# HOMEWORK 1. NUMERICAL METHOD

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(1) Find intervals containing solutions to the following equations.
  **a:** $x - 2^{-x} = 0$
  **b:** $2x\cos(2x) - (x+1)^2 = 0$

(2) Find

$$\max_{a\le x\le b} f(x)$$
$$\min_{a\le x\le b} f(x)$$
$$\max_{a\le x\le b} |f(x)|$$

for the following functions and intervals.
  **a:** $f(x) = (2 - e^x + 2x)/3,\ [0,1]$
  **b:** $f(x) = (4x-3)/(x^2-2x),\ [0.5, 1.5]$

(3) Suppose $f \in C[a,b]$ and $x_1$ and $x_2$ are in $[a,b]$. Show that a number $\xi$ exists between $x_1$ and $x_2$ with

$$f(\xi) = \frac{f(x_1)+f(x_2)}{2}$$

(4) Assume you have $n$ values $x_i$.
  (a) Evaluate the sample mean

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

  (b) Evaluate the sample variance
    (i) You can evaluate the sample variance using a **two-pass algorithm**, ie

$$\mathrm{Var}(x) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

In this case, you need to evaluate the mean first, so you have to loop through the $x_i$ once to get the mean and a second time time to get the sample variance.

    (ii) You can also evaluate the sample variance using the **one-pass algorithm**, ie

$$\mathrm{Var}(x) = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right) - \overline{x}^2$$

In this case, you can compute the sums of the $x_i$ and the $x_i^2$ values at the same time and then perform only one subtraction at the end.

Write two functions, one for each algorithm, and test them on the two cases below:

$$\begin{aligned} x_i &\in \{0, 0.01, 0.02, ..., 0.09\} \\ x_i &\in \{123456789.0, 123456789.01, ..., 123456789.09\} \end{aligned}$$