The Cardinality-Constrained approach applied to manufacturing problems

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Motivation: Uncertainty in manufacturing

The data from the machines might be **unknown**

DATA → SYSTEM → PERFORMANCE

Machines' failures

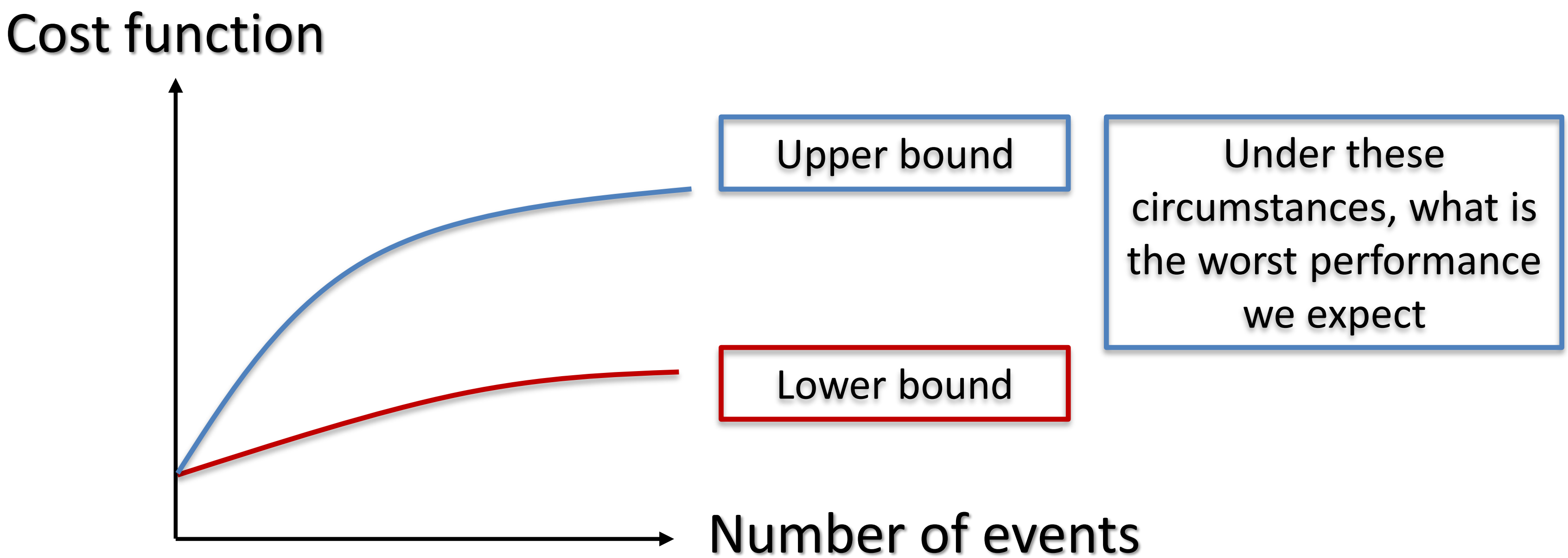
System configuration

Impact on the solution (e.g. performance decreasing and/or cost increasing)

Motivation: Uncertainty in manufacturing

Number of disruptive events

DATA → SYSTEM → PERFORMANCE



CARDINALITY-CONSTRAINED APPROACH

The approach permits to select a constraint over which up to Γ coefficients go to the maximum value, where those coefficients are the ones causing the worst impact

Bertsimas & Sim
"The price of robustness"
Operations Research
(2004)

The approach has been rarely[1] applied to manufacturing problems in the literature.

1. Moreira, 2015

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The Cardinality-Constrained approach

$$\max \sum_i c_i x_i$$

$s.t.$

$$\begin{cases} \sum_j \tilde{a}_{ij} x_j \leq b_i \; \forall i \\ x_i \leq u_i \; \forall i \\ x_i \geq 0 \; \forall i \end{cases}$$

ASSUMPTION:
At maximum Γ_i coefficients $\tilde{a}_{ij}$ go to the maximum value together $\forall i$

WHICH ONES?	THE ONES CAUSING THE WORST IMPACT

Γ_i IS THE CARDINALITY

The Cardinality-Constrained approach

MAXIMIZATION PROBLEM

$$\sum_j \bar{a}_{ij} x_j + max_{|S_i| = \Gamma_i} \left\{ \sum_{j \in S_i} \hat{a}_{ij} x_j \right\} \leq b_i \; \forall i$$

Sub-problem: worst case selection

$$max \sum_j \hat{a}_{ij} x_j z_{ij}$$

$$s.t. \begin{cases} \sum_j z_{ij} \leq \Gamma_i \\ z_{ij} \leq 1 \; \forall j \\ z_{ij} \geq 0 \; \forall j \end{cases}$$

Dual

$$min \; \Gamma_i z_i + \sum_j p_{ij}$$

$$s.t. \begin{cases} z_i + p_{ij} \geq \hat{a}_{ij} x_j \; \forall j \\ z_i \geq 0 \; \forall j \\ p_{ij} \geq 0 \; \forall j \end{cases}$$

It is possible to reduce all to one single optimization problem!

Advantages

- **Linear formulation.**
- **Intuitive meaning of the approach.**
- **No need of probability distributions.**
- **Ability to "*tune*" the number of events (level of robustness)**
- **One single Mathematical Programming Problem.**
- **Overcomes Stochastic Programming drawbacks.**

Our work

We have evaluated the applicability and behavior of the Cardinality-Constrained Approach in 3 relevant manufacturing problems, on which it has been never applied.

PART TYPE SELECTION

MACHINE LOADING

BUFFER ALLOCATION

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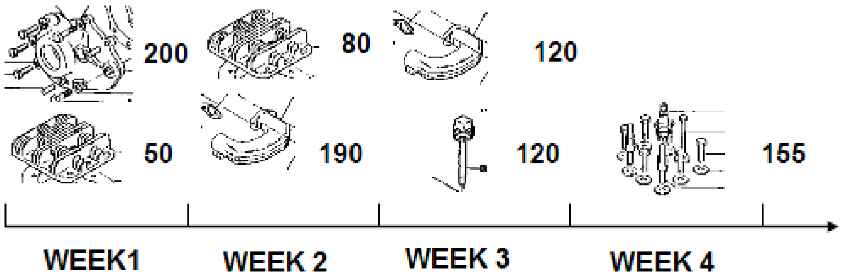
The Part Type Selection Problem

- Order quantity
- Importance of products
- Tools required
- Processing times (tools,…)
- Available time

⇨ BATCH

Which products will be produced?



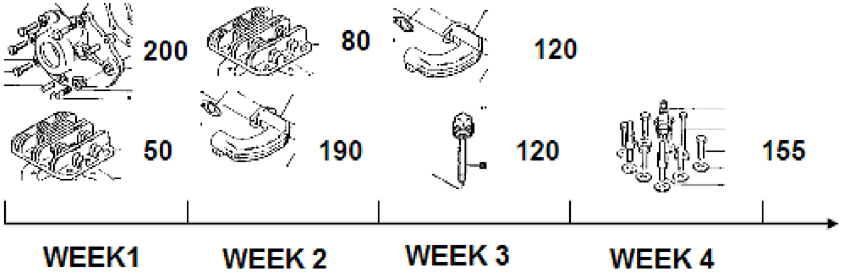
UNCERTAINTY

Uncertain processing times have the most critical impact on the solution (due date, customer satisfaction, penalties,…)

- Order quantity
- Importance of products
- Tools required
- Processing times (tools,…)
- Available time

⇨ BATCH

Which products will be produced?



DETERMINISTIC ILP MODEL:
maximize the size of the batch

$$\begin{aligned}
\max \quad & \sum_i w_i y_i \\
s.t. \quad & \sum_j c_j x_j \leq S \\
& \sum_j T_j \leq M \\
& \sum_i t_{i,j} q_i y_i \leq T_j \quad \forall j \\
& a_{i,j}\, y_i \leq x_j \quad \forall i,j
\end{aligned}$$

Hwan & Shogan
“Modelling and solving an FMS part selection problem”
International Journal of Production Research (1989)

ROBUST COUNTERPART:

$$\sum_i \bar{t}_{i,j} q_i y_i + \max_{S_j} \left\{ \sum_i \hat{t}_{i,j} q_i y_i \right\} \leq T_j \quad \forall j$$

$$\begin{aligned}
\max \quad & \sum_i w_i y_i \\
s.t. \quad & \sum_j c_j x_j \leq S \\
& \sum_j T_j \leq M \\
& a_{i,j}\, y_i \leq x_j \quad \forall i,j \\
& \sum_i t_{i,j} q_i y_i + \Gamma_j\, z_j + \sum_i p_{i,j} \leq T_j \quad \forall j \\
& z_j + p_{i,j} \geq \hat{t}_{i,j}\, q_i\, y_i
\end{aligned}$$

The Part Type Selection Problem

CASE STUDY: Screws production facility

- Use the robust Part Type Selection model for supporting Production Planning decisions
- Evaluate the worst impact of disruptive events on the production
- Prioritize products in a batch

TESTS

Impact of disruptions on processing times

- 2X
- 10X

Importance of products

- Importance ↑ - due date ↓ (linear)
- Same importance (constant weights)

Cardinality of disruptive events $[\, \Gamma \,]$

- 0 (deterministic case); 1; 2; 3; 4; 5

RESULTS

10X disruptions

linear weights

BATCH: extract from the solution

$$y_i = \begin{cases} 1 \; included \; in \; the \; batch \\ 0 \; not \; included \; in \; the \; batch \end{cases}$$

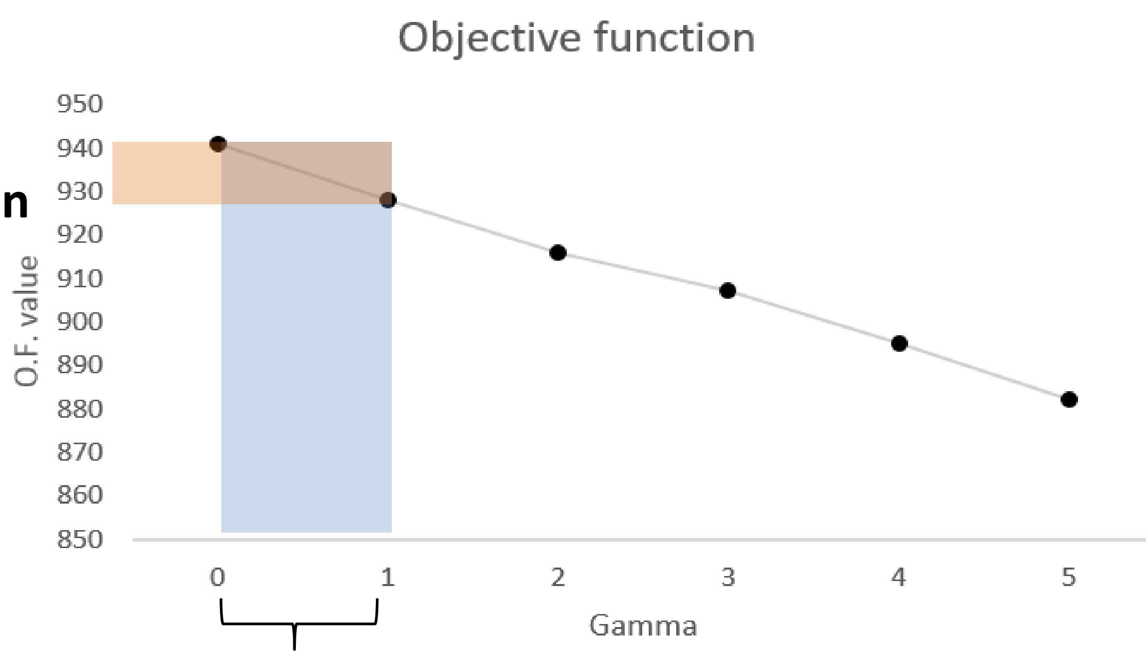
		Cardinality					
Part Code	Weights	0	1	2	3	4	5
36	10	1	1	1	1	1	1
37	9	1	1	1	1	1	1
38	9	1	1	1	1	1	1
39	8	1	1	1	1	1	0
40	7	1	1	1	1	1	1
41	10	1	1	1	1	1	1
42	11	1	1	1	1	1	1
43	3	1	1	0	0	0	0
44	4	1	1	1	1	1	0
45	3	1	1	1	1	1	1

RESULTS

Evaluation of how much disruptive events influence the detriment of the objective function

TOOL FOR PIANIFICATION

Detriment of objective function



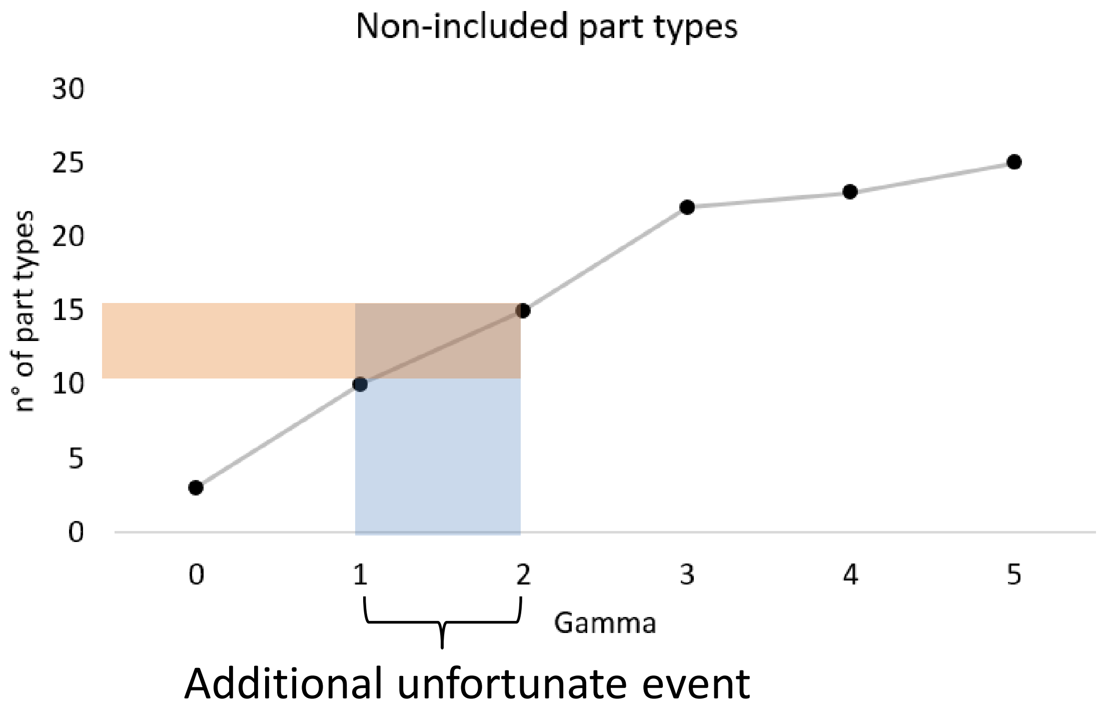
Additional unfortunate event

The Part Type Selection Problem

RESULTS

Evaluation of how much disruptive events influence the composition of the batch ⇨ TOOL FOR PIANIFICATION

Lost part types
Evaluation of how many part types are not included in the batch



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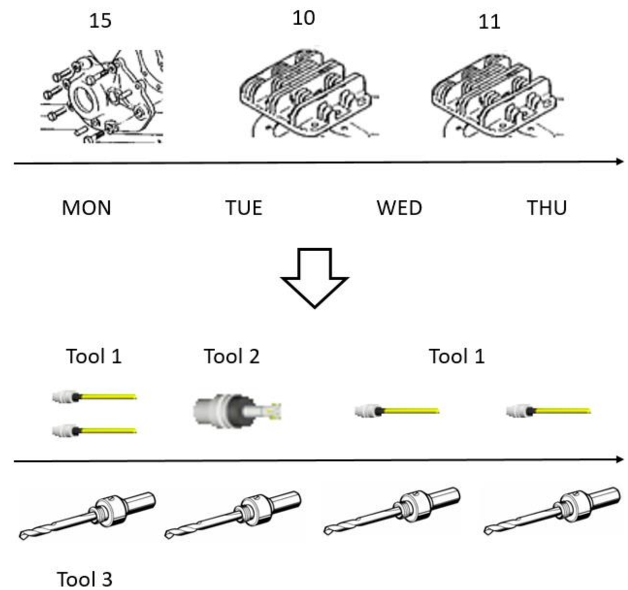
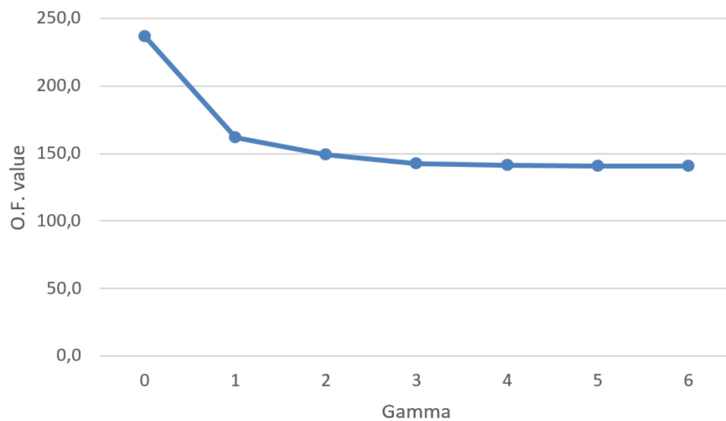
The Machine Loading Problem

- Product demand
- Costs/Profits
- Tools required
- Processing times
- Available time

- Production
- Tools to load on machines

UNCERTAINTY





Same considerations are valid in this case

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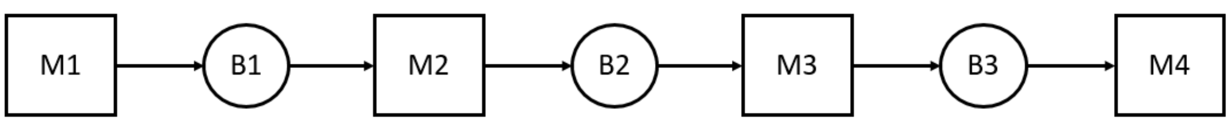
The Buffer Allocation Problem

- Cost of buffer space
- Processing times
- Failures' data/variability of processing times

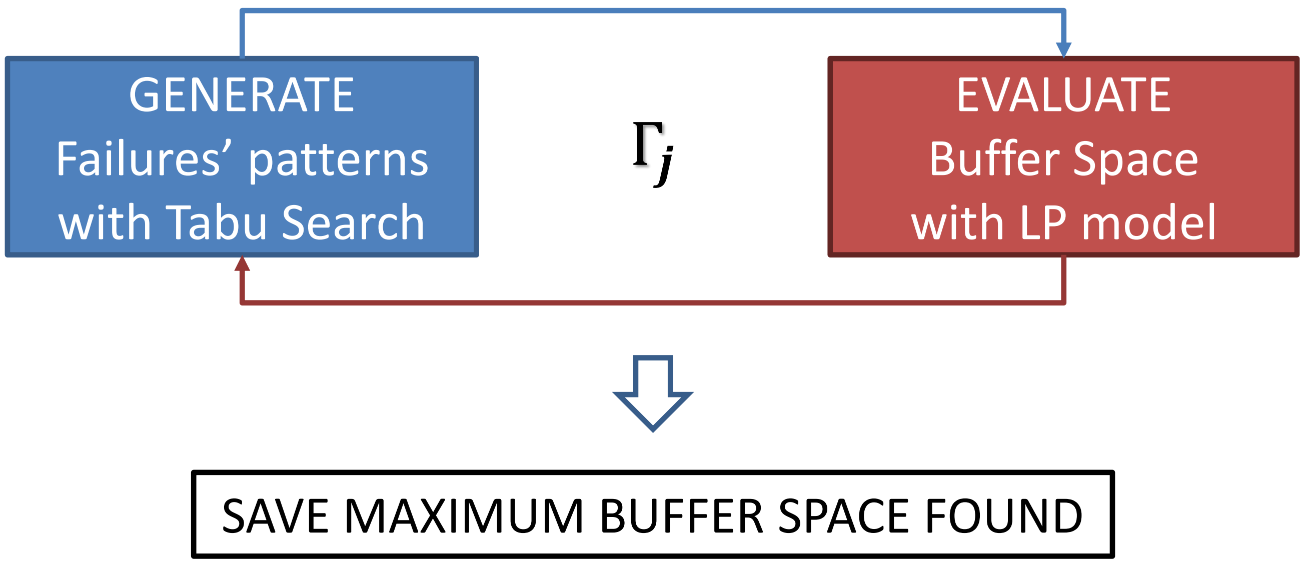
BUFFER CONFIGURATION

- **In the analytical form, it was not possible to group the constraints by machine ($\forall j$)**
- **Other formulations resulted in non-linear dual problems**



Matheuristic Tabu Search algorithm for each fixed cardinality

- Generation of failures' patterns with a boolean *Failures' matrix*.
- Evaluation of the buffer space given the pattern
- Output: maximum buffer space obtained (over N iterations)



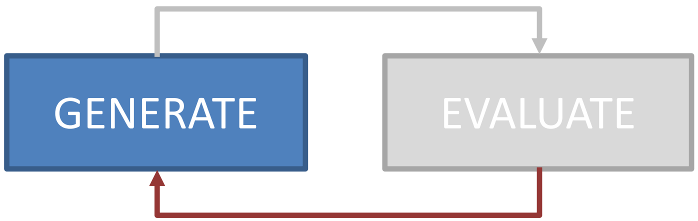
The Buffer Allocation Problem

GENERATE → EVALUATE

Alfieri & Matta
"Mathematical programming formulations for approximate simulation of multistage production systems."
European Journal of Operational Research (2009)

Failure's Matrix ⇒ SIMULATION-OPTIMIZATION MODEL ⇒ BUFFER SPACE

The Buffer Allocation Problem



$$\hat{z}_{ij} = \begin{bmatrix} 1 & 1 & 0 & \dots & 0 \\ 1 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 1 \end{bmatrix}$$

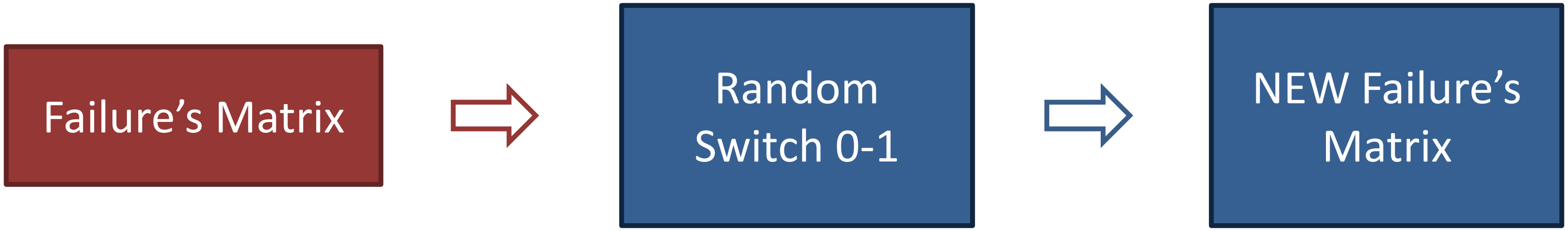
1: increment of processing times

0: nominal processing times

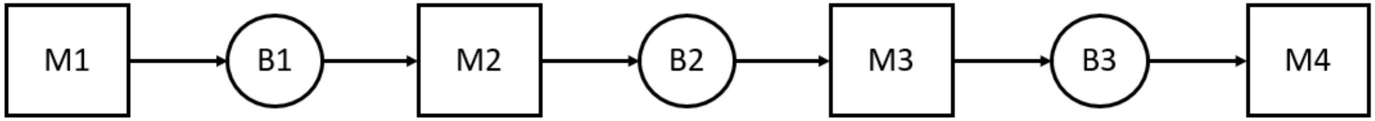
Cardinality constraint

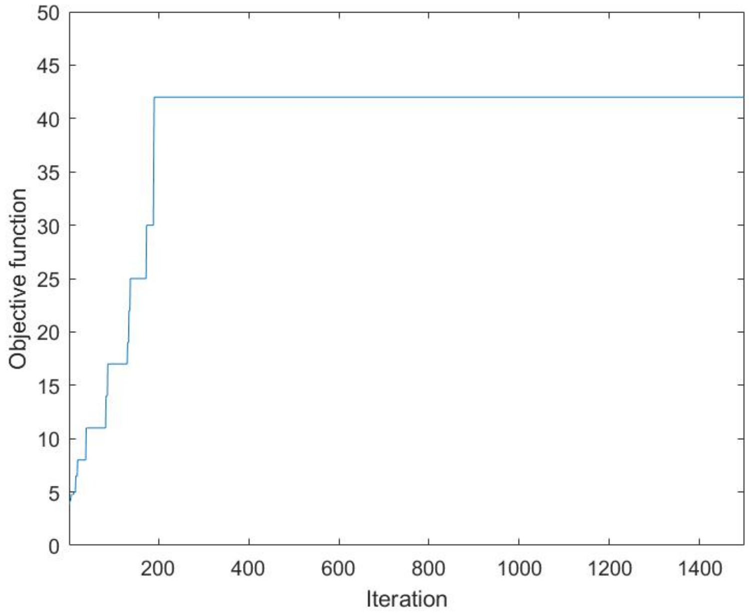
$$\sum_{i} \hat{z}_{ij} = \Gamma_j \ \forall j$$

NOTE: Cardinality is respected while generating new failures' patterns



The cardinality vector Γ represents the number of failures on the machines of the flow line





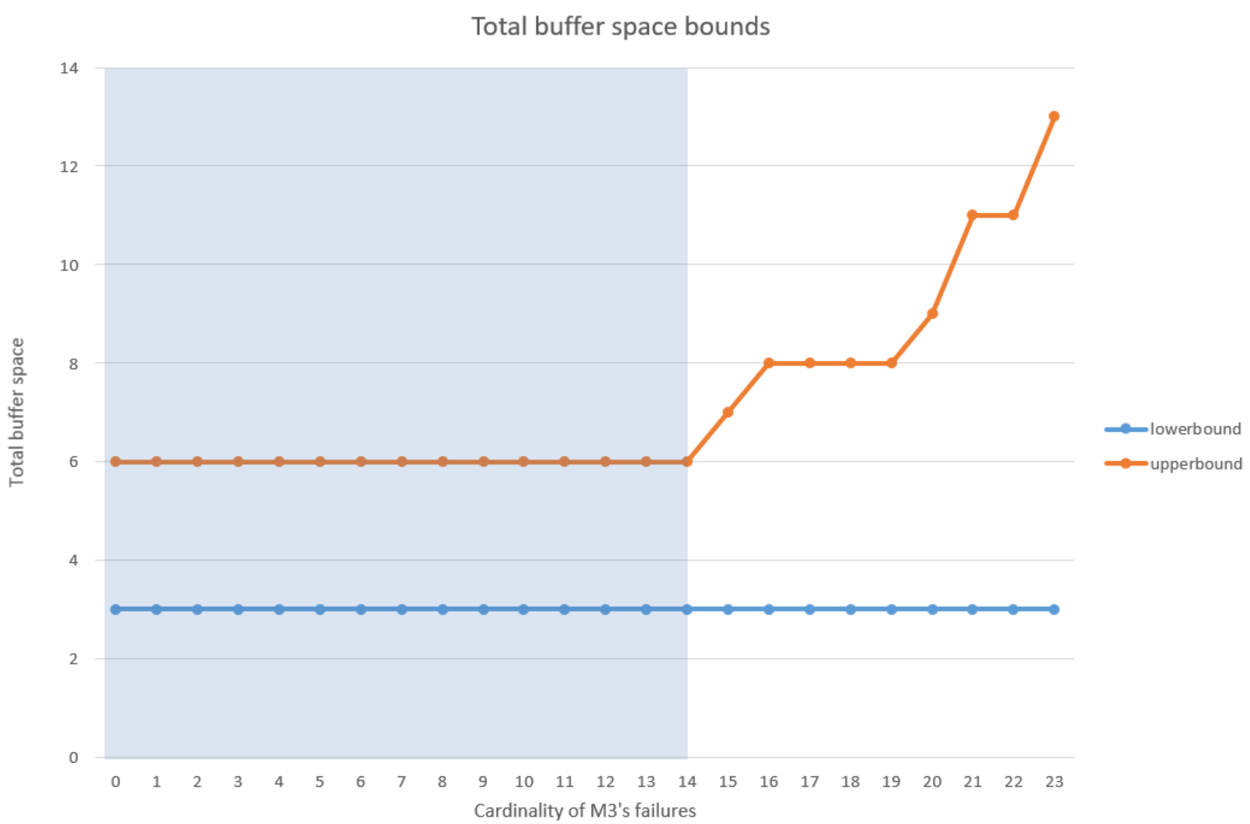
$$\boldsymbol{\Gamma} = [10\ 10\ \Gamma_3\ 10]$$

Disruptions on M3

- $\Gamma_3 \in [0, 23]$
- Each run: 1500 iterations

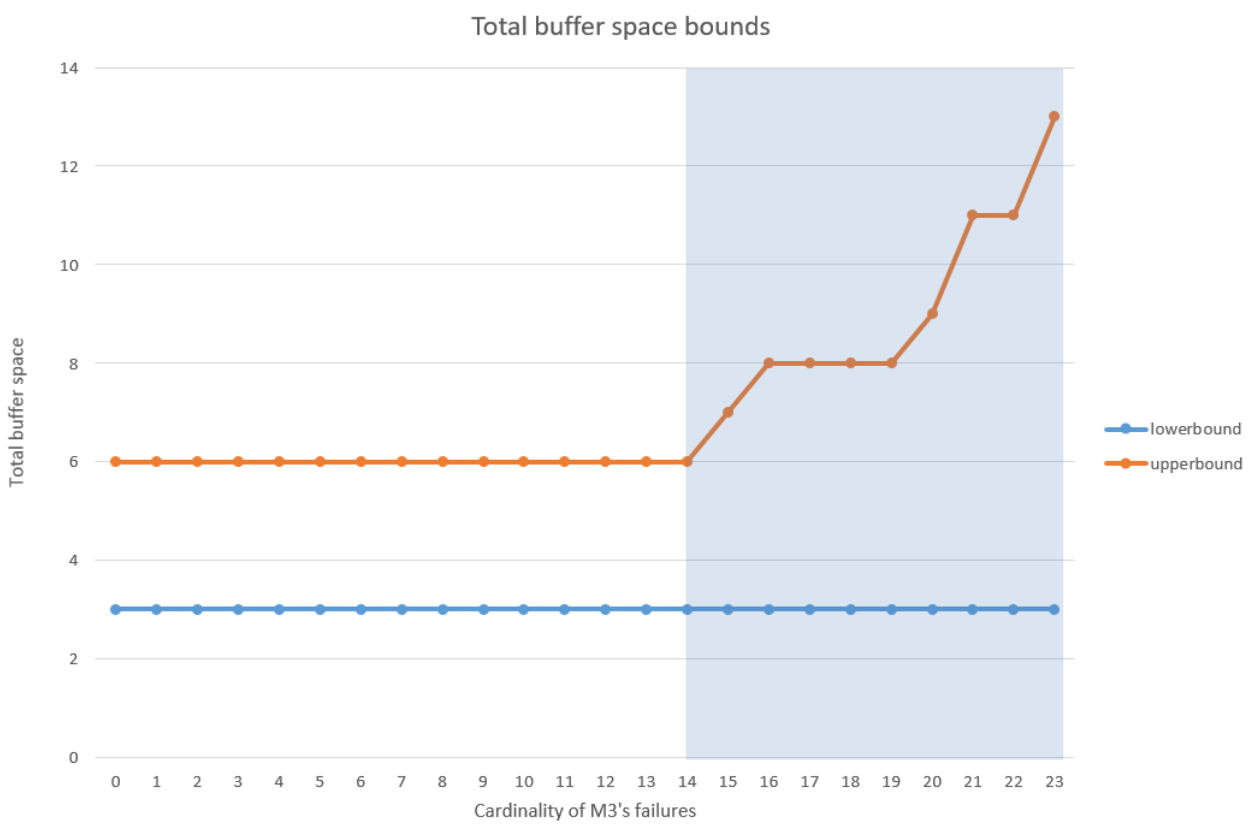
The Buffer Allocation Problem

Until $\Gamma_3 = 14$ there is no increment in buffer space



The Buffer Allocation Problem

After $\Gamma_3 = 14$ the buffer space is increased to cover from failures on M3



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Conclusions

CARDINALITY-CONSTRAINED APPROACH

PART TYPE SELECTION	✓	ROBUST ILP
MACHINE LOADING	✓	ROBUST MILP
BUFFER ALLOCATION	✓	ROBUST MATHEURISTIC

✓ **Applicability in the industrial practice**

THANK YOU