POLITECNICO
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A cardinality-constrained approach for robust machine loading problems

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About

This work develops a robust formulation of an MLP, based on the cardinality-constrained approach, to evaluate the optimal solution in the presence of a given number of fluctuations of the processing times. The applicability of the model in the practice has been tested on a case study.

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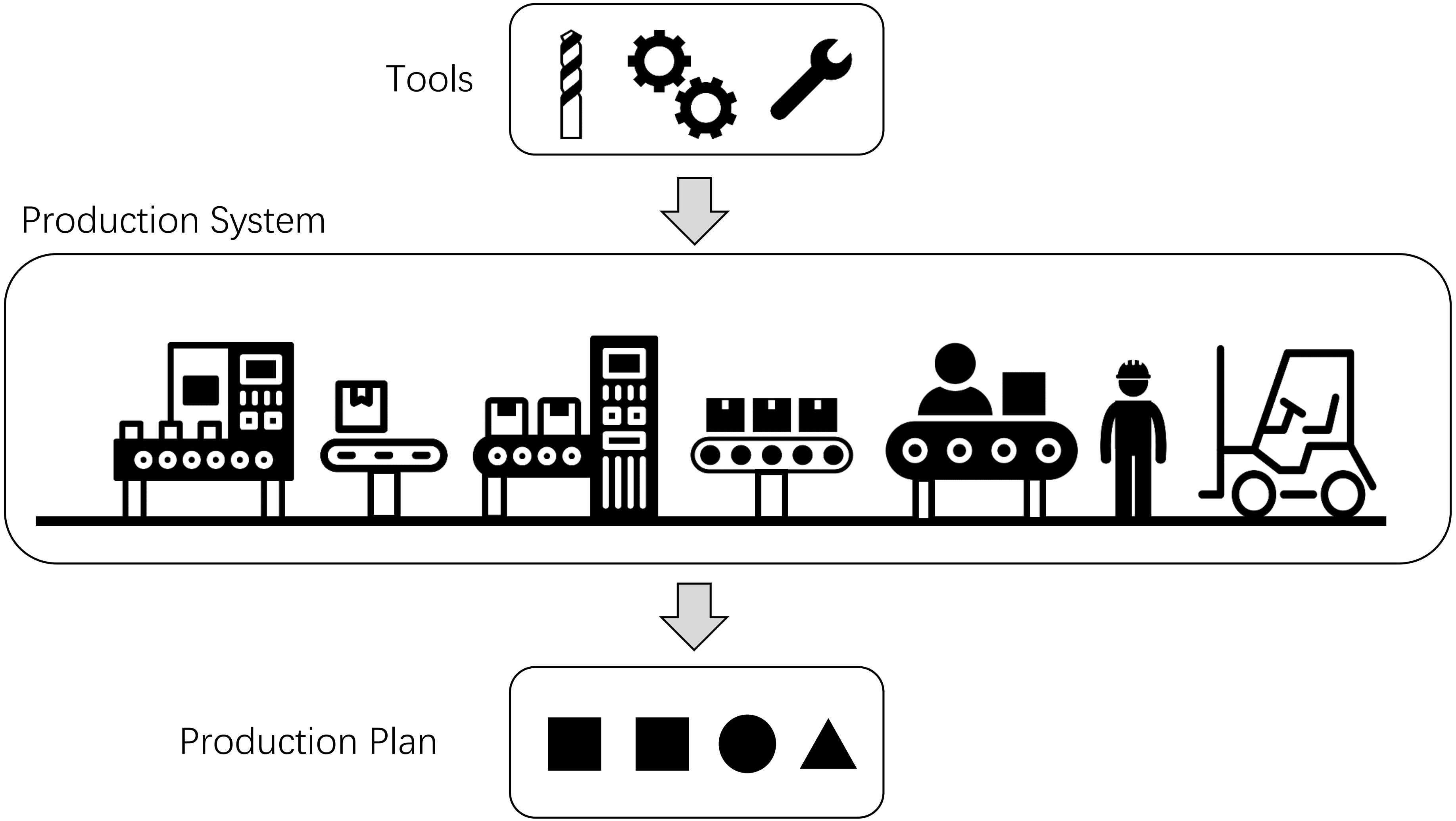
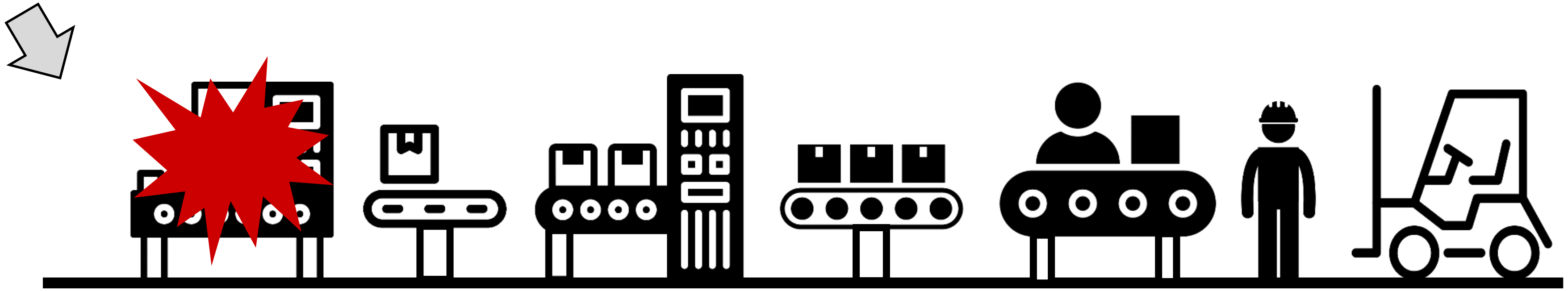
The industrial problem



The industrial problem

Time to repair a machine: 5 ± 2 min



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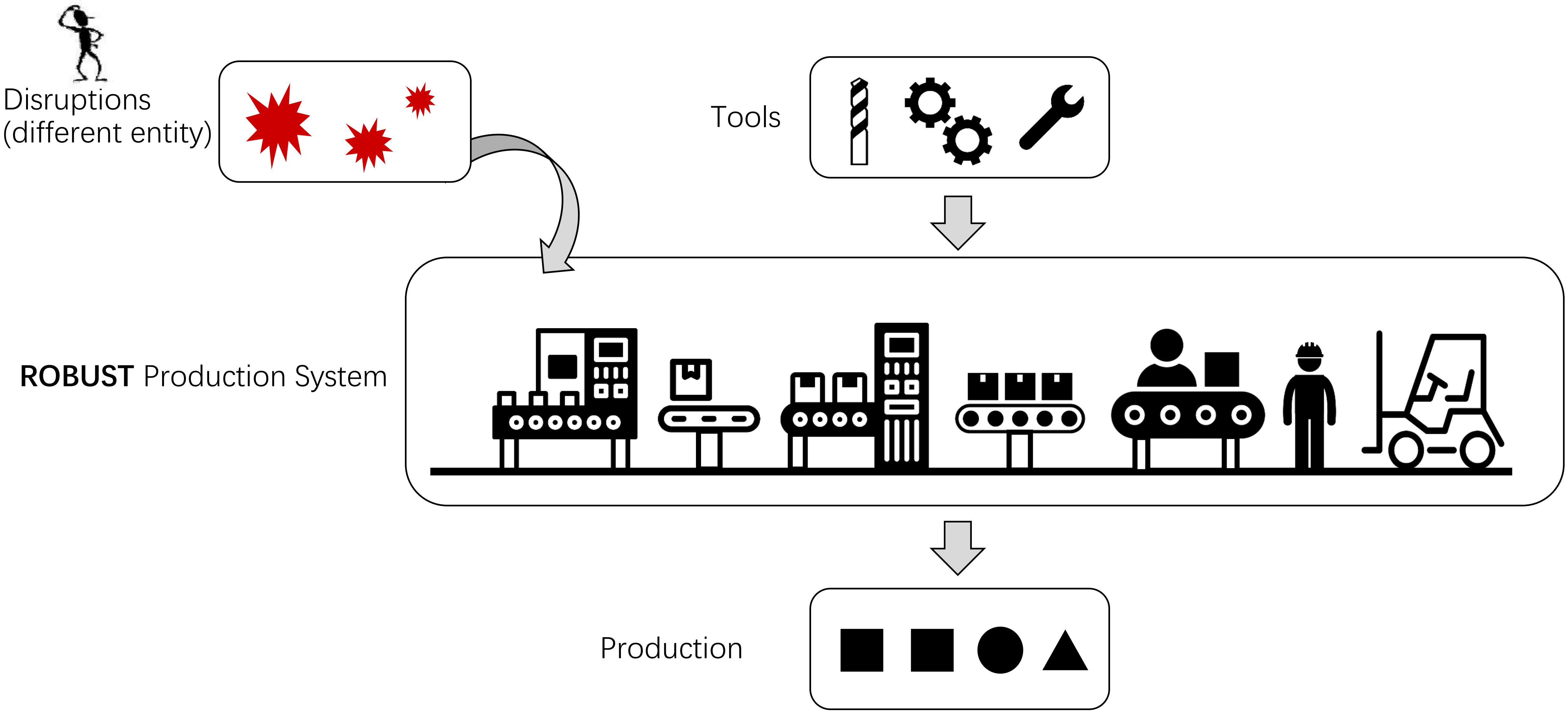
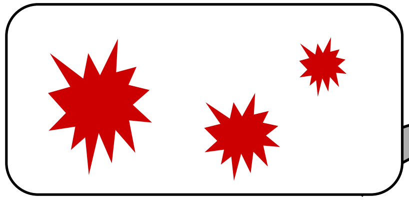
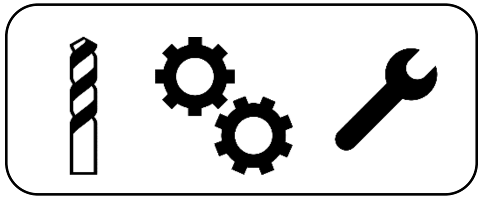


The industrial problem

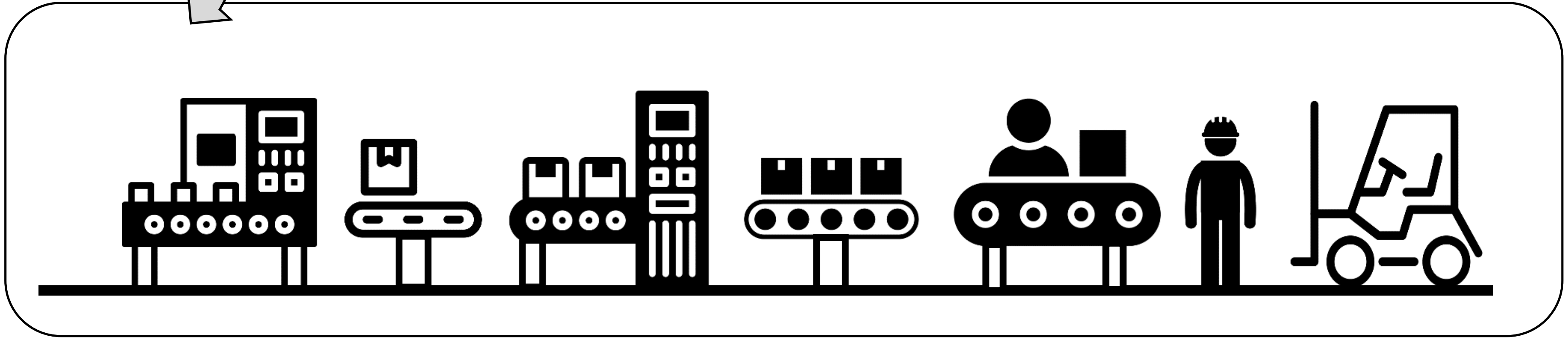
ALL Disruptions



Tools

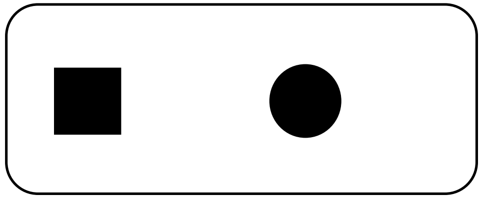


ROBUST Production System



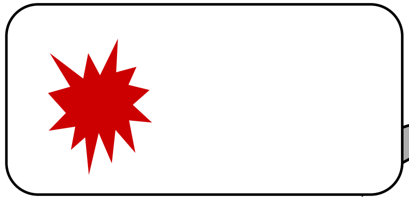
SOLUTION IS TOO CONSERVATIVE

Production

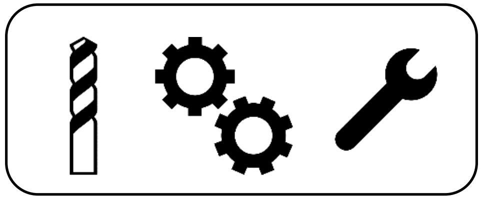


The industrial problem

SOME Disruptions



Tools



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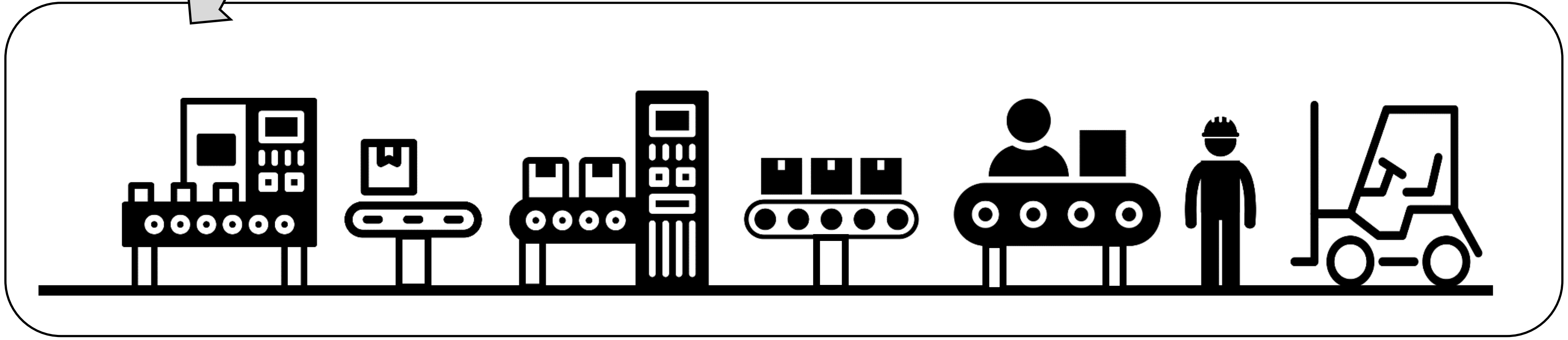
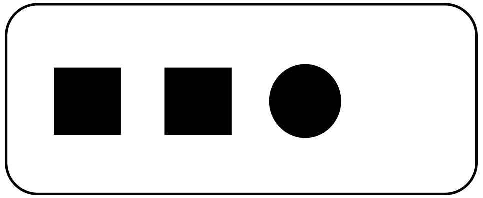


THIS IS A GOOD COMPROMISE

Production



Motivation: uncertainty in manufacturing

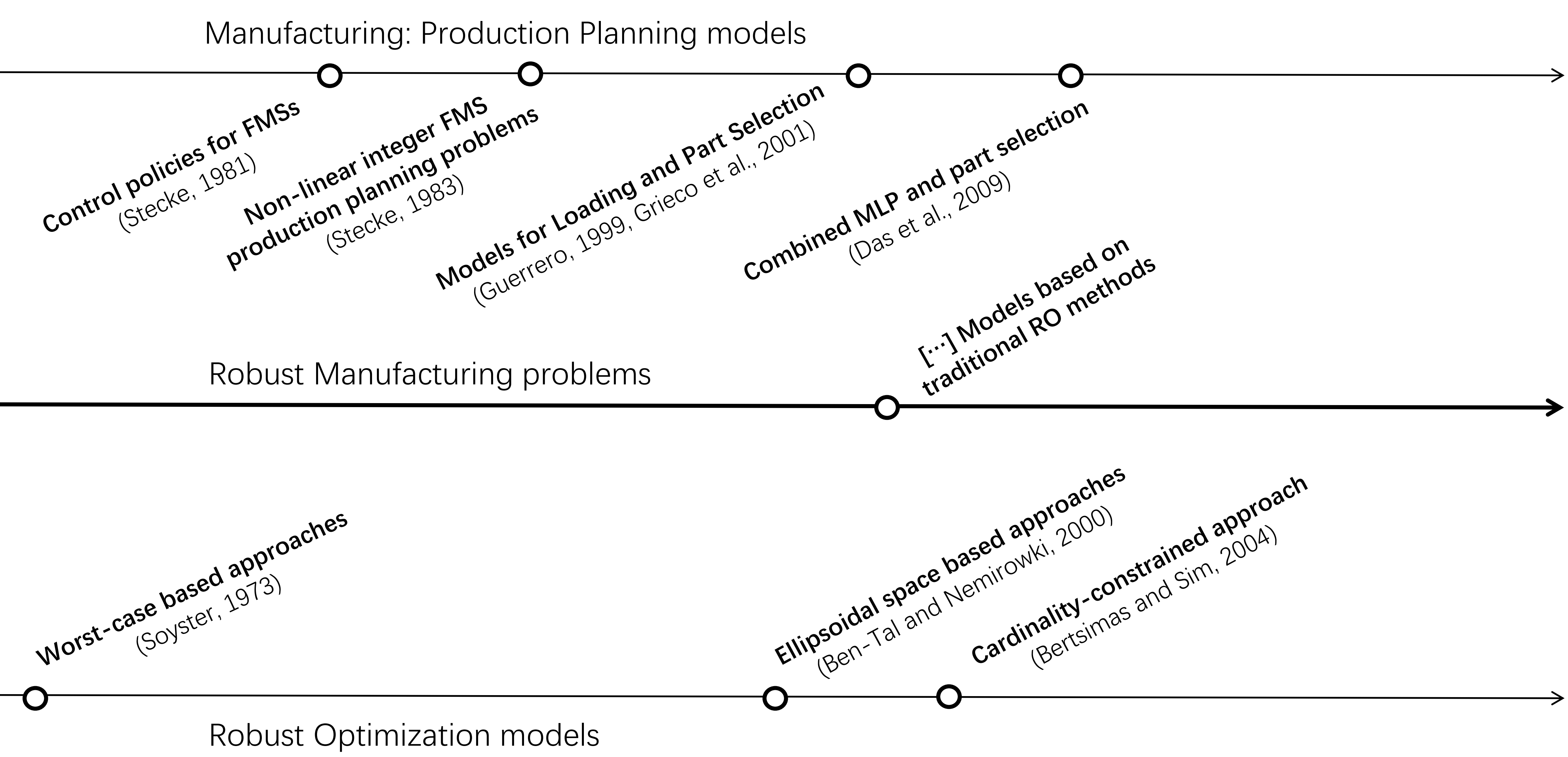
Manufacturing: Production Planning models

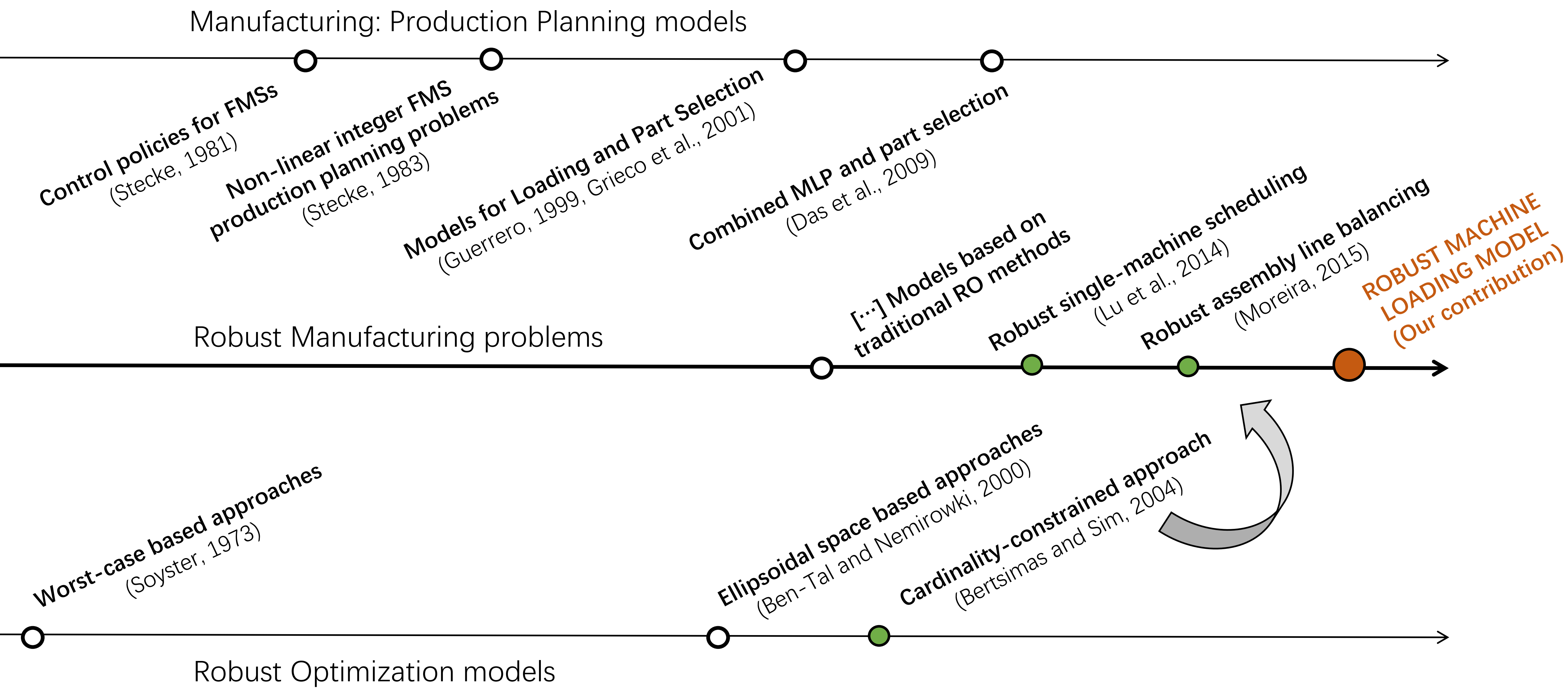
- **Control policies for FMSs** (Stecke, 1981)
- **Non-linear integer FMS production planning problems** (Stecke, 1983)
- **Models for Loading and Part Selection** (Guerrero, 1999, Grieco et al., 2001)
- **Combined MLP and part selection** (Das et al., 2009)

- **Worst-case based approaches** (Soyster, 1973)
- **Ellipsoidal space based approaches** (Ben-Tal and Nemirowki, 2000)
- **Cardinality-constrained approach** (Bertsimas and Sim, 2004)

Robust Optimization models

Motivation: uncertainty in manufacturing

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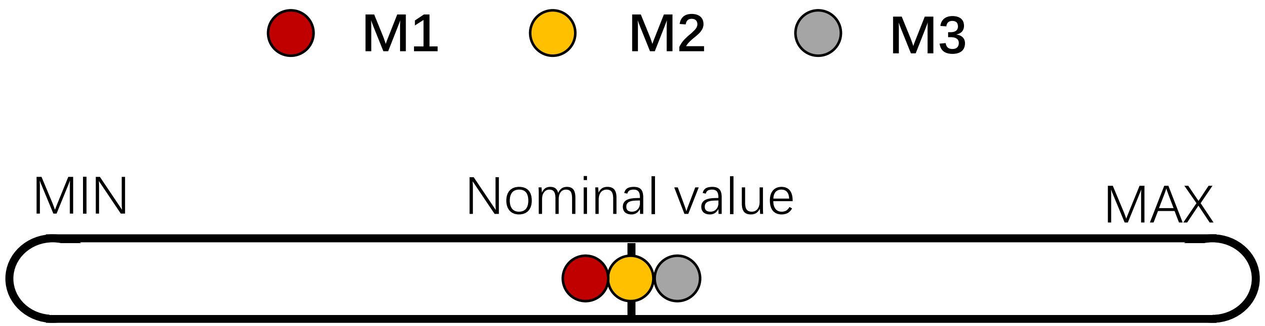
Combined MLP and part selection
(Das et al., 2009)
[···] Models based on traditional RO methods
Robust single-machine scheduling
(Lu et al., 2014)
Robust assembly line balancing
(Moreira, 2015)
ROBUST MACHINE LOADING MODEL
(Our contribution)
Robust Manufacturing problems
Worst-case based approaches
(Soyster, 1973)
Ellipsoidal space based approaches
(Ben-Tal and Nemirowki, 2000)
Cardinality-constrained approach
(Bertsimas and Sim, 2004)
Robust Optimization models

The cardinality-constrained approach

Time to repair machines: 5 ± 2 min

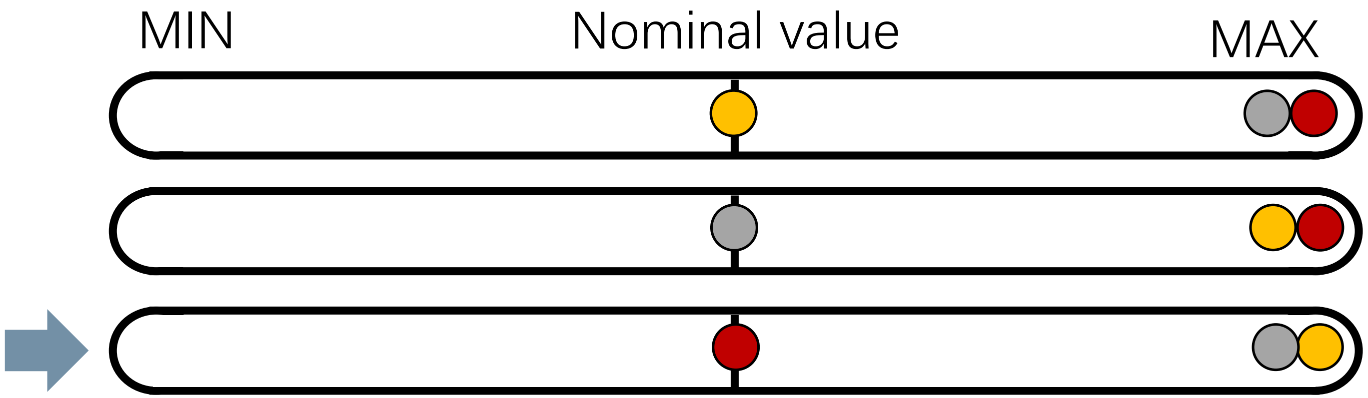


CLASSICAL ROBUST OPTIMIZATION

- Find the system design that withstands this situation
- Conservative solutions are usually found this way

CARDINALITY-CONSTRAINED

- Only a subset of parameters vary to their maximum value
- Combinatorial nature of the problem: which of the combinations is to be chosen?
  The one that causes the worst effect.



The cardinality-constrained approach

Starting from a classical Mathematical Programming problem, we assume we can provide a range for its coefficients, then: $\tilde{a}_{ij} \in [\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}]$

$$\max \sum_i c_i x_i$$

$s.t.$

Random coefficients

$$\begin{cases} \sum_j \tilde{a}_{ij} x_j \leq b_i \; \forall i \\ x_i \leq u_i \; \forall i \\ x_i \geq 0 \; \forall i \end{cases}$$

ASSUMPTION At maximum Γ_i coefficients $\tilde{a}_{ij}$ go to the maximum value $\bar{a}_{ij} + \hat{a}_{ij}$ together $\forall i$

Γ_i IS THE CARDINALITY

The approach permits to select a constraint over which up to Γ coefficients go to the maximum value, where those coefficients are the ones causing the worst impact

WHICH ONES? → THE ONES CAUSING THE WORST IMPACT

Bertsimas & Sim, "*The price of robustness*", Operations Research (2004)

The cardinality-constrained approach

$$\max \sum_i c_i x_i$$

s.t.

$$\begin{cases} \sum_j \tilde{a}_{ij} x_j \le b_i \ \forall i \\ x_i \le u_i \ \forall i \\ x_i \ge 0 \ \forall i \end{cases}$$

INNER MAXIMIZATION PROBLEM

$$\sum_j \bar{a}_{ij} x_j + \max_{|S_i| = \Gamma_i} \left\{ \sum_{j \in S_i} \hat{a}_{ij} x_j \right\} \le b_i \ \forall i$$

Sub-problem:
worst case selection

$$\max \sum_j \hat{a}_{ij} x_j z_{ij}$$

s.t.

$$\begin{cases} \sum_j z_{ij} \le \Gamma_i \\ z_{ij} \le 1 \ \forall j \\ z_{ij} \ge 0 \ \forall j \end{cases}$$

Dual

$$\min \ \Gamma_i z_i + \sum_j p_{ij}$$

s.t.

$$\begin{cases} z_i + p_{ij} \ge \hat{a}_{ij} x_j \ \forall j \\ z_i \ge 0 \ \forall j \\ p_{ij} \ge 0 \ \forall j \end{cases}$$

It is possible to reduce all to one single optimization problem!

Why we have chosen this approach

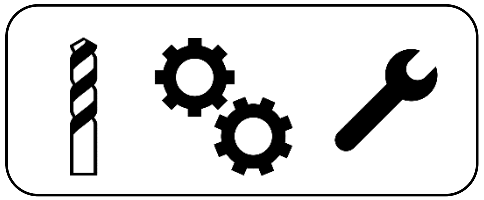
- ✓ Linear formulation.
- ✓ Intuitive meaning of the approach.
- ✓ No need of probability distributions.
- ✓ Ability to “*tune*” the number of events (level of robustness)
- ✓ One single Mathematical Programming Problem.
- ✓ Overcomes Stochastic Programming drawbacks.

➡ LET US BUILD A ROBUST MLP

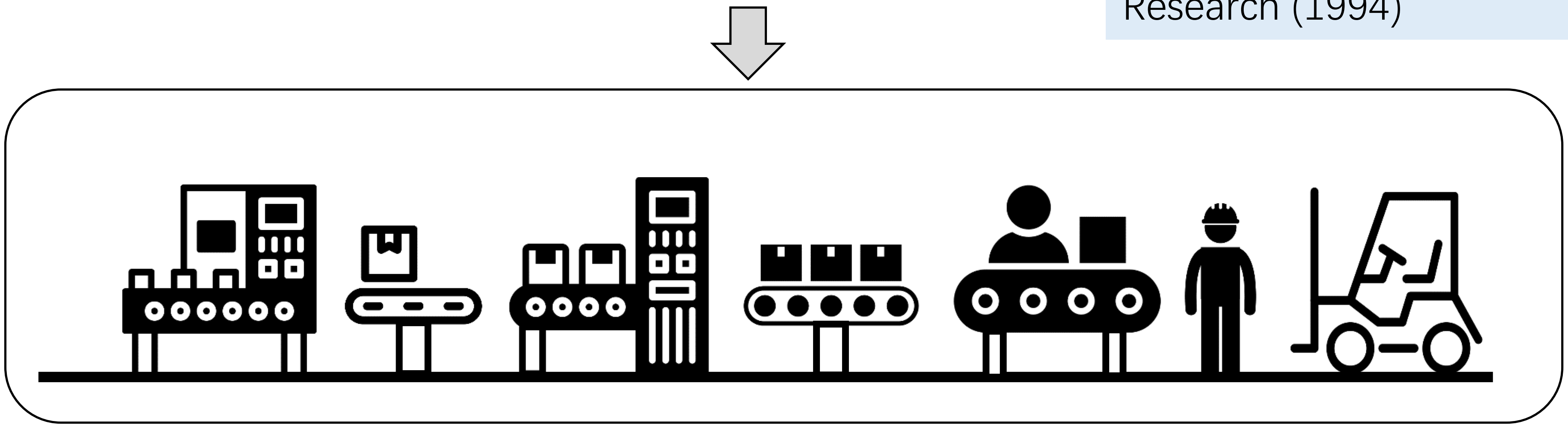
Assumptions

Sodhi, Askin, Sen, “Multiperiod Tool and Production Assignment in Flexible Manufacturing Systems”, International Journal of Production Research (1994)

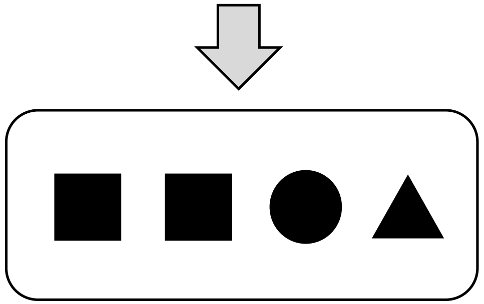
Tools



Production System



Production

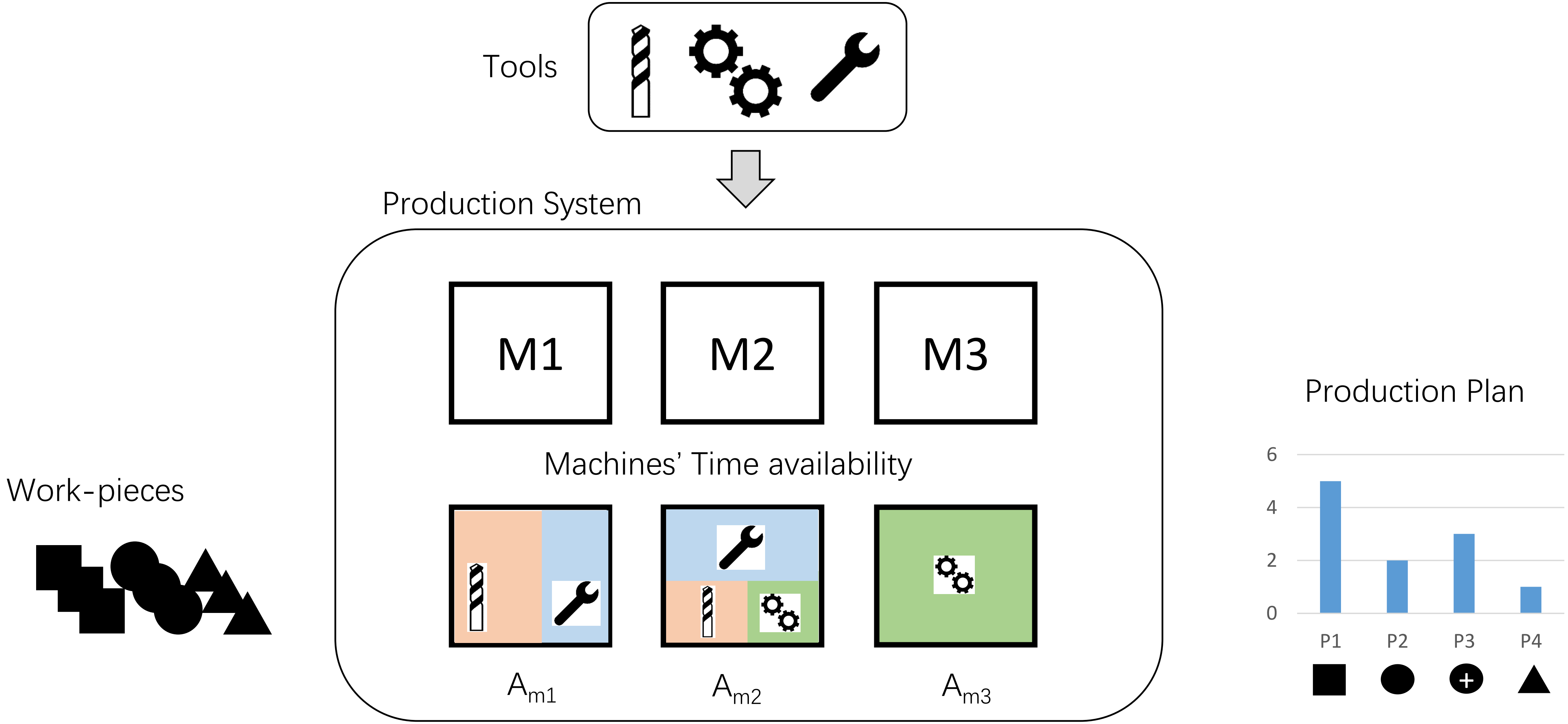


Assumptions

- The time horizon corresponds to the production planning period
- A single time period is the time interval between two tool changeovers.
- No tool transportation system is present. Thus, it is not possible to change tool without stopping the production.
- In each time period t, a machine m can process work-pieces with a certain time availability A_{mt} (shift)
- The model does not consider workload balancing among machines
- Tool storage capacity is limited
- The batch of part types has been already selected in a previous step. The MLP model assigns production quantities, which are assumed to be continuous variables (partial production is allowed).

Sodhi, Askin, Sen, "Multiperiod Tool and Production Assignment in Flexible Manufacturing Systems", International Journal of Production Research (1994)

Assumptions



The Machine Loading Problem

Sodhi, Askin, Sen, "Multiperiod Tool and Production Assignment in Flexible Manufacturing Systems", International Journal of Production Research (1994)

$$\max \quad \sum_i \sum_t w_i x_{it} - \sum_i C_i S_i - \sum_i \sum_t h_{it} x_{it} \qquad (1)$$

subject to:

$$\sum_j L_{jmt} k_j \le K_m \quad \forall m,t \qquad (2)$$

$$\sum_t x_{it} = D_i - S_i \quad \forall i \qquad (3)$$

$$\sum_i O_{ij} x_{it} \le \sum_m p_{jmt} \quad \forall j,t \qquad (4)$$

$$p_{jmt} \le L_{jmt} A_{mt} \quad \forall j,m,t \qquad (5)$$

$$\sum_j p_{jmt} \le A_{mt} \quad \forall m,t \qquad (6)$$

$$\sum_m L_{jmt} \le \alpha_j \quad \forall j,t \qquad (7)$$

$$L_{jmt} \in \{0,1\} \quad \forall j,m,t$$

$$p_{jmt} \ge 0 \ \forall j,m,t; \ x_{it} \ge 0 \ \forall i,t; \ S_i \ge 0 \ \forall i$$

p_{jmt}	time spent for production on the tool $j \in J$ of machine $m \in M$ in period $t \in T$
S_i	shortage of production of product type $i \in I$
L_{jmt}	Boolean variable that is 1 when tool $j \in J$ is loaded on machine $m \in M$ in period $t \in T$
v_{ijt}	time spent for production of product type $i \in I$ over tool $j \in J$ in period $t \in T$
x_{it}	quantity of product type $i \in I$ produced in period $t \in T$
α_j	number of tool copies available for tool $j \in J$
A_{mt}	available time for production on machine $m \in M$ in period $t \in T$
C_i	shortage cost per product type $i \in I$
D_i	total demand of product type $i \in I$
k_j	number of slots required by tool $j \in J$
K_m	total slots available in the slot magazine of machine $m \in M$
O_{ij}	processing time of one unit of product $i \in I$ on tool $j \in J$
h_{it}	holding cost per part of product type $i \in I$ in period $t \in T$ over the remaining time horizon
w_i	total earning per part for the production of product type $i \in I$

The Machine Loading Problem

max $$\sum_i \sum_t w_i x_{it} - \sum_i C_i S_i - \sum_i \sum_t h_{it} x_{it} \qquad (1)$$

subject to:

$$\sum_j L_{jmt} k_j \le K_m \quad \forall m,t \qquad (2)$$

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$$p_{jmt} \ge 0 \ \forall j,m,t; \ x_{it} \ge 0 \ \forall i,t; \ S_i \ge 0 \ \forall i$$

- Maximization of the total profit related to the production of products
- Minimizing the storage and stocking costs

The Machine Loading Problem

$$\max \quad \sum_i \sum_t w_i x_{it} - \sum_i C_i S_i - \sum_i \sum_t h_{it} x_{it} \qquad (1)$$

subject to:

$$\sum_j L_{jmt} k_j \le K_m \quad \forall m,t \qquad (2)$$

Limit the number of tools that can be loaded on machines, as the tool slots on machine m are limited to K_m slots.

$$\sum_t x_{it} = D_i - S_i \quad \forall i \qquad (3)$$

$$\sum_i O_{ij} x_{it} \le \sum_m p_{jmt} \quad \forall j,t \qquad (4)$$

$$p_{jmt} \le L_{jmt} A_{mt} \quad \forall j,m,t \qquad (5)$$

$$\sum_j p_{jmt} \le A_{mt} \quad \forall m,t \qquad (6)$$

$$\sum_m L_{jmt} \le \alpha_j \quad \forall j,t \qquad (7)$$

Limit the maximum number of tool copies available for each *j-th* tool.

$$L_{jmt} \in \{0,1\} \quad \forall j,m,t$$

$$p_{jmt} \ge 0 \ \forall j,m,t; \ x_{it} \ge 0 \ \forall i,t; \ S_i \ge 0 \ \forall i$$

The Machine Loading Problem

$$\max \quad \sum_i \sum_t w_i x_{it} - \sum_i C_i S_i - \sum_i \sum_t h_{it} x_{it} \qquad (1)$$

subject to:

$$\sum_j L_{jmt} k_j \le K_m \quad \forall m,t \qquad (2)$$

$$\sum_t x_{it} = D_i - S_i \quad \forall i \qquad (3)$$

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Compute the production shortage (difference with the product demand)

The Machine Loading Problem

$$\max \quad \sum_i \sum_t w_i x_{it} - \sum_i C_i S_i - \sum_i \sum_t h_{it} x_{it} \qquad (1)$$

$$\text{subject to:} \quad \sum_j L_{jmt} k_j \le K_m \quad \forall m,t \qquad (2)$$

$$\sum_t x_{it} = D_i - S_i \quad \forall i \qquad (3)$$

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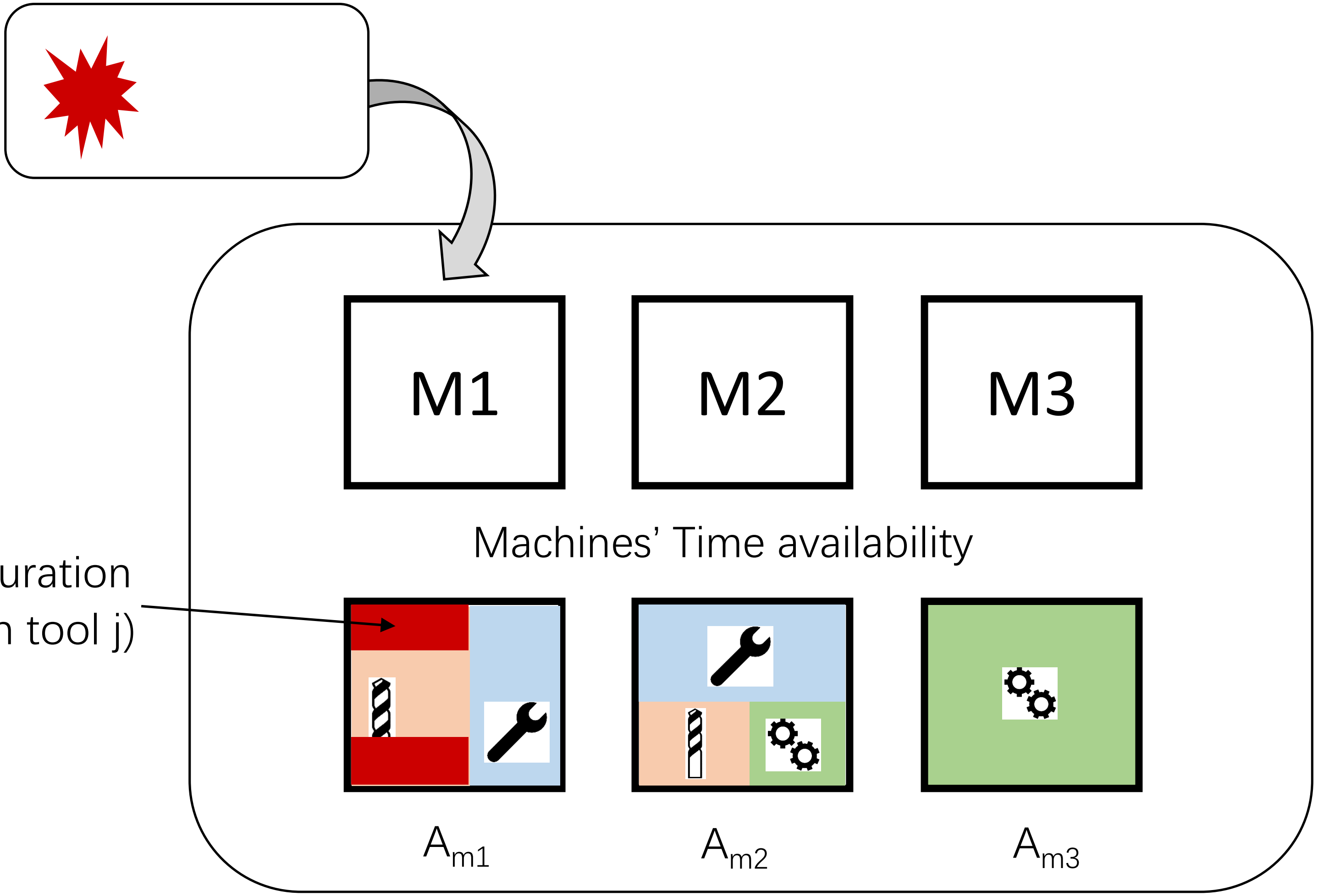
Guarantee that all the production is made within the time availability of the machines and respecting the loading of tools.

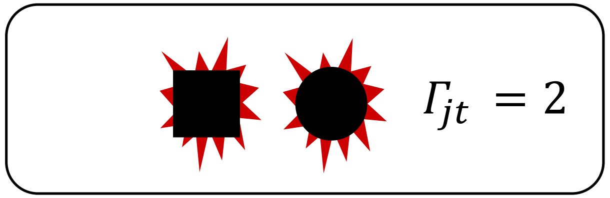
Assumptions: modeling the uncertainty

SOME
Disruptions



$\mathbf{FD_{ij}}$: failure duration (part type i on tool j)

<u>DEFINE</u> Γ_{jt} as the number of product types which are affected by failures on tool j over time period t



<u>ASSUME</u> that the disruption will affect the production of all the workpieces, once an affected part type is selected.

Assumptions: modeling the uncertainty

		$\boldsymbol{O_{ij}}$ **[TU/part]**		**TOTAL TIME [TU/part]**
		1 TU/part		10
$\Gamma_{jt} = 1$				13

$\mathrm{FD_{ij}} = 4$ time units

1 failure lasting
4 time units

It is equivalent to:		1.3 TU/part		13

Then we assume: $$\hat{O}_{ij} = \frac{\sum_t \bar{O}_{ij}\, x_{it} + \Gamma_{jt}\,(FD_{ij} - \bar{O}_{ij})}{\sum_t \bar{O}_{ij}\, x_{it}}$$

$\hat{O}_{ij} = 0.3$ time units

$\delta_{ij} = \hat{O}_{ij}/\bar{O}_{ij} = 30\%$

We can use $$\hat{O}_{ij} = \frac{\bar{O}_{ij}\, D_i + \Gamma_{jt}\,(FD_{ij} - \bar{O}_{ij})}{\bar{O}_{ij}\, D_i}$$ to avoid non-linearities

How to estimate the cardinality and the intervals?

We could estimate the **processing time range** $[\bar{O} - \hat{O}, \bar{O} + \hat{O}\ ]$

with $\widehat{O_{ij}} \approx MTTR_j$

We can estimate the **cardinality paramerers** from

$$\Gamma_{jt} \leftarrow MTBF_j$$

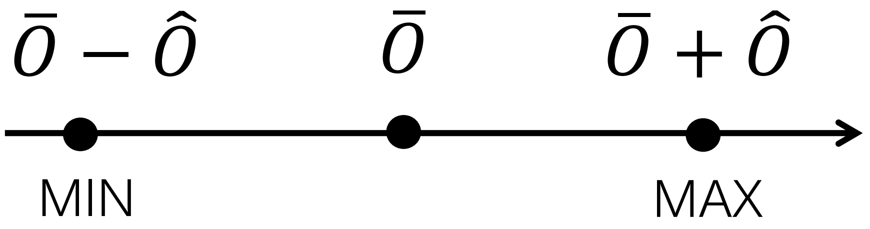
or from historical data available in the company

Assumptions: modeling the uncertainty

The solution rely on the availability of time

$$\sum_i O_{ij} x_{it} \le \sum_m p_{jmt} \quad \forall j,t$$

Assume processing time are lying on an interval



Construction of a robust constraint

$$\sum_i \bar{O}_{ij} x_{it} + \max_{U_{jt}} \left\{ \sum_{i \in U_{jt}} \hat{O}_{ij} x_{it} \right\} \le \sum_m p_{jmt} \quad \forall j,t$$

$$\begin{aligned} \max \quad & \sum_i \hat{O}_{ij} \tilde{x}_{it} z_{ijt} \\ \text{subject to:} \quad & \sum_i z_{ijt} \le \Gamma_{jt} \quad \forall j,t \\ & 0 \le z_{ijt} \le 1 \quad \forall i,j,t \end{aligned}$$

The INNER MAXIMIZATION PROBLEM selects the worst combination of events over a constrained number (cardinality), imposed by the user

Robust Model Formulation

Starting from the inner maximization problem, we can write its dual equivalent

PRIMAL

$$\begin{aligned} \max \quad & \sum_i \hat{O}_{ij} \tilde{x}_{it} z_{ijt} \\ \text{subject to:} \quad & \sum_i z_{ijt} \le \Gamma_{jt} \quad \forall j,t \\ & 0 \le z_{ijt} \le 1 \quad \forall i,j,t \end{aligned}$$

$$\Leftrightarrow$$

DUAL

$$\begin{aligned} \min \quad & \Gamma_{jt} r_{jt} + \sum_i q_{ijt} \\ \text{subject to:} \quad & r_{jt} + q_{ijt} \ge \hat{O}_{ij} \tilde{x}_{it} \quad \forall i,j,t \\ & r_{ij} \ge 0 \ \forall i,j; \ q_{ijt} \ge 0 \ \forall i,j,t \end{aligned}$$

At optimum, the objective functions of primal and dual problem coincide, thus:

$$\Gamma_{jt} r^*_{jt} + \sum_i q^*_{ijt} = \sum_i \hat{O}_{ij} x^*_{it} z^*_{ijt}$$

This is valid for any $\boldsymbol{x_{it}}$ thus also for $\boldsymbol{x^*_{it}}$

Robust Model Formulation

The Robust MLP becomes:

$$\max \quad \sum_i \sum_t w_i x_{it} - \sum_i C_i S_i - \sum_i \sum_t h_{it} x_{it}$$

subject to:

$$\sum_j L_{jmt} k_j \le K_m \quad \forall m,t$$

$$\sum_t x_{it} = D_i - S_i \quad \forall i$$

$$p_{jmt} \le L_{jmt} A_{mt} \quad \forall j,m,t$$

$$\sum_i \bar{O}_{ij} x_{it} + \Gamma_{jt} r_{jt} + \sum_i q_{ijt} \le \sum_m p_{jmt} \forall j,t$$

$$r_{jt} + q_{ijt} \ge \hat{O}_{ij} x_{it} \quad \forall i,j,t$$

$$\sum_j p_{jmt} \le A_{mt} \quad \forall m,t$$

$$\sum_m L_{jmt} \le \alpha_j \ \forall j,t$$

p_{jmt}	time spent for production on the tool $j \in J$ of machine $m \in M$ in period $t \in T$
q_{ijt}, r_{ij}	auxiliary dual-variables
S_i	shortage of production of product type $i \in I$
L_{jmt}	Boolean variable that is 1 when tool $j \in J$ is loaded on machine $m \in M$ in period $t \in T$
v_{ijt}	time spent for production of product type $i \in I$ over tool $j \in J$ in period $t \in T$
z_{ijt}	auxiliary primal variable
x_{it}	quantity of product type $i \in I$ produced in period $t \in T$
α_j	number of tool copies available for tool $j \in J$
Γ_{jt}	cardinality parameter of tool $j \in J$ in period $t \in T$
A_{mt}	available time for production on machine $m \in M$ in period $t \in T$
C_i	shortage cost per product type $i \in I$
D_i	total demand of product type $i \in I$
k_j	number of slots required by tool $j \in J$
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O_{ij}	processing time of one unit of product $i \in I$ on tool $j \in J$
h_{it}	holding cost per part of product type $i \in I$ in period $t \in T$ over the remaining time horizon
w_i	total earning per part for the production of product type $i \in I$

Case study

We have tested the robust MLP using a real case from the literature (Das et al. 2009)

- 12 product types, 12 tools types, 5 time periods (9-hour shifts, so A_{mt} = 540 min)
- For each product type, the weight, the shortage cost, and the holding cost are wi = w = 30 €/unit, Ci = C = 40 €/unit, and hit = h = 0, respectively
- Each product requires between 2 and 5 tool types

Assumptions

- Tool slots capacity is not affecting the problem solution.
- Only one copy of each tool type is available
- Since holding costs are null, the arrangement of production over time is pattern-less. Thus, quantities have been aggregated over the time period.

*The model has been solved on a computer equipped with processor Intel Core i7 @2.5Ghz and 8GB of installed RAM, using IBM ILOG CPLEX v12.5. The computational times are very short, ranging between 5.13 s and 5.82 s for all runs.

Case study

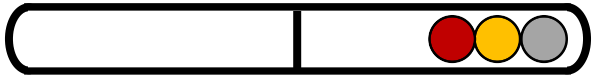
Product data				Tool slots	
Product	D_i [part]	PT_i [min/part]		Tools	k_j
1	160	14		1	4
2	4	11.9		2	2
3	8	11.9		3	2
4	8	7.7		4	2
5	40	7.9		5	2
6	4	14.8		6	1
7	4	11.3		7	1
8	20	7.8		8	1
9	20	8.3		9	1
10	8	6.5		10	1
11	8	20.3		11	3
12	4	15.4		12	3

Product	Nominal processing times ($\bar{O}_{ij}$) [min/part]											
	Tools											
Prod. Types	4.61	2	2.74	4.73	0	0	0	0	0	0	0	0
	2.46	2.51	0	4.19	2.7	0	0	0	0	0	0	0
	4.4	0	0	2.86	4.62	0	0	0	0	0	0	0
	0	0	0	0	0	3.12	1.95	2.64	0	0	0	0
	0	0	0	0	0	0	0	0	4.29	3.68	0	0
	0	0	0	0	0	0	0	0	3.15	5.39	2.18	4.14
	0	0	0	0	0	0	0	0	0	2.7	2.44	6.18
	0	0	1.92	2.59	3.3	0	0	0	0	0	0	0
	0	0	3.04	5.28	0	0	0	0	0	0	0	0
	4.45	2.1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	5.42	4	0	0	0	0	5.48	5.47
	0	0	0	0	0	0	0	4.22	3.62	3.92	3.72	0


DETERMINISTIC SOLUTION
1
0,5
0
1 2 3 4 5 6 7 8 9 10 11 12
ROBUST SOLUTION (Γ =1)
1
0,5
0
1 2 3 4 5 6 7 8 9 10 11 12

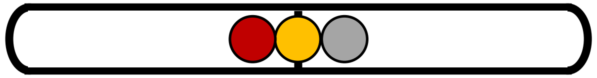
Results

(a) Deterministic case				(b) $x_i^*(\delta = 0.1)$				
i	D_i	PT_i	x_i^*	Γ=1	Γ=2	Γ=3	Γ=4	Γ=5
1	160	14	124.8	103.8	108.6	107.6	107.4	107.3
2	4	11.9	4	4	4	4	4	4
3	8	11.9	8	8	8	8	8	8
4	8	7.7	8	8	8	8	8	8
5	40	7.9	40	40	40	40	40	40
6	4	14.8	0	4	0	0	0	0
7	4	11.3	4	4	4	4	4	4
8	20	7.8	20	20	20	20	20	20
9	20	8.3	20	20	20	20	20	20
10	8	6.5	8	8	8	8	8	8
11	8	20.3	0	0	0	0	0	0
12	4	15.4	0	2.6	0	0	0	0

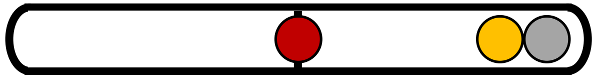


SOYSTER (1973)

DETERMINISTIC



CARDINALITY-CONSTRAINED



Results

Deterministic Case

(a) Deterministic case					
i	D_i	w_i	C_i	PT_i	x_i^*
1	160	30	40	14	124.8
2	4	30	40	11.9	4
3	8	30	40	11.9	8
4	8	30	40	7.7	8
5	40	30	40	7.9	40
6	4	30	40	14.8	0
7	4	30	40	11.3	4
8	20	30	40	7.8	20
9	20	30	40	8.3	20
10	8	30	40	6.5	8
11	8	30	40	20.3	0
12	4	30	40	15.4	0

The products that require less production time are selected first
[REMINDER: the production quantities are continuous variables]

Results

(a) Deterministic case				(b) $x_i^*(\delta = 0.1)$					(c) $x_i^*(\delta = 0.5)$					(d) $x_i^*(\delta = 1)$				
i	D_i	PT_i	x_i^*	Γ=1	Γ=2	Γ=3	Γ=4	Γ=5	Γ=1	Γ=2	Γ=3	Γ=4	Γ=5	Γ=1	Γ=2	Γ=3	Γ=4	Γ=5
1	160	14	124.8	103.8	108.6	107.6	107.4	107.3	63.3	58.5	62	61.2	60.8	37.2	27.4	30.6	29.4	28.8
2	4	11.9	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
3	8	11.9	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
4	8	7.7	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
5	40	7.9	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
6	4	14.8	0	4	0	0	0	0	4	4	0	0	0	4	4	0	0	0
7	4	11.3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
8	20	7.8	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
9	20	8.3	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
10	8	6.5	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
11	8	20.3	0	0	0	0	0	0	2.7	0	0	0	0	4.5	1.8	0	0	0
12	4	15.4	0	2.6	0	0	0	0	4	3.5	0	0	0	4	4	0	0	0

The demand for some products is always satisfied (2, 3, 4, 5, 7, 8, 9, 10)

Results

(a) Deterministic case					(d) $x_i^*(\delta = 1)$				
i	D_i	PT_i	x_i^*		Γ=1	Γ=2	Γ=3	Γ=4	Γ=5
1	160	14	**124.8**		**37.2**	**27.4**	**30.6**	**29.4**	**28.8**
2	4	11.9	4		4	4	4	4	4
3	8	11.9	8		8	8	8	8	8
4	8	7.7	8		8	8	8	8	8
5	40	7.9	40		40	40	40	40	40
6	4	14.8	**0**		**4**	**4**	**0**	**0**	**0**
7	4	11.3	4		4	4	4	4	4
8	20	7.8	20		20	20	20	20	20
9	20	8.3	20		20	20	20	20	20
10	8	6.5	8		8	8	8	8	8
11	8	20.3	0		4.5	1.8	0	0	0
12	4	15.4	0		4	4	0	0	0

- For certain cardinality values, the ranking changes and forces the saturation of the demand of other products (for example, product 6 over product 1)
- This is due to the selection of the worst combination of constrained events, which is done by solving the master problem.

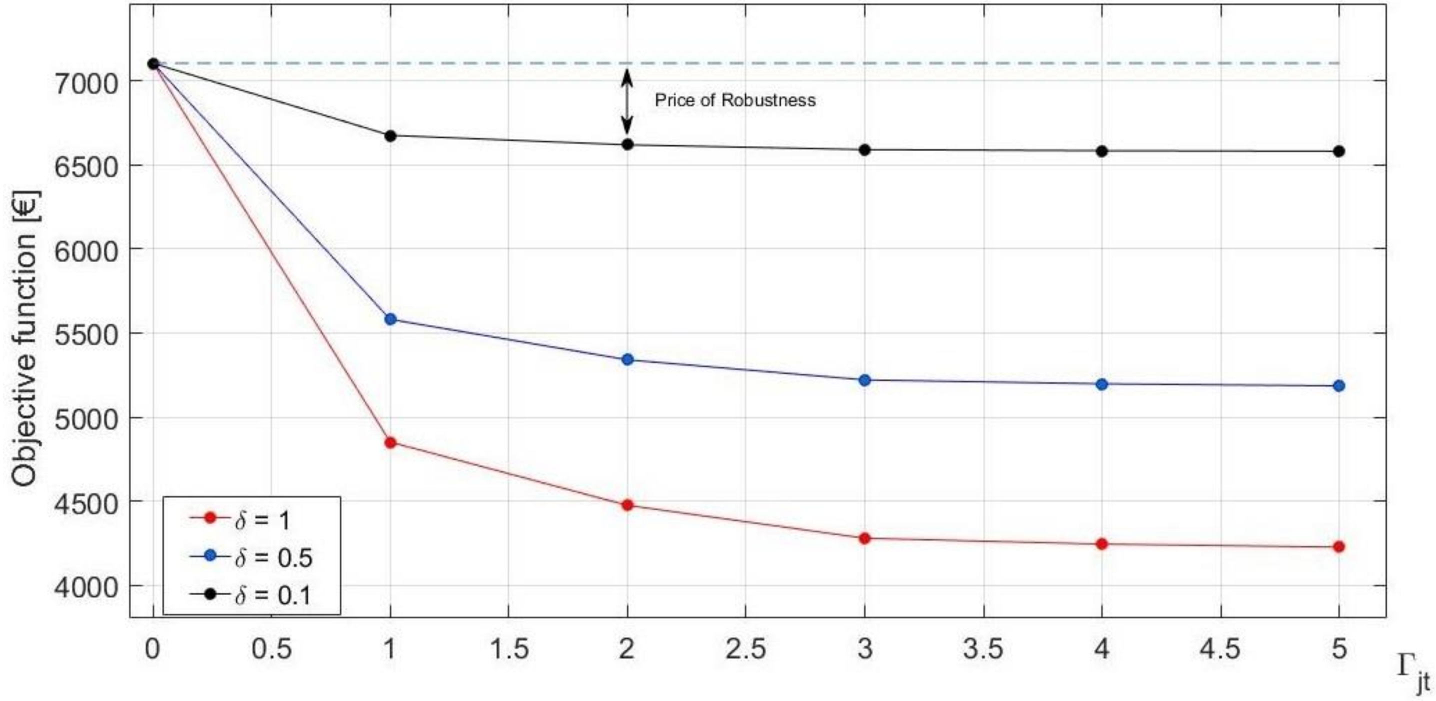
Results

(a) Deterministic case				(d) $x_i^*(\delta = 1)$				
i	D_i	PT_i	x_i^*	Γ=1	Γ=2	Γ=3	Γ=4	Γ=5
1	160	14	124.8	37.2	27.4	30.6	29.4	28.8
2	4	11.9	4	4	4	4	4	4
3	8	11.9	8	8	8	8	8	8
4	8	7.7	8	8	8	8	8	8
5	40	7.9	40	40	40	40	40	40
6	4	14.8	0	4	4	0	0	0
7	4	11.3	4	4	4	4	4	4
8	20	7.8	20	20	20	20	20	20
9	20	8.3	20	20	20	20	20	20
10	8	6.5	8	8	8	8	8	8
11	8	20.3	0	4.5	1.8	0	0	0
12	4	15.4	0	4	4	0	0	0

In other cases, the ranking is such only till an equilibrium condition, after which the original priority ranking is maintained.

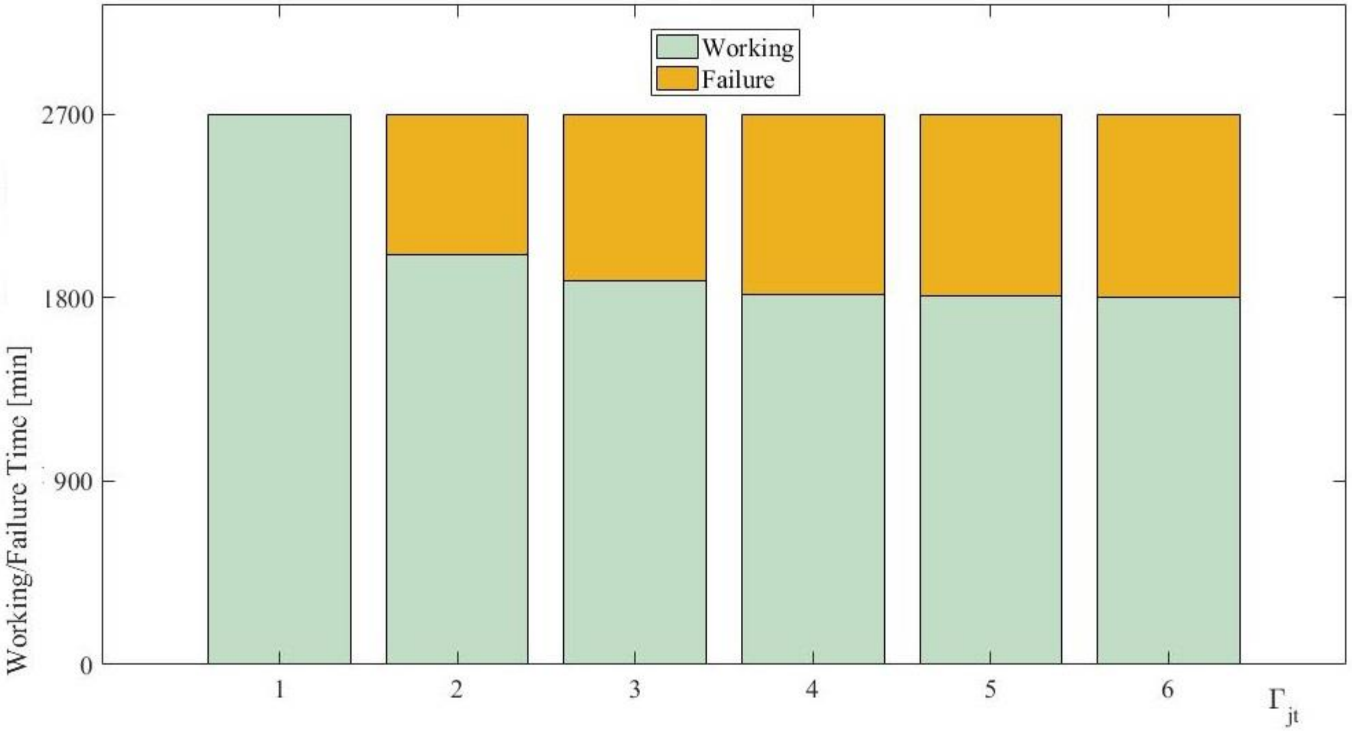
Results

$\delta_{ij} = \hat{O}_{ij}/\bar{O}_{ij}$



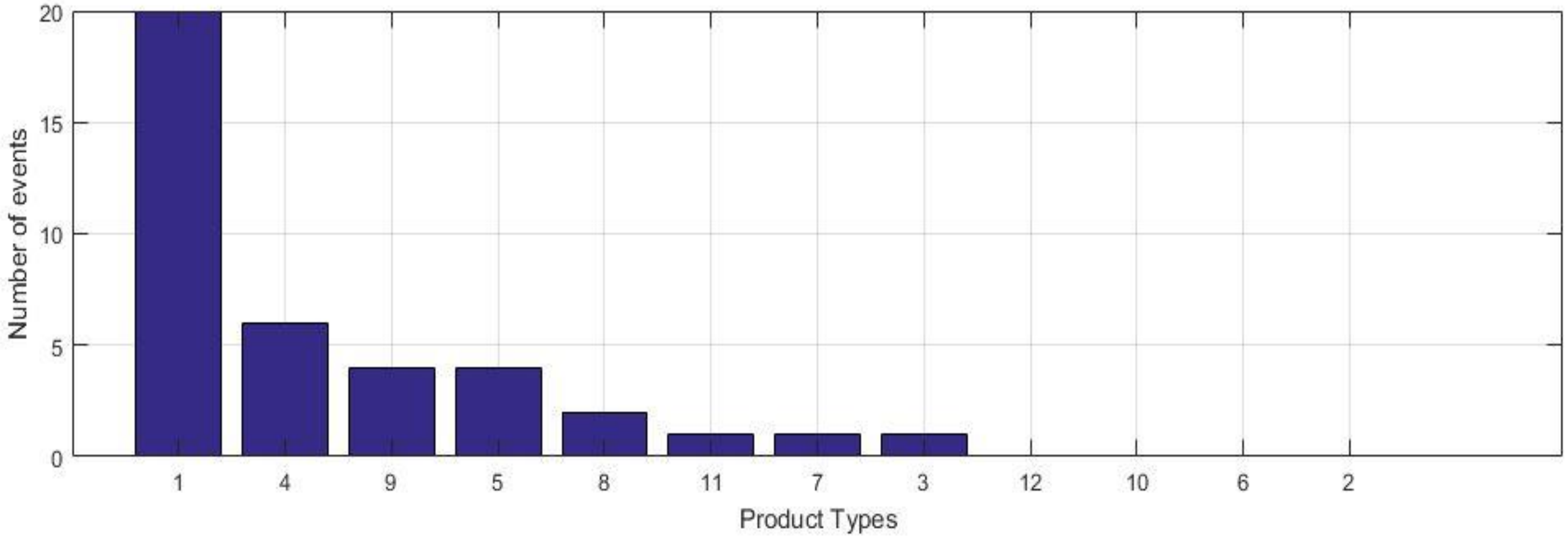
- Given a certain δ, the O.F. decreases as the cardinality increases.
- With increasing cardinality, since the time availability is limited and saturated, the total number of parts produced decreases together with the O.F.
- For any Γ, the distance between the OF of the deterministic solution and the one of the robust solution is the price of robustness. The marginal price of robustness decreases in cardinality.

Results



- Machine avaliability is always saturated.
- Failure time increases in cardinality, and less time is left for production
- With maximum cardinality, we obtain the expected ratio failure/working time (given by δ = 0.5)

Results



- For a practical application, it is possible to evaluate the products that are mostly subject to risk.
- This is done by evaluating how many events occur on a specific part type.
- The user can react, for example by providing multiple copies of tools for the production of critical products.

Final Remarks and Future Developments

What to do next

- **Application** of the robust MLP to a real case (on-going)
- **Comparison** with other existing methodologies
- **Validation** of the approach with evaluation of the performance of robust solutions using parameters' realization's scenarios.
- **Extension** of cardinality sets. It is possible to consider sets over single work-pieces.
- **Sensitivity analisys** on holding costs.

Managerial Insights

- The computation of the price of robustness can help the user to choose which production plan to implement
- The analisys can provide indications over which products are more risky: the user can prevent the most distructive events by focusing on critical tools

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