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FACULTAD DE CIENCIAS FÍSICO MATEMÁTICAS

Tópicos de Mécanica Cuántica
Guia 2

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27 de noviembre de 2020

1. Demostrar que la lagrangiana de interacción de Klein-Gordon es invariante de norma, posee simetria U(1) y obtener las corrientes de Noether asociadas a las respectivas transformaciones

- Invariante de norma.
  Se tiene que:
$$\mathcal{L} = (\partial^\mu \phi)^* (\partial_\mu \phi)$$
  aplicando la transformaciónes:
$$\partial^\mu \to \partial^\mu - igA^\mu \qquad \phi \to e^{i\alpha}\phi$$
  se tiene lo siguiente:
$$\mathcal{L} = \left(\partial^\mu(e^{i\theta}\phi) - ig\left(A^\mu + \frac{1}{g}\partial^\mu\theta\right)e^{i\theta}\right)^* \left(\partial_\mu(e^{i\theta}\phi) - ig\left(A_\mu + \frac{1}{g}\partial_\mu\theta\right)e^{i\theta}\right)$$
  como:
$$\begin{aligned}\partial^\mu &= \left\langle \partial^0, \partial^j \right\rangle = \langle \partial_{ct}, -\nabla\rangle \\ \partial_\mu &= \langle \partial_0, \partial_j\rangle = \langle \partial_{ct}, \nabla\rangle\end{aligned}$$
  entonces:
$$\begin{aligned}\mathcal{L} =& \left(\left\langle \partial_{ct}e^{-i\theta}\phi^*, -\nabla(e^{-i\theta}\phi^*)\right\rangle + ig\left\langle A^0, A^j\right\rangle e^{-i\theta}\phi^* + i\left\langle \partial_{ct}\theta, -\nabla\theta\right\rangle e^{-i\theta}\phi^*\right) \\ & \left(\left\langle \partial_{ct}e^{i\theta}\phi, \nabla(e^{i\theta}\phi)\right\rangle - ig\left\langle A_0, A_j\right\rangle e^{i\theta}\phi^* - i\left\langle \partial_{ct}\theta, \nabla\theta\right\rangle e^{i\theta}\phi\right) \\ =& \left(\left\langle \frac{1}{c}(e^{-i\theta}\partial_t\phi^* - ie^{-i\theta}\phi^*\partial_t\theta), -e^{-i\theta}\nabla\phi^* + ie^{-i\theta}\phi^*\nabla\theta\right\rangle + ig\left\langle A^0, A^j\right\rangle e^{-i\theta}\phi^*\right. \\ & \left. + i\left\langle \frac{1}{c}\partial_t\theta, -\nabla\theta\right\rangle e^{-i\theta}\phi^*\right) \\ & \left(\left\langle \frac{1}{c}(e^{i\theta}\partial_t\phi + ie^{i\theta}\phi\partial_t\theta), e^{i\theta}\nabla\phi + ie^{i\theta}\phi\nabla\theta\right\rangle - ig\left\langle A_0, A_j\right\rangle e^{i\theta}\phi\right. \\ & \left. + i\left\langle \frac{1}{c}\partial_t\theta, \nabla\theta\right\rangle e^{i\theta}\phi\right) \\ =& \left(\left\langle \frac{1}{c}e^{-i\theta}\partial_t\phi^*, -e^{-i\theta}\nabla\phi^*\right\rangle + ig\left\langle A^0, A^j\right\rangle e^{-i\theta}\phi^*\right)\left(\left\langle \frac{1}{c}e^{i\theta}\partial_t\phi, e^{i\theta}\nabla\phi\right\rangle - ig\left\langle A_0, A_j\right\rangle e^{i\theta}\phi\right) \\ =& \left(\left\langle \frac{1}{c}\partial_t\phi^*, -\nabla\phi^*\right\rangle + ig\left\langle A^0, A^j\right\rangle \phi^*\right)\left(\left\langle \frac{1}{c}\partial_t\phi, \nabla\phi\right\rangle - ig\left\langle A_0, A_j\right\rangle \phi\right) \\ =& \left(\partial^\mu\phi^* + igA^\mu\phi^*\right)\left(\partial_\mu\phi - igA_\mu\phi\right) \\ =& \left(D^\mu\phi\right)^*\left(D_\mu\phi\right)\end{aligned}$$

- Corrientes de Noether.
  Se tiene que:
$$\delta\mathcal{L} = 0$$
  entonces:
$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\delta(\partial^\mu\phi) + \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi^*)}\delta(\partial^\mu\phi^*)$$

donde

$$\delta\phi = i\alpha\phi \qquad \delta(\partial^\mu\phi) = i\alpha\partial_\mu\phi$$

entonces

$$\begin{aligned}
\delta\mathcal{L} =& i\alpha\frac{\partial\mathcal{L}}{\partial\phi}\phi + i\alpha\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\partial_\mu\phi - i\alpha\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi^*)}\partial_\mu\phi^* \\
=& i\alpha\frac{\partial\mathcal{L}}{\partial\phi}\phi - i\alpha\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\right)\phi + i\alpha\partial^\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\phi\right) - i\alpha\partial^\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi^*)}\phi^*\right) \\
=& i\alpha\left[\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\right)\right]\phi + i\alpha\partial^\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\phi - \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi^*)}\phi^*\right] \\
=& \partial^\mu\left(i\alpha\left[\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\phi - \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi^*)}\phi^*\right]\right)
\end{aligned}$$

con lo cual obtenemos que:

$$\partial^\mu\left(i\alpha\left[\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\phi - \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi^*)}\phi^*\right]\right) = 0$$

donde

$$j_\mu = i\alpha\left[\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)}\phi - \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi^*)}\phi^*\right]$$

calculando las derivadas parciales se tiene que:

$$\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)} = -\partial_\mu\phi^* \qquad \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi^*)} = -\partial_\mu\phi$$

por lo tanto:

$$j_\mu = i\alpha\left[\phi^*\partial_\mu\phi - \phi\partial\phi^*\right]$$

2. Obtener la expresión para los términos de la energía de un sistema descrito por la ecuación de Schrödinger originalmente degenerado al primer orden de la teoría de perturbaciones.
3. Obtener la expresión cuántica para el campo electromagnético a partir de la lagrangiana electromagnética clásica.