Tarea 5 - Métodos numéricos
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Problema 1

A lighthouse L is located on a small island 5 km north of a point A on a straight east-west shoreline. A cable is to be laid from L to point B on the shoreline 10 km east of A. The cable will be laid through the water in a straight line from L to a point C on the shoreline between A and B, and from there to B along the shoreline. (see Figure 1). The part of the cable lying in the water costs $5.000/km, and the part along the shoreline costs $3, 000/km. Where should C be chosen to minimize the total cost of the cable?

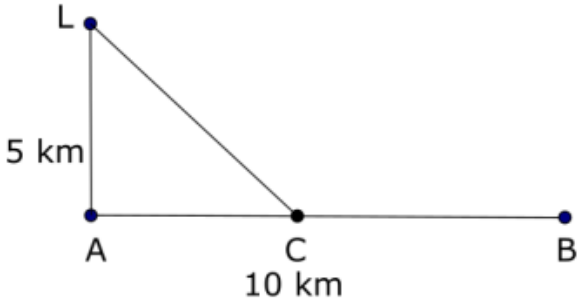

Figura 1: Problema 1

Llamnado x al segmento de recta $\overline{AC}$, y a $\overline{LC}$, entonces el segmento $\overline{CB}$ puede ser calculado como $10 - x$. Las definiciones antes mencionadas se encuentran en la figura 2.

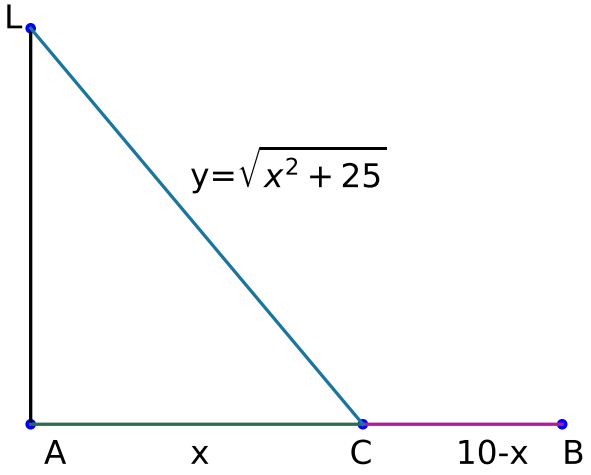

Figura 2: Representación de las definiciones de cada segmento de linea.

Definiendo la función de costo de cada linea se obtiene la función 1.

$$C(x) = 3000(10 - x) + 5000\sqrt{x^2 + 25} \tag{1}$$

Realizando la derivada con respecto a x de le función 1 para encontrar los valores críticos.

$$\frac{dC(x)}{dx} = -3000 + \frac{5000(x)}{\sqrt{x^2+25}}$$

Encontrando los valores criticos se obtiene lo siguiente:

$$\begin{aligned} -3000 + \frac{5000(x)}{\sqrt{x^2+25}} &= 0 \\ \frac{5000(x)}{\sqrt{x^2+25}} &= 3000 \\ 5x &= 3\sqrt{x^2+25} \\ 25x^2 &= 9(x^2+25) \\ 13x^2 - 225 &= 0 \\ (\sqrt{13}x - 15)(\sqrt{13}x + 15) &= 0 \\ x_1 &= 15/\sqrt{13} \\ x_2 &= -15/\sqrt{13} \end{aligned}$$

La solución x_2 es despreciada, ya que su sentido físico no admisible, por lo tanto el punto C debe estar a 4.16km del punto A.

Problema 2

Implement the following algorithms: Bisection, Newton, and Secant meth- ods for optimization in 1D.

Problmea 3

Problema 3a

Find the minimum value and minimum point of the function 2 on the interval [-1, 1] using the previous implemented algorithms. Compare the results in terms of number of iterations.

$$f(x) = -sin(x) + x^2 + 1 \tag{2}$$

Problema 3b

Compare and comment the results obtained for each algorithm on the interval [-1, 1] with function 3.

$$f(x) = sin(x) - x^2 + 1 \tag{3}$$