Formas relativistas

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-u^2/c^2}} = m\gamma\vec{u} = m\vec{U} \qquad \text{donde } \vec{U} = \frac{d\vec{x}}{d\tau}$$

Ahora, determinaremos $\xi(u)$

La conexión entre K, K' para las velocidades ②

① $$(U_d)_x = -\frac{c\beta \operatorname{Sen}\theta'}{\gamma(1-\beta^2\operatorname{Cos}\theta')} \qquad (U_d)_z = \frac{c\beta(1-\operatorname{Cos}\theta')}{1-\beta^2\operatorname{Cos}\theta'}$$

para θ' pequeños $\operatorname{Cos}\theta' \simeq 1 - \frac{\theta'^2}{2}$ ③

$$(U_d)_z = \frac{c\beta[1-(1-\theta'^2/2)]}{[1-\beta^2(1-\theta'^2/2)]}$$ Usando ③ en ②

Usando ③ en ① y tomando el cuadrado

$$(U_d)_x^2 = \frac{c^2\beta^2\theta'^2}{\gamma^2(1-\beta^2)^2\left(1+\frac{\theta^2}{2(1-\beta^2)}\right)^2} \cong \frac{c^2\beta^2\theta'^2}{\gamma^2(1-\beta^2)^2} + \theta(\theta^4) + \dots$$

$$(U_d)_z^2 = \frac{c^2\beta^2(\theta'^2/2)^2}{\left[1-\beta^2+\frac{\beta^2\theta'^2}{2}\right]^2} \sim \theta(\theta^4)+\theta(\theta^6)$$

$$(U_d)^2 = (U_d)_x^2 + (U_d)_z^2 = \underbrace{\left[\frac{c^2\beta^2\theta'^2}{(1-\beta^2)}\right]}_{\eta} = \eta$$

$$\boxed{(U_d)^2 = \eta}$$

De la misma forma $(U_c)_x, (U_c)_z$

$$\Rightarrow \boxed{U_c^2 = (U_c)_x^2 + (U_c)_z^2 = U_a^2 - \frac{\eta}{\gamma_a^3}}$$

$$\gamma_a = \frac{1}{\sqrt{1-u_a^2/c^2}}$$ Ejercicio Propuesto

Regresamos a la conservación de la energía visto en K

$$\xi(u_a) + \xi(0) = \xi(u_c) + \xi(u_d)$$

donde $\boxed{u_c^2 = u_a^2 - \frac{\eta}{\gamma_a^3}}$

Expandiendo

$$\xi(u_c(\eta)) = \xi(u_c(\eta))\Big|_{\eta=0} + \left[\frac{\partial\xi(u_c)}{\partial u_c^2}\cdot\frac{\partial u_c^2}{\partial\eta}\right]\Bigg|_{\eta=0}\cdot\eta + \theta(\eta^2)$$

with $\theta(\eta^2) \to 0$

$$\boxed{\xi(u_c(\eta)) = \xi(u_a) + \eta\left[\frac{\partial\xi(u_c)}{\partial u_c^2}\left(\frac{\partial u_c^2}{\partial\eta}\right)\right]\Bigg|_{\eta=0}}$$

$$\xi(u_d(\eta)) = \xi(u_d(\eta))\Big|_{\eta=0} + \eta\left[\frac{\partial\xi(u_d)}{\partial u_d^2}\cdot\frac{\partial u_d^2}{\partial\eta}\right]\Bigg|_{\eta=0} + \dots$$

$$\boxed{\xi(u_d(\eta)) = \xi(0) + \eta\left[\frac{\partial\xi(u_d)}{\partial u_d^2}\cdot\frac{\partial u_d^2}{\partial\eta}\right]\Bigg|_{\eta=0} + \dots}$$

Usando estas aproximaciones en la conservación de la energía

$$\xi(u_a) + \xi(0) = \xi(u_a) + \eta\left[\frac{\partial\xi(u_c)}{\partial u_c^2}\cdot\frac{\partial u_c^2}{\partial\eta}\right]\Bigg|_{\eta=0} + \xi(0) + \eta\left[\frac{\partial\xi(u_d)}{\partial u_d^2}\cdot\frac{\partial u_d^2}{\partial\eta}\right]\Bigg|_{\eta=0}$$

$$0 = -\frac{\cancel{\eta}}{\gamma_a^3}\left(\frac{\partial\xi(u_c)}{\partial u_c^2}\right)\Bigg|_{\eta=0} + \cancel{\eta}\left[\frac{\partial\xi(u_d)}{\partial u_d^2}\cdot 1\right]\Bigg|_{\eta=0}$$

$$0 = -\frac{1}{\gamma_a^3}\frac{\partial\xi(u_a)}{\partial u_a^2} + \left(\frac{\partial\xi(u_d)}{\partial u_d^2}\right)\Bigg|_{\eta=0}$$

$$\boxed{0 = -\frac{1}{\gamma_a^3}\frac{\partial\xi(u_a)}{\partial u_a^2} + \frac{\partial\xi(0)}{\partial u_d^2}}$$

$$\frac{\partial\xi(0)}{\partial u^2} = \frac{m}{2}$$

$$\boxed{\frac{\partial\xi(u_a)}{\partial u_a^2} = \frac{m}{2}\gamma_a^3 = \frac{m}{2\left[1-\frac{u_a^2}{c^2}\right]^{3/2}}}$$

$\xi(u_a)$ $\qquad u_a = u$

$$\xi(u) = \frac{mc^2}{(1-u^2/c^2)^{1/2}} + \left[\xi(0) - mc^2\right]$$

$$\int_{\xi(0)}^{\xi(u)} d\xi(u_a) = \int_0^u \frac{m}{2}\frac{du'^2}{\left(1-\frac{u'^2}{c^2}\right)^{3/2}}$$

$$\xi(u) = \frac{mc^2}{(1-u^2/c^2)} + \xi(0) - mc^2$$

Se define la energía cinética

$$\boxed{T(u) = \xi(u) - \xi(0) = mc^2\left[\frac{1}{\left[1-\frac{u^2}{c^2}\right]^{1/2}} - 1\right]}$$

$\lim u \ll c$ $\qquad T(u) = mc^2\left[1 + \frac{1}{2}\frac{u^2}{c^2} - 1\right]$

$$= \frac{1}{2}mu^2$$

$$\vec{P} = \frac{m\vec{u}}{\sqrt{1-u^2/c^2}} \quad \text{y} \quad \xi(u) = \frac{mc^2}{\left(1-\frac{u^2}{c^2}\right)^{1/2}} + [\xi(0) - mc^2]$$

$K^0 \to \gamma\gamma$ (i) $\qquad E_\gamma + E_\gamma = \xi_K(0)$

$K^0 \to \pi^0\pi^0$ $\qquad E_K(0) = E_K^\pi + E_K^\pi$
$= 2E_K^\pi$

$$\boxed{E_K^\pi = \frac{\xi_K(0)}{2}}$$

Pero $E_{cin}^\pi = T_\pi + \xi_\pi(0) =$

$$T_\pi = \frac{\xi_K(0)}{2} - \xi_\pi(0)$$ Se mide en (i)

Se mide T_π

$$\xi_\pi(0) = \frac{\xi_K(0)}{2} - T_\pi$$

$$\boxed{\xi(0) = mc^2}$$

$$\Rightarrow \xi(u) = \frac{mc^2}{\sqrt{1-u^2/c^2}}$$

Entonces $\boxed{\vec{P} = \frac{m\vec{u}}{\sqrt{1-u^2/c^2}}}$ $\quad \boxed{\xi(u) = \frac{mc^2}{\sqrt{1-u^2/c^2}}}$

$$\Rightarrow \vec{u} = \frac{c^2\vec{P}}{E}$$

Se puede construir el 4-vector

$$E/c, \vec{P} \longrightarrow \left[\frac{mc}{\sqrt{1-u^2/c^2}}, \frac{m\vec{u}}{\sqrt{1-u^2/c^2}}\right]$$

$$= [m\gamma c, m\vec{U}] \qquad \vec{U} = \vec{u}\gamma$$

P_0, $\vec{P}$

La longitud del 4-vector momento

$$\boxed{P_0P_0 - \vec{P}\cdot\vec{P} = P_0'P_0' - \vec{P}'\cdot\vec{P}'}$$

K $\qquad K'$

$$x_0x_0 - \vec{x}\cdot\vec{x} = x_0x_0' - \vec{x}\cdot\vec{x}$$

$(x_0, \vec{x}) =$

$E = \xi$

$$P_0^2 - \vec{P}\cdot\vec{P} = \left(\frac{E}{c}\right)^2 - \vec{P}\cdot\vec{P}$$

$$= \frac{m^2c^4}{(1-u^2/c^2)}\cdot\frac{1}{c^2} - \frac{m^2u^2}{(1-u^2/c^2)}$$

$$= m^2c^2 = P_0'^2 - \vec{P}'\cdot\vec{P}'$$

m = masa en reposo

$$\boxed{P_0^2 - \vec{P}\cdot\vec{P} = m^2c^2} \qquad P_0 = E/c$$

También $\boxed{E^2 = m^2c^4 + p^2c^2}$

$[E] =$ GeV, MeV, eV

$c = 1$

$$[m] = \frac{\text{GeV}}{c^2}, \frac{\text{MeV}}{c^2}, \frac{\text{eV}}{c^2}$$

$$\boxed{P_0^2 - \vec{P}\cdot\vec{P} = m^2}$$

$$\boxed{E^2 = m^2 + p^2}$$