

OPTIMIZATION. HOMEWORK 3

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- (1) Is the set $S = \{a \in \mathbb{R}^k \mid p(0) = 1, |p(t)| \leq 1 \text{ for } t \in [\alpha, \beta]\}$, where

$$p(t) = a_1 + a_2 t + \cdots + a_k t^{k-1},$$

convex?

- (2) Suppose f is convex, $\lambda_1 > 0$ and $\lambda_2 \leq 0$ with $\lambda_1 + \lambda_2 = 1$, and let $x_1, x_2 \in \text{dom } f$. Show that the inequality

$$f(\lambda_1 x_1 + \lambda_2 x_2) \geq \lambda_1 f(x_1) + \lambda_2 f(x_2)$$

always holds.

- (3) Show that the following function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is convex.

$$f(x) = -\exp(-g(x))$$

where $g : \mathbb{R}^n \rightarrow \mathbb{R}$ has a convex domain and satisfies

$$\begin{bmatrix} \nabla^2 g(x) & \nabla g(x) \\ \nabla^T g(x) & 1 \end{bmatrix} \succeq 0$$

for $x \in \text{dom } g$.

- (4) Show that $f(x, y) = x^2/y$, $y > 0$ is convex.
 (5) Find all the values of the parameter a such that $[1, 0]^T$ is the minimizer or maximizer of the function

$$f(x_1, x_2) = a^3 x_1 e^{x_2} + 2a^2 \log(x_1 + x_2) - (a + 2)x_1 + 8ax_2 + 16x_1 x_2.$$

- (6) Consider the sequence $x_k = 1 + 1/k!$, $k = 0, 1, \dots$. Does this sequence converge linearly to 1? Justify your response.
 (7) Show that $f(\mathbf{x}) = \log \sum_{i=1}^n \exp(x_i)$ is convex.
 (8) Show that $f(x) = \log \sum_{i=1}^n \exp(g_i(x)) : \mathbb{R} \rightarrow \mathbb{R}$ is convex if $g_i : \mathbb{R} \rightarrow \mathbb{R}$ are convex
 (9) Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a differentiable function. Show that if f is convex over a nonempty convex set C then

$$(\nabla f(x) - \nabla f(y))^T (x - y) \geq 0, \forall x, y \in C$$