# Cr-48

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## 1 Introduction

The nucleus we are going to study is Cr-48. It has a half-life of 21.56 h [2] It has 24 protons and 24 neutrons: it is an even-even nucleus and the g.s. is $0^+$.

The ground state is expected to be with all the nucleons occupying the lowest possible orbits: 4 protons in the $f_{7/2}$ shell and 4 neutrons in the $f_{7/2}$ shell. Moreover the g.s. will be dominated by this configuration since the $f_{7/2}$ is isolated from the neighbouring shells.
The information on the spectrum is taken both from NNDC website and [1]. The spectrum presents 2 rotational bands: one with positive parity states, one with negative parity states:

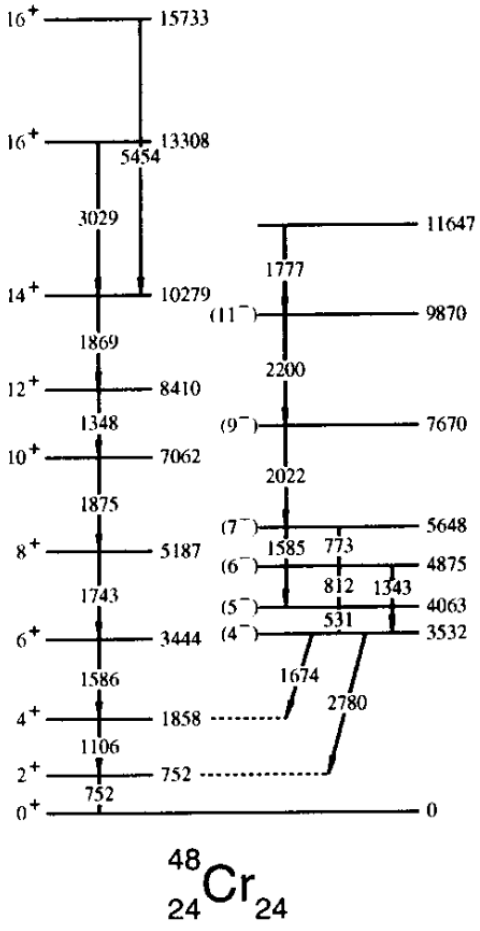


Figure 1: Spectrum of $^{48}Cr$ from [1]

The interaction I have used is KB3G (Kuo-Brown residual interaction) and the valence space is composed by the fp shell, as a consequence we will be able to describe only positive states since for negative states excitations from the core to higher states are required. We will focus on the positive band, which is the yrast band.
Because of Pauli principle the band termination will happen at a state with $(\frac{7}{2}+\frac{5}{2}+\frac{3}{2}+\frac{1}{2})\cdot 2 = 16$ As a consequence, we will look for the first $0^+, 2^+, ...16^+$ states.

Using KB3G interaction we found the following values reported in Table 1 and compared to experimental values found on NNDC.org:

| $N$ | $J^P$ | $E[MeV]$ | $E_X[keV]$ | $E_{sper}[keV]$ |
|---|---|---|---|---|
| 1 | $0^+$ | $-100.535$ | 0 | 0 |
| 2 | $2^+$ | $-99.761$ | 774 | 752.15(13) |
| 3 | $4^+$ | $-98.794$ | 1741 | 1858.4(22) |
| 4 | $6^+$ | $-97.281$ | 3253 | 3444.8(4) |
| 5 | $8^+$ | $-95.627$ | 4908 | 5188.4(5) |
| 6 | $10^+$ | $-93.834$ | 6701 | 7064(7) |
| 7 | $12^+$ | $-92.383$ | 8152 | 8411.9(8) |
| 8 | $14^+$ | $-90.32$ | 10215 | 10280.9(9) |
| 9 | $16^+$ | $-87.098$ | 13437 | 13310(9) |

Table 1: Comparison between experimental energy levels and the ones computed with kshell code

As one can see shell model describes very precisely this scheme.
In order to verify the rotational behaviour of this band we plot the energy of the levels computed with KB3G and the experimental values as a function of the angular momentum:

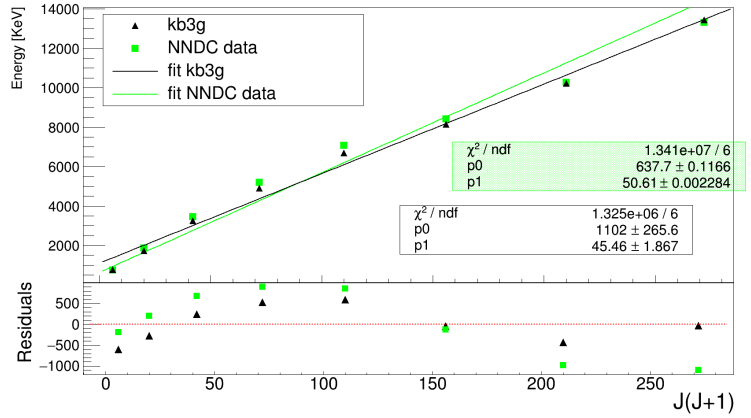


Figure 2: Energy of the levels of the yrast band vs J(J+1)

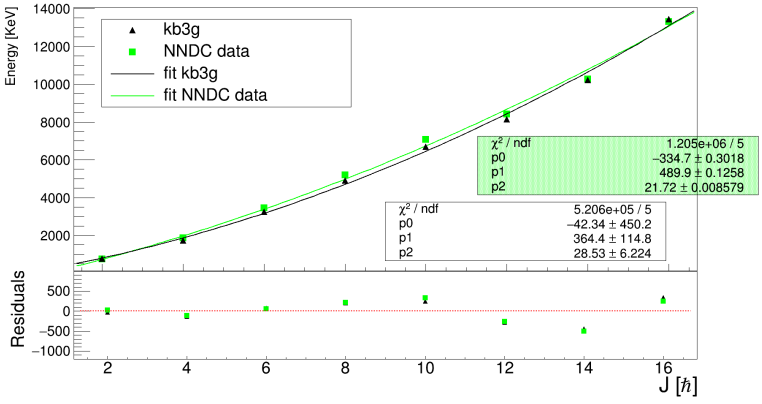


Figure 3: Energy of the levels of the yrast band vs J

As one can see from Figure 2 and Figure 3, the behaviour of this band is typical of rotational bands. However, going higher in spin the linear dependence in Figure 2 becomes less and less accurate: this is due to the fact that increasing spin, pairs are broken and in the end the shape is almost spherical. This will be confirmed by the analysis of the wavefunctions. The ratio $R = \frac{E(4^+)}{E(2^+)}$ gives $R_{\text{exp}} = 2.47$, that means it is an intermediate situation between pure vibration ($R = 2$) and pure rotation ($R = 3.3$)

### 1.1 Backbending

Considering the E2 transitions and plotting the spin of the initial state as a function of the energies of the gamma rays emitted in the transition, we find:

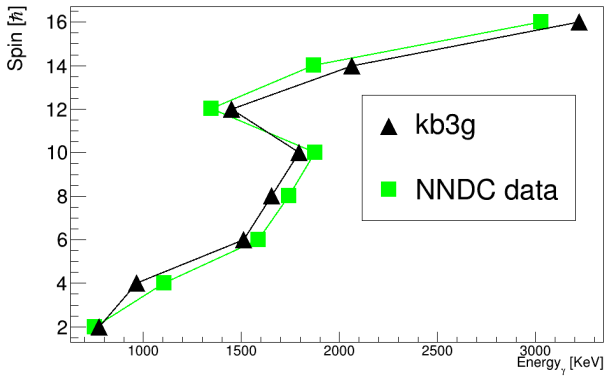


Figure 4: Spin vs energy of gamma energies

As one can see from Figure 4, the code is also able to correctly predict the backbending starting at $J = 10^+$.

## 2 Transition prob

Now we want to compare the values of the transition probabilities given by kshell code with the experimental values found at NNDC.org[1]. We used the following relation to link the B(E2) values from $e^2fm^4$ to $wu$ in order to compare the data.

$$B(E\lambda) = \frac{e^2}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 r_0^2 A^{2\lambda/3} \underset{A=48}{=} 10.36\,\mathrm{e^2fm^4} \tag{1}$$

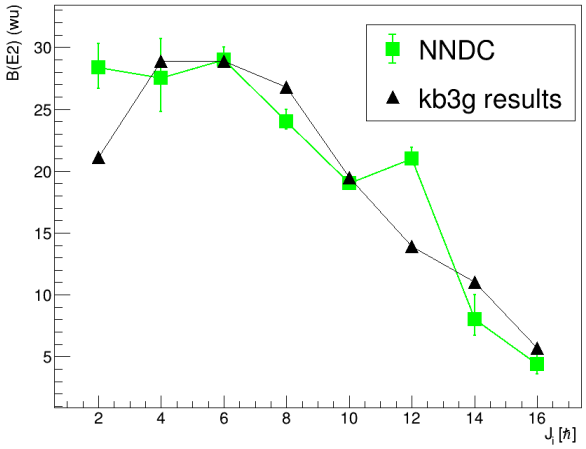


Figure 5: B(E2) vs J in the yrast band

| $J_i$ | $J_f$ | $B(E2)[e^2fm^4]$ | $B(E2)[wu]$ | $B(E2)_{sper}[wu]$ |
|---|---|---|---|---|
| $2^+$ | $0^+$ | 218.6 | 21.1 | $28.4\,(^{+19}_{-17})$ |
| $4^+$ | $2^+$ | 299.3 | 28.9 | $27.5\,(^{+32}_{-27})$ |
| $6^+$ | $4^+$ | 299.7 | 28.9 | $29\,(^{+10}_{-6})$ |
| $8^+$ | $6^+$ | 278.1 | 26.8 | $24\,(^{+10}_{-6})$ |
| $10^+$ | $8^+$ | 201.6 | 19.5 | $19\,(^{+7}_{-4})$ |
| $12^+$ | $10^+$ | 143.6 | 13.9 | $21\,(^{+9}_{-5})$ |
| $14^+$ | $12^+$ | 114.4 | 11.0 | $8.0\,(^{+20}_{-13})$ |
| $16^+$ | $14^+$ | 59.5 | 5.7 | $4.4\,(^{+11}_{-8})$ |

Table 2: Experimental and computed B(E2) values

As one can see from Figure 6, the computed values are very close to the experimeental ones. The B(E2) values are high at low spins, which is a clear sign of collectivity, but they decrease with increasing spin: the nucleus is rotating changing the moment of inertia. As the spin increases Coriolis antipariring is acting and it breaks pairs. Because of this phenomenon at the maximum spin, the shape is not prolate anymore but more spherical (all pairs broken). This will be clear also looking at the deformation parameter.

[1]The value of B(E2) found in NNDC for the B(E2,$12^+ \to 10^+$) is not line with the expected trend and with the results published in [1]

# 3 Quadrupole

From the BE(2) or from the spectroscopic quadrupole moment $Q_2$, one can get the intrinsic quadrupole moment $Q_0$ and thus the deformation parameter $\beta$ using the following relations:

$$Q_2 = Q_0 \frac{3K^2 - J(J+1)}{(2J+3)(J+1)} \tag{2}$$

$$Q_0 = \sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} ZeR_0^2\beta \tag{3}$$

| $J$ | $Q_2[efm^2]$ | $Q_0[efm^2]$ | $\beta$ |
|---|---|---|---|
| 2 | −28.88 | 101.09 | 0.29 |
| 4 | −38.47 | 105.80 | 0.31 |
| 6 | −38.98 | 97.45 | 0.28 |
| 8 | −38.84 | 92.25 | 0.27 |
| 10 | −23.71 | 54.54 | 0.16 |
| 12 | −5.80 | 13.04 | 0.04 |
| 14 | −6.26 | 13.86 | 0.04 |
| 16 | −7.51 | 16.42 | 0.05 |

These values of the deformation parameter are compatible with the ones found in [1]. From the sign of $\beta$ we can deduce a prolate deformation. The deformation parameter $\beta$ ranges from 0.28 near the ground state to 0.1 as the band termination at $J = 16^+$ is approached, when the valence nucleons have aligned to the maximum spin in the $1f_{7/2}$ shell.

# 4 Study of wavefunctions

In this last section we are going to have a look at the occupation numbers of the wavefunctions given by the kshell code. Of course, the occupation numbers for protons and neutrons will be the same, being an N=Z nucleus

| $J^\pi$ | $0f_{7/2}$ | $1p_{3/2}$ | $0f_{5/2}$ | $1p_{1/2}$ |
|---|---|---|---|---|
| $0^+$ | 3.331 | 0.302 | 0.295 | 0.071 |
| $2^+$ | 3.293 | 0.36 | 0.269 | 0.077 |
| $4^+$ | 3.32 | 0.352 | 0.251 | 0.077 |
| $6^+$ | 3.359 | 0.349 | 0.217 | 0.075 |
| $8^+$ | 3.434 | 0.31 | 0.19 | 0.067 |
| $10^+$ | 3.594 | 0.207 | 0.158 | 0.041 |
| $12^+$ | 3.734 | 0.102 | 0.141 | 0.023 |
| $14^+$ | 3.794 | 0.057 | 0.134 | 0.016 |
| $16^+$ | 3.91 | 0.026 | 0.058 | 0.006 |

Table 3: Experimental and computed B(E2) values

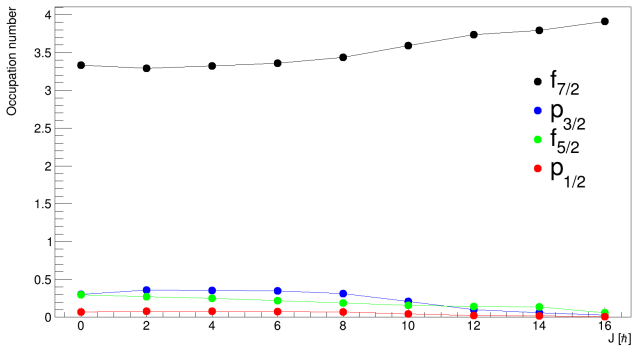


Figure 6: Occupation numbers vs J in the yrast band

As one can see from the Table 3, going higher in energy the shells with lowest $l$ give less contribution to the total wavefunction since it will be more convenient to occupy orbitals with higher $l$. The dominant term is the $f_{\frac{7}{2}}$. This is in line with our expectations since the energy gap between the shell immediately above is large.

# References


[1] F. Brandolini, S.M. Lenzi, D.R. Napoli, R.V. Ribas, H. Somacal, C.A. Ur, D. Bazzacco, J.A. Cameron, G. de Angelis, M. De Poli, C. Fahlander, A. Gadea, S. Lunardi, G. Martínez-Pinedo, N.H. Medina, C. Rossi Alvarez, J. Sánchez-Solano, and C.E. Svensson. Precise dsam lifetime measurements in 48cr and 50cr as a test of large scale shell model calculations. *Nuclear Physics A*, 642(3):387–406, 1998.

[2] Jun Chen. Nuclear data sheets for a=48. *Nuclear Data Sheets*, 179:1–382, 2022.