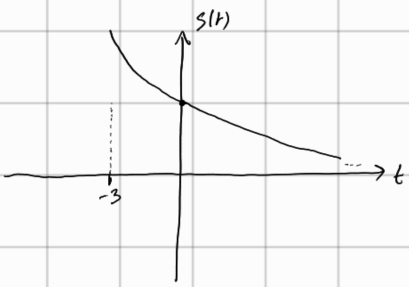
ALCUNE CONSIDERAZIONI:

- É possibile che una piccola parte degli esercizi sia scorretta
- Alcuni esercizi possono essere risolti in modi diversi, ma ugualmente corretti
- Gli esercizi sono svolti in ordine cronologico, quindi piú si va in fondo nel file piú sará probabile che con piú esperienza alle spalle saranno corretti.

BUON LAVORO!

1)

$$s(t) = e^{-t} u(t+3)$$

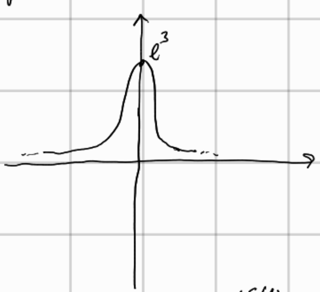


$$s(t) = e^{-(t+3)+3} u(t+3) = e^{3} e^{-(t+3)} u(t+3)$$

$$\mathcal{E}_s = \int_{-3}^{+\infty} e^{-t}\, dt = \left.\frac{e^{-t}}{-1}\right|_{-3}^{+\infty} = e^{3}$$

$$S(f) = e^{3} \cdot \frac{e^{6\pi j f}}{1 + 2\pi j f}$$

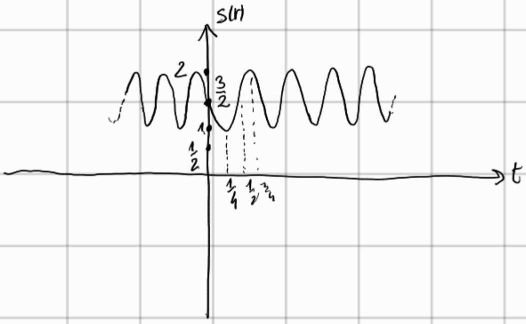
$$|S(f)| = \frac{e^{3}}{\sqrt{1 + 4\pi^2 f^2}}$$



$$\angle S(f) = 6\pi f - \operatorname{atan}(2\pi f)$$

2)

$$s(t) = 1 + \cos^2\left(\pi t + \frac{\pi}{4}\right) = 1 + \frac{1}{2} + \frac{1}{2}\cos\left(2\pi t + \frac{\pi}{2}\right) = \frac{3}{2} + \frac{1}{2}\cos\left(2\pi t + \frac{\pi}{2}\right)$$



$T = 1$

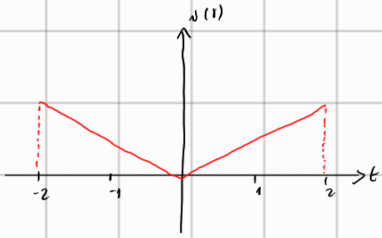
$$P_s = \frac{1}{1}\int_0^1 [s(t)]^2 dt = \frac{9}{4} + \frac{1}{4}\int_0^1 \cos^2\left(2\pi t + \frac{\pi}{2}\right) dt = \frac{9}{4} + \frac{1}{4}\left[\frac{1}{2} + \frac{1}{2}\int_0^1 \cos(4\pi t + \pi)\, dt\right] = \frac{9}{4} + \frac{1}{8} = \frac{19}{8}$$

$E_s = \infty$

$$S(f) = \frac{3}{2}\delta(f) + \frac{1}{4}\left[\delta(f-1)e^{j\frac{\pi}{2}} + \delta(f+1)e^{-j\frac{\pi}{2}}\right]$$

3) $s(t)=\sum_{K=-\infty}^{+\infty}\left[\Pi\left(\frac{t}{4}-K\right)-\Delta\left(\frac{t}{2}-2K\right)\right]=$

$$=\sum_{K=-\infty}^{+\infty}\left[\Pi\left(\frac{t-4K}{4}\right)-\Delta\left(\frac{t-4K}{2}\right)\right]$$



$s(t)=\Pi\left(\frac{t}{4}\right)-\Delta\left(\frac{t}{2}\right)$ Periodo $=4$

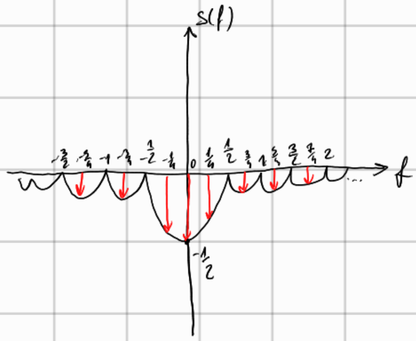
$$S(t)=s(t)*\sum_{K=-\infty}^{+\infty}\delta(t-4K)$$

$$I(f)=4\,\mathrm{sinc}(4f)-2\,\mathrm{sinc}^2(2f)$$

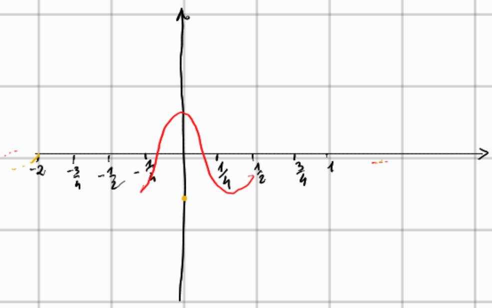
$$S(f)=\left(4\,\mathrm{sinc}(4f)-2\,\mathrm{sinc}^2(2f)\right)\cdot\frac{1}{4}\sum_{K=-\infty}^{+\infty}\delta\left(f-\frac{K}{4}\right)$$

$$S(f)=\sum_{K=-\infty}^{+\infty}\left(\cancel{\mathrm{sinc}(K)}-\frac{1}{2}\,\mathrm{sinc}^2\left(\frac{K}{2}\right)\right)\delta\left(f-\frac{K}{4}\right)$$

$$=\frac{1}{2}\sum_{K=-\infty}^{+\infty}\mathrm{sinc}^2\left(\frac{K}{2}\right)\delta\left(f-\frac{K}{2}\right)$$

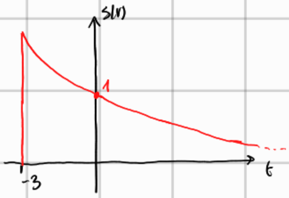


$\mathrm{Sinc}(4f)-\frac{1}{2}\mathrm{Sinc}^2(2f)$

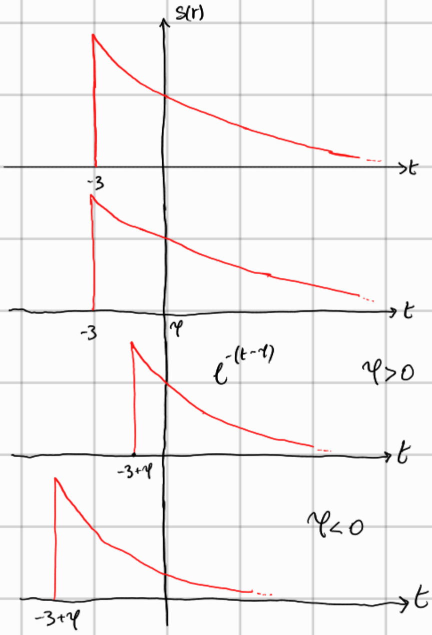


1)

$$s(t) = e^{-t}\, u(t+3)$$

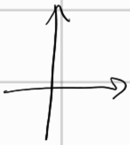


$$r_s(\tau) = \int_{-\infty}^{+\infty} s(t)\, s^*(t-\tau)\, dt$$



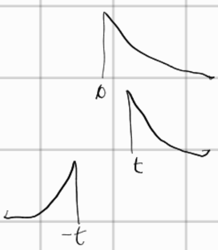
$$r_s(\tau) = \int_{-3+\tau}^{+\infty} e^{-t} \cdot e^{-t} e^{\tau}\, dt = e^{\tau} \int_{-3+\tau}^{+\infty} e^{-2t}\, dt = e^{\tau}\, \frac{e^{-2t}}{-2}\Big|_{-3+\tau}^{+\infty} = \frac{e^{\tau}}{2}\, e^{6-2\tau} = \frac{e^{6-\tau}}{2}$$

$$r_s(\tau) = \int_{-3}^{+\infty} e^{-t} e^{-t} e^{\tau}\, dt = e^{\tau}\, \frac{e^{-2t}}{-2}\Big|_{-3}^{+\infty} = \frac{e^{\tau} \cdot e^{6}}{2} = \frac{e^{6+\tau}}{2}$$



$h(t-\tau)$

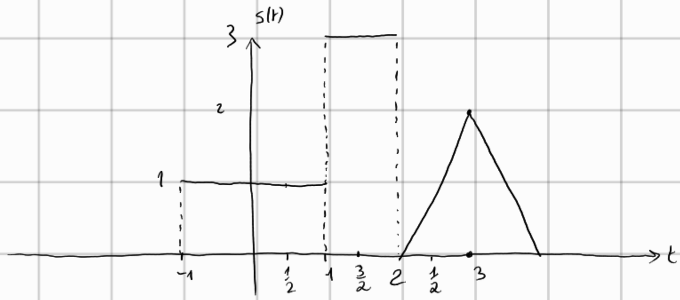
$h(-(\tau-t))$



1)

a) $s(t) = \Pi\left(\frac{t}{2}\right) + 3\Pi\left(\frac{2t-3}{2}\right) + 2\Delta(t-3) =$

$= \Pi\left(\frac{t}{2}\right) + 3\Pi\left(t-\frac{3}{2}\right) + 2\Delta(t-3)$



Segnale di energia $\Rightarrow P_s = 0$

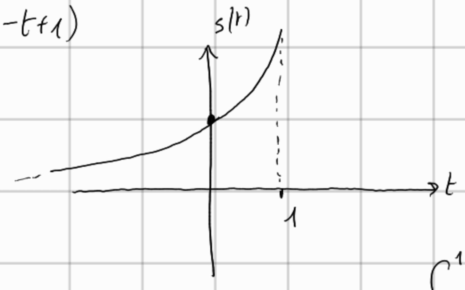
$\mathcal{E}_s = 2 + 9 + 2 \cdot \frac{4 \cdot 1}{3} = 11 + \frac{8}{3} = \frac{41}{3}$

$F[s(t)] = 2\,\mathrm{sinc}(2f) + 3e^{-3\pi j f}\,\mathrm{sinc}(f) + 2e^{-6\pi j f}\,\mathrm{sinc}^2(f)$

b)

$s(t) = e^{2t}u(-t+1)$

$-t+1 \geq 0$
$t < 1$



Segnale di energia: $P_s = 0$.

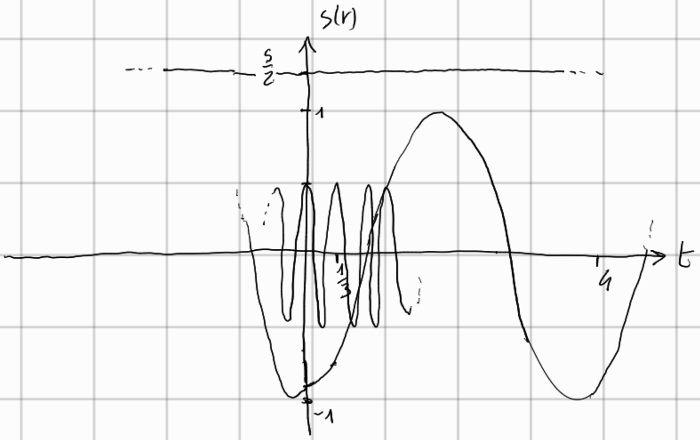
$\mathcal{E}_s = \int_{-\infty}^{1} e^{4t}\,dt = \left.\frac{e^{4t}}{4}\right|_{-\infty}^{1} = \frac{e^4}{4}$

$$F[s(t)] = \int_{-\infty}^{1} e^{2t} e^{-2\pi j f t} dt = \int_{-\infty}^{1} e^{(2-2\pi j f)t} dt = \left. \frac{e^{(2-2\pi j f)t}}{(2-2\pi j f)} \right|_{-\infty}^{1} = \frac{e^{2-2\pi j f}}{2-2\pi j f}$$

c)

$$s(t) = 2 - \cos\left(\frac{\pi}{2}t + \frac{1}{10}\right) + \cos^2(3\pi t) = 2 - \cos\left(\frac{\pi}{2}t + \frac{1}{10}\right) + \frac{1}{2} + \frac{1}{2}\cos(6\pi t) =$$

$$= \frac{5}{2} - \cos\left(\frac{\pi}{2}t + \frac{1}{10}\right) + \frac{1}{2}\cos(6\pi t)$$



Segnale di potenza $\Rightarrow E_s = \infty$

Periodo minimo = 4

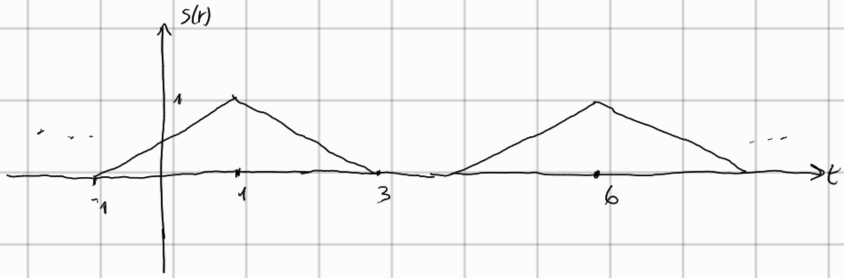
$$P_s = \frac{1}{4}\int_0^4 |s(t)|^2 dt = \frac{1}{\cancel{4}}\left[\cancel{4}\cdot\frac{25}{4} + \cancel{4}\cdot\frac{1}{2} + \frac{1}{8}\cdot\cancel{4}\right] = \frac{50}{8} + \frac{4}{8} + \frac{1}{8} = \frac{55}{8}$$

$$F[s(t)] = \frac{5}{2}\delta(f) + \frac{1}{4}\left[\delta(f-3) + \delta(f+3)\right] - \frac{1}{2}\left[\delta\left(f-\frac{1}{4}\right)e^{j/10} + \delta\left(f+\frac{1}{4}\right)e^{-j/10}\right]$$

d)

$$s(t) = \sum_{k=-\infty}^{+\infty} \Delta\left(\frac{t-1-5k}{2}\right)$$

$$\hat{s}(t) = \Delta\left(\frac{t-1}{2}\right)$$



Periodo = 5

$$s(t) = \hat{s}(t) * \sum_{k=-\infty}^{+\infty} \delta(t-5k)$$

$$S(f) = \frac{1}{5} \hat{S}(f) \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{5}\right)$$

$$\hat{S}(f) = e^{-2\pi j f} \cdot \frac{2}{5} \operatorname{sinc}^2(2f) \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{5}\right)$$

$$\mathcal{E}_s = +\infty$$

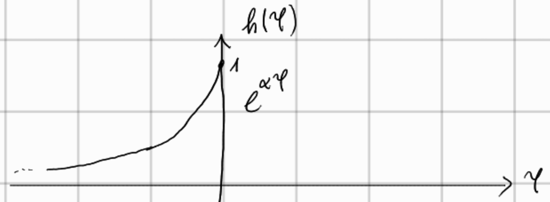
$$P = \int_0^5 |s(t)|^2 dt = \frac{1}{5} \cdot 2\left(\frac{2 \cdot 1}{3}\right) = \frac{4}{15}$$

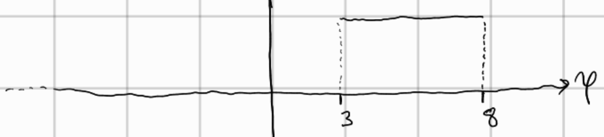
2)

$h(t) = e^{\alpha t} u(-t)$

$s(t) = u(t-3) - u(t-8)$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, s(t-\tau)\, d\tau$$





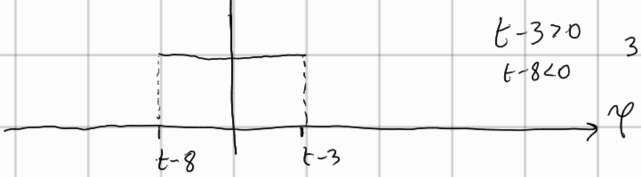
$t-3<0 \quad t<3$

$$y(t) = \int_{t-8}^{t-3} e^{\alpha\tau} dt = \frac{e^{\alpha\tau}}{\alpha}\Big|_{t-8}^{t-3} = \frac{e^{\alpha t}\left(e^{-3\alpha} - e^{-8\alpha}\right)}{\alpha}$$

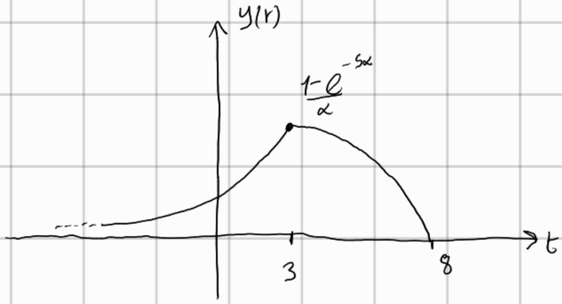
$t-3>0$

$t-8<0$ $\quad 3<t<8$

$$y(t) = \int_{t-8}^{0} e^{\alpha\tau} d\tau \quad \frac{e^{\alpha\tau}}{\alpha}\Big|_{t-8}^{0} = \frac{1-e^{\alpha t - 8\alpha}}{\alpha}$$



Se $t>8$, $y(t)=0$



3) $h(t) = 2e^{-\alpha(t-1)} u(t-1)$

$x(t) = \frac{1}{2} - \cos(\pi t) + 5\cos^2(5t-4) =$

$= 3 - \cos(\pi t) + \frac{5}{2}\cos(10t-8)$

Sia $f(t) = e^{-\alpha t} u(t)$. $F(f) = \frac{1}{\alpha + 2\pi j f}$

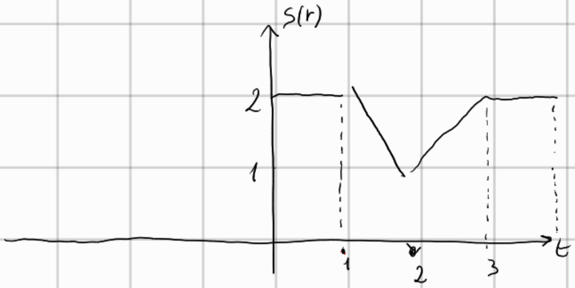
$H(f) = \frac{2e^{-2\pi j f}}{\alpha + 2\pi j f}$

Considero $x(t)$. Combinaz. di seni. Costante ha $f_0 = 0$. $\cos(\pi t)$ ha $f = \frac{1}{2}$, $\cos(10t-8)$ ha $f = \frac{5}{\pi}$.

$y(t) = 3 \cdot \frac{2}{\alpha} - |H(\frac{1}{2})| \cos(\pi t + \angle H(\frac{1}{2})) + \frac{5}{2} |H(\frac{5}{\pi})| \cos(10t - 8 + \angle H(\frac{5}{\pi}))$.

2K18)

a) $s(t) = 2\Pi\left(\frac{t}{4} - \frac{1}{2}\right) - \Delta(t-2) = 2\Pi\left(\frac{t-2}{4}\right) - \Delta(t-2)$



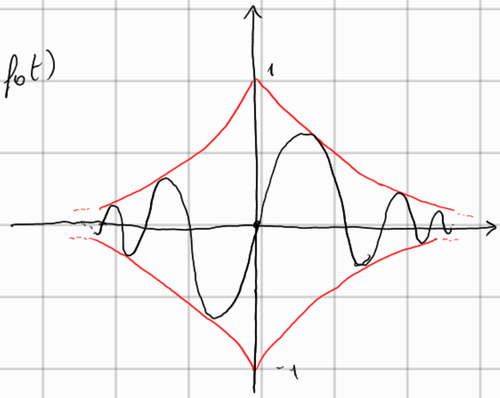
$P_s = 0$

$E_s = 4 + 4 + 2\left(\frac{1}{3} + 2\right) = 8 + \frac{14}{3} = \frac{38}{3}$

$S(f) = 2e^{-4\pi jf}\, 4\,\mathrm{sinc}(4f) - e^{-4\pi jf}\,\mathrm{sinc}^2(f)$

b)

$s(t) = e^{-|t|} \sin(2\pi f_0 t)$



$P_s = 0$, segnale di energia.

Pari

$$\mathcal{E}_s = \int_{-\infty}^{+\infty} |s(t)|^2 dt = 2\int_0^{+\infty} e^{-2t}\sin^2(2\pi f_0 t)\,dt = 2\int_0^{+\infty} e^{-2t}\left(\frac{1}{2} - \frac{1}{2}\cos(4\pi f_0 t)\right)dt = \int_0^{+\infty} e^{-2t}\left(1-\cos(4\pi f_0 t)\right)dt =$$

$$= \int_0^{+\infty} e^{-2t}dt - \int_0^{+\infty}\frac{e^{-2t}}{2}\left(e^{j4\pi f_0 t}+e^{-j4\pi f_0 t}\right)dt = \left.\frac{e^{-2t}}{-2}\right|_0^{+\infty} - \int_0^{+\infty}\frac{e^{-2t}}{2}e^{j4\pi f_0 t}dt - \int_0^{+\infty}\frac{e^{-2t}}{2}e^{-j4\pi f_0 t}dt =$$

$$= \frac{1}{2} - \int_0^{+\infty}\frac{e^{(-2+j4\pi f_0)t}}{2}dt - \int_0^{+\infty}\frac{e^{(-2-j4\pi f_0)t}}{2}dt = \frac{1}{2} - \left.\frac{e^{(-2+j4\pi f_0)t}}{2(-2+j4\pi f_0)}\right|_0^{+\infty} - \left.\frac{e^{(-2-j4\pi f_0)t}}{2(-2-j4\pi f_0)}\right|_0^{+\infty} =$$

$$= \frac{1}{2} - \frac{1}{2(2-j4\pi f_0)} - \frac{1}{2(2+j4\pi f_0)} =$$

$$= \frac{1}{2} - \frac{4}{2(4+16\pi^2 f_0^2)} = \frac{4+16\pi^2 f_0^2-4}{2(4+16\pi^2 f_0^2)} = \frac{16\pi^2 f_0^2}{2(4+16\pi^2 f_0^2)}$$

$$s(t) = e^{-|t|}\underbrace{\sin(2\pi f_0 t)}_{\cos(2\pi f_0 t - \frac{\pi}{2})}$$

$S(f) = F[e^{-|t|}] * F[\sin(2\pi f_0 t)]$

$f(t) = e^{-|t|} = e^{-t}u(t) + e^{-(-t)}u(-t)$

$$F(f) = \frac{1}{1+j2\pi f} + \frac{1}{1-j2\pi f} = \frac{2}{1+4\pi^2 f^2}$$

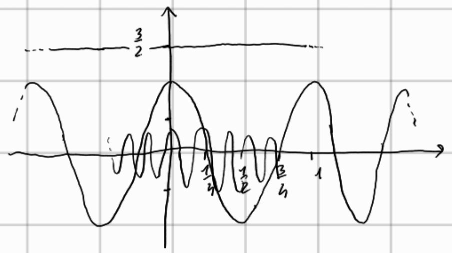
$g(t) = \cos(2\pi f_0 t - \frac{\pi}{2})$ $\quad G(f) = \frac{1}{2}\delta(f-f_0)e^{-j\frac{\pi}{2}} + \frac{1}{2}\delta(f+f_0)e^{+j\frac{\pi}{2}} = -\frac{j}{2}\delta(f-f_0) + \frac{j}{2}\delta(f+f_0)$

$$S(f) = -\frac{j}{2}\cdot\frac{2}{1+4\pi^2(f-f_0)^2} + \frac{j}{2}\cdot\frac{2}{1+4\pi^2(f+f_0)^2} = j\left(\frac{1}{1+4\pi^2(f+f_0)^2} - \frac{1}{1+4\pi^2(f-f_0)^2}\right)$$

c)

$$s(t) = 1 + \cos(2\pi t) + \frac{1}{2} - \frac{1}{2}\cos(8\pi t + 4) =$$

$$= \frac{3}{2} + \cos(2\pi t) - \frac{1}{2}\cos(8\pi t + 4)$$



Periodo minimo: 1

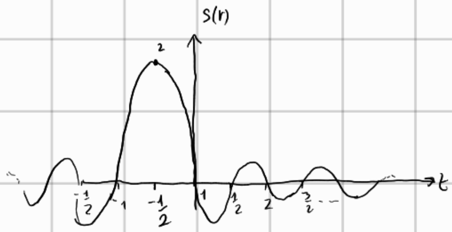
$\mathcal{E}_s = +\infty$

$$P_s = \frac{9}{4} + \frac{1}{2} + \frac{1}{8} = \frac{18}{8} + \frac{4}{8} + \frac{1}{8} = \frac{23}{8}$$

$$S(f) = \frac{3}{2}\delta(f) + \frac{1}{2}\delta(f-1) + \frac{1}{2}\delta(f+1) - \frac{1}{4}\left[\delta(f-4)e^{4j} + \delta(f+4)e^{-4j}\right]$$

d)

$$s(t) = 2\,\mathrm{sinc}(2t+1) = 2\,\mathrm{sinc}\left(2\left(t+\frac{1}{2}\right)\right)$$



Sia $f(t) = \Pi(t)$. Allora $F(f) = \mathrm{sinc}\, f$.

Ma per la simmetria, $F[F[t]] = f(-f) = f(f)$, perché pari.

$\Rightarrow F[\mathrm{sinc}\, t] = \Pi(f)$

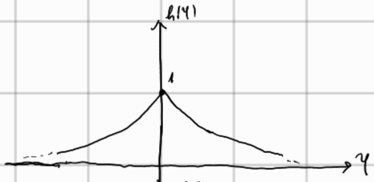
$$S(f) = \frac{2}{2} e^{\pi j f}\, \Pi\left(\frac{f}{2}\right) = e^{\pi j f}\, \Pi\left(\frac{f}{2}\right)$$

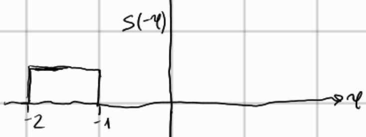
$$|S(f)|^2 = \Pi\left(\frac{f}{2}\right)^2 \Rightarrow \mathcal{E}_s = \int_{-1}^{1} 1\, df = 2$$

2) $h(t) = e^{-\alpha|t|}$

$s(t) = u(t-1) - u(t-2)$

$$y(t) = \int_{-\infty}^{+\infty} h(\varphi) s(t-\varphi)\, d\varphi$$



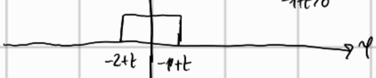


$1 - e^{-\alpha}$

$-1+t<0 \quad t<1$

$$y(t) = \int_{-2+t}^{-1+t} e^{\alpha\varphi}\, d\varphi = \left.\frac{e^{\alpha\varphi}}{\alpha}\right|_{-2+t}^{-1+t} = \frac{e^{\alpha t}(e^{-\alpha} - e^{-2\alpha})}{\alpha}$$

$-2+t<0$
$-1+t>0$
$1<t<2$

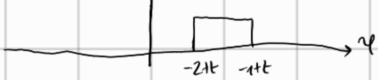


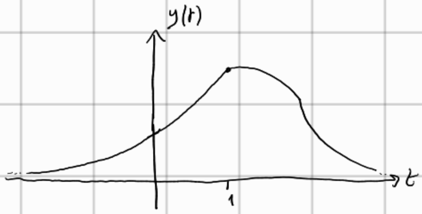
$$y(t) = \int_{t-2}^{0} e^{\alpha\varphi}\, d\varphi + \int_{0}^{t-1} e^{-\alpha\varphi}\, d\varphi = \left.\frac{e^{\alpha\varphi}}{\alpha}\right|_{t-2}^{0} + \left.\frac{e^{-\alpha\varphi}}{-\alpha}\right|_{0}^{t-1} = \frac{1 - e^{\alpha t - 2\alpha} - e^{-\alpha t + \alpha} + 1}{\alpha} = \frac{2 - e^{-\alpha t + \alpha} - e^{\alpha t - 2\alpha}}{\alpha} =$$

$$= \frac{2 - e^{-\alpha(t-1)} - e^{\alpha(t-2)}}{\alpha}$$

$-2+t>0 \quad t>2$

$$y(t) = \int_{t-2}^{t-1} e^{-\alpha\varphi}\, d\varphi = \left.\frac{e^{-\alpha\varphi}}{-\alpha}\right|_{t-2}^{t-1} = \frac{e^{-\alpha(t-2)} - e^{-\alpha(t-1)}}{\alpha} = \frac{e^{-\alpha t}(e^{2\alpha} - e^{\alpha})}{\alpha}$$





$$F\left[\cos(2\pi f_0 t + \varphi)\right] =$$

$$F\left[\cos\left(2\pi f_0\left(t + \frac{\varphi}{2\pi f_0}\right)\right)\right] =$$

$$= e^{-j(2\pi f_0(-\frac{\varphi}{2\pi f_0}))} \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right] =$$

$$= \frac{1}{2} e^{+\frac{f}{f_0} j\varphi}\left[\delta(f-f_0) + \delta(f+f_0)\right] =$$

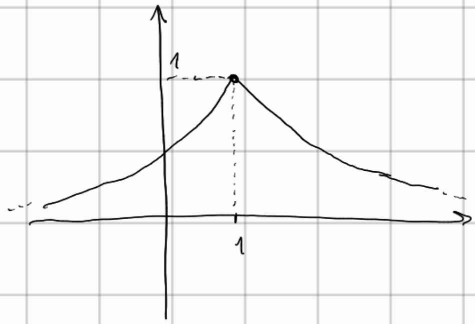
$$\frac{1}{2}\left[e^{j\varphi}\delta(f-f_0) + e^{-j\varphi}\delta(f+f_0)\right]$$

$$s(t) = e^{-3t}\cos(2\pi f_0 t)\, u(t)$$

2k16)

1) $s(t) = e^{-|t-1|} =$

$= e^{-(t-1)} u(t-1) + e^{-(-(t-1))} u(-(t-1))$

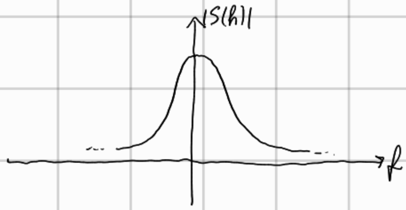


Sia $f(t) = e^{-|t|} = e^{-t}u(t) + e^{-(-t)}u(-t)$

Allora $F(f) = \frac{1}{1+2\pi j f} + \frac{1}{1-2\pi j f} = \frac{2}{1+4\pi^2 f^2}$

Quindi:

$S(f) = \frac{e^{-2\pi j f}\cdot 2}{1+4\pi^2 f^2} \Rightarrow |S(f)| = \frac{2}{1+4\pi^2 f^2} \qquad \angle S(f) = -2\pi f$



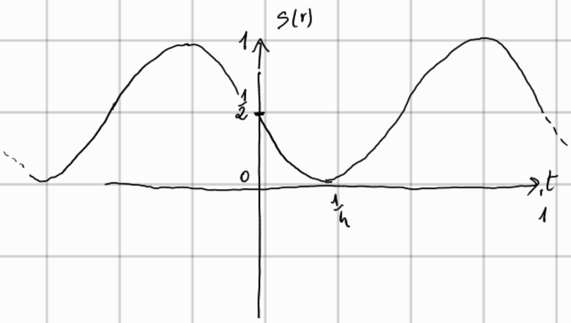
$P_s = 0$, segnale di energia.

$\mathcal{E}_s = \int_{-\infty}^{+\infty} e^{-2|t-1|}\,dt \Rightarrow y = t-1 \Rightarrow \mathcal{E}_s = \int_{-\infty}^{+\infty} \overset{\text{pari}}{e^{-2|y|}}\,dy = 2\int_{0}^{+\infty} e^{-2y}\,dy = 2\left.\frac{e^{-2y}}{-2}\right|_{0}^{+\infty} = 1$

$dt = dy$

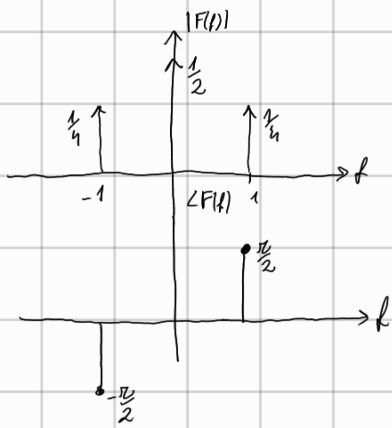
2)

$s(t) = 1 - \sin^2\left(\pi t + \frac{\pi}{4}\right) = \cos^2\left(\pi t + \frac{\pi}{4}\right) = \frac{1}{2} + \frac{1}{2}\cos\left(2\pi t + \frac{\pi}{2}\right)$



$\mathcal{E}_s = +\infty$

$P_s = \frac{1}{4} + \frac{1}{4}\cdot\frac{1}{2} = \frac{3}{8}$



$F[s(t)] = \frac{1}{2}\delta(f) + \frac{1}{4}\left[e^{j\frac{\pi}{2}}\delta(f-1) + e^{-j\frac{\pi}{2}}\delta(f+1)\right] = \frac{1}{2}\delta(f) + \frac{j}{4}\delta(f-1) - \frac{j}{4}\delta(f+1)$

3)

$$s(t) = \sum_{k=-\infty}^{+\infty} \left[\Pi\left(\frac{t-4k}{4}\right) - \Delta(t-4k) \right]$$

$$\delta(t) = \Pi\left(\frac{t}{4}\right) - \Delta(t)$$

$$I(f) = 4\,\text{sinc}(4f) - \text{sinc}^2(f)$$

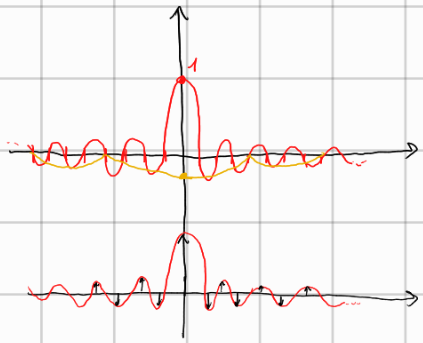
Periodo = 4

$$s(t) = \delta(t) * \sum_{k=-\infty}^{+\infty} \delta(t-4k)$$

$$S(f) = \left(4\,\text{sinc}(4f) - \text{sinc}^2(f)\right) \cdot \frac{1}{4} \sum_{k=-\infty}^{+\infty} \delta\left(t - \frac{k}{4}\right)$$

Disegno $\text{sinc}(4f) - \dfrac{\text{sinc}^2(f)}{4}$

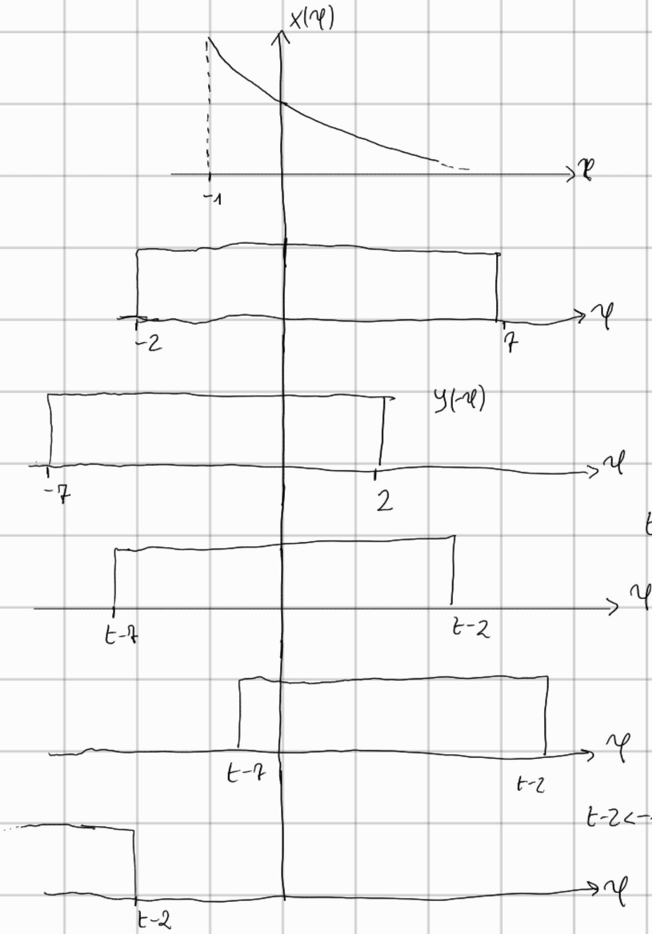
$4f = k \Rightarrow f = \frac{k}{4}$



$\mathcal{E}_s = +\infty$

$$P_s = \frac{1}{4}\left(2 \cdot 1 + 2\,\frac{1}{3}\right) = \frac{1}{2} + \frac{2}{12} = \frac{8}{12}$$

h) $x(t) = e^{-t}\mu(t+1)$ $\qquad$ $y(t) = \mu(t+2) - \mu(t-7)$



$t-7 < -1 \Rightarrow t < 6$

$t-2 > -1 \qquad t > 1$

$$e - e^{-4}$$

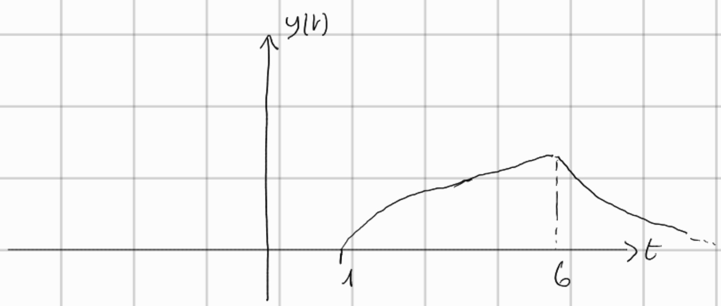
$$y(t) = \int_{-1}^{t-2} e^{-\tau} d\tau = \left.\frac{e^{-\tau}}{-1}\right|_{-1}^{t-2} = \frac{e^{-(t-2)} - e}{-1} = e - e^{-(t-2)}$$

$t-7 > -1 \Rightarrow t > 6$

$$y(t) = \int_{t-7}^{t-2} e^{-\tau} d\tau = \left.\frac{e^{-\tau}}{-1}\right|_{t-7}^{t-2} = e^{-(t-7)} - e^{-(t-2)} = e^{-t}(e^{7} - e^{2})$$

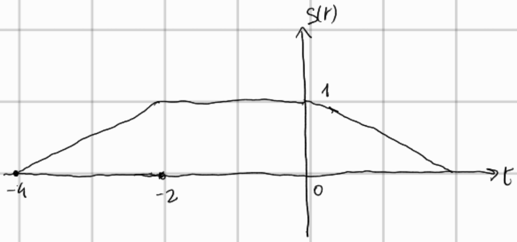
$t-2 < -1 \qquad t < 1$

$y(t) = 0$



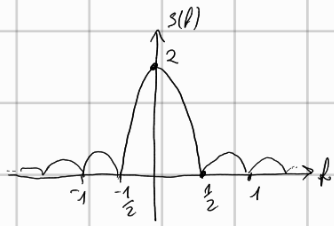
1)

a) $s(t) = \Delta\left(\frac{t}{2}\right) + \Delta\left(\frac{t}{2}+1\right) = \Delta\left(\frac{t}{2}\right) + \Delta\left(\frac{t+2}{2}\right)$



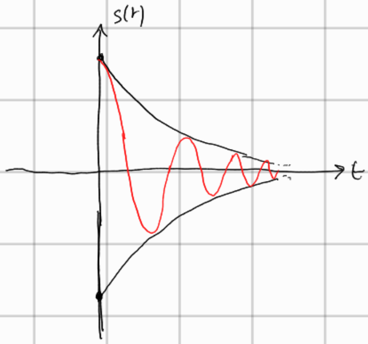
$\mathcal{E}_s = 2\cdot 1 + \frac{2}{3} + \frac{2}{3} = \frac{6+2+2}{3} = \frac{10}{3}$

$F[s(t)] = 2\,\mathrm{sinc}^2(2f) + e^{4\pi j f}\, 2\,\mathrm{sinc}^2(2f) = 2\,\mathrm{sinc}^2(2f)\left(1+e^{4\pi j f}\right)$



b)

$s(t) = 3e^{-3t}\cos(2\pi f_0 t)\, u(t)$



$\mathcal{E}_s = \int_0^{+\infty} 9e^{-6t}\cos^2(2\pi f_0 t)\,dt =$

$= \int_0^{+\infty} 9e^{-6t}\left(\frac{1}{2} + \frac{1}{2}\cos(4\pi f_0 t)\right)dt = \frac{9}{2}\int_0^{+\infty} e^{-6t}\,dt + \frac{9}{4}\int_0^{+\infty} e^{-6t}\left(e^{j4\pi f_0 t} + e^{-j4\pi f_0 t}\right)dt =$

$= \frac{9}{2}\left.\frac{e^{-6t}}{-6}\right|_0^{+\infty} + \frac{9}{4}\int_0^{+\infty} e^{(-6+j4\pi f_0)t}\,dt + \frac{9}{4}\int_0^{+\infty} e^{(-6-j4\pi f_0)t}\,dt =$

$\frac{9}{12} + \frac{9}{4}\cdot\left.\frac{e^{(-6+j4\pi f_0)t}}{-6+j4\pi f_0}\right|_0^{+\infty} + \frac{9}{4}\cdot\left.\frac{e^{(-6-j4\pi f_0)t}}{-6-j4\pi f_0}\right|_0^{+\infty} =$

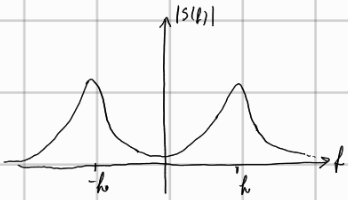
$= \frac{9}{12} + \frac{9}{4}\left(\frac{1}{6-j4\pi f_0} + \frac{1}{6+j4\pi f_0}\right) = \frac{9}{12} + \frac{9}{4}\left(\frac{12}{36+16\pi^2 f_0^2}\right) =$

$$= \frac{9}{12} + \frac{27}{\underset{4(9+4\pi^2 f_0^2)}{36+16\pi^2 f_0^2}} = \frac{81+36\pi^2 f_0^2+81}{12\,(9+4\pi^2 f_0^2)} = \frac{162+36\pi^2 f_0^2}{12\,(9+4\pi^2 f_0^2)}$$

$$F[s(t)] = F[3e^{-3t}u(t)] * F[\cos(2\pi f_0 t)] =$$

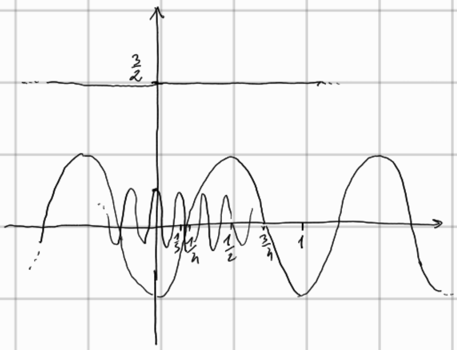
$$= \frac{3}{3+2\pi j f} * \left[\frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)\right] = \frac{\frac{3}{2}}{3+2\pi j(f-f_0)} + \frac{\frac{3}{2}}{3+2\pi j(f+f_0)} = \frac{\frac{3}{2}(6+4\pi j f)}{9+12\pi j f - 4\pi^2(f-f_0)(f+f_0)} =$$

$$= \frac{3\,(3+2\pi j f)}{9+12\pi j f - 4\pi^2(f-f_0)(f+f_0)}$$



c)

$$s(t) = 1-\cos(2\pi t)+\cos^2(5\pi t) = \frac{3}{2}-\cos(2\pi t)+\frac{1}{2}\cos(10\pi t)$$

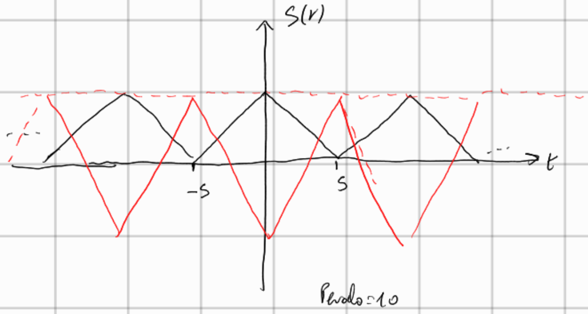


$$P_s = \frac{9}{4}+\frac{1}{2}+\frac{1}{8} = \frac{18}{8}+\frac{4}{8}+\frac{1}{8} = \frac{23}{8}$$

$$F[s(t)] = \frac{3}{2}\delta(f) - \frac{1}{2}\delta(f-1) - \frac{1}{2}\delta(f+1) + \frac{1}{4}\delta(f-5) + \frac{1}{4}\delta(f+5)$$

d)

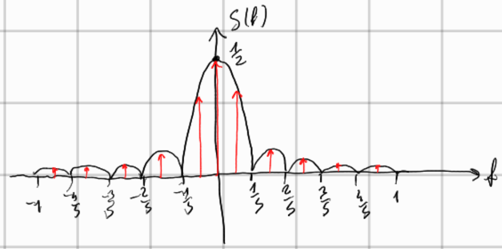
$$s(t) = 1-2\sum_{k=-\infty}^{+\infty}\Delta\left(\frac{t-10k}{5}\right)$$



$$s(t) = \Delta\left(\frac{t}{5}\right)$$

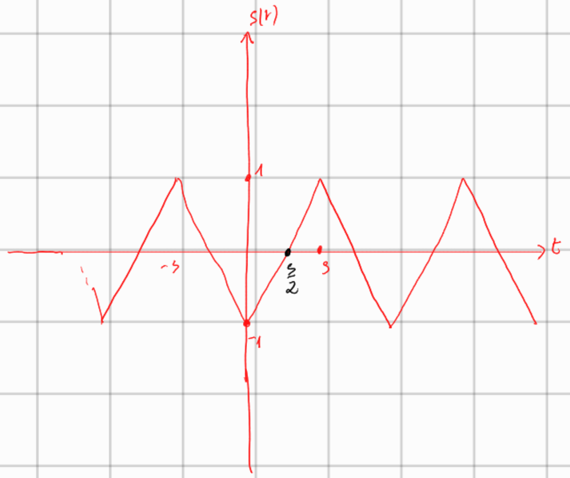
$$S(f) = I(f)\cdot\frac{1}{T}\sum_{k=-\infty}^{+\infty}\delta\left(f-\frac{k}{T}\right) \qquad I(f) = 5\,\mathrm{sinc}^2(5f)$$

$$S(f) = \frac{1}{2}\,\mathrm{sinc}^2(5f)\sum_{k=-\infty}^{+\infty}\delta\left(f-\frac{k}{10}\right) = \frac{1}{2}\sum_{k=-\infty}^{+\infty}\mathrm{sinc}^2\left(\frac{k}{2}\right)\delta\left(f-\frac{k}{10}\right)$$



$s(t) = 1 - 2\Delta\left(\frac{t}{5}\right)$ Periodo $= 10$

$I(f) = \delta(f) - 10\,\text{sinc}^2(5f)$



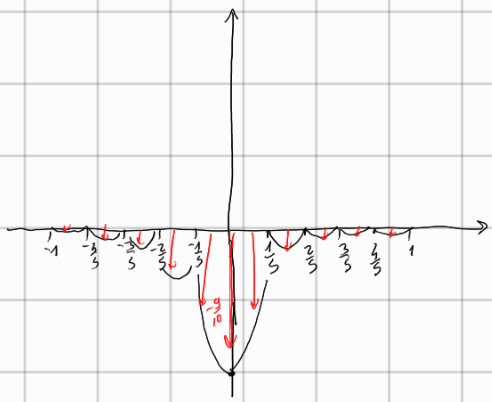
$S(f) = I(f) \cdot \frac{1}{10} \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{10}\right)$

$P_s = \frac{5}{2} \cdot 1 \cdot \frac{1}{3} \cdot 4 = \frac{10}{3} \cdot \frac{1}{10} = \frac{1}{3}$

$= \left(\frac{\delta(f)}{10} - \text{sinc}^2(5f) \right) \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{10}\right) =$

$= \frac{\delta(f)}{10} - \text{sinc}^2(5f) \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{10}\right) =$

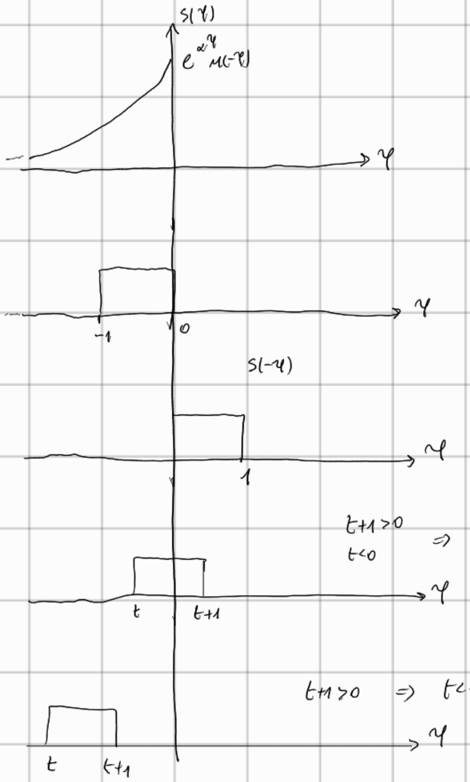
$= \frac{\delta(f)}{10} - \sum_{k=-\infty}^{+\infty} \text{sinc}^2\left(\frac{k}{2}\right) \delta\left(f - \frac{k}{10}\right)$



2)

$h(t) = e^{\alpha t} u(-t)$ $\qquad$ $s(t) = u(t+1) - u(t)$

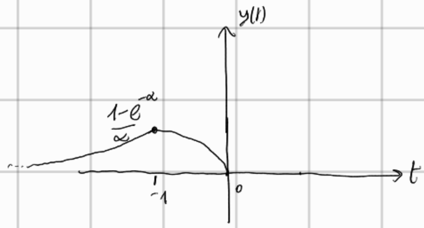
$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, s(t-\tau)\, d\tau$$



$t > 0$ $\qquad$ $y(t) = 0$

$t+1 > 0$
$t < 0$ $\Rightarrow$ $-1 < t < 0$ $\qquad$ $y(t) = \int_t^0 e^{\alpha\tau} d\tau = \frac{e^{\alpha\tau}}{\alpha}\Big|_t^0 = \frac{1-e^{\alpha t}}{\alpha}$

$t+1 > 0 \Rightarrow t < -1$ $\qquad$ $y(t) = \int_t^{t+1} e^{\alpha\tau} d\tau = \frac{e^{\alpha\tau}}{\alpha}\Big|_t^{t+1} = \frac{e^{\alpha(t+1)} - e^{\alpha t}}{\alpha} = \frac{e^{\alpha t}(e^{\alpha}-1)}{\alpha}$



3)

$$X(f) = 3\delta(f) - \frac{1}{2}\delta\left(f-\frac{1}{2}\right) - \frac{1}{2}\,\delta\left(f+\frac{1}{2}\right) + \frac{5}{4}\delta\left(f-\frac{5}{\pi}\right)e^{-j8} + \frac{5}{4}\delta\left(f+\frac{5}{\pi}\right)e^{j8}$$

$$h(t) = 2e^{-\alpha(t-1)}\,u(t-1)$$

$$x(t) = \frac{1}{2} - \cos(\pi t) + 5\cos^2(5t-4) = 3 - \cos(\pi t) + \frac{5}{2}\cos(10t-8)$$

$10 = 2\pi f$

$f = \frac{5}{\pi}$

Sia $f(t) = 2e^{-\alpha t}u(t)$

allora $F[f(t)] = \dfrac{2}{\alpha + 2\pi j f}$. Quindi, $H(f) = \dfrac{e^{-2\pi j f}\cdot 2}{\alpha + 2\pi j f}$

Ho sistema lineare, la risposta in uscita a un segnale sinusoidale é:

$$|H(f)| = \frac{2}{\sqrt{\alpha^2 + 4\pi^2 f^2}} = \frac{2}{\alpha\sqrt{1+\left(\frac{f}{f_0}\right)^2}}$$

$$y(t) = 3\,|H(0)| - |H(\tfrac{1}{2})|\cos\left(\pi t + \angle H(\tfrac{1}{2})\right) + $$
$$+ \frac{5}{2}\,|H(\tfrac{5}{\pi})|\cos\left(10t - 8 + \angle H(\tfrac{5}{\pi})\right)$$

$f_0 = \dfrac{\alpha}{2\pi}$

$$\angle H(f) = -2\pi f - \operatorname{atan}\left(\frac{2\pi f}{\alpha}\right)$$

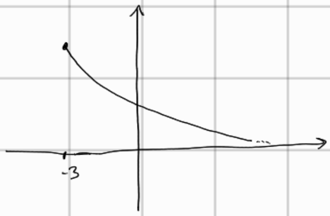
$|H(0)| = \dfrac{2}{\alpha}$

$$|H(\tfrac{1}{2})| = \frac{2}{\alpha\sqrt{1+\left(\frac{1}{2f_0}\right)^2}} \qquad \angle H(\tfrac{1}{2}) = -\pi - \operatorname{atan}\left(\frac{\pi}{\alpha}\right)$$

$$|H(\tfrac{5}{\pi})| = \frac{2}{\alpha\sqrt{1+\left(\frac{5}{\pi f_0}\right)^2}} \qquad \angle H(\tfrac{5}{\pi}) = -10 - \operatorname{atan}\left(\frac{10}{\alpha}\right)$$

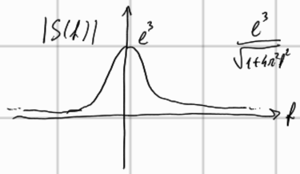
1)

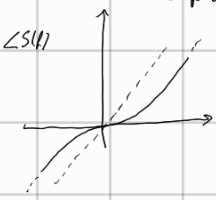
$$s(t) = e^{-t}\,u(t+3)$$



$$s(t) = e^{-(t+3-3)}\,u(t+3) = e^{3}e^{-(t+3)}\,u(t+3)$$

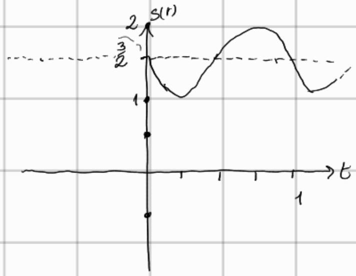
$$S(f) = \frac{e^{3}\cdot e^{6\pi j f}}{1+2\pi j f} \qquad \mathcal{E}_s = \int_{-\infty}^{+\infty} e^{-2t}\,u(t+3)\,dt = \int_{-3}^{+\infty} e^{-2t}\,dt = \left.\frac{e^{-2t}}{-2}\right|_{-3}^{+\infty} = \frac{e^{6}}{2}$$



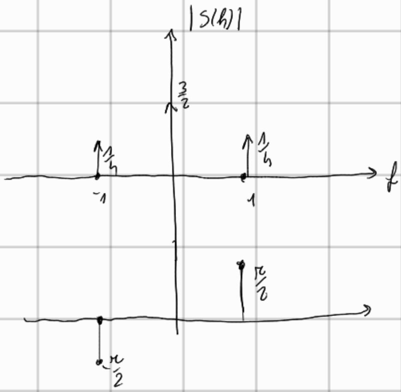


2)

$$s(t) = 1+\cos^2\!\left(\pi t+\frac{\pi}{4}\right) = 1+\frac{1}{2}+\frac{1}{2}\cos\!\left(2\pi t+\frac{\pi}{2}\right) = \frac{3}{2}+\frac{1}{2}\cos\!\left(2\pi t+\frac{\pi}{2}\right)$$



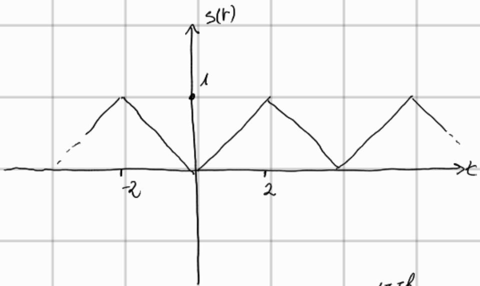
$$S(f) = \frac{3}{2}\delta(f) + \frac{1}{4}\left[e^{+j\frac{\pi}{2}}\delta(f-1) + e^{-j\frac{\pi}{2}}\delta(f+1)\right] = \frac{3}{2}\delta(f) + \frac{j}{4}\delta(f-1) - \frac{j}{4}\delta(f+1)$$



$$P_s = \frac{9}{4}+\frac{1}{4}\cdot\frac{1}{2} = \frac{18}{8}+\frac{1}{8} = \frac{19}{8}$$

3)

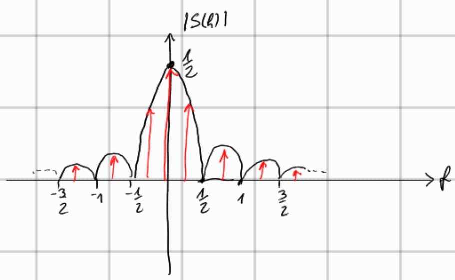
$$s(t) = \sum_{k=-\infty}^{+\infty}\left[\Pi\!\left(\frac{t-4k}{4}\right) - \Delta\!\left(\frac{t-4k}{2}\right)\right]$$



$$P = \frac{1}{4}\left(2\cdot\frac{2}{3}\right) = \frac{1}{3}$$

$$\delta(t) = \Delta\!\left(\frac{t-2}{2}\right) \qquad I(f) = 2\,e^{-4\pi j f}\,\mathrm{sinc}^2(2f)$$

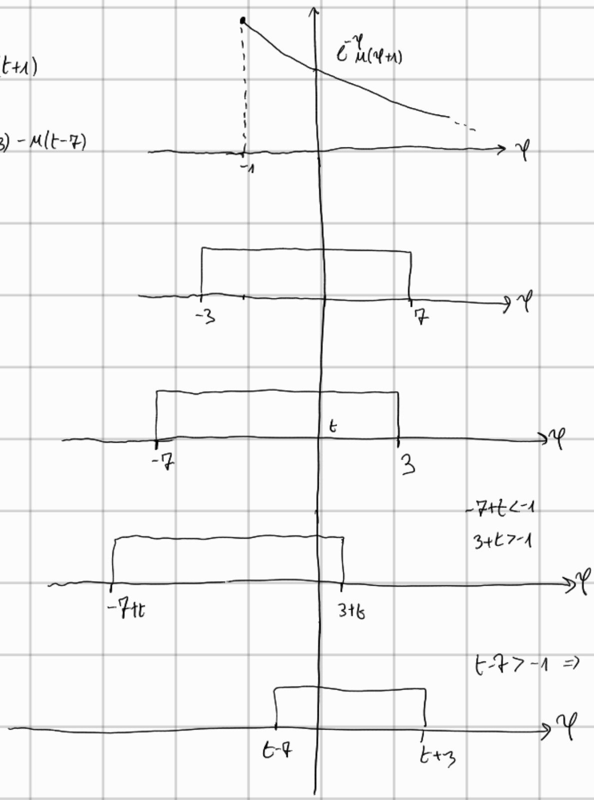
$$s(t) = \sum_{k=-\infty}^{+\infty} \delta(t-kT) \qquad T=4$$



$$S(f) = I(f)\frac{1}{T}\sum_{k=-\infty}^{+\infty}\delta\!\left(f-\frac{k}{T}\right) = \frac{e^{-4\pi j f}}{2}\mathrm{sinc}^2(2f)\sum_{k=-\infty}^{+\infty}\delta\!\left(f-\frac{k}{4}\right) = \frac{1}{2}\sum_{k=-\infty}^{+\infty} \overbrace{e^{-\pi j k}\,\mathrm{sinc}^2\!\left(\frac{k}{2}\right)}^{\to\, C_k}\delta\!\left(f-\frac{k}{4}\right)$$

3) $h(t) = e^{-t}\mu(t+1)$

$s(t) = \mu(t+3) - \mu(t-7)$



$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, s(t-\tau)\, d\tau$$

$-7+t < -1$

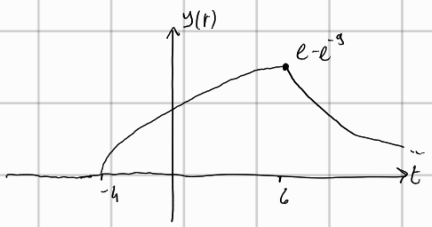
$3+t > -1$

$-4 < t < 6$

$$y(t) = \int_{-1}^{3+t} e^{-\tau} d\tau = \frac{e^{-\tau}}{-1}\Big|_{-1}^{3+t} = \frac{e^{-t-3} - e^{1}}{-1} = e - e^{-t-3}$$

$t-7 > -1 \Rightarrow t > 6$

$$y(t) = \int_{t-7}^{t+3} e^{-\tau} d\tau = \frac{e^{-\tau}}{-1}\Big|_{t-7}^{t+3} = e^{-t+7} - e^{-t-3} = e^{-t}(e^{7} - e^{-3})$$

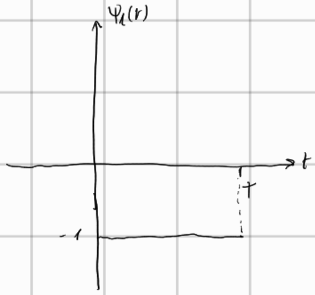


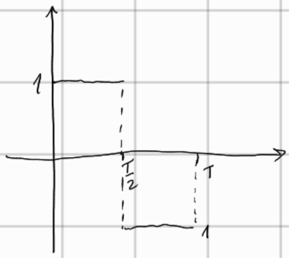
6)

$$s(t) = \sin\left(\frac{4\pi}{T} t\right) \qquad t \in [0,T]$$

$$\psi_1(t) = -u(t) + u(t-T) \qquad \psi_2(t) = u(t) - 2u\left(t-\frac{T}{2}\right) + u(t-T)$$

$$c_k^0 = \frac{\int_0^T s(t)\,\psi_k(t)\,dt}{\int_0^T \psi_k^2(t)\,dt}$$





• $\int_0^T \psi_1(t)^2\,dt = T$ • $\int_0^T \psi_2(t)^2\,dt = T$

$$\int_0^T \psi_1(t)\,\psi_2(t)\,dt = \int_0^{\frac{T}{2}} (-1)\,dt + \int_{\frac{T}{2}}^T dt = 0 \Rightarrow \text{ortogonali.}$$

$$c_1 = \frac{1}{T}\int_0^T s(t)\,\psi_1(t)\,dt = -\frac{1}{T}\int_0^T \sin\left[\left(\frac{4\pi}{T}\right)t\right] dt = \frac{1}{T}\,\frac{\cos\left(\frac{4\pi}{T}t\right)}{\frac{4\pi}{T}}\Bigg|_0^T = \frac{1}{T}\,\frac{T}{4\pi}\left[\cos(4\pi) - \cos(0)\right] = 0$$

$$c_2 = \frac{1}{T}\int_0^T s(t)\,\psi_2(t)\,dt = \frac{1}{T}\int_0^{\frac{T}{2}} \sin\left(\frac{4\pi}{T}t\right)dt - \frac{1}{T}\int_{\frac{T}{2}}^{T} \sin\left(\frac{4\pi}{T}t\right)dt =$$

$$= \frac{1}{T}\,T\,\frac{-\cos\left(\frac{4\pi}{T}t\right)}{4\pi}\Bigg|_0^{\frac{T}{2}} + \frac{1}{T}\,T\,\frac{\cos\left(\frac{4\pi}{T}t\right)}{4\pi}\Bigg|_{\frac{T}{2}}^{T} =$$

$$= \frac{1}{4\pi}\left(\cancel{\cos(0)} - \cancel{\cos(2\pi)}\right) + \frac{1}{4\pi}\left[\cancel{\cos(4\pi)} - \cancel{\cos(2\pi)}\right] = 0$$

7)

$$s(t) = \mathrm{sinc}(3t)$$

Sia $s(t) = \Pi\left(\frac{t}{3}\right)$, allora $S(f) = 3\,\mathrm{sinc}(3f)$

Per la simmetria:

$$\mathcal{F}[S(t)] = s(-f) \Rightarrow \mathcal{F}[3\,\mathrm{sinc}(3t)] = \Pi\left(-\frac{f}{3}\right) = \Pi\left(\frac{f}{3}\right)$$

$$\Rightarrow S(f) = \frac{1}{3}\Pi\left(\frac{f}{3}\right) \Rightarrow \mathcal{E}_s = |S(f)|^2 = \frac{1}{9}\Pi\left(\frac{f}{3}\right) \Rightarrow R_s(\tau) = \mathcal{F}^{-1}\left[\mathcal{E}_s(f)\right] = \frac{1}{9}\cdot 3\,\mathrm{sinc}(3\tau) = \frac{1}{3}\mathrm{sinc}(3\tau)$$

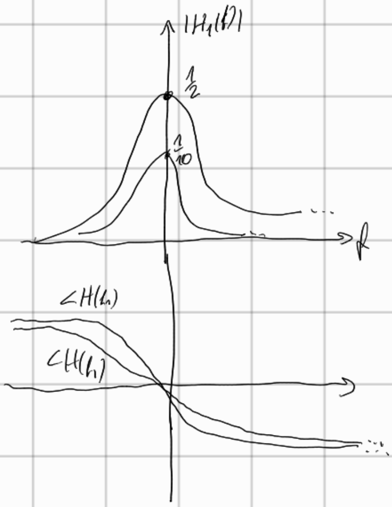
$h_1(t) = 5e^{-10t}\, u(t) \qquad h_2(t) = \frac{1}{2} e^{-5t}\, u(t)$

$$H(f) = \left[\frac{5}{10+2\pi j f}\right]\left[\frac{1}{2(5+2\pi j f)}\right]$$

$(1+\frac{1}{5}\pi j f) \qquad 1+\frac{2}{5}\pi j f$

$$|H(f)| = \frac{5}{10\sqrt{1+\left(\frac{f\pi}{5}\right)^2}} \cdot \frac{1}{10\sqrt{1+\left(\frac{2}{5}\pi f\right)^2}}$$

$$\angle H(f) = -\operatorname{atan}\left(\frac{\pi}{5} f\right) - \operatorname{atan}\left(\frac{2}{5}\pi f\right)$$



$x(t) = \cos(2\pi t) + 2\sin(6\pi t)$

$$\angle H(1) = -\operatorname{atan}\left(\frac{\pi}{5}\right) - \operatorname{atan}\left(\frac{2}{5}\pi\right)$$

$$|H(1)| = \frac{1}{20} \cdot \frac{1}{\sqrt{1+\left(\frac{\pi}{5}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{2}{5}\pi\right)^2}}$$

$$y(t) = |H(1)|\cos(2\pi t + \angle H(1)) + 2|H(3)|\sin(6\pi t + \angle H(3))$$