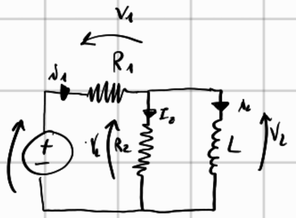
ALCUNE CONSIDERAZIONI:

- É possibile che una piccola parte degli esercizi sia scorretta
- Alcuni esercizi possono essere risolti in modi diversi, ma ugualmente corretti
- Gli esercizi sono svolti in ordine cronologico, quindi più si va in fondo nel file più sarà probabile che con più esperienza alle spalle saranno corretti.

BUON LAVORO!



$i_L(0) = 0{,}4\,A$

$L = 0{,}1\,H$

$R_1 = 1000\,\Omega$

$R_2 = 200\,\Omega$

$E = 220\,V \ (t>0)$

NOTA: Ci evolviamo in regime costante

Studio circuito con equazione.

$$E - v_1 - v_L = 0$$

$$i_2 + i_L - i_1 = 0$$

$$v_1 = E - v_L = E - L\frac{di_L}{dt}$$

$$G_2 L\frac{di_L}{dt} + i_L - G_1 E + G_1 L\frac{di_L}{dt} = 0$$

$$\begin{cases} \dfrac{di_L}{dt} L(G_1+G_2) + i_L = G_1 E \\ i_L(0^+) = 0{,}4\,A \end{cases}$$

SOLUZIONE GENERALE:

$$i_L(t) = A e^{-\frac{t}{\tau}} + i_{L\,cost}$$

$$\Rightarrow \frac{di_L}{dt} = -\frac{A}{\tau} e^{-\frac{t}{\tau}}$$

$$-L(G_1+G_2)\frac{A}{\tau} e^{-\frac{t}{\tau}} = G_1 E - A e^{-\frac{t}{\tau}} - i_{L\,cost}$$

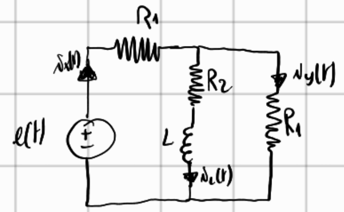
$$\Rightarrow \tau = L(G_1+G_2)$$

$i_{L\,cost} = G_1 E$ $\qquad$ $i_L(0^+) = A + G_1 E$

$\Rightarrow A$ lo ricavo

$$i_L(t) = (0{,}4 - G_1 E)\, e^{-\frac{t}{L(G_1+G_2)}} + G_1 E$$

1.4)



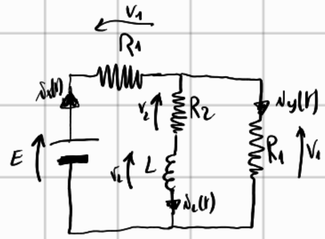
$$e(t)=\begin{cases}10\,V & t<0\\ -10\,V & t>0\end{cases}$$

$R_1=10\,\Omega$

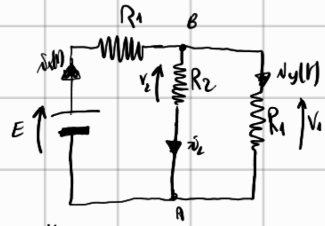
$R_2=20\,\Omega$

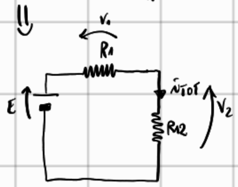
$L=2\,mH$

Se ho $t<0$ Generatore diventa batteria



Ci troviamo in una situaz. costante: $V_L=0$ perché $\frac{di_L}{dt}=0$

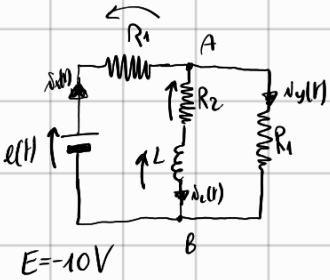




$$R_{12}=\left(\frac{1}{10}+\frac{1}{20}\right)^{-1}=\frac{20}{3}\,\Omega$$

$$i_{tot}=\frac{E}{R_{12}+R_1}=\frac{10}{\frac{20}{3}+10}=10\cdot\frac{3}{50}=\frac{3}{5}A$$

$$i_L=i_{tot}\frac{R_1}{R_1+R_2}=\frac{\frac{3}{5}\cdot 10}{30}=\frac{1}{5}A=i_L(0^-)=i_L(0^+)$$



$E = -10V$

$$\begin{cases} i_x - i_y - i_L = 0 \\ E - V_1 - V_2 - V_L = 0 \\ E - V_1 - V_{01} = 0 \end{cases}$$

$E - i_x R_1 - i_L R_2 - V_L = 0$

$E - i_x R_1 - i_y R_1 = 0$

$\Rightarrow \quad i_x = +i_L + i_y$

$E - i_L R_1 - i_y R_1 - i_L R_2 - V_L = 0$

$$i_y = \frac{E}{R_1} - i_L - i_L \frac{R_2}{R_1} - \frac{V_L}{R_1}$$

$\Rightarrow \quad i_L R_2 + V_L - i_y R_1 = 0$

$\Rightarrow \quad i_L R_2 + V_L - E + i_L R_1 + i_L R_2 + V_L = 0$

$2 i_L R_2 + i_L R_1 + 2V_L - E = 0$

$$\begin{cases} 2L \dfrac{d i_L}{dt} + i_L (2R_2 + R_1) = E \\ i_L(0^+) = \dfrac{1}{5} A \end{cases}$$

$i_L(t) = A e^{-\frac{t}{\tau}} + i_{L cost}$

$$\frac{d i_L}{dt} = -\frac{A}{\tau} e^{-\frac{t}{\tau}}$$

$$\Rightarrow \quad -\frac{2LA}{\tau} e^{-\frac{t}{\tau}} = E - (2R_2 + R_1) A e^{-\frac{t}{\tau}} - (2R_2 + R_1) i_{L cost}$$

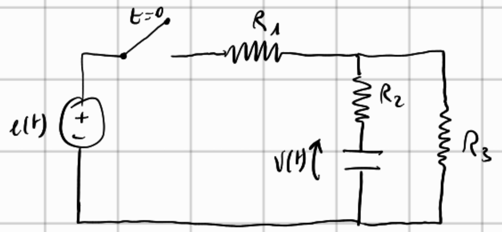
$$\Rightarrow \quad i_{L cost} = \frac{E}{2R_2 + R_1}$$

$$\tau = \frac{2L}{2R_2 + R_1} \qquad i_L(0^+) = A + \frac{E}{2R_2 + R_1}$$

$$\Rightarrow \quad i_L(t) = \left(\frac{1}{5} - \frac{E}{2R_2 + R_1}\right) e^{-\frac{(2R_2 + R_1)t}{2L}} + \frac{E}{2R_2 + R_1}$$

$$\Rightarrow \quad i_L(t) = \frac{2}{5} e^{-12500t} - \frac{1}{5}$$

1.5)



$e(t)=10\cos(\omega t) = \sqrt{2}\cdot\frac{10}{\sqrt{2}}\cos(\omega t)$

$\omega=100$ rad/s

$R_1=20\Omega$ $R_2=5\Omega$

$R_3=10\Omega$ $C=1mF$

$\tau=?$

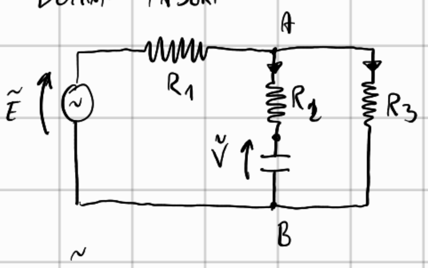
$V_c(t)=?$ se $t>0$

Se $t<0$ circuito aperto.

$\Rightarrow$ $\tilde{V}_c=0$ e $V(t)=0$ perché costante

Regime sinusoidale. Condizioni iniziali a 0.

Dominio FASORI



$\tilde{U}_B=0$

Impedenza totale di AB $=\left(\frac{1}{R_2}-\frac{j}{C\omega}\right)=\left(\frac{C\omega-R_2 j}{C\omega R_2}\right)$

$$\Rightarrow \tilde{U}_A\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{C\omega R_2}{C\omega-R_2 j}\right)=\frac{\tilde{E}}{R_1}$$

$$\tilde{U}_A\left(\frac{3}{20}+\frac{1/2}{\frac{1}{10}-5j}\right)=\frac{\sqrt{2}}{4}$$

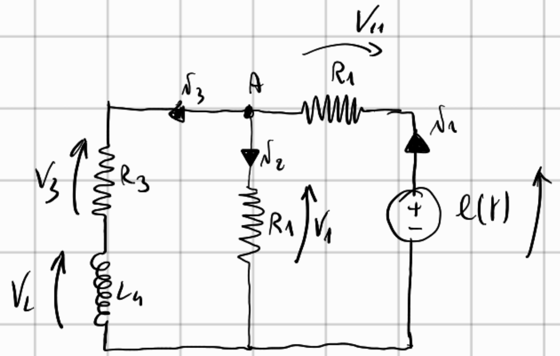
$$\frac{1}{2}\cdot\frac{\frac{1}{10}+5j}{\frac{1}{100}+25}=\frac{1}{2}\cdot\frac{2501}{100}\left(\frac{1}{10}+5j\right)$$

$$\tilde{U}_A\left(\frac{3}{20}+\frac{2501}{2000}+\frac{2501}{40}j\right)=\frac{\sqrt{2}}{4}$$

$$\tilde{U}_A\left(1,4+62,5j\right)=0,35$$

$$\tilde{U}_A = \frac{0{,}35}{1{,}4+62{,}5j} = \frac{0{,}35(1{,}4-62{,}5j)}{2+3906} = \frac{0{,}5}{3908} - \frac{21{,}9}{3908}j = 0{,}0001 - 0{,}006j$$

Quesito 2)



$e(t) = 10\sqrt{2}\sin(1000t)\cdot u(t)$ V

$R_1 = 2\,\Omega \quad R_3 = 1\,\Omega \quad L_4 = 1\,mH$

Trova energia immagazz. in induttore

$$\begin{cases} e - V_{11} - V_1 = 0 \\ e - V_{11} - V_3 - V_L = 0 \\ i_1 - i_2 - i_L = 0 \end{cases}$$

$\Rightarrow \quad i_1 = i_L + i_2$

$e - R_1 i_1 - R_1 i_2 = 0$

$e - R_1 i_1 - i_L R_3 - V_L = 0$

$\Rightarrow \; e - R_1 i_L - R_1 i_2 - R_1 i_2 = 0 \Rightarrow R_1 i_2 = \frac{1}{2}(e - R_1 i_L)$

$R_1 i_2 - i_L R_3 - V_L = 0$

$e - R_1 i_L - 2 R_3 i_L - 2 V_L = 0$

$e - R_1 i_L - 2 R_3 i_L - 2 L \frac{d i_L}{dt} = 0$

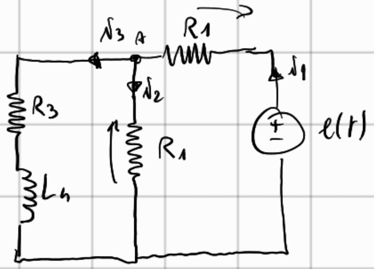
$$\begin{cases} \frac{d i_L}{dt} + \frac{i_L}{L}(R_1 + 2R_3) = \frac{e}{L} \\ i_L(0^+) = 0 \end{cases}$$

$e(t) = 10\sqrt{2} \sin(1000t) \cdot u(t)$ V

$R_1 = 2\,\Omega \quad R_3 = 1\,\Omega \quad L_4 = 1\,mH$

$$\begin{cases} f'(t) + 4000 f(t) = \sqrt{2} \cdot 10000 \sin(1000t) \\ f(0) = 0 \end{cases}$$

ESERCIZIO 2



$e(t) = 10\sqrt{2}\sin(1000t)\cdot u(t)$ V

$R_1 = 2\,\Omega$ $\quad R_3 = 1\,\Omega$ $\quad L_4 = 1$ mH

Note: Quando ho $t<0$, regime stazionario e tutto è spento $\Rightarrow i_L = 0$

$\Rightarrow i_L(0^-) = i_L(0^+) = 0$

Scrivo equazione differenziale:

$i_2 + i_L = i_1$ $\qquad\qquad i_2 = i_1 - i_L$

$e - R_1 i_1 - R_1 i_2 = 0 \quad \Rightarrow \quad R_1 i_1 = e - R_1 i_2 = e - R_1 i_1 + R_1 i_L$

$e - i_L R_3 - L\dfrac{di_L}{dt} - R_1 i_1 = 0 \qquad \Downarrow$

$R_1 i_1 = \dfrac{e + R_1 i_L}{2}$

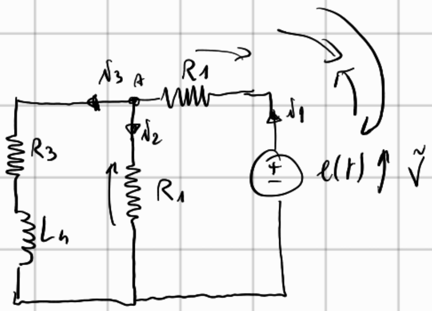
$\rightarrow \; e - i_L R_3 - L\dfrac{di_L}{dt} - \dfrac{e + R_1 i_L}{2} = 0$

$\dfrac{e}{2} = i_L R_3 + \dfrac{R_1 i_L}{2} + L\dfrac{di_L}{dt}$

$$\begin{cases} \dfrac{di_L}{dt} + \left(R_3 + \dfrac{R_1}{2}\right)\dfrac{1}{L}\, i_L = \dfrac{e}{2L} \\ i_L(0^+) = 0 \end{cases}$$

$i_G(t) = \sqrt{2}\, I_L^+ \sin(100t + \varphi^+)$
perché si va in regime.

Trovo $i_g(t)$



$e(t) = 10\sqrt{2}\sin(1000t)\cdot u(t)\ V$

$R_1 = 2\,\Omega \quad R_3 = 1\,\Omega \quad L_n = 1\,mH$

$\dot{Z}_L = j\omega L \quad \dot{Z}_3 = R_3 \quad \dot{Z}_{R_1} = R_1$

$\tilde{V} - \tilde{U}_A ?$

$t > 0 \Rightarrow \tilde{V} = 10 e^{j0}$

$$\tilde{U}_A\left(\frac{1}{R_3 + j\omega L} + \frac{1}{R_1} + \frac{1}{R_1}\right) = \frac{\tilde{V}}{R_1}$$

$$\tilde{U}_A\left(\frac{1}{1+j} + 1\right) = \frac{\tilde{V}}{2}$$

$$\frac{1}{1+j} = \frac{1-j}{1+1} = \frac{1}{2} - j\frac{1}{2}$$

$$\Rightarrow \tilde{U}_A\left(\frac{3}{2} - j\frac{1}{2}\right) = 5$$

$$\Rightarrow \tilde{U}_A = \frac{5}{\frac{3}{2} - j\frac{1}{2}} = \frac{10}{3-j} = \frac{30}{10} + \frac{10j}{10} = 3 + j$$

$$\tilde{I}_3 = \tilde{U}_A \cdot \frac{1}{1+j} = \tilde{U}_A\left(\frac{1}{2} - j\frac{1}{2}\right) = \frac{1}{2}(3+j)(1-j) =$$

$$\frac{1}{2}(3 - 3j + j + 1) = \frac{1}{2}(4 - 2j) = 2 - j$$

$\tilde{I}_2 = \tilde{U}_A \frac{1}{R_1} = \left(\frac{3}{2} + \frac{1}{2}j\right) A \qquad \tilde{I}_1 = (\tilde{V} - \tilde{U}_A)\frac{1}{R_1} = \frac{7}{2} - \frac{j}{2}$

$\tilde{I}_3 = (2 - j)\,A$

$\Rightarrow |\tilde{I}_3| = \sqrt{65} \qquad \arg(\tilde{I}_3) = \sin^{-1}\left(\frac{1}{\sqrt{65}}\right)$

$i_g(t) = \sqrt{130}\sin\left(100t + \sin^{-1}\left(\frac{1}{\sqrt{65}}\right)\right)$

$i(t) = A e^{-\frac{R_{eq}t}{L}} + \sqrt{130}\sin\left(100t + \sin^{-1}\left(\frac{1}{\sqrt{65}}\right)\right)$

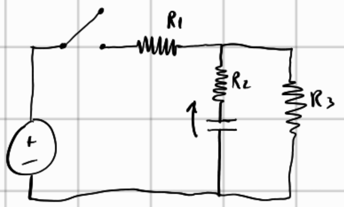
$i(0^+) = 0 \Rightarrow A + \sqrt{130}\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{65}}\right)\right)$

$A = -\sqrt{2}$

$i(t) = -\sqrt{2}\, e^{-\frac{R_{eq}t}{L}} + \sqrt{130}\sin\left(100t + \sin^{-1}\left(\frac{1}{\sqrt{65}}\right)\right)$

Calcoli da rifare con $\tilde{I}_3$ corretto

1.5)



$e(t) = 10\cos(\omega t)$

$\omega = 100\ rad/s$

$R_1 = 20\Omega$ $R_2 = 5\Omega$

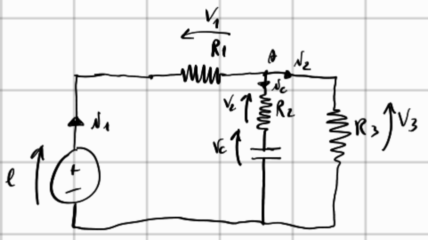
$R_3 = 10\Omega$ $C = 1mF$

Se circuito aperto, non c'è corrente e tensione nel condensatore.

$V_c = 0$ $i_c = 0$

$\downarrow$

$V_c(0^-) = 0 = V_c(0^+)$ condiz. iniziale



$$\begin{cases} i_1 = i_2 + i_c \Rightarrow i_2 = i_1 - i_c \\ e - R_1 i_1 - R_2 i_c - V_c = 0 \\ e - R_1 i_1 - R_3 i_2 = 0 \end{cases}$$

$\Rightarrow$ $e - R_1 i_1 - R_3 i_1 + R_3 i_c = 0$

$$i_1 = \frac{e + R_3 i_c}{R_1 + R_3}$$

$$\Rightarrow \quad V_c + R_2 i_c + R_1 \cdot \frac{e + R_3 i_c}{R_1 + R_3} = e$$

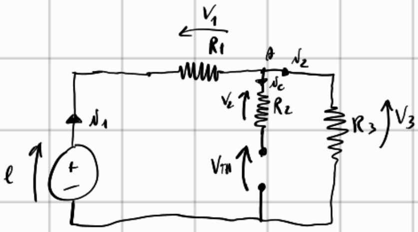
$$(R_1 + R_3)V_c + R_2(R_1 + R_3)i_c + R_1 e + R_1 R_3 i_c = e(R_1 + R_3)$$

$$C(R_1R_3 + R_1R_2 + R_2R_3)\frac{dV_c}{dt} + (R_1 + R_3)V_c = eR_3$$

$$\begin{cases} \dfrac{dV_c}{dt} + \dfrac{R_1 + R_3}{C(R_1R_2 + R_1R_3 + R_2R_3)} V_c = \dfrac{eR_3}{C(R_1R_2 + R_1R_3 + R_2R_3)} \\ V_c(0^-) = V_c(0^+) = 0 \end{cases}$$

Integrale particolare:

$$V_c^g(t) = \sqrt{2}\, V_c^+ \cos(\omega t + \varphi^+)$$



$e(t) = 10\cos(\omega t)$

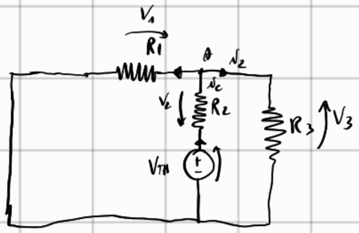
$\omega = 100\ rad/s$

$R_1 = 20\Omega \quad R_2 = 5\Omega$

$R_3 = 10\Omega \quad C = 1mF$

$V_{TH} = V_3$

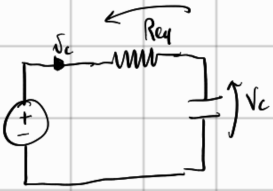
$$V_3 = \frac{e \cdot R_3}{R_1 + R_3} = \frac{10}{3}\cos(\omega t)$$



$\Downarrow \quad R_{eq} = (R_1 // R_3) + R_2$

$$R_1 // R_3 \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_3}\right)^{-1} = \frac{20}{3}\Omega$$

$$R_{123} = \frac{35}{3}\Omega$$



$e - R i_c - V_c = 0$

$$e = CR\frac{dV_c}{dt} + V_c$$

$$\begin{cases} \frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{e}{RC} \\ V(0^-) = V(0^+) = 0 \end{cases}$$

$V_c(t) = \sqrt{2}\, V_0 \cos(\omega t + \varphi)$

$$\tilde{V} = \tilde{E} \cdot \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{10}{3\sqrt{2}} \cdot \frac{-10j}{-10j + \frac{35}{3}} = \frac{36\sqrt{2}}{3 \cdot 17} - \frac{42\sqrt{2}}{3 \cdot 17} j = 1 - \frac{7}{6} j$$

$$|\tilde{V}| = \sqrt{9 + \frac{49}{4}} = 1{,}5$$

$$\arg(\tilde{V}) = \cos^{-1}\left(\frac{1}{1{,}5}\right) = -0{,}84$$

$$\tilde{V} = 1{,}5\, e^{-0{,}84 j}$$

$\Rightarrow V_c(t) = \sqrt{2} \cdot 1{,}5 \cos(\omega t - 0{,}84)$

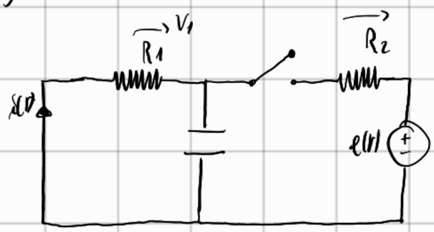
$$v(t) = v^{go}(t) + v^{g}(t) =$$

$$= A e^{-\frac{t}{RC}} + \sqrt{2} \cdot 1{,}5 \cos(\omega t - 0{,}84)$$

$$v(0) = A + \sqrt{2}\, 1{,}5 \cos(-0{,}84)$$

$$\Rightarrow A = -1{,}4$$

1.10)



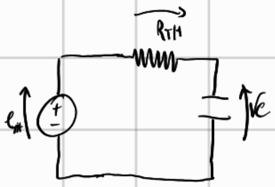
$e(t) = \sin(100t)\ V$

$R_1 = 15\Omega \quad R_2 = 10\Omega$

$C = 1mF \quad V(0^+) = 1V$

$\Rightarrow V_{TH} = \dfrac{e R_1}{R_1 + R_2} = \dfrac{3}{5}\sin(100t)$

$R_{TH} = \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)^{-1} = \left(\dfrac{1}{10} + \dfrac{1}{15}\right)^{-1} = \left(\dfrac{3}{30} + \dfrac{2}{30}\right)^{-1} = 6\Omega$



$$\begin{cases} \dfrac{dv_c}{dt} + \dfrac{v_c}{RC} = \dfrac{e_{TH}}{RC} \\ v(0^+) = 1V \end{cases}$$

$v_c^g(t) = \sqrt{2}\, V_0 \sin(\omega t + \varphi)$

Rete fasori:

$\tilde{V}_c = \tilde{E}_{TH} \cdot \dfrac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R_{TH}} = \dfrac{3\sqrt{2}}{10} \cdot \dfrac{-10j}{-10j + 6} = \dfrac{3\sqrt{2}(-j)}{6 - 10j} = \dfrac{15\sqrt{2}}{68} - \dfrac{9\sqrt{2}}{68} j = 0{,}3 - 0{,}19j$

$|\tilde{V}_c| = 0{,}36$

$\arg(\tilde{V}_c) = -\cos^{-1}\left(\dfrac{0{,}3}{0{,}36}\right) = -0{,}59$

$v_c(t) = 0{,}51 \sin(100t - 0{,}59)$

$v_c^T(t) = A e^{-\frac{t}{RC}} + 0{,}51 \sin(100t - 0{,}59)$

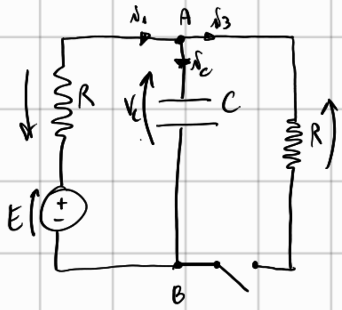
$v_c^T(0) = 1 = A + 0{,}51 \sin(-0{,}59) \Rightarrow$

$A = 1 - 0{,}51 \sin(-0{,}59) = 1{,}28$

$v(t) = 1{,}28\, e^{-167t} + 0{,}51 \sin(100t - 0{,}59)$

$i_1(t) = -\dfrac{1}{R_1} \cdot v(t) = -0{,}085 e^{-167t} - 0.034 \sin(100t - 0{,}59)\ A$

1.20.1)



$R = 10\Omega$

$C = 2mF$

$E = 10V$

$t > 0$ chiuso

$t < 0$ aperto

$t > 0 \Rightarrow$

$$\begin{cases} E - Ri_1 - V_c = 0 \\ E - Ri_1 - Ri_3 = 0 \\ i_3 = i_1 - i_c \end{cases}$$

$\Rightarrow E - Ri_1 - Ri_1 + Ri_c$

$\Rightarrow Ri_1 = \frac{E + Ri_c}{2}$

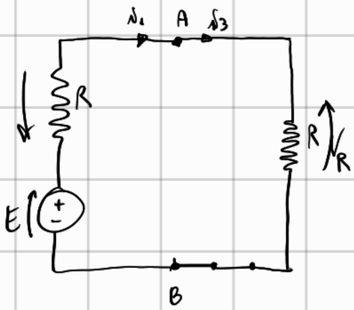
$i_c = c\frac{dV_c}{dt}$

$E - \frac{E + Ri_c}{2} - V_c = 0$

$\frac{E}{2} = V_c + \frac{Ri_c}{2}$

$$\begin{cases} \frac{dV_c}{dt} + \frac{2}{RC}V_c = \frac{E}{RC} \\ V_c(0^-) = V_c(0^+) = E \end{cases}$$

$V_c^{g}(t) = 5V$



$V_R = E\frac{\cancel{R}}{2\cancel{R}} = 5V$

$\Rightarrow V_c^{g}(t) = 5V$

$V_c^{go}(t) = Ae^{-\frac{t}{RC}}$

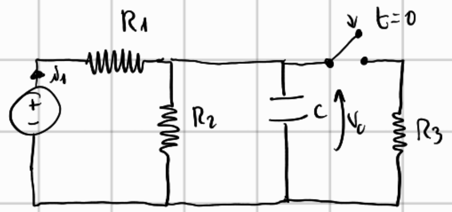
$V_c(t) = Ae^{-\frac{t}{RC}} + 5$

$V_c(0^+) = 10 \Rightarrow 10 = 5 + A \Rightarrow A = 5$

$V_c(t) = 5e^{-\frac{t}{RC}} + 5$

$i_c(t) = c\frac{dV_c}{dt} = -\frac{5}{R}e^{-\frac{t}{RC}}$

1.7)



$E = 220V$ $C = 1F$

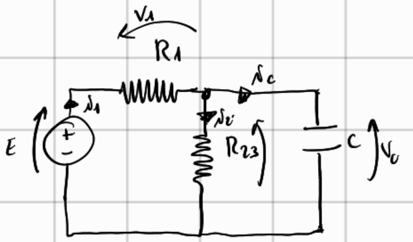
$R_1 = R_3 = 1k\Omega$ $R_2 = 500\,\Omega$

Se t aperto, regime stazionario.

$\Rightarrow i_c = 0$ $i_2 = \frac{E}{R_1 + R_2} = \frac{220}{1500} A \cong 0{,}15\,A$

$V_2 = V_C = i_2 R_2 = 73{,}3V$

NOW: Tutto chiuso:



$$R_{23} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{3}{1000}\right)^{-1} = \frac{1000}{3}\,\Omega$$

$$\begin{cases} E - R_1 i_1 - R_{23} i_2 = 0 \\ E - R_1 i_1 - V_C = 0 \\ i_1 = i_2 + i_c \end{cases}$$

$R_1 i_1 = E - R_{23} i_1 + R_{23} i_c$

$\Rightarrow i_1 (R_1 + R_{23}) = E + R_{23} i_c$

$\Rightarrow E - \frac{R_1}{R_1 + R_{23}} (E + R_{23} i_c) - V_c = 0$

$(R_1 + R_{23}) E - R_1 E - R_1 R_{23} i_c - (R_1 + R_{23}) V_c = 0$

$R_1 R_{23} C \frac{dV_c}{dt} + (R_1 + R_{23}) V_c = R_{23} E$

$$\Rightarrow \begin{cases} \frac{dV_c}{dt} + \frac{R_1 + R_{23}}{R_1 R_{23} C} V_c = \frac{E}{R_1 C} \\ V(0^-) = V(0^+) = 73{,}3V \end{cases}$$

$$V_{23} = \frac{E R_{23}}{R_1 + R_{23}} = \frac{220 \cdot \frac{1000}{3}}{\frac{1000}{3} + 1000} = \frac{220}{\cancel{3}} \cdot \cancel{1000} \cdot \frac{\cancel{3}}{\cancel{4000}_4} = 55\,V$$

$v_c^g(t) = 55\,V$ perché regime stazionario.

$$v_c(t) = A e^{-\frac{t}{R_{eq} C}} + 55$$

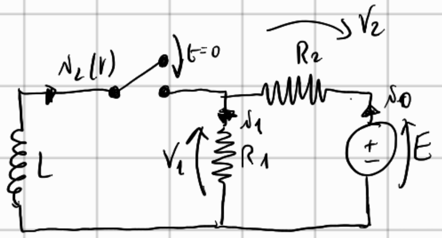
$$R_{eq} = \left(\frac{R_1 R_{23}}{R_1 + R_{23}}\right) = \frac{1000 \cdot \frac{1000}{3}}{1000 + \frac{1000}{3}} = \frac{1000^2}{\cancel{3}} \cdot \frac{\cancel{3}}{4000} = \frac{1000}{4} = 250\,\Omega$$

$$v_c(0) = A + 55 = 73{,}3\,V$$

$$A = 18{,}3\,V$$

$$v_c(t) = 18{,}3\, e^{-\frac{t}{R_{eq}}} + 55\,V$$

1.11)



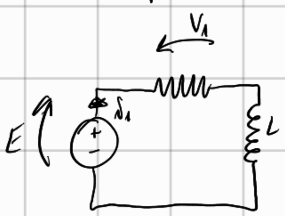
$R_1=5\Omega \quad R_2=4\Omega \quad L=1mH$

$E=10V \quad i_L(0)=2A$

$t>0$:

$$V_1=V_{TH}=E\cdot\frac{R_1}{R_1+R_2}=5,55V$$

$$R_{eq}=R_1//R_2=\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1}=\left(\frac{1}{5}+\frac{1}{4}\right)^{-1}=2,22\Omega$$



$$\begin{cases} E=Ri_L+L\dfrac{di_L}{dt} \\ i_L(0)=2A \end{cases}$$

Si va in regime stazionario.

$V_L=0 \Rightarrow i_L=V_1G=EG=2,5A$

$i_L^p(t)=2,5A$

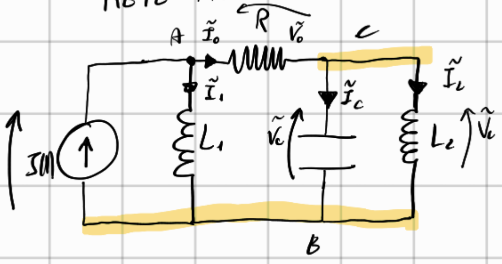
$i_L^{go}(t)=Ae^{-\frac{Rt}{L}}+2,5$

$i_L(0)=-2=A+2,5 \Rightarrow A=-4,5$

$$\Rightarrow i_L(t)=-4,5e^{-\frac{Rt}{L}}+2,5$$

2.3)

RETE FASORIALE



$J(t) = 10\sqrt{2}\sin(100t + 0,35)\,A$

$R = 4\Omega \quad C = 3mF$

$L_1 = 2mH \quad L_2 = 5mH$

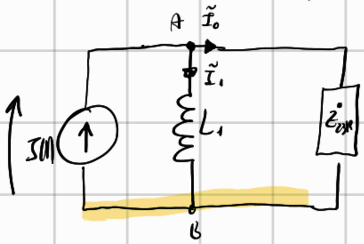
$\dot{Z}_1 = j\omega L_1$ $\qquad\qquad \dot{Z}_1 = 0,2j$

$\dot{Z}_2 = j\omega L_2 \qquad \dot{Z}_R = R$ $\qquad \dot{Z}_2 = 0,5j$

$\dot{Z}_3 = \frac{-j}{\omega C}$ $\qquad\qquad \dot{Z}_3 = -3,3j$

$\dot{Z}_R = 4$

$$\dot{Z}_{32} = \left(\frac{1}{j\omega L_2} + j\omega C\right)^{-1} = \left(\frac{1}{\dot{Z}_2} + \frac{1}{\dot{Z}_3}\right)^{-1} = -0,59j$$



$$\dot{Z}_{23R} = 4 - 0,59j$$

$$\dot{Z}_{123R} = \left(\frac{1}{\dot{Z}_{23R}} + \frac{1}{\dot{Z}_1}\right)^{-1} = 0,01 + 0,2j$$

$$\tilde{I}_0 = \frac{\tilde{J}\,\frac{1}{\dot{Z}_{23R}}}{\dot{Z}_{123R}^{-1}} = -0,21 + 0,45j$$

$$\tilde{I}_L = \frac{\tilde{I}_0 \cdot \frac{1}{\dot{Z}_2}}{\dot{Z}_{23R}^{-1}} = 3,84 + 1,15j$$

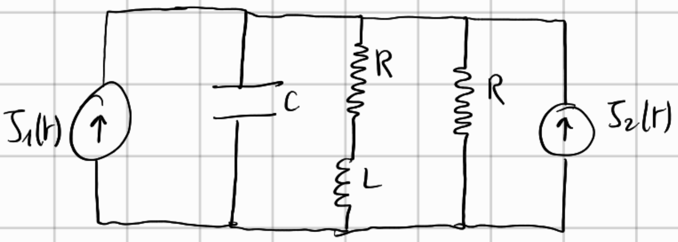
$$= 4e^{0,3j}$$

$$\tilde{V}_L = j\omega L_2 \tilde{I}_L = -0,6 + 1,9j = 2e^{1,87j}$$

$$P = \tilde{V}_L \cdot \check{\tilde{I}}_L = (3,81 - 1,2j)(-0,6 + 1,9j) =$$

$$P = 8e^{-\frac{2}{3}j} = +8j$$

2.4)



$J_1(t) = 10\cos(1000t)\ A = 10\sin(1000t - \frac{\pi}{2})\ A$

$J_2(t) = 10\sin(1000t)\ A$

$R = 2\,\Omega$

$L = 2\,mH$

$C = 1\,mF$

$\dot{Z}_C = \frac{-j}{\omega C} = -j$

$\dot{Z}_L = j\omega L = 2j$

$\dot{Z}_R = 2$

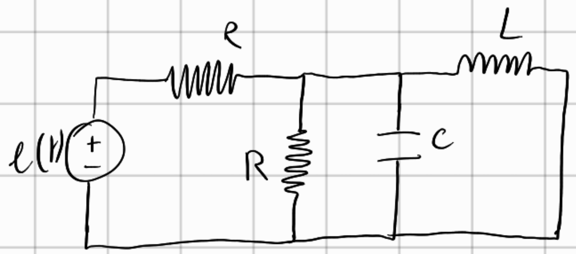
$\Rightarrow$ $\dot{Z}_{RL} = 2 + 2j$

$\dot{Z}_{RRLC} = \left(\frac{1}{\dot{Z}_{RL}} + \frac{1}{\dot{Z}_C} + \frac{1}{\dot{Z}_R}\right)^{-1} = \frac{2}{3} - \frac{2}{3}j$

$$\tilde{I}_L = (\tilde{J}_1 + \tilde{J}_2) \cdot \frac{\frac{1}{\dot{Z}_{RL}}}{\dot{Z}_{RRLC}^{-1}} = -2{,}36 - 2{,}36j$$

$i_L(t) = \sqrt{2}\,\frac{10}{3}\cos(1000t - \frac{3}{4}\pi)$

2.5)



$e(t) = 5\sqrt{2}\,\sin\left(1000t + \frac{\pi}{3}\right)\,V$

$R = 0{,}21\,\Omega \qquad L = 1{,}12\,mH$

$C = 1{,}23\,mF$

Potenza complessa e istant. assorb. da parallelo RC.

$\dot{Z}_R = 0{,}21\,\Omega$

$\dot{Z}_C = -1{,}23j\,\Omega \qquad \dot{Z}_{RC} = \left(\frac{1}{\dot{Z}_R} + \frac{1}{\dot{Z}_C}\right)^{-1} = 0{,}2 + 0{,}035j$

$\dot{Z}_L = 1{,}12j\,\Omega \qquad \dot{Z}_{RLC} = \left(\frac{1}{\dot{Z}_{RC}} + \frac{1}{\dot{Z}_L}\right)^{-1} = 0{,}21 + 3{,}52j$

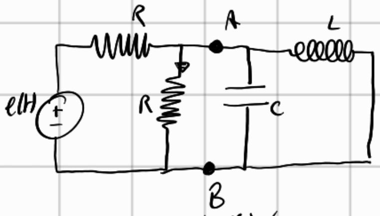
$\dot{Z}_{TOT} = \dot{Z}_{RLC} + \dot{Z}_R = 0{,}42 + 3{,}52j$

$\tilde{I}_{TOT} = \frac{\tilde{E}}{\dot{Z}_{TOT}} = 1{,}41 \cdot e^{-0{,}4j}$

$\tilde{V}_C = \frac{\tilde{E} \cdot \dot{Z}_{RLC}}{\dot{Z}_{TOT}} = 5e^{1{,}1j} \qquad \tilde{I}_{RC} = \frac{\tilde{I}_{TOT}\,\dot{Z}_L}{\dot{Z}_{RLC}} = 0{,}42 - 0{,}15j = 0{,}45\,e^{-0{,}34j}$

$P = \tilde{V}_C \cdot \overset{*}{I}_{RC} = \frac{9}{4}\,e^{1{,}44j}$

2.5)



$e(t)=\sqrt{2}5\sin(1000t+\frac{\pi}{3})V$

$R=0{,}21\,\Omega$

$L=0{,}00112\,H$

$C=0{,}00123\,F$

$$\tilde{U}_A\underbrace{\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{jwL}-\frac{j}{wC}\right)}_{\lambda=9{,}5-1{,}7j}=\frac{\tilde{E}}{R}$$

$$\tilde{U}_A=\frac{\tilde{E}}{R\lambda}=0{,}84+2{,}32j$$

$$\tilde{I}_R=\frac{\tilde{U}_A}{R}=4+11j$$

$$\tilde{I}_c=\tilde{U}_A\,jwC=-2{,}85+3$$

$$\dot{P}_1=\tilde{V}_c\cdot\check{\tilde{I}}_c=-7{,}45j\ VaR$$

$$\dot{P}_2=28.9\,W$$

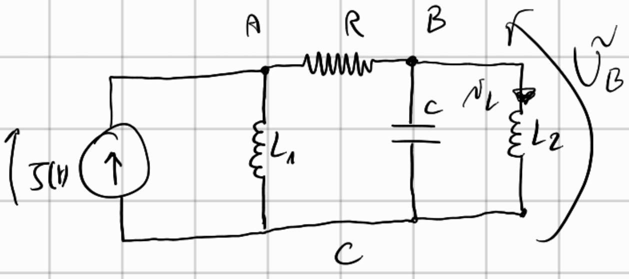
$i_R(t)=11{,}7\cdot\sqrt{2}\sin(1000t+1{,}22)$

$v_R(t)=2{,}47\cdot\sqrt{2}\sin(1000t+1{,}22)$

$i_c(t)=c\frac{dv_R}{dt}=3\sqrt{2}\sin(1000t+2{,}80)$

$P_{ist}(t)=v_R(t)i_R(t)+v_c(t)i_c(t)=57{,}8\sin^2(1000t+1{,}22)+14{,}8\sin(1000t+1{,}22)\cdot\sin(1000t+2{,}8)$

2.3)



$\dot{Z}_2 = jwL_2 = 0{,}5j\,\Omega$ $\quad \dot{Z}_C = -\frac{10}{3}j\,\Omega$

$\dot{Z}_1 = jwL_1 = 0{,}2j\,\Omega$ $\quad \dot{Z}_R = 4\,\Omega$

$\tilde{U}_B\left(\frac{1}{Z_C} + \frac{1}{Z_2} + \frac{1}{R}\right) - \frac{\tilde{U}_A}{R} = 0$ $\qquad \lambda = 0{,}25 - 1{,}7j$

$\tilde{U}_A\left(\frac{1}{R} + \frac{1}{Z_1}\right) - \frac{\tilde{U}_B}{R} = \tilde{J}$ $\qquad \gamma = 0{,}25 - 5j$

$\Rightarrow \quad \tilde{U}_B\lambda - \tilde{U}_A\beta = 0$

$\tilde{U}_A\gamma - \tilde{U}_B\beta = \tilde{J}$

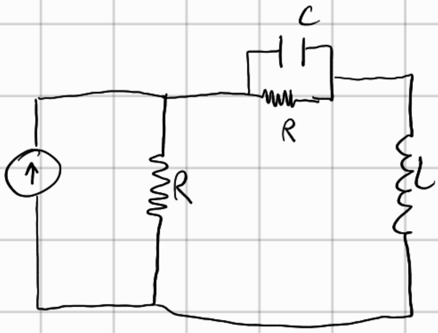
$\Rightarrow \tilde{U}_A = \frac{\tilde{U}_B\lambda}{\beta}$

$\Rightarrow \tilde{U}_B\frac{\gamma\lambda}{\beta} - \tilde{U}_B\beta = \tilde{J}$ $\qquad \longrightarrow \tilde{U}_B = \frac{\tilde{J}}{\varphi} = \frac{10e^{0{,}35j}}{34{,}65e^{-2{,}94j}} = 0{,}29\,e^{-3j}$

$\Rightarrow \underbrace{\left(\frac{\gamma\lambda}{\beta} - \beta\right)}_{=\varphi} = -34 - 6{,}7j$

$\tilde{I}_{L2} = \dot{Z}_2\tilde{U}_B = 0{,}14\,e^{-1{,}42j}$

2.1)



$i(t) = 10\sin(2t)\,A$

$R = 2\Omega$

$L = 1H$

$C = 0{,}25F$

$\dot{Z}_R = 2\Omega \quad \dot{Z}_L = 2j\,\Omega \quad \dot{Z}_C = -2j\,\Omega$

$\dot{Z}_{RC} = \left(\frac{1}{2} - \frac{1}{2j}\right)^{-1} = 1 - j$

$\dot{Z}_{RCL} = 1 + j$

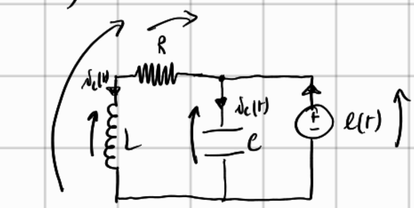
$\dot{Z}_{eq} = \left(\frac{1}{1+j} + \frac{1}{2}\right)^{-1} = \frac{4}{5} + \frac{2}{5}j$

$\tilde{V} = \dot{Z}_{eq}\tilde{I} = \left(\frac{4}{5} + \frac{2}{5}j\right) \cdot \frac{10}{\sqrt{2}} = \frac{8}{\sqrt{2}} + \frac{4}{\sqrt{2}}j = 4\sqrt{2} + 2\sqrt{2}j$

$\tilde{J} = 5\sqrt{2}$

$\Rightarrow \dot{P} = \tilde{V} \cdot \check{\tilde{I}} = 40\,W + 20j\,VaR$

2.2)



$e(t) = 10\cos(1000t)$ V

$R = 10\,\Omega \quad L = 20\,\text{mH}$

$C = 0{,}1\,\text{mF}$

$\dot{Z}_C = -10j\,\Omega \quad \dot{Z}_L = 20j \quad \dot{Z}_R = 10\,\Omega$

$\dot{Z}_{RL} = 10 + 20j\,\Omega$

$$\tilde{I}_C = \frac{\tilde{E}}{\dot{Z}_C} = \frac{5\sqrt{2}}{-10j} = \frac{\sqrt{2}}{2}j = \frac{\sqrt{2}}{2}e^{j\frac{\pi}{2}}$$

$$\tilde{V}_L = \frac{\tilde{E}\cdot\dot{Z}_L}{\dot{Z}_{RL}} = \frac{5\sqrt{2}\cdot 20j}{10+20j} = \frac{5\sqrt{2}\cdot 2j}{1+2j} = \frac{\overset{2}{\cancel{10}}\sqrt{2}(1-2j)j}{\cancel{5}} = (2\sqrt{2} - 4\sqrt{2}j)j = 4\sqrt{2} + 2\sqrt{2}j$$

$$\tilde{I}_L = \frac{\tilde{V}_L}{\dot{Z}_L} = \frac{1}{20j}(4\sqrt{2} + 2\sqrt{2}j) = \frac{\sqrt{2}}{5j} + \frac{\sqrt{2}}{10} = +\frac{\sqrt{2}}{10} - \frac{\sqrt{2}}{5}j$$

$i_C(t) = \cos\left(1000t + \frac{\pi}{2}\right)$ A

$i_L(t) = \frac{\sqrt{5}}{5}\cos(1000t - 1{,}11)$ A

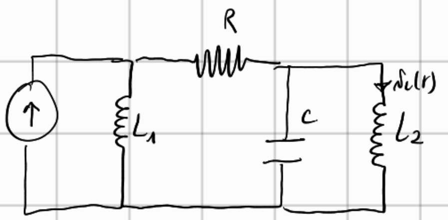
Alternative Route

$\dot{Z}_{eq} = 5 - 15j\,\Omega$

$$\tilde{I}_{TOT} = \frac{\tilde{E}}{\dot{Z}_{eq}} = \frac{5\sqrt{2}}{5-15j} = \frac{\sqrt{2}}{1-3j} = \frac{\sqrt{2}(1+3j)}{10} = \frac{\sqrt{2}}{10} + \frac{3\sqrt{2}}{10}j$$

$$\tilde{I}_L = \tilde{I}_{TOT} - \tilde{I}_C = \frac{\sqrt{2}}{10} + \frac{3\sqrt{2}}{10}j - \frac{\sqrt{2}}{2}j = \frac{\sqrt{2}}{10} - \frac{\cancel{2}\sqrt{2}}{\cancel{10}_5}j = \frac{\sqrt{2}}{10} - \frac{\sqrt{2}}{5}j$$

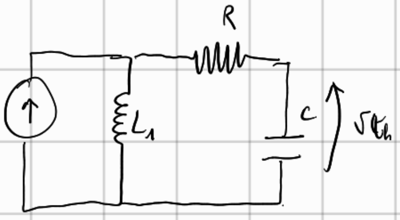
2.3)



$s(t) = 10\sqrt{2}\sin(100t + 0{,}35)\,A$

$R = 4\,\Omega \quad C = 3\,mF$

$L_1 = 2\,mH \quad L_2 = 5\,mH$



$\dot{Z}_{L_2} = \frac{1}{2}j$

$\dot{Z}_R = 4\,\Omega \quad \dot{Z}_C = -\frac{10}{3}j \quad \dot{Z}_{L_1} = +\frac{1}{5}j$

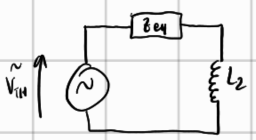
$\dot{Z}_{RC} = 4 - \frac{10}{3}j$

$\dot{Z}_{TOT} = \left(\frac{1}{4-\frac{10}{3}j} + \frac{1}{\frac{1}{5}j}\right)^{-1} \cong 0{,}2j$!

$\tilde{V} = \dot{Z}_{TOT}\tilde{J} = 10e^{0{,}35j} \cdot \frac{1}{5}e^{\frac{\pi}{2}j} = 2e^{1{,}92j}$

$\tilde{V}_{TH} = \frac{\tilde{V}\dot{Z}_C}{\dot{Z}_{RC}} = 0{,}6 + 1{,}1j = 1{,}28e^{j}$

$\dot{Z}_{eq} = \left(\frac{1}{Z_R + Z_{L_1}} + \frac{1}{Z_C}\right)^{-1} = 1{,}721 - 1{,}985j$



$\tilde{I}_L = \frac{\tilde{V}_{TH}}{\dot{Z}_{eq} + \dot{Z}_{L_2}} = 1{,}5 - j$

$\tilde{V}_{L_2} = \frac{\tilde{V}_{TH}\dot{Z}_L}{\dot{Z}_{eq} + \dot{Z}_{L_2}} = 0{,}5 + 0{,}77j$

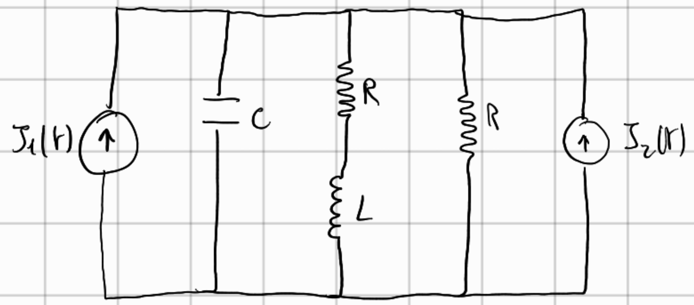
$\dot{P} = 1{,}66j$ VaR

$\tilde{I}_L = 1{,}8\,e^{-0{,}59j} \quad i_L(t) = 2{,}55\sin(100t - 0{,}59)$

$\tilde{V}_L = 0{,}92\,e^{j} \quad v_L(t) = 1{,}27\sin(100t + 1)$

$p(t) = i_L(t)v_L(t) = 3{,}24\sin(100t - 0{,}59)\sin(100t + 1)$

2.4)



$J_1(t) = 10\cos(1000t)$ A

$J_2(t) = 10\sin(1000t)$ A

$R = 2\,\Omega$

$L = 2\,mH$ $C = 1\,mF$

SOVRAPPOSIZ:

$\tilde{J}_1 = 5\sqrt{2}$

$\tilde{J}_2 = 5\sqrt{2}\, e^{-\frac{\pi}{2}j}$

$\dot{Z}_{RL} = \dot{Z}_R + \dot{Z}_L = 2 + 2j$

$$\tilde{I}_L^1 = \tilde{J}_1 \cdot \frac{\frac{1}{\dot{Z}_{RL}}}{\frac{1}{\dot{Z}_{RL}} + \frac{1}{\dot{Z}_R} + \frac{1}{\dot{Z}_C}} = -\frac{5\sqrt{2}}{3} j$$

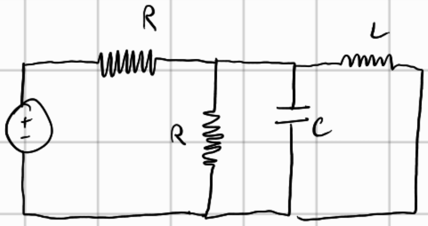
$$\tilde{I}_L^2 = -\frac{5\sqrt{2}}{3}$$

$$\tilde{I}_L^{TOT} = -\frac{5\sqrt{2}}{3} - \frac{5\sqrt{2}}{3} j = \frac{10}{3} e^{-\frac{3}{4}\pi}$$

$$i_L(t) = \frac{10\sqrt{2}}{3}\cos\left(1000t - \frac{3}{4}\pi\right) = \frac{10\sqrt{2}}{3}\sin\left(1000t - \frac{\pi}{4}\right)$$

Good!

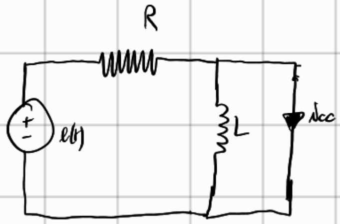
2.5)



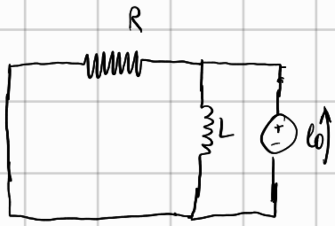
$e(t) = 5\sqrt{2}\sin\left(1000t+\frac{\pi}{3}\right)$ V

$R = 0{,}21\,\Omega$ $L = 1{,}12$ mH

$C = 1{,}23$ mF



$\tilde{I}_{cc} = \frac{\tilde{E}}{R} = \frac{5e^{\frac{\pi}{3}}}{0{,}21} = 23{,}81\, e^{\frac{\pi}{3}}$

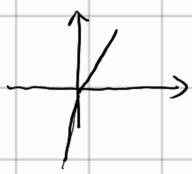


$\dot{q}_{eq} = \left(\frac{1}{\dot{Z}_R} + \frac{1}{\dot{Z}_L}\right) = \left(\frac{1}{0{,}21} + \frac{1}{j\cdot 1{,}12}\right) = 4{,}76 - 0{,}89j$

68,78

$\dot{Z}_R = 0{,}21\,\Omega$ $\dot{Z}_C = \frac{-j}{1{,}23} = -0{,}813j$

$\dot{q}_{eq} = \left(\frac{1}{\dot{Z}_R} + \frac{1}{\dot{Z}_C} + \dot{q}_N\right) = 9{,}52 + 0{,}34j$



$\tilde{I}_R = \frac{\dot{J}_{cc}\,\dot{q}_R}{\dot{q}_{eq}} = 6{,}315 + 10{,}09j =$

$\tilde{V}_R = \tilde{I}_R \dot{Z}_R = 1{,}326 + 2{,}119j = 2{,}5e^{j}$ $\longrightarrow v_R(t) = 3{,}536\sin(1000t+1)$ V

$\tilde{I}_C = \tilde{V}_R \cdot \frac{1}{\dot{Z}_C} = -2{,}61 + 1{,}63j = 3{,}07e^{2{,}58j}$

$i_R(t) = 16{,}83\sin(1000t+1)$ A

$i_C(t) = 4{,}342\sin(1000t+2{,}58)$ A

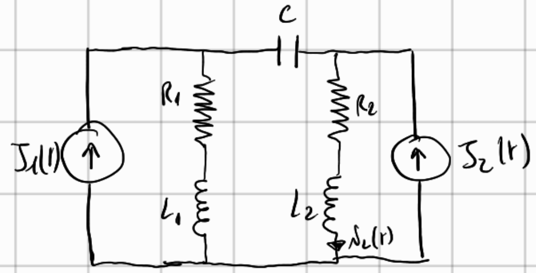
$\dot{P}_{c_1} = \tilde{V}_R \check{I}_C = -7{,}69j$ VaR

$\dot{P}_{c_2} = 29{,}75$ W

$p(t) = i_R^2(t)R + v_R(t)i_C(t) = 59{,}48\sin^2(1000t+1) +$

$+\, 59{,}51\sin(1000t+1)\sin(1000t+2{,}58)$

2.7)



$J_1(t) = 4\cos(4t)$ A

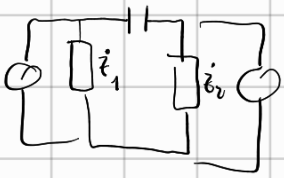
$J_2(t) = 2\cos(4t - \frac{2}{3}\pi)$ A

$R_1 = R_2 = 2\Omega$

$L_1 = L_2 = 1$H

$C = 2$F

Sovrapposizione



Spengo J_2.

$\dot{Z}_{CRL} = \dot{Z}_C + \dot{Z}_R + \dot{Z}_L = -\frac{j}{8} + 2 + 4j = 2 + \frac{31}{8}j$

$$\tilde{I}_L^1 = \frac{\tilde{J}_1 \cdot \dot{Z}_{R_1L_1}}{\dot{Z}_{R_1L_1} + \dot{Z}_{CRL}} = \frac{2\sqrt{2} \cdot (2+4j)}{2+4j+2+\frac{31}{8}j} \simeq 1{,}432 \text{ A}$$

Spengo J_1.

$\dot{Z}_{CRL_1} = 2 + \frac{31}{8}j$ $\qquad$ $\dot{Z}_{L_2R_2} = 2+4j$

$$\dot{I}_L^2 = \frac{\sqrt{2}\, e^{-\frac{2}{3}\pi} \cdot (2+\frac{31}{8}j)}{2+\frac{31}{8}j+2+4j} = -0{,}353 - 0{,}602j$$

$\tilde{I}_{TOT} = \tilde{I}_L^1 + \tilde{I}_L^2 = 1{,}079 - 0{,}602j$

$\tilde{V}_{L_2} = \tilde{I}_{TOT} \dot{Z}_L = 4j(1{,}079 - 0{,}602j) = 2{,}41 + 4{,}32j$ $\quad$ (3,40

6,109

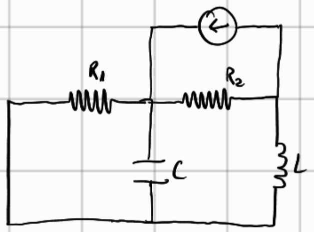
$P_R = \tilde{V}_{L_2} \tilde{I}_{TOT} \cdot \sin(\frac{\pi}{2}) = 6{,}112j$ VaR

angolo di ritardo

$P_1 = \tilde{I}_{TOT} \cdot \tilde{V}_R \cos(0) = 3{,}053$ W

angolo di ritardo

2.8)

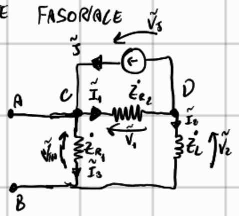


$J(t) = 2\sqrt{2}\cos(20t + 0{,}23)$ A

$R_1 = 12\,\Omega \quad R_2 = 2\,\Omega$

$L = 0{,}2\,H \quad C = 0{,}1\,F$

RETE FASORIALE



$$\tilde{I}_3 = \frac{\tilde{J} \cdot \dot{Z}_{R_2}}{\dot{Z}_{R_2} + \dot{Z}_{R_1} + \dot{Z}_L} = 0{,}274 - 0{,}0133j$$

$$\tilde{V}_{TH} = \tilde{I}_3 \dot{Z}_{R_1} = 3{,}29 - 0{,}159j$$

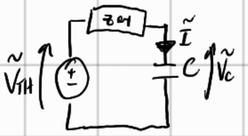
$$\tilde{U}_C \left(\frac{1}{\dot{Z}_{R_1} + \dot{Z}_L} + \frac{1}{\dot{Z}_{R_2}} \right) = \tilde{J}$$

$$\tilde{U}_C = \frac{\tilde{J}}{\frac{1}{\dot{Z}_{R_1} + \dot{Z}_L} + \frac{1}{\dot{Z}_{R_2}}} = 3{,}346 + 0{,}94j$$

$$\tilde{V}_{TH} = \frac{\tilde{U}_C \cdot \dot{Z}_{R_1}}{\dot{Z}_{R_1} + \dot{Z}_L} = \frac{\frac{24}{7} e^{0{,}23j}}{12 + 4j} = 3{,}29 - 0{,}159j$$

$$\dot{Z}_{R_2L} = \dot{Z}_{R_2} + \dot{Z}_{L_2} = 2 + 4j\ \Omega$$

$$\dot{Z}_{eq} = \left(\frac{1}{\dot{Z}_{R_1}} + \frac{1}{\dot{Z}_{R_2L}} \right)^{-1} = 2{,}5 + 2{,}717j\ \Omega$$



$$\tilde{I} = \frac{\tilde{V}}{\dot{Z}_{eq} + \dot{Z}_C} = 0{,}705 - 0{,}689j$$

$$\tilde{V}_0 = \tilde{I} \cdot \dot{Z}_C = -0{,}345 - 0{,}353j$$

$$\dot{P}_C = \tilde{V}_C \cdot \tilde{I}_C = -0{,}486\ \text{VaR}$$

$$i_C(t) = 0{,}986\sqrt{2}\cos(20t - 0{,}774)$$

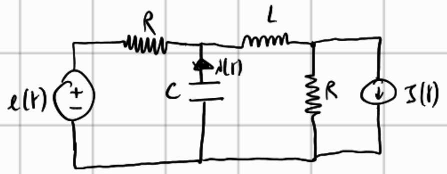
$$V_C(t) = \sqrt{2}\, 0{,}496 \cos(20t - 2{,}344)$$

$$P(t) = 0{,}978 \cos(20t - 0{,}774)\cos(20t - 2{,}344)$$

$$= \frac{1}{2} 0{,}978 \left[\cos(40t - 3{,}12) + \cancel{\cos\left(\frac{\pi}{2}\right)} \right]$$

Manca un meno

2.9)



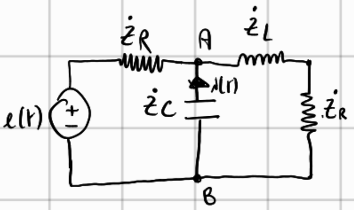
$J(t) = 2\sqrt{2}\sin(2\pi f t + 0{,}12)\,A$

$e(t) = 10\sqrt{2}\cos(2\pi f t)\,V$

$f = 50\,Hz$

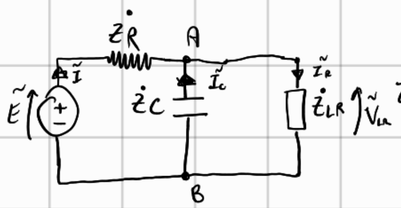
$R = 1\,\Omega \quad C = 1\,mF \quad L = 3\,mH$

SOVRAPPOSIZIONE:



$\dot{Z}_R = 1\,\Omega \quad \dot{Z}_L = 100\pi \cdot 3 \cdot 10^{-3} J = \frac{3}{10}\pi J$

$\dot{Z}_C = \frac{-J}{100\pi 10^{-3}} = -\frac{10}{\pi} J$



$\dot{Z}_{LR} = 1 + \frac{3}{10}\pi J$

$\left(\frac{1}{\dot{Z}_{LR}} + \frac{1}{\dot{Z}_C}\right)^{-1} = 1{,}683 + 0{,}588 J$

$\tilde{U}_A \left(\frac{1}{\dot{Z}_C} + \frac{1}{\dot{Z}_R} + \frac{1}{\dot{Z}_{LR}}\right) = \frac{\tilde{E}}{\dot{Z}_R}$

$I_{TOT} = 3{,}556 - 0{,}779 J$

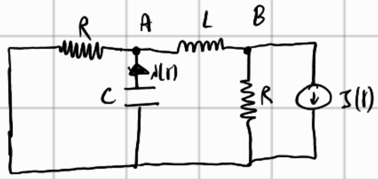
$I = \frac{I_{TOT} Z_{LR}}{Z_{LR} + Z_C} =$

$\tilde{U}_A = \frac{\tilde{E}}{\dot{Z}_R}\left(\frac{1}{\dot{Z}_C} + \frac{1}{\dot{Z}_R} + \frac{1}{\dot{Z}_{LR}}\right)^{-1} = 6{,}443 + 0.779 J$

$\tilde{I}_C = -\tilde{U}_A \cdot \dot{Z}_C = -2{,}48 + 20{,}51 J$

$\dot{P} = 134 J\ VAR$

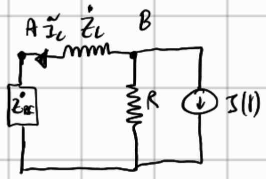
SOVRAPPOSIZIONE 2



$\Rightarrow$

$v(t) = 20{,}575\sqrt{2}\cos(2\pi f t + 1{,}366)$
$+ 1{,}035\sqrt{2}\sin(2\pi f t + 0.0939)$

$\dot{Z}_{RC} = \left(\frac{1}{Z_R} + \frac{1}{Z_C}\right)^{-1} = \left(1 + \frac{\pi}{10} J\right)^{-1} = 0{,}91 - 0{,}286 J$



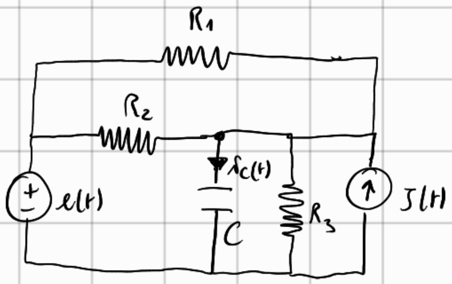
$\tilde{I}_L = -\tilde{J} \cdot \frac{\dot{Z}_R}{\dot{Z}_R + \dot{Z}_L + \dot{Z}_{RC}} = -0{,}968 + 0{,}207 J$

$\tilde{I}_C = -\tilde{I}_L \cdot \frac{\dot{Z}_R}{\dot{Z}_{RC}} = 1{,}03 + 0.097 J$

$\tilde{V}_C = \tilde{I}_C \dot{Z}_C = 0{,}3088 - 3{,}279 J$

$\dot{P} = -3.407 J$

ES. 2)



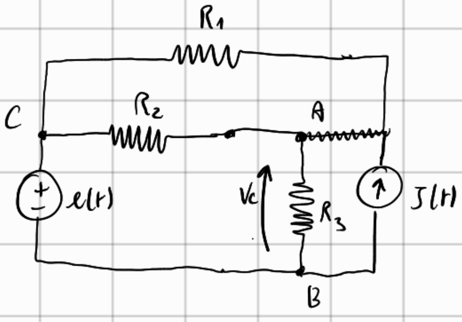
$e(t) = E_0 = 2V$

$J(t) = \begin{cases} 1A & se\ t<0 \\ -1A & se\ t>0 \end{cases}$

$R_1 = 4\Omega \quad R_2 = 4\Omega \quad R_3 = 2\Omega \quad C = 2F$

Se $t<0 \rightarrow$ Circuito è in regime stazionario.

$\Rightarrow i_C = 0 \rightarrow$ Studio rete.

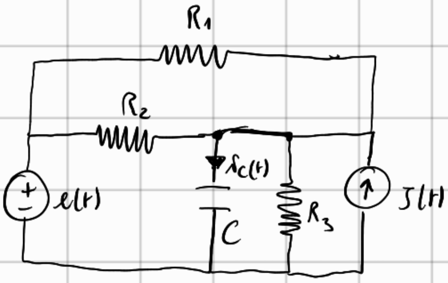


$U_B = 0 \qquad U_C = 2V$

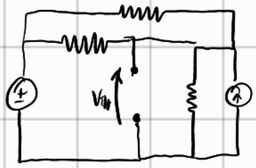
$U_A(G_3 + G_2 + G_1) - 2G_2 = J$

$$U_A = \frac{J + 2G_2}{G_1 + G_2 + G_3} = 1{,}5V = V_C(0^-) = V_C(0^+)$$

Se $t>0$, $J = -J_0 \Rightarrow U_A = -0{,}5V$

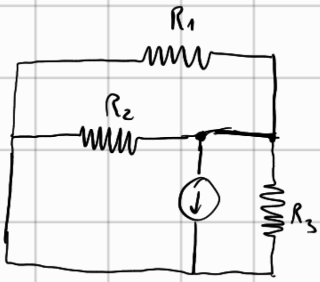


Trovo equivalente Thevenin:



Già studiato, $V_{TH} = 1{,}5V$

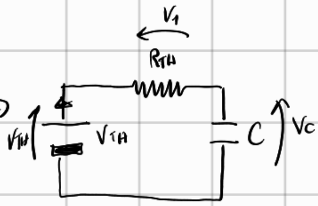
R_{TH}?



$\Rightarrow R_{12} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = 2\Omega$

$R_{123} = \left(\frac{1}{2} + \frac{1}{2}\right)^{-1} = 1\Omega = R_{TH}$

$\Rightarrow$



$V_{TH} - R_{TH} I - V_C = 0$

$$\begin{cases} R_T C \dfrac{dV_C}{dt} - V_C = V_{TH} \\ V_C(0^+) = 1{,}5V \end{cases}$$

$V_C^{\,p}(t) = -0{,}5V$

$V_C^{\,go}(t) = A e^{-\frac{t}{R_T C}}$

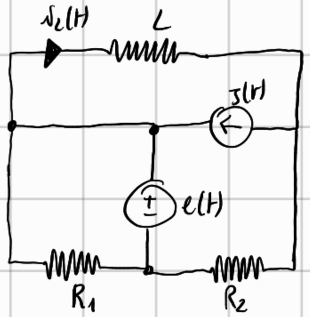
$V_C(t) = A e^{-\frac{t}{RC}} - 0{,}5$

$V_C(0^+) = A - 0{,}5 = 1{,}5V$

$A = 2V$

$V_C(t) = 2 e^{-\frac{t}{2}} - 0{,}5V$

ES. 1)

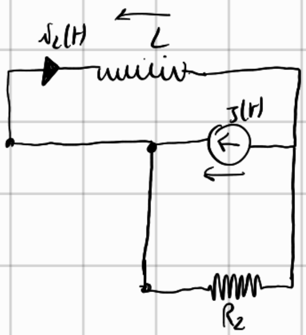


$e(t) = E_0 = 2V$

$J(t) = \sqrt{2}\cos(1000t + \frac{\pi}{4})$

$L = 1mH$ $\quad R_1 = R_2 = 1\Omega$

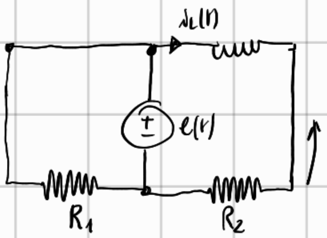
Sovrapposizione effetti: Spengo $e(t)$



$$\tilde{I}_L = \frac{\tilde{J}\cdot \dot{Z}_{R_2}}{\dot{Z}_{R_2}+\dot{Z}_L} = \frac{e^{\frac{\pi}{4}j}\cdot 1}{1+j} = \frac{\sqrt{2}}{2}$$

$i_L(t) = \cos(1000t)$

Spengo $J(t)$



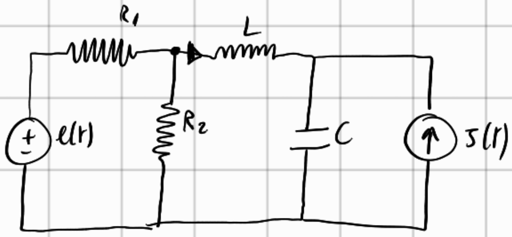
Circuito in regime stazionario

$V_L = 0$

$i_L(t) = \frac{E_0}{R_2} = 2A$ $\quad P_{R_1} = G_1E^2 = 4W$ potenza media

$i_L(t) = 2 + \cos(1000t)$

ES. 1)

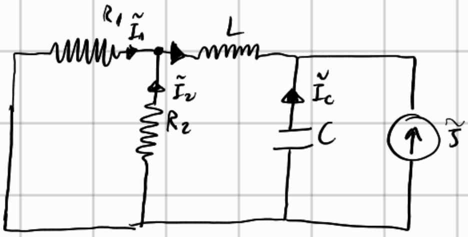


$e(t)=8\cos(\omega t)$ $\quad J(t)= \sin(\omega t)$

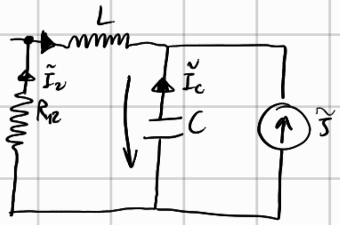
$\omega=1\,krad/s$

$L=1\,mH$ $\quad C=0{,}5\,mF$ $\quad R_1=R_2=2\,\Omega$

SOVRAPPROSIZIONE:



$\Downarrow$



$\dot Z_c = -2j\ \Omega$

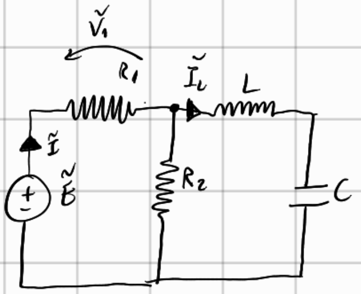
$\dot Z_L = j\ \Omega$

$R_{12}=1\,\Omega$

$$\tilde I_2 = -\frac{\tilde J \cdot \dot Z_c}{\dot Z_c+\dot Z_L+\dot Z_{12}} = \frac{\sqrt2 j}{1-j} = -\frac{\sqrt2}{2}+\frac{\sqrt2}{2}j = e^{\frac{3\pi}{4}j} \Rightarrow \tilde I_c = -\tilde I_2 - \tilde J = \cancel{\frac{\sqrt2}{2}} - \frac{\sqrt2}{2}j - \cancel{\frac{\sqrt2}{2}} = -\frac{\sqrt2}{2}j$$

$$\tilde V_c = \tilde I_c \tilde Z_c = -\sqrt2\,V = 2\sin(\omega t - \pi)$$

$$v_L^1(t) = \sqrt2 \sin\left(\omega t + \frac{3\pi}{4}\right)$$



$$\tilde U_A\left(\frac{1}{Z_2}+\frac{1}{Z_L+Z_C}+\frac{1}{Z_1}\right) = \frac{\tilde E}{Z_1}$$

$$\tilde U_A\left(\frac12 + j + \frac12\right) = \frac{\tilde E}{2} \qquad \tilde U_A = \frac{\tilde E}{2(1+j)} = \frac{4\sqrt2}{2(1+j)} = \sqrt2-\sqrt2 j = 2e^{-\frac{\pi}{4}j} \Rightarrow V_J(t)=2\sqrt2\cos\left(\omega t - \frac{\pi}{4}\right)$$

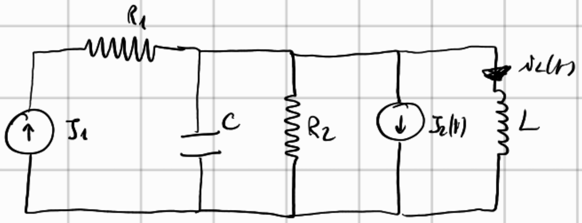
$$\tilde{I}_L = \tilde{U}_A \cdot \dot{Z}_L = \sqrt{2}(1-j)\,j = \sqrt{2}\,j + \sqrt{2} = \sqrt{2} + \sqrt{2}\,j = 2e^{\frac{\pi}{4}j}$$

$$i_L^2(t) = 2\sqrt{2}\cos\left(\omega t + \frac{\pi}{4}\right)$$

$$i_1(t) = \sqrt{2}\sin\left(\omega t + \frac{3\pi}{4}\right) + 2\sqrt{2}\cos\left(\omega t + \frac{\pi}{4}\right)$$

$$= 3\sqrt{2}\cos\left(\omega t + \frac{\pi}{4}\right)$$

ES. 1



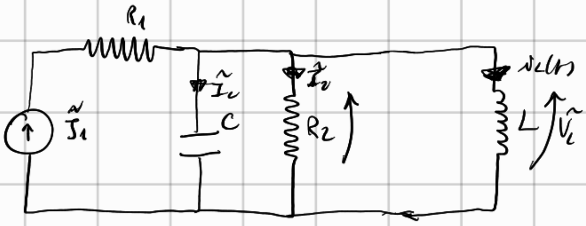
$J_1 = 3\sin(\omega_1 t)$

$J_2 = 6\cos(\omega_2 t)$

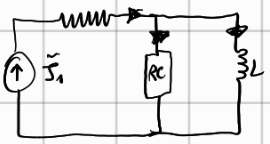
$\omega_1 = 1\,rad/s \quad \omega_2 = 2\,rad/s$

$L = 1H \quad C = 1F \quad R_1 = 1\Omega \quad R_2 = \frac{2}{3}\Omega$

SOVRAPPOSIZIONE EFFETTI



$\Downarrow$



$$\dot Z_{RC} = \left(\frac{1}{\dot Z_{R_2}} + \frac{1}{\dot Z_C}\right)^{-1} = \left(\frac{3}{2} + j\right)^{-1} = \frac{6}{13} - \frac{4}{13}j$$

$$\tilde I_L^1 = \frac{\tilde J\, \dot Z_{RC}}{\dot Z_{RC} + \dot Z_L} = \frac{\frac{3}{\sqrt2}\cdot\left(\frac{6}{13} - \frac{4}{13}j\right)}{\frac{6}{13} - \frac{4}{13}j + j} = -\sqrt2 j = \sqrt2 e^{-\frac{\pi}{2}j}$$

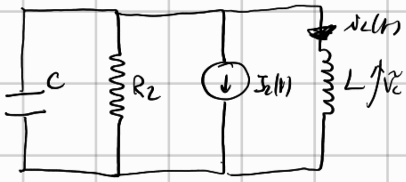
$$\tilde I_{RC} = \tilde J - \tilde I_L = \frac{3}{\sqrt2} + \sqrt2 j$$

$$\tilde V_L = \sqrt2\, V$$

$$\tilde I_2^1 = \tilde I_{RC} \cdot \frac{\frac{1}{\dot Z_{RC}}}{\frac{1}{\dot Z_{RC}} + \frac{1}{\dot Z_C}} = 3\sqrt2 - \frac{5}{4}\sqrt2 j$$

$$i_L^1(t) = 2\sin\left(\omega_1 t - \frac{\pi}{2}\right) = -2\cos(\omega_1 t)$$

$$V_C^1(t) = 2\sin(\omega_1 t)$$



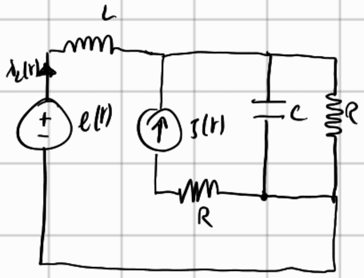
$\dot Z_C = \frac{-j}{2}\Omega \quad \dot Z_{R_2} = \frac{2}{3}\Omega \quad \dot Z_L = 2j \quad \dot Z_{RC} = \left(2j + \frac{3}{2}\right)^{-1} = \frac{6}{25} - \frac{8}{25}j$

$$\tilde I_L^2 = -\frac{\tilde J_2 \cdot \dot Z_{RC}}{\dot Z_{RC} + \dot Z_L} = -\frac{\frac{6}{\sqrt2}\cdot\left(\frac{6}{25} - \frac{8}{25}j\right)}{\frac{6}{25} - \frac{8}{25}j + 2j} = +\frac{\sqrt2}{2} + \frac{\sqrt2}{2}j$$

$$i_L^2(t) = \sqrt2\cos\left(\omega_2 t + \frac{\pi}{4}\right)$$

$$i_{TOT}(t) = 2\sin\left(\omega_1 t - \frac{\pi}{2}\right) + \sqrt2\cos\left(\omega_2 t + \frac{3}{4}\pi\right)$$

ESERCIZIO 1)

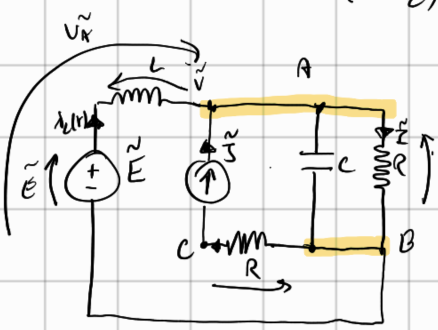


$e(t) = 2\sqrt{2}\cos(\omega t - \frac{\pi}{4})$ $\quad J(t) = \sin(\omega t)$

$\omega = 1\,rad/s$

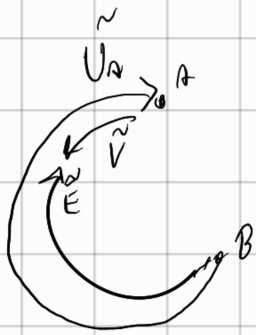
$L = 1H \quad C = 1F \quad R = 1\Omega$

$J(t) = \sin(\omega t) = \cos(\omega t - \frac{\pi}{2})$



$\tilde{J} = \frac{\sqrt{2}}{2} e^{-\frac{\pi}{2} j}$

$= -\frac{\sqrt{2}}{2} j$



$\tilde{U}_A + \tilde{V} - \tilde{E} = 0$

$\dot{Z}_L = j\,\Omega \quad \dot{Z}_C = \frac{1}{j\omega C} = -j\,\Omega$

$\dot{Z}_R = 1\,\Omega$

$\tilde{U}_B = 0$

$\tilde{U}_A\left(\frac{1}{\dot{Z}_C} + \frac{1}{\dot{Z}_R} + \frac{1}{\dot{Z}_L}\right) = \tilde{J} + \frac{\tilde{E}}{\dot{Z}_L}$

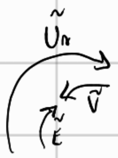
$\frac{\tilde{U}_C}{\dot{Z}_R} = -\tilde{J}$

$\tilde{U}_A = \tilde{J} - j\tilde{E}$

$\tilde{U}_A = \tilde{J} - j\tilde{E} = \frac{\sqrt{2}}{2} e^{-\frac{\pi}{2} j} - j2e^{-\frac{\pi}{4} j} = -\frac{\sqrt{2}}{2} j - j2e^{-\frac{\pi}{4} j} = -\frac{\sqrt{2}}{2} - \frac{3}{2}\sqrt{2} j$

$\tilde{U}_C = -\tilde{J}\dot{Z}_R = -\tilde{J} = +\frac{\sqrt{2}}{2} j$

$\tilde{E} = 2e^{-\frac{\pi}{4} j}$

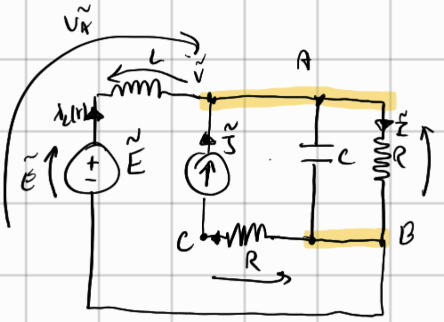


$\tilde{E} - \tilde{V} - \tilde{U}_A = 0$

$\tilde{V} = \tilde{E} - \tilde{U}_A = 2e^{-\frac{\pi}{4} j} + \frac{\sqrt{2}}{2} + \frac{3}{2}\sqrt{2} j = \frac{3}{2}\sqrt{2} + \frac{\sqrt{2}}{2} j$

$$\tilde V = \tilde I \dot Z_L \Rightarrow \tilde I = \frac{\tilde V}{\dot Z_L} = \frac{\frac{3}{2}\sqrt2 + \frac{\sqrt2}{2} j}{j} = \frac{\sqrt2}{2} - \frac32\sqrt2\, j = \sqrt5\, e^{-1,25 j}$$

$$\Rightarrow i_L(t) = \sqrt{10}\cos(\omega t - 1,25)$$



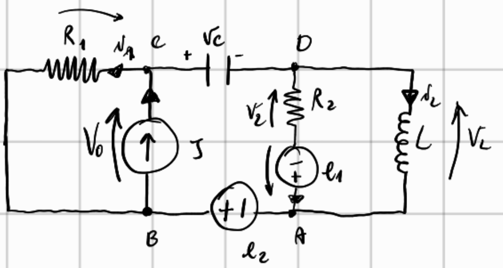
$$\tilde I_1 = \frac{\tilde U_A}{Z_R} = -\frac{\sqrt2}{2} - \frac32\sqrt2\, j$$

$$\tilde U_C = \frac{\sqrt2}{2} j$$

$$\dot P_1 = \tilde U_A \check{\tilde I}_1 = \left(-\frac{\sqrt2}{2} - \frac32\sqrt2\, j\right)\left(-\frac{\sqrt2}{2} + \frac32\sqrt2\, j\right) = 5\,W$$

$$\dot P_2 = -\tilde U_C \cdot \check{\tilde J} = \left(-\frac{\sqrt2}{2} j\right)\left(+\frac{\sqrt2}{2} j\right) = +\frac12\,W$$

1)



$J(t) = \sin(\omega t) = \cos\left(\omega t - \frac{\pi}{2}\right)$

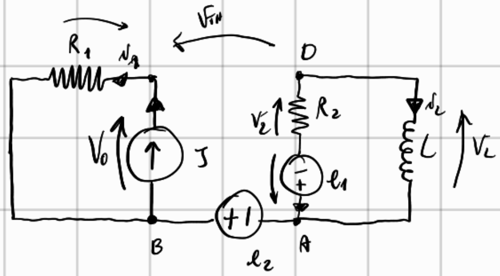
$e_1(t) = 2\cos(\omega t)$

$e_2(t) = \cos(\omega t)$

$L = 2H \quad C = 1F \quad R_1 = 1\Omega \quad R_2 = 2\Omega$

$\dot{Z}_L = 2j\,\Omega \quad \dot{Z}_C = -j\,\Omega \quad \dot{Z}_1 = 1\Omega \quad \dot{Z}_2 = 2\Omega$

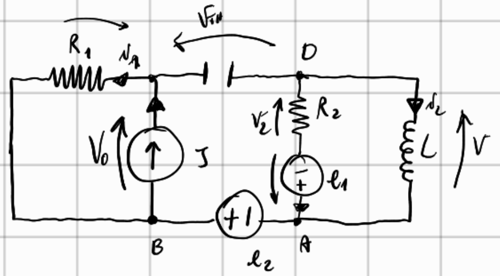
$\dot{Y}_L = -\frac{j}{2}S \quad \dot{Y}_C = j\,S \quad \dot{Y}_1 = 1S \quad \dot{Y}_2 = \frac{1}{2}S$

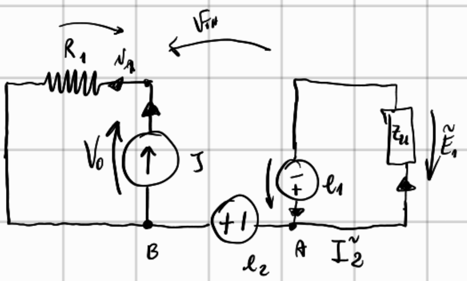


$\tilde{I}_1 = \tilde{J} \quad \tilde{V}_0 = R_1\tilde{I}_1 = \frac{\sqrt{2}}{2}e^{-\frac{\pi}{2}j} = -\frac{\sqrt{2}}{2}j$

Serie R_2-L:

$\dot{Z}_{2L} = \dot{Z}_2 + \dot{Z}_L = 2 + 2j$



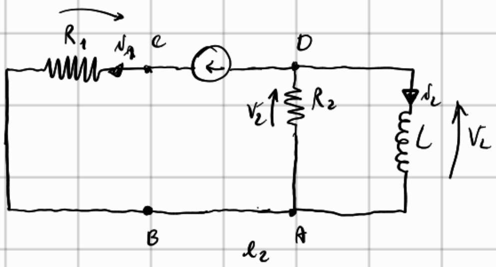


$$\tilde{I}_2 = \tilde{E}_1 \dot{Z}_{2L} = \sqrt{2}(2+2j) = 2\sqrt{2} + 2\sqrt{2}j$$

$$\tilde{V}_2 = \frac{\tilde{E} \cdot \dot{Z}_2}{\dot{Z}_{L2}} = \frac{\sqrt{2} \cdot 2}{2+2j} = \frac{\sqrt{2}}{1+j} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j$$

$$\tilde{V}_{TH} = \tilde{V}_0 + \tilde{E}_2 + \tilde{E}_1 - \tilde{V}_2 = -\cancel{\frac{\sqrt{2}}{2}}j + \cancel{\frac{\sqrt{2}}{2}} + \sqrt{2} - \cancel{\frac{\sqrt{2}}{2}} + \cancel{\frac{\sqrt{2}}{2}j} = \sqrt{2}\ V$$

$\dot{R}_{TH}$?



$$\dot{Z}_{eq} = \dot{Z}_1 + \left(\frac{1}{Z_2} + \frac{1}{Z_L}\right)^{-1} = 2 + j\ \Omega$$

$\Rightarrow$ 

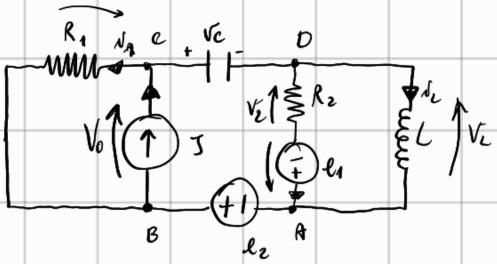
$\tilde{V} = \sqrt{2}V \qquad \dot{Z}_{TH} = 2+j\ \Omega$

$$\tilde{V}_T = \frac{\tilde{V} \cdot \dot{Z}_c}{\dot{Z}_c + \dot{Z}_{TH}} = \frac{-\sqrt{2}j}{2} = -\frac{\sqrt{2}}{2}j \longrightarrow \tilde{I}_c = \tilde{V}_c \dot{Y}_c = \frac{\sqrt{2}}{2} A$$

$$\Rightarrow \tilde{V}_T = \frac{\sqrt{2}}{2} e^{-\frac{\pi}{2}} \Rightarrow V_T(t) = \cos(\omega t - \frac{\pi}{2})$$

$\tilde{I}_1 = \tilde{J} - \tilde{I}_c = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j$

$\tilde{V}_1 = \tilde{I}_1 \dot{Z}_1 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j \qquad \Rightarrow \dot{P} = \tilde{V}\tilde{I}^* = 1\ W$



$J(t) = \sin(\omega t) = \cos\left(\omega t - \frac{\pi}{2}\right)$

$e_1(t) = 2\cos(\omega t)$

$e_2(t) = \cos(\omega t)$

$L = 2H \quad C = 1F \quad R_1 = 1\Omega \quad R_2 = 2\Omega$

$\dot{Z}_L = 2j\,\Omega \quad \dot{Z}_C = -j\,\Omega \quad \dot{Z}_1 = 1\Omega \quad \dot{Z}_2 = 2\Omega$

$\dot{Y}_L = -\frac{j}{2}S \quad \dot{Y}_C = j\,S \quad \dot{Y}_1 = 1S \quad \dot{Y}_2 = \frac{1}{2}S$

$\tilde{U}_A = 0 \qquad \tilde{U}_B = \frac{\sqrt{2}}{2}V$

$$\begin{cases} \tilde{U}_C(\dot{Y}_1 + \dot{Y}_C) - \frac{\sqrt{2}}{2}\dot{Y}_1 - \tilde{U}_D\dot{Y}_C = \tilde{J} \\ \tilde{U}_D(\dot{Y}_2 + \dot{Y}_L + \dot{Y}_C) - \tilde{U}_C\dot{Y}_C = -\tilde{E}_1\dot{Y}_2 \end{cases}$$

$$\begin{cases} \tilde{U}_C(1+j) - \frac{\sqrt{2}}{2} - \tilde{U}_D j = -\frac{\sqrt{2}}{2}j \\ \tilde{U}_D\left(\frac{1}{2} - \frac{j}{2} + j\right) - \tilde{U}_C j = -\frac{\sqrt{2}}{2} \end{cases}$$

$\tilde{U}_D j = \tilde{U}_C(1+j) + \frac{\sqrt{2}}{2}j - \frac{\sqrt{2}}{2}$

$\rightarrow \tilde{U}_D = -\tilde{U}_C(j-1) + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j$

(1) $\tilde{U}_D = \tilde{U}_C - \tilde{U}_C j + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j$

(2) $\frac{1}{2}\tilde{U}_D(1+j) - \tilde{U}_C j = -\frac{\sqrt{2}}{2}$

$\Rightarrow \tilde{U}_D(1+j) - 2j\tilde{U}_C = -\sqrt{2}$

$\tilde{U}_D + j\tilde{U}_D - 2j\tilde{U}_C = -\sqrt{2}$

(1) $\rightarrow$ (2)

$\tilde{U}_C - \cancel{\tilde{U}_C j} + \cancel{\frac{\sqrt{2}}{2}} + \frac{\sqrt{2}}{2}j + \cancel{j\tilde{U}_C} + \tilde{U}_C + \frac{\sqrt{2}}{2}j - \cancel{\frac{\sqrt{2}}{2}} - 2\tilde{U}_C j = -\sqrt{2}$

$2\tilde{U}_C - 2j\tilde{U}_C = -\sqrt{2} - \sqrt{2}j$

$\tilde{U}_C(1-j) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j$

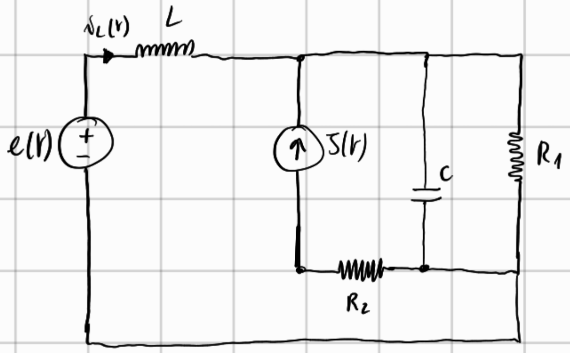
$\tilde{U}_C = -\frac{\sqrt{2}}{2}j$

$\tilde{I}_1 = (\tilde{U}_C - \tilde{U}_B)\dot{Y}_1 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j$

$\tilde{U}_D = -\cancel{\frac{\sqrt{2}}{2}j} - \cancel{\frac{\sqrt{2}}{2}} + \cancel{\frac{\sqrt{2}}{2}} + \cancel{\frac{\sqrt{2}}{2}j} = 0\,V$

$P_{arr} = Re(\check{V}_1 \check{I}_1^*) = 1W$

1)



$e(t) = 2\sqrt{2}\cos(\omega t - \frac{\pi}{4})$ e $J(t) = \sin(\omega t) = \cos(\omega t - \frac{\pi}{2})$

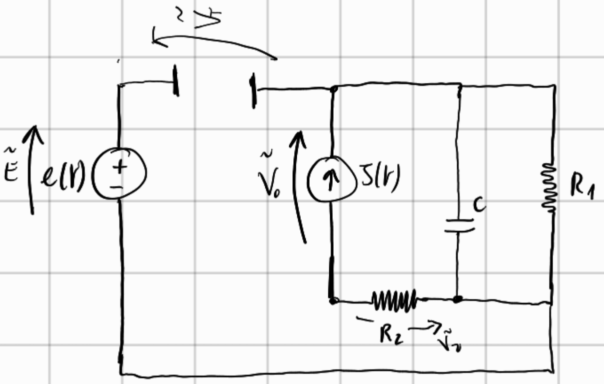
$\omega = 1\,rad/s$

$L = 1H$ $C = 1F$ $R_1 = 1\Omega$ $R_2 = 1\Omega$

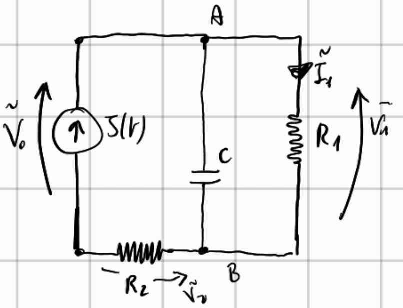
$\dot{Z}_L = j\Omega$ $\dot{Z}_C = -j\Omega$ $\dot{Z}_1 = 1\Omega$ $\dot{Z}_2 = 1\Omega$

$\dot{Y}_L = -j\,S$ $\dot{Y}_C = j\,S$ $\dot{Y}_1 = 1\,S$ $\dot{Y}_2 = 1\,S$

$\tilde{E} = 2e^{-\frac{\pi}{4}j} = \sqrt{2} - \sqrt{2}j$ $\tilde{J} = -\frac{\sqrt{2}}{2}j$



$\tilde{V}_T = \tilde{E} + \tilde{V}_2 - \tilde{V}_0$



$\tilde{V}_2 = \tilde{J}\dot{Z}_2 = -\frac{\sqrt{2}}{2}J$

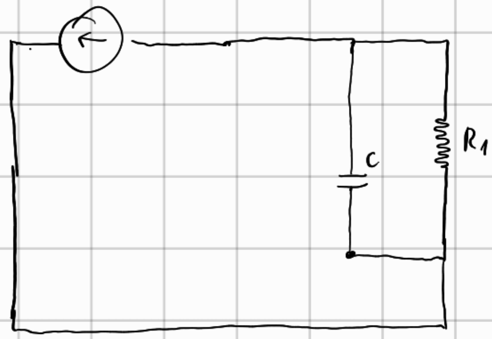
$\tilde{I}_1 = \frac{\tilde{J}\,\dot{Z}_C}{\dot{Z}_1 + \dot{Z}_C} = \frac{-\frac{\sqrt{2}}{2}J\cdot(-J)}{1-J} = \frac{-\frac{\sqrt{2}}{2}}{1-J} = -\frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4}J$

$\tilde{V}_1 = \tilde{I}_1\dot{Z}_1 = -\frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4}J$

$\tilde{V}_1 + \tilde{V}_2 - \tilde{V}_0 = 0$

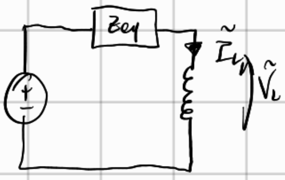
$\tilde{V}_0 = \tilde{V}_1 + \tilde{V}_2 = -\frac{\sqrt{2}}{4} - \frac{3}{4}\sqrt{2}J$

$\tilde{V}_T = \tilde{E} + \tilde{V}_2 - \tilde{V}_0 = \sqrt{2} - \sqrt{2}J - \frac{\sqrt{2}}{2}J + \frac{\sqrt{2}}{4} + \frac{3}{4}\sqrt{2}J = \frac{5}{4}\sqrt{2} - \frac{3}{4}\sqrt{2}J$



$\dot{Z}_{eq} = (\dot{Y}_1 + \dot{Y}_C)^{-1} = (1+J)^{-1} = \frac{1}{2} - \frac{1}{2}J$

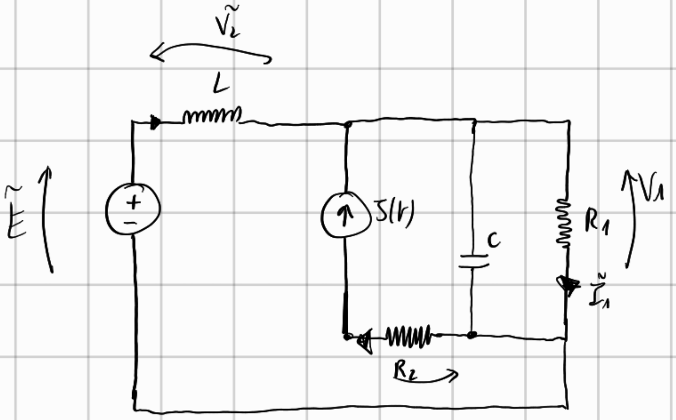
$\Downarrow$



$\tilde{I}_L = \frac{\tilde{E}_T}{\dot{Z}_{eq} + \dot{Z}_L} = \frac{\frac{5}{4}\sqrt{2} - \frac{3\sqrt{2}}{4}J}{\frac{1}{2} + \frac{1}{2}J} = \frac{\sqrt{2}}{2} - 2\sqrt{2}J = \frac{\sqrt{34}}{2}e^{-1,36J}$

$i_L(t) = \sqrt{17}\cos(\omega t - 1,36)\ A$

$\tilde{V}_L = \dot{Z}_L\tilde{I}_L = 2\sqrt{2} + \frac{\sqrt{2}}{2}J$



$e(t) = 2\sqrt{2}\cos(\omega t - \frac{\pi}{4})$ e $J(t) = \sin(\omega t) = \cos(\omega t - \frac{\pi}{2})$

$\omega = 1 \text{ rad/s}$

$L = 1H$ $C = 1F$ $R_1 = 1\Omega$ $R_2 = 1\Omega$

$\dot{Z}_L = j\Omega$ $\dot{Z}_C = -j\Omega$ $\dot{Z}_1 = 1\Omega$ $\dot{Z}_2 = 1\Omega$

$\dot{Y}_L = -j\,s$ $\dot{Y}_C = j\,s$ $\dot{Y}_1 = 1\,s$ $\dot{Y}_2 = 1\,s$

$\tilde{E} = 2e^{-\frac{\pi}{4}j} = \sqrt{2} - \sqrt{2}j$ $\tilde{J} = -\frac{\sqrt{2}}{2}j$

$\tilde{V}_1 + \tilde{V}_L - \tilde{E} = 0$

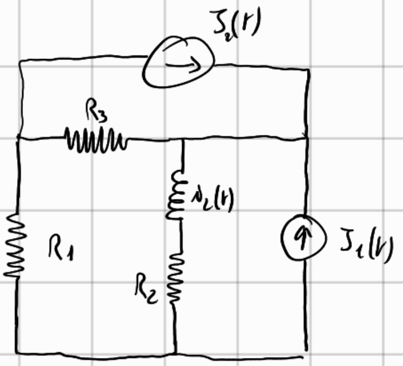
$\tilde{V}_1 = \tilde{E} - \tilde{V}_L = \sqrt{2} - \sqrt{2}j - 2\sqrt{2} - \frac{\sqrt{2}}{2}j = -\sqrt{2} - \frac{3}{2}\sqrt{2}j$

$\tilde{I}_1 = \dot{Z}_1\tilde{V}_1 = -\sqrt{2} - \frac{3}{2}\sqrt{2}j$

$\tilde{I}_2 = \tilde{J}$ $P_1 = Re(\tilde{V}_1\check{\tilde{I}}_1) = \frac{13}{2}W$

$\tilde{V}_2 = \dot{Z}_2\tilde{J}$ $P_2 = Re(\tilde{V}_2\check{\tilde{I}}_2) = \frac{1}{2}W$

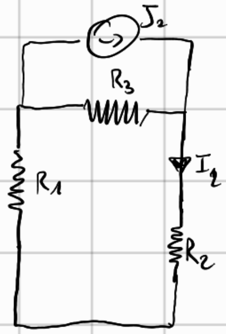
1)



$$J_1(t) = \begin{cases} 0 & t<0 \\ \sin(t) & t>0 \end{cases} \qquad J_2(t) = \begin{cases} 1 & t<0 \\ 0 & t>0 \end{cases}$$

$$R_1 = R_2 = R_3 = 1\,\Omega \qquad L = 1\,H$$

Se $t<0$, $v_L=0$ perché regime

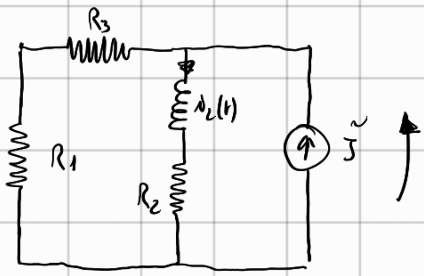


$$I_2 = \frac{J_2 R_3}{R_1+R_2+R_3} = \frac{J_2}{3} = \frac{1}{3}\,A$$

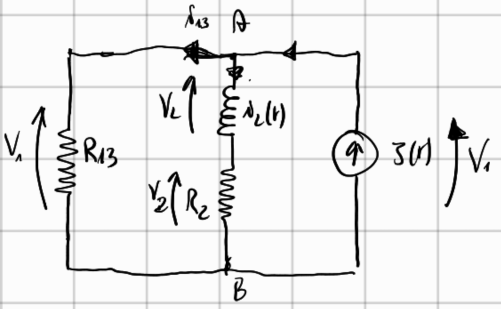
$$i_L(0^-) = i_L(0^+) = \frac{1}{3}\,A$$

Se ho $t>0$

FASORI



$$\tilde{I}_L = \frac{\tilde{J}\cdot(\dot{Z}_1+\dot{Z}_3)}{\dot{Z}_1+\dot{Z}_3+\dot{Z}_2+\dot{Z}_L} = \frac{\tilde{J}\cdot(2)}{3+j} = \frac{\sqrt{2}}{3+j} = \frac{3\sqrt{2}}{10} - \frac{\sqrt{2}}{10}j = \frac{\sqrt{5}}{5}e^{-0,32j} = \frac{\sqrt{10}}{5}\sin(t-0,32)$$



$R_{13} = 2\,\Omega$

$$V_1 - V_L - V_2 = 0$$

$$J = i_L + i_{13}$$

$$i_{13} R_{13} - V_L - i_L R_2 = 0$$

$i_{13} = J - i_L$

$$R_{13} J - R_{13} i_L - L\frac{di_L}{dt} - i_L R_2 = 0$$

$$L\frac{di_L}{dt} + i_L (R_{13} + R_2) = R_{13} J$$

$$\begin{cases} \dfrac{di_L}{dt} + i_L \dfrac{(R_{13}+R_2)}{L} = \dfrac{R_{13}}{L} J \\ i_L(0^+) = \dfrac{1}{3} A \end{cases}$$

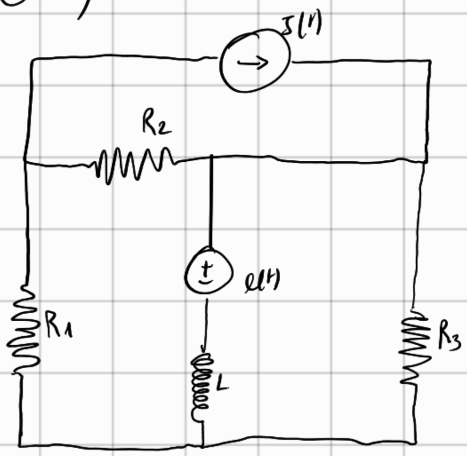
$$i_g(t) = \frac{\sqrt{10}}{5} \sin(t - 0{,}32)$$

$$i(t) = A e^{-\frac{t}{\tau}} + i_g(t)$$

$$i(0) = \frac{1}{3} \Rightarrow A + 0{,}2 = \frac{1}{3} \Rightarrow A = 0{,}13$$

$$i_1(t) = J(t) - i_L(t)$$

2)

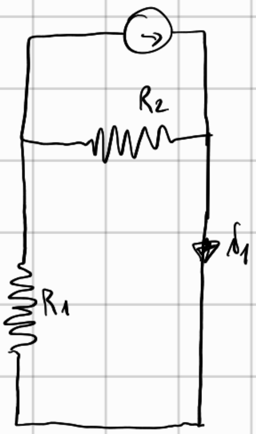


$$e(t)=\begin{cases}0 & t<0\\ \sin t & t>0\end{cases} \qquad J(t)=\begin{cases}1 & t<0\\ 0 & t>0\end{cases}$$

$R_1=R_2=1\,\Omega \quad R_3=2\,\Omega \quad L=1\,H$

Se $t<0$ regime stazionario

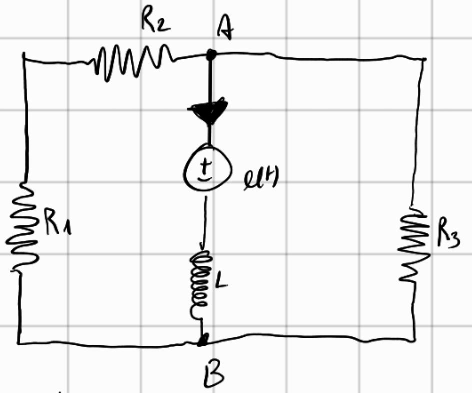
$\Rightarrow V_L=0$



$$I_1=\frac{J\cdot R_2}{R_1+R_2}=\frac{1}{2}A$$

Se $t>0$ studio regime sinusoidale

$t>0$



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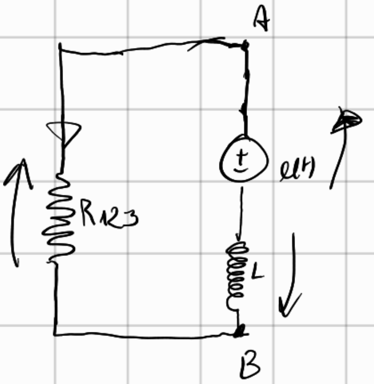


$$R_{123} = \left(\frac{1}{R_{12}} + \frac{1}{R_3}\right)^{-1} = 1\,\Omega$$

$$\tilde{I} = \frac{\tilde{E}}{\dot{Z}_{123} + \dot{Z}_L} = \frac{\frac{\sqrt{2}}{2}}{1+j} = \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4}j = \frac{1}{2}e^{-\frac{\pi}{4}j}$$

$$i_L(t) = \frac{\sqrt{2}}{2}\sin\left(t - \frac{\pi}{4}\right)$$

Nel dominio del tempo:

$$e - R_{eq}\, i_L - V_L = 0$$

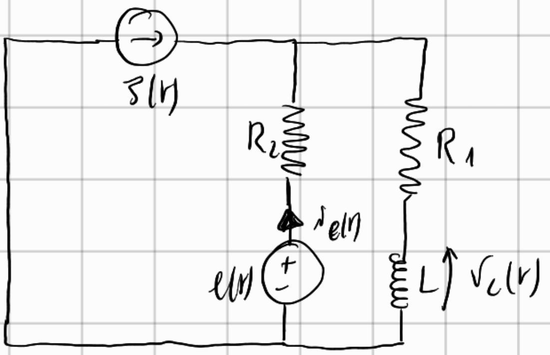
$$\begin{cases} \frac{d i_L}{dt} + \frac{R_{eq}}{L} i_L = e \\ i_L(0) = \frac{1}{2} \end{cases}$$

$i_g(t) = \frac{\sqrt{2}}{2}\sin\left(t - \frac{\pi}{4}\right)$

$i_{g0}(t) = A e^{-\frac{R}{L}t}$ $\qquad$ $i_L(t)$ prende negativa

$$\Rightarrow i(0) = A - \frac{1}{2} = \frac{1}{2} \Rightarrow A = 1$$

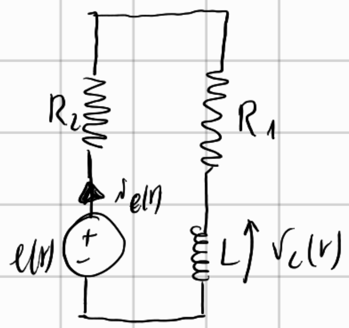
ESERCIZIO 2)



$e(t) = 12\,u(-t)$ V

$J(t) = 4\,u(t)$ A $\qquad R_1 = R_2 = 2\,\Omega \quad L = 1\,H$

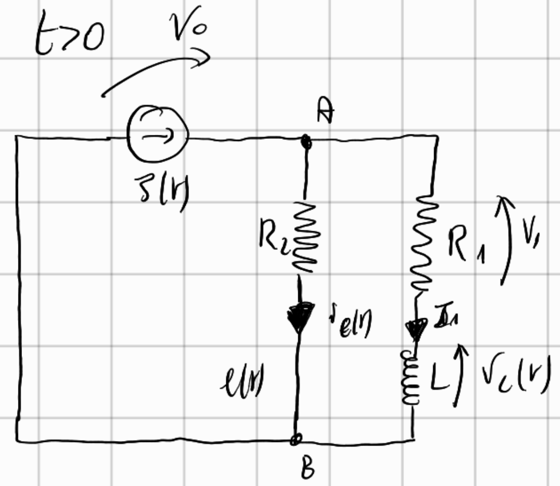
Se $t<0 \Rightarrow$



$v(t)=0 \quad \Rightarrow \quad I = \dfrac{E}{R_1+R_2} = 3A$

$i(0^-) = 3A$

Se $t>0$



Regime costante: $\rightarrow V=0 \rightarrow$

$$I_1 = \frac{J_1 R_2}{R_1+R_2} = \frac{4\cdot 2}{4} = 2A$$

$$V_2 - V_1 - V_C = 0 \qquad R_2 i_e - R_1 i_L - V_L = 0$$

$$J = i_e + i_1 \qquad R_2 J - R_2 i_L - R_1 i_L - V_L = 0$$

$$i_e = J - i_1 \qquad L\frac{di_L}{dt} + i_L(R_1+R_2) = R_2 J$$

$$\begin{cases} \dfrac{di_L}{dt} + i_L\dfrac{(R_1+R_2)}{L} = \dfrac{R_2}{L}J \\ i_L(0^+) = 3\ A \end{cases}$$

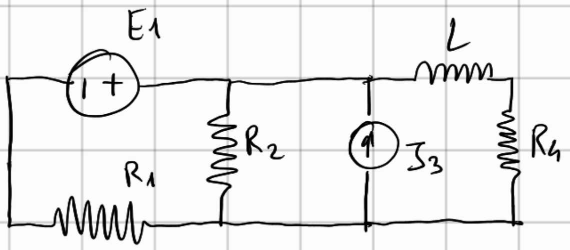
$$\tau = \frac{R_1+R_2}{L}$$

$$I_g(t) = 2A$$

$$i_L(t) = Ae^{-\frac{t}{\tau}} + 2$$

$$A = 1$$

1)



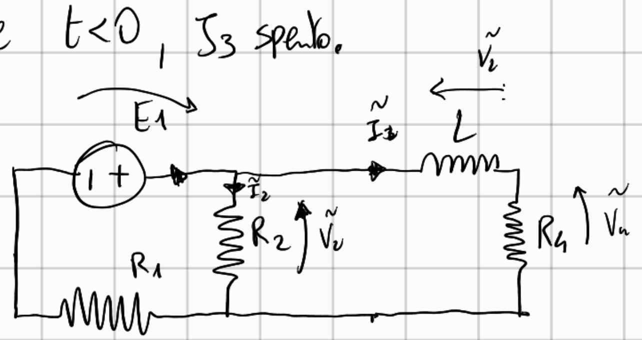
$R_4 = 5k\Omega \qquad R_1 = R_2 = 10k\Omega$

$L = 10mH$

$E_1(t) = 100\sqrt{2}\, \sin(10^5 t)\, V$

$J_3(t) = 10\, u(t)\, mA$

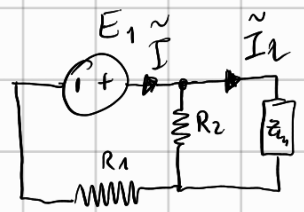
Se $t<0$, J_3 spento.



$\dot{Z}_{R_1} = 10k\Omega \qquad \dot{Z}_{R_2} = 10k\Omega$

$\dot{Z}_{R_4} = 5k\Omega \qquad \dot{Z}_L = j\,(10 \cdot 10^{-3} \cdot 10^5) = 1000\, j\ \Omega$

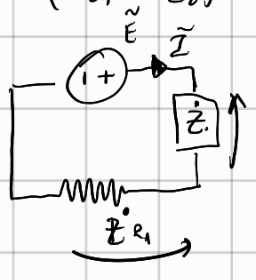
$\Rightarrow$



25,247
2,473 j

$\dot{Z}_{L4} = 5k + 1k\, j\ \Omega$

$\dot{Z}_{L42} = \left(\frac{1}{\dot{Z}_{L4}} + \frac{1}{\dot{Z}_2}\right)^{-1} = 3362{,}83 + 442{,}478\, j$

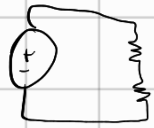


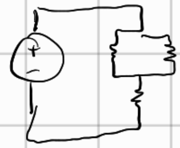
$\tilde{I} = \frac{\tilde{E}}{\dot{Z}_{L42} + \dot{Z}_{R_1}} = \frac{100}{\dot{Z}_{L42} + \dot{Z}_{R_1}} = 7{,}475 \cdot 10^{-3} - 2{,}475 \cdot 10^{-4}\, j$

$\tilde{I}_L = \frac{\tilde{I} \cdot \dot{Z}_{R_2}}{\dot{Z}_{R_2} + \dot{Z}_{L4}} = 4{,}95 \cdot 10^{-3} - 4{,}95 \cdot 10^{-4}\, j = 4{,}975 \cdot 10^{-3}\, e^{-0{,}1 t}$

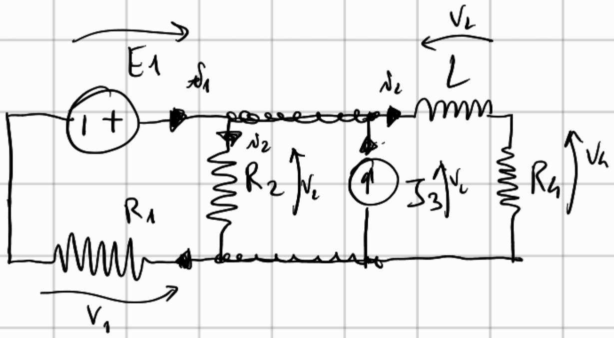
$i_L(t) = \sqrt{2}\cdot 4{,}975\cdot 10^{-3}\sin\left(10^5 t - 0{,}1\right)$

$t<0$





$t>0$:



$$\begin{cases} i_1 - i_2 - i_L + J_3 = 0 \\ E_1 - V_2 - V_1 = 0 \\ V_2 - V_L - V_4 = 0 \end{cases}$$

$$i_2 = \frac{E_1}{R_2} - \frac{i_1 R_1}{R_2}$$

$$\Rightarrow \quad E_1 - i_2 R_2 - i_1 R_1 = 0 \quad \rightarrow \quad i_1 = i_2 + i_L - J_3$$

$$i_2 R_2 - V_L - i_L R_4 = 0 \qquad i_1 = \frac{E_1}{R_2} - \frac{i_1 R_1}{R_2} + i_L - J_3$$

$$E - i_1 R_1 - V_L - i_L R_4 = 0$$

$$i_1\left(1+\frac{R_1}{R_2}\right) = \frac{E_1}{R_2} + i_L - J_3$$

$$i_1 = \left(\frac{E_1}{R_2} + i_L - J_3\right)\left(1+\frac{R_1}{R_2}\right)^{-1}$$

$$E - R_1\left(\frac{E_1}{R_2} + i_L - J_3\right)\left(1+\frac{R_1}{R_2}\right)^{-1} - V_L - i_L R_4 = 0$$

$$E\left(1+\frac{R_1}{R_2}\right) - \frac{R_1 E_1}{R_2} - R_1 i_L + R_1 J_3 - V_L\left(1+\frac{R_1}{R_2}\right) - i_L R_4\left(1+\frac{R_1}{R_2}\right) = 0$$

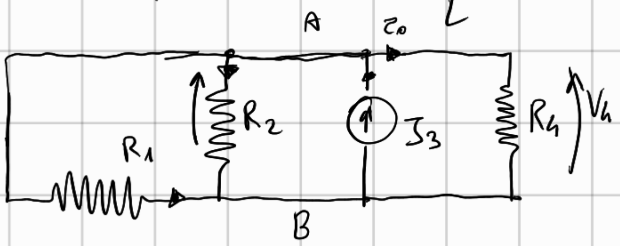
$$i_L \underbrace{\left(+R_1 + R_4 + \frac{R_4 R_1}{R_2}\right)}_{R_{eq} = 205\,\Omega} + V_L \underbrace{\left(1+\frac{R_1}{R_2}\right)}_{2} = E \underbrace{\left(1+\frac{R_1}{R_2}\right)}_{2} - \frac{R_1 E_1}{R_2} + R_1 J_3$$

$$i_L R_{eq} + 2 V_L = 2E - E_1 + 100$$

$$2L\frac{di_L}{dt} + i_L R_{eq} = E_1 + 100$$

$$\Rightarrow \begin{cases} \dfrac{di_L}{dt} + \dfrac{i_L R_{eq}}{2L} = \dfrac{E_1}{2L} + \dfrac{50}{L} \\ \\ i_L(0^+) = \sqrt{2} \cdot 4{,}975 \cdot 10^{-3} \sin(-0{,}1) \end{cases}$$

Sovrapposizione effetti:



$V_L = 0$ corto circuito

$$U_A (G_4 + G_2 + G_1) = J$$

$$U_A = \frac{J}{G_4 + G_2 + G_1} = 25\ V$$

$$I_0 = V_4 G_4 = U_A G_4 = 5 \cdot 10^{-3}\ A$$

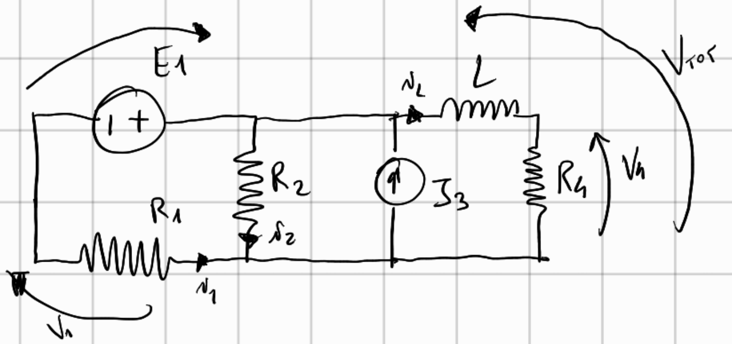
$$i_L^{g}(t) = \sqrt{2} \cdot 4{,}975 \cdot 10^{-3} \sin(10^5 t - 0{,}1) + I_0$$

$$i_L^{go}(t) = A e^{-\frac{R_{eq} t}{2L}} + V_c^{g}(t)$$

$$i_L(0) = A + \sqrt{2}\, 4{,}975 \cdot \cancel{10^{-3}} \sin(-0{,}1) + I_0 = \sqrt{2}\, 4{,}975 \cancel{10^{-3}} \sin(-0{,}1)$$

$$A = -5 \cdot 10^{-3}$$

$$i_L(t) = -\frac{1}{200} e^{-\frac{R_{eq} t}{2L}} + 7 \cdot 10^{-3} \sin(10^5 t - 0{,}1)$$



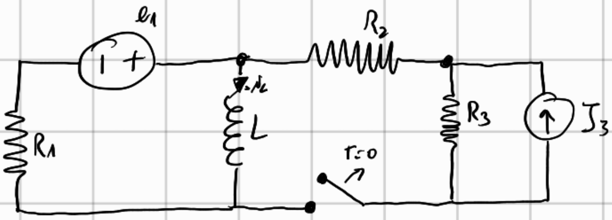
$$V_L(t) = L\frac{di_L}{dt} = \frac{R_{eq}}{4} e^{-\frac{R_{eq}}{2L}t} + 7\cos(10^5 t - 0{,}1)$$

$$V_4(t) = i_L(t) R_4 = -\frac{R_4}{200} e^{-\frac{R_{eq}}{2L}t} + R_4 7 \cdot 10^{-3} \sin(10^5 t - 0{,}1)$$

$$V_{TOT}(t) = V_4(t) + V_L(t) = \left(-\frac{R_4}{200} + \frac{R_{eq}}{4}\right) e^{-\frac{R_{eq}}{2L}t} + 35\sin(10^5 t - 0{,}1) + 7\cos(10^5 t - 0{,}1)$$

$$V_1 = V_{TOT} - E_1$$

2)



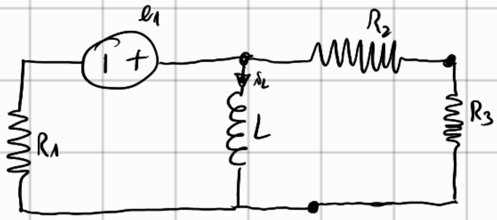
$R_1 = 10\,\Omega \quad R_2 = R_3 = 5\,\Omega$

$e_1(t) = \sqrt{2}\, E_{10} \cos(\omega t)$

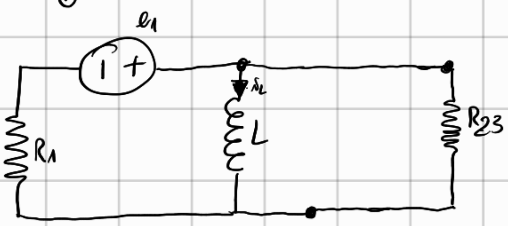
$E_{10} = 220\,V \quad \omega = 2\pi\, 50\ rad/s$

Se $t<0$ interruttore chiuso:

Sovrapposizione effetti: Fasori



$\Downarrow$



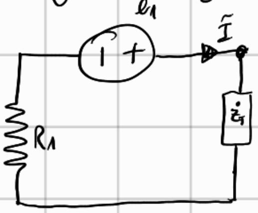
$R_{23} = 10\,\Omega$

$\dot{Z}_L = j\omega L = 100\pi \cdot 1 \cdot 10^{-3} j = \frac{\pi}{10} j\ \Omega$

$\dot{Z}_{R_1} = 10\,\Omega \quad \dot{Z}_{23} = 10\,\Omega$

$\dot{Z}_{L23} = (\dot{Y}_L + \dot{Y}_{23})^{-1} = 9{,}9 \cdot 10^{-3} + 0{,}314 j$

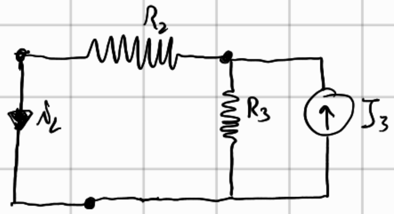
$\Downarrow \longrightarrow \tilde{E}$



$\tilde{I} = \dfrac{\tilde{E}}{\dot{Z}_T + \dot{Z}_1} = 22 - 0{,}69 j$

$\tilde{I}_L = \dfrac{\tilde{I}\, \dot{Z}_{23}}{\dot{Z}_{23} + \dot{Z}_L} = 22 - 1{,}38 j = 22 e^{-0{,}06 j}$

$i_L(t) = 22\sqrt{2} \cos(\omega t - 0{,}06)\ A$



V_L corto circuito

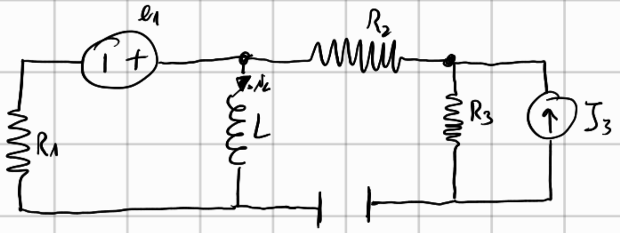
$$I_L = \frac{J_3 \cdot R_3}{R_2 + R_3} = \frac{10 \cdot 10^{-3} \cdot 5}{10} = 5 \cdot 10^{-3}\,A = 5\,mA$$

$$i_L(t) = 22\sqrt{2}\cos(\omega t - 0{,}06) + 5 \cdot 10^{-3}$$

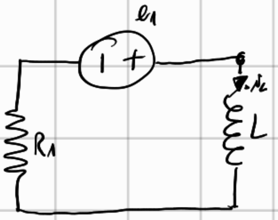
$t<0$

$$i_L(0^+) = 31\,A$$

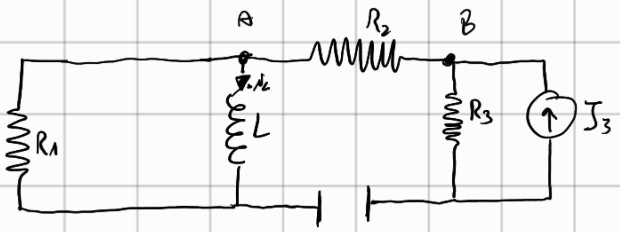
ORA STUDIO $t>0$.



Sovrapposizione:



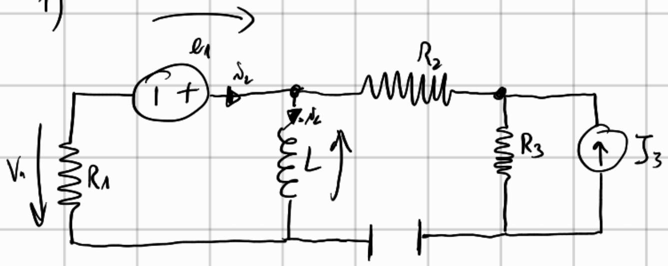
$$\tilde{I}_L = \frac{\tilde{E}_1}{\dot{Z}_L + \dot{Z}_1} = 22 - 0{,}69\,j =$$



$$i_L(t) = 0$$

$$i_L'(t) = 22\sqrt{2}\cos(\omega t - 0{,}031)\ A$$

1)



$$e_1 - V_L - i_L R_1 = 0$$

$$\begin{cases} e_1 = L\dfrac{d i_L}{dt} + i_L R_1 \\ \\ i(0^+) = 31\ A \end{cases}$$

$$i_L(t) = A\, e^{-\frac{Lt}{R}} + 22\sqrt{2}\cos(\omega t - 0{,}031)$$

$$i_L(0^+) = A + 31{,}1 = 31$$

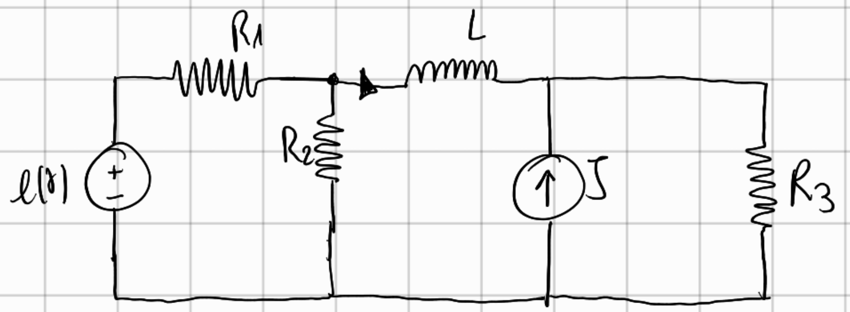
$$A = -0{,}1$$

$$i_L(t) = -\frac{1}{10}\, e^{-10^{-3}t} + 22\sqrt{2}\cos(\omega t - 0{,}031)$$

$$i_L(10^{-2}) = -31{,}2\ A$$

$$E = \frac{1}{2} L i^2 = 0{,}49\ J$$

6)



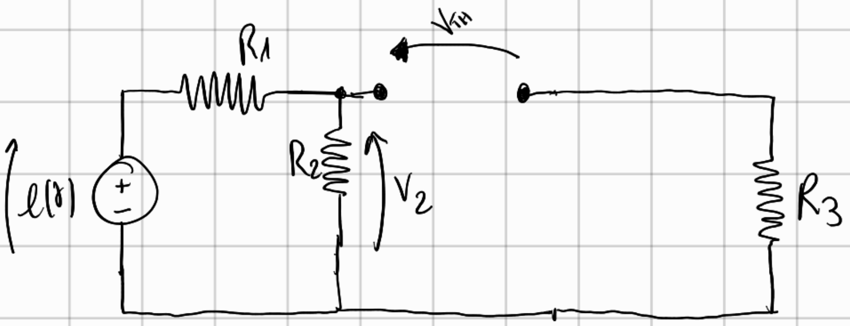
$$e(t) = \begin{cases} 10\cos(50t) & t<0 \\ 0 & t \geq 0 \end{cases}$$

$J = 5A$

$R_1 = R_2 = 40\,\Omega$

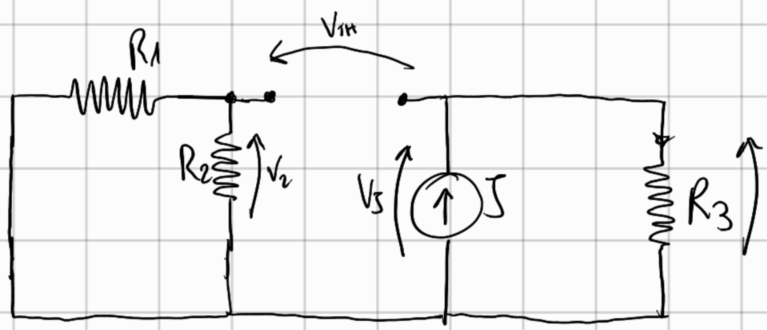
$R_3 = 20\,\Omega \quad L = 500\,mH$

Calcolo equivalente di Thevenin:



$V_{TH} = V_2$

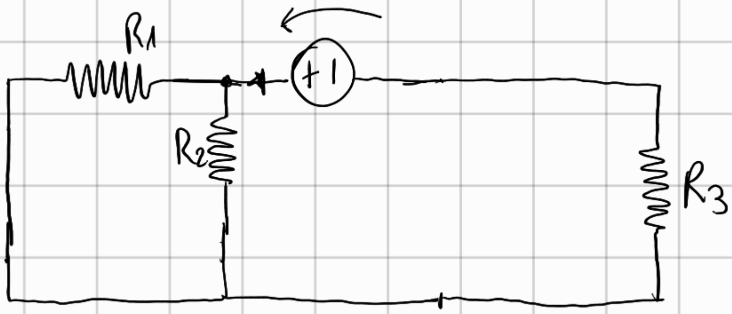
$$V_2 = \frac{e(t)\,R_2}{R_1+R_2} = \frac{e(t)}{2} = 5\cos(50t)\,u(t)$$



$V_J + V_{TH} - V_2 = 0$

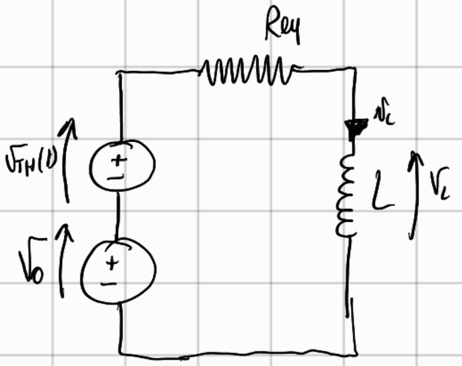
$V_J = -V_{TH} \quad \rightarrow \quad V_J = JR_3 = 100V$

$V_{TH} = -100\,V$



$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} + R_3 = 40\,\Omega$$

$\Rightarrow$



$v_{TH}(t) = 5\cos(50t)u(t)$

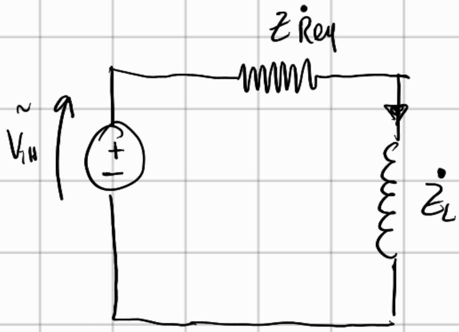
$V_{TH} = -100\,V$

$L = 500\,mH$

$t \leq 0$ uso sovrapposizione :

$\dot{Z}_L = 25j$

$\dot{Z}_R = 40$



$$\tilde{I}_L = \frac{\tilde{V}_{TH}}{\dot{Z}_R + \dot{Z}_L} = \frac{\frac{5}{2}\sqrt{2}}{40+25j} = 0,075\,e^{-0,56j}$$

$\Rightarrow i_L^1(t) = 0,11\cos(50t - 0,56)$ A

CASO 2: $V_L = 0 \Rightarrow I_L = \frac{V_0}{R_{eq}} = \frac{-100}{40} = -\frac{5}{2}$ A

$i_L(t) = 0,11\cos(50t - 0,56) - \frac{5}{2}$ A

$i_L(0^-) = -2,41$ A

$V_0 + V_{TH} - R i_L - V_L = 0$

Se $t>0$, si spegne sinusoidale.

$i_L^P(t) = -\frac{5}{2}$ A già studiato

$$\begin{cases} L\frac{di_L}{dt} + Ri_L = V_0 + V_{TH} \\ i_L(0^+) = -2{,}41 \text{ A} \end{cases}$$

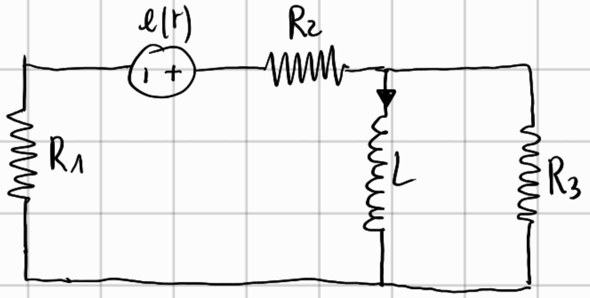
$$i_L^{go}(t) = Ae^{-\frac{t}{\tau}}$$

$$i_L(t) = Ae^{-\frac{t}{\tau}} - \frac{5}{2}$$

$$i_L(0) = A - \frac{5}{2} = -2{,}41 \Rightarrow A = 0{,}1$$

$$i_L(t) = 0{,}1e^{-\frac{R_0 t}{L}} - \frac{5}{2} \text{ A}$$

2)



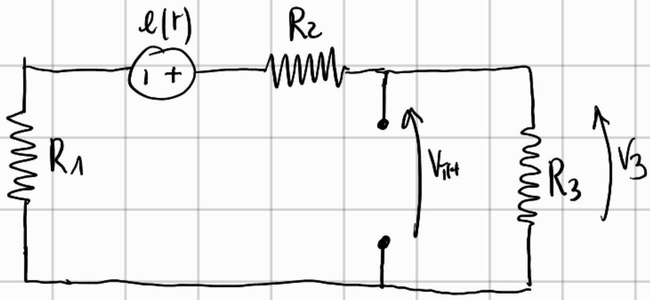
$$e(t) = \begin{cases} 2\cos(100t)\ V & t<0 \\ 0\ V & t \geq 0 \end{cases}$$

$R_1 = R_2 = 10\Omega$

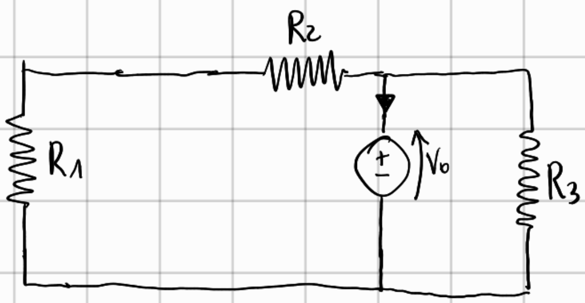
$R_3 = 5\Omega$

$L = 100\,mH$

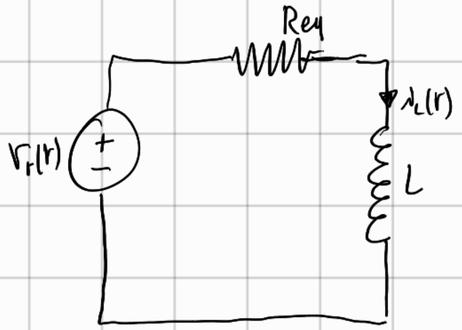
THEVENIN



$$V_3 = \frac{e(t) R_3}{R_1+R_2+R_3} = \frac{2\cos(100t)\,\cancel{5}}{\cancel{25}\,5} = \frac{2}{5}\cos(100t)\,u(t) = V_{TH}$$



$$R_{eq} = \left(\frac{1}{R_1+R_2} + \frac{1}{R_3}\right)^{-1} = 4\Omega$$



$$\dot{Z}_L = 100 \cdot 100 \cdot 10^{-3} = 10j\ \Omega$$

$t<0 \Rightarrow \quad \tilde{I}_L = \dfrac{\tilde{V}}{\dot{Z}_R + \dot{Z}_L} = \dfrac{\frac{\sqrt{2}}{5}}{4+10j} = 0{,}026\, e^{-1{,}2 j}$

$i_L(t) = 0{,}037 \cos(100t - 1{,}2)\ A$

Se $t>0$ si spegne generatore. $\Rightarrow i_L \to 0$ all'infinito.

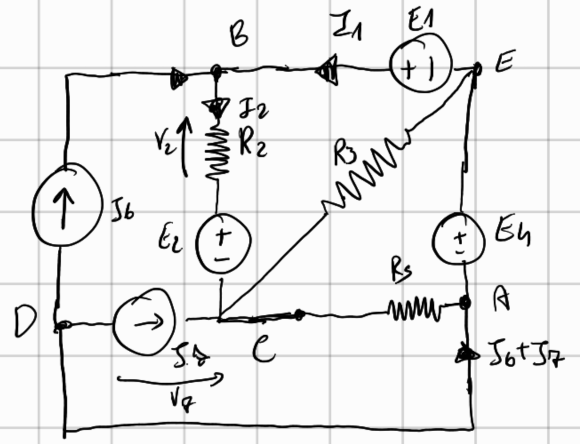
$i^g(t) = 0$

$$\begin{cases} \dfrac{d i_L}{dt} + \dfrac{R_{eq}}{L} i_L = 0 \\ i_L(0^+) = 0{,}0134\ A \end{cases}$$

$i_L(t) = A e^{-\frac{t}{\tau}} \Rightarrow i_L(0) = A = 0{,}0134\ A$

$i_L(t) = 0{,}0134\, e^{-\frac{R}{L}t}$

1)



$R_2 = 2k\Omega \quad R_3 = 6k\Omega \quad R_5 = 5k\Omega$

$E_1 = 8V \quad E_2 = 32V \quad E_4 = 12V$

$J_6 = 2mA \quad J_7 = 3mA$

$U_A = 0 \quad U_D = 0$

$U_E = 12V \qquad E_1 = U_B - U_E \Rightarrow U_B = 12 + 8 = 20\,V$

$$U_C\left(G_5 + G_3 + G_2\right) = J_7 - E_2G_2$$

$$U_C = \frac{J_7 - E_2G_2}{G_2 + G_3 + G_5} = \frac{285}{13}\,V$$

$$U_C - U_D = \frac{285}{13}\,V$$

$$(U_B - U_C) - V_2 - E_2 = 0 \Rightarrow -V_2 = E_2 - (U_B - U_C) = \frac{441}{13}\,V$$

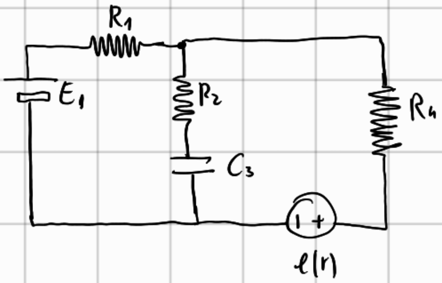
$$= V_2 = \frac{-441}{13}\,V$$

$I_2 = V_2 G_2 = -0{,}017A$

$I_1 + J_6 - I_2 = 0$

$I_1 = I_2 - J_6 = -0{,}019\,A = -20mA$

2)



$E_1 = 12V$

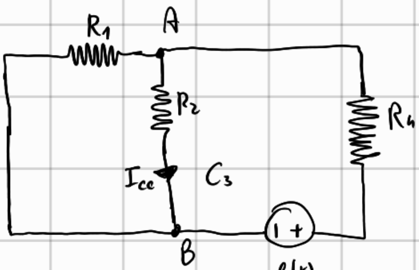
$e_4(t) = \sqrt{2}\, E_{10} \cos(\omega_1 t) \cdot u(-t)$

$E_{10} = 10V \qquad \omega_1 = 1\, krad/s$

$R_1 = R_2 = R_4 = 10\,\Omega$

$C_3 = 100\,\mu F$

Trovo equivalente di Norton:

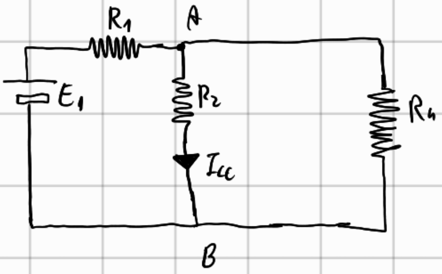


Solo la parte variabile nel tempo:

$$U_A(G_1 + G_2 + G_4) = \frac{e(t)}{R_4}$$

$$U_A = \frac{e(t)\, G_4}{G_1 + G_2 + G_4} = \frac{e(t)}{3} = \frac{\sqrt{2}\, 10}{3} \cos(\omega_1 t) u(-t)$$

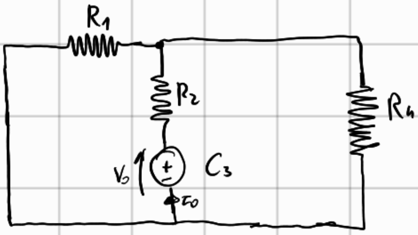
$$I_{cc} = \frac{U_A}{R_A} = \frac{\sqrt{2}}{3} \cos(\omega_1 t) u(-t)$$



$$U_A(G_1+G_2+G_4) = E_1 G_1$$

$$U_A = \frac{E_1 G_1}{G_1+G_2+G_4} = 4\,V$$

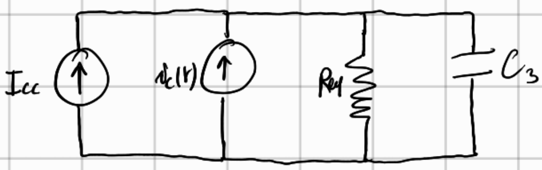
$$I_{cc} = 4 \cdot G_2 = \frac{4}{10}\,A = \frac{2}{5}\,A$$



$$R_{14} = \left(\frac{1}{R_1}+\frac{1}{R_4}\right)^{-1} = 5\,\Omega$$

$$R_{124} = R_{14}+R_2 = 15\,\Omega = R_{eq} \qquad G_N = \frac{1}{15}\,S$$

$\Rightarrow$



$$i_c(t) = \frac{\sqrt{2}}{3}\cos(\omega_1 t)\,u(-t)\ A$$

$$I_{cc} = \frac{2}{5}\,A \qquad C_3 = 100\,\mu F$$

$$G_N = \frac{1}{15}\,S$$

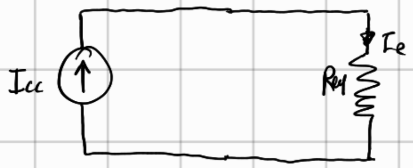
Studio per $t<0$: Sovrapposizione effetti: $\qquad Z_C = \frac{-j}{\omega C} = -10j$

$$\tilde{I}_R = \frac{\tilde{J}\cdot \dot{Y}_R}{\dot{Y}_R+\dot{Y}_C} = \frac{\frac{1}{3}\cdot\frac{1}{15}}{\frac{1}{15}+\frac{1}{10}j} = \frac{4}{39}-\frac{2}{13}j$$

$\tilde{V} = \dot{z}_R \tilde{I}_R = \frac{20}{13} - \frac{30}{13}J = 2,77\, e^{-0,98 J}$

$v_1(t) = 3,92 \cos(\omega t - 0,98)\ V$

Sovrapposizione:



$V_R = R_{eq} I_{cc} = 6V$ che è lo stesso di quando si spegne:

$$\begin{cases} \frac{dv_0}{dt} + \frac{v_0}{R_{eq}C} = \frac{I_{cc} + i_e}{RC} \\ v(0^+) = 8,18\ V \end{cases}$$

$v^P(t) = 6V$

$v^{go}(t) = Ae^{-\frac{t}{RC}}$

$v(t) = Ae^{-\frac{t}{RC}} + 6V$

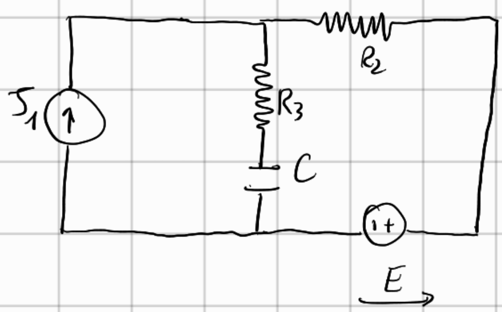
$v(0) = A + 6 = 8,18 \Rightarrow A = 2,18\ A$

$v_c(t) = 2,18\, e^{-\frac{t}{RC}} + 6$

$i(t) = \frac{-2,18}{RC} \cdot 10^{-4}\, e^{-\frac{t}{RC}} = -0,15\, e^{-\frac{t}{RC}}$

$P = v_c(t) \cdot (-i(t)) = (2,18\, e^{-\frac{t}{RC}} + 6) \cdot (+0,15\, e^{-\frac{t}{RC}})$ $\square$

1)

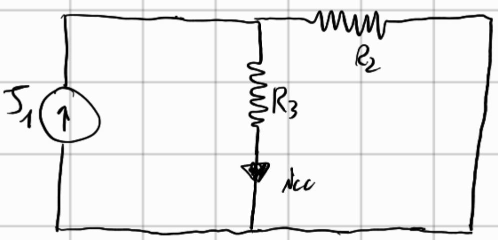


$J_1(t) = \sqrt{2} \cdot 10 \sin(100t)$ mA

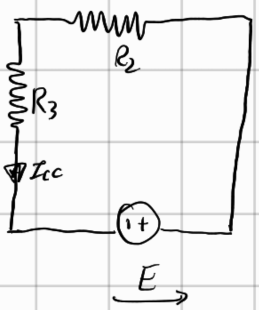
$E_2 = 12\, u(t)$ V

$R_2 = R_3 = 500\,\Omega$ $\quad C_3 = 50\,\mu F$

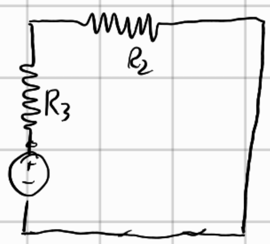
EQUIVALENTE NORTON:



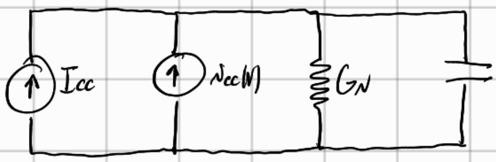
$$I_{cc}(t) = \frac{J_1(t)\, R_2}{R_2 + R_3} = \frac{J_1(t)}{2}$$



$$I_{cc} = \frac{E}{R_2 + R_3} = 0{,}012\,A = 12\, u(t)\ mA$$



$R_{eq} = R_{32} = 1000\,\Omega \Rightarrow G_N = \frac{1}{1000}$ S



$i_{cc}(t) = 5\sqrt{2}\sin(100t)$ mA

$I_{cc} = 12\,u(t)$ mA

$G_N = 1$ mS

Se $t<0$:

$\dot{Y}_N = 10^{-3}$ S

$\dot{Y}_c = 5\cdot 10^{-3} j$ S

$\tilde{J} = 5$

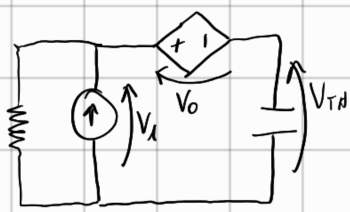
$$\tilde{I}_1 = \frac{\tilde{J}\,\dot{Y}_N}{\dot{Y}_c + \dot{Y}_N} = \frac{5\cdot\cancel{10^{-3}}\cdot 10^{-3}}{\cancel{10^{-3}} + 5\cdot\cancel{10^{-3}}j} = (0{,}192 - 0{,}962j)\cdot 10^{-3}$$

$$\tilde{V}_c = R_N \cdot \tilde{I}_1 = 0{,}192 - 0{,}962j = 0{,}98\,e^{-1{,}37j}$$

$$\Rightarrow V_c(t) = 1{,}39\sin(100t - 1{,}37)$$

$$V_c(0^-) = V_c(0^+) = -1{,}36\ V$$

Se $t>0$: Studio con sovrapposizione contributo di I_{cc}.



$V_{TH} = V_1 - V_0$

Condensatore si comporta come aperto

$$V_c^p = R_N I_{cc} = 12\ V$$

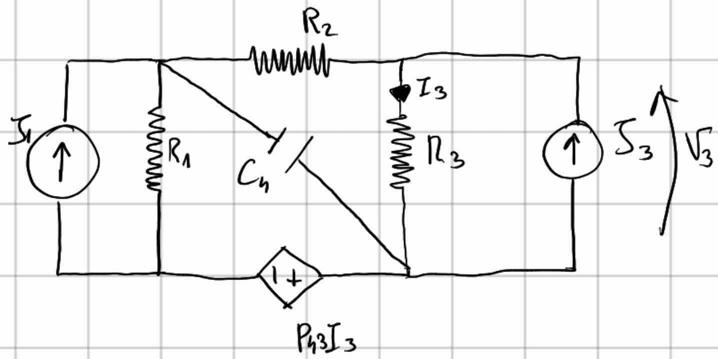
$$\begin{cases} \dfrac{dv_c}{dt} + \dfrac{v_c}{RC} = \dfrac{J_1 + J_{cc}}{C} \\ \\ v_c(0^+) = -1{,}36\ V \end{cases}$$

$$v_c(t) = A e^{-\frac{t}{RC}} + 1{,}39 \sin(100t - 1{,}37) + 12$$

$$v_c(0) = A + 12 - \cancel{1{,}36} = -\cancel{1{,}36}$$

$$A = -12\ V$$

$$v_c(t) = -12 e^{-\frac{t}{RC}} - 1{,}39 \sin(100t - 1{,}37) + 12$$



$R_1 = 50\,\Omega$ $R_3 = 100\,\Omega$ $R_2 = 200\,\Omega$

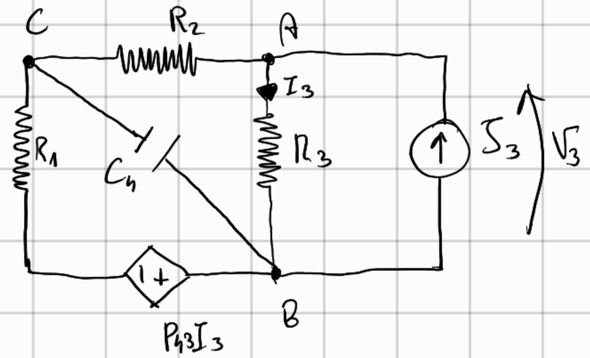
$P_{43} = 75\,\Omega$ $C_4 = 10\,mF$

$J_1(t) = 10^{-3}\sqrt{2}\,\sin(100t)\ mA$

$J_3(t) = 5\cdot 10^{-3}\sqrt{2}\,\sin(1000t)\ mA$

$\dot{Y}_1 = \frac{1}{50}\,S$ $\dot{Y}_2 = \frac{1}{200}\,S$

$\dot{Y}_3 = \frac{1}{100}\,S$ $\dot{Y}_C = j\omega C = 10j\ S$



$$\tilde{U}_A(\dot{Y}_3 + \dot{Y}_2) - \tilde{U}_C\,\dot{Y}_2 = \tilde{J}_3$$

$$\tilde{U}_C(\dot{Y}_1 + \dot{Y}_C + \dot{Y}_2) - \tilde{U}_A\,\dot{Y}_2 = -P_{43}\,\tilde{U}_A\,\dot{Y}_3\,\dot{Y}_1$$

$$\Rightarrow\ \tilde{U}_A\left(\frac{3}{200}\right) - \tilde{U}_C\cdot\frac{1}{200} = 5$$

$$\tilde{U}_C\left(\frac{1}{40} + 10j\right) - \tilde{U}_A\cdot\frac{1}{200} = -75\cdot\tilde{U}_A\cdot\frac{1}{100}\cdot\frac{1}{50}$$

$$\Rightarrow\ 3\tilde{U}_A - \tilde{U}_C = 1000$$

$$\tilde{U}_C(5 + 2000j) - \tilde{U}_A = -3\tilde{U}_A$$

$3\tilde{U}_A - \tilde{U}_C = 1000$

$5\tilde{U}_C + 2000j\tilde{U}_C + 2\tilde{U}_A = 0$

$$\begin{cases} \tilde{U}_C = 3\tilde{U}_A - 1000 \\ 15\tilde{U}_A - 5000 \quad + 6000j\tilde{U}_A + \; - 2000000j + 2\tilde{U}_A = 0 \end{cases}$$

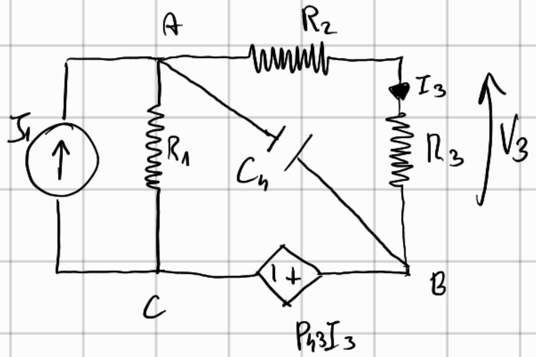
$\tilde{U}_A(17 + 6000j) = 5000 + 2000000j$

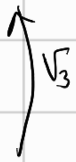
$= 333{,}3 + 0{,}11j$

$U_C = 0{,}33j$

$(\tilde{U}_A - \tilde{U}_B)\dot{Y}_3 = 3{,}33 + 1{,}1 \cdot 10^{-3} j = \tilde{I}_3$

$\Rightarrow \quad \tilde{V}_3 = \tilde{U}_A$





$Y_C = \omega C j = j\,S$

$$\tilde{U}_A(\dot{Y}_{23} + \dot{Y}_C + \dot{Y}_1) - \tilde{U}_B(\dot{Y}_{23} + \dot{Y}_C) = \tilde{J}_1$$

$$\tilde{U}_C = 0 \qquad \tilde{U}_B = P_{43}\tilde{I}_3 = P_{43}\frac{\tilde{U}_A}{3}\dot{Y}_{23}$$

$$\dot{Y}_{23} = \frac{1}{300}\,S \qquad \tilde{U}_B = \frac{75}{300}\tilde{U}_A$$

$$\tilde{U}_A\left(\frac{1}{300} + j + \frac{1}{50}\right) - \left(\frac{75}{300} + j\right)\frac{1}{3}\tilde{U}_A = 1$$

$$\tilde{U}_A(1 + 300j + 6) - \frac{75}{3}\tilde{U}_A - 100j\tilde{U}_A = 300$$

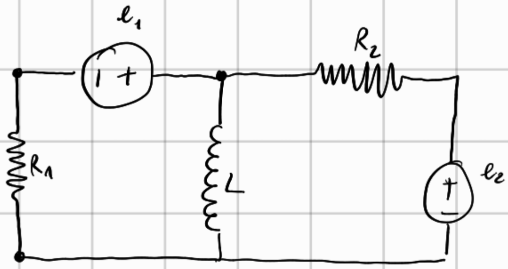
$$\tilde{U}_A(200j - 18) = 300$$

$$\tilde{U}_A = -0{,}13 - 1{,}49j$$

$$\tilde{V}_3 = \frac{\tilde{U}_A \cdot \dot{Z}_3}{\dot{Z}_3 + \dot{Z}_2} = -0{,}045 - 0{,}5j$$

$$\tilde{I}_3 = \tilde{V}_3\dot{Y}_3 = -4{,}5\cdot 10^{-4} - 5\cdot 10^{-3}j$$

2)



$e_1(t)=\sqrt{2}E_{10}\cos(\omega t)$

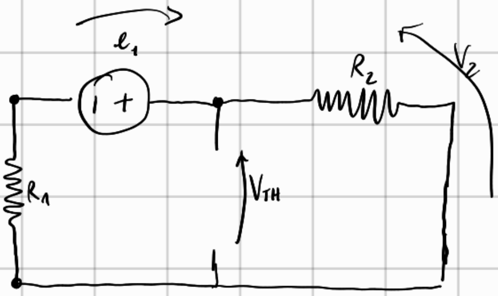
$E_{10}=220\,V$ $\quad \omega=2\pi 50\,rad/s$

$e_2(t)=100u(t)\,V$

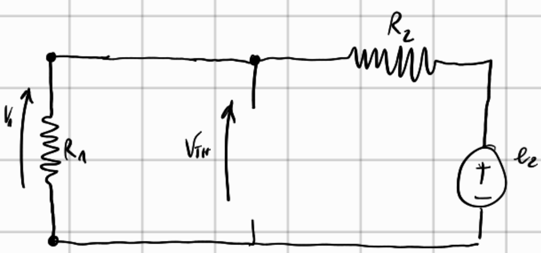
$R_1=R_2=10\,\Omega$

$L=1\,mH$

EQUIVALENTE DI THEVENIN

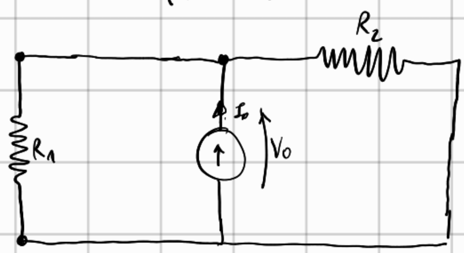


$$V_2=\frac{e_1 R_2}{R_1+R_2}=110\sqrt{2}\cos(\omega t)=V_{TH}$$

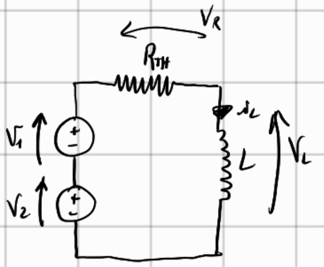


$$V_1=\frac{e_2 R_1}{R_1+R_2}=50u(t)\,V$$

R_{eq}: $R_1 /\!/ R_2=\left(\frac{1}{10}+\frac{1}{10}\right)^{-1}=5\,\Omega$



$\Rightarrow$



$v_1(t) = 50\,u(t)\ V$

$v_2(t) = 110\sqrt{2}\cos(\omega t)\ V$

$R_{TH} = 5\ \Omega$

$L = 1\ mH \qquad \dot{Z}_L = j\omega L = \frac{\pi}{10} j\ \Omega$

$t<0$: Studio con i fasori:

$$\tilde{I}_L = \frac{\tilde{V}_2}{\dot{Z}_R + \dot{Z}_L} = 21{,}9 - 1{,}38j$$

$$i_L(t) = 21{,}96\sqrt{2}\cos(\omega t - 0{,}06)$$

$$i_L(0^+) = i_L(0^-) = 31\ A$$

$t>0$: già studiato regime fasoriale.

Studio regime stazionario:

Induttore si comporta da corto:

$$I_L = \frac{V_1}{R_{TH}} = 10\ A$$

$$\Rightarrow \begin{cases} \frac{d i_L}{dt} + \frac{R}{L} i_L = \frac{v_1 + v_2}{L} \\ i_L(0^+) = 31\ A \end{cases}$$

$$i_L(t) = A e^{-\frac{R}{L}t} + 10 + 31{,}06\cos(\omega t - 0{,}06)$$

$$i_L(0) = A + 41 = 31$$

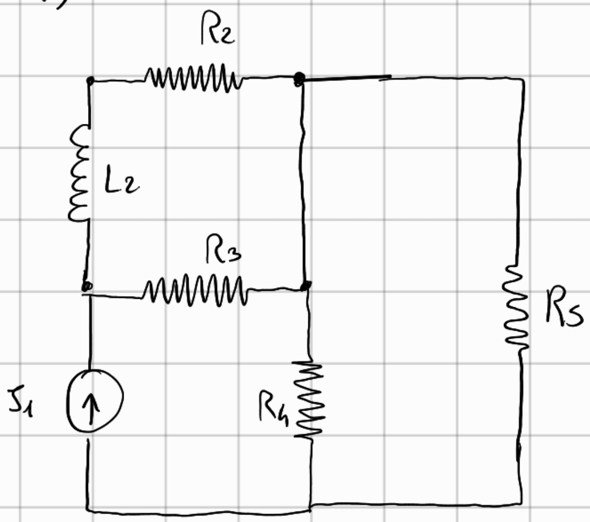
$$\Rightarrow A = -10\ A$$

$$\Rightarrow i_L(t) = -10 e^{-\frac{R}{L}t} + 10 + 31{,}06\cos(\omega t - 0{,}06)\ A$$

Con $\frac{R}{L} = 5000\ Hz$

$$\Rightarrow i_L\left(\tfrac{1}{100}\right) = -21\ A \quad \Rightarrow \mathcal{E}(t_0) = \frac{1}{2} L i^2 = 0{,}22\ J$$

1)



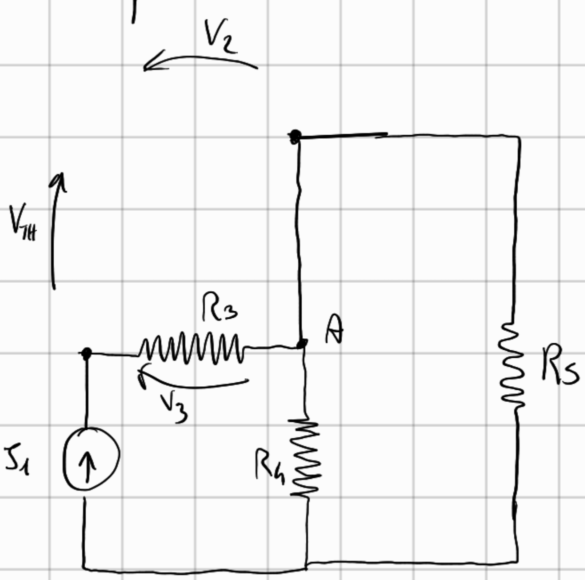
$J_1(t) = \sqrt{2}\, J_{10} \sin(\omega_1 t)\, u(-t)$

$J_{10} = 1A \quad \omega_1 = 1 krad/s$

$L_2 = 1H \quad R_2 = 1\Omega$

$R_3 = R_4 = R_5 = 2\Omega$

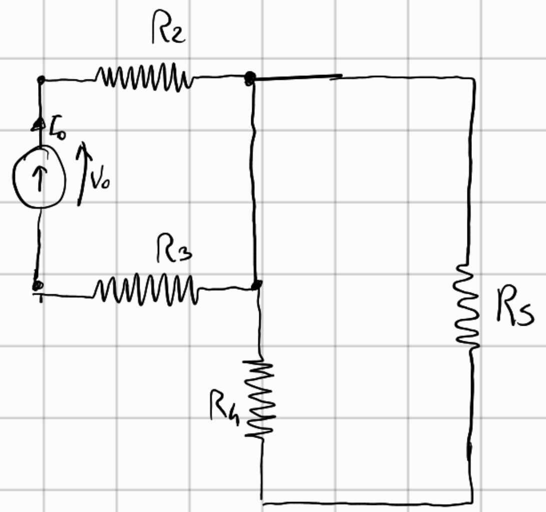
Trovo equivalente Thevenin:



$V_{TH} - V_2 + V_3 = 0 \Rightarrow V_{TH} = -V_3$

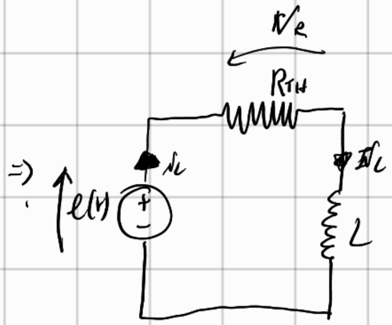
$V_3 = R_3 J_1 = 2\sqrt{2} \sin(\omega t)\, u(-t)\ V$

$V_{TH} = -2\sqrt{2} \sin(\omega t)\, u(-t)$



R_{45} in parallelo con corto circuito.

$\Rightarrow R_{eq} = R_2 + R_3 = 3\,\Omega$



$e(t) = -2\sqrt{2}\sin(\omega t)\,u(-t) = +2\sqrt{2}\sin(\omega t + \pi)\,u(-t)$

$t<0 \rightarrow$ Regime sinusoidale. $\dot{Z}_L = 1000j$

$$\tilde{I}_L = \frac{\tilde{e}}{\dot{Z}_R + \dot{Z}_L} = \frac{-2}{3+1000j} = -6\cdot10^{-6} + 2\cdot10^{-3}j$$

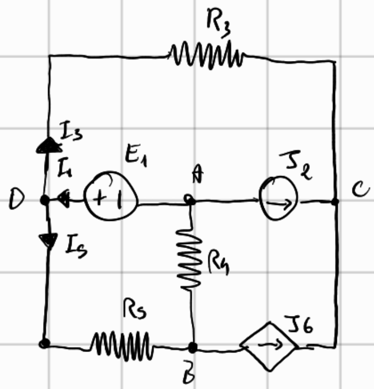
$i_L(t) = 2\sqrt{2}\cdot10^{-3}\sin(\omega t + \pi)$ $t<0$

$i_L(0^-) = 0$

Per $t>0$ stessa cosa.

$$\begin{cases} \dfrac{di_L}{dt} + \dfrac{R}{L} i_L = 0 \\ i_L(0^+) = 0 \end{cases} \Rightarrow i_L(t) = 0\ A$$

1)



$E_1 = 2V \quad J_2 = 1A \quad J_6 = \alpha N_1 \quad \alpha = 1$

$R_3 = 1\Omega \quad R_4 = 3\Omega \quad R_5 = 2\Omega$

Pass ohne E_1:

$I_1 = I_3 + I_5$

$U_A = 0$ $\qquad$ $I_1 = (U_D - U_B)G_5 + (U_D - U_C)G_3$

$U_D = E_1 = 2V$ $\qquad$ $I_1 = (2 - U_B)G_5 + (2 - U_C)G_3$

$$\begin{cases} U_B(G_5 + G_4) - U_D G_5 = -J_6 \\ U_C G_3 - U_D G_3 = J_2 + J_6 \end{cases}$$

$$\begin{cases} U_B \frac{5}{6} - \frac{U_D}{2} = -\left(1 - \frac{U_B}{2} + 2 - U_C\right) \\ U_C - 2 = 1 + \left(1 - \frac{U_B}{2} + 2 - U_C\right) \end{cases}$$

$$\Rightarrow \begin{cases} 5U_B \cancel{-6} = \cancel{-6} + 3U_B - 12 + 6U_C \\ 2U_C - 4 = 2 + 2 - U_B + 4 - 2U_C \end{cases}$$

$$\begin{cases} 2U_B - 6U_C = -12 \\ U_B + 4U_C = 12 \end{cases}$$

$U_B = \frac{12}{7}V \quad U_C = \frac{18}{7}V$