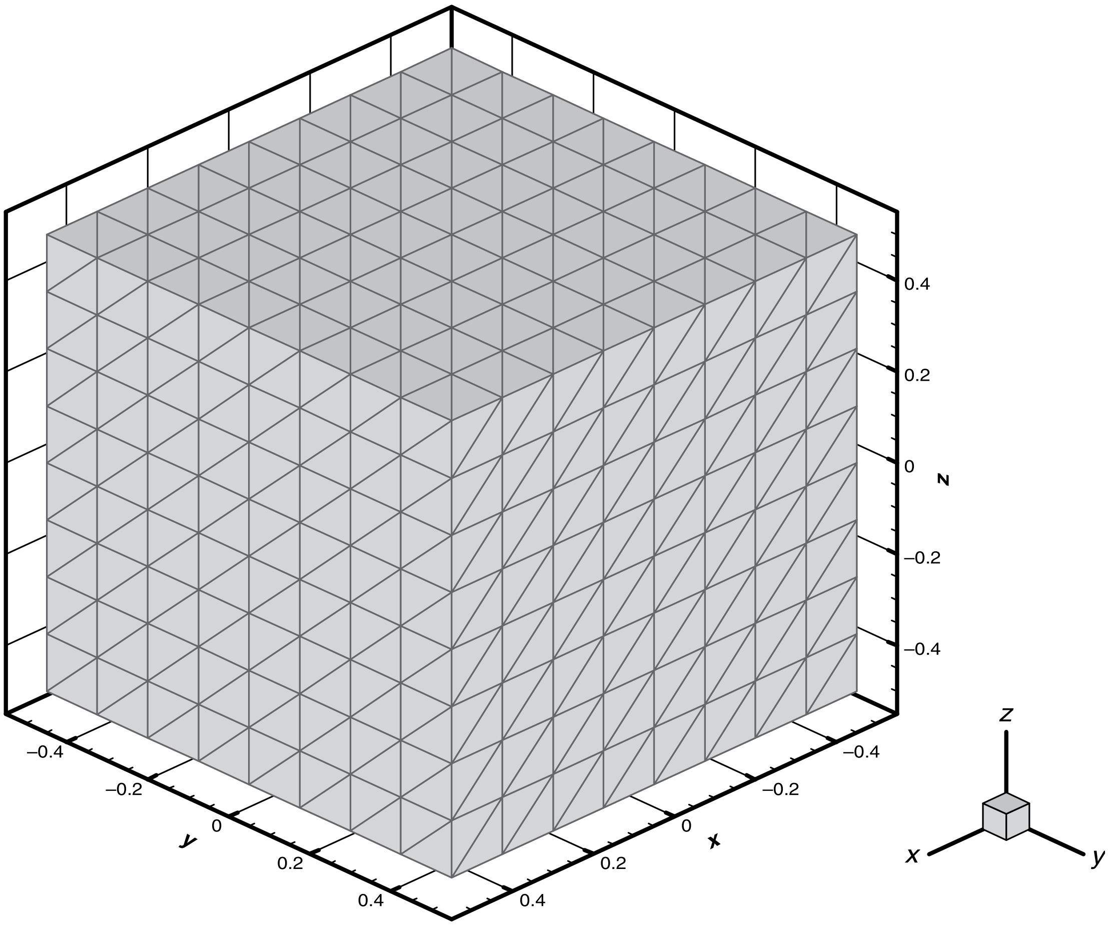

Figure 2.15. A cube meshed with 768 elements and with linear potential in the x direction.

2.12.3 Potential Field in a Cube

Two 3D examples are given next using a BEM program with constant surface elements [59]. A cube is shown in Figure 2.15, which is a simple interior problem used to show the accuracy of the 3D code with constant elements. The cube has an edge length = 1 and is applied with a linear potential $\phi(x, y, z) = x$ on all surfaces. The normal derivative q for this problem should be 1 on the surface at $x = 0.5$ and -1 on the surface at $x = -0.5$.

Table 2.2 shows the results obtained with the conventional BEM and by using the CBIE, HBIE, and dual BIE for BEM meshes with increasing numbers of elements. One can conclude from these results that the HBIE and the dual BIE are equally as accurate as the CBIE. Note that constant triangular elements are used in this study. If linear or quadratic elements were applied, a few elements should have been sufficient for obtaining results with a similar level of accuracy because of the specified linear field.

2.12.4 Electrostatic Field Outside a Conducting Sphere

A single conducting sphere model (Figure 2.16) is shown next. This is a simple exterior problem with curved boundaries. The conducting sphere has a radius

Table 2.2. *Results for the cube with a linear potential in the x direction*

Model		Normal derivative at (0.5, 0, 0)		
Elem/edge	Total DOFs	CBIE	HBIE	Dual BIE
2	48	1.08953	1.07225	1.06800
4	192	0.99124	1.00624	0.99754
8	768	0.99825	1.00438	0.99894
12	1728	0.99908	1.00327	0.99934
16	3072	0.99942	1.00260	0.99953
20	4800	0.99959	1.00216	0.99963
24	6912	0.99969	1.00185	0.99970
Exact Value			1.00000	

$a = 1$, and a constant electric potential $\phi_0 = 1$ is applied on its surface. The analytical solution of the electric field outside the sphere is $\phi = (a/r)\phi_0$, with r being the distance from the center of the sphere, which gives a charge density on the surface equal to 1, assuming the dielectric constant $\varepsilon = 1$.

Table 2.3 gives the BEM results of the charge density at the point (1, 0, 0) on the surface of the sphere. For this problem, the dual BIE is slightly less accurate than the CBIE because of the curved surface that cannot be represented accurately by constant elements and can cause the evaluations of hypersingular integrals to be less accurate [59].

Several numerical examples for solving both 2D and 3D potential problems are presented in this subsection. Constant elements are used for all the examples, and reasonably accurate BEM solutions are obtained. Linear or quadratic elements can be applied to improve the accuracy of the BEM solutions (see problems). These examples are used again in the next chapter on

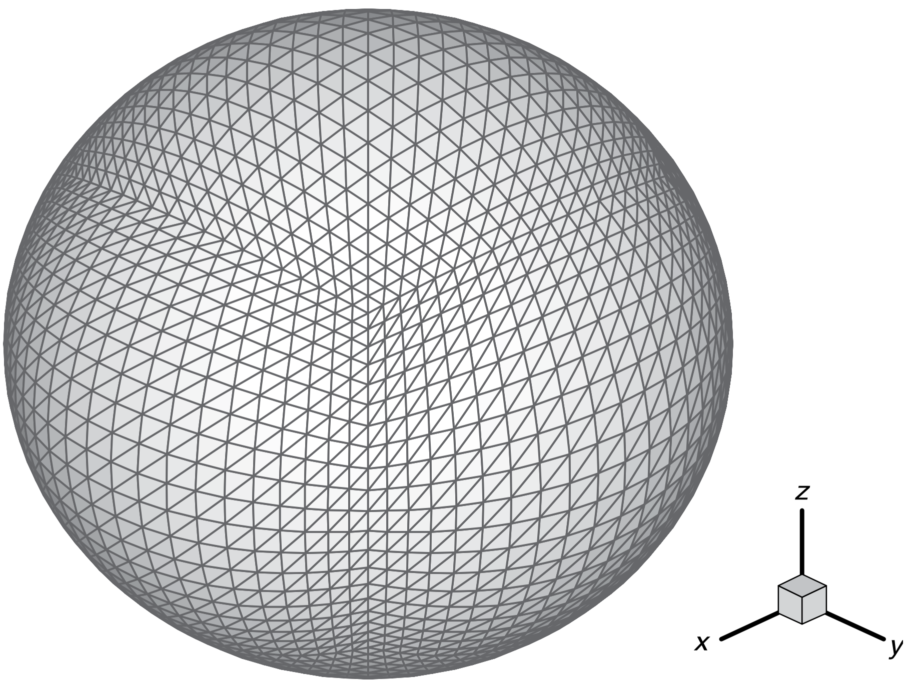

Figure 2.16. A spherical perfect conductor meshed with 4800 elements.

Table 2.3. *Results for the single perfect conducting sphere*

Model DOFs	Charge density at (1, 0, 0)	
	CBIE	Dual BIE
768	0.98749	0.95086
1728	0.99377	0.96609
3072	0.99634	0.97431
4800	0.99761	0.97937
6912	0.99832	0.98278
Exact Value	1.00000	

the fast multipole solution techniques to demonstrate the computational efficiencies of the fast multipole BEM for solving large-scale problems.

2.13 Summary

In this chapter, the BIE formulations for solving potential problems are presented. It is shown that the partial differential equation (Poisson equation or Laplace equation) can be transformed into BIEs with the help of the fundamental solution and the Green's identity. Both the conventional BIE and the hypersingular BIE formulations are discussed. Weakly singular forms of these BIEs are also presented to show that singular integrals in the BIE formulations and therefore their BEM solutions can be avoided altogether if the integral terms are arranged properly. The discretization procedures are discussed with constant, linear, and quadratic line elements for 2D problems and with linear and quadratic surface elements for 3D problems. Programming for the BEM using the conventional approach is discussed briefly, and several numerical examples are presented.

This chapter is the basis for all other chapters dealing with fast multipole solution techniques for potential, elasticity, Stokes flow, and acoustic wave problems. The basic ideas, BIE formulations, BEM discretization procedures, programming, and solutions for those problems are similar to these discussed in this chapter. Therefore, it is very important to understand all of the material covered in this chapter before moving on to the following chapters.

Problems

2.1. Show that $G(\mathbf{x}, \mathbf{y})$ given by Eq. (2.5) does satisfy Eq. (2.4); that is, $\nabla^2 G(\mathbf{x}, \mathbf{y}) = 0$, for $r \neq 0$; and near $r = 0$, $-\nabla^2 G(\mathbf{x}, \mathbf{y})$ behaves like a $\delta(\mathbf{x}, \mathbf{y})$ function. For example, $-\int_{V_\varepsilon} \nabla^2 G(\mathbf{x}, \mathbf{y}) dV(\mathbf{y}) = 1$, where V_ε is a circular region centered at $\mathbf{x}$ with radius ε.