Tarea 03 - Optimización

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Problema 01

Is the set $S = \{a \in \mathbb{R}^K | p(0) = 1, |p(t)| \leq 1 for t \in [\alpha, \beta]\}$ **where** $p(t) = a_1 + a_2 t + \cdots + a_k t^{k-1}$ **convex?**

Problema 02

Suppose f **is convex,** $\lambda_1 > 0$ **and** $\lambda_2 \leq 0$ **with** $\lambda_1 + \lambda_2 = 1$**, and let** $x_1, x_2 \in$ **dom** f**. Show that the inequality**

$$f(\lambda_1 x_1 + \lambda_2 x_2) \geq \lambda_1 f(x_1) + \lambda_2 f(x_2)$$

always holds.

Problema 03

Show that the following function $f : \mathbb{R}^n \to \mathbb{R}$ **is convex.**

$$f(x) = -exp(-g(x))$$

where $g : \mathbb{R}^n \to \mathbb{R}$ **has convex domain and satisfaces**

$$\begin{bmatrix} \nabla^2 g(x) & \nabla g(x) \\ \nabla^T g(x) & 1 \end{bmatrix} \succeq 0$$

for $x \in$ **dom** g

Problema 04

Show that $f(x, y) = x^2/y, y > 0$ **is convex.**

Problema 05

Find all the values of the parameter a such that $[1, 0]^T$ **is the minimizer or maximizer of the function.**

$$f(x_1, x_2) = a^3 x_1 e^{x_2} + 2a^2 log(x_1 + x_2) - (a + 2)x_1 + 8ax_2 + 16x_1 x_2$$

Problema 06

Consider the sequence $x_k = 1+1/k!, k = 0, 1, \ldots$. Does this sequence converge linearly to 1? Justify your response.

Problema 07

Show that

$$f(x) = log\left(\sum_{i=1}^{n} exp(x_i)\right)$$

is convex

Problema 08

Show that

$$f(x) = log\left(\sum_{i=1}^{n} exp(g_i(x_i))\right)$$

is convex if $g_i : \mathbb{R} \to \mathbb{R}$ are convex.

Problema 09

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a differentiable function. Show that if f is convex over a nonempty convex set C then

$$(\nabla f(x) - \nabla f(y))^T (x - y) \geq 0, \qquad \forall x, y \in C$$