ALCUNE CONSIDERAZIONI:

- É possibile che una piccola parte degli esercizi sia scorretta
- Alcuni esercizi possono essere risolti in modi diversi, ma ugualmente corretti
- Gli esercizi sono svolti in ordine cronologico, quindi più si va in fondo nel file più sará probabile che con più esperienza alle spalle saranno corretti.

BUON LAVORO!

5)

$f_c = 15\,kHz$

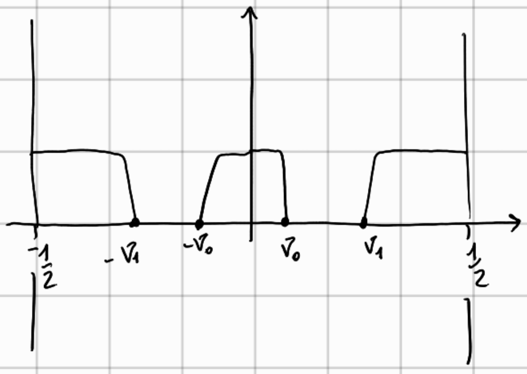
$\nu = \frac{f}{f_c} \Rightarrow \nu_0 = \frac{f_0}{f_c} = \frac{2}{15}$

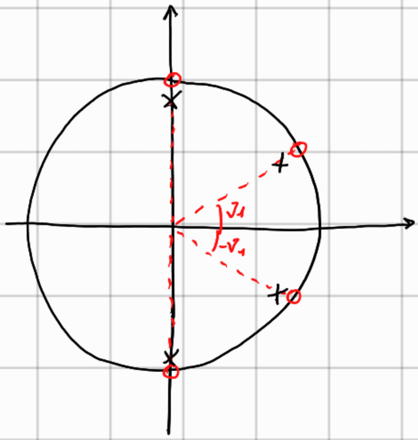
$$h[n] = \int_{-\frac{1}{2}}^{-\frac{2}{15}} e^{2\pi j \nu n}\, d\nu + \int_{\frac{2}{15}}^{\frac{1}{2}} e^{2\pi j \nu n}\, d\nu =$$

$$= \left.\frac{e^{2\pi j \nu n}}{2\pi j n}\right|_{-\frac{1}{2}}^{-\frac{2}{15}} + \left.\frac{e^{2\pi j \nu n}}{2\pi j n}\right|_{\frac{2}{15}}^{\frac{1}{2}} = \frac{e^{-\frac{4}{15}\pi j n} - e^{-\pi j n} + e^{\pi j n} - e^{\frac{4}{15}\pi j n}}{2\pi j n} =$$

$$= \cancel{\frac{\sin(\pi n)}{\pi n}}^{\to 0} - \frac{\sin\left(\frac{4}{15}\pi n\right)}{\pi n} = -\frac{4}{15}\,\mathrm{sinc}\left(\frac{4}{15} n\right)$$

3) $f_c = 20\,kHz$ $\nu_0 = \frac{f_0}{f_c} = \frac{3}{20}$ $\nu_1 = \frac{5}{20} = \frac{1}{4}$





$$H(z) = \frac{(z - e^{j2\pi\nu_0})(z - e^{-j2\pi\nu_0})(z+j)(z-j)}{(z - p_1 e^{j2\pi\nu_0})(z - p_1 e^{-j2\pi\nu_0})(z + p_2 j)(z - p_2 j)} =$$

$$= \frac{(z^2+1)(z^2 - 2\cos(2\pi\nu_0)z + 1)}{(z^2 + p_2^2)(z^2 - 2p_1\cos(2\pi\nu_0)z + p_1^2)} =$$

$$= \frac{z^4 - 2\cos(2\pi\nu_0)z^3 + z^2 + z^2 - 2\cos(2\pi\nu_0)z + 1}{z^4 - 2p_1\cos(2\pi\nu_0)z^3 + p_1^2 z^2 + p_2^2 z^2 - 2p_1 p_2^2\cos(2\pi\nu_0)z + p_1^2 p_2^2} =$$

$$= \frac{z^{-4} - 2\cos(2\pi\nu_0)z^{-3} + 2z^{-2} - 2\cos(2\pi\nu_0)z^{-1} + 1}{p_1^2 p_2^2 z^{-4} - 2p_1 p_2^2\cos(2\pi\nu_0)z^{-3} + (p_1^2 + p_2^2)z^{-2} - 2p_1\cos(2\pi\nu_0)z^{-1} + 1}$$

6)

$s(t) \,/\, S(f) = \Pi\left(\frac{f}{2B}\right)$, $B = 1000\ Hz$ $\qquad$ $f_c > 2 f_{max}$? No!

$f_c = 1500$ campioni/s $\Rightarrow$ $f_c = 1500\ Hz$

$s(t) = 2B\,\mathrm{sinc}(2Bt)$

$\nu f_c = f$

$S_a\left(\frac{\nu}{T}\right) = \Pi\left(\frac{\nu}{2TB}\right) = \Pi\left(\frac{3}{2}\nu\right)$

$$S(\nu) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} S_a\left(f - \frac{m}{T}\right) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} S_a\left(\frac{\nu - m}{T}\right)$$

$$s[m] = \int_{-\frac{1}{2}}^{\frac{1}{2}} S(\nu) e^{j2\pi\nu m} d\nu = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{T} e^{j2\pi\nu m} d\nu = \frac{1}{T} \left.\frac{e^{j2\pi\nu m}}{j2\pi m}\right|_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{\sin(\pi m)}{\pi T m} = \frac{1}{T}\mathrm{sinc}(\pi m)$$

$$s_r(t) = \sum_{m=-\infty}^{+\infty} \mathrm{sinc}(\pi m)\,\mathrm{sinc}\left(\frac{t - mT}{T}\right)$$

5)

$f_c = 15KHz \qquad f = 2000Hz$

$n \neq 0$

$$h[n] = \int_{-\frac{1}{2}}^{-\frac{2}{15}} e^{2\pi j \nu n} d\nu + \int_{\frac{2}{15}}^{\frac{1}{2}} e^{+2\pi j \nu n} d\nu = \left.\frac{e^{2\pi j \nu n}}{2\pi j n}\right|_{-\frac{1}{2}}^{-\frac{2}{15}} + \left.\frac{e^{2\pi j \nu n}}{2\pi j n}\right|_{\frac{2}{15}}^{\frac{1}{2}} = \frac{e^{-\frac{4}{15}\pi j n} - e^{-\pi j n}}{2\pi j n} +$$

$$+ \frac{e^{\pi j n} - e^{\frac{4}{15}\pi j n}}{2\pi j n} =$$

Se $n=0$: $h[0] = -\frac{2}{15} + \frac{1}{2} + \frac{1}{2} - \frac{2}{15} =$

$1 - \frac{4}{15} = \frac{11}{5}$

$$= \frac{e^{\pi j n} - e^{-\pi j n}}{2\pi j n} - \frac{e^{\frac{4}{15}\pi j n} - e^{-\frac{4}{15}\pi j n}}{2\pi j n} =$$

$$= \cancel{\text{sinc}(\pi n)} - \frac{\sin\left(\frac{4}{15}\pi n\right)}{\pi n} =$$

$$= -\frac{4}{15}\,\text{sinc}\left(\frac{4}{15} n\right)$$

$$h[n] = \frac{11}{15}\delta[n] - \frac{4}{15}\,\text{sinc}\left(\frac{4}{15} n\right)$$

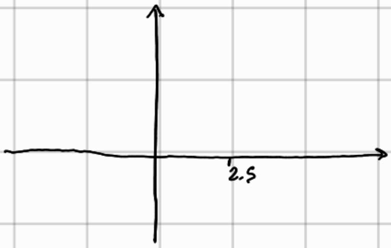
6)

$x(t) = \sin^2(5t) = \frac{1}{2} - \frac{1}{2}\cos(10t)$ $\qquad f_c = 2.5\,Hz$

$\downarrow$
$2\pi f = 10$
$\frac{5}{\pi}$

$f_c > 2 f_{max}$?

$S_a(f) = \frac{1}{2}\delta(f) - \frac{1}{4}\delta(f - \frac{5}{\pi}) - \frac{1}{4}\delta(f + \frac{5}{\pi})$



$S_r(f) = I(f)\, S_{DTFT}(fT)$ $\qquad I(f) = F[\operatorname{sinc}\frac{t}{T}] = \frac{1}{T}\Pi\left(\frac{f}{\frac{1}{T}}\right)$

$S_{DTFT}(f) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} S_a(f - \frac{m}{T})$

$I(f) = T\,\Pi\left(\frac{f}{1/T}\right)$

$S_r(f) = \frac{2}{5}\cdot\frac{5}{2}\left[\frac{1}{2}\delta(f) - \frac{1}{4}\delta(f + \frac{5}{\pi} - 2.5) - \frac{1}{4}\delta(f - \frac{5}{\pi} + 2.5)\right) =$

$S_r(t) = \frac{1}{2} - \frac{1}{2}\cos\left(2\pi\left(-\frac{5}{\pi} + 2.5\right)t\right) = \sin^2\left((2.5\pi - 5)t\right)$

6)

$$x(t) = \cos(2\pi t) - \frac{1}{2}\cos(3\pi t) \qquad f_c = 2.2\,Hz$$

$f_{max} = \frac{3}{2}\,Hz \Rightarrow 2f_{max} = 3\,Hz > f_c$. Ci saranno perdite.

$$S_r(f) = I(f)\,S_{DTFT}(fT) \qquad \text{con } I(f) = F\left[\text{sinc}\left(\tfrac{t}{T}\right)\right] = T\,\Pi\left(\frac{f}{1/T}\right)$$

$$S_a(f) = \frac{1}{2}\delta(f-1) + \frac{1}{2}\delta(f+1) - \frac{1}{4}\delta\left(f-\frac{3}{2}\right) - \frac{1}{4}\delta\left(f+\frac{3}{2}\right)$$

$$S_{DTFT}(fT) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} S_a\left(f - \frac{m}{T}\right)$$

Caso $m=0$:

$$S_a(f) = \frac{1}{2}\delta(f-1) + \frac{1}{2}\delta(f+1) - \frac{1}{4}\cancel{\delta\left(f-\frac{3}{2}\right)} - \frac{1}{4}\cancel{\delta\left(f+\frac{3}{2}\right)}$$

$$S_a\left(f-\frac{1}{T}\right) = \frac{1}{2}\cancel{\delta(f-3.2)} + \frac{1}{2}\cancel{\delta(f-1.2)} - \frac{1}{4}\cancel{\delta(f-3.7)} - \frac{1}{4}\delta(f-0.7)$$

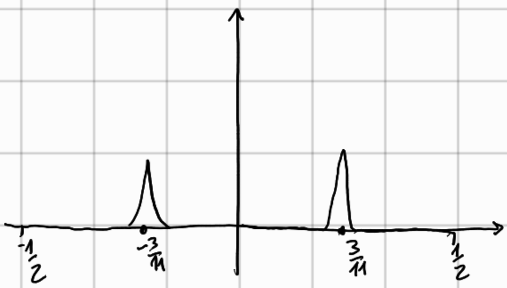
$$S_a\left(f+\frac{1}{T}\right) = \frac{1}{2}\cancel{\delta(f-1.2)} + \frac{1}{2}\cancel{\delta(f+3.2)} - \frac{1}{4}\delta(f+0.7) - \frac{1}{4}\cancel{\delta(f+3.7)}$$

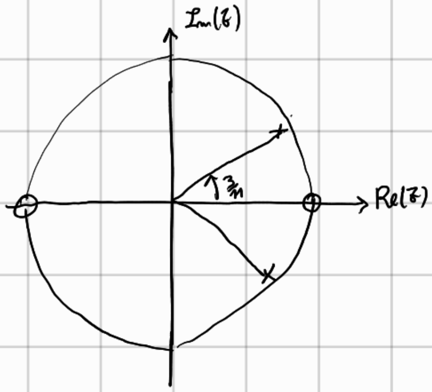
Mi fermo a $[-1.1, 1.1]$ Hz.

$$S_r(f) = I(f)\,S_{DTFT}(fT) = \frac{\cancel{T}}{\cancel{T}}\left[\frac{1}{2}\delta(f-1) + \frac{1}{2}\delta(f+1) - \frac{1}{4}\delta(f+0.7) - \frac{1}{4}\delta(f-0.7)\right]$$

$$S_r(t) = \cos(2\pi t) - \frac{1}{2}\cos(2\pi\cdot 0.7\,t)$$

5)





$f_c = 22kHz$

$$\nu_0 = \frac{6}{22} = \frac{3}{11}$$

$$H(z) = \frac{(z-1)(z+1)}{(z-\rho e^{j2\pi\nu_0})(z-\rho e^{-j2\pi\nu_0})} = \frac{z^2-1}{z^2-2\rho\cos(2\pi\nu_0)z+\rho^2} =$$

$$= \frac{1-z^{-2}}{1-2\rho\cos(2\pi\nu_0)z^{-1}+\rho^2 z^{-2}}$$

$$y[n] = 2\rho\cos(2\pi\nu_0)\,y[n-1] - \rho^2 y[n-2] +$$
$$+ x[n] - x[n-2]$$

$$T_X = \frac{N_c T_c}{N_p} \qquad \frac{1}{T_x} = \frac{N_p}{N_c T_c} \Rightarrow \frac{N_p}{(N_c+1)T_c} = \frac{N_p}{N_c T_c + T_c} =$$

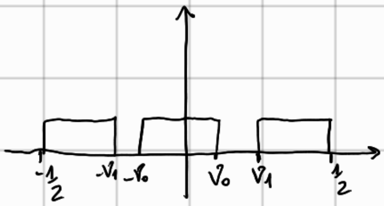
$$\Downarrow$$

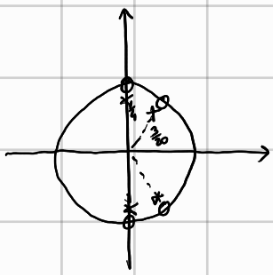
$$\Delta T_x = \frac{T_c}{N_p} \qquad \frac{N_p}{T_c} ?$$

$$\frac{\Delta T_x}{T_x} = \frac{\Delta F_x}{F_x} = \frac{1}{N_c}$$

$$\Delta F_x = \frac{1}{N_c T_x}$$

5)





$$H(z) = \frac{(z-j)(z+j)(z-e^{-2\pi j\frac{3}{20}})(z+e^{-2\pi j\frac{3}{20}})}{(z-\rho j)(z+\rho j)(z-\rho_1 e^{-2\pi j\frac{3}{20}})(z+\rho_1 e^{-2\pi j\frac{3}{20}})} = \frac{(z^2+1)(z^2-2\cos(\frac{3\pi}{10})z+1)}{(z^2+\rho^2)(z^2-2\rho_1\cos(\frac{3}{10}\pi)z+\rho_1^2)} =$$

$$= \frac{z^4-2\cos(\frac{3}{10}\pi)z^3+2z^2-2\cos(\frac{3}{10}\pi)z+1}{z^4-2\rho_1\cos(\frac{3}{10}\pi)z^3+(\rho_1^2+\rho^2)z^2-2\rho_1\rho^2\cos(\frac{3}{10}\pi)z+\rho^2\rho_1^2}$$

$$H(z) = \frac{1-2\cos(\frac{3}{10}\pi)z^{-1}+2z^{-2}-2\cos(\frac{3}{10}\pi)z^{-3}+z^{-4}}{1-2\rho_1\cos(\frac{3}{10}\pi)z^{-1}+(\rho_1^2+\rho^2)z^{-2}-2\rho_1\rho^2\cos(\frac{3}{10}\pi)z^{-3}+\rho_1^2\rho^2 z^{-4}}$$

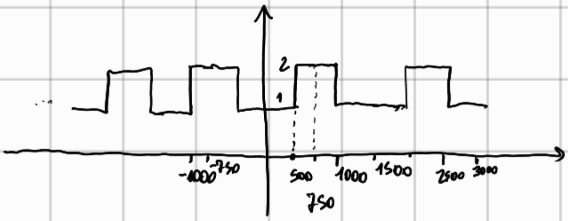
$$y[n] = 2\rho_1\cos\left(\frac{3}{10}\pi\right)y[n-1]-(\rho_1^2+\rho^2)y[n-2]+2\rho_1\rho^2\cos\left(\frac{3}{10}\pi\right)y[n-3]-\rho_1^2\rho^2y[n-4]+$$

$$+\ x[n]-2\cos\left(\frac{3}{10}\pi\right)x[n-1]+2x[n-2]-2\cos\left(\frac{3}{10}\pi\right)x[n-3]+x[n-4]$$

6) $S(f) = \Pi\left(\frac{f}{2B}\right)$ $\quad B = 1000 Hz$

$f_c = 1500 Hz$

$$S_{DTFT}(fT) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} S_a\left(f - \frac{m}{T}\right) = \frac{1}{T}\left[\Pi\left(\frac{f-1500}{2000}\right) + \Pi\left(\frac{f}{2000}\right) + \Pi\left(\frac{f+1500}{2000}\right) + \dots\right]$$



$S_r(f) = I(f)\, S_{DTFT}(fT)$ $\qquad I(f) = T\,\Pi\left(\frac{f}{\frac{1}{T}}\right) = \frac{1}{1500}\Pi\left(\frac{f}{1500}\right)$

$$T\cdot\frac{1}{T}\cdot\Pi\left(\frac{f}{1500}\right)\left[\Pi\left(\frac{f-1500}{2000}\right) + \Pi\left(\frac{f}{2000}\right) + \Pi\left(\frac{f+1500}{2000}\right) + \dots\right] =$$

$$= 2\,\Pi\left(\frac{f}{1500}\right) - \Pi\left(\frac{f}{1000}\right)$$

$S_r(t) = 2\cdot 1500\ \mathrm{sinc}(1500t) - 1000\ \mathrm{sinc}(1000t)$

6) $x(t) = \cos(10000\pi t) + \cos(2000\pi t) \qquad f_c = 6200\,Hz$

$$X_a(f) = \frac{1}{2}\delta(f-5000) + \frac{1}{2}\delta(f+5000) + \frac{1}{2}\delta(f-1000) + \frac{1}{2}\delta(f+1000)$$

$$\frac{1}{2T} = 3100\,Hz$$

$$S_{DTFT}(fT) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} S_a\left(f-\frac{m}{T}\right) = \frac{1}{T}\cdot\frac{1}{2}\Big[\cancel{\delta(f-5000)} + \cancel{\delta(f+5000)} + \delta(f-1000) + \delta(f+1000) +$$

$$+ \cancel{\delta(f-11200)} + \delta(f-1200) + \cancel{\delta(f-7200)} + \cancel{\delta(f-5200)} +$$

$$+ \delta(f+1200) + \cancel{\delta(f+11200)} + \cancel{\delta(f+5200)} + \cancel{\delta(f+7200)} +$$

$$\ldots\Big]$$

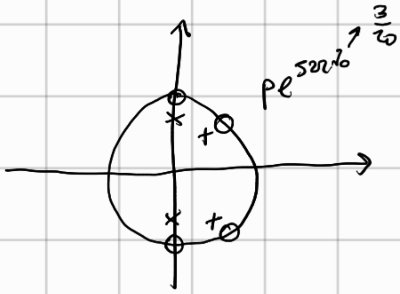
$$S_r(f) = T\,\Pi\left(\frac{f}{\frac{1}{T}}\right) S_{DTFT}(fT) = T\cdot\frac{1}{T}\cdot\frac{1}{2}\Big[\delta(f-1000) + \delta(f+1000) + \delta(f-1200) + \delta(f+1200)\Big]$$

$$S_r(t) = \cos(2000\pi t) + \cos(2400\pi t)$$

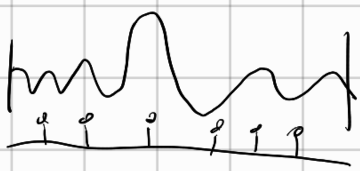
5)

$f \in [3000, 5000]$ kHz

$f_c = 20$ kHz



5)



$h_W[n] = h[n]\, w[n]$

$h_{FIR}[n] = h_W[n-M]$

$T_X N_P = T_C N_C$

$T_X = \frac{T_C N_C}{N_P}$

$\Delta T_X = \frac{T_C}{N_P}$

$\Gamma_{T_X} = \frac{1}{N_C}$

$\frac{\Delta T_X}{T_X} = \frac{\Delta F_X}{F_X}$

$\Downarrow$

$\Delta T_X = \Delta F_X \cdot T_X^2$

0.000001

$1 \cdot 10^{-6}$

$\frac{1}{mHz} = \frac{1}{10^{-3} s^{-1}} =$

6)

$x(t) = \sin^2(5t) = \frac{1}{2} - \frac{1}{2}\cos(10t)$ $\quad 2\pi f = 10 \Rightarrow f = \frac{5}{\pi} \simeq 1.59$

$f_c = 2.5\,Hz \Rightarrow \frac{1}{2T} = 1.25\,Hz$ $\quad S_a(f) = \frac{1}{2}\delta(f) - \frac{1}{4}\delta(f-\frac{5}{\pi}) - \frac{1}{4}\delta(f+\frac{5}{\pi})$

$X_{DTFT}(fT) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} S_a(f - \frac{n}{T})$

$S_a(f) = \frac{1}{2}\delta(f) - \frac{1}{4}\delta(f-1.59) - \frac{1}{4}\delta(f+1.59)$

$S_a(f-2.5) = \frac{1}{2}\delta(f-2.5) - \frac{1}{4}\delta(f-4.09) - \frac{1}{4}\delta(f-0.91)$

$S_a(f+2.5) = \frac{1}{2}\delta(f+2.5) - \frac{1}{4}\delta(f+0.91) - \frac{1}{4}\delta(f+4.09)$

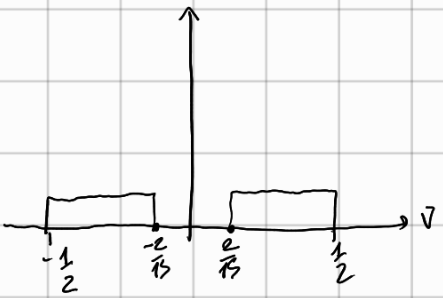
$S_r(f) = T\,\Pi\left(\frac{f}{1/T}\right) X_{DTFT}(fT) = \frac{1}{2}\delta(f) - \frac{1}{4}\delta(f-0.91) - \frac{1}{4}\delta(f+0.91)$

$S_r(t) = \frac{1}{2} - \frac{1}{2}\cos(5.72t)$

5)

$f_c = 15kHz$

$\nu_0 = \frac{2}{15}$



$$h[n] = \int_{\frac{2}{15}}^{\frac{1}{2}} e^{2\pi j \nu n}\, d\nu + \int_{-\frac{1}{2}}^{-\frac{2}{15}} e^{2\pi j \nu n}\, d\nu = \frac{e^{2\pi j \nu n}}{2\pi j n}\Bigg|_{\frac{2}{15}}^{\frac{1}{2}} + \frac{e^{2\pi j \nu n}}{2\pi j n}\Bigg|_{-\frac{1}{2}}^{-\frac{2}{15}} =$$

$n \neq 0$

$$= \frac{e^{\pi j n} - e^{\frac{4}{15}\pi j n} + e^{-\frac{4}{15}\pi j n} - e^{-\pi j n}}{2\pi j n} =$$

$n=0 \Rightarrow \frac{1}{2} - \frac{2}{15} + \left(-\frac{2}{15} + \frac{1}{2}\right) = 1 - \frac{4}{15}$

$$= \operatorname{sinc}(\pi n) - \frac{\sin\left(\frac{4}{15}\pi n\right)}{\pi n} =$$

$$= -\frac{4}{15}\operatorname{sinc}\left(\frac{4}{15} n\right)$$

$$h[n] = \delta[n] - \frac{4}{15}\operatorname{sinc}\left(\frac{4}{15} n\right)$$

$$h_0[n-M] = h_W[n-M]$$