OPTIMIZATION. HOMEWORK 4

OSCAR DALMAU

Comments:

- Please, follow the general guidelines provided at the beginning of the course.
- In this homework, we expect the student to draw conclusions about the experimental performance of the algorithms. So, feel free to include all the analysis tools learned in statistics.

List of problems:

(1) Implement the steepest descent algorithm with fixed step size $\alpha > 0$ (for example: $\alpha = 0.001$, α is a parameter of the algorithm) and Newton's algorithm

(2) Obtain the minimum of the following functions using the previous algorithms with the starting point $\boldsymbol{x}^0$ provided below. Additionally, run or execute the algorithm for a randomly selected starting point $\boldsymbol{x}^0$. Plot (k, f_k) and $(k, \|\boldsymbol{g}_k\|)$ for each function.

- Rosembrock function, for $n = 2$ and $n = 100$

$$\begin{aligned}
f(\boldsymbol{x}) &= \sum_{i=1}^{n-1} \left[100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2\right] \\
\boldsymbol{x}^0 &= [-1.2, 1, 1, \cdots, 1, -1.2, 1]^T \\
\boldsymbol{x}^* &= [1, 1, \cdots, 1, 1]^T \\
f(\boldsymbol{x}^*) &= 0
\end{aligned}$$

- Wood function

$$\begin{aligned}
f(\boldsymbol{x}) &= 100(x_1^2 - x_2)^2 + (x_1 - 1)^2 + (x_3 - 1)^2 + 90(x_3^2 - x_4)^2 \\
&\quad 10.1[(x_2 - 1)^2 + (x_4 - 1)^2] + 19.8(x_2 - 1)(x_4 - 1) \\
\boldsymbol{x}^0 &= [-3, -1, -3, -1]^T \\
\boldsymbol{x}^* &= [1, 1, 1, 1]^T \\
f(\boldsymbol{x}^*) &= 0
\end{aligned}$$

(3) Apply the steepest descent algorithm in item (1) to obtain the minimum of $f(\boldsymbol{x})$ for $\eta \sim \mathcal{N}(0, \sigma)$ and $\lambda, \sigma > 0$. Plot (t_i, y_i) and $(t_i, x_i^*(\lambda))$ in the same figure.

$$
\begin{aligned}
f(\boldsymbol{x}) &= \sum_{i=1}^{n} (x_i - y_i)^2 + \lambda \sum_{i=1}^{n-1} (x_{i+1} - x_i)^2 \\
y_i &= t_i^2 + \eta, \; t_i = \frac{2}{n-1}(i-1) - 1, \; i = 1, 2, \cdots, n.
\end{aligned}
$$

consider the following cases $\lambda \in \{1, 10, 1000\}$ with $n = 128$.