La ecuación de Klein-Gordon

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m_0^2c^2}{\hbar^2}\right]\Psi = 0$$

$$\left[\Box + \frac{m_0^2c^2}{\hbar^2}\right]\Psi = 0$$

se puede escribir en forma de la ecuación de Schrödinger

Sea $\Psi = \varphi + \chi$ (1) y asumamos que se cumple

$$i\hbar\frac{\partial\Psi}{\partial t} = m_0c^2[\varphi - \chi] \quad (2)$$

Usando (1) y (2) en la ec. de K.G

$$\frac{1}{c^2}\frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}\psi\right] = \frac{1}{c^2}\frac{\partial}{\partial t}\left[\frac{m_0 c^2}{i\hbar}(\varphi-\chi)\right]$$

$$= \frac{1}{\cancel{c^2}}\,\frac{m_0 \cancel{c^2}}{i\hbar}\,\frac{\partial}{\partial t}\left[\varphi-\chi\right]$$

$$= -\frac{m_0 i}{\hbar}\left(\frac{\partial}{\partial t}\varphi - \frac{\partial}{\partial t}\chi\right)$$

$$\left[\nabla^2 - \frac{m_0^2 c^2}{\hbar^2}\right]\psi = \left[\nabla^2(\varphi+\chi) - \frac{m_0 c^2}{\hbar^2}(\varphi+\chi)\right]$$

Luego

$$-\frac{m_0 i}{\hbar}\left[\frac{\partial}{\partial t}\varphi - \frac{\partial}{\partial t}\chi\right] = \nabla^2\varphi \quad \nabla^2\chi$$
$$- \frac{m_0^2 c^2}{\hbar^2}\varphi - \frac{m_0^2 c^2}{\hbar^2}\chi$$

$$-m_0 i\hbar\left[\frac{\partial}{\partial t}\varphi - \frac{\partial}{\partial t}\chi\right] = \hbar^2\nabla^2\left[\varphi+\chi\right]$$
$$- m_0^2 c^2\varphi - m_0^2 c^2\chi$$

$$i\hbar\left[\frac{\partial}{\partial t}\varphi - \frac{\partial}{\partial t}\chi\right] = -\frac{\hbar^2}{m_0}\nabla^2(\varphi+\chi] + m_0c^2\varphi + m_0c^2\chi$$

$$i\hbar\frac{\partial\varphi}{\partial t} - i\hbar\frac{\partial}{\partial t}\chi = -\frac{\hbar^2}{2m_0}\nabla^2(\varphi+\chi) - \frac{\hbar^2}{2m_0}\nabla^2(\varphi+\chi) + m_0c^2\varphi + m_0c^2\varphi$$

Ecuaciones acopladas

$$i\hbar\frac{\partial\varphi}{\partial t} = -\frac{\hbar^2}{2m_0}\nabla^2(\varphi+\chi) + m_0c^2\varphi \qquad (a)$$

$$i\hbar\frac{\partial}{\partial t}\chi = \frac{\hbar^2}{2m_0}\nabla^2(\varphi+\chi) - m_0c^2\chi \qquad (b)$$

Sumando (a) + (b)

$$i\hbar\frac{\partial}{\partial t}(\varphi+\chi) = m_0c^2(\varphi-\chi)$$

recuperamos la ec. (2)

Sea ahora la resta (a) - (b)

$$i\hbar\left[\frac{\partial}{\partial t}\varphi - \frac{\partial}{\partial t}\chi\right] = -\frac{\hbar^2}{m_0}\nabla^2(\varphi+\chi) + m_0^2c^2(\varphi+\chi)$$

$$i\hbar\frac{\partial}{\partial t}\left[\varphi-\chi\right] = -\frac{\hbar^2}{m_0}\nabla^2(\varphi+\chi) + m_0^2c^2(\varphi+\chi)$$

pero $(\varphi-\chi) = \frac{i\hbar}{m_0c^2}\frac{\partial}{\partial t}\Psi$

$$i\hbar\frac{\partial}{\partial t}\left[\frac{i\hbar}{m_0c^2}\frac{\partial}{\partial t}\Psi\right] = -\frac{\hbar^2}{m_0}\nabla^2\Psi + m_0^2c^2\Psi$$

$$-\frac{\hbar^2}{m_0c^2}\frac{\partial^2\Psi}{\partial t^2} = -\frac{\hbar^2}{m_0}\nabla^2\Psi + m_0^2c^2\Psi$$

$$\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = \nabla^2\Psi - \frac{m_0^2c^2}{\hbar^2}\Psi$$

Recuperamos la ecuación de Klein-Gordon

Las ecuaciones acopladas

$$i\hbar \frac{\partial}{\partial t}\varphi = -\frac{\hbar^2}{2m_0}\nabla^2(\varphi+\chi) + m_0 c^2 \varphi$$

$$i\hbar \frac{\partial}{\partial t}\chi = \frac{\hbar^2}{2m_0}\nabla^2(\varphi+\chi) - m_0 c^2 \chi$$

se pueden acomodar como sigue

$\Psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$ y usando $\hat{\tau}_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\hat{\tau}_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$\hat{\tau}_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $\mathbb{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Se satisface que $\hat{\tau}_i^2 = \mathbb{1}$ Mostrar

Además $\tau_i \tau_j + \tau_j \tau_i = i \tau_k$ $i, j, k = 1, 2, 3$ ciclico

anticonmutador $\{\tau_i, \tau_j\} = i \tau_k$

Luego, escribimos

$$i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$$

donde $\hat{H} = (\hat{\tau}_3 + i\,\hat{\tau}_2) \frac{\hat{p}^2}{2m_0} + \hat{\tau}_3 m_0 c^2$

$$\hat{H} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \frac{\hat{p}^2}{2m_0} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} m_0 c^2$$

Es fácil mostrar que $\hat{H}^2 = c^2 \hat{p}^2 + m_0^2 c^4$

$$\hat{H}^2 = \left[\begin{pmatrix} +1 & +1 \\ -1 & -1 \end{pmatrix} \frac{\hat{p}^2}{2m_0} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} m_0 c^2 \right]^2$$

$$= \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \left(\frac{\hat{p}^2}{2m_0} \right)^2 + \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{\hat{p}^2}{2m_0} m_0 c^2$$

$$+ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} m_0 c^2 \frac{\hat{p}^2}{2m_0} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} m_0^2 c^4$$

$$= \begin{pmatrix} 1-1 & 1-1 \\ -1+1 & -1+1 \end{pmatrix} \left(\frac{\hat{p}^2}{2m_0} \right)^2 + \begin{pmatrix} 1 & -1 \\ -1 & +1 \end{pmatrix} \frac{\hat{p}^2}{2m_0} m_0 c^2 +$$

$$+\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cancel{m_0} c^2 \frac{p^2}{2\cancel{m_0}} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} m_0^2 c^4$$

$$H^2 = \left[\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right] \frac{\hat{p}^2 c^2}{2} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} m_0^2 c^4$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \frac{\hat{p}^2 c^2}{2} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} m_0^2 c^4$$

$$\hat{H}^2 = \mathbb{1}\, \hat{p}^2 c^2 + \mathbb{1}\, m_0^2 c^4$$

Así entonces $i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$

$$\left[i\hbar \frac{\partial}{\partial t} \Psi - \hat{H} \Psi \right] = 0$$

$$\left[i\hbar \frac{\partial}{\partial t} - \hat{H} \right] \Psi = 0$$

Veamos si cada componente de Ψ satisface K.G.

$$\left[i\hbar\frac{\partial}{\partial t}+\hat{H}\right]\left[i\hbar\frac{\partial}{\partial t}-\hat{H}\right]\psi=0$$

$$=\left[i^2\hbar^2\frac{\partial^2}{\partial t^2}-\hat{H}\right]\Psi=0$$

$$\left[i^2\hbar^2\frac{\partial^2}{\partial t^2}-c^2\hat{p}^2-m_0^2c^4\right]\Psi=0$$

$$\left[-\hbar^2\frac{\partial^2}{\partial t^2}-c^2\left[i\hbar\nabla\right]^2-m_0^2c^4\right]\Psi=0$$

$$\left[-\hbar^2\frac{\partial^2}{\partial t^2}+\hbar^2c^2\nabla^2-m_0^2c^4\right]\Psi=0$$

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2+\frac{m_0^2c^2}{\hbar^2}\right]\Psi=0$$

Se satisface la ec. de K.G.

Analizamos ahora la densidad de carga

$$\rho' = \frac{ie\hbar}{2m_0c^2}\left[\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right]$$

pero $\dfrac{\partial\psi}{\partial t} = \dfrac{m_0c^2}{i\hbar}(\varphi - \chi)$

$$\rho' = \frac{ie\hbar}{2m_0c^2}\left[\psi^*\frac{m_0c^2}{i\hbar}(\varphi-\chi) - \psi\frac{m_0c^2}{-i\hbar}(\varphi^*-\chi^*)\right]$$

$$= \frac{\cancel{ie\hbar}}{2\cancel{m_0c^2}}\frac{\cancel{m_0c^2}}{\cancel{i\hbar}}\left[\psi^*(\varphi-\chi) + \psi(\varphi^*-\chi^*)\right]$$

$$= \frac{e}{2}\left[[\varphi^*+\chi^*][\varphi-\chi] + (\varphi+\chi)(\varphi^*-\chi^*)\right]$$

$$\rho' = e\left[\varphi^*\varphi - \chi^*\chi\right]$$

en términos de Ψ

$$\rho' = e\,\Psi^\dagger\tau_3\Psi$$

De manera similar (probar)

$$j' = \frac{e\hbar}{2m_0 i}\left[\Psi^{\dagger}\hat{\tau}_3[\hat{\tau}_3 + i\tau_2]\nabla\Psi^{\dagger} - (\nabla\Psi^{\dagger})\hat{\tau}_3(\hat{\tau}_3 + i\hat{\tau}_2)\Psi \right]$$

Pero la normalización de la carga

$$\int \rho' \, dx^3 = \pm e$$, entonces

$$\int \Psi \hat{\tau}_3 \Psi \, dx^3 = \pm 1 = \int (\varphi\varphi^* - \chi\chi^*) \, dx^3$$

Consideremos nuevamente partícula libre, en esta representación

Sea $\Psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix} = A\begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix} \exp\left[\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - Et)\right]$

Usemos en $i\hbar \frac{\partial}{\partial t}\Psi = \hat{H}\Psi$

a) $i\hbar \frac{\partial}{\partial t} \Psi = i\hbar A_0 \begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix} \exp\left[\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - Et)\right] \frac{\partial}{\partial t}\left[-\frac{i}{\hbar}Et\right]$

$$= \Psi\left[-\frac{i}{\hbar}E\right] i\hbar$$

b) $\hat{H}\Psi = \left(\begin{bmatrix} +1 & 1 \\ -1 & -1 \end{bmatrix}\frac{\hat{p}^2}{2m_0} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} m_0 c^2\right)\begin{pmatrix} \varphi \\ \chi \end{pmatrix}$

$\Rightarrow$

$$E\begin{bmatrix} \varphi \\ \chi \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\frac{p^2}{2m_0}\begin{pmatrix} \varphi \\ \chi \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} m_0 c^2 \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$$

De aquí obtenemos

$$E\varphi_0 = \frac{p^2}{2m_0}(\varphi_0 + \chi_0) + m_0 c^2 \varphi_0$$

$$E\chi_0 = -\frac{p^2}{2m_0}(\varphi_0 + \chi_0) - m_0 c^2 \varphi_0$$

φ_0 y χ_0 están determinadas por las soluciones a

$$\left[E - \frac{p^2}{2m_0} - m_0 c^2\right]\varphi_0 - \frac{p^2}{2m_0}\chi_0 = 0$$

$$\left(\frac{p^2}{2m_0}\right)\varphi_0 + \left(E + \frac{p^2}{2m_0} + m_0 c^2\right]\chi_0 = 0$$

Para resolver este sistema de ecuaciones acopladas, se debe cumplir que

$$\det\begin{bmatrix} E - (p^2/2m_0) - m_0 c^2 & -p^2/2m_0 \\ p^2/2m_0 & E + p^2/2m_0 + m_0 c^2 \end{bmatrix} = 0$$

$$\Rightarrow \quad E^2 - \left(\frac{p^2}{2m_0} + m_0 c^2\right)^2 + \left(\frac{p^2}{2m_0}\right)^2 = 0$$

$\Rightarrow$

$$E = \pm c\sqrt{p^2 + m_0^2 c^2} = \pm E_p.$$

Recuperamos los valores de la energía.

Soluciones:

Para $\underline{E = +E_p}$

$$\Psi^{(+)}(p) = A_{(+)} \begin{pmatrix} \varphi_0^{(+)} \\ \chi_0^{(+)} \end{pmatrix} \exp\left[\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - E_p \cdot t)\right]$$

$$= \begin{pmatrix} \varphi^{(+)}(p) \\ \chi^{(+)}(p) \end{pmatrix}$$

Usando E_p en las ec. acopladas

$$\left[E_p - \frac{p^2}{2m_0} - m_0c^2\right]\varphi_0 - \frac{p^2}{2m_0}\chi_0 = 0$$

$$\frac{p^2}{2m_0}\varphi_0 + \left[E_p + \frac{p^2}{2m_0} + m_0c^2\right]\chi_0 = 0$$

Sumando ambas ecuaciones

$$\left[E_p - m_0c^2\right]\varphi_0 + \left[E_p + m_0c^2\right]\chi_0 = 0$$

$$\Rightarrow \frac{(E_p - m_0c^2)}{(E_p + m_0c^2)} = -\frac{\chi_0}{\varphi_0} \rightarrow \begin{array}{l} \chi_0 = m_0c^2 - E_p \\ \varphi_0 = m_0c^2 + E_p \end{array}$$

Usando la normalización

$$\int \Psi^{\dagger} \hat{\tau}_3 \Psi \, dx^3 = +1 = \int (\varphi\varphi^* - \chi\chi^*) dx^3$$

$$\Rightarrow A_{(+)} = \frac{1}{\sqrt{E_p L^3}} \frac{1}{\sqrt{2 m_0 c^2}}$$

$$1 = \int \Big[A_{(+)} \varphi_0^{(+)} \cancel{e^{i\xi}} A_{(+)}^{\dagger} \varphi_0^{(+)} \cancel{e^{-i\xi}}$$

$$- A_{(+)} \chi_0^{(+)} \cancel{e^{i\xi}} A_{+}^{\dagger} \chi_0^{(+)} \cancel{e^{-i\xi}} \Big] dV$$

$$1 = \int \Big[|A_{(+)}|^2 \varphi_0^{(+)} \varphi_0^{(+)\dagger} - |A_{(+)}|^2 \chi_0^{(+)} \chi_0^{(+)\dagger} \Big] dV$$

$$1 = |A_{(+)}|^2 \int \left[(m_0 c^2 + E_p)^2 - (m_0 c^2 - E_p)^2 \right] dV$$

$$1 = |A_{(+)}|^2 L^3 \left[(m_0 c^2 + E_p)^2 - (m_0 c^2 - E_p)^2 \right]$$

De forma similar (Ejercicio propuesto)

$$E = -E_p$$

$$\psi^{(-)}(p) = A_{(-)} \begin{pmatrix} \varphi_0^{(-)} \\ \chi_0^{(-)} \end{pmatrix} \exp\left[i(\vec{p}\cdot\vec{x} + E t)\right]$$

$$= \begin{bmatrix} \varphi_0^{(-)}(p) \\ \chi_0^{(-)}(p) \end{bmatrix}$$

con $\begin{bmatrix} \varphi_0^{(-)} \\ \chi_0^{(-)} \end{bmatrix} = \begin{bmatrix} m_0 c^2 - E_p \\ m_0 c^2 + E_p \end{bmatrix}$ y

$$A_{(-)} = \frac{1}{\sqrt{4 m_0 c^2}} \frac{1}{\sqrt{L^3 E_p}}$$

* En el límite no relativista

$$E_p = c\left[p^2 + m_0^2 c^2\right]^{1/2} = c m_0 c\left[1 + p^2/m_0^2 c^2\right]^{1/2}$$

$$E_p \simeq m_0 c^2\left[1 + \frac{1}{2}\frac{p^2}{m_0^2 c^2}\right] = m_0 c^2 + \frac{p^2}{2 m_0}$$

En esta situación

$$\begin{pmatrix} A_{(+)}\,\varphi_0^{(+)} \\ A_{(+)}\,\chi_0^{(+)} \end{pmatrix} = \frac{1}{\sqrt{L^3}} \begin{bmatrix} (m_0c^2 + E_p)/\sqrt{E_p\,4m_0c^2} \\ (m_0c^2 - E_p)/\sqrt{E_p\,4m_0c^2} \end{bmatrix}$$

$$\simeq \frac{1}{\sqrt{L^3}} \begin{bmatrix} 2m_0c^2/2m_0c^2 \\ [-p^2/2m_0]\,/2m_0c^2 \end{bmatrix}$$

$$= \frac{1}{\sqrt{L^3}} \begin{bmatrix} 1 \\ -\frac{1}{4}\left(\frac{v}{c}\right)^2 \end{bmatrix} \xrightarrow[\frac{v}{c}\to 0]{\lim} \frac{1}{\sqrt{L^3}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

De forma análoga

$$\begin{bmatrix} A_{(-)}\,\varphi_0^{(-)} \\ A_{(-)}\,\chi_0^{(-)} \end{bmatrix} \sim \frac{1}{\sqrt{L^3}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Conjugación de Cargo

Revisemos nuestra solución

$$\psi^{(-)}(\underline{p}) = A_{(-)} \begin{bmatrix} \varphi_0^{(-)} \\ \chi_0^{(-)} \end{bmatrix} \exp\left[\frac{i(\vec{p}\cdot\vec{x} + E_p t)}{\hbar} \right]$$

$$= A_{(-)} \begin{bmatrix} m_0 c^2 - E_p \\ m_0 c^2 + E_p \end{bmatrix} \exp\left[\frac{i}{\hbar} (\vec{p}\cdot\vec{x} + E_p t) \right]$$

Si hacemos $\vec{p} \longrightarrow -\vec{p}$

$$\psi^{(-)}(-\vec{p}) = A_{(-)} \begin{bmatrix} m_0 c^2 - E_p \\ m_0 c^2 + E_p \end{bmatrix} \exp\left[\frac{i}{\hbar} (-\vec{p}\cdot\vec{x} + E_p t) \right]$$

$$= A_{(-)} \begin{bmatrix} m_0 c^2 - E_p \\ m_0 c^2 + E_p \end{bmatrix} \exp\left[-\frac{i}{\hbar} (\vec{p}\cdot\vec{x} - E_p t) \right]$$

$$= A_{(-)} \begin{bmatrix} m_0 c^2 - E_p \\ m_0 c^2 + E_p \end{bmatrix} \left[\exp\left[\frac{+i}{\hbar} (\vec{p}\cdot\vec{x} - E_p t) \right] \right]^*$$

$$\Psi^{(-)}(-\vec{p}) = A_{(-)} \begin{bmatrix} \chi_0^{(+)} \\ \varphi_0^{(+)} \end{bmatrix} \left[\exp\left[\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - E_p \cdot t)\right]\right]^*$$

$$= \begin{bmatrix} \chi^{*(+)}(+\vec{p}) \\ \varphi^{*(+)}(+\vec{p}) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \varphi^* \\ \chi^* \end{bmatrix}$$

$$= \tau_1 \left[\Psi^{(+)}(\vec{p})\right]^*$$

$$\boxed{\begin{aligned} \varphi_0^{(+)} &= m_0 c^2 + E_p \\ \chi_0^{(+)} &= m_0 c^2 - E_p \end{aligned}}$$

Se interpreta entonces lo siguiente

si $\Psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$ representa una carga positiva

entonces $\Psi_c = \tau_1 \Psi^* = \begin{bmatrix} \chi^* \\ \varphi^* \end{bmatrix} \equiv \hat{C}\Psi\hat{C}^{-1}$

representa una partícula con carga negativa

Se denomina, estado conjugado de carga de Ψ

Explícitamente, la operación conjugación de carga, implica las siguientes transformaciones

$$\varphi_0^{(+)} \longrightarrow \chi_0^{(-)} \qquad +E_p \longrightarrow -E_p$$

$$\chi_0^{(+)} \longrightarrow \varphi_0^{(-)}$$

$$\vec{p} \longrightarrow -\vec{p}$$

Si llamamos Ψ la partícula, entonces

Ψ_c es la antipartícula

$\pi^- \to$ partícula , $\pi^+ \to$ antipartícula

Partículas neutras también caben en esta descripción

$$\Psi_c = \tau_1 \Psi^* = \alpha \Psi \qquad \alpha \in \mathbb{R}.$$

Esto significa que $\Psi = \varphi + \chi$ es real si

$$\operatorname{Im} \varphi = -\operatorname{Im} \chi$$

Si es partícula neutra, ent-

$\Psi_c = \alpha \Psi$ también debe satisfacer que

$$\mathrm{Im}(\alpha \varphi) = -\mathrm{Im}(\alpha \chi)$$

Esto significa que $\alpha \in \mathbb{R}$.

Además, de $\Psi_c = \tau_1 \Psi^* = \alpha \Psi$

Tomando $(\Psi_c)_c = \Psi$

$$(\Psi_c)_c = \tau_1 [\alpha \Psi]^* = \tau_1 \alpha \Psi^*$$

$$= \alpha [\tau_1 \Psi^*] = \alpha [\alpha \Psi] = \alpha^2 \Psi = \Psi$$

Entonces $\alpha^2 = 1 \rightarrow \alpha = \pm 1$

Existen 2 tipos de partículas neutras

a) Paridad positiva de conjugación de carga

$$\Psi_c = \tau_1 \Psi^* = \Psi \qquad [\varphi^* = \chi]$$

b) Paridad negativa de conjugación de carga

$\alpha = -1$

$$\Psi_c = \tau_1 \Psi^* = -\Psi \quad (\varphi^* = -\chi)$$

Interacción de una partícula de spin cero con un campo electromagnético.