Invariancia de Norma (Gauge)

Las ecuaciones de Maxwell son invariantes ante la transformación

$$\boxed{A_\mu' = A_\mu + \partial_\mu \chi(x)} \quad (1)$$

$\chi(x)$ = función escalar

¿Qué consecuencias tiene en la ec. de K.G?

$$g^{\mu\nu}\left[i\hbar\partial_\nu - \frac{e}{c}A_\nu\right]\left[i\hbar\partial_\mu - \frac{e}{c}A_\mu\right]\Psi =$$

$$= m_0^2 c^2 \Psi$$

Si aplicamos la ec. (1)

$$\boxed{g^{\mu\nu}\left[i\hbar\partial_\nu - \frac{e}{c}A'_\nu\right]\left[i\hbar\partial_\mu - \frac{e}{c}A'_\mu\right]\Psi = m_0^2c^2\Psi}$$

usando ec. 1

$$g^{\mu\nu}\left[i\hbar\partial_\nu - \frac{e}{c}A_\nu - \frac{e}{c}\partial_\nu\chi\right]\left[i\hbar\partial_\mu - \frac{e}{c}A_\mu - \frac{e}{c}\partial_\mu\chi\right]\Psi = m_0^2c^2\Psi$$

Veamos $\left[i\hbar\partial_\mu - \frac{e}{c}A_\mu - \frac{e}{c}\partial_\mu\chi\right]\Psi =$

$$i\hbar\partial_\mu\Psi - \frac{e}{c}A_\mu\Psi - \frac{e}{c}(\partial_\mu\chi)\Psi =$$

Si $\Psi' = e^{\left\{\frac{ie}{\hbar c}\chi\right\}}\Psi$

$$i\hbar\, e^{-\frac{ie}{\hbar c}\chi}\, e^{\frac{ie}{\hbar c}\chi}\,\partial_\mu\Psi - \frac{e}{c}A_\mu\, e^{-\frac{ie}{\hbar c}\chi} e^{\frac{ie}{\hbar}\chi}\Psi$$

$$- e^{-\frac{ie}{\hbar c}\chi}(\partial_\mu\chi)\, e^{\frac{ie}{\hbar c}\chi}\Psi$$

$$\Rightarrow e^{-\frac{iex}{\hbar c}}\Big[\underline{i\hbar e^{\frac{ie}{\hbar c}x}\partial_\mu \psi} - \frac{e}{c}A_\mu \psi'$$

$$\underline{- \frac{\cancel{e}}{\cancel{c}}\,\frac{\hbar \cancel{c}}{i\cancel{e}}\left[\frac{ie}{\hbar c}\partial_\mu x\right] \underbrace{e^{\frac{iex}{\hbar c}}\psi}}\Big]$$

$$= e^{-\frac{iex}{\hbar c}}\Big[\underline{i\hbar e^{iex/\hbar c}\partial_\mu \psi} + i\hbar\left[\psi' \partial_\mu\left[\frac{iex}{\hbar c}\right]\right]$$

$$- \frac{e}{c}A_\mu \psi'\Big]$$

$$= e^{-\frac{iex}{\hbar c}}\left[i\hbar\,\partial_\mu\left[e^{iex/\hbar c}\psi\right] - \frac{e}{c}A_\mu \psi'\right]$$

$$\Rightarrow e^{-\frac{iex}{\hbar c}}\left[i\hbar\,\partial_\mu \psi' - \frac{e}{c}A_\mu \psi'\right]$$

Aplicamos segundo operador

$$\left(i\hbar\partial_\nu - \frac{e}{c}A_\nu - \frac{e}{c}\partial_\nu x\right)\left[e^{\frac{-iex}{\hbar c}}\left[i\hbar\partial_\mu\psi' - \frac{e}{c}A_\mu\psi'\right]\right]$$

$$\left[i\hbar\partial_\nu - \frac{e}{c}A_\nu - \frac{\cancel{e}}{\cancel{c}}\frac{\hbar\cancel{c}}{i\cancel{e}}\left[\frac{ie}{\hbar c}\partial_\nu x\right]\right]\left[\ldots\ldots\right]$$

$$= i\hbar\left\{\partial_\nu\left[e^{-iex/\hbar c}\right]\right\}\left[i\hbar\partial_\mu\psi' - \frac{e}{c}A_\mu\psi'\right]$$

$$+ i\hbar\, e^{-iex/\hbar c}\,\partial_\nu\left[i\hbar\partial_\mu\psi' - \frac{e}{c}\partial_\mu\psi'\right]$$

$$-\frac{e}{c}A_\nu e^{\frac{-iex}{\hbar c}}\left[i\hbar\partial_\mu\psi' - \frac{e}{c}A_\mu\psi'\right]$$

$$\cancel{+}i\hbar\left(\partial_\nu\left(\frac{ie}{\hbar c}x\right)\right)e^{-iex/\hbar c}\left[i\hbar\partial_\mu\psi' - \frac{e}{c}A_\mu\psi'\right]$$

$$= i\hbar\, e^{-iex/\hbar c}\,\partial_\nu\left[\frac{-ie}{\hbar c}x\right]\left[i\hbar\partial_\mu\psi' - \frac{e}{c}A_\mu\psi'\right]$$

$$+ e^{-iex/\hbar c}\, i\hbar\,\partial_\nu\left[i\hbar\partial_\mu\psi' - \frac{e}{c}A_\mu\psi'\right] + \ldots$$

$$-\frac{e}{c} A_\nu\, e^{-ie\chi/\hbar c}\left[i\hbar\,\partial_\mu \psi' - e/c\, A_\mu \psi'\right]$$

$$-\frac{e}{c}(\partial_\nu \chi)\, e^{-ie\chi/\hbar c}\left[i\hbar\,\partial_\mu \psi' - \frac{e}{c} A_\mu \psi'\right]$$

$$= e^{-ie\chi/\hbar c}\Big[i\hbar\,\partial_\nu\left(i\hbar\,\partial_\mu \psi' - e/c\, A_\mu \psi'\right)$$

$$-\frac{e}{c} A_\nu\left[i\hbar\,\partial_\mu \psi' - e/c\, A_\mu \psi'\right]\Big]$$

$$= e^{-ie\chi/\hbar c}\left[\left(i\hbar\,\partial_\nu - \frac{e}{c} A_\nu\right)\left(i\hbar\,\partial_\mu \psi' - \frac{e}{c} A_\mu \psi'\right)\right]$$

Regresando a la ec. de K. G.

$$\underline{e^{-ie\chi/\hbar c}}\; g^{\mu\nu}\left[\left(i\hbar\,\partial_\nu - \frac{e}{c} A_\nu\right)\left(i\hbar\,\partial_\mu - \frac{e}{c} A_\mu\right)\psi'\right] =$$

$$= m_0^2 c^2 \psi$$

$$\boxed{g^{\mu\nu}\left[\left(i\hbar\,\partial_\nu - \frac{e}{c} A_\nu\right)\left(i\hbar\,\partial_\mu - \frac{e}{c} A_\mu\right)\psi'\right] = m_0^2 c^2 \psi'}$$

donde $\boxed{\psi' = \exp\left[\frac{ie}{\hbar c}\chi\right]\psi}$

Las observables son funciones de $\psi'\psi'^*$

Se dice que la ec. de K.G es gauge invariant para el acoplamiento mínimo

$$\left(P_\nu - \frac{e}{c}A'_\nu\right)\psi\exp\left[\frac{ie}{\hbar c}\chi\right] =$$

$$= \left[P_\nu - \frac{e}{c}A_\nu - \frac{e}{c}\partial\chi\right]\psi\exp\left[\frac{ie}{\hbar c}\chi\right] =$$

$$= \left[P_\nu - \frac{e}{c}A_\nu\right]\psi\exp\left[\frac{ie}{\hbar c}\chi\right]$$

Problemas propuestos

1. Sea $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ ✓

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

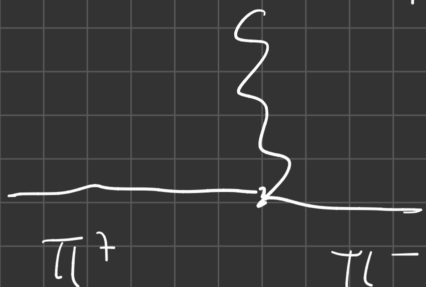
a) Determinar las ecuaciones que satisface el campo A_ν (Ec. de Euler Lagrange).

b) Determinar el tensor $T^\mu{}_\nu$ $(T^{\mu\nu})$

2.- $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

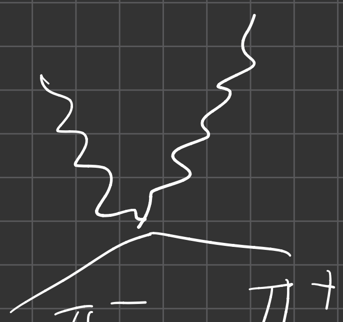
$$+ \frac{1}{2m_0} \left[\left(i\hbar \partial_\mu - \frac{e}{c} A_\mu\right) \psi^* \times \left(-i\hbar \partial^\mu - \frac{e}{c} A^\mu\right) \psi \right]$$

$$- m_0^2 c^2 \psi^* \psi$$



$\pi^+ A_\mu \pi^-$

$\pi^- \pi^+ A_\mu A_\nu$



a) Usando ec. de Euler-Lagrange para campos, determinar que ψ satisface

$$\left(P^{\mu} - \frac{e}{c} A^{\mu}\right)\left(P_{\mu} - \frac{e}{c} A_{\mu}\right)\psi = m_0^2 c^2 \psi$$

b) Mostrar que la ecuación para A_{μ}

$$\partial^{\mu} F_{\mu\nu} = J_{\nu} = \frac{ie\hbar}{2m_0}\left[\begin{array}{l}\psi^{*}\left[\partial_{\nu} + \frac{ie}{\hbar c} A_{\nu}\right]\psi \\ -\psi\left(\partial_{\nu} - \frac{ie}{\hbar c} A_{\nu}\right)\psi^{*}\end{array}\right]$$

3. Mostrar que

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \text{ es invariante}$$

ante 1 $\quad A_{\mu}' = A_{\mu} + \partial_{\mu} \chi$

$$\mathcal{L} = \mathcal{L}' = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} F_{\mu\nu}' F^{\mu\nu'}$$

$$F^{\mu\nu'} = \partial^{\mu} A^{\nu'} - \partial^{\nu} A^{\mu'}$$

$$\mathcal{L}' = -\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} \quad ① \quad F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu$$

$$A'_\mu = A_\mu + \partial_\mu \chi$$

$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$