ALCUNE CONSIDERAZIONI:

- É possibile che una piccola parte degli esercizi sia scorretta
- Alcuni esercizi possono essere risolti in modi diversi, ma egualmente corretti
- Gli esercizi sono svolti in ordine cronologico, quindi più si va in fondo nel file più sará probabile che con più esperienza alle spalle saranno corretti.

BUON LAVORO!

1)

$$X(t) = A(t) + 1 + \cos\left(2\pi f_0 t + \Theta + \frac{\pi}{10}\right)$$

$$P_A(f) = \Lambda\left(\frac{f-f_0}{2B}\right) + \Lambda\left(\frac{f+f_0}{2B}\right) \qquad \Theta \propto \mathcal{U}(0, 2\pi) \qquad f_0 \gg B.$$

$$R_X(\tau) = R_A(\tau) + R_B(\tau)$$

↑ vista l'indipendenza tra A e Θ

$$B(t) = 1 + \cos\left(2\pi f_0 t + \Theta + \frac{\pi}{10}\right)$$

$$R_B(t,\tau) = E\left[B(t)B(t-\tau)\right] = \frac{1}{2\pi}\int_0^{2\pi}\left(1+\cos\left(2\pi f_0 t + \theta + \frac{\pi}{10}\right)\right)\left(1+\cos\left(2\pi f_0(t-\tau)+\theta+\frac{\pi}{10}\right)\right)d\theta =$$

$$\frac{1}{2\pi}\left[\int_0^{2\pi} 1\, d\theta + \cancelto{0}{\int_0^{2\pi}\cos\left(2\pi f_0 t + \theta + \frac{\pi}{10}\right)d\theta} + \cancelto{0}{\int_0^{2\pi}\cos\left(2\pi f_0(t-\tau)+\theta+\frac{\pi}{10}\right)d\theta} + \right.$$

$$\left. + \int_0^{2\pi}\cos\left(2\pi f_0 t + \theta + \frac{\pi}{10}\right)\cos\left(2\pi f_0(t-\tau)+\theta+\frac{\pi}{10}\right)d\theta\right] =$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$

$$= 1 + \frac{1}{4\pi}\cancelto{0}{\int_0^{2\pi}\cos\left(2\pi f_0(2t-\tau)+2\theta+\frac{\pi}{5}\right)d\theta} + \frac{1}{4\pi}\int_0^{2\pi}\cos(2\pi f_0\tau)\,d\theta =$$

$$= 1 + \frac{1}{2}\cos(2\pi f_0 \tau)$$

Quindi, $P_B(f) = \delta(f) + \frac{1}{4}\delta(f-f_0) + \frac{1}{4}\delta(f+f_0)$

$$P_X(f) = P_A(f) + P_B(f) = \Lambda\left(\frac{f-f_0}{2B}\right) + \Lambda\left(\frac{f+f_0}{2B}\right) + \delta(f) + \frac{1}{4}\delta(f-f_0) + \frac{1}{4}\delta(f+f_0)$$

$$R_A(\tau) = \mathcal{F}^{-1}\left[P_A(f)\right] = 2B\,\mathrm{sinc}^2(2B\tau)\,e^{2\pi j f_0 \tau} + 2B\,\mathrm{sinc}^2(2B\tau)\,e^{-2\pi j f_0\tau}$$

$$R_X(\tau) = 4B\,\mathrm{sinc}^2(2B\tau)\cos(2\pi f_0\tau) + 1 + \frac{1}{2}\cos(2\pi f_0\tau)$$

2) $P_A = \int_{-\infty}^{+\infty} P_A(f)\,df = 1 \cdot \frac{4B}{2} + \frac{1 \cdot 4B}{2} = 4B$

$\Rightarrow \alpha = \frac{4B}{\frac{3}{2}} = \frac{8}{3}B$

$P_B = R_B(0) = \frac{3}{2}$

1)

$$Y(t) = X(t)\cos^2(2\pi f_0 t) + C$$

$X(t)$ SSL, $P_X(f) = \Delta\left(\frac{f}{2B}\right)$ $\qquad f_0 \gg 2B$

C indipendente da X. $\quad C \propto N(c, 1, 2)$

Stazionarietà di $Y(t)$: lo è perché C e X sono indipendenti.

Sia $B(t) = X(t)\cos^2(2\pi f_0 t)$

$E[C^2] - \overset{1}{\mu^2} = \sigma^2 = 4 \Rightarrow E[C^2] = 5$

$$R_Y(\tau) = E[Y(t)Y(t-\tau)] = E[(B(t)+C)(B(t-\tau)+C)] = R_B(\tau) + E[C^2] + \overset{0,\ \text{sono indipendenti}}{E[B(t)\cdot C]} + \overset{0}{E[B(t-\tau)C]}$$

1)

$$Y(t) = X(t)\cos^2(2\pi f_0 t) + C$$

• X e C indipendenti

$R_X(f) = \Delta\left(\frac{f}{2B}\right)$ $\qquad C \propto N(c, 1, 2)$ $\qquad E[C^2] = \sigma_c^2 + \mu^2 = 5$

$$Y(t) = \frac{X(t)}{2} + \frac{X(t)}{2}\cos(4\pi f_0 t) + C$$

spettro di potenza non ha $\delta(f)$ in $0 \Rightarrow E[X(t)] = 0$

$$E[Y(t)] = \cancel{\frac{E[X(t)]}{2}} + \cancel{\frac{E[X(t)]}{2}}\cos(4\pi f_0 t) + E[C] = 1$$

$$R_Y(t,\tau) = E\left[\left(\frac{X(t)}{2} + \frac{X(t)}{2}\cos(4\pi f_0 t) + C\right)\left(\frac{X(t-\tau)}{2} + \frac{X(t-\tau)}{2}\cos(4\pi f_0(t-\tau)) + C\right)\right] =$$

$$= \frac{1}{4}E[X(t)X(t-\tau)] + \frac{\cos(4\pi f_0 t)}{4}E[X(t)X(t-\tau)] + \frac{\cos(4\pi f_0(t-\tau))}{4}E[X(t)X(t-\tau)] +$$

$$+ \frac{1}{4}\cos(4\pi f_0 t)\cos(4\pi f_0(t-\tau))E[X(t)X(t-\tau)] + E[C^2] =$$

$$= \frac{1}{4}R_X(\tau) + \frac{1}{4}\cos(4\pi f_0 t)R_X(\tau) + \frac{1}{4}\cos(4\pi f_0(t-\tau))R_X(\tau) + \frac{1}{4}\cos(4\pi f_0 t)\cos(4\pi f_0(t-\tau))R_X(\tau) + 5 =$$

$$= \frac{R_X(\tau)}{4}\left(1 + \cos(4\pi f_0 t) + \cos(4\pi f_0(t-\tau)) + \cos(4\pi f_0 t)\cos(4\pi f_0(t-\tau))\right) + 5$$

$$\cos(4\pi f_0(t-\tau))(1+\cos(4\pi f_0 t))$$

$$[1+\cos(4\pi f_0 t)][1+\cos(4\pi f_0(t-\tau))]$$

2)

$$R_X(\tau) = \Delta(\tau)$$

$$h(t) = \delta(t-1) + \frac{1}{2}\delta(t-2) \qquad R_{YX}(t,\tau)?$$

$$P_X(f) = \mathrm{sinc}^2(f)$$

$$H(f) = e^{-2\pi j f} + \frac{1}{2} e^{-4\pi j f} = e^{-2\pi j f}\left(1 + \frac{e^2}{2}\right) \qquad |H(f)| = 1 + \frac{e^2}{2} \qquad |H(f)|^2 = 1 + \frac{e^4}{4} + e^2$$

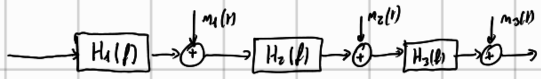
$$P_Y(f) = \mathrm{sinc}^2(f) + \frac{e^4}{4}\mathrm{sinc}^2(f) + e^2\,\mathrm{sinc}^2(f)$$

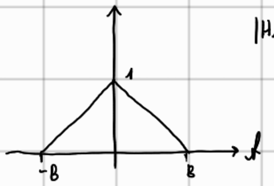
$$R_Y(\tau) = \Delta(\tau) + \frac{e^4}{4}\Delta(\tau) + e^2\Delta(\tau)$$

$$t-\alpha-t+\tau$$

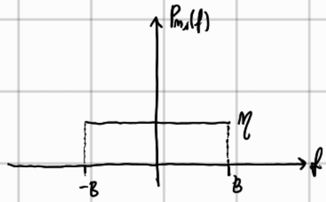
$$R_{YX}(t,\tau) = E\left[Y(t)X(t-\tau)\right] = E\left[\int_{-\infty}^{+\infty} h(\alpha)X(t-\alpha)\,d\alpha\, X(t-\tau)\right] = \int_{-\infty}^{+\infty} E\left[X(t-\alpha)X(t-\tau)\right]h(\alpha)\,d\alpha =$$

$$\int_{-\infty}^{+\infty} R_X(\tau-\alpha)\,h(\alpha)\,d\alpha = (R_X * h)(\tau) = \Delta(\tau-1) + \frac{1}{2}\Delta(\tau-2)$$



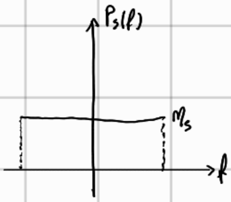


$|H_1(f)|^2 = \Delta\left(\frac{f}{B}\right)$



$P_n = 9\,mV^2 \qquad \eta = \frac{P_n}{2B}$

$P_s(f) = 8\cdot 10^{-4}\,mV^2/Hz \quad |f| \in [0, B]$



Quindi, $P_{rs}(f) = \Delta\left(\frac{f}{B}\right)^3 P_s(f)$

$$P_{rs}(f) = \begin{cases} \eta_s\left(1 - \frac{f^3}{B^3} + \frac{3f^2}{B^2} - 3\frac{f}{B}\right) & 0 < f < B \\ \eta_s\left(1 + \frac{f^3}{B^3} + \frac{3f^2}{B^2} + 3\frac{f}{B}\right) & -B < f < 0 \end{cases}$$

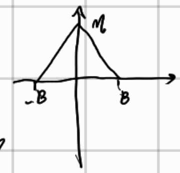
$$\Delta\left(\frac{f}{B}\right) = \begin{cases} -\frac{f}{B} + 1 & 0 < f < B \\ +\frac{f}{B} + 1 & -B < f < 0 \end{cases}$$

$$P_{rs} = \int_0^B \eta_s\,df + \int_{-B}^0 \eta_s\,df + \eta_s\int_{-B}^0 \frac{f^3}{B^3}\,df - \eta_s\int_0^B \frac{f^3}{B^3}\,df + \eta_s\int_{-B}^B \frac{3f^2}{B^2}\,df + \eta_s\int_0^B \left(-\frac{3f}{B}\right)df + \eta_s\int_{-B}^0 \frac{3f}{B}\,df =$$

$$= 2B\eta_s + \eta_s\frac{f^4}{4B^3}\Big|_{-B}^0 - \eta_s\frac{f^3}{4B^3}\Big|_0^B = 2B\eta_s - \frac{\eta_s B}{4} - \frac{\eta_s B}{4} = 2B\eta_s - \frac{\eta_s B}{2} =$$

$$= 2B\eta_s + 2B\eta_s - \eta_s\left(\frac{3}{2}B\right) + \frac{3}{2}\eta_s B = 4B\eta_s = 40\cdot 10^3\cdot 8\cdot 10^{-4} = 32\,mV^2$$

$P_{n_1}(f) = \Delta\left(\frac{f}{B}\right)^2 \eta \qquad P_{n_1} = \int_{-B}^{B} \eta\,\Delta\left(\frac{f}{B}\right)^2 df =$



$= \eta\left(\frac{B}{3} + \frac{B}{3}\right) = \frac{2}{3}B\eta$

$\hookrightarrow \frac{2}{3}B\cdot\frac{P_n}{2B} = \frac{P_n}{3}$

$\Rightarrow \frac{P_s}{P_n} = \frac{32}{16.5} = 1.94$

$P_{n_2}(f) = \Delta\left(\frac{f}{B}\right)\eta \qquad P_{n_2} = B\eta = \frac{P_n}{2}$

$P_{n_3}(f) = P_n$

$P_n = \frac{P_n}{3} + \frac{P_n}{2} + P_n = \frac{2P_n + 3P_n + 6P_n}{6} = \frac{11}{6}P_n = \frac{11}{6}\cdot 9 = \frac{11}{2}3 = 16.5\,mV^2$

1)

$$R_X(\tau) = \Delta(\tau)$$

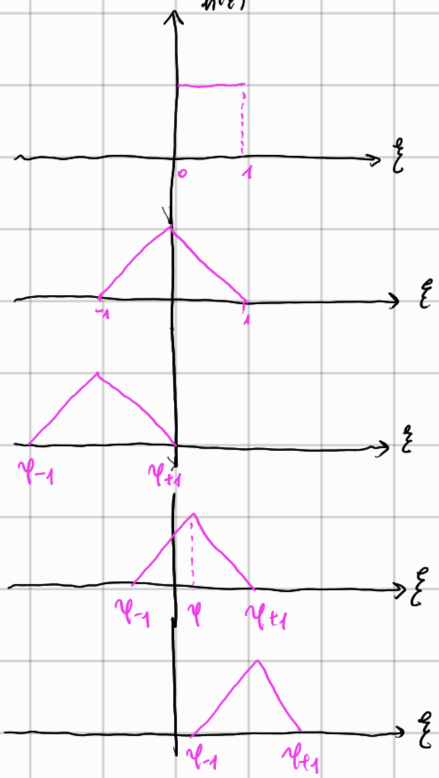
$$h(t) = u(t) - u(t-1)$$

Valutare $R_{YX}(\tau)$

$$E\left[Y(t)X(t-\tau)\right] = E\left[\int_{-\infty}^{+\infty} h(\xi)x(t-\xi)d\xi \; x(t-\tau)\right] = E\left[\int_{-\infty}^{+\infty} h(\xi)x(t-\xi)\,x(t-\tau)\,d\xi\right] =$$

$$= \int_{-\infty}^{+\infty} h(\xi)R_x(\tau-\xi)d\xi = R_{YX}(\tau)$$

$$\overset{\shortparallel}{(h * R_x)(\tau)}$$



$\tau+1<0$ $\quad R_{yx}(\tau)=0$

$\tau<-1$

$\tau-1<0$

$\tau+1<0 \Rightarrow \quad -1<\tau<1$

$\tau-1>0 \quad\quad 1<\tau<2$

$\tau-1<1$

$\tau>2 \quad\quad R_{yx}(\tau)=0$

$$R_{yx}(\tau) = \begin{cases} \tau - \frac{\tau^2}{2} + \frac{1}{2} & \tau\in[-1,1] \\ 4+\tau^2-4\tau & \tau\in[1,2] \end{cases}$$

$$R_{yx}(\tau) = \frac{\tau}{2}(1+1-\tau) + \frac{1}{2} = \tau - \frac{\tau^2}{2} + \frac{1}{2}$$

$$R_{yx}(\tau) = (2-\tau)(2-\tau) = 4+\tau^2-4\tau$$

$$R_{yx}(\tau) = \int_0^{\tau}(\xi+1-\tau)d\xi + \int_{\tau}^{\tau+1}(-\xi+\tau+1)d\xi =$$

1) $Y(t) = X(t-2) + X(t)\sin^2(8\pi f_0 t)$

$P_X(f) = \Delta\left(\frac{f-2}{2}\right) + \Delta\left(\frac{f+2}{2}\right) \quad f_0 \gg 4$

$R_X(\tau) = F^{-1}[P_X(f)] = e^{4\pi j t}\, 2\,\mathrm{sinc}^2(2\tau) + e^{-4\pi j t}\, 2\,\mathrm{sinc}^2(2\tau)$

$R_Y(t,\tau) \triangleq E\left[\left(X(t-2) + X(t)\sin^2(8\pi f_0 t)\right)\left(X(t-2-\tau) + X(t-\tau)\sin^2(8\pi f_0(t-\tau))\right)\right] =$

$= E\left[X(t-2)X(t-2-\tau)\right] + E\left[X(t-2)X(t-\tau)\right]\sin^2(8\pi f_0(t-\tau)) +$

$+ \sin^2(8\pi f_0 t)\, E\left[X(t)X(t-2-\tau)\right] + \sin^2(8\pi f_0 t)\sin^2(8\pi f_0(t-\tau))\, E\left[X(t)X(t-\tau)\right] =$

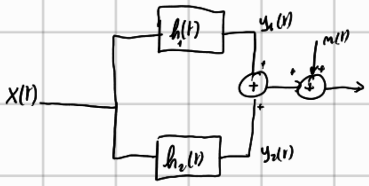
$= R_X(\tau) + R_X(\tau-2)\sin^2(8\pi f_0(t-\tau)) + \sin^2(8\pi f_0 t)\, R_X(\tau+2) + \sin^2(8\pi f_0 t)\sin^2(8\pi f_0(t-\tau))\, R_X(\tau)$

2)

$$y(t) = (h_1 * x)(t) + (h_2 * x)(t) + n(t)$$

$$R_y(t,\tau) = E\left[y(t)\, y^*(t-\tau)\right] = E\left[\left(\int_{-\infty}^{+\infty} h_1(\xi)x(t-\xi)\,d\xi + \int_{-\infty}^{+\infty} h_2(\eta)x(t-\eta)\,d\eta + n(t)\right)\left(\int_{-\infty}^{+\infty} h_1^*(\mu)x^*(t-\tau-\mu)\,d\mu + \int_{-\infty}^{+\infty} h_2^*(\alpha)x^*(t-\tau-\alpha)\,d\alpha + n^*(t-\tau)\right)\right] =$$

$$= E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)h_1^*(\mu)x(t-\xi)x^*(t-\tau-\mu)\,d\xi\, d\mu + \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\right.$$



$y_1(t) = (h_1 * x)(t)$ $\qquad P_{y_1}(f) = |H_1(f)|^2 P_x(f)$

$P_{y_2}(f) = |H_2(f)|^2 P_x(f)$

CALCOLO:

$$E\left[y_1(t)y_2^*(t-\tau)\right] = E\left[\int_{-\infty}^{+\infty} h_1(\xi)x(t-\xi)\,d\xi \int_{-\infty}^{+\infty} h_2^*(\eta)x^*(t-\tau-\eta)\,d\eta\right] =$$

$$= E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)h_2^*(\eta)x(t-\xi)x^*(t-\tau-\eta)\,d\xi\, d\eta\right] =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)h_2^*(\eta)\,R_x(\tau-(\eta-\xi))\,d\xi\, d\eta \qquad \eta-\xi = y$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \underbrace{h_1(\eta-y)h_2^*(\eta)}_{R^*_{h_1h_2}(y)} R_x(\tau-y)\,dy\,d\eta =$$

$$= \int_{-\infty}^{+\infty} R^*_{h_1h_2}(y)R_x(\tau-y)\,dy = \left(R^*_{h_1h_2} * R_x\right)(\tau)$$

NOTA: $R_{y_2y_1}(\tau) = R^*_{y_1y_2}(-\tau) = \left(R_{h_1h_2} * R_x^*\right)(-\tau)$

$$R_{y_0}(\tau) = E\left[y_0(t)\,y_0(t-\tau)\right] = E\left[y_1(t)y_1^*(t-\tau)\right] + E\left[y_1(t)y_2^*(t-\tau)\right] + E\left[y_2(t)y_2^*(t-\tau)\right] + E\left[y_2(t)y_1^*(t-\tau)\right] =$$

$$= R_{y_1}(\tau) + R_{y_1y_2}(\tau) + R_{y_2y_1}(\tau) + R_{y_2}(\tau)$$

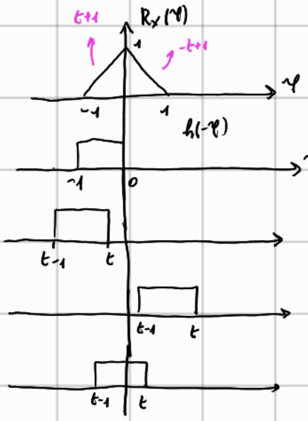
2)

$$R_x(\tau) = \Delta(\tau)$$

$$h(t) = u(t) - u(t-1)$$

$$R_{yx}(t,\tau) = E\left[x(t)\int_{-\infty}^{+\infty} h^*(\xi)\, x^*(t-\tau-\xi)\, d\xi\right] = E\left[\int_{-\infty}^{+\infty} x(t) h^*(\xi)\, x^*(t-\tau-\xi)\, d\xi\right] = \int_{-\infty}^{+\infty} h(\xi) R_x(\tau+\xi)\, d\xi = \int_{-\infty}^{+\infty} h(\xi) R_x^*(-\tau-\xi)\, d\xi = (h * R_x^*)(-\tau)$$

Calcolo $(h * R_x^*)(t) = (h * R_x)(t)$



$t-1<-1$, $t>-1$ $\Rightarrow$ $-1<t<0$

$$S(\tau) = \int_{-1}^{t} (\tau+1)\, d\tau = \left(\frac{\tau^2}{2} + \tau\right)\Big|_{-1}^{t} = \frac{t^2-1}{2} + t+1 = \frac{t^2}{2} + t + \frac{1}{2}$$

$1<t<2$

$$S(\tau) = \int_{t-1}^{1} (-\tau+1)\, d\tau = \left(-\frac{\tau^2}{2} + \tau\right)\Big|_{t-1}^{1} = \left(\frac{-1+(t-1)^2}{2} + 1 - t + 1\right) = -\cancel{\frac{1}{2}} + \frac{t^2}{2} + \cancel{\frac{1}{2}} - t + 2 - t = \frac{t^2}{2} - 2t + 2$$

$t-1>-1$, $t<1$ $\quad 0<t<1$

$$S(\tau) = \int_{t-1}^{0} (\tau+1)\, d\tau + \int_{0}^{t} (-\tau+1)\, d\tau = \left(\frac{\tau^2}{2} + \tau\right)\Big|_{t-1}^{0} + \left(-\frac{\tau^2}{2} + \tau\right)\Big|_{0}^{t} = -\frac{(t-1)^2}{2} - \cancel{t} + 1 - \frac{t^2}{2} + \cancel{t} = -\frac{t^2}{2} + \frac{1}{2} + t - \frac{t^2}{2} =$$

$$= -t^2 + t + \frac{1}{2}$$

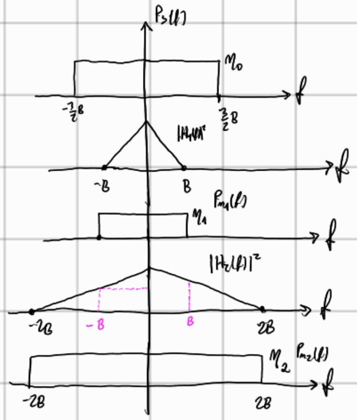
$$R_{yx}(\tau) = \begin{cases} \frac{\tau^2}{2} - \tau + \frac{1}{2} & \tau \in [0,1] \\ -\tau^2 - \tau + \frac{1}{2} & \tau \in [-1,0] \\ \frac{\tau^2}{2} + 2\tau + 2 & \tau \in [-2,-1] \\ 0 & \text{altrove} \end{cases}$$

2)

$|H_1(f)|^2 = \Delta\left(\frac{f}{B}\right) \qquad |H_2(f)|^2 = \Delta\left(\frac{f}{2B}\right) \qquad B = 10\,KHz$

$P_{m_1}(f) = \eta \quad \forall f \in [-B, B] \qquad P_{m_2}(f) = \eta \quad \forall f \in [-2B, 2B]$

$P_S(f) = \eta_0 \quad \forall f \in \left[-\frac{3}{2}B, \frac{3}{2}B\right]$



$P_x(f) = |H_1(f)|^2|H_2(f)|^2 P_S(f) + |H_2(f)|^2 P_{m_1}(f) + P_{m_2}(f)$

$$P_x = \underbrace{\int_{-B}^{B} |H_1(f)|^2|H_2(f)|^2 P_S(f)\,df}_{0} + \underbrace{\int_{-B}^{B} |H_2(f)|^2 P_{m_1}(f)\,df}_{1} + P_{m_2}$$

1 $\displaystyle\int_{-B}^{B} |H_2(f)|^2 P_{m_1}(f)\,df = \eta_1 2 \int_0^B |H_2(f)|^2 df = 2\eta_1 \int_0^B \left(-\frac{f}{2B}+1\right) df = 2\eta_1\left[\left(1-\frac{1}{2}\right)+1\right]\cdot\frac{B}{2} = \frac{3}{2}\eta_1 B$

0 $\displaystyle\int_{-B}^{B} |H_1(f)|^2|H_2(f)|^2 P_S(f)\,df = 2\eta_0 \int_0^B \left(-\frac{f}{2B}+1\right)\left(-\frac{f}{B}+1\right) df = 2\eta_0\left[\int_0^B \frac{f^2}{2B^2}df + \int_0^B df - \int_0^B \frac{f}{B}df - \int_0^B \frac{f}{2B}df\right] = 2\eta_0\left[\frac{f^3}{6B^2}\Big|_0^B + B - \frac{f^2}{2B}\Big|_0^B - \frac{f^2}{4B}\Big|_0^B\right] =$

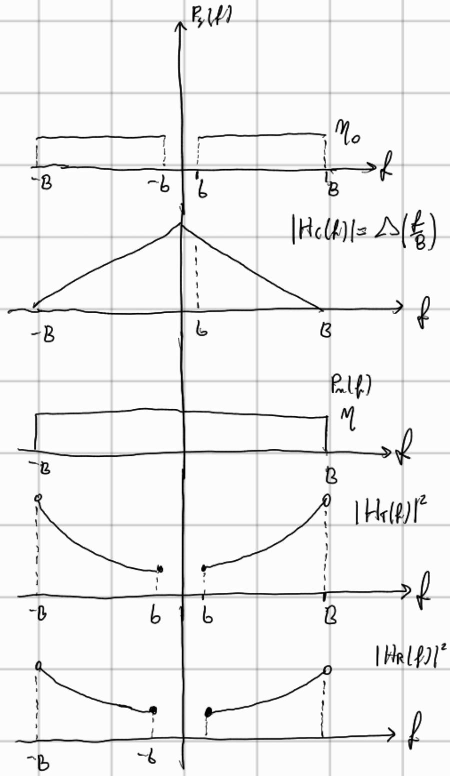
$= 2\eta_0\left[\frac{B}{6} + B - \frac{B}{2} - \frac{B}{4}\right] = 2\eta_0\left[\frac{B}{6} - \frac{3}{4}B + B\right] = \frac{5}{6}\eta_s B$

$P_x = \frac{5}{6}\eta_s B + \frac{3}{2}\eta_1 B + 4\eta_2 B$

$P_m = \frac{3}{2}\eta_1 B + 4\eta_2 B = \alpha$

$$\left(\frac{P_s}{P_m}\right) = \frac{\frac{5}{6}\eta_s}{\alpha} = \frac{3}{2}\frac{\eta_0}{\alpha} + 1$$

3)



$$\eta_0 = \frac{P_s^{rms\,2}}{2(B-b)}$$

$$\eta = \frac{P_n^{rms\,2}}{2B}$$

$$P_x(f) = |H_c(f)|^2 P_s(f) + P_n(f)$$

$$P_x = 2\eta_0 \int_b^B \left(-\frac{f}{B}+1\right)^2 df + 2B\eta$$

$$P_x = 2\eta_0 \left(\frac{(B-b)\cdot\left(-\frac{b}{B}+1\right)^2}{3}\right) + 2B\eta = \frac{2\eta_0}{3}\left[(B-b)\left(\frac{b^2}{B^2}+1-\frac{2b}{B}\right)\right] + 2B\eta =$$

$$= \frac{2\eta_0}{3}\underbrace{\left[\frac{b^2}{B}-\frac{b^3}{B^2}+B-b-2b+\frac{2b^2}{B}\right]}_{\alpha} + 2B\eta =$$

$$= \frac{2}{3}\eta_0\alpha + 2B\eta$$

$$\left(\frac{P_x}{P_n}\right) = \frac{\frac{2}{3}\eta_0\alpha + 2B\eta}{2B\eta} = \frac{\eta_0\alpha}{B\eta} + 1$$

$$|H_T(f)|^2 \equiv \frac{K\alpha}{|H_c(f)|} = \begin{cases} \dfrac{K\alpha}{-\frac{f}{B}+1} & f \in [b, B] \\[2ex] \dfrac{K\alpha}{\frac{f}{B}+1} & f \in [-B, -b] \end{cases}$$

$$|H_R(f)|^2 \equiv \frac{\alpha}{K|H_c(f)|} = \begin{cases} \dfrac{\alpha}{\left(-\frac{f}{B}+1\right)K} & f \in [b, B] \\[2ex] \dfrac{\alpha}{\left(\frac{f}{B}+1\right)K} & f \in [-B, -b] \end{cases}$$

4) $x(t)$ come ingresso

$$r_y(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} y(t)\,y^*(t-\tau)\,dt =$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left[\int_{-\infty}^{+\infty} h(\xi)\,x(t-\xi)\,d\xi \int_{-\infty}^{+\infty} h^*(\eta)\,x^*(t-\tau-\eta)\,d\eta \right] dt =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left[\overbrace{\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t-\xi)\,x^*(t-\tau-\eta)\,dt}^{*} \right] h(\xi)\,h^*(\eta)\,d\xi\,d\eta =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} r_x(\tau-(\xi-\eta))\,h(\xi)\,h^*(\eta)\,d\xi\,d\eta \qquad \boxed{\xi-\eta = y}$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} r_x(\tau-y)\,\underbrace{h(\xi)\,h^*(\xi-y)\,d\xi}\,dy =$$

$$\int_{-\infty}^{+\infty} r_x(\tau-y)\,r_h(y)\,dy = (r_x * r_h)(\tau) \Rightarrow P_y(f) = |H(f)|^2 P_x(f)$$

*

$$r_x(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\,x^*(t-\tau)\,dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t_1)\,x^*(t_2)\,dt_1 = r_x(\underbrace{t_1-t_2}_{\tau})$$

$$t-\tau = t_2$$
$$t = t_1$$

1) $Y(t) = X(t) + X(t-\Delta) + X(t-2\Delta)$

$P_x(f) = \Delta\left(\frac{f}{B}\right) \qquad \Delta \ll \frac{1}{B}$

$E[Y(t)] = 0$, $P_x(f)$ non ha delta in 0.

$R_y(t,\tau) = E\left[\left(X(t) + X(t-\Delta) + X(t-2\Delta)\right)\left(X(t-\tau) + X(t-\tau-\Delta) + X(t-\tau-2\Delta)\right)\right] =$

$= R_x(\tau) + R_x(\tau+\Delta) + R_x(\tau+2\Delta) + R_x(\tau-\Delta) + R_x(\tau) + R_x(\tau+\Delta) +$

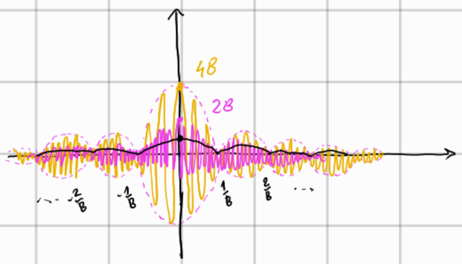
$+ R_x(\tau-2\Delta) + R_x(\tau-\Delta) + R_x(\tau) =$

$= 3R_x(\tau) + 2R_x(\tau+\Delta) + 2R_x(\tau-\Delta) + R_x(\tau-2\Delta) + R_x(\tau+2\Delta)$

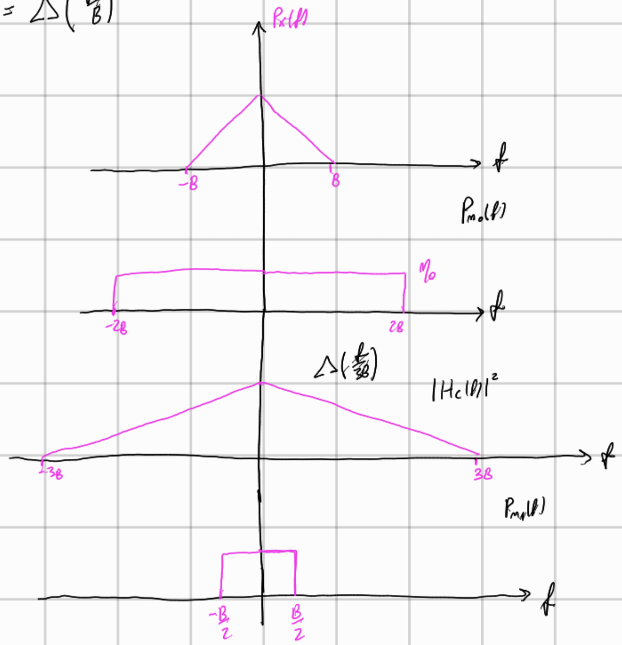
$P_y(f) = 3P_x(f) + 2e^{-2\pi jf\Delta} P_x(f) + 2e^{-2\pi jf\Delta} P_x(f) + e^{-2\pi jf(2\Delta)} P_x(f) + e^{+2\pi jf(2\Delta)} P_x(f)$

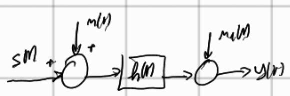
$= 3P_x(f) + 4\cos(2\pi\Delta f) P_x(f) + 2\cos(2\pi 2\Delta f) P_x(f)$

$P_x(f) = B\,\mathrm{sinc}^2(Bf)$



2) $P_X(f) = \Delta\left(\frac{f}{B}\right)$





$$P_y(f) = |H_1(f)|^2 P_s(f) + |H_1(f)|^2 P_{n_0}(f) + P_{n_1}(f)$$

calcolo

$$P_{S_R} = \int_{-\infty}^{+\infty} |H_1(f)|^2 P_s(f)\,df = 2\int_0^B \left(-\frac{f}{B}+1\right)\left(-\frac{f}{3B}+1\right)df = 2\left[\int_0^B \frac{f^2}{3B^2}df - \int_0^B \frac{f}{B}df - \int_0^B \frac{f}{3B}df + \int_0^B df\right] = 2\left[\frac{f^3}{9B^2}\Big|_0^B - \frac{f^2}{2B}\Big|_0^B - \frac{f^2}{6B}\Big|_0^B + B\right] =$$

$$= 2\left[\frac{B}{9} - \frac{B}{2} - \frac{B}{6} + B\right] = 2\left[\frac{2B - 9B - 3B + 18B}{18}\right] =$$

$$= \frac{8}{9}B$$

$$P_{N_0} = 2\eta_0 \int_0^{2B}\left(-\frac{f}{3B}+1\right)df = 2\eta_0\left[-\frac{f^2}{6B}\Big|_0^{2B} + 2B\right] = 2\eta_0\left[2B - \frac{4}{6}B\right] = \eta_0 \frac{8}{3}B$$

$$P_{n_1} = B\eta_1$$

$$\frac{P_Y}{P_N} = \frac{\frac{8}{9}B + \frac{8}{3}\eta_0 B + \eta_1 B}{\frac{8}{3}\eta_0 B + \eta_1 B}$$

NOTA: $P_X(f) = P_S(f) + P_{n_0}(f) = \begin{cases} \left(-\frac{f}{B}+1\right) + \eta_0 & f \in [0, B] \\ \left(\frac{f}{B}+1\right) + \eta_0 & f \in [-B, 0] \end{cases}$

$$|H_r^0(f)|^2 \equiv \begin{cases} \frac{1}{-\frac{f}{3B}+1} \cdot \left(\frac{\eta_1}{-\frac{f}{B}+1+\eta_0}\right)^{1/2} & f \in [0, B/2] \\ 1 & |f| \in [B/2, B] \\ \frac{1}{\frac{f}{3B}+1} \cdot \left(\frac{\eta_1}{-\frac{f}{B}+1+\eta_0}\right)^{1/2} & f \in \left[-\frac{B}{2}, 0\right] \end{cases}$$

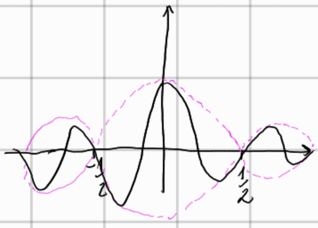
$$|H_R(f)|^2 = \begin{cases} \frac{1}{-\frac{f}{3B}+1} \cdot \left(\frac{-\frac{f}{B}+1+\eta_0}{\eta_1}\right)^{1/2} & f \in \left[0, \frac{B}{2}\right] \\ 1 & |f| \in \left[\frac{B}{2}, B\right] \\ \frac{1}{\frac{f}{3B}+1} \cdot \left(\frac{\frac{f}{B}+1+\eta_0}{\eta_1}\right)^{1/2} & f \in \left[-\frac{B}{2}, 0\right] \end{cases}$$

1) $Y(t) = X(t-2) + X(t)\sin^2(8\pi f_0 t)$

$f_0 \gg 4$

$P_x(f) = \Delta\left(\frac{f-2}{2}\right) + \Delta\left(\frac{f+2}{2}\right)$

$R_x(\tau) = 4\cos(2\pi 2\tau)\,\mathrm{sinc}^2(2\tau)$



$E[Y(t)] = 0$, perché non ho δ in 0.

$R_Y(t,\tau) = E\left[\left(X(t-2) + \frac{X(t)}{2} - \frac{X(t)}{2}\cos(16\pi f_0 t)\right)\left(X(t-\tau-2) + \frac{X(t-\tau)}{2} - \frac{X(t-\tau)}{2}\cos(16\pi f_0(t-\tau))\right)\right] =$

$= R_x(\tau) + \frac{1}{2}R_x(\tau-2) - \frac{1}{2}R_x(\tau-2)\cos(16\pi f_0(t-\tau)) +$

$+ \frac{1}{2}R_x(\tau+2) + \frac{1}{4}R_x(\tau) - \frac{1}{4}R_x(\tau)\cos(16\pi f_0(t-\tau)) +$

$- \frac{1}{2}R_x(\tau+2)\cos(16\pi f_0 t) - \frac{1}{4}R_x(\tau)\cos(16\pi f_0 t) + \frac{R_x(\tau)}{4}\cos(16\pi f_0 t)\cos(16\pi f_0(t-\tau))$

Ciclostazionario su periodo $\frac{1}{8f_0}$.

$\overline{R_x}(\tau) = R_x(\tau) + \frac{1}{2}R_x(\tau-2) + \frac{1}{2}R_x(\tau+2) + \frac{1}{4}R_x(\tau) + \frac{R_x(\tau)}{8}\cos(16\pi f_0\tau)$

$= \frac{5}{4}R_x(\tau) + \frac{1}{2}R_x(\tau-2) + \frac{1}{2}R_x(\tau+2) + \frac{R_x(\tau)}{8}\cos(16\pi f_0\tau)$

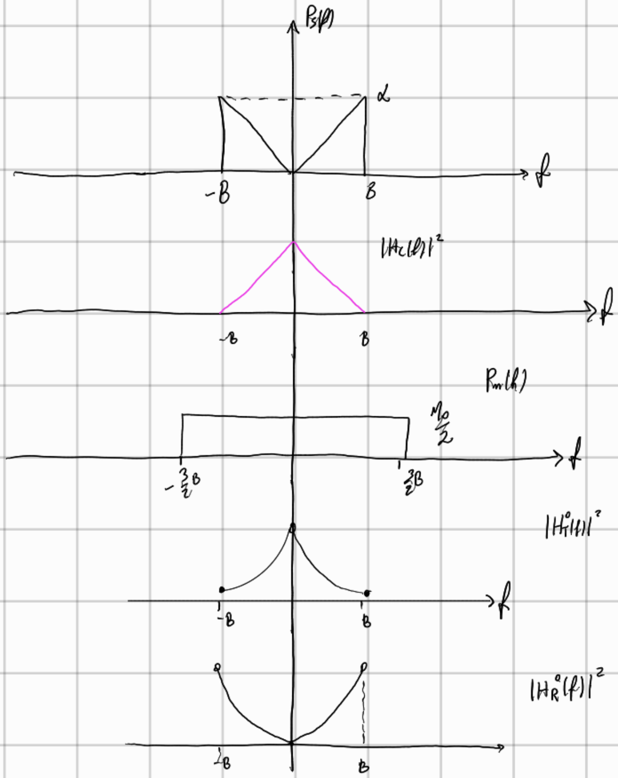
$\overline{P_x}(f) = \frac{5}{4}P_x(f) + \frac{1}{2}e^{-2\pi j 2f}P_x(f) + \frac{1}{2}e^{-2\pi j(-2f)}P_x(f) + \frac{1}{8}\left(P_x(f-8f_0) + P_x(f+8f_0)\right)$

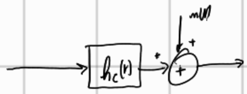
$= \frac{5}{4}P_x(f) + \cos(2\pi 2f)P_x(f) + \frac{1}{8}P_x(f-8f_0) + \frac{1}{8}P_x(f+8f_0)$

3)

$$P_S(f) = \alpha\left[\Pi\left(\frac{f}{2B}\right) - \Delta\left(\frac{f}{B}\right)\right]$$

$$|H_C(f)|^2 = \Delta\left(\frac{f}{B}\right) \qquad P_n(f) = \frac{N_0}{2}\Pi\left(\frac{f}{3B}\right)$$



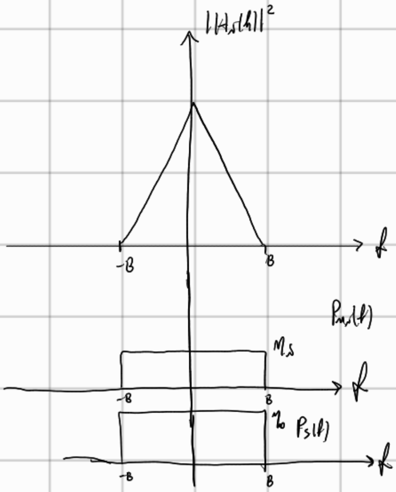


$P_S = \alpha B$

$P_N = 3\frac{N_0 B}{2}$

$$\frac{P_S}{P_N} = \frac{\alpha B}{\frac{N_0 B}{2}} = \frac{2\alpha}{N_0}$$

3)



$P_s = g_m V^2 \qquad \eta_s = \dfrac{P_s}{2B}$

$P_{S_R} = 2\eta_0 \cdot \dfrac{B \cdot 1}{4} = \dfrac{\eta_0 B}{2} \qquad P_{m_1} = 2\eta_s \dfrac{B}{3} \qquad P_{m_2} = 2\eta_s \dfrac{B}{2} \qquad P_{m_3} = 2B\eta_s$

$$\frac{S}{N} = \frac{\eta_0 B/2}{2\eta_s\left(B+\frac{B}{2}+\frac{B}{3}\right)} = \frac{\eta_0 \cancel{B}}{4\eta_s \cancel{B}\left(\frac{11}{6}\right)} = \frac{6}{44}\,\frac{\eta_0}{\eta_s}$$

1)

$$Y(t) = X(t)\cos^2(2\pi f_0 t) + C$$

$X(t)$ SSL , $P_x(f) = \Delta\left(\frac{f}{2B}\right)$

C indipendente da $X(t)$. $\quad C \propto N(c, 1, 2)$

$$E[Y(t)] = E[X(t)]\cos^2(2\pi f_0 t) + E[C] = 1$$

$$R_y(t,\tau) = E\left[\left(X(t)\cos^2(2\pi f_0 t) + C\right)\left(X(t-\tau)\cos^2(2\pi f_0(t-\tau)) + C\right)\right] =$$

$\sigma_c^2 + \mu_c^2 = 5$

$$= R_x(\tau)\left(\frac{1}{2} + \frac{1}{2}\cos(4\pi f_0 t)\right)\left(\frac{1}{2} + \frac{1}{2}\cos(4\pi f_0(t-\tau))\right) + E[C^2] =$$

$$= \frac{R_x(\tau)}{4} + \frac{R_x(\tau)}{4}\cos(4\pi f_0 t) + \frac{R_x(\tau)}{4}\cos(4\pi f_0(t-\tau)) + \frac{R_x(\tau)}{4}\frac{1}{2}\left(\cos(4\pi f_0 \tau) + \cos(4\pi f_0(2t-\tau)\right) + 5$$

- Ciclostazionario: medio su $\frac{1}{2f_0}$.

$$\bar{R}_x(\tau) = \frac{R_x(\tau)}{4} + \frac{R_x(\tau)}{8}\cos(4\pi f_0 \tau) + 5$$

$$P_x(f) = \frac{P_x(f)}{4} + \frac{1}{16}P_x(f-2f_0) + \frac{1}{16}P_x(f+f_0) + 5\delta(f)$$

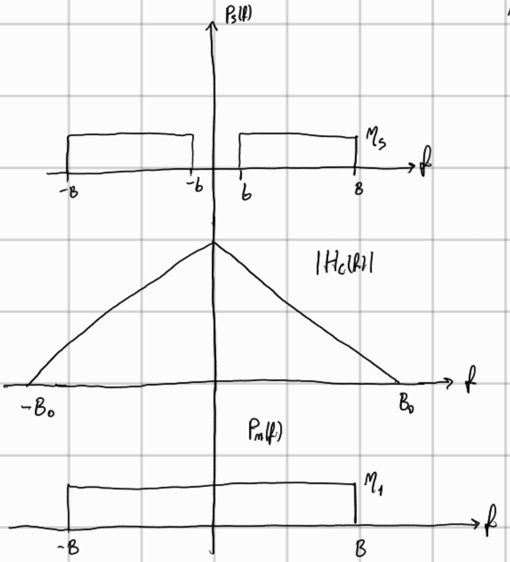
$Y(t)$ é SSL.

2) $R_x(\tau) = \Delta(\tau)$ $\qquad$ $h(t) = \delta(t-1) + \frac{1}{2}\delta(t-2)$

$$R_{yx}(\tau) = E\left[y(t)X(t-\tau)\right] = E\left[\int_{-\infty}^{+\infty} h(\xi)X(t-\xi)\,d\xi\, X(t-\tau)\right] = \int_{-\infty}^{+\infty} h(\xi)R_x(\tau-\xi)\,d\xi = (h * R_x)(\tau)$$

$$R_{yx}(\tau) = \Delta(\tau-1) + \frac{1}{2}\Delta(\tau-2)$$

4)



$$\eta_s = \frac{P_s^{RMS\,2}}{2(B-b)}$$

$$\eta_1 = \frac{P_m^{RMS\,2}}{2B}$$

$$\frac{P_s}{P_N} = ?$$

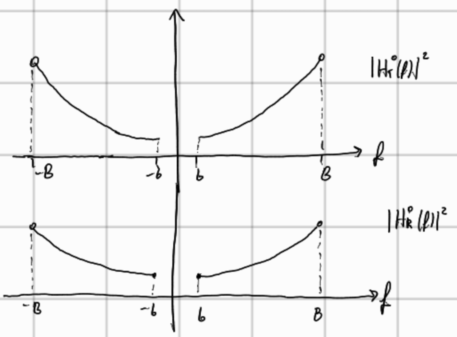
$$P_s(f) = |H_c(f)|^2 P_s(f)$$

$$P_s = 2\eta_s \int_b^B \left(-\frac{f}{B_0}+1\right)^2 df = 2\eta_s \left[\int_b^B \frac{f^2}{B_0^2}\,df + \int_b^B df - 2\int_b^B \frac{f}{B_0}\,df \right] = 2\eta_s \left[\frac{f^3}{3B_0^2}\Big|_b^B + (B-b) - \frac{f^2}{B_0}\Big|_b^B \right] = 2\eta_s \left[\underbrace{\frac{B^3-b^3}{3B_0^2} + (B-b) - \frac{(B^2-b^2)}{B_0}}_{\alpha} \right]$$

$$P_N(f) = 2B\eta_1 \quad \Rightarrow \quad \frac{P_s}{P_N} = \frac{2\eta_s\alpha}{2\eta_1 B} = \frac{\eta_s\alpha}{\eta_1 B}$$

$$|H_T^o(f)|^2 = \frac{\alpha K}{|H_c(f)|} \left(\overbrace{\frac{P_m(f)}{P_s(f)}}^{\frac{\eta_1}{\eta_s}} \right)^{\frac{1}{2}} \equiv \begin{cases} \dfrac{1}{\left(-\frac{f}{B_0}+1\right)} & f\in[b,B] \\[2ex] \dfrac{1}{\frac{f}{B_0}+1} & f\in[-B,-b] \end{cases}$$

$$|H_R^o(f)|^2 = \frac{\alpha}{K|H_c(f)|} \left(\overbrace{\frac{P_s(f)}{P_m(f)}}^{\frac{\eta_s}{\eta_1}} \right)^{\frac{1}{2}} \equiv \begin{cases} \dfrac{1}{\left(-\frac{f}{B_0}+1\right)} & f\in[b,B] \\[2ex] \dfrac{1}{\frac{f}{B_0}+1} & f\in[-B,-b] \end{cases}$$



4)

$$R_{y_1y_2}(t,\varphi) = E\left[\int_{-\infty}^{+\infty} h_1(\xi)x_1(t-\xi)\,d\xi \int_{-\infty}^{+\infty} h_2^*(\eta)x_2^*(t-\varphi-\eta)\,d\eta\right] =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)\,h_2^*(\eta)\,R_{x_1x_2}(\varphi-(\xi-\eta))\,d\xi\,d\eta =$$

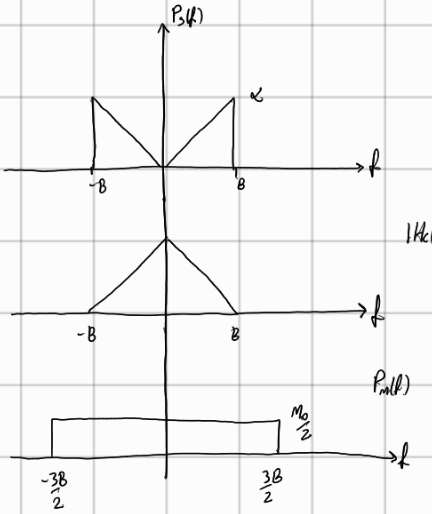
$\xi-\eta=y$

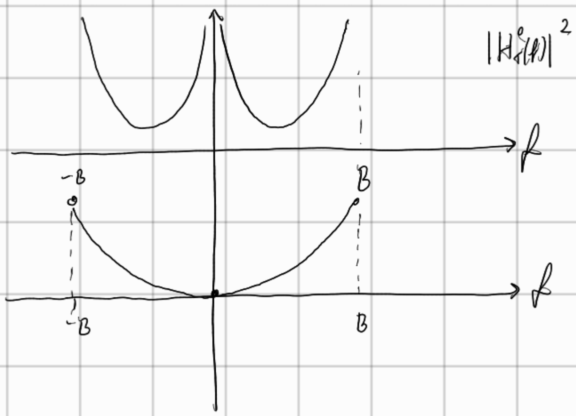
$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)\,h_2^*(\xi-y)\,R_{x_1x_2}(\varphi-y)\,d\xi\,dy =$$

$$= \int_{-\infty}^{+\infty} r_{h_1h_2}(y)\,R_{x_1x_2}(\varphi-y)\,dy = (r_{h_1h_2} * R_{x_1x_2})(\varphi)$$

3)

$$P_s(f) = \alpha\left[\Pi\left(\frac{f}{2B}\right) - \Delta\left(\frac{f}{B}\right)\right]$$





$\frac{P_s}{P_N} = ?$

$$P_{s_R}(f) = |H_c(f)|^2 P_s(f) = 2\int_0^B \left(-\frac{f}{B}+1\right)\left(\frac{\alpha f}{B}\right) df = 2\alpha\int_0^B \left(-\frac{f^2}{B^2} + \frac{f}{B}\right) df = 2\alpha\left[-\frac{f^3}{3B^2}\Big|_0^B + \frac{f^2}{2B}\Big|_0^B\right] = 2\alpha\left[-\frac{B}{3} + \frac{B}{2}\right] = \frac{\alpha}{3}B$$

$$P_n = \frac{3}{2}\eta_0 B$$

$$\frac{P_s}{P_N} = \frac{2\alpha}{9\eta_0}$$

$$|H_T(f)|^2 = \frac{\alpha K}{|H_c(f)|^2}\left(\frac{P_n(f)}{P_s(f)}\right)^{\frac{1}{2}} = \begin{cases} \dfrac{1}{\sqrt{\left(-\frac{f}{B}+1\right)\frac{f\alpha}{B}}} & f \in [0, B] \\[2ex] \dfrac{1}{\sqrt{\left(\frac{f}{B}+1\right)\left(-\frac{f\alpha}{B}\right)}} & f \in [-B, 0] \end{cases}$$

$$|H_R^{\circ}(f)|^2 = \begin{cases} \sqrt{\dfrac{f/B\,\alpha}{\left(-\frac{f}{B}+1\right)}} & f \in [0, B] \\[2ex] \sqrt{\dfrac{-f/B\,\alpha}{\left(\frac{f}{B}+1\right)}} & f \in [-B, 0] \end{cases}$$

1)

$$Y(t) = X(t)\cos(2\pi f_0 t) - X(t-\Delta)\sin^2(2\pi f_0 t)$$

$X(t)$ SSL

$P_x(f) = \Delta\left(\frac{f}{B}\right) \quad f_0 \gg B$

$E[Y(t)] = 0$, no impulsi in $P_x(0)$.

$$R_y(t,\tau) = E\left[\left(X(t)\cos(2\pi f_0 t) - X(t-\Delta)\sin^2(2\pi f_0 t)\right)\left(X(t-\tau)\cos(2\pi f_0(t-\tau)) - X(t-\tau-\Delta)\sin^2(2\pi f_0(t-\tau))\right)\right] =$$

$$= R_x(\tau)\cos(2\pi f_0 t)\cos(2\pi f_0(t-\tau)) - R_x(\tau-\Delta)\cos(2\pi f_0(t-\tau))\left(\frac{1}{2} - \frac{1}{2}\cos(4\pi f_0 t)\right) +$$
$$- R_x(\tau+\Delta)\cos(2\pi f_0 t)\left(\frac{1}{2} - \frac{1}{2}\cos(4\pi f_0(t-\tau))\right) + R_x(\tau)\left(\frac{1}{2} - \frac{1}{2}\cos(4\pi f_0 t)\right)\left(\frac{1}{2} - \frac{1}{2}\cos(4\pi f_0(t-\tau))\right) =$$

$$= \frac{R_x(\tau)}{2}\left(\cos(2\pi f_0(2t-\tau)) + \cos(2\pi f_0 \tau)\right) - \frac{R_x(\tau-\Delta)}{2}\cos(2\pi f_0(t-\tau)) + \frac{1}{2}R_x(\tau-\Delta)\cos(4\pi f_0 t) +$$
$$- \frac{R_x}{2}(\tau+\Delta)\cos(2\pi f_0 t) + \frac{R_x}{2}(\tau+\Delta)\left(\frac{1}{2}\cos(2\pi f_0(3t-2\tau)) + \frac{1}{2}\cos(2\pi f_0(t-2\tau))\right) +$$
$$\frac{R_x(\tau)}{4} - \frac{R_x}{2}(\tau)\cos(4\pi f_0(t-\tau)) - \frac{1}{2}R_x(\tau)\cos(4\pi f_0 t) + \frac{1}{4}R_x(\tau)\left(\frac{1}{2}\cos(4\pi f_0(2t-\tau)) + \frac{1}{2}\cos(4\pi f_0 \tau)\right)$$

Ciclostazionario. Calcolo $\bar{R}_y(\tau)$ integrando e mediando sul periodo $\frac{1}{f_0}$.

$$\bar{R}_y(\tau) = \frac{R_x(\tau)}{2}\cos(2\pi f_0 \tau) + \frac{R_x(\tau)}{4} + \frac{1}{8}R_x(\tau)\cos(4\pi f_0 \tau)$$

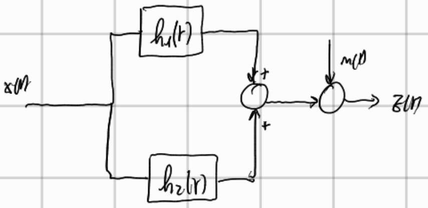
$$\bar{P}_y(f) = \frac{P_x}{4}(f-f_0) + \frac{P_x}{4}(f+f_0) + \frac{P_x(f)}{4} + \frac{1}{16}P_x(f-2f_0) + \frac{1}{16}P_x(f+2f_0)$$

- Risultato coerente per operazione di modulazione sulla frequenza f_0.

2)

$$z(t) = (h_c * x)(t) + n(t)$$

$$h_c(t) = h(t) + h(t-\Delta)$$



$$R_z(t,\varphi) = E\left[\left((h_1 * x)(t) + (h_2 * x)(t) + n(t)\right)\left((h_1 * x)(t-\varphi) + (h_2 * x)(t-\varphi) + n(t-\varphi)\right)^*\right]$$

$$= \overset{1}{E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)x(t-\xi)h_1^*(\eta)x^*(t-\varphi-\eta)\,d\xi\,d\eta\right]} + \overset{2}{E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)x(t-\xi)h_2^*(\eta)x^*(t-\varphi-\eta)\,d\xi\,d\eta\right]} +$$

$$+ \overset{3}{E\left[\int_{-\infty}^{+\infty} h_1(\xi)x(t-\xi)n(t-\varphi)\,d\xi\right]} + \overset{4}{E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_2(\xi)x(t-\xi)h_1^*(\eta)x^*(t-\varphi-\eta)\,d\xi\,d\eta\right]} +$$

$$+ \overset{5}{E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_2(\xi)x(t-\xi)h_2^*(\eta)x(t-\varphi-\eta)\,d\xi\,d\eta\right]} + \overset{6}{E\left[\int_{-\infty}^{+\infty} h_2(\xi)x(t-\xi)n(t-\varphi)\,d\xi\right]} +$$

$$+ \overset{7}{E\left[\int_{-\infty}^{+\infty} h_1^*(\xi)x^*(t-\varphi-\xi)n(t)\,d\xi\right]} + \overset{8}{E\left[\int_{-\infty}^{+\infty} h_2^*(\xi)x^*(t-\varphi-\xi)n(t)\,d\xi\right]} + R_n(\varphi) =$$

1

$$E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)x(t-\xi)h_1^*(\eta)x^*(t-\varphi-\eta)\,d\xi\,d\eta\right] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)h_1^*(\eta)R_x(\varphi-(\xi-\eta))\,d\xi\,d\eta = \qquad \xi-\eta=\alpha$$

$$= \int_{-\infty}^{+\infty} h_1(\xi)h_1^*(\xi-\alpha)R_x(\varphi-\alpha)\,d\xi\,d\alpha = (r_{h_1} * R_x)(\varphi)$$

2 $= (r_{h_1h_2} * R_x)(\varphi)$

3 $= 0$, incoerenti

4 $= (r_{h_2h_1} * R_x)(\varphi)$

5 $= (r_{h_2} * R_x)(\varphi)$

6, 7, 8 $= 0$, incoerenti

$$R_y(\varphi) = (r_{h_1} * R_x)(\varphi) + (r_{h_1h_2} * R_x)(\varphi) + (r_{h_2h_1} * R_x)(\varphi) + (r_{h_2} * R_x)(\varphi) + R_n(\varphi)$$

$$P_y(f) = |H(f)|^2P_x(f) + e^{2\pi jf\Delta}|H(f)|^2P_x(f) + e^{-2\pi jf\Delta}|H(f)|^2P_x(f) + |H(f)|^2P_x(f) + P_n(f) =$$

$$= 2|H(f)|^2P_x(f) + 2\cos(2\pi f\Delta)|H(f)|^2P_x(f) + P_n(f)$$

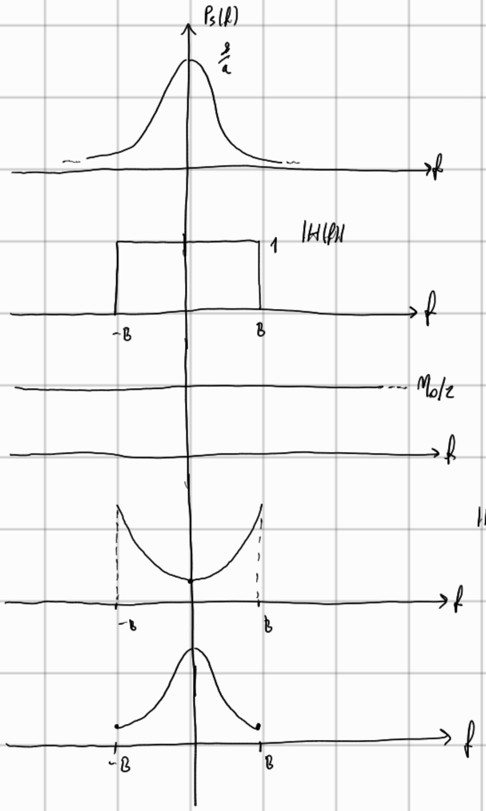
$$r_{h_1h_2}(\varphi) = \int_{-\infty}^{+\infty} h(t)h(t-\Delta-\varphi)\,dt = r_h(\varphi+\Delta)$$

$$r_{h_2h_1}(\varphi) = \int_{-\infty}^{+\infty} h(t-\Delta)h(t-\varphi)\,dt = r_h(\varphi-\Delta)$$

$y = t-\Delta$

3) $R_s(\tau) = e^{-a|\tau|}$

$$P_s(f) = \frac{1}{a+2\pi j f} + \frac{1}{a-2\pi j f} = \frac{2a}{a^2+4\pi^2 f^2}$$



$$|H_T(f)|^2 = \frac{\eta_0^{\frac{1}{2}}(a^2+4\pi^2 f^2)^{\frac{1}{2}}}{2\sqrt{a}}$$

$$|H_R(f)|^2 = \frac{2\sqrt{a}}{\eta_0^{\frac{1}{2}}(a^2+4\pi^2 f^2)^{\frac{1}{2}}}$$

1)

$$Y(t) = X(t-2) + X(t)\sin^2(8\pi f_0 t)$$

$$P_X(f) = \Delta\left(\frac{f-2}{2}\right) + \Delta\left(\frac{f+2}{2}\right) \quad f_0 >> 4 \qquad X(t) \text{ SSL}$$

$$E[Y(t)] = 0$$

$$R_Y(t,\tau) = E\left[\left(X(t-2) + X(t)\sin^2(8\pi f_0 t)\right)\left(X(t-2-\tau) + X(t-\tau)\sin^2(8\pi f_0(t-\tau))\right)\right] =$$

$$= R_X(\tau) + \frac{R_X(\tau-2)}{2}\left(1-\cos(16\pi f_0(t-\tau))\right) + \frac{R_X(\tau+2)}{2}\left(1-\cos(16\pi f_0 t)\right) + \frac{R_X(\tau)}{4}\left(1-\cos(16\pi f_0 t)\right)\left(1-\cos(16\pi f_0(t-\tau))\right) =$$

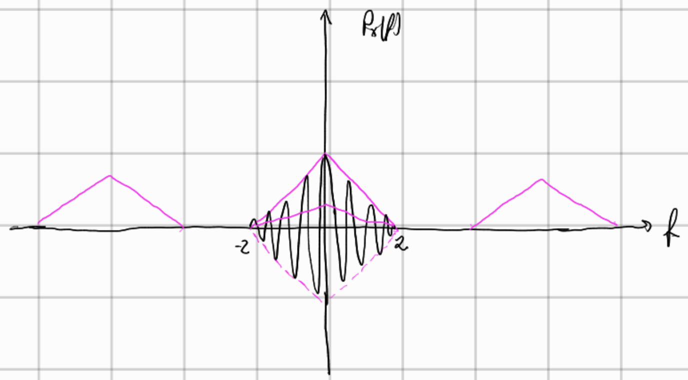
$$= R_X(\tau) + \frac{R_X}{2}(\tau-2) - \frac{R_X}{2}(\tau-2)\cos(16\pi f_0(t-\tau)) + \frac{R_X(\tau+2)}{2} - \frac{R_X(\tau+2)}{2}\cos(16\pi f_0 t) + \frac{R_X(\tau)}{4}\Big(1-\cos(16\pi f_0 t) - \cos(16\pi f_0(t-\tau)) +$$

$$+ \frac{1}{2}\left(\cos(16\pi f_0(2t-\tau)) + \cos(16\pi f_0\tau)\right)\Big)$$

Ciclostazionario: media su periodo $\frac{1}{8f_0}$

$$\bar{R}_X(\tau) = R_X(\tau) + \frac{R_X(\tau-2)}{2} + \frac{R_X(\tau+2)}{2} + \frac{R_X(\tau)}{4} + \frac{R_X(\tau)}{8}\cos(16\pi f_0\tau)$$

$$P_X(f) = P_X(f) + P_X(f)\cos(2\pi\cdot 2f) + \frac{P_X(f)}{4} + \frac{1}{16}\left(P_X(f-8f_0) + P_X(f+8f_0)\right)$$



2) $y(t) = (h_1 * x)(t) + (h_2 * x)(t) + n(t)$

$$E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)\, x(t-\xi)\, h_1^*(\eta)\, x^*(t-\varphi-\eta)\, d\xi\, d\eta\right] =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)\, h_1^*(\eta)\, R_x(\varphi-(\xi-\eta))\, d\xi\, d\eta = \qquad \xi-\eta = y$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)\, h_1^*(\xi-y)\, R_x(\varphi-y)\, d\xi\, dy =$$

$$= \int_{-\infty}^{+\infty} r_{h_1}(y)\, R_x(\varphi-y)\, dy = (r_{h_1} * R_x)(\varphi) \qquad \square$$

Stessa dimostrazione se ho $r_{h_1h_2}$

$$\Rightarrow R_y(t,\varphi) = E[y(t)\, y^*(t-\varphi)] = (r_{h_1} * R_x)(\varphi) + (r_{h_1h_2} * R_x)(\varphi) + (r_{h_2h_1} * R_x)(\varphi) + (r_{h_2} * R_x)(\varphi) + R_n(\varphi)$$

NOTA: $\int_{-\infty}^{+\infty} h_1(t)\, h_2^*(t-\varphi)\, dt = r_{h_1h_2}(\varphi)$

$$F[r_{h_1h_2}(\varphi)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)\, h_2^*(\xi-\varphi)\, e^{-2\pi j f\varphi}\, d\varphi\, d\xi =$$

$$= \int_{-\infty}^{+\infty} h_1(\xi) \int_{-\infty}^{+\infty} h_2^*(\xi-\varphi)\, e^{-2\pi j f\varphi}\, d\varphi\, d\xi = \int_{-\infty}^{+\infty} h_1(\xi) \int_{-\infty}^{+\infty} h_2^*(y)\, e^{-2\pi j f(\xi-y)}\, dy\, d\xi =$$

$\xi-\varphi = y$

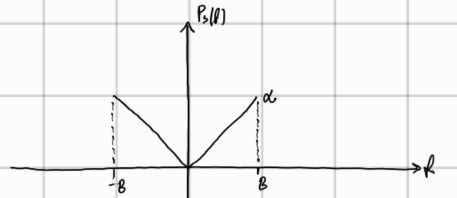
$$= H_1(f)\, H_2^*(f)$$

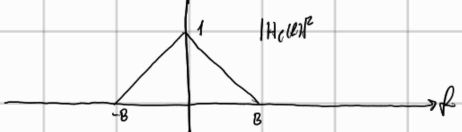
$$P_y(f) = |H_1(f)|^2 P_x(f) + H_1(f) H_2^*(f) P_x(f) + H_1^*(f) H_2(f) P_x(f) + |H_2(f)|^2 P_x(f) + P_n(f) =$$

$$|H_1(f)|^2 P_x(f) + |H_1(f)||H_2(f)|\, e^{j(\angle H_1 - \angle H_2)} P_x(f) + |H_1(f)||H_2(f)|\, e^{-j(\angle H_1 - \angle H_2)} P_x(f) + |H_2(f)|^2 P_x(f) + P_n(f) =$$

$$= |H_1(f)|^2 P_x(f) + |H_1(f)||H_2(f)| \cos(\angle H_1(f) - \angle H_2(f))\, P_x(f) + |H_2(f)|^2 P_x(f) + P_n(f)$$

3) $P_s(f) = \alpha\left[\Pi\left(\frac{f}{2B}\right) - \Lambda\left(\frac{f}{B}\right)\right]$





$$P_s(f) = \begin{cases} \frac{\alpha}{B} f & f \in [0,B] \\ -\frac{\alpha}{B} f & f \in [-B,0] \end{cases}$$

$$|H_c(f)|^2 = \begin{cases} -\frac{f}{B}+1 & f \in [0,B] \\ \frac{f}{B}+1 & f \in [-B,0] \end{cases}$$

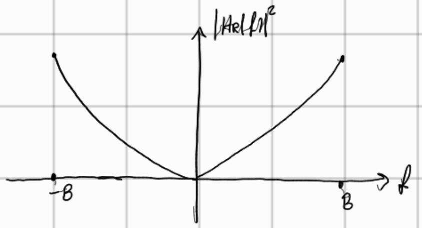
$\frac{P_{S_R}}{P_N} = ?$

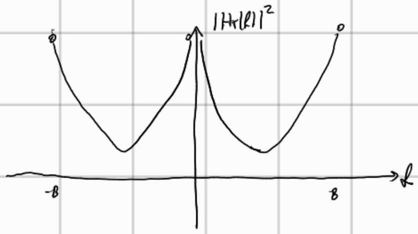
$$P_s = \int_{-\infty}^{+\infty} P_s(f)|H_c(f)|^2 df = 2\int_0^B \left(\frac{\alpha}{B}f\right)\left(-\frac{f}{B}+1\right) df = \frac{2\alpha}{B}\left[-\frac{f^3}{3B}\Big|_0^B + \frac{f^2}{2}\Big|_0^B\right] = \frac{2\alpha}{B}\left[-\frac{B^2}{3} + \frac{B^2}{2}\right] = -\frac{2}{3}\alpha B + \alpha B = \frac{1}{3}\alpha B$$

$P_N = \frac{3\eta_0 B}{2}$ $\Rightarrow$ $\frac{P_{S_R}}{P_N} = \frac{\frac{1}{3}\alpha B}{\frac{3\eta_0 B}{2}} = \frac{2}{9}\frac{\alpha}{\eta_0}$

$$|H_R(f)|^2 = \begin{cases} \frac{\alpha}{K\sqrt{-\frac{f}{B}+1}}\left(\frac{2\frac{\alpha}{B}f}{\eta_0}\right)^{\frac{1}{2}} & f \in [0,B] \\ \frac{\alpha}{K\sqrt{\frac{f}{B}+1}}\left(\frac{-2\frac{\alpha}{B}f}{\eta_0}\right)^{\frac{1}{2}} & f \in [-B,0] \end{cases}$$

$$|H_T(f)|^2 = \begin{cases} \frac{\alpha K}{\sqrt{-\frac{f}{B}+1}} \cdot \left(\frac{\eta_0}{2\frac{\alpha}{B}f}\right)^{\frac{1}{2}} & f \in [0,B] \\ \frac{\alpha K}{\sqrt{\frac{f}{B}+1}}\left(\frac{\eta_0}{-2\frac{\alpha}{B}f}\right) & f \in [-B,0] \end{cases}$$





1)

$S(t) = A + X(t) + X(t)\cos(\pi B t + \theta)$

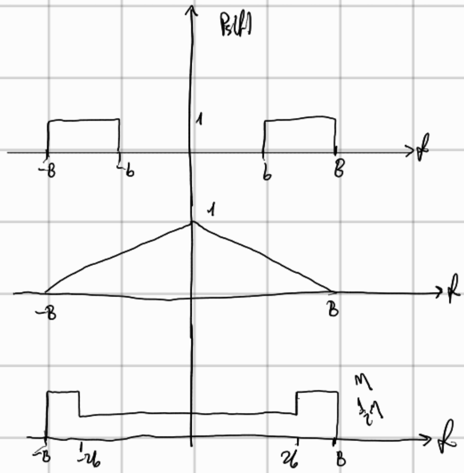
$E[Y(t)] = A$ se X e Θ scorrelate

$R_Y(t, \tau) = E[(A + X$

4)

$P_S = \Pi\left(\frac{f}{2B}\right) - \Pi\left(\frac{f}{2b}\right)$ $\qquad b \ll B$

$|H_c(f)|^2 = \Delta\left(\frac{f}{B}\right)$ $\qquad P_n = \eta\left(\Pi\left(\frac{f}{2B}\right) - \frac{1}{2}\Pi\left(\frac{f}{4b}\right)\right)$



$$|H_c(f)|^2 = \begin{cases} -\frac{f}{B}+1 & f \in [b, B] \\ \frac{f}{B}+1 & f \in [-B, -b] \end{cases}$$

$P_{SR}(f) = |H_c(f)|^2 P_S(f)$

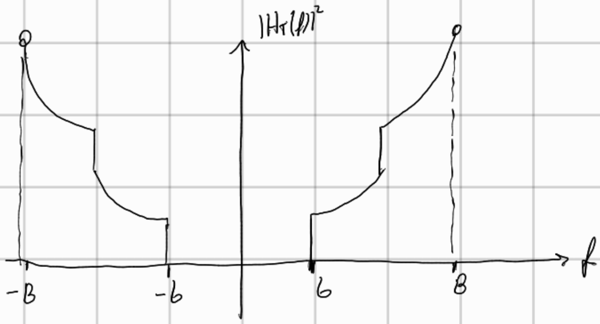
$$P_{SR} = 2\int_b^B \left(-\frac{f}{B}+1\right)df = 2\left[-\frac{f^2}{2B}\Big|_b^B + B-b\right] = 2\left[-\frac{(B^2-b^2)}{2B} + B-b\right] = \left(-\frac{b}{B}+1\right)(B-b)$$

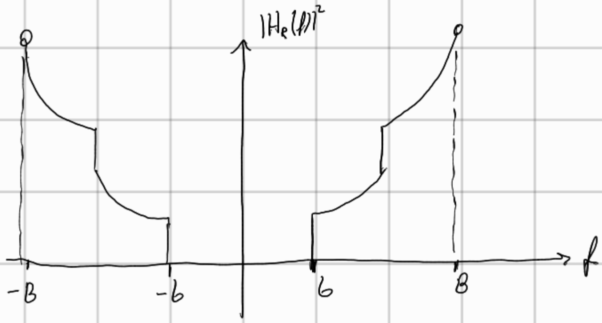
$P_n = \eta 2b + \eta(B-2b) = B\eta$

$$\frac{P_{SR}}{P_N} = \frac{(B-b)(B-b)}{\eta B^2} = \frac{B^2+b^2-2bB}{\eta B}$$

$$|H_T(f)|^2 = \begin{cases} \frac{K\alpha}{\left(-\frac{f}{B}+1\right)^{\frac12}} \cdot \frac{1}{2}\eta & f \in [b, 2b] \\ \frac{K\alpha}{\left(-\frac{f}{B}+1\right)^{\frac12}}\eta & f \in [2b, B) \\ \frac{K\alpha}{\left(\frac{f}{B}+1\right)^{\frac12}} \frac{1}{2}\eta & f \in [-2b, -b] \\ \frac{K\alpha}{\left(\frac{f}{B}+1\right)^{\frac12}}\eta & f \in (-B, -2b] \end{cases}$$

$$|H_R(f)|^2 = \begin{cases} \frac{2K}{\eta\alpha\left(-\frac{f}{B}+1\right)^{\frac12}} & f \in [b, 2b] \\ \frac{K}{\alpha\left(-\frac{f}{B}+1\right)^{\frac12}} & f \in [2b, B) \\ \frac{2K}{\eta\alpha\left(\frac{f}{B}+1\right)^{\frac12}} & f \in [-2b, -b] \\ \frac{K}{\alpha\left(\frac{f}{B}+1\right)^{\frac12}} & f \in (-B, -2b] \end{cases}$$





3)

$$Y(t) = A - \frac{1}{2} X(t) \sin^2\left(7\pi f_0 t + \frac{\pi}{3}\right)$$

$$A \propto (\cdot, \mu_A, \sigma_A^2) = (\cdot, 1, 2)$$

$$P_{X_1}(f) = \Delta\left(\frac{f-f_0}{B}\right) + \Delta\left(\frac{f+f_0}{B}\right) \qquad R_X(\tau) = e^{2\pi j f_0 t} B\,\text{sinc}(Bt) + e^{-2\pi j f_0 t} B\,\text{sinc}(Bt) =$$

$$= 2B\,\text{sinc}(Bt)\cos(2\pi f_0 t)$$

$X(t)$ SSL

$$E[Y(t)] = E[A]$$

$$R_Y(t,\tau) = E[Y(t)Y(t-\tau)] = E\left[\left(A - \frac{1}{2}X(t)\sin^2\left(7\pi f_0 t + \frac{\pi}{3}\right)\right)\left(A - \frac{1}{2}X(t-\tau)\sin^2\left(7\pi f_0(t-\tau) + \frac{\pi}{3}\right)\right)\right] =$$

$$= E[A^2] + 0 + 0 + \frac{1}{4}R_X(\tau)\cdot\frac{1}{2}\left(1 - \cos\left(14\pi f_0 t + \frac{2}{3}\pi\right)\right)\frac{1}{2}\left(1 - \cos\left(14\pi f_0(t-\tau) + \frac{2}{3}\pi\right)\right) =$$

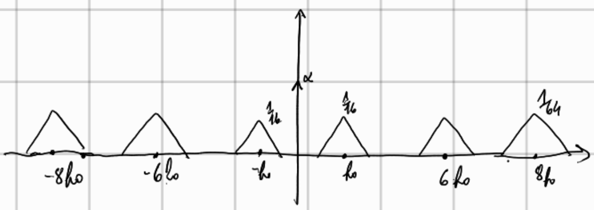
$$= E[A^2] + \frac{1}{16}R_X(\tau)\left(1 - \cos\left(14\pi f_0 t + \frac{2}{3}\pi\right) - \cos\left(14\pi f_0(t-\tau) + \frac{2}{3}\pi\right) + \cos\left(14\pi f_0 t + \frac{2}{3}\pi\right)\cos\left(14\pi f_0(t-\tau) + \frac{2}{3}\pi\right)\right) =$$

$$E[A^2] + \frac{1}{16}R_X(\tau)\left(1 - \cos\left(14\pi f_0 t + \frac{2}{3}\pi\right) - \cos\left(14\pi f_0(t-\tau) + \frac{2}{3}\pi\right) + \frac{1}{2}\cos\left(14\pi f_0(2t-\tau) + \frac{4}{3}\pi\right) + \frac{1}{2}\cos(14\pi f_0 \tau)\right)$$

Ciclostazionario. Medio su $\frac{1}{7f_0}$

$$\overline{R_Y}(\tau) = \underset{\alpha}{E[A^2]} + \frac{1}{16}R_X(\tau) + \frac{1}{32}R_X(\tau)\cos(14\pi f_0 \tau)$$

$$\overline{P_Y}(f) = \alpha\,\delta(f) + \frac{1}{16}P_X(f) + \frac{1}{64}P_X(f - 7f_0) + \frac{1}{64}P_X(f + 7f_0)$$



1)

$$Y(t) = X(t-2) + X(t)\sin^2(8\pi f_0 t) \qquad P_x(f) = \Lambda\left(\frac{f-2}{2}\right) + \Lambda\left(\frac{f+2}{2}\right) \quad (f_0 \gg 1)$$

$$E[Y(t)] = 0$$

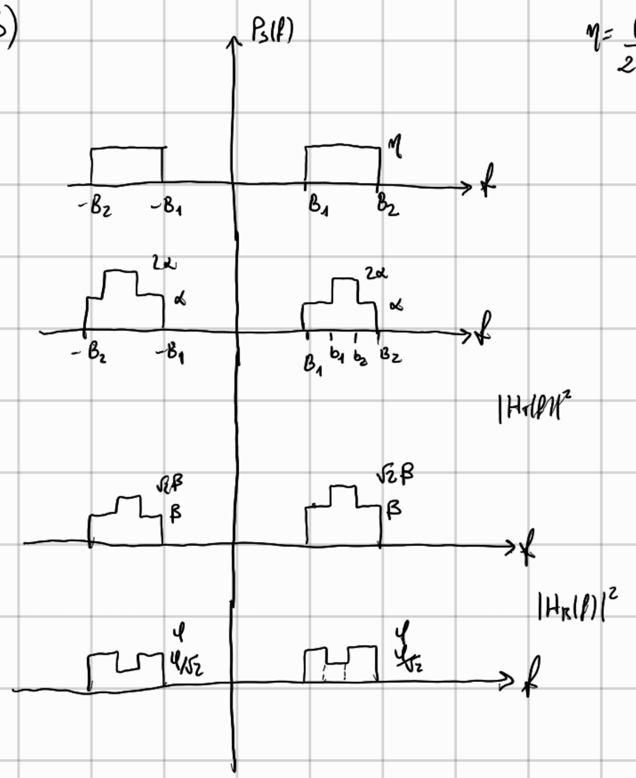
$$R_Y(t,\tau) = E\left[\left(X(t-2) + X(t)\sin^2(8\pi f_0 t)\right)\left(X(t-2-\tau) + X(t-\tau)\sin^2(8\pi f_0(t-\tau))\right)\right] =$$

$$R_x(\tau) + R_x(\tau-2)\sin^2(8\pi f_0(t-\tau)) + R_x(\tau+2)\sin^2(8\pi f_0 t) + R_x(\tau)\frac{1}{2}\left(1-\cos(16\pi f_0 t)\right)\left(1-\cos(16\pi f_0(t-\tau))\right) =$$

$$R_x(\tau) + \frac{R_x(\tau-2)}{2}\left(1-\cos(16\pi f_0(t-\tau))\right) + \frac{R_x(\tau+2)}{2} - \frac{R_x(\tau+2)}{2}\cos(16\pi f_0 t) + \frac{R_x(\tau)}{2} - \frac{R_x(\tau)}{2}\cos(16\pi f_0 t) - \frac{R_x(\tau)}{2}\cos(16\pi f_0(t-\tau)) + \frac{R_x(\tau)}{2}\cos(16\pi f_0 t)\cos(16\pi f_0(t-\tau)) =$$

$$= R_x(\tau) + \frac{R_x(\tau-2)}{2} - \frac{R_x(\tau-2)}{2}\cos(16\pi f_0(t-\tau)) + \frac{R_x(\tau+2)}{2} - \frac{R_x(\tau+2)}{2}\cos(16\pi f_0 t) + \frac{R_x(\tau)}{2} - \frac{R_x(\tau)}{2}\cos(16\pi f_0 t) - \frac{R_x(\tau)}{2}\cos(16\pi f_0(t-\tau)) + \frac{R_x(\tau)}{4}\cos(16\pi f_0(2t-\tau)) + \frac{R_x(\tau)}{4}\cos(16\pi f_0 \tau)$$

3)



$$\eta = \frac{P_S}{2(B_2-B_1)} \qquad P_N = \alpha(B_1-b_1) + \alpha(B_2-b_2) + 2\alpha(b_2-b_1)$$

$$|H_T(f)|^2 = Kb\left(\frac{P_m(f)}{P_S(f)}\right)^{\frac{1}{2}} = \begin{cases} Kb\left(\frac{\alpha}{\eta}\right)^{\frac{1}{2}} & |f| \in [B_1, b_1] \\ Kb\left(\frac{2\alpha}{\eta}\right)^{\frac{1}{2}} & |f| \in [b_1, b_2] \\ Kb\left(\frac{\alpha}{\eta}\right)^{\frac{1}{2}} & |f| \in [b_2, B_2] \end{cases}$$

$$|H_R(f)|^2 = \frac{b}{K}\cdot\left(\frac{P_S(f)}{P_m(f)}\right)^{\frac{1}{2}} = \begin{cases} \frac{b}{K}\left(\frac{\eta}{\alpha}\right)^{\frac{1}{2}} & |f| \in [B_1, b_1] \\ \frac{b}{K}\left(\frac{\eta}{2\alpha}\right)^{\frac{1}{2}} & |f| \in [b_1, b_2] \\ \frac{b}{K}\left(\frac{\eta}{\alpha}\right)^{\frac{1}{2}} & |f| \in [b_2, B_2] \end{cases}$$

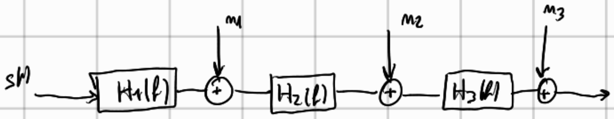
$$P_{SR}(f) = b^2 P_S(f)$$

$$P_{SR} = 2b^2\eta(B_2-B_1) = b^2 P_S$$

$$P_{mo}(f) = \begin{cases} \alpha\varphi & |f| \in [B_1, b_1] \\ 2\alpha\frac{\varphi}{\sqrt{2}} & |f| \in [b_1, b_2] \\ \alpha\varphi & |f| \in [b_2, B_2] \end{cases}$$

$$P_{mo} = 2\left(\alpha\varphi(b_1-B_1) + \alpha\varphi(b_2-B_2) + \alpha\varphi\sqrt{2}(b_2-b_1)\right)$$

3)



$$|H_1(f)|^2 = \Delta\left(\frac{f}{B}\right) = \begin{cases} -\frac{f}{B}+1 & f\in[0,B] \\ \frac{f}{B}+1 & f\in[-B,0] \end{cases}$$

$P_{n_i} = g_{n_i} V^2$

$\eta_i = \frac{P_{n_i}}{2B}$

$P_s(f) = 8 \cdot 10^{-4} = \eta_0 \quad f\in[-B,B]$

$P_{S_R}(f) = |H_1(f)|^6 P_S(f)$

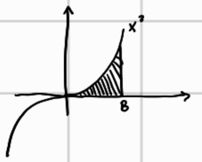
$$P_{S_R} = 2\int_0^B \left(-\frac{f}{B}+1\right)^3 \eta_0 \, df = 2\eta_0 \left(\frac{B \cdot 1}{4}\right) = \frac{\eta_0 B}{2}$$

$$\Rightarrow \quad \frac{P_{S_R}}{P_{N_R}} = \frac{\frac{\eta_0 B}{2}}{\frac{2}{3}\eta_i B + \eta_i B + 2\eta_i B} = \frac{\frac{\eta_0}{2}}{\frac{2+3+6}{3}\eta_i} = \frac{3}{22}\frac{\eta_0}{\eta_i} = 0.24 = -6.12\,dB$$

$$P_{n_{1R}}(f) = |H_1(f)|^4 P_{n_1}(f) \Rightarrow P_{n_{1R}} = 2\eta_i \left(\frac{B \cdot 1}{3}\right) = \frac{2}{3}\eta_i B$$

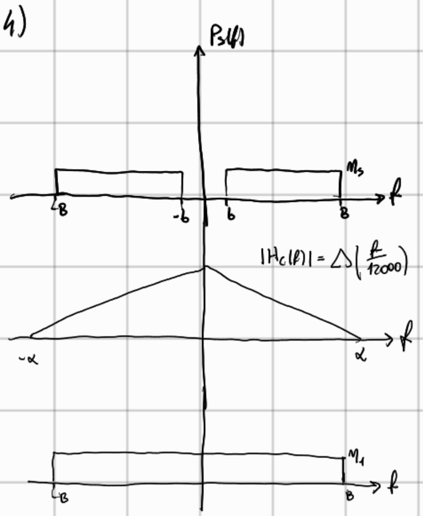
$$P_{n_{2R}}(f) = |H_1(f)|^2 P_{n_2}(f) \Rightarrow P_{n_{2R}} = 2\eta_i \left(\frac{B}{2}\right) = \eta_i B$$

$P_{n_{3R}} = 2B\eta_i$



$$A = \int_0^B x^3 dx = \frac{x^4}{4}\Big|_0^B = \frac{B^4}{4} = \frac{B^3 B}{4}$$

4)



$P_s^{RMS} = 10\,mV$

$\eta_s = \frac{P_s^{RMS\,2}}{2(B-b)} \cong 0.05$

$$|H_c(f)| = \begin{cases} -\frac{f}{\alpha}+1 & f\in[0,\alpha] \\ \frac{f}{\alpha}+1 & f\in[-\alpha,0] \end{cases}$$

$P_{n_1}^{RMS} = 5\,mV$ $\quad \eta_1 = \frac{P_{n_1}^{RMS\,2}}{2B}$

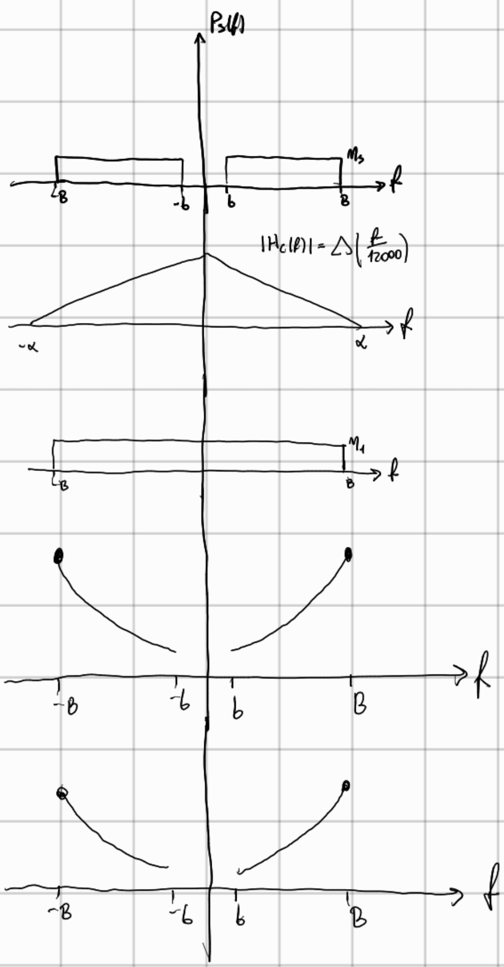
$P_y(f) = |H(f)|^2 P_x(f)$

$$\frac{f^2}{\alpha^2}+1-\frac{2f}{\alpha}$$

$$P_y = 2\eta_s\int_b^B \left(-\frac{f}{\alpha}+1\right)^2 df = 2\eta_s\cdot\left[\frac{f^3}{3\alpha^2}\Big|_b^B + (B-b) - \frac{f^2}{\alpha}\Big|_b^B\right] =$$

$$2\eta_s\left[+\frac{B^3-b^3}{3\alpha^2}+(B-b)-\frac{B^2-b^2}{\alpha}\right] = 38.8\,mV^2$$

$P_n = 25\,mV^2$ $\quad \frac{P_{s_0}}{P_N} = 1.55$



$$|H_c(f)| = \begin{cases} -\frac{f}{\alpha}+1 & f\in[0,\alpha] \\ \frac{f}{\alpha}+1 & f\in[-\alpha,0] \end{cases}$$

$$|H_R(f)|^2 = \frac{K\beta}{|H_c(f)|}\left(\frac{P_n(f)}{P_s(f)}\right)^{\frac{1}{2}} = \begin{cases} \frac{K\beta}{-\frac{f}{\alpha}+1}\left(\frac{\eta_1}{\eta_s}\right)^{\frac{1}{2}} & f\in[b,B] \\ \frac{K\beta}{\frac{f}{\alpha}+1}\left(\frac{\eta_1}{\eta_s}\right)^{\frac{1}{2}} & f\in[-B,-b] \end{cases}$$

$$|H_T(f)|^2 = \frac{K}{\beta\,|H_c(f)|}\left(\frac{P_s(f)}{P_n(f)}\right)^{\frac{1}{2}} = \begin{cases} \frac{K}{(-\frac{f}{\alpha}+1)\beta}\left(\frac{\eta_s}{\eta_1}\right)^{\frac{1}{2}} & f\in[b,B] \\ \frac{K}{\beta(\frac{f}{\alpha}+1)}\left(\frac{\eta_s}{\eta_1}\right)^{\frac{1}{2}} & f\in[-B,-b] \end{cases}$$

4)

$$R_{y_1y_2}(\tau) = E[y_1(t)\, y_2(t-\tau)] = E\left[\int_{-\infty}^{+\infty} h_1(\xi) X_1(t-\xi)\, d\xi \int_{-\infty}^{+\infty} h_2(\eta) X_2(t-\tau-\eta)\, d\eta\right] =$$

$$= E\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)\, h_2(\eta)\, X_1(t-\xi)\, X_2(t-\tau-\eta)\, d\xi\, d\eta\right] =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_1(\xi)\, h_2(\eta)\, R_{x_1x_2}(\tau-(\xi-\eta))\, d\xi\, d\eta \qquad \xi-\eta = y$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \underline{h_1(\xi)\, h_2(\xi-y)}\, R_{x_1x_2}(\tau-y)\, \underline{d\xi}\, dy =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} r_{h_1h_2}(y)\, R_{x_1x_2}(\tau-y)\, dy = (r_{h_1h_2} * R_{x_1x_2})(\tau)$$

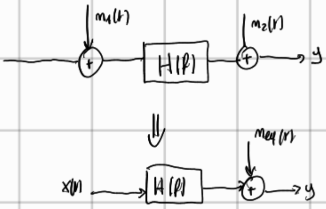
2) $R_x(\tau) = \Delta(\tau)$

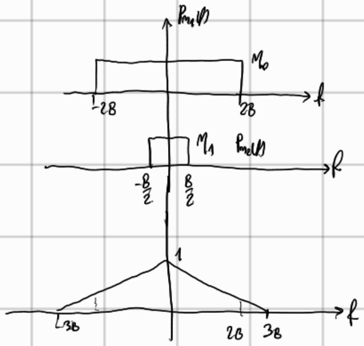
$h(t) = \delta(t-1) + \frac{1}{2}\delta(t-2) \Rightarrow R_{yx}(\tau) = \Delta(\tau-1) + \frac{1}{2}\Delta(\tau-2)$

$$R_{yx}(t,\tau) = E\left[\int_{-\infty}^{+\infty} h(\xi) X(t-\xi)\, d\xi \cdot X(t-\tau)\right] = \int_{-\infty}^{+\infty} h(\xi) R_x(\tau-\xi)\, d\xi = (R_x * h)(\tau)$$

2)

$P_X(f) = \Delta\left(\frac{f}{B}\right)$ $\quad P_{n_1}(f) = \eta_0 \quad f \in [-2B, 2B]$ $\quad |H_c(f)|^2 = \Delta\left(\frac{f}{3B}\right)$ $\quad P_{n_2}(f) = \eta_1 \quad f \in \left[-\frac{B}{2}, \frac{B}{2}\right]$





$n_{eq}(t) = n_2(t) + (h * n_1)(t)$ $\quad$ Se rumori sono scorrelati:

$$P_{n_{eq}}(f) = P_{n_2}(f) + |H(f)|^2 P_{n_1}(f) = \begin{cases} \eta_1 + \eta_0\left(-\frac{f}{3B} + 1\right) & f \in \left[0, \frac{B}{2}\right] \\ \eta_0\left(-\frac{f}{3B} + 1\right) & f \in \left(\frac{B}{2}, 2B\right] \\ \eta_1 + \eta_0\left(\frac{f}{3B} + 1\right) & f \in \left[-\frac{B}{2}, 0\right] \\ \eta_0\left(\frac{f}{3B} + 1\right) & f \in \left[-2B, -\frac{B}{2}\right] \end{cases}$$

$$P_{n_{eq}} = B\eta_1 + \eta_0 2 \cdot 2B\left(1 + \left(-\frac{2}{3} + 1\right)\right) \cdot \frac{1}{2} = B\eta_1 + 2\eta_0 B\left(\frac{4}{3}\right) = B\eta_1 + \frac{8}{3} B\eta_0$$

$$P_{S_R}(f) = |H(f)|^2 P_S(f) = \Delta\left(\frac{f}{B}\right) \Delta\left(\frac{f}{3B}\right)$$

$$P_{S_R} = 2\int_0^B \left(-\frac{f}{B} + 1\right)\left(-\frac{f}{3B} + 1\right) df = 2\int_0^B \left(\frac{f^2}{3B^2} - \frac{f}{B} - \frac{f}{3B} + 1\right) df = 2\left[\frac{f^3}{9B^2}\Big|_0^B - \frac{f^2}{2B}\Big|_0^B - \frac{f^2}{6B}\Big|_0^B + B\right] = 2\left[\frac{B}{9} - \frac{B}{2} - \frac{B}{6} + B\right] =$$

$$= 2B\left[\frac{2\cancel{B} - 9\cancel{B} - 3\cancel{B} + 18\cancel{B}}{18}\right] = \frac{16}{18}B = \frac{8}{9}B$$

$$\frac{P_{SR}}{P_N} = \frac{\frac{8}{9}B}{B\eta_1 + \frac{8}{3}B\eta_0} = \frac{\frac{8}{3}}{3\eta_1 + 8\eta_0} = \frac{8}{9\eta_1 + 24\eta_0}$$

$$P_S(f) = \begin{cases} -\frac{f}{B} + 1 & f \in [0, B] \\ \frac{f}{B} + 1 & f \in [-B, 0] \end{cases}$$

$$P_{n_{eq}}(f) = P_{n_2}(f) + |H(f)|^2 P_{n_1}(f) = \begin{cases} \eta_1 + \eta_0\left(-\frac{f}{3B} + 1\right) & f \in \left[0, \frac{B}{2}\right] \\ \eta_0\left(-\frac{f}{3B} + 1\right) & f \in \left(\frac{B}{2}, 2B\right] \\ \eta_1 + \eta_0\left(\frac{f}{3B} + 1\right) & f \in \left[-\frac{B}{2}, 0\right] \\ \eta_0\left(\frac{f}{3B} + 1\right) & f \in \left[-2B, -\frac{B}{2}\right] \end{cases}$$

$$|H_T(f)|^2 = \frac{K\alpha}{|H_c(f)|}\left(\frac{P_m(f)}{P_S(f)}\right)^{\frac{1}{2}} = \begin{cases} \dfrac{K\alpha}{\left(-\frac{f}{3B}+1\right)^{\frac{1}{2}}}\left(\dfrac{\eta_1+\eta_0\left(-\frac{f}{3B}+1\right)}{-\frac{f}{B}+1}\right)^{\frac{1}{2}} & f\in\left[0,\frac{B}{2}\right] \\[2ex] \dfrac{K\alpha}{\left(-\frac{f}{3B}+1\right)^{\frac{1}{2}}}\left(\dfrac{\eta_0\left(-\frac{f}{3B}+1\right)}{-\frac{f}{B}+1}\right)^{\frac{1}{2}} & f\in\left[\frac{B}{2},B\right] \\[2ex] \dfrac{K\alpha}{\left(\frac{f}{3B}+1\right)^{\frac{1}{2}}}\left(\dfrac{\eta_1+\eta_0\left(\frac{f}{3B}+1\right)}{\frac{f}{B}+1}\right)^{\frac{1}{2}} & f\in\left[-\frac{B}{2},0\right] \\[2ex] \dfrac{K\alpha}{\left(\frac{f}{3B}+1\right)^{\frac{1}{2}}}\left(\dfrac{\eta_0\left(f/3B+1\right)}{\frac{f}{B}+1}\right)^{\frac{1}{2}} & f\in\left[-B,-\frac{B}{2}\right] \end{cases}$$

$$|H_R(f)|^2 = \frac{\alpha}{K|H_c(f)|}\left(\frac{P_S(f)}{P_m(f)}\right)^{\frac{1}{2}} = \begin{cases} \dfrac{\alpha}{K\left(-\frac{f}{3B}+1\right)^{\frac{1}{2}}}\left(\dfrac{-\frac{f}{B}+1}{\eta_1+\eta_0\left(-\frac{f}{3B}+1\right)}\right)^{\frac{1}{2}} & f\in\left[0,\frac{B}{2}\right] \\[2ex] \dfrac{\alpha}{K\left(-\frac{f}{3B}+1\right)^{\frac{1}{2}}}\left(\dfrac{-\frac{f}{B}+1}{\eta_0\left(-\frac{f}{3B}+1\right)}\right)^{\frac{1}{2}} & f\in\left[\frac{B}{2},B\right] \\[2ex] \dfrac{\alpha}{K\left(\frac{f}{3B}+1\right)^{\frac{1}{2}}}\left(\dfrac{+\frac{f}{B}+1}{\eta_1+\eta_0\left(\frac{f}{3B}+1\right)}\right)^{\frac{1}{2}} & f\in\left[-\frac{B}{2},0\right] \\[2ex] \dfrac{\alpha}{K\left(\frac{f}{3B}+1\right)^{\frac{1}{2}}}\left(\dfrac{\frac{f}{B}+1}{\eta_0\left(\frac{f}{3B}+1\right)}\right)^{\frac{1}{2}} & f\in\left[-B,-\frac{B}{2}\right] \end{cases}$$