POLITECNICO
MILANO 1863

Spacecraft Attitude Dynamics

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SRP and magnetic disturbance torques

Task 1: Implement approximate orbit models

Define your spacecraft orbit with respect to the Earth in the body frame

$$r_N = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} \cos\theta \\ \sin\theta\cos i \\ \sin\theta\sin i \end{bmatrix} \qquad R = \frac{a(1-e^2)}{1+e\cos\theta} \qquad \dot{\theta} = \frac{n(1+e\cos\theta)^2}{(1-e^2)^{3/2}},$$

$$n - Earth$$

$$\hat{r}_B = A_{B/N}\hat{r}_N$$

Define the Sun direction with respect to the spacecraft in the body frame:

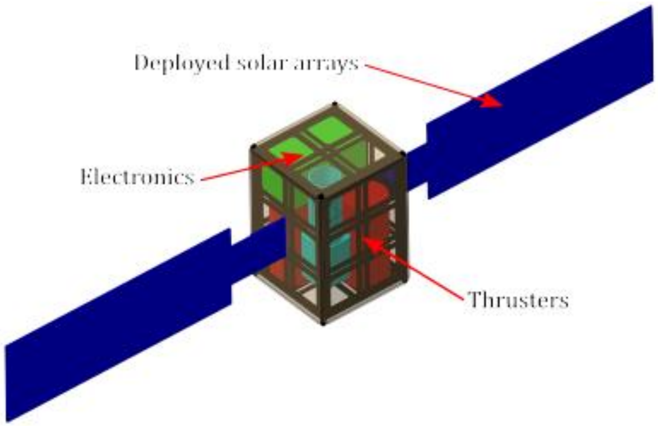
$$\underline{S}_N = R_S \begin{bmatrix} \cos n^{sun}\, t \\ \sin n^{sun}\, t\cos\varepsilon \\ \sin n^{sun}\, t\sin\varepsilon \end{bmatrix}$$

Sun direction with respect to the spacecraft can be approximated by the Sun direction with respect to the Earth since $R_S \gg R$

$$n^{sun} = 2\pi/T, T = 1year, \varepsilon = 23.45°$$

$$\hat{S}_B = A_{B/N}\hat{S}_N$$

Task 2: model the geometry of the spacecraft, including solar panels and deployed elements



$$J_{depl} = \begin{bmatrix} 100.9 & 0 & 0 \\ 0 & 25.1 & 0 \\ 0 & 0 & 91.6 \end{bmatrix} \cdot 10^{-2}[Kgm^2].$$

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Define the orientation of each outer panel

Spacecraft main body
n1=[1;0;0]; n4=-n1;
n2=[0;1;0]; n5=-n2;
n3=[0;0;1]; n6=-n3;

Solar panels
n7=[1;0;0]; n8=-n7;
n9=[1;0;0]; n10=-n9;
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Task 3: Add the SRP disturbance torque

$$P = \frac{F_e}{c} \qquad F_e = 1358\ W/m\text{^}2$$

$$\bar{F}_i = -PA_i(\hat{S}_B \cdot \widehat{N}_{Bi})\left[(1-\rho_s)\hat{S}_B + (2\rho_s(\hat{S}_B \cdot \widehat{N}_{Bi}) + \frac{2}{3}\rho_d)\widehat{N}_{Bi}\right]$$

Normal to the surface in the body frame $\widehat{N}_{Bi}$

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Spacecraft main body

n1=[1;0;0]; n4=-n1;
n2=[0;1;0]; n5=-n2;
n3=[0;0;1]; n6=-n3;
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$$\rho_s = 0.5, \rho_d = 0.1$$

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Solar panels

n7=[1;0;0]; n8=-n7;
n9=[1;0;0]; n10=-n9;
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$$\rho_s = 0.1, \rho_d = 0.1$$

$$T_{SRP} = \begin{cases} \sum_{i=1}^{n} \bar{r}_i \times \bar{F}_i & \hat{S}_B \cdot \widehat{N}_{Bi} > 0 \\ 0 & \hat{S}_B \cdot \widehat{N}_{Bi} < 0 \end{cases}$$

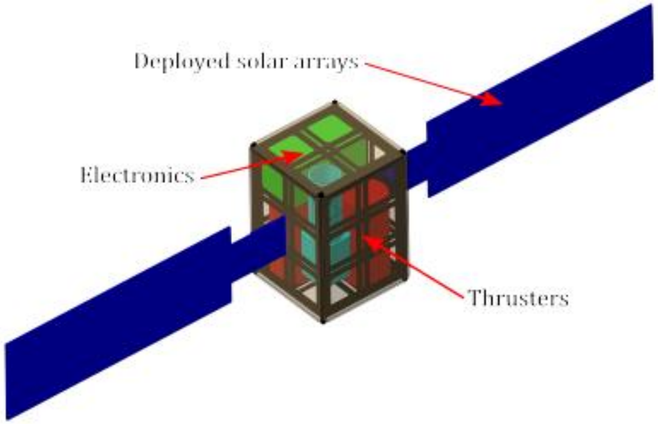
Can assume the CoM displaced from geometric centre

Task 4: Update the gravity gradient torque with your new elliptic orbit for the defined spacecraft geometry

$$I_x\dot{\omega}_x + \left(I_z - I_y\right)\omega_z\omega_y = \frac{3Gm_t}{R^3}\left(I_z - I_y\right)c_3c_2$$
$$I_y\dot{\omega}_y + \left(I_x - I_z\right)\omega_x\omega_z = \frac{3Gm_t}{R^3}\left(I_x - I_z\right)c_1c_3$$
$$I_z\dot{\omega}_z + \left(I_y - I_x\right)\omega_y\omega_x = \frac{3Gm_t}{R^3}\left(I_y - I_x\right)c_2c_1$$

$$J_{depl} = \begin{bmatrix} 100.9 & 0 & 0 \\ 0 & 25.1 & 0 \\ 0 & 0 & 91.6 \end{bmatrix} \cdot 10^{-2} [Kgm^2].$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = A_{B/L} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$



$$A_{L/N} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix}$$

Task 5: Simple magnetic disturbance model

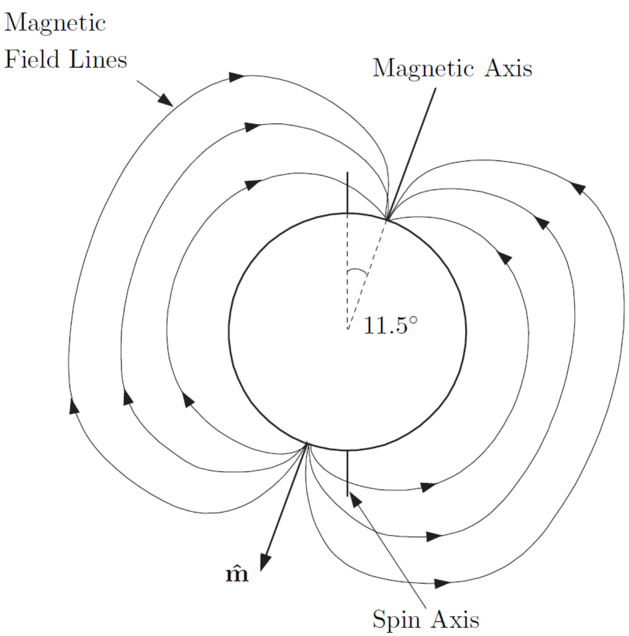
1. Compute magnetic field in the inertial frame

$$\underline{b}_N = \frac{R^3 H_0}{r^3}\left[3(\underline{\hat{m}} \cdot \underline{\hat{r}})\underline{\hat{r}} - \underline{\hat{m}}\right]$$

$$H_0 = \left((g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2\right)^{1/2}$$

$$\hat{m} = \begin{bmatrix} \sin 11.5^\circ \cos \omega_\oplus t \\ \sin 11.5^\circ \sin \omega_\oplus t \\ \cos 11.5^\circ \end{bmatrix}$$

		IGRF 1995		IGRF 2000	
n	m	g_n^m	h_n^m	g_n^m	h_n^m
1	0	-29682	-	-29615	-
1	1	-1789	5318	-1728	5186
2	0	-2197	-	-2267	-
2	1	3074	-2356	3072	-2478
2	2	1685	-425	1672	-458
3	0	1329	-	1341	-
3	1	-2268	-263	-2290	-227
3	2	1249	302	1253	296
3	3	769	-406	715	-492
4	0	941	-	935	-
4	1	782	262	787	272
4	2	291	-232	251	-232
4	3	-421	98	-405	119
4	4	116	-301	110	-304



2. Compute magnetic field in the body fixed frame

$$\underline{b}_B = A_{B/N}\underline{b}_N$$

3. Compute the parasitic magnetic torque

$$\underline{M} = \underline{m} \times \underline{b} \qquad \underline{m} = [0.01 \quad 0.05 \quad 0.01]^T Am^2$$

Note IGRF data is in nT