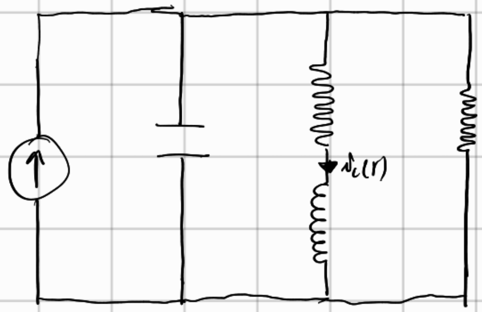
ALCUNE CONSIDERAZIONI:

- É possibile che una piccola parte degli esercizi sia scorretta
- Alcuni esercizi possono essere risolti in modi diversi, ma ugualmente corretti
- Gli esercizi sono svolti in ordine cronologico, quindi più si va in fondo nel file più sará probabile che con più esperienza alle spalle saranno corretti.

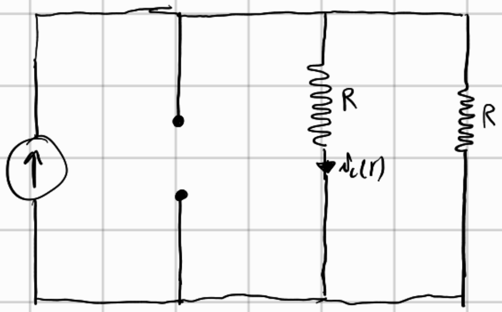
BUON LAVORO!



$L = 10\mu H \quad R = 2\Omega \quad C = 5\mu F$

$J(t) = 20\, u(-t)$ A

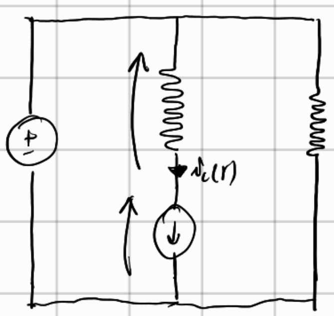
Se $t<0$, a regime ho stazionario:



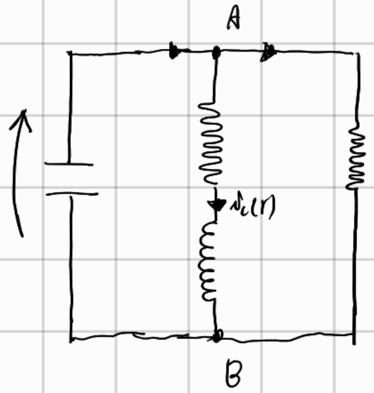
$$i_L(t) = \frac{J(t) \cdot R}{R+R} = 10A$$

$i_L(0^-) = i_L(0^+) = 10A$

$V_C(0^-) = i_L R = 20V$



$V_L(0^+) = V_C(0^-) - R i_L = 0V$



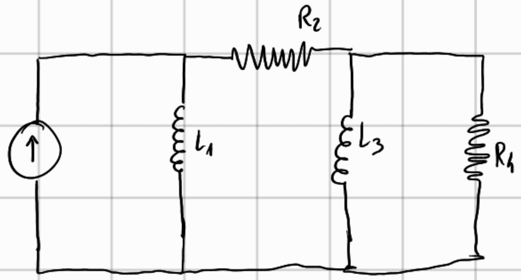
$$v_C = R i_L + L\frac{di_L}{dt}$$

$$i_L(t) + C\frac{dv_C}{dt} + \frac{v_C}{R} = 0$$

$$i_L(t) + CR\frac{di_L}{dt} + CL\frac{d^2 i_L}{dt^2} + i_L + \frac{L}{R}\frac{di}{dt} = 0$$

$$\begin{cases} \dfrac{d^2 i_L}{dt^2} + \left(\dfrac{R}{L} + \dfrac{1}{RC}\right)\dfrac{di_L}{dt} + \dfrac{2}{CL} i_L = 0 \\ i_L(0^+) = 10\,A \\ \left.\dfrac{di_L}{dt}\right|_0 = 0\,A \end{cases}$$

3)



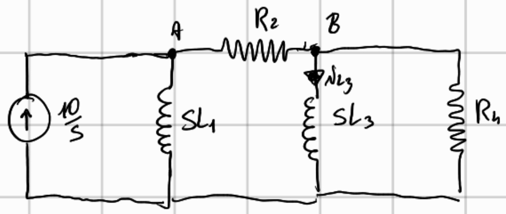
$J(t) = 10\,u(t)$ mA

$R_4 = 1\,\Omega \quad R_2 = 3\,\Omega$

$L_1 = L_3 = 2H$

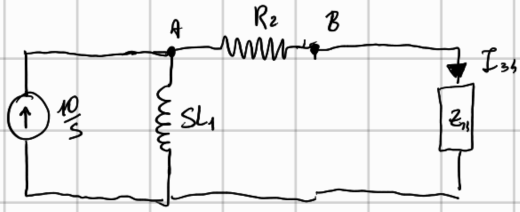
Tutto scarico per $t<0$

Rete di Laplace:



$$\Rightarrow Z_{34} = \left(\frac{1}{sL_3}+\frac{1}{R_4}\right)^{-1} = \left(\frac{R_4+sL_3}{R_4\, sL_3}\right)^{-1} = \frac{R_4 L_3 s}{R_4 + L_3 s}$$

⇓



$$I_{34} = \frac{J \cdot sL_1}{sL_1 + \frac{R_4 L_3 s}{R_4 + L_3 s}} = \frac{20}{2s + \frac{2s}{1+2s}} = \frac{20}{\frac{2s+4s^2+2s}{1+2s}} = \frac{20+40s}{4s^2+4s} = \frac{5+10s}{s^2+s}$$

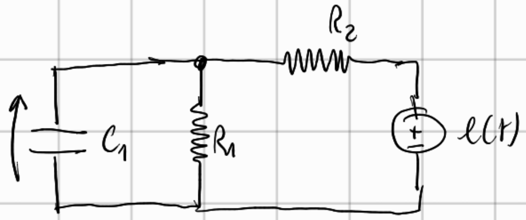
$$I_{L_3} = \frac{I_{34} \cdot R_4}{R_4 + sL_3} = \frac{5+10s}{s^2+s} \cdot \frac{1}{1+2s} = \frac{5\cancel{(1+2s)}}{(s^2+s)\cancel{(1+2s)}} = \frac{5}{s(s+1)}$$

Antitrasform:

$$\frac{5}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = \frac{As+A+Bs}{s(s+1)} \Rightarrow \quad A=5 \quad B=-5$$

$$\| \quad \frac{5}{s} - \frac{5}{s+1} \Rightarrow \quad i_{L_3}(t) = 5u(t) - 5e^{-t}u(t)$$

3)



$R_1 = R_2 = 10\,k\Omega$

$C = 10\,\mu F \quad V_1(0^-) = 1.5\,V$

$e(t) = 10\left(u(t) - u(t - 10ms)\right)\ V$

$$E(S) = \frac{10}{S} - \frac{10}{S} e^{-10^{-2}s}$$

$\Rightarrow$

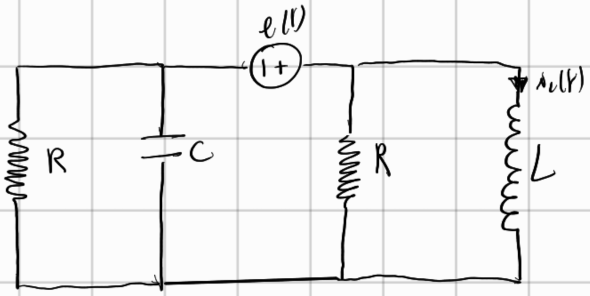
$$U_A\left(cs + \frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E(s)}{R_2} + c\cancel{s}\cdot\frac{V_c(0^-)}{\cancel{s}}$$

$$U_A\left(10^{-5}s + 2\cdot10^{-3}\right) = E\cdot10^{-3} + 10^{-5}\cdot1.5$$

$$U_A = \frac{10^{-3}E + 1.5\cdot10^{-5}}{2\cdot10^{-3} + 10^{-5}s} = \frac{E + 1.5\cdot10^{-2}}{2 + 10^{-3}s} =$$

$$= \frac{10}{S(2+10^{-3}s)} - \frac{10e^{-10^{-2}s}}{S(2+10^{-2}s)} + \frac{1.5\cdot10^{-2}}{2+10^{-3}s}$$

7)



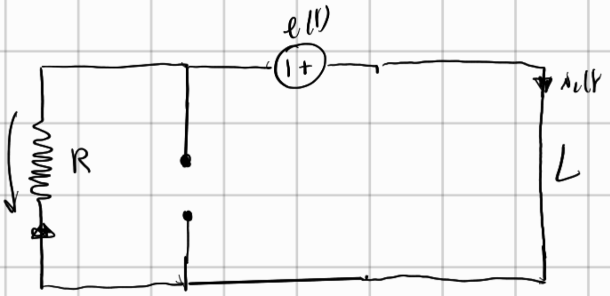
$$e(t) = \begin{cases} -1\,V & t<0 \\ 1\,V & t\geq 0 \end{cases}$$

$R = 10\,\Omega$

$L = 5\,mH$

$C = 100\,\mu F$

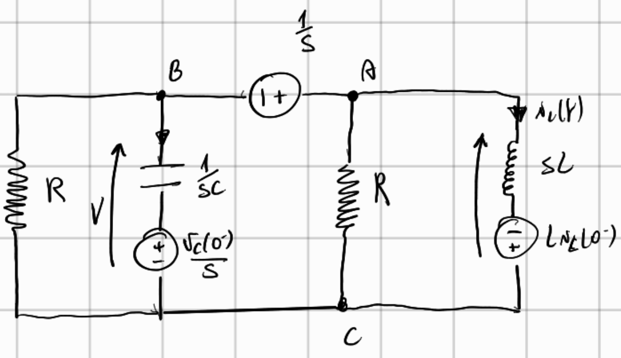
Se $t<0$ a regime:



$i_L = \frac{e(t)}{R} = -\frac{1}{10}\,A$ $\quad v_C = e(t) = -1\,V$

Condiz. iniziali.

Studio per $t\geq 0$ con Laplace:



$$U_C\left(\frac{1}{R}+sC+\frac{1}{sL}+\frac{1}{R}\right)+\frac{1}{s}\cdot\frac{1}{R} = -\frac{v_C(0^-)}{\cancel{s}}\cdot\cancel{s}C+\frac{i_L(0^-)}{s}$$

$$U_C\left(\frac{2}{R}+sC+\frac{1}{sL}\right) = -\frac{1}{10s}+C-\frac{1}{10s}$$

$$U_C\left(\frac{2}{10}+sC+\frac{1}{sL}\right) = -\frac{1}{5s}+C$$

$$U_C = \frac{\frac{-1+scs}{5s}}{\frac{1}{5}+sc+\frac{1}{sL}} = \frac{\frac{-1+scs}{5s}}{\frac{1}{sL}\left(L+s^2Lc+1\right)} = \frac{\frac{-1+5cs}{5\cancel{s}}\cdot \cancel{s}L}{s^2Lc+L+1} =$$

$$= \frac{\frac{-L+sLcs}{s}}{Lc\left(s^2+\frac{1}{c}+\frac{1}{Lc}\right)} = \frac{-L+sLcs}{sLc\left(s^2+\frac{1}{c}+\frac{1}{Lc}\right)} = \frac{-1+5cs}{5c\left(s^2+\frac{1}{c}+\frac{1}{Lc}\right)}$$

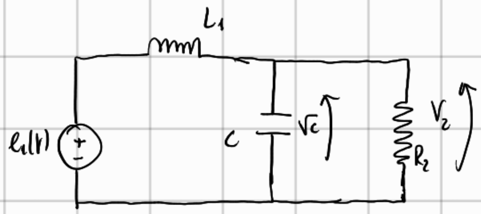
$R = 10\Omega$

$L = 5mH$

$C = 100\mu F$

$$s^2+\frac{1}{c}+\frac{1}{Lc} = s^2+10^4$$

3)



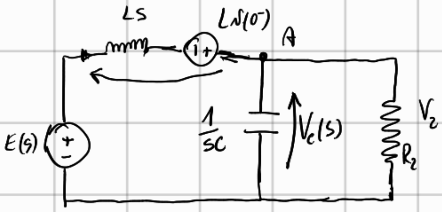
$R_1 = 100\,\Omega$ $\quad R_2 = 50\,\Omega$

$C = 10\,\mu F$ $\quad v_c(0^-) = 0V$

$L = 100\,\mu H$ $\quad i_L(0^-) = 10\,mA$

$e(t) = 10u(t)$ $\qquad$ Trovare v_2

TRASFORMO CON LAPLACE



$$U_A\left(sC + \frac{1}{R_2} + \frac{1}{Ls}\right) = \left(L\,i(0^-) + E(s)\right)\cdot\frac{1}{Ls}$$

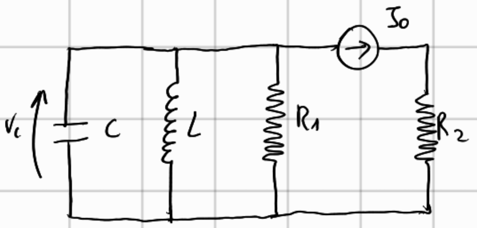
$$U_A\left(\frac{R_2LCs^2 + Ls + R_2}{R_2Ls}\right) = \frac{i(0^-)}{s} + \frac{10}{Ls^2}$$

$5\cdot 10^{-3}$

$$U_A = \frac{\frac{10^{-2}+10}{s(1+10^{-4}s)}\cdot 50\cdot 10^{-4}s}{10^{-9}s^2 + 10^{-4}s + 50} = \frac{\frac{5\cdot 10^{-5} + 5\cdot 10^{-2}}{1+10^{-4}s}}{10^{-9}s^2 + 10^{-4}s + 50} =$$

$$\underline{5\cdot 10^{-5} + 5\cdot 10^{-2}}$$

3)



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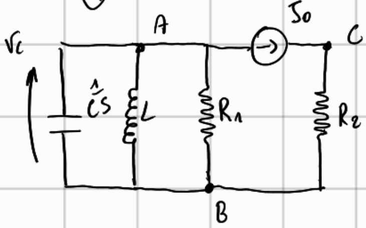


$R_1 = R_2 = 10\,\Omega$

$L = 10\,mH$

$C = 1\,\mu F$

$J(t) = 10(u(t) - u(t-1))\,mA$

$$U_A\left(Cs + \frac{1}{Ls} + \frac{1}{R_1}\right) = -J_0$$

$$U_C\left(\frac{1}{R_2}\right) = J_0$$

$$U_A = \frac{-J_0}{Cs + \frac{1}{Ls} + \frac{1}{R_1}}$$

$$H^{J_0}_{v_C}(s) = \frac{-1}{Cs + \frac{1}{Ls} + \frac{1}{R_1}} = \frac{-1}{\frac{1}{Ls}\left(CLs^2 + 1 + \frac{Ls}{R_1}\right)} = \frac{-Ls}{CLs^2 + \frac{L}{R_1}s + 1} = \frac{-s}{C\left(s^2 + \frac{1}{R_1 C}s + \frac{1}{LC}\right)}$$

$$s^2 + \frac{1}{R_1 C}s + \frac{1}{LC} = 0$$

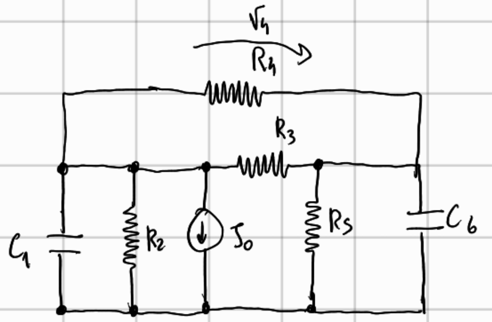
POLI: $s_1 = -11270 = -1.1 \cdot 10^4\ Hz$

$s_2 = -88729 = -8.8 \cdot 10^4\ Hz$

$$J(s) = \frac{10}{s} - \frac{10}{s}e^{-10^{-3}s}$$

$$V_C(s) = H^{J_0}_{v_C}(s)\,J(s) = \frac{10e^{-10^{-3}s} - 10}{C\left(s^2 + \frac{1}{R_1 C}s + \frac{1}{LC}\right)}$$

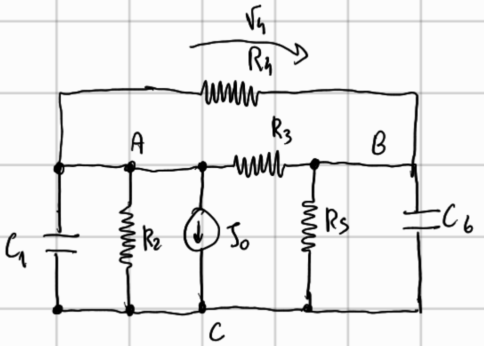
3)



$R_2 = 1\Omega \quad R_3 = 2\Omega \quad R_4 = R_5 = 3\Omega$

$C_1 = 3\,mF \quad v_{C_1}(0^-) = 1V \quad C_6 = 1\,mF \quad v_{C_6}(0^-) = 0V$

$J(t) = 10\,u(t)\ mA$



Condiz. iniziali spente per trovare f. di trasferimento

$$\begin{cases} U_A\left(\frac{1}{R_4} + \frac{1}{R_3} + sC_1 + \frac{1}{R_2}\right) - U_B\left(\frac{1}{R_3} + \frac{1}{R_4}\right) = -J_0 \\ U_B\left(\frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_5} + sC_6\right) - U_A\left(\frac{1}{R_3} + \frac{1}{R_4}\right) = 0 \end{cases}$$

$$\begin{cases} U_A\left(\frac{1}{3} + \frac{1}{2} + 1 + 3\cdot 10^{-3}s\right) - U_B\left(\frac{1}{3} + \frac{1}{2}\right) = -J_0 \\ U_B\left(\frac{1}{3} + \frac{1}{3} + 1 + 10^{-3}s\right) - U_A\left(\frac{1}{2} + \frac{1}{3}\right) = 0 \end{cases}$$

$$\begin{cases} U_A\left(\frac{11}{6} + 3\cdot 10^{-3}s\right) - U_B\cdot\frac{5}{6} = -\frac{10}{s} \\ U_B\left(\frac{5}{3} + 10^{-3}s\right) - U_A\frac{5}{6} = 0 \end{cases}$$

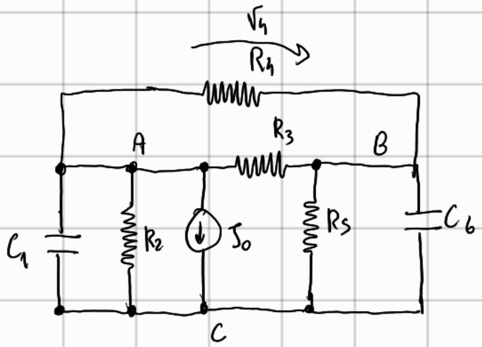
$$U_A = \frac{6}{5}U_B\left(\frac{5}{3} + 10^{-3}s\right) \Rightarrow U_A = 2U_B + \frac{3}{25}\cdot 10^{-2}U_B s$$

$$\left(2U_B + \frac{3}{2500}U_B s\right)\cdot\left(\frac{11}{6} + \frac{3}{1000}s\right) - U_B\frac{5}{6} = -\frac{10}{s}$$

$$\frac{11}{3}U_B + \frac{3}{500}U_B s + \frac{11}{5000}U_B s + \frac{9U_B}{25\cdot 10^5}s^2 - \frac{5}{6}U_B = -\frac{10}{s}$$

$$\frac{17}{6}U_B + \frac{41}{5000}U_B s + \frac{9}{25\cdot 10^5}U_B s^2 = -\frac{10}{s}$$

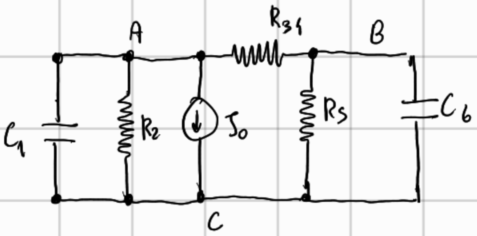
$$U_B = \frac{-10}{s\left(3.6\cdot 10^{-6}s^2 + 8.2\cdot 10^{-3}s + 2.83\right)}$$



$\Downarrow$

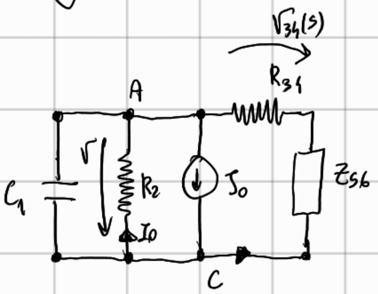
$$R_{34} = \left(\frac{1}{R_3}+\frac{1}{R_4}\right)^{-1} = \frac{6}{5}\,\Omega$$

$$V_{34}(s) = V_4(s)$$



$$Z_{56} = \left(\frac{1}{R_5} + sC_6\right)^{-1} = \left(\frac{1}{3} + 3\cdot 10^{-3}s\right)^{-1} = \frac{1}{\frac{1}{3} + C_6 s}$$

$\Downarrow$



$$I_0(s) = \frac{J_0\cdot 1/R_2}{\frac{1}{Z_{56}} + \frac{1}{R_2} + sC_1}$$

$$V(s) = I_0 R_2 = \frac{J_0}{\frac{1}{Z_{56}} + \frac{1}{R_2} + sC_1} = \frac{J_0}{\frac{1}{3} + (C_6+C_1)s + 1}$$

$$V_{34}(s) = \frac{V(s)\cdot R_{34}}{R_{34}+Z_{56}} = \frac{V(s)\,1/Z_{56}}{\frac{1}{Z_{56}} + \frac{1}{R_{34}}} = \frac{J_0\cdot\left(\frac{1}{3}+C_6 s\right)}{\left(\frac{4}{3}+(C_6+C_1)s\right)\cdot\left(\underbrace{\frac{5}{6}+\frac{1}{3}}_{\frac{7}{6}}+C_6 s\right)} = \frac{J_0\left(\frac{1}{3}+C_6 s\right)}{\frac{14}{9} + \frac{7}{6}(C_6+C_1)s + \frac{4}{3}C_6 s + C_6(C_1+C_6)s^2} =$$

$$\frac{J_0\left(\frac{1}{3}+C_6 s\right)}{\frac{14}{9}+\frac{7}{6}(C_6+C_1)s+\frac{4}{3}C_6 s+C_6(C_1+C_6)s^2} =$$

$$x = -750 - j\,50\sqrt{1335}$$

$$x = -750 + j\,50\sqrt{1335}$$

$$= \frac{J_0\left(\frac{1}{3}+10^{-3}s\right)}{4\cdot 10^{-6}s^2+6\cdot 10^{-3}s+1.56} =$$

$$= \frac{\left(\frac{4\cdot 10^{6}}{3}+4\cdot 10^{3}s\right)J_0}{s^2+1.5\cdot 10^{3}s+3.9\cdot 10^{6}}$$

$$H_{V_C}^{J_0}(s) = \frac{1.3\cdot 10^{6}+4\cdot 10^{3}s}{s^2+1.5\cdot 10^{3}s+3.9\cdot 10^{6}}$$

Se metto condizioni iniziali, il calcolo risulta essere lo stesso per la $H^{J_0}(s)$, perché la struttura circuitale rimane uguale. ⇒ CAMBIA solo J_0 che diventa generatore impulsivo.

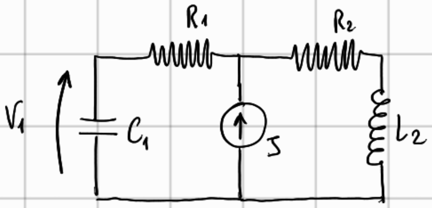
$$V_C(s) = H_{V_C}^{J_0}(s)\cdot\left(J_0 + C\,V_{C_1}(0^-)\right)$$

$$V_C(s) = \frac{1.3\cdot 10^{6}}{s\left(s^2+1.5\cdot 10^{3}s+3.9\cdot 10^{6}\right)} + \frac{4\cdot 10^{3}s}{\left(s^2+1.5\cdot 10^{3}s+3.9\cdot 10^{6}\right)} +$$

$$+ \frac{3.9\cdot 10^{3}+12s}{s^2+1.5\cdot 10^{3}s+3.9\cdot 10^{6}} \Rightarrow$$

$$V_C(s) = \frac{1.3\cdot 10^{6}}{s\left(s^2+1.5\cdot 10^{3}s+3.9\cdot 10^{6}\right)} + \frac{3.9\cdot 10^{3}+(12+4\cdot 10^{3})s}{\left(s^2+1.5\cdot 10^{3}s+3.9\cdot 10^{6}\right)}$$

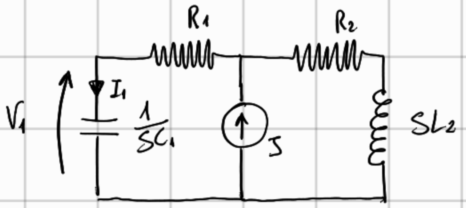
3)



$R_1 = 10\,\Omega$

$L_2 = 100\,mH$

$J(t) = 10u(t)\ mA$



$$I_1(s) = \frac{J_0 \cdot (R_2 + sL_2)}{R_2 + sL_2 + R_1 + \frac{1}{sC_1}}$$

$$V_1 = I_1(s) \cdot \frac{1}{sC_1} = \frac{J_0 (R_2 + sL_2) \cdot \frac{1}{sC_1}}{R_1 + R_2 + sL_2 + \frac{1}{sC_1}}$$

$$H^{J_0}_{V_1}(s) = \frac{(R_2 + sL_2) \cdot \cancel{\frac{1}{sC_1}}}{\cancel{\frac{1}{sC_1}} (R_1C_1s + R_2C_1s + C_1L_2s^2 + 1)} = \frac{R_2 + sL_2}{C_1L_2\left(s^2 + \frac{R_1}{L_2}s + \frac{R_2}{L_2}s + \frac{1}{C_1L_2}\right)} =$$

$$= \frac{\frac{R_2}{C_1L_2} + \frac{s}{C_1}}{\left(s^2 + \frac{R_1+R_2}{L_2}s + \frac{1}{C_1L_2}\right)}$$

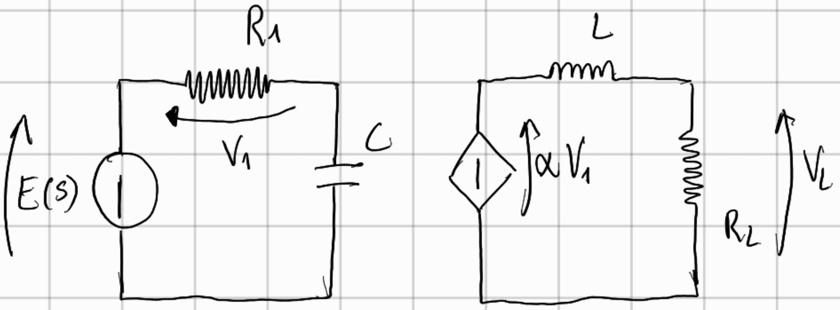
Polinomio: $s^2 + \frac{R_1+R_2}{L_2}s + \frac{1}{C_1L_2}$

Poli: 100Hz
3000Hz

$$\begin{cases} \frac{R_1+R_2}{L_2} = 3100 \\ \frac{1}{C_1L_2} = 300000 \end{cases} \Rightarrow \begin{cases} 10 + R_2 = 310 \\ \frac{1}{C_1} = 30000 \end{cases} \Rightarrow \begin{matrix} R_2 = 300\ \Omega \\ C_1 = \frac{1}{30000}F = 3.33 \cdot 10^{-5}F = 33.3\,\mu F \end{matrix}$$

$$V_1(s) = \frac{\frac{R_2}{C_1L_2} + \frac{s}{C_1}}{\left(s^2 + \frac{R_1+R_2}{L_2}s + \frac{1}{C_1L_2}\right)} \cdot \frac{10}{s} = \frac{\frac{10R_2}{sC_1L_2} + \frac{10}{C_1}}{s^2 + \frac{R_1+R_2}{L_2}s + \frac{1}{C_1L_2}}$$

4)



$R_1 = 100\,\Omega$

$R_L = 50\,\Omega$

$C = 10\,\mu F$

$L = 1\,mH$

$\alpha = 0{,}5$

$e(t) = 10\sqrt{2}\,\sin(100t)\,u(t)$

$H^{E}_{V_L}(S)$ da trovare

$$V_L(s) = \frac{\alpha V_1 R_L}{R_L + sL} \quad \text{con } V_1 = \frac{E(S)R_1}{R_1 + \frac{1}{CS}} = \frac{E(S)R_1}{\frac{R_1CS+1}{CS}} = \frac{E(S)R_1CS}{R_1CS+1}$$

$$V_1 = \frac{E(S)R_1CS}{R_1CS+1}$$

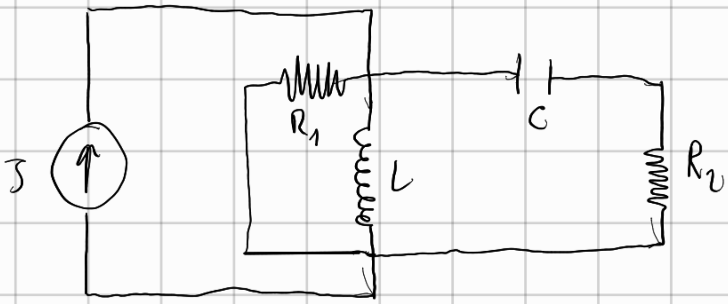
$$i = C\frac{dV}{dt} \Rightarrow F = \frac{[s]}{[\Omega]}$$

$$V_L(s) = \frac{\alpha R_1R_2\,C E(s)\,s}{(R_L+sL)(1+R_1CS)} = \frac{\alpha R_1R_2 C E(S) s}{R_L + R_1R_LCS + R_1LCs^2 + sL} =$$

$$\frac{\alpha R_1R_2CSE(S)}{R_1LCS^2 + (R_1R_2C+L)s + R_L} = \frac{\alpha \cancel{R_1}R_L\cancel{C}E(S)s}{\cancel{R_1}L\cancel{C}\left(S^2+\left(\frac{R_2}{L}+\frac{1}{LC}\right)s+\frac{R_L}{R_1LC}\right)} =$$

$$= \frac{\alpha R_L E(S)\cdot s}{L\left(s^2+\left(\frac{R_2}{L}+\frac{1}{LC}\right)s+\frac{R_L}{R_1LC}\right)} \Rightarrow H^{E}_{V_L}(S) = \frac{\frac{\alpha R_L s}{L}}{\left(s^2+\left(\frac{R_2}{L}+\frac{1}{LC}\right)s+\frac{R_L}{R_1LC}\right)}$$

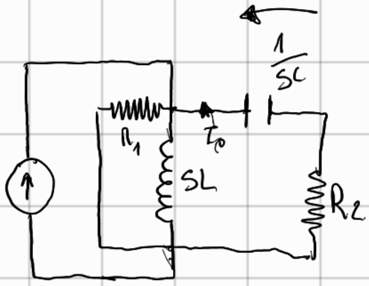
$$E(s) = \frac{10\sqrt{2} \cdot 100}{s^2 + 100^2}$$



$R_1 = 1\Omega \quad R_2 = 2\Omega \quad C = 10\mu F$

$L = 2mH$

$J(t) = 5\delta(t)\ A$



$$\left(R_2 + \frac{1}{SC}\right)^{-1} = \left(\frac{SCR_2+1}{SC}\right)^{-1} = \frac{SC}{1+R_2SC}$$

$$I_0(S) = \frac{J\cdot\left(\frac{SC}{1+R_2CS}\right)}{\frac{SC}{1+R_2CS} + \frac{1}{R_1} + \frac{1}{SL}} = \frac{JSC + \frac{J}{R_2}}{\frac{1}{SL}\left(CLS^2 + \frac{SL}{R} + 1\right)} = \frac{S^2LCJ + \frac{JSL}{R_2}}{CL\left(S^2 + \frac{S}{RC} + \frac{1}{CL}\right)} = \frac{S^2J + \frac{JS}{RC}}{S^2 + \frac{S}{RC} + \frac{1}{CL}}$$

$\frac{1}{R} = \frac{3}{2}$ $$V_C(S) = \frac{1}{SC} I_0 = \frac{\frac{SJ}{C} + \frac{J}{RC^2}}{S^2 + \frac{S}{RC} + \frac{1}{CL}} = \frac{\frac{J}{C}S + \frac{J}{RC^2}}{S^2 + \frac{S}{RC} + \frac{1}{CL}}$$

POLINOMIO:

$$S^2 + 150000S + 5\cdot 10^7 = 0$$

$S_1 = -334\ Hz$

$S_2 = -149666\ Hz$

$$V_c(s) = \frac{\frac{I}{C}s + \frac{I}{RC^2}}{s^2 + \frac{s}{RC} + \frac{1}{CL}} = \frac{5\cdot 10^5 s + \frac{5}{2}\cdot 10^{10}}{(s+334)(s+149666)} = \frac{5\cdot 10^5 s + 2{,}5\cdot 10^9}{(s+334)(s+149666)}$$

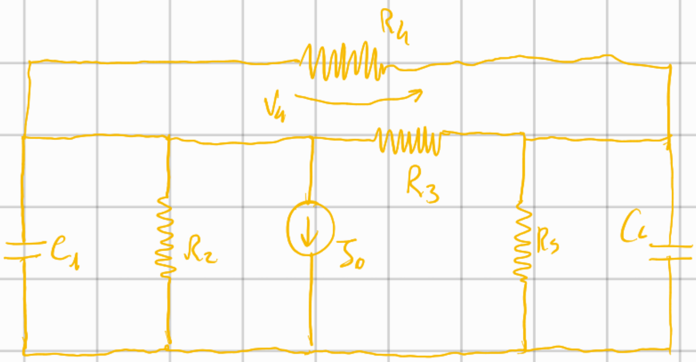
$$V_c(s) = \frac{A}{(s+334)} + \frac{B}{(s+149666)}$$

$$A = \lim_{s\to -334} V_c(s)(s+334) = 166260$$

$$B = \lim_{s\to -149666} V_c(s)\cdot(s+149666) = 333706$$

$$V_c(s) = \frac{166260}{s+334} + \frac{33706}{s+149666}$$

$$V(t) = 166260\, e^{-334t} + 33706\, e^{-149666t}$$



$R_2 = 1\,\Omega \quad R_3 = 2\,\Omega \quad R_4 = R_5 = 3\,\Omega$

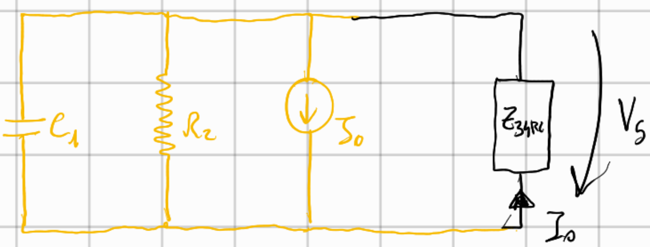
$C_1 = 3\,mF \quad V_{C_1}(0^-) = 1\,V \quad V_{C_6}(0^-) = 0\,V$

$C_6 = 1\,mF$

$J(t) = 10\,u(t)\ mA$

$$R_{34} = \frac{6}{5}\,\Omega$$

$$Z_{R_5C_6} = \left(\frac{1}{R_5} + sC_6\right)^{-1} = \frac{R_5}{R_5C_6s+1}$$



$$Z_{34RC} = \frac{R_5 + R_{34}R_5C_6s + R_{34}}{R_5C_6s+1}$$

$$I_0 = \frac{J_0 \cdot \frac{1}{Z_{34RC}}}{\frac{1}{Z_{34RC}} + \frac{1}{R_2} + C_1 s} = \frac{J_0 \cdot \frac{R_5C_6s+1}{R_5 + R_{34}R_5C_6s + R_{34}}}{\frac{R_5C_6s+1}{R_5 + R_{34}R_5C_6s + R_{34}} + \frac{1}{R_2} + C_1 s}$$

$$\frac{J_0 \cdot \dfrac{R_5C_6S+1}{R_5+R_{34}R_5C_6S+R_{34}}}{\dfrac{R_5C_6S+1}{R_5+R_{34}R_5C_6S+R_{34}}+\dfrac{1}{R_2}+C_1S} = \frac{J_0(1+R_5C_6S)}{R_5C_6S+1+\dfrac{R_5+R_{34}R_5C_6S+R_{34}}{R_2}+C_1S(R_5+R_{34}R_5C_6S+R_{34})}$$

$$= \frac{J_0(1+R_5C_6S)R_2}{R_2R_5C_6S+R_2+R_5+R_{34}R_5C_6S+R_{34}+C_1S(R_5+R_{34}R_5C_6S+R_{34})R_2}$$

$$= \frac{J_0(1+3\cdot10^{-3}s)}{1+3\cdot10^{-3}s+3+6\cdot10^{-3}s+1.2+(4.2+3.6\cdot10^{-3}s)SC_1} =$$

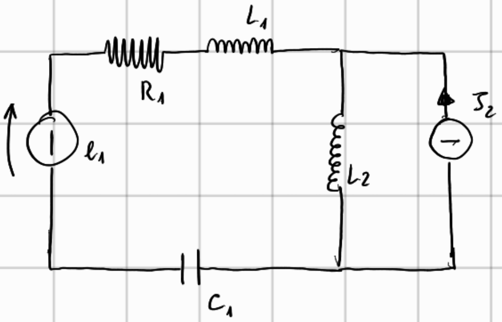
$$= \frac{J_0(1+3\cdot10^{-3}s)}{5.2+9\cdot10^{-3}s+12.6\cdot10^{-3}s+10.8\cdot10^{-6}s} =$$

$$= \frac{J_0(1+3\cdot10^{-3}s)}{5.2+21.6\cdot10^{-3}s+10.8\cdot10^{-6}s}$$

$$V_4(s) = \frac{V_5(s)\cdot R_{34}}{R_{34}+Z_{34RC}} = \frac{J_0(1.2+3.6\cdot10^{-3}s)}{5.2+21.6\cdot10^{-3}s+10.8\cdot10^{-6}s}\cdot\frac{1}{1.2+\dfrac{4.2+3.6\cdot10^{-3}s}{1+3\cdot10^{-3}s}} =$$

$$\frac{J_0(1.2+3.6\cdot10^{-3}s)}{5.2+21.6\cdot10^{-3}s+10.8\cdot10^{-6}s}\cdot\frac{1+3\cdot10^{-3}s}{4.2+3.6\cdot10^{-3}s+1.2+3.6\cdot10^{-3}s}$$

3)



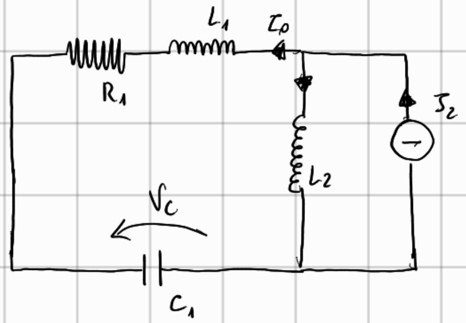
$R_1 = 2k\Omega$ $\qquad e_1(t) = 10\left(u(t) - u(t-\tau)\right) V$

$L_1 = 1mH$ $\qquad J_2(t) = 10 \sin(\omega t)\, u(t)$

$L_2 = 5mH$

$C_1 = 5\mu F$

Calcolo $H_{V_c}^{J_2}(s)$:

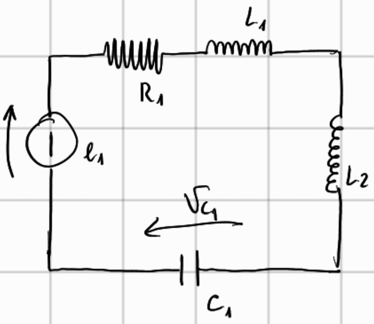


$$I_0 = \frac{J_2 \cdot sL_2}{sL_2 + R_1 + sL_1 + \frac{1}{sC_1}} = \frac{J_2 L_2 s}{\frac{1}{sC_1}\left((L_1+L_2)C_1 s^2 + R_1 C_1 s + 1\right)}$$

$$V_C = I_0 \cdot \frac{1}{sC_1} = \frac{J_2 L_2 s}{(L_1+L_2)C_1 s^2 + R_1 C_1 s + 1} =$$

$$= \frac{\frac{J_2 L_2}{(L_1+L_2)C_1} s}{s^2 + \frac{R_1}{L_1+L_2} s + \frac{1}{(L_1+L_2)C_1}} \qquad (1)$$

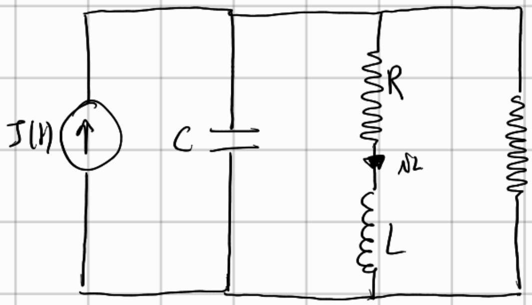
$$H(s) = \frac{\frac{L_2}{(L_1+L_2)C_1} s}{s^2 + \frac{R_1}{L_1+L_2} s + \frac{1}{(L_1+L_2)C_1}}$$



$$V_{C_1} = \frac{-e_1 \cdot \frac{1}{C_1 s}}{\frac{1}{C_1 s} + R_1 + (L_1 + L_2)s} = \frac{-e_1}{C_1(L_1+L_2)s^2 + R_1 C_1 s + 1} = \frac{\frac{-e_1}{C_1(L_1+L_2)}}{s^2 + \frac{R_1}{L_1+L_2}s + \frac{1}{C_1(L_1+L_2)}}$$

$V_C(s) =$

2.3)



$J(t) = 20\,u(-t)$

$R = 2\,\Omega$

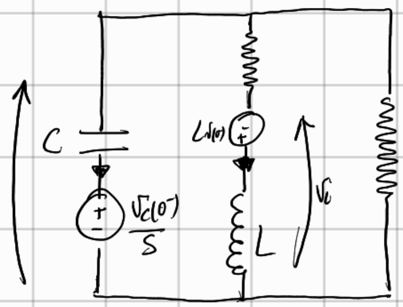
$L = 10\,\mu H$

$C = 5\,\mu F$

In $t<0$ circuito in regime stat.

$i_L = 10A = i_L(0^-) = i_L(0^+)$

$v_c(t) = 20V$



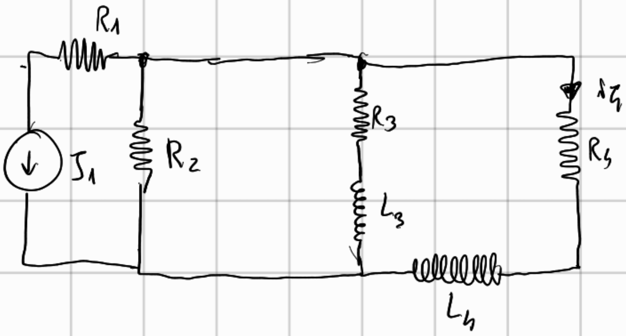
$$U_A\left(\frac{1}{R} + \frac{1}{R+sL} + sC\right) = \frac{v_c(0^-)}{s}\cdot sC - \frac{L i(0^-)}{R+sL}$$

$$U_A\left(\frac{R+sL+R+R^2Cs+RLCs^2}{R(R+sL)}\right) = \frac{RC\,v_c(0^-) + LC\,v_c(0^-)s^2 - L i(0^-)}{R+sL}$$

$$U_A = \frac{20RC + 20LCs^2 - 10L}{\frac{2R+sL+R^2Cs+RLCs^2}{R}} = \frac{20R^2L + 20RLCs^2 - 10L}{RLCs^2 + (L+R^2C)s + 2R}$$

$$= \frac{8\cdot10^{-4} + 2\cdot10^{-9}s^2 - 10^{-4}}{10^{-10}s^2 + 3\cdot10^{-5}s + 4}$$

1)



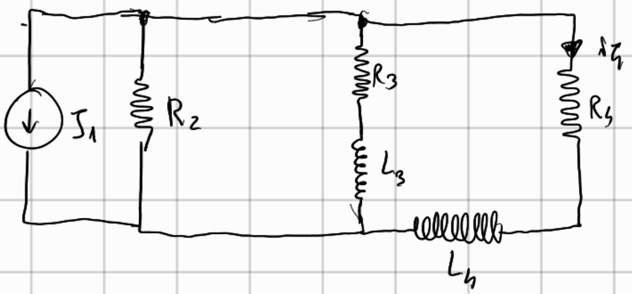
$$Z_{R_4L_4} = R_4 + SL$$

$$Y_{R_4L_4} = \frac{1}{R_4+SL}$$

$R_1 = R_2 = 10\Omega \quad R_3 = 5\Omega \quad R_4 = 10\Omega$

$L_3 = 0.1H \quad L_4 = 0.5H$

RETE EQUIVALENTE:



$$I_4 = -\frac{J_1 \cdot \left(\frac{1}{R_4+SL_4}\right)}{\frac{1}{R_4+SL_4} + \frac{1}{R_3+SL_3} + \frac{1}{R_2}} = -\frac{J_1\left(\cancel{\frac{1}{R_4+SL_4}}\right)}{\left(\cancel{\frac{1}{R_4+SL_4}}\right)\cdot\left(1+\frac{R_4+SL_4}{R_3+SL_3}+\frac{R_4+SL_4}{R_2}\right)} =$$

$$= \frac{-J_1}{\frac{R_2R_3 + R_2L_3S + R_2R_4 + R_2L_4S + (R_3+SL_3)(R_4+SL_4)}{R_2(R_3+SL_3)}}$$

$$= \frac{-J_1(R_2R_3 + R_2L_3S)}{\underbrace{R_2R_3}_{50} + \underbrace{R_2R_4}_{100} + (\underbrace{R_2L_3}_{1} + \underbrace{R_2L_4}_{5})S + \underbrace{R_3R_4}_{50} + \underbrace{R_3L_4}_{2.5}S + \underbrace{R_4L_3}_{1}S + \underbrace{L_3L_4}_{0.05}S^2} =$$

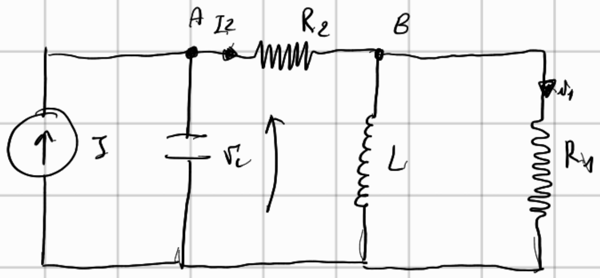
$$= \frac{-J_1(R_2R_3 + R_2L_3S)}{0.05S^2 + 9.5S + 200} =$$

$$= \frac{-J_1(50+S)}{\frac{1}{20}S^2 + 9.5S + 200} = \frac{-J_1(1000+20S)}{S^2 + 190S + 4000}$$

$S^2 + 190S + 4000 = 0$

$x_1 = -165{,}8\ Hz$

$x_2 = -24\ Hz$



$$J(s) = \frac{10^{-2}}{s} - \frac{10^{-2}}{s} e^{-10^{-2} s}$$

$R_1 = 5k\Omega \quad R_2 = 10k\Omega \quad L_1 = 1mH$

$C = 100nF \qquad J(t) = 10u(t) - 10u(t-10^{-2}) \; mA$

$H^{J}_{I_1}(s) = ?$

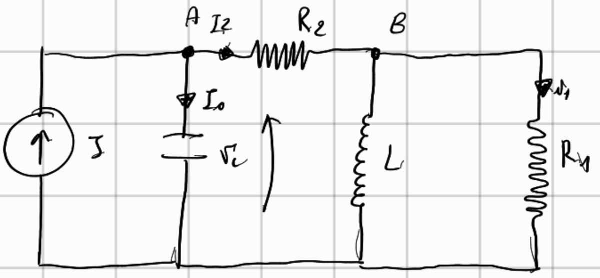
$$Z_{LR_1} = \left(\frac{1}{R_1} + \frac{1}{sL}\right)^{-1} = \left(\frac{sL+R}{R_1 sL}\right)^{-1} = \frac{R_1 L s}{R_1 + Ls}$$

$$I_2 = \frac{J \cdot \frac{1}{Cs}}{\frac{1}{Cs} + R_2 + \frac{R_1 L s}{R_1 + Ls}} = \frac{J \cdot \frac{1}{Cs}}{\frac{R_1 + Ls + R_1 R_2 C s + R_2 L C s^2 + R_1 L C s^2}{Cs(R_1 + Ls)}} =$$

$$= \frac{J(R_1 + Ls)}{LC(R_1+R_2)s^2 + (R_1 R_2 C + L)s + R_1}$$

$$I_1 = \frac{sLJ}{LC(R_1+R_2)s^2 + (R_1 R_2 C + L)s + R_1}$$

$$H_1^J(s) = \frac{\frac{1}{C(R_1+R_2)} s}{s^2 + \frac{(R_1 R_2 C + L)s}{R_1 + R_2} + \frac{R_1}{LC(R_1+R_2)}}$$



$$Z_{R_2LR_1} = \frac{R_1LS}{R_1+LS} + R_2 = \frac{R_1R_2 + R_2LS + R_1LS}{R_1+LS}$$

$$I_0 = \frac{J \cdot SC}{SC + \frac{R_1+LS}{R_1R_2+R_2LS+R_1LS}}$$

$$V_C(S) = \frac{J(R_1R_2+R_2Ls+R_1Ls)}{R_1R_2CS+R_2LCS^2+R_1LCS^2+R_1+LS} = \frac{J(R_1R_2+R_2Ls+R_1Ls)}{LC(R_1+R_2)S^2+(R_1R_2C+L)S+R_1} =$$

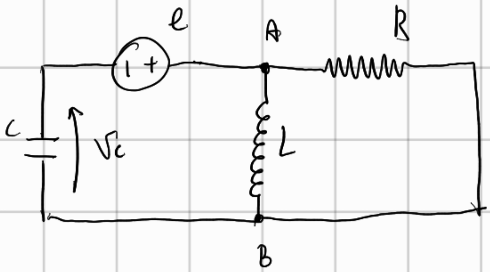
$$= \frac{J\left(\frac{R_1R_2+R_2Ls+R_1Ls}{LC(R_1+R_2)}\right)}{S^2+\frac{(R_1R_2C+L)}{LC(R_1+R_2)}S+\frac{R_1}{LC(R_1+R_2)}} =$$

$$= \frac{J \cdot \left(\frac{1}{3}\cdot 10^{14} + 10^7 s\right)}{S^2 + 3.33\cdot 10^{15} S + 3.3\cdot 10^9}$$

$$S^2 + 3.3\cdot 10^{15} S + 3.3\cdot 10^9 = 0$$

$$S_1 = -3.3\cdot 10^{15}\ Hz \quad S_2 = -1\cdot 10^{-6}\ Hz$$

3)



$C = 1\mu F$

$$U_A\left(\frac{1}{sL}+\frac{1}{R}+cs\right)=ecs$$

$$U_A=\frac{eCs}{\frac{1}{sL}+\frac{1}{R}+Cs}=\frac{eCs}{\frac{1}{sL}\left(1+\frac{L}{R}s+LCs^2\right)}=\frac{eLCs^2}{1+\frac{L}{R}s+LCs^2}=\frac{es^2}{s^2+\frac{1}{RC}s+\frac{1}{LC}}$$

$$V_c=U_A-e=\frac{\cancel{es^2}-\cancel{es^2}-\frac{e}{RC}s-\frac{e}{LC}}{s^2+\frac{1}{RC}s+\frac{1}{LC}}=-e\,\frac{\frac{1}{RC}s+\frac{1}{LC}}{s^2+\frac{1}{RC}s+\frac{1}{LC}}$$

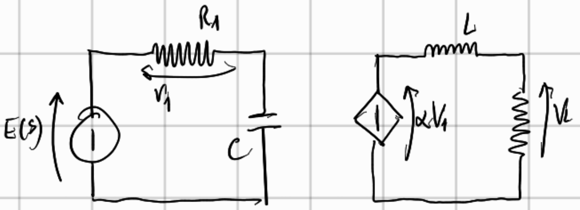
$$H(s)=-\frac{\frac{1}{RC}s+\frac{1}{LC}}{s^2+\frac{1}{RC}s+\frac{1}{LC}}$$

$$s^2+\frac{1}{RC}s+\frac{1}{LC}=0$$

$$\begin{cases}-\frac{1}{RC}=s_1+s_2\\ \frac{1}{LC}=s_1s_2\end{cases}\Rightarrow\begin{cases}-\frac{1}{R}=-5000\cdot10^{-6}\Rightarrow\frac{1}{R}=5\cdot10^{-3}\Rightarrow R=\frac{1}{5}\cdot10^{3}\\ \frac{1}{L}=4\cdot10^{6}\cdot10^{-6}=4\Rightarrow L=\frac{1}{4}H\end{cases}$$

$$(s+1000)(s+4000)=s^2+5000s+4000000=0$$

3)



$R_1 = 100\,\Omega \quad R_L = 50\,\Omega$

$C = 10\,\mu F \quad L = 1\,mH$

$\alpha = 0.5$

$e(t) = 10\,u(t - 5ms)$

$$V_L(s) = \frac{\alpha V_1 \cdot R_L}{R_L + sL}$$

$$V_1 = \frac{E(s)\, R_1}{R_1 + \frac{1}{CS}} = \frac{E(s) R_1}{\frac{1 + R_1 CS}{CS}} = \frac{E(s)\, R_1 CS}{1 + R_1 CS}$$

$$V_L(s) = \frac{\alpha E(s) R_1 CS \cdot R_L}{(1 + R_1 CS)(R_L + SL)} = \frac{\alpha E(s)\, R_1 R_L CS}{R_L + SL + R_1 R_L CS + R_1 CLS^2} = \frac{\alpha E(s) \cancel{R_1} R_L \cancel{C} S}{\cancel{R_1} \cancel{C} L \left(S^2 + \frac{R_L}{L} S + \frac{1}{R_1 C} S + \frac{R_L}{R_1 CL}\right)} =$$

$$= \frac{\frac{\alpha E R_L}{L} S}{S^2 + \left(\frac{R_L}{L} + \frac{1}{R_1 C}\right) S + \frac{R_L}{R_1 CL}} = \frac{2.5 \cdot 10^4 S\, E(S)}{S^2 + 5.1 \cdot 10^4 S + 5 \cdot 10^7}$$

$S^2 + 5.1 \cdot 10^4 S + 5 \cdot 10^7 = 0$

$S_1 = -1000\,Hz$

$S_2 = -50000\,Hz$

$$E(S) = \frac{10}{S} e^{-5 \cdot 10^{-3} S}$$

$$V_L(s) = \frac{2.5 \cdot 10^5\, e^{-5 \cdot 10^{-3} S}}{(S + 1000)(S + 50000)} = 2.5 \cdot 10^5 e^{-5 \cdot 10^{-3} S} \left(\frac{A}{S + 1000} + \frac{B}{S + 50000}\right)$$

Poli: $A = \lim_{S \to -1000} \frac{}{S + 50000} = \frac{1}{51000}$

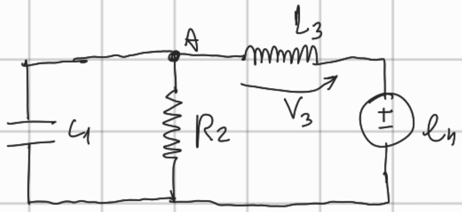
$B = \frac{1}{51000}$

$$\Rightarrow V_L(s) = \frac{2.5 \cdot 10^5}{51000} e^{-5 \cdot 10^{-3} S} \left(\frac{1}{S + 1000} + \frac{1}{S + 50000}\right) = 4{,}9\, e^{-5 \cdot 10^{-3} S} \left(\frac{1}{S + 1000} + \frac{1}{S + 50000}\right) \Rightarrow$$

$$V_L(t) = 4{,}9\, e^{-1000(t - 5 \cdot 10^{-3})} + 4.9\, e^{-50000(t - 5 \cdot 10^{-3})}$$

$$V_L(5.05 \cdot 10^{-3}) = 6.97\,V$$

3)



$R_2 = 3\Omega \quad C_1 = 3F \quad L_3 = 1H$

$v_{C_1}(0^-) = 0V \quad i_L(0^-) = 10mA$

$e(t) = 10t\, u(t)\ V$

$$U_A\left(C_1 s + \frac{1}{R_2} + \frac{1}{sL_3}\right) = \frac{E_4}{sL_3}$$

$$U_A = \frac{E_4/\cancel{s}L_3}{\frac{1}{\cancel{s}L_3}\left(C_1L_3s^2 + \frac{L_3}{R_2}s + 1\right)} = \frac{E_4/C_1L_3}{s^2 + \frac{1}{R_2C_1}s + \frac{1}{C_1L_3}}$$

$$U_A + V_3 - e_4 = 0$$

$$V_3 = E_4 - U_A \Rightarrow \qquad V_3(s) = \frac{E_4s^2 + \frac{E_4}{R_2C_1}s + \cancel{\frac{E_4}{C_1L_3}} - \cancel{\frac{E_4}{C_1L_3}}}{s^2 + \frac{1}{R_2C_1}s + \frac{1}{C_1L_3}}$$

$$H^E_{V_L}(s) = \frac{s^2 + \frac{s}{R_2C_1}}{s^2 + \frac{1}{R_2C_1}s + \frac{1}{C_1L_3}}$$

$$V_L(s) = H^E_{V_L}(s)\left(E_4(s) + L\, i_L(0^-)\right) = H^E_{V_L}(s)\left(\frac{10}{s^2} + 10^{-2}\right) - 10^{-2} =$$

$$\frac{10 + \frac{10}{R_2C_1s}}{s^2 + \frac{1}{R_2C_1}s + \frac{1}{C_1L_3}} + \frac{10^{-2}s^2 + \frac{10^{-2}s}{R_2C_1}}{s^2 + \frac{1}{R_2C_1}s + \frac{1}{C_1L_3}} - 10^{-2} =$$

$$= \frac{10}{s^2 + \frac{1}{9}s + \frac{1}{3}} + \frac{10}{9}\cdot\frac{1}{s\left(s^2 + \frac{1}{9}s + \frac{1}{3}\right)} + \frac{\cancel{10^{-2}s^2} + \cancel{\frac{10^{-2}}{9}s} - \cancel{10^{-2}s^2} - \cancel{\frac{10^{-2}}{9}s} - \frac{10^{-2}}{3}}{s^2 + \frac{1}{9}s + \frac{1}{3}} =$$

$$= \frac{10 - \frac{10^{-2}}{3}}{s^2 + \frac{1}{9}s + \frac{1}{3}} + \frac{10}{9}\cdot\frac{1}{s\left(s^2 + \frac{1}{9}s + \frac{1}{3}\right)}$$

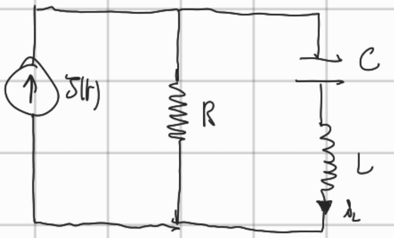
$$9.99 \cdot \frac{1}{s^2 + \frac{1}{9}s + \frac{1}{3}} + \frac{10}{3s} + \frac{10}{9} \cdot \frac{-3s - \frac{1}{3}}{s^2 + \frac{1}{9}s + \frac{1}{3}} =$$

$$= 9.63 \cdot \frac{1}{s^2 + \frac{1}{9}s + \frac{1}{3}} + \frac{10}{3s} - \frac{10}{3} \frac{s}{s^2 + \frac{1}{9}s + \frac{1}{3}}$$

$$\omega_n^2 = \frac{1}{3} \Rightarrow \omega_n = \frac{\sqrt{3}}{3} \qquad 2\xi \frac{\sqrt{3}}{3} = \frac{1}{9} \Rightarrow \sqrt{3}\,\xi = \frac{1}{6} \Rightarrow \xi = \frac{\sqrt{3}}{18}$$

$$v(t) = 9.63 \cdot \frac{1}{0.57} e^{-\frac{1}{18}t} \sin(0.57t)\,u(t) + \frac{10}{3} u(t) +$$

$$- \frac{10}{3}\left(-0.097 e^{-\frac{1}{18}t} \sin(0.57t) + 0.99\, e^{-\frac{1}{18}t} \cos(0.57t) \right) u(t)$$



$J(t) = 5t\left(\ell(t) - \ell(t-2)\right)$

$J(t) = 5t\left(u(t) - u(t-2)\right) =$

$5t\,u(t) - 5t\,u(t-2) = 5t\,u(t) - 5(t-2+2)u(t-2) =$

$5t\,u(t) - 5(t-2)u(t-2) - 10u(t-2)$

$J(s) = \frac{5}{s^2} - \frac{5}{s^2}e^{-2s} - \frac{10}{s}e^{-2s}$

$I_L(s) = \frac{J(s)R}{R + sL + \frac{1}{Cs}} = \frac{J(s)R}{\frac{1}{Cs}\left(LCs^2 + RCs + 1\right)} = \frac{CR\,J(s)\,s}{LC\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)} = \frac{\frac{R}{L}s\,J(s)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$

Polinomio car.:

$s^2 + s + 1 = 0$

$I_L(s) = \frac{s\,J(s)}{s^2+s+1} = \frac{5}{s(s^2+s+1)} - 5e^{-2s}\cdot\frac{1}{s(s^2+s+1)} - \frac{10e^{-2s}}{s^2+s+1}$

$\frac{1}{s(s^2+s+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+s+1} \Rightarrow As^2 + As + A + Bs^2 + Cs$

$A + B = 0 \quad B = -1$

$A + C = 0 \quad C = -1$

$A = 1$

$\frac{1}{s(s^2+s+1)} = \frac{1}{s} - \frac{s}{s^2+s+1} - \frac{1}{s^2+s+1}$

$\omega_n^2 = 1 \Rightarrow \omega_n = 1$

$\xi = \frac{1}{2}$

$$f_1(t) = u(t) \qquad f_3(t) = \frac{1}{\frac{1}{2}\sqrt{3}} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right) u(t)$$

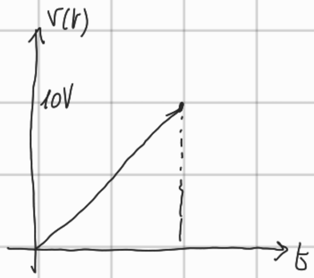
$$f_2(t) = -\frac{\sqrt{3}}{3} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)^{u(t)} + e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) u(t)$$

$$I_L(s) = \frac{5}{s(s^2+s+1)} - 5e^{-2s} \frac{1}{s(s^2+s+1)} - \frac{10e^{-2s}}{s^2+s+1}$$

$$i_L(t) = 5u(t) + \frac{5}{3}\sqrt{3}\, e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right) - 5e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{10\sqrt{3}}{3} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)$$

$$-5u(t-2) - \frac{25}{3}\sqrt{3}\, e^{-\frac{1}{2}(t-2)} \sin\left(\frac{\sqrt{3}}{2}(t-2)\right) + 5e^{-\frac{1}{2}(t-2)} \cos\left(\frac{\sqrt{3}}{2}(t-2)\right) + \frac{2\sqrt{3}}{3} e^{-\frac{1}{2}(t-2)} \sin\left(\frac{\sqrt{3}}{2}(t-2)\right)$$

2)

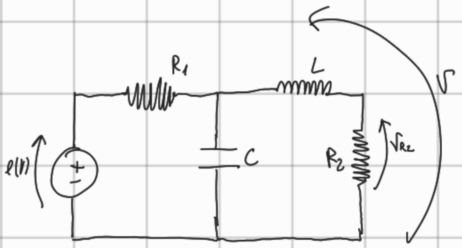


$$v(t) = 10t\left(u(t) - u(t-1)\right)$$

$$v(t) = 10t\,u(t) - 10(t-1+1)\,u(t-1) =$$

$$= 10t\,u(t) - 10(t-1)u(t-1) - 10u(t-1)$$

$$E(s) = \frac{10}{s^2} - \frac{10}{s^2}e^{-s} - \frac{10}{s}e^{-s}$$



$R_1 = 1k\Omega$ $R_2 = 1k\Omega$

$L = 50mH$ $C = 5\mu F$

$$\dot{Z}_{LR_2} = R_2 + Ls$$

$$\dot{Z}_{LR_2C} = \left(\frac{1}{R_2+Ls} + Cs\right)^{-1} = \left(\frac{1+R_2Cs+LCs^2}{R_2+Ls}\right)^{-1} = \frac{R_2+Ls}{LCs^2+R_2Cs+1}$$

$$V(s) = \frac{E(s)\cdot\dfrac{R_2+Ls}{LCs^2+R_2Cs+1}}{R_1+\dfrac{R_2+Ls}{LCs^2+R_2Cs+1}} = \frac{E(s)\,(R_2+Ls)}{R_1LCs^2+R_1R_2Cs+R_1+R_2+Ls}$$

$$V_2(s) = \frac{V(s)\cdot R_2}{R_2+Ls} = \frac{E(s)R_2}{R_1LCs^2+R_1R_2s+Ls+R_1+R_2} = \frac{\dfrac{E(s)R_2}{R_1LC}}{s^2+\dfrac{R_2}{L}s+\dfrac{1}{R_1C}s+\dfrac{1}{LC}+\dfrac{R_2}{R_1LC}} \Rightarrow$$

$$H(s) = \frac{\dfrac{R_2}{R_1LC}}{s^2+\left(\dfrac{R_2}{LC}+\dfrac{1}{R_1C}\right)s+\dfrac{R_2}{R_1LC}+\dfrac{1}{LC}}$$

SBAGLIATO

$\Downarrow$

$$H(s) = \frac{4\cdot10^6}{s^2+(4\cdot10^3+200)s+4\cdot10^6+4\cdot10^6} = \frac{4\cdot10^6}{s^2+(4\cdot10^3+200)s+8\cdot10^6}$$

$S_1 = -4\cdot 10^9\ Hz$ $\qquad \frac{10}{s^2} - \frac{10}{s^2}e^{-s} - \frac{10}{s}e^{-s}$

$S_2 = -2\cdot 10^{-3}\ Hz$

$$H(s) = \frac{4\cdot 10^6}{(s+4\cdot 10^9)(s+2\cdot 10^{-3})}$$

$$V_{R_2}(s) = H(s)E(s) = \frac{4\cdot 10^7(1-e^{-s})}{s^2(s+4\cdot 10^9)(s+2\cdot 10^{-3})} - \frac{4\cdot 10^7 e^{-s}}{s(s+4\cdot 10^9)(s+2\cdot 10^{-3})}$$

$$\frac{1}{s^2(s+4\cdot 10^9)(s+2\cdot 10^{-3})} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+4\cdot 10^9} + \frac{D}{s+2\cdot 10^{-3}}$$

$$A\left(\cancel{s^3} + \cancel{(4\cdot 10^9+200)s^2} + \cancel{8\cdot 10^6 s}\right) + B\left(\cancel{s^2} + \cancel{(4\cdot 10^9+200)s} + 8\cdot 10^6\right) +$$

$$C\left(\cancel{s^3} + \cancel{2\cdot 10^{-3}s^2}\right) + D\left(\cancel{s^3} + \cancel{4\cdot 10^9 s^2}\right)$$

$$A + C + D = 0$$

$$A(4\cdot 10^9+200) + B + 2\cdot 10^{-3}C + D\cdot 4\cdot 10^9 = 0$$

$$8\cdot 10^6 A + (4\cdot 10^9+200)B = 0$$

$$8\cdot 10^6 B = 1$$

$A = -6.25\cdot 10^{-5}$ $\qquad B = 8\cdot 10^{-6}$

$C = -3.12\cdot 10^{-12}$ $\qquad D = 6.25\cdot 10^{-5}$

$$\frac{1}{s(s+4\cdot10^{9})(s+2\cdot10^{-3})} = \frac{A}{s} + \frac{B}{s+4\cdot10^{9}} + \frac{C}{s+2\cdot10^{-3}}$$

$$A\left(s^2+(4\cdot10^9+200)s+8\cdot10^6\right)+B\left(s^2+2\cdot10^{-3}s\right)+C\left(s^2+4\cdot10^9 s\right)$$

$A+B+C=0$

$(4\cdot10^9+200)A+2\cdot10^{-3}B+4\cdot10^9C=0$

$A=8\cdot10^{-6}$

$A=-6.25\cdot10^{-5}$ $B=8\cdot10^{-6}$

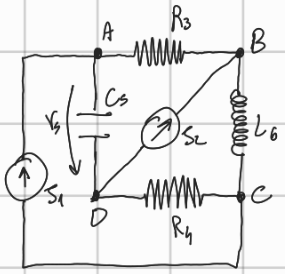
$C=-3.12\cdot10^{-12}$ $D=6.25\cdot10^{-5}$

$A=8\cdot10^{-6}$ $B=6.25\cdot10^{-15}$ $C=-1.25\cdot10^{-7}$

$$\frac{4\cdot10^7(1-e^{-s})}{s^2(s+4\cdot10^9)(s+2\cdot10^{-3})} = (1-e^{-s})\left(-\frac{2500}{s}+\frac{320}{s^2}-\right.$$

SBAGLIATO

1)

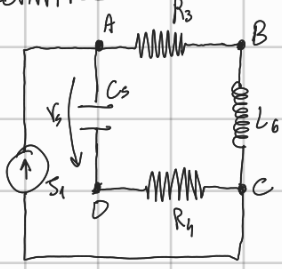


$$J_1(t) = 200\cdot10^{-3}\,u(t) = \frac{1}{5}u(t)$$

$$J_2(t) = \frac{100\cdot10^{-3}}{1\cdot10^{-3}}\,t\,u(t) = 100\,t\,u(t)$$

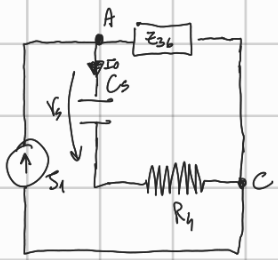
$R_3 = R_4 = 20\Omega \quad C_5 = 100mF \quad L_6 = 10\mu H$

SOVRAPPOSIZIONE



$\Downarrow$

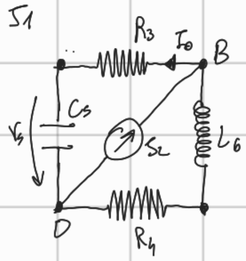
$$\dot{Z}_{36} = R_3 + sL_6$$



$$I_0 = \frac{J_1 Z_{36}}{Z_{36} + R_4 + \frac{1}{sC_5}} = \frac{J_1(R_3 + sL_6)}{\frac{1}{sC_5}\left(1 + R_4C_5s + R_3C_5s + L_6C_5s^2\right)}$$

$$V_5(s) = -I_0\frac{1}{sC_5} = \frac{-J_1(R_3 + sL_6)}{L_6C_5s^2 + (R_4C_5 + R_3C_5)s + 1}$$

SPENGO J_1



$$I_0^2 = \frac{J_2 (R_4 + sL_6)}{R_4 + sL_6 + R_3 + \frac{1}{sC_5}}$$

$$V_5 = -\frac{1}{sC_5} \cdot I_0^2 = -\frac{J_2 (R_4 + sL_6)}{1 + (R_3C_5 + R_4C_5)s + C_5L_6 s^2}$$

$$V_5(s) = -\frac{1}{C_5L_6s^2 + (R_3C_5 + R_4C_5)s + 1}\left(J_2(R_4 + sL_6) + J_1(R_3 + sL_6)\right)$$

$R_3 = R_4 = 20\Omega$ $C_5 = 100nF$ $L_6 = 10\mu H$

$$V_5(s) = -\frac{1}{C_5L_6s^2 + (R_3C_5 + R_4C_5)s + 1}\left(\frac{100}{s^2}(R_4 + sL_6) + \frac{1}{5s}(R_3 + sL_6)\right)$$

$$= -\frac{1}{10^{-12}s^2 + 4\cdot 10^{-6}s + 1}\left(\frac{2000}{s^2} + \frac{10^{-3}}{s} + \frac{1}{4s} + 2\cdot 10^{-6}\right)$$

$$= -\frac{10^{12}}{s^2 + 4\cdot 10^6 s + 10^{12}}\left(\frac{2000}{s^2} + \frac{251}{1000s} + 2\cdot 10^{-6}\right)$$

$$= -\frac{2\cdot 10^{15}}{s^2(s^2 + 4\cdot 10^6 s + 10^{12})} - \frac{251\cdot 10^9}{s(s^2 + 4\cdot 10^6 s + 10^{12})} - \frac{2\cdot 10^6}{s^2 + 4\cdot 10^6 s + 10^{12}}$$

$$\frac{1}{s^2(s^2+4\cdot10^6 s+10^{12})} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2.67\cdot10^5} + \frac{D}{s+3.73\cdot10^6}$$

$$A(\cancel{s^3}+4\cdot\cancel{10^6}s^2+\cancel{10^{12}}s)+B(\cancel{s^2}+4\cdot10^6 s+10^{12})+C(\cancel{s^3}+3.73\cdot\cancel{10^6}s^2)+D(\cancel{s^3}+2.67\cdot\cancel{10^5}s^2)=1$$

$$\begin{cases} \cancel{A}+C+D=0 \\ \cancel{4\cdot10^6A}+B+3.73\cdot10^6C+2.67\cdot10^5D=0 \\ \cancel{10^{12}A}=0 \\ 10^{12}B=1 \end{cases}$$

$A=0$ $B=10^{-12}$ $C=-2.9\cdot10^{-19}$ $D=2.9\cdot10^{-19}$

$$= \frac{10^{-12}}{s^2} - \frac{2.9\cdot10^{-19}}{s+2.67\cdot10^5} + \frac{2.9\cdot10^{-19}}{s+3.73\cdot10^6}$$

1)

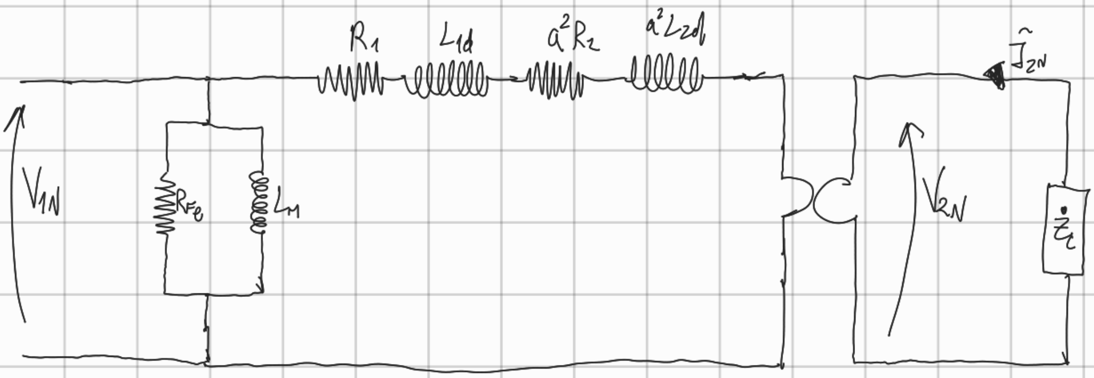
$P = 15 kVA$

$V_{1N} = 2300 V$ $V_{2N} = 230 V$

$I_{CA} = 0.21 A$ $P_{CA} = 40 W$

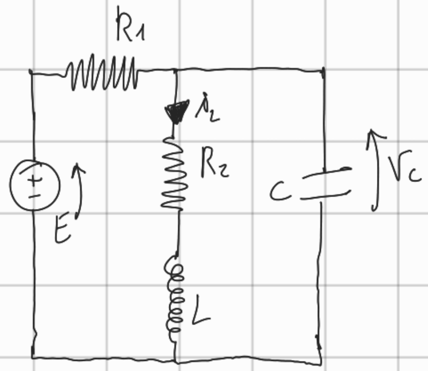
$V_{CC} = 37 V$ $P_{CC} = 100 W$

$Z_C = (3+2j)\,\Omega$



$I_{2N} = 15 kVA$

3)



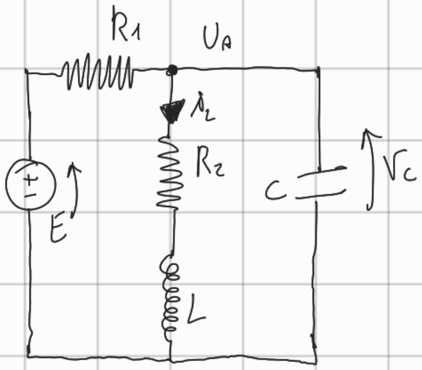
$E = 10\,u(t)$

$R_1 = 10K\Omega \quad R_2 = 10K\Omega$

$L = 10mH \qquad i_L(0^-) = 0\,A$

$C = 1\mu F \qquad V_C(0^-) = 1V$

Uso sovrapposizione effetti: E acceso.

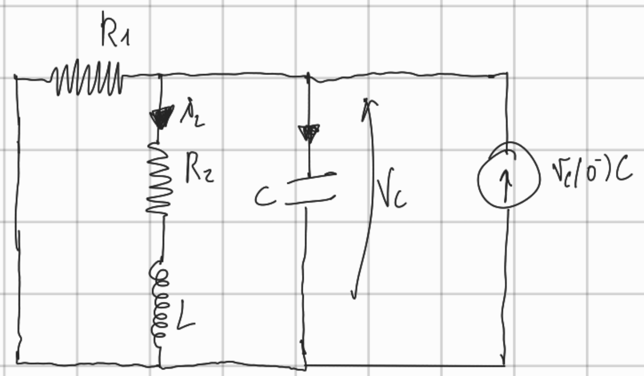


$$U_A\left(\frac{1}{R_1} + \frac{1}{R_2+SL} + SC\right) = \frac{E}{R_1}$$

$$U_A\left(\frac{R_2+SL+R_1+R_1R_2CS+R_1LCS^2}{R_1R_2+R_1LS}\right) = \frac{E}{R_1}$$

$$U_A = \frac{E\,(R_2+Ls)}{R_1LCs^2+(R_1R_2C+L)s+R_1+R_2}$$

$$I_L(S) = \frac{U_A(S)}{R_2+LS} = \frac{E}{R_1LCS^2+(R_1R_2C+L)S+R_1+R_2}$$



$$I_L(S) = \frac{C V_c(0^-) \cdot \left(\frac{1}{R_2 + SL}\right)}{\frac{1}{R_2+SL} + \frac{1}{R_1} + SC} =$$

$$I_L^2(S) = \frac{C V_c(0^-) \cdot \frac{1}{R_2+SL}}{\frac{R_1 + R_2 + SL + R_1R_2CS + R_1LCS^2}{(R_2+SL)R_1}} = \frac{R_1 C V_c(0^-)}{R_1LCS^2 + (R_1R_2C+L)S + R_1 + R_2}$$

$$I_L(S) = \frac{1}{R_1LCs^2 + (R_1R_2C+L)s + R_1 + R_2}\left(E + R_1 C V_c(0^-)\right)$$

$$= \frac{\frac{1}{R_1LC}}{S^2 + \frac{(R_1R_2C+L)}{R_1LC}S + \frac{R_1+R_2}{R_1LC}}\left(\frac{10}{S} + R_1 C V_c(0^-)\right) =$$

$$= \frac{10^5}{S(S^2 + 1.0001\cdot 10^6 s + 2\cdot 10^8)} + \frac{100}{S^2 + 1.0001\cdot 10^6 s + 2\cdot 10^8}$$

$S^2 + 1.0001\cdot 10^6 S + 2\cdot 10^8 = 0$

$S_1 \cong -200\ Hz$

$S_2 \cong -10^6\ Hz$

a) $$\frac{1}{s(s+200)(s+10^6)} = \frac{A}{s} + \frac{B}{s+200} + \frac{C}{s+10^6}$$

$$A = \lim_{s \to 0} \frac{1}{(s+200)(s+10^6)} = 5 \cdot 10^{-9}$$

$$B = \lim_{s \to -200} \frac{1}{s(s+10^6)} = -5.001 \cdot 10^{-9}$$

$$C = \lim_{s \to -10^6} \frac{1}{s(s+200)} = 1.0002 \cdot 10^{-12}$$

b) $$\frac{1}{(s+200)(s+10^6)} = \frac{A}{s+200} + \frac{B}{s+10^6}$$

$$A = \lim_{s \to -200} \frac{1}{s+10^6} = 1.0002 \cdot 10^{-6}$$

$$B = \lim_{s \to -10^6} \frac{1}{s+200} = -1.0002 \cdot 10^{-6}$$

$$I_L(s) = 10^5 \left(\frac{5 \cdot 10^{-9}}{s} - \frac{5.001 \cdot 10^{-9}}{s+200} + \frac{1.0002 \cdot 10^{-12}}{s+10^6} \right) + 100 \left(\frac{1.0002 \cdot 10^{-6}}{s+200} - \frac{1.0002 \cdot 10^{-6}}{s+10^6} \right) =$$

$$= \frac{5 \cdot 10^{-4}}{s} - \frac{5.001 \cdot 10^{-4}}{s+200} + \frac{1.0002 \cdot 10^{-7}}{s+10^6} + \frac{1.0002 \cdot 10^{-4}}{s+200} - \frac{1.0002 \cdot 10^{-4}}{s+10^6}$$

$$i(t) = 5 \cdot 10^{-4} u(t) - 5.001 \cdot 10^{-4} e^{-200t} + 1.0002 \cdot 10^{-7} e^{-10^6 t} + 1.0002 \cdot 10^{-4} e^{-200t} - 1.0002 \cdot 10^{-4} e^{-10^6 t}$$

1)

$$\int_\gamma \frac{e^z+e^{-z}}{z^4}\,dz \qquad \gamma=\{z\in\mathbb{C} \mid |z|=1\}$$

(PICCOLO CAMEO DI ANALISI 2)

$g(z)=e^z+e^{-z}$ olomorfa in $\mathbb{C}$

$z_0=0 \qquad n=3$

$$g^{(3)}(0)=\frac{3!}{2\pi i}\int_\gamma f(z)\,dz$$

$$\int_\gamma f(z)\,dz=\frac{2\pi i}{6}\left(e^0-e^0\right)=0$$

2)

$$\int_\gamma \frac{z^2+1}{z(z-8)}\,dz \qquad \gamma:\{z\in\mathbb{C} \mid |z-3|=4\}$$

$g(z)=\frac{z^2+1}{z-8}$ olomorfa nell'interno di γ.

$z_0=0 \qquad n=0$

$$g(0)=\frac{1}{2\pi i}\int_\gamma f(z)\,dz \rightarrow \int_\gamma f(z)\,dz=2\pi i\, g(0)=-\frac{\pi i}{4}$$

3)

$u(x,y)=ax^2+y^2$

$\Delta u=0=2a+2=0 \qquad a=-1$

$u(x,y)=y^2-x^2$

$u_y=2y$

$$\begin{cases} u_x = V_y \\ u_y = -V_x \end{cases} \Rightarrow \nabla V(-u_y, u_x)$$

$$\Rightarrow V = \int (-2y)\,dx = -2xy + C(y)$$

$$V_y = -2x + C'(y) = -2x$$

$$C(y) = C$$

$$f(x+iy) = y^2 - x^2 + i(-2xy + c)$$

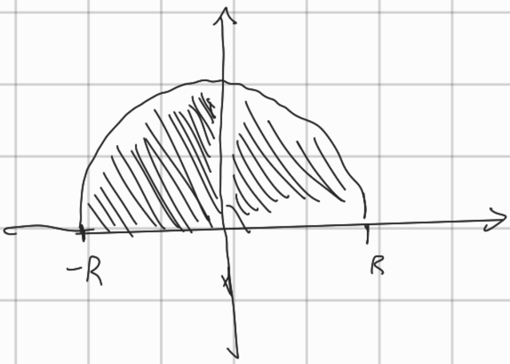
4)

$$\int_{-\infty}^{+\infty} \frac{\sin^2(\pi x)}{x^2+1}\,dx = \int_{-\infty}^{+\infty} \frac{\frac{1}{2}(1-\cos(2\pi x))}{x^2+1}\,dx =$$

$$\frac{1}{2}\int_{-\infty}^{+\infty} \frac{1}{x^2+1}\,dx - \underbrace{\frac{1}{2}\int_{-\infty}^{+\infty} \frac{\cos(2\pi x)}{x^2+1}\,dx}$$

$$\int_{-\infty}^{+\infty} \frac{\cos(2\pi x)}{x^2+1}\,dx = Re\left(\int_{-\infty}^{+\infty} \frac{e^{2\pi i x}}{x^2+1}\,dx\right)$$

$$f(z) = \frac{e^{2\pi i z}}{z^2+1}$$



$$\Gamma_R = [-R, R] \cup \gamma_R^+(0)$$

$$\int_{\Gamma_R} f(z)\,dz = \int_{\mathbb{R}} \frac{e^{2\pi i x}}{x^2+1}\,dx + \int_{\gamma_R^+(0)} f(z)\,dz$$

$\to 0$ se $R \to +\infty$

Grande cerchio

$R \to +\infty$

$\|$

$2\pi i \operatorname{Res}(f(z), i)$

$$\operatorname{Res}(f(z), i) = \lim_{z \to i} \frac{e^{2\pi i z}}{(z+i)} = \frac{e^{-2\pi}}{2i}$$

$$\int_{\mathbb{R}} \frac{e^{2\pi i x}}{x^2+1}\,dx = \cancel{2}\pi \cancel{i} \cdot \frac{e^{-2\pi}}{\cancel{2i}} = \pi e^{-2\pi}$$

$$\int_{-\infty}^{+\infty} \frac{1}{x^2+1}\,dx =$$

$$f(z) = \frac{1}{z^2+1}$$

$$\Gamma_R = [-R, R] \cup \gamma_R^+(0)$$

$$\int_{\Gamma_R} f(z)\,dz = \int_{[-R,R]} f(z)\,dz + \cancel{\int_{\gamma_R^+(0)} f(z)\,dz} \to 0$$

se $R \to +\infty$

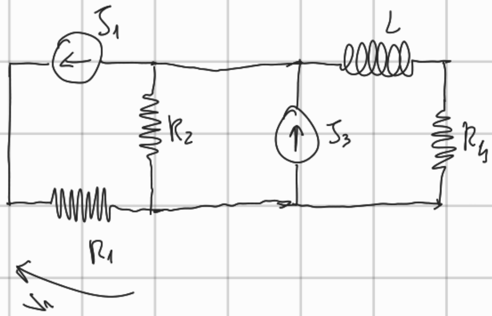
$\Downarrow$

$$2\pi i \operatorname{Res}(f(z), i) = 2\pi i \cdot \frac{1}{2i} = \pi$$

$$\frac{1}{2}\int_{-\infty}^{+\infty}\frac{1}{x^2+1}\,dx-\frac{1}{2}\underbrace{\int_{-\infty}^{+\infty}\frac{\cos(2\pi x)}{x^2+1}\,dx}=$$

$$=\frac{1}{2}\pi-\frac{1}{2}\left(\frac{\pi}{e^{2\pi}}\right)=\frac{1}{2}\pi\left(1-\frac{1}{e^{2\pi}}\right)=\frac{1}{2}\pi\left(\frac{e^{2\pi}-1}{e^{2\pi}}\right)$$

3)

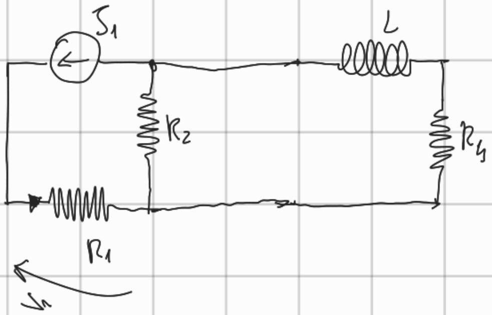


$R_4 = 5K\Omega$ $\quad R_2 = R_1 = 10K\Omega$ $\quad L = 10mH$

$J_1(t) = 10 \cdot 10^{-3}\, u(t)$ A

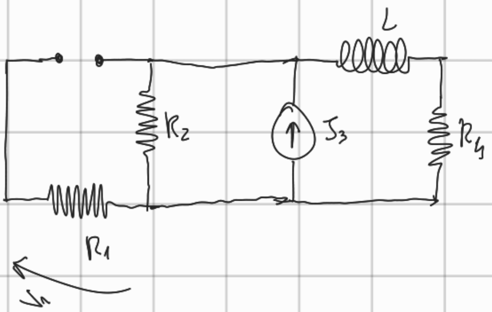
$J_3(t) = 100\sqrt{2}\, \sin(\omega t) u(t) \cdot 10^{-3}$ A

Spego J_3:



$V_1(s) = R_1 J_1 \Rightarrow H^{J_1}_{R_1}(s) = R_1$

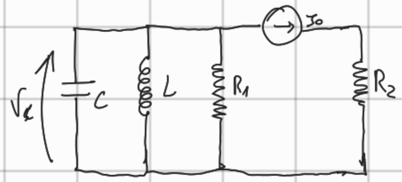
2a RICHIESTA



$V_1 = 0 \Rightarrow H(s) = 0$

$V_1^{TOT}(s) = H_1(s) J_1 + H_2(s) J_2 = R_1 J_1 = \frac{100}{s}$

3)



$R_1 = R_2 = 10\,\Omega$

$L = 10\,mH \qquad C = 1\,\mu F$

$J(t) = 10\left(u(t) - u(t - 1ms)\right)\,mA$

$J(t) = 10^{-2}u(t) + 10^{-2}u(t - 1ms);$

$J(s) = \frac{10}{s} + \frac{10}{s}e^{-10^{-3}s}$

$Z_{TOT} = \left(sC + \frac{1}{sL} + \frac{1}{R_1}\right)^{-1} = \left(\frac{R_1LCs^2 + R_1 + sL}{R_1Ls}\right)^{-1} = \frac{R_1Ls}{R_1LCs^2 + sL + R_1}$

$V_c(s) = J(s)Z_{TOT} = \frac{\cancel{R_1L}s}{\cancel{R_1L}C\left(s^2 + \frac{s}{R_1C} + \frac{1}{LC}\right)}\left(\frac{10}{s} + \frac{10}{s}e^{-10^{-3}s}\right) =$

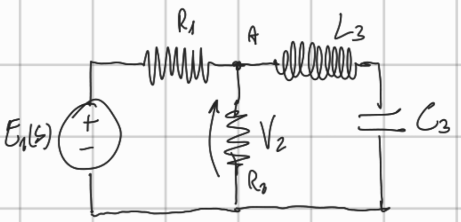
$= \frac{\frac{10^{-2}}{C} + \frac{10^{-2}}{C}e^{-10^{-3}s}}{s^2 + \frac{s}{R_1C} + \frac{1}{LC}}$

$s^2 + \frac{s}{R_1C} + \frac{1}{LC} = 0$

$s_1 = -9{,}9 \cdot 10^4\,Hz$

$s_2 = -1\,KHz$

3)



$R_1 = R_2 = 10\Omega$

$L_3 = 3H \quad C_3 = 2F$

$$U_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{sL_3 + \frac{1}{sC_3}}\right) = \frac{E_1}{R_1}$$

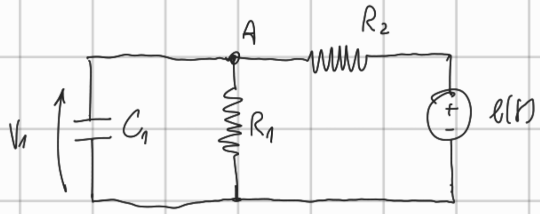
$$\frac{1}{\frac{C_3L_3s^2+1}{sC_3}} = \frac{sC_3}{1+C_3L_3s^2}$$

$$\frac{R_2 + R_2C_3L_3s^2 + R_1 + R_1C_3L_3s^2 + R_1R_2C_3s}{R_1R_2(1+C_3L_3s^2)}$$

$$U_A = \frac{E_1}{\cancel{R_1}} \cdot \frac{\cancel{R_1}R_2(1+C_3L_3s^2)}{(R_2C_3L_3 + R_1C_3L_3)s^2 + R_1R_2C_3s + R_1 + R_2}$$

$$H(s) = \frac{R_2 + R_2C_3L_3s^2}{(R_2C_3L_3 + R_1C_3L_3)\left(s^2 + \frac{R_1R_2C_3}{(R_2+R_1)C_3L_3}s + \frac{1}{C_3L_3}\right)}$$

3)



$R_1 = R_2 = 10k\Omega$ $C = 10\mu F$ $V_1(0^-) = 1.5V$

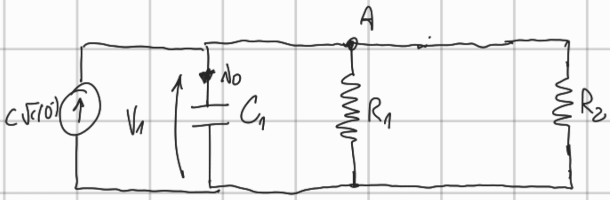
$e(t) = 10(u(t) - u(t - 10ms))\ V$

Spengo cond. iniziale:

$$U_A\left(\frac{1}{R_1} + SC_1 + \frac{1}{R_2}\right) = \frac{E}{R_2}$$

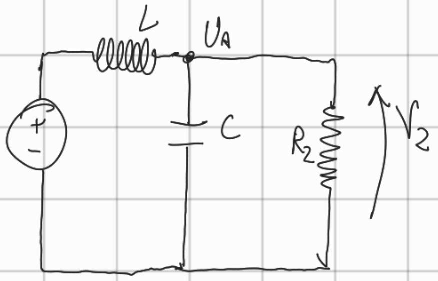
$$U_A\left(\frac{R_2 + R_1R_2CS + R_1}{R_1R_2}\right) = \frac{E}{R_2}$$

$$U_A = \frac{R_1E}{R_1R_2CS + R_1 + R_2}$$



$$I_0 = \frac{CV_c(0^-) \cdot C_1S}{\frac{1}{R_1} + \frac{1}{R_2} + C_1S} = \frac{CV_c(0^-) \cdot C_1SR_1R_2}{R_1R_2CS + R_1 + R_2}$$

3)



$R_1 = 100\,\Omega \quad R_2 = 50\,\Omega$

$C = 10\,\mu F \quad v_C(0^-) = 0\,V$

$L = 100\,\mu H \quad i_L(0^-) = 10\,mA$

$e(t) = 10\,u(t)$

$H_e^{v_2}(s) = ?$

$$\tilde{U}_A\left(Cs + \frac{1}{Ls} + \frac{1}{R_2}\right) = \frac{E}{Ls}$$

$$\tilde{U}_A\left(\frac{R_2LCs^2 + R_2 + Ls}{R_2Ls}\right) = \frac{E}{Ls}$$

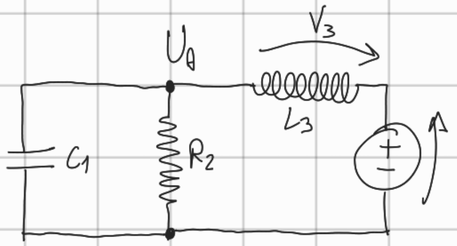
$$\tilde{U}_A = \frac{ER_2}{R_2LCs^2 + R_2 + Ls}$$

$$H_e^{v_2}(s) = \left(\frac{R_2}{R_2LCs^2 + Ls + R_2}\right)$$

Con le condiz. iniziali generatore impulsivo é nella stessa posizione di E.

$$V_2(s) = H(s)\left(E - L\,i_L(0^-)\right)$$

3)



$U_A - E + V_3$

$R_2 = 3\,\Omega \quad C_1 = 3F \quad v_{C_1}(0^-) = 0V$

$L_3 = 1H \quad i_2(0^-) = 10mA$

$e(t) = 10t\,u(t)\ V$

Calcolo $H(s)$ ignorando condiz. iniziali:

$$U_A\left(\frac{1}{sL_3} + \frac{1}{R_2} + C_1 s\right) = \frac{E}{sL_3}$$

$$U_A\left(\frac{R_2 + L_3 s + R_2 L_3 C_1 s^2}{L_3 R_2 s}\right) = \frac{E}{sL_3}$$

$$U_A = \frac{E R_2}{R_2 L_3 C_1 s^2 + L_3 s + R_2}$$

$$V_3^2 = E - U_A = E\left(1 - \frac{R_2}{R_2 L_3 C_1 s^2 + L_3 s + R_2}\right)$$

$$H_E(s) = 1 - \frac{R_2}{R_2 L_3 C_1 s^2 + L_3 s + R_2}$$

Caso generatore condiz. iniziali uguale.

$$V_3^1 = -U_A = -\frac{L i_L(0^-) R_2}{R_2 L_3 C_1 s^2 + L_3 s + R_2}$$

$$V_3(S) = E - \frac{E R_2}{R_2 L_3 C_1 S^2 + L_3 s + R_2} - \frac{L i_L(0^-) R_2}{R_2 L_3 C_1 S^2 + L_3 S + R_2}$$

$$V_3(s) = \frac{10}{S^2} - \frac{3}{s^2(9s^2+s+3)} - \frac{3 \cdot 10^{-2}}{9s^2+s+3} =$$

$$= \frac{10}{S^2} + \frac{1/3}{s^2\left(s^2+\frac{s}{9}+\frac{1}{3}\right)} - \frac{10^{-2}}{s^2+\frac{s}{9}+\frac{1}{3}}$$

$$9s^2+s+3=0$$

Poli complessi e coniugati

$$\frac{1}{s^2\left(s^2+\frac{1}{9}s+\frac{1}{3}\right)} = \frac{As+B}{s^2} + \frac{Cs+D}{s^2+\frac{1}{9}s+\frac{1}{3}}$$

$$(As+B)\left(s^2+\frac{1}{9}s+\frac{1}{3}\right) + (Cs+D)s^2 = 1$$

$$\cancel{A}s^3 + \cancel{\frac{A}{9}}s^2 + \frac{A}{3}s + \cancel{B}s^2 + \frac{B}{9}s + \frac{B}{3} + \cancel{C}s^3 + \cancel{D}s^2 = 1$$

$$\begin{cases} A+C=0 \\ \frac{A}{9}+B+D=0 \\ \frac{A}{3}+\frac{B}{9}=0 \quad \Rightarrow \quad \frac{A}{3}+\frac{1}{3}=0 \qquad A=-1 \\ B=3 \end{cases}$$

$$A=-1 \qquad C=1 \qquad -\frac{1}{9}+3=-D$$

$$\frac{26}{9}=-D \qquad D=-\frac{26}{9}$$

$\Rightarrow$

$$V(s) = \frac{10}{s^2} + \frac{1}{3}\left(\frac{-s+3}{s^2} + \frac{s-\frac{26}{9}}{s^2+\frac{1}{9}s+\frac{1}{3}}\right) - \frac{10^{-2}}{s^2+\frac{1}{9}s+\frac{1}{3}}$$

$$\frac{10}{s^2} \Rightarrow 10t\,u(t)$$

$$\frac{3}{s^2} \Rightarrow 3t\,u(t)$$

$$-\frac{1}{s} \Rightarrow -u(t)$$

$$\frac{1}{s^2+\frac{1}{9}s+\frac{1}{3}} \Rightarrow \quad 1.75\, e^{-\frac{t}{18}} \sin(0.57t)\, u(t)$$

$$\omega_m^2 = \frac{1}{3} \Rightarrow \omega_m = \frac{\sqrt{3}}{3}$$

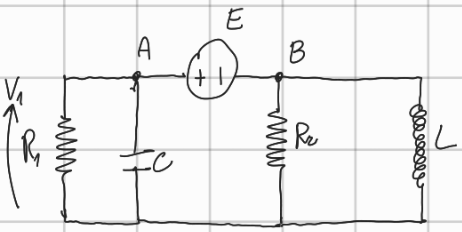
$$2\delta \frac{\sqrt{3}}{3} = \frac{1}{9} \quad \Rightarrow \sqrt{3}\,\delta = \frac{1}{6} \qquad \delta = \frac{\sqrt{3}}{18}$$

$$\left[1.75\, e^{-\frac{t}{18}} \sin(0.57t)\, u(t)\right]' =$$

$$-0.1\, e^{-\frac{t}{18}} \sin(0.57t)\, u(t) + \quad e^{-\frac{t}{18}} \cos(0.57t)\, u(t)$$

$$v(t) = \Big[10t + t - \frac{1}{3} - 0.03\, e^{-\frac{t}{18}} \sin(0.57t) + 0.33\, e^{-\frac{t}{18}} \cos(0.57t) +$$
$$- 1.7 e^{-\frac{t}{18}} \sin(0.57t)\Big]\, u(t)$$

3)



$R_1 = R_2 = 0.5\,\Omega$

$C = 100\,\mu F \qquad v_c(0^-) = 1\,V$

$L = \frac{1}{3}\,mH \qquad e(t) = 10\,u(t - 5ms)\,V$

$H_E^{V_1}(s) = ?$

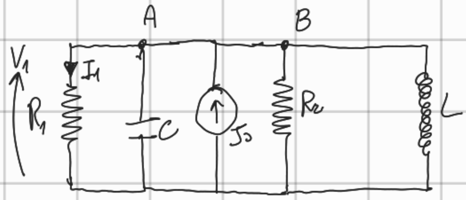
$$Z_{1C} = \left(\frac{1}{R} + sC\right)^{-1} = \left(\frac{1 + R_1 C s}{R_1}\right)^{-1} = \frac{R_1}{1 + R_1 C s}$$

$$Z_{2L} = \left(\frac{1}{R} + \frac{1}{sL}\right)^{-1} = \left(\frac{sL + R}{RLs}\right)^{-1} = \frac{RLs}{R + sL}$$

$$V_1(s) = \frac{E \cdot \frac{\cancel{R}}{1 + RCs}}{\frac{\cancel{R}}{1 + RCs} + \frac{\cancel{R}Ls}{R + sL}} = \frac{\frac{E}{1 + RCs}}{\frac{R + sL + Ls + RLCs^2}{(R + sL)(1 + RCs)}} =$$

$$= \frac{E(R + sL)}{RLCs^2 + 2Ls + R}$$

CASO COND. INIZIALI



$$V_1(s) = \frac{J_0}{\frac{1}{R} + \frac{1}{R} + sC + \frac{1}{sL}} = \frac{J_0}{\frac{2Ls + RLCs^2 + R}{RLs}} = \frac{J_0 RLs}{RLCs^2 + 2Ls + R}$$

$$V(s) = \frac{E(R+sL)}{RLCs^2+2Ls+R} + \frac{J_0 RLs}{RLCs^2+2Ls+R} =$$

$$= \frac{E\frac{(R+sL)}{RLC}}{s^2+\frac{2}{RC}s+\frac{1}{LC}} + \frac{\frac{J_0 s}{C}}{s^2+\frac{2}{RC}s+\frac{1}{LC}} =$$

$$= \frac{1}{s}e^{-5\cdot10^{-3}s} \cdot \frac{\frac{0.5+3.3\cdot10^{-7}s}{1.67\cdot10^{-12}}}{s^2+40000s+3\cdot10^{10}} + \frac{s}{s^2+40000s+3\cdot10^{10}} =$$

$$= \frac{e^{-5\cdot10^{-3}s}}{s} \frac{(3\cdot10^{11}+198\cdot10^{19}s)}{s^2+40000s+3\cdot10^{10}} + \frac{s}{s^2+40000s+3\cdot10^{10}} =$$

$$= \frac{e^{-5\cdot10^{-3}s}\cdot 3\cdot10^{11}}{s(s^2+40000s+3\cdot10^{10})} + \frac{e^{-5\cdot10^{-3}s}\cdot \overset{B}{\overset{\parallel}{1.98\cdot10^{19}}}s}{s^2+40000s+3\cdot10^{10}} + \frac{s}{s^2+40000s+3\cdot10^{10}}$$

$$s^2+40000s+3\cdot10^{10}=0$$

Poli complessi e coniugati

$$\frac{1}{s(s^2+40000s+3\cdot10^{10})} = \frac{a}{s} + \frac{bs+c}{s^2+40000s+3\cdot10^{10}}$$

$$\cancel{a}s^2+40000as+3\cdot10^{10}a+\cancel{b}s^2+sc=0$$

$$\begin{cases} a+b=0 \\ 40000a+c=0 \\ a=\frac{1}{3}10^{-10} \end{cases} \qquad \begin{aligned} b&=-\frac{1}{3}10^{-10} \\ c&=-1.33\cdot10^{-6} \end{aligned}$$

$$= \frac{1}{3}\cdot 10^{-10}\cdot\frac{1}{S} - \frac{1}{3}\cdot 10^{-10}\frac{S}{S^2+40000S+3\cdot 10^{10}} - \frac{1.33\cdot 10^{-6}}{S^2+40000S+3\cdot 10^{10}}$$

$$\mathcal{L}^{-1}\left[\frac{1}{S^2+40000S+3\cdot 10^{10}}\right] =$$

$$W_n^2 = 3\cdot 10^{10} \Rightarrow W_n = 173205$$

$$W_n \cdot 2\psi = 40000$$

$$\psi = \frac{20000}{W_n} = 0.12$$

$$f(t) = 5.8\cdot 10^{-6} e^{-20784t} \sin(172414t)u(t) = g(t)$$

$$f'(t) = -0.12 e^{-20784t}\sin(172414t) + e^{-20784t}\cos(172414t) = h(t)$$

$$\mathcal{L}^{-1}\left[\frac{1}{S(S^2+40000S+3\cdot 10^{10})}\right] = \frac{1}{3}\cdot 10^{-10}u(t) + 4\cdot 10^{-12}e^{-20784t}\sin(172414t)$$

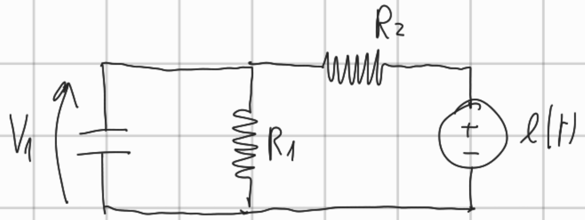
$$-\frac{1}{3}\cdot 10^{-10} e^{-20784t}\cos(172414t) +$$

$$-7.714\cdot 10^{-12} e^{-20784t}\sin(172414t) = f(t)$$

$$V(S) = \frac{e^{-5\cdot 10^{-3}S}\cdot 8{,}35\cdot 10^{11}}{S(S^2+40000S+3\cdot 10^{10})} + \frac{e^{-5\cdot 10^{-3}S}\cdot 5.5\cdot 10^{13}}{S^2+40000S+3\cdot 10^{10}} + \frac{S}{S^2+40000S+3\cdot 10^{10}}$$

$$V(t) = 3\cdot 10^{11} f(t-5ms) + 1.98\cdot 10^{19} g(t-5ms) + h(t)$$

3)



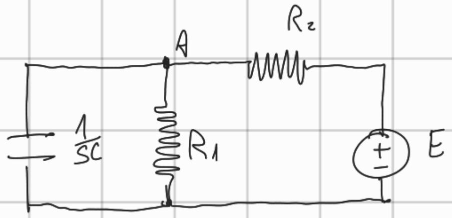
$R_1 = R_2 = 10\,k\Omega$

$C = 10\,\mu F \quad v_1(0^-) = 1.5\,V$

$e(t) = 10u(t) - 10u(t - 10ms)\,V$

$$E(s) = \frac{10}{s} - \frac{10}{s} e^{-10^{-2}s}$$

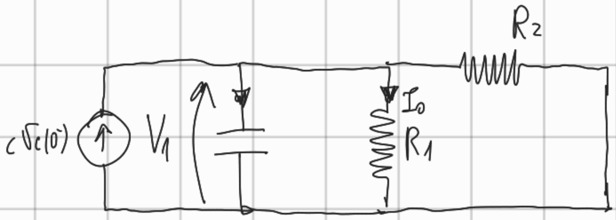
LAPLACE: Uso sovrapp. degli effetti:



$$V_A\left(sC + \frac{1}{R} + \frac{1}{R}\right) = \frac{E}{R}$$

$$V_A\left(\frac{2 + RCs}{R}\right) = \frac{E}{R}$$

$$V_A = \frac{E}{2 + RCs}$$



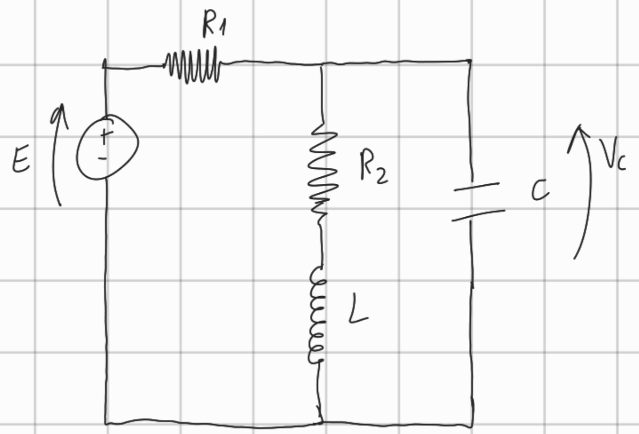
$$V_1 = \frac{cV_c(0^-)}{sC + \frac{2}{R}} = \frac{cV_c(0^-)}{\frac{2+RCs}{R}} = \frac{RCV_c(0^-)}{RCs+2}$$

$$V(s) = \frac{E}{RCs+2} + \frac{RCV_c(0^-)}{RCs+2} =$$

$$= \frac{E}{RC\left(s+\frac{2}{RC}\right)} + \frac{\cancel{RC}V_c(0^-)}{\cancel{RC}\left(s+\frac{2}{RC}\right)} =$$

$$= \frac{Es}{R} \cdot \frac{1}{s+\frac{2}{RC}} + \frac{V_c(0^-)}{s+\frac{2}{RC}}$$

③

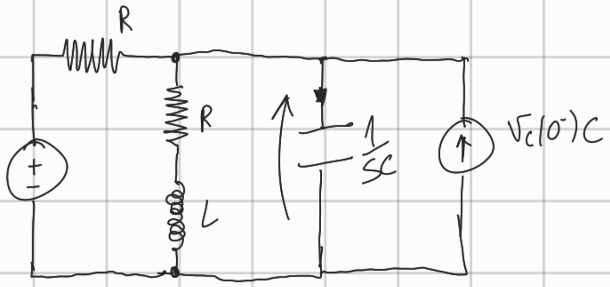


$R_1 = 10K\Omega \quad R_2 = 10K\Omega$

$L = 10mH \quad i_L(0^-) = 0$

$C = 1\mu F \quad V_c(0^-) = 1V$

$e(t) = 10u(t)$



$$U_A\left(\frac{1}{R} + \frac{1}{R+SL} + SC\right) = V_c(0^-)C + \frac{E}{R}$$

$$U_A\left(\frac{R+SL+R+RCS(R+SL)}{R(R+SL)}\right) = V_c(0^-)C + \frac{E}{R}$$

$$U_A\left(\frac{2R+SL+R^2CS+RLCS^2}{R^2+RLS}\right) = V_c(0^-)C + \frac{E}{R}$$

$$U_A = \frac{(V_c(0^-)C + E/R)R(R+LS)}{RLCS^2+(L+R^2C)S+2R}$$

$$I_A = (SL+R)U_A = \frac{RC\,V_C(0^-)}{RLC\left(S^2+\frac{(L+R^2C)}{RLC}S+\frac{2}{LC}\right)} + \frac{E}{RLC\left(S^2+\frac{(L+R^2C)}{RLC}S+\frac{2}{LC}\right)} =$$

$$= \frac{V_C(0^-)/L}{S^2+\frac{L+R^2C}{RLC}S+\frac{2}{LC}} + \frac{E/RLC}{S^2+\frac{(L+R^2C)}{RLC}S+\frac{2}{LC}}$$

$$\frac{L+R^2C}{RLC} = 1000100 \qquad \frac{2}{LC} = 2\cdot 10^8$$

$$S^2 + 1000100S + 2\cdot10^8 = 0$$

$$S_{1_2} = \begin{cases} -200 \\ -10^6 \text{ circa} \end{cases}$$

$$V_C(0^-)/L = 100 \qquad E/RLC = \frac{10^5}{S}$$

$$\frac{1}{(S+200)(S+10^6)} = \frac{A}{S+200} + \frac{B}{S+10^6}$$

$$A = \frac{1}{999800} \approx 10^{-6} \qquad B = -\frac{1}{999800} \approx -10^{-6}$$

$$\frac{1}{S(S+200)(S+10^6)} = \frac{A}{S} + \frac{B}{S+200} + \frac{C}{S+10^6}$$

$$A = \frac{1}{2\cdot10^8} \qquad B = -5.001\cdot10^{-9} \qquad C = 10^{-12}$$

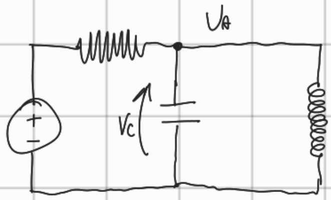
$$\downarrow$$

$$5\cdot10^{-9}$$

$$V(s) = \frac{10^{-4}}{s+200} - \frac{10^{-4}}{s+10^{6}} + \frac{5 \cdot 10^{-4}}{s} - \frac{5 \cdot 10^{-4}}{s+200} + \frac{10^{-7}}{s+10^{6}}$$

$$V(t) = 10^{-4} e^{-200t} - 10^{-4} e^{-10^{6}t} + 5 \cdot 10^{-4} u(t) - 5 \cdot 10^{-4} e^{-200t} + 10^{-8} e^{-10^{6}t}$$

3)



$R_1 = 100\,\Omega$ $\quad R_2 = 50\,\Omega$

$C = 10\,\mu F$ $\quad V_C(0^-) = 1\,V$

$L = 1\,mH$ $\quad i_L(0^-) = 0\,A$

$e(t) = \sin(1000t)\,u(t)\ V$

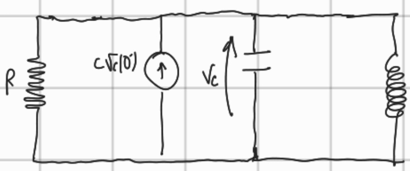
SOVRAPP. EFFETTI:

$$U_A\left(\frac{1}{R} + SC + \frac{1}{SL}\right) = \frac{E}{R}$$

$$U_A\left(\frac{SL + LRCS^2 + R}{SLR}\right) = \frac{E}{R}$$

$$U_A = \frac{ESL}{LRCS^2 + SL + R} = \frac{SE/_{RC}}{S^2 + \frac{S}{RC} + \frac{1}{LC}}$$

CASO 2: CONDIZ. INIZ.



$$V_C = \frac{CV_C(0^-)}{\frac{1}{R} + SC + \frac{1}{SL}} = \frac{CV_C(0^-)\cdot SLR}{LRCS^2 + SL + R} = \frac{RLCS\,V_C(0^-)}{LRCS^2 + SL + R} =$$

$$= \frac{SV_C(0^-)}{S^2 + \frac{S}{RC} + \frac{1}{LC}}$$

$$V_C(S) = \frac{S}{S^2 + \frac{S}{RC} + \frac{1}{LC}}\left(\frac{E}{RC} + V_C(0^-)\right)$$

$$V_C(s) = \frac{s}{s^2+1000s+10^8}\left(1000 \cdot \frac{1000}{s^2+10^6} + 1\right)$$

$$10^6 \cdot \frac{s}{(s^2+10^6)(s^2+1000s+10^8)}$$

$$= \frac{As+B}{s^2+10^6} + \frac{Cs+D}{s^2+1000s+10^8}$$

$$(As+B)(s^2+1000s+10^8) + (Cs+D)(s^2+10^6) =$$

$$As^3 + 1000As^2 + 10^8As + Bs^2 + 1000Bs + 10^8B +$$

$$+ Cs^3 + 10^6Cs + Ds^2 + 10^6D = 0$$

$$\begin{cases} A+C=0 \\ 1000A + B + D = 0 \\ 10^8A + 1000B + 10^6C = 10^6 \\ 10^8B + 10^6D = 0 \end{cases}$$

$$A \cong \frac{1}{100} \qquad B \cong \frac{1}{10}$$

$$C \cong -\frac{1}{100} \qquad D = -10.2$$

$$= \frac{10^{-2}s}{s^2+10^6} + \frac{10^{-1}}{s^2+10^6} - \frac{10^{-2}s}{s^2+1000s+10^8} - \frac{10.2}{s^2+1000s+10^8}$$

$$V(s) = \underbrace{\frac{10^{-2}s}{s^2+10^6}}_{f_2(t)} + \underbrace{\frac{10^{-1}}{s^2+10^6}}_{f_1(t)} + \frac{0.99\,s}{s^2+1000s+10^8} - \underbrace{\frac{10.2}{s^2+1000s+10^8}}_{f_3(t)}$$

$$f_1(t) = 10^{-4} \sin(1000\,t)$$

$$f_2(t) = 10^{-5} \cdot 1000 \cos(1000t) = 10^{-2} \cos(1000t)$$

$$10^8 = \omega_n^2 \Rightarrow \omega_n = 10^4$$

$$1000 = \omega_n 2 \delta \Rightarrow \quad \delta = \frac{1000}{2 \cdot 10^4} = \frac{1}{20}$$

$$f_3(t) = -10.2 \left(10^{-4} e^{-500t} \sin(10000t) \right) u(t)$$

$$f_4(t) = 0.99 \left(-\frac{1}{20} e^{-500t} \sin(10000t) + e^{-500t} \cos(10000t) \right) u(t)$$