

Tarea N°2

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November 4, 2020

Problema 1

Sea

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (1)$$

donde

$$F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) \quad (2)$$

- Determinar las ecuaciones que satisface el campo A_ν .

Se tiene que la ecuación de Euler-Lagrange para campos es la siguiente:

$$\nabla_\nu \left(\frac{\partial \mathcal{L}}{\partial (\nabla_\nu A_\mu)} \right) = \frac{\partial \mathcal{L}}{\partial A_\mu} \quad (3)$$

Como la ecuación 1 no depende del campo A_μ , entonces:

$$\frac{\partial \mathcal{L}}{\partial A_\mu} = 0$$

por lo tanto, la ecuación 3 se escribe de la siguiente manera:

$$\nabla_\nu \left(\frac{\partial \mathcal{L}}{\partial (\nabla_\nu A_\mu)} \right) = 0 \quad (4)$$

Calculando $\frac{\partial \mathcal{L}}{\partial(\nabla_\nu A_\mu)}$ se tiene que :

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial(\nabla_\nu A_\mu)} &= \frac{\partial}{\partial(\nabla_\nu A_\mu)} \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) \\ &= \frac{\partial}{\partial(\nabla_\nu A_\mu)} ((\partial_\mu A_\nu - \partial_\nu A_\mu) F^{\mu\nu}) \\ &= F^{\mu\nu}\end{aligned}$$

entonces

$$\nabla_\nu \left(\frac{\partial \mathcal{L}}{\partial (\nabla_\nu A_\mu)} \right) = \nabla_\nu F^{\mu\nu}$$

por lo tanto el campo A_ν debe cumplir la siguiente ecuación:

$$\nabla_\nu F^{\mu\nu} = 0$$

Problema 2

- Determinar el tensor $T^\mu{}_\nu(T^{\mu\nu})$

Se tiene que

$$T^{\mu}_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu} A_{\nu})} - g^{\mu}_{\nu} \mathcal{L} \qquad T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu} A_{\nu})} - g^{\mu\nu} \mathcal{L}$$

Calculando $T^\mu{}_\nu(T^{\mu\nu})$

$$\begin{aligned}T^\mu{}_\nu(T^{\mu\nu}) &= \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} - g^\mu{}_\nu \mathcal{L} \right) \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} - g^{\mu\nu} \mathcal{L} \right) \\&= (F^\mu{}_\nu \partial_\nu A_\mu)(F^{\mu\nu} \partial_\nu A_\mu) - (F^\mu{}_\nu \partial_\nu A_\mu)(g^{\mu\nu} \mathcal{L}) \\&\quad - (g^\mu{}_\nu \mathcal{L})(F^{\mu\nu} \partial_\nu A_\mu) + (g^\mu{}_\nu \mathcal{L})(g^{\mu\nu} \mathcal{L}) \\&= (F^\mu{}_\nu \partial_\nu A_\mu)(F^{\mu\nu} \partial_\nu A_\mu)\end{aligned}$$

por lo tanto

$$T^{\mu}{}_{\nu}(T^{\mu\nu}) = (F^{\mu}{}_{\nu}\partial_{\nu}A_{\mu})(F^{\mu\nu}\partial_{\nu}A_{\mu})$$