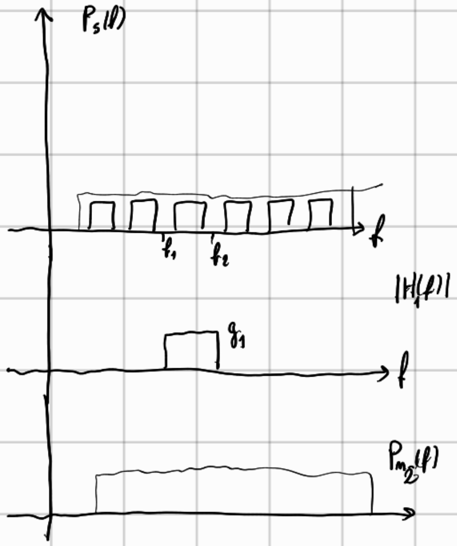


$$y(t) = \overbrace{(h_4 * h_3 * h_2 * h_1 * s)(t)}^{n_s(t)} + (h_4 * h_3 * h_2 * h_1 * m_0)(t) + (h_4 * h_3 * h_2 * m_1)(t) +$$

$$(h_4 * h_3 * m_2)(t) + (h_4 * m_3)(t) + m_4(t)$$

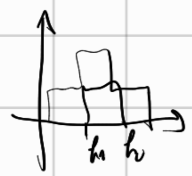
$$P_y(f) = |H_1(f)|^2|H_3(f)|^2|H_2(f)|^2|H_4(f)|^2 P_s(f) + |H_1(f)|^2|H_3(f)|^2|H_2(f)|^2|H_4(f)|^2 P_{m_4}(f) +$$
$$|H_1(f)|^2|H_3(f)|^2|H_2(f)|^2 P_{m_1}(f) + |H_1(f)|^2|H_3(f)|^2 P_{m_2}(f) + |H_1(f)|^2 P_{m_3}(f) + P_{m_4}(f)$$

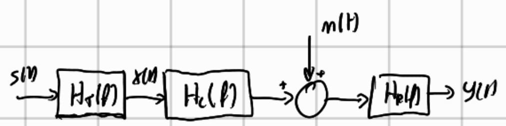
$$\frac{P_s}{P_m} = \frac{(g_1 \cancel{g_2 g_3 g_4})^2 M}{(g_1 \cancel{g_2 g_3 g_4})^2 M_0 + \frac{(\cancel{g_2 g_3 g_4})^2 M_1}{g_1^2} + \frac{(\cancel{g_3 g_4})^2 M_2}{(g_1 g_2)^2} + \frac{\cancel{g_4^2} M_3 + M_4}{(g_1 g_2 g_3)^2} \; g_1^2 g_2^2 g_3^2 g_4^2}$$



$$P_y(f) = |H_1(f)|^2|H_2(f)|^2 P_s(f) + |H_1(f)|^2|H_2(f)|^2 P_m(f) + |H_2(f)|^2 P_m(f) + P_m(f)$$

$$g_1^2 g_2^2$$





$$y(t) = (h_T * h_C * h_R * s)(t) + (h_R * m)(t)$$

$$r_s(t) = \alpha s(t - t_0)$$

$$|H_T(f)|\,|H_C(f)|\,|H_R(f)| = K$$

$$\frac{P_M P_x}{P_e} = \frac{\int_{-\infty}^{+\infty} |H_R(f)|^2 P_m(f)\,df \int_{-\infty}^{+\infty} |H_T(f)|^2 P_S(f)\,df}{k^2 P_S}$$

$$\left|\int_{-\infty}^{+\infty} x(r)\, y^*(r)\, dr\right|^2 \leq \int_{-\infty}^{+\infty} |x(r)|^2\, dr \int_{-\infty}^{+\infty} |y^*(r)|^2\, dr$$

$$\begin{cases} |H_R^o(f)|\, P_m(f)^{\frac{1}{2}} = \alpha\, |H_T^o(f)|\, P_S(f)^{\frac{1}{2}} \\ |H_R^o(f)|\,|H_c(f)|\,|H_T^o(f)| = k \end{cases}$$

$$\frac{K}{\alpha\, |H_c(f)|} \left(\frac{P_m(f)}{P_S(f)}\right)^{\frac{1}{2}} = |H_T^o(f)|^2$$

$$|H_R^o(f)| = \frac{k}{|H_c(f)|\,|H_T^o(f)|}$$

$$|H_R^o(f)|^2 \frac{|H_c(f)|\, K}{\alpha} \left(\frac{P_m(f)}{P_S(f)}\right)^{\frac{1}{2}} = K^2$$

$$\frac{\alpha K}{|H_c(f)|} \left(\frac{P_S(f)}{P_m(f)}\right)^{\frac{1}{2}}$$

1)

$$Y(t) = X(t)\cos(2\pi f_0 t) - X(t-\Delta)\sin^2(2\pi f_0 t)$$

$$X(t) \,/\, P_X(f) = \Delta\left(\frac{f}{B}\right) \qquad f_0 \gg 2B$$

$$E[Y(t)] = E\left[X(t)\cos(2\pi f_0 t)\right] - E\left[X(t-\Delta)\sin^2(2\pi f_0 t)\right] =$$

$$= 0 \text{ perché } E[X(t)] = 0$$

$$R_Y(t) = E\left[\left(X(t)\cos(2\pi f_0 t) - X(t-\Delta)\left(\frac{1}{2} - \frac{1}{2}\cos(4\pi f_0 t)\right)\right)\left(X(t-\tau)\cos(2\pi f_0(t-\tau)) - X(t-\tau-\Delta)\left(\frac{1}{2}-\frac{1}{2}\cos(4\pi f_0(t-\tau))\right)\right)\right]$$

$$= \overbrace{\cos(2\pi f_0 t)\cos(2\pi f_0(t-\tau))R_X(\tau)}^{1} - \overbrace{\cos(2\pi f_0 t)\left(\frac{1}{2}-\frac{1}{2}\cos(4\pi f_0(t-\tau))\right)R_X(\tau+\Delta)}^{2} +$$

$$- \underbrace{\cos(2\pi f_0(t-\tau))\left(\frac{1}{2}-\frac{1}{2}\cos(4\pi f_0 t)\right)R_X(\tau-\Delta)}_{3} + \underbrace{\left(\frac{1}{2}-\frac{1}{2}\cos(4\pi f_0 t)\right)\left(\frac{1}{2}-\frac{1}{2}\cos(4\pi f_0(t-\tau))\right)R_X(\tau)}_{4}$$

Ciclicamente stazionario su periodo $\frac{1}{f_0}$.

Calcolo $\bar{R}_Y(\tau) = f_0 \int_0^{T_0} R_Y(t,\tau)\,dt =$

1) $$f_0\int_0^{T_0} \frac{1}{2}\left(\cos(2\pi f_0(2t-\tau)) + \cos(2\pi f_0 \tau)\right)R_X(\tau)\,dt = \frac{\cos(2\pi f_0 \tau)R_X(\tau)}{2}$$

2) $$f_0\int_0^{T_0} \frac{1}{2}\left(\cos(2\pi f_0 t)\cos(4\pi f_0(t-\tau))R_X(\tau+\Delta)\right)dt =$$

$$= \frac{f_0}{4}\int_0^{T_0}\left(\cos(6\pi f_0 t - 4\pi f_0\tau) + \cos(-2\pi f_0 t + 4\pi f_0\tau)\right)R_X(\tau+\Delta)\,dt = 0$$

3) 0

4) $$\frac{1}{4}R_X(\tau) + \frac{1}{4}f_0\int_0^{T_0}\frac{1}{2}\left(\cos(4\pi f_0(2t-\tau)) + \cos(4\pi f_0\tau)\right)R_X(\tau)\,dt =$$

$$= \frac{1}{4}R_X(\tau) + \frac{1}{8}\cos(4\pi f_0\tau)R_X(\tau)$$

$$\bar{R}_Y(\tau) = \frac{1}{2}\cos(2\pi f_0\tau)R_X(\tau) + \frac{1}{4}R_X(\tau) + \frac{1}{8}\cos(4\pi f_0\tau)R_X(\tau)$$

$$\bar{P}_y(f) = \frac{1}{4} P_x(f-f_0) + \frac{1}{4} P_x(f+f_0) + \frac{1}{4} P_x(f) + \frac{1}{16} P_x(f-2f_0) + \frac{1}{16} P_x(f+f_0)$$

b)

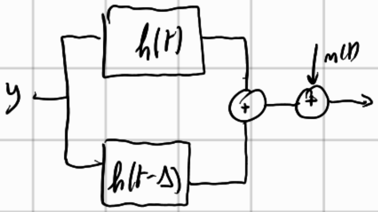
$$y(t) = x(t)\cos(2\pi f_0 t) - x(t-\Delta)\overbrace{\sin^2(2\pi f_0 t)}^{\frac{1}{2}-\frac{1}{2}\cos(4\pi f_0 t)}$$

$$Y(f) = \frac{1}{2}X(f-f_0) + \frac{X(f+f_0)}{2} - \frac{1}{2}e^{-2\pi j f\Delta}X(f) + \frac{1}{4}X(f-2f_0) + \frac{1}{4}X(f+2f_0)$$

2)

$$z(t) = (h_c * x)(t) + n(t)$$

$$h_c(t) = h(t) + h(t-\Delta) \qquad R_z(t,\tau) = ?$$



$$P_z(f) = |H_c(f)|^2 P_x(f) + P_n(f)$$

$$|H_c(f)|^2 = (H_1(f) + H_2(f))(H_1^*(f) + H_2^*(f)) =$$

$$= (H(f) + H(f)e^{-2\pi j f\Delta})(H^*(f) + H^*(f)e^{2\pi j f\Delta}) =$$

$$= |H(f)|^2 + |H(f)|^2 e^{-2\pi j f\Delta} + |H(f)|^2 e^{2\pi j f\Delta} + |H(f)|^2 =$$

$$2|H(f)|^2 + 2|H(f)|^2 \cos(2\pi f\Delta)$$

$$P_z(f) = (2|H(f)|^2 + 2|H(f)|^2 \cos(2\pi f\Delta)) P_x(f) + P_n(f)$$

$$R_z(\tau) = 2(r_h * R_x)(\tau) + 2((\delta(t-\Delta) + \delta(t+\Delta)) * r_h * R_x)(\tau) + R_n(\tau)$$

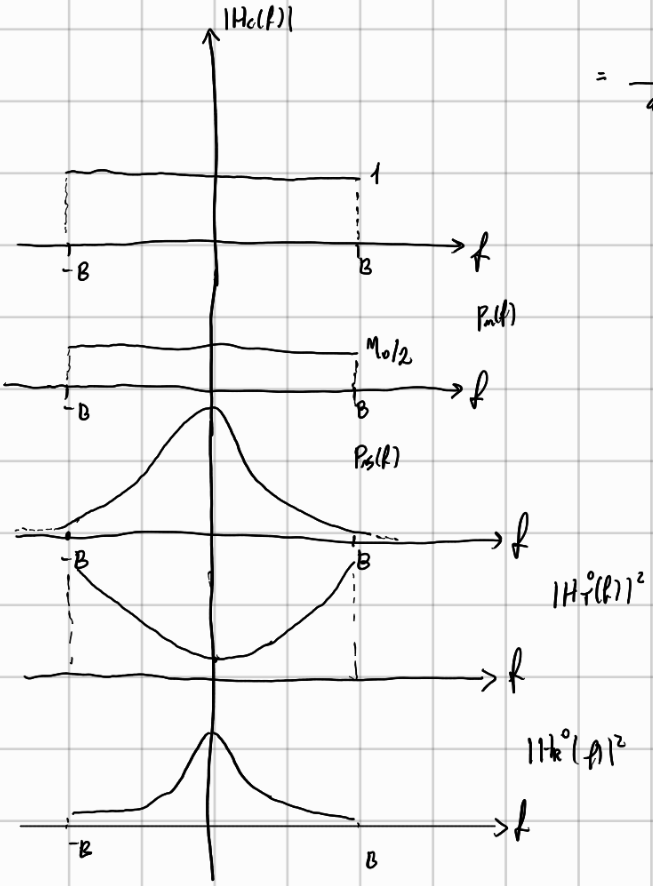
$$2(r_h * R_x)(\tau) + 2(r_h * R_x)(\tau-\Delta) + 2(r_h * R_x)(\tau+\Delta) + R_n(\tau)$$

3) $R_s(\tau) = e^{-a|\tau|}$

$R_s(\tau) = e^{-a\tau}u(\tau) + e^{a\tau}u(-\tau)$

$e^{-a(-\tau)}u(-(-\tau))$

$$= \frac{1}{a+2\pi jf} + \frac{1}{a-2\pi jf} = \frac{2a}{a^2+4\pi^2f^2}$$



$$|H_i^o(f)|^2 = \frac{\alpha}{|H_c(f)|}\left(\frac{P_n(f)}{P_s(f)}\right)^{\frac{1}{2}} = \alpha\left(\frac{N(a^2+4\pi^2f^2)}{4a}\right)^{\frac{1}{2}}$$

$$|H_R^o(f)|^2 = \beta\left(\frac{4a}{N(a^2+4\pi^2f^2)}\right)^{\frac{1}{2}}$$

3)

$$S(t) = A + x(t) + X(t)\cos(\pi B t + \Theta)$$

$$R_s(t,\tau) = E\Big[\big(A + X(t) + X(t)\cos(\pi B t + \Theta)\big)\big(A + X(t-\tau) + X(t-\tau)\cos(\pi B(t-\tau)+\Theta)\big)\Big] =$$

$$= A^2 + A\cdot 0 + A E\big[X(t-\tau)\cancel{\cos(B(t-\tau)+\Theta)}^{0}\big] + A\cdot 0 + R_x(\tau) + E\big[X(t)X(t-\tau)\cancel{\cos(\pi B(t-\tau)+\Theta)}^{0}\big] +$$

$$+ A E\big[X(t)\cancel{\cos(\pi B t + \Theta)}^{0}\big] + E\big[X(t)X(t-\tau)\cancel{\cos(\pi B t + \Theta)}^{0}\big] + E\big[X(t)X(t-\tau)\cos(\pi B t + \Theta)\cos(\pi B(t-\tau)+\Theta)\big] =$$

$$= A^2 + R_x(\tau) + \frac{R_x(\tau)}{2} E\big[\cos(\pi B(2t-\tau) + 2\Theta) + \cos(\pi B \tau)\big] =$$

$$= A^2 + R_x(\tau) + \frac{R_x(\tau)}{2}\cos(\pi B \tau)$$

$$P_s(f) = A^2\delta(f) + P_x(f) + \frac{1}{4}P_x\left(f - \frac{B}{2}\right) + \frac{1}{4}P_x\left(f + \frac{B}{2}\right)$$

4)

$$R_{y_1y_2}(\tau) = E[y_1(t)\, y_2^*(t-\tau)] = E\left[\int_{-\infty}^{+\infty} h_1(\xi)\, x_1(t-\xi)\, d\xi \int_{-\infty}^{+\infty} h_2^*(\eta)\, x_2^*(t-\tau-\eta)\, d\eta\right] =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)\, h_2^*(\eta)\, R_{x_1x_2}(\tau+\eta-\xi)\, d\xi\, d\eta =$$

$$\xi - \eta = y$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)\, h_2^*(\xi - y)\, R_{x_1x_2}(\tau - y)\, d\xi\, dy =$$

$$= \int_{-\infty}^{+\infty} r_{h_1h_2}(y)\, R_{x_1x_2}(\tau - y)\, dy = (r_{h_1h_2} * R_{x_1x_2})(\tau)$$