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SPEED CONTROL OF A SE-DC MOTOR FOR TRACTION UNIT

Report

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DYNAMICS OF ELECTRICAL MACHINES AND DRIVES - 2020/2021

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1 System model

1.1 Motion characteristics

A DC separately excited motor is used to move an ATM tramway vehicle "Carelli 1928" with the following characteristics:

Motor rated speed	314 rad/s
Efficiency	0.9
Armature circuit time constant	10 ms
Excitation circuit rated voltage	120 V
Excitation circuit rated current	1 A
Excitation circuit time constant	1 s

Table 1: Carachteristics of the vehicle *Carelli 1928*

The tram should accelerate from 0 to 60 km/h in 25 s. So that means having the following nominal acceleration a_n:

$$a_n = \frac{\Delta v}{\Delta t} = \frac{60\ km/h - 0\ km/h}{25\ s} \cdot \frac{1000\ m}{1\ km} \cdot \frac{1\ h}{3600\ s} = 0.667\ m/s^2 \tag{1}$$

The tramway mass is 10 T and it should be considered 200 people as the tramway trainload with a standard weight of 80 kg. Then, the total mass m_t is:

$$m_t = 10\ T + 200 \cdot 80\ kg = 26000\ kg \tag{2}$$

The friction force is proportional to the speed and at rated speed (60 km/h or 314 rad/s) is 1/3 of traction force. The power balance is used in order to relate the traction and the friction forces and is stablished as the nominal power; remembering that $v_n = 60\ km/h = 16.7\ m/s$. Thus:

$$P_{friction} = \frac{1}{3} P_{traction} \tag{3}$$

$$P_{traction} = F_{traction} \cdot v_n = (m_t \cdot a_n) \cdot v_n = 289\ kW \tag{4}$$

So the nominal power is:

$$P_n = P_{traction} + P_{friction} = \frac{4}{3}P_{traction} = 385\ kW \tag{5}$$

1.2 Electrical characteristics

In order to compute the nominal excitation torque the rated armature current is needed, so the following power balance is given computing the efficiency:

$$P_n = \eta \cdot i_n \cdot V_n \tag{6}$$

Where

$$i_n = \frac{P_n}{\eta \cdot V_n} = 535\ A \tag{7}$$

Also, the rated torque is:

$$T_n = \frac{P_n}{\Omega_n} = 920\ N \tag{8}$$

Furthermore, as $T_e = K \cdot i_e \cdot i_a$ it is possible to get the motor coefficient K, having $i_e = 1\ A$:

$$K = \frac{T_n}{i_n \cdot i_e} = 1.72\ N \cdot m/A^2 \tag{9}$$

Assuming that the loss of efficiency in the power is due to the Joule losses, then it is said that:

$$(1-\eta)V_n \cdot i_n = R_a \cdot i_n^2 \tag{10}$$

Then,

$$R_a = \frac{(1-\eta)V_n}{i_n} = 0.112\ \Omega \tag{11}$$

Taking into account that $\tau_a = 0.01\ s$, then L_a can be computed:

$$L_a = R_a \cdot \tau_a = 0.112\ mH \tag{12}$$

Finally, another power balance can be determined to get the constant emf E value:

$$\eta V_n \cdot i_n = E_n \cdot i_n$$

So,

$$E_n = \eta V_n = 540\ V \tag{13}$$

In the other hand, for the excitation circuit (with $\tau_e = 1\ s$ $V_{en} = 120\ V$ and $i_{en} = 1\ A$, the value of the resistance R_e and the inductance L_e are obtained as follows:

$$R_e = \frac{V_{en}}{i_{en}} = 120\ \Omega \tag{14}$$

$$L_e = R_e \cdot \tau_e = 120\ H \tag{15}$$

1.3 Mechanical characteristics

The mechanical characteristics of the system are described by the equation:

$$J_{eq} \cdot \dot{\Omega} + \beta \cdot \Omega = T_e - T_l$$

Where T_e is given as an output of the motor, T_l can be described by a profile and J_{eq} and β can be obtained as follows. Having the energy balance based on the linear and the rotating elementes of the system:

$$\frac{1}{2} \cdot m_t \cdot v_n^2 = \frac{1}{2} \cdot J_{eq} \cdot \Omega_n^2 \tag{16}$$

So the equivalent inertia J_{eq} seen from the rotor of the motor is:

$$J_{eq} = m_t \cdot \frac{v_n^2}{\Omega_n^2} = 73.5 \ kg \cdot m^2 \tag{17}$$

The friction coefficient can be obtained in this way, remembering that the friction torque is 1/3 of the traction torque ($T_{traction} = T_n$):

$$T_{friction} = \Omega \cdot \beta = \frac{1}{3} \cdot T_{traction} \tag{18}$$

So,

$$\beta = \frac{1}{3} \cdot \frac{T_n}{\Omega_n} = 0.977 \ N \cdot m \cdot s \tag{19}$$

2 Load torque and speed profiles

As shown in the Figure 1, where $\theta = arctan(\frac{slope\%}{100})$, the force F_L is equal to $F_L = m_t \cdot g \cdot sin(\theta)$. So, the value of that force depends on the value of $theta$, which can be described through a profile.

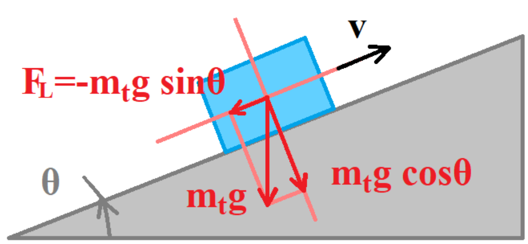

Figure 1: Free body diagram of the vehicle

The related load torque is:

$$T_L = \frac{P_L}{\Omega} = \frac{F_L \cdot v}{\Omega} = \frac{m_t \cdot g \cdot sin(\theta) \cdot v}{\Omega} \tag{20}$$

Thus, the profile was built using the following logic where the values of the speeds (v and Ω) were taken from the output of the whole systems and the value of the slope was obtained based on logical ports that depend on the actual covered distance:

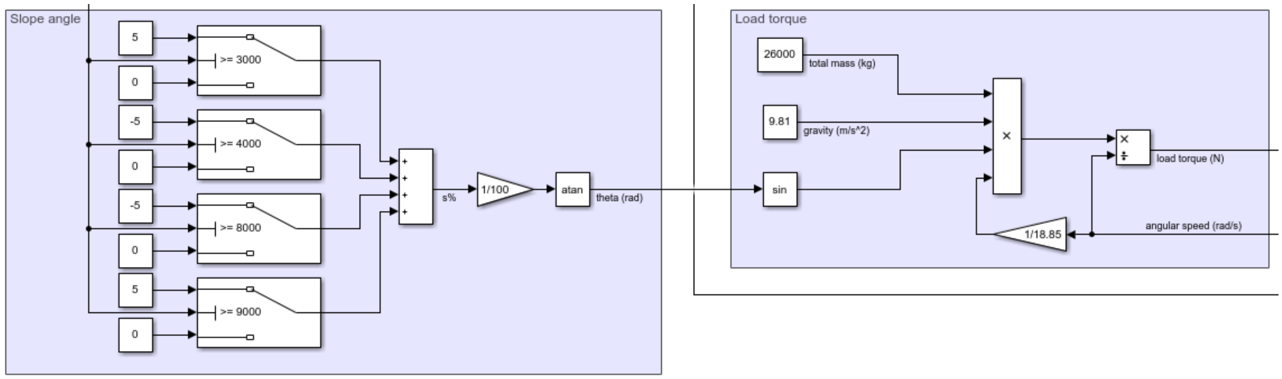

Figure 2: Load torque profile generator with feedback

So the load torque profile T_L is:

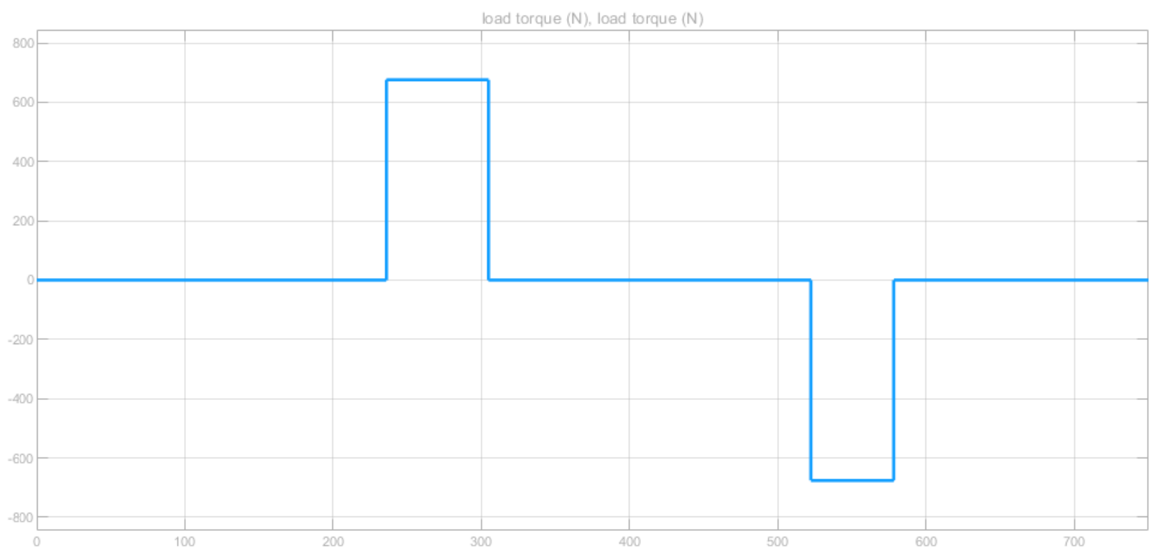

Figure 3: Load torque profile

Also, the relationship $J_{eq} \cdot \dot{\Omega} + \beta \cdot \Omega = T_e - T_L$ is hold, so:

$$\Omega = \frac{1}{J_{eq} \cdot s + \beta} \cdot (T_e - T_L) \tag{21}$$

In the same way, the angular speed profile was built based on three characteristics: the value of the distance to be covered in each profile section, the value of linear speed required and the actual at-the-time covered distance (called *cumulative distance*). Through some logical conditions, the linear speed profile is generated and, after a gain, the angular speed profile too.

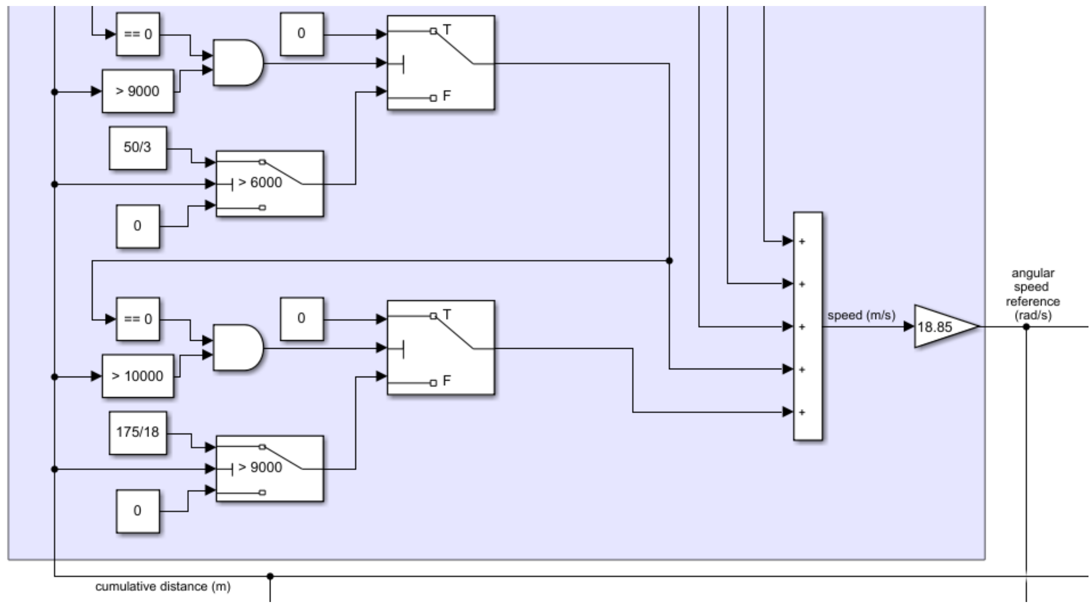

Figure 4: Section of the angular speed profile generator with feedback

Hence, the angular speed reference fulfills both the profile and the covered distance, because it integrates the output linear speed (via the angular speed) to get the real covered distance (taking into account that due to the system structure the covered distance in a section is not exactly the linear speed reference times the time):

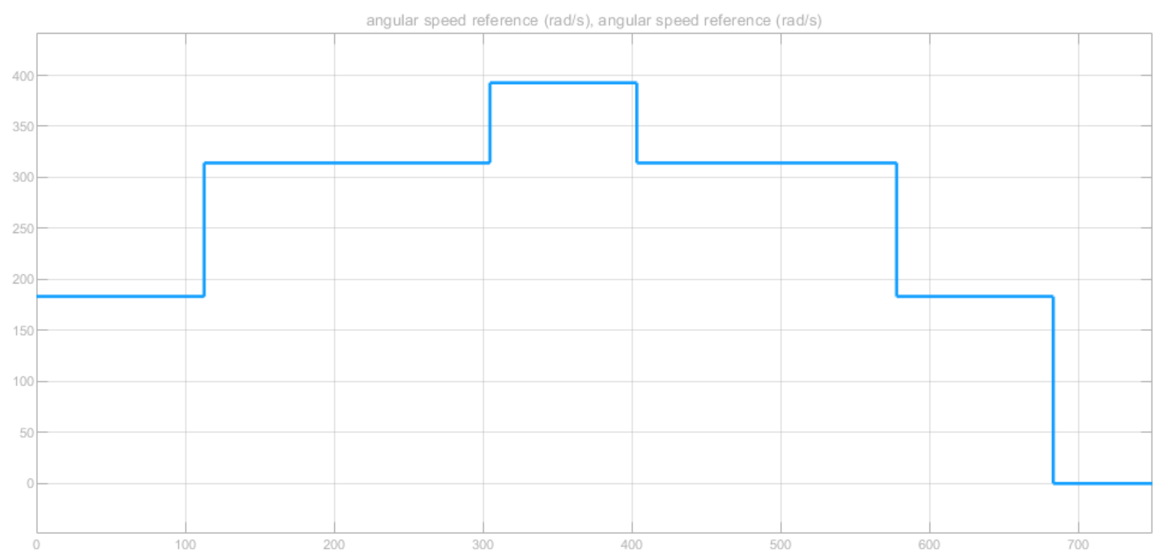

Figure 5: Angular speed profile

3 Control scheme

Finally, the implemented control scheme is:

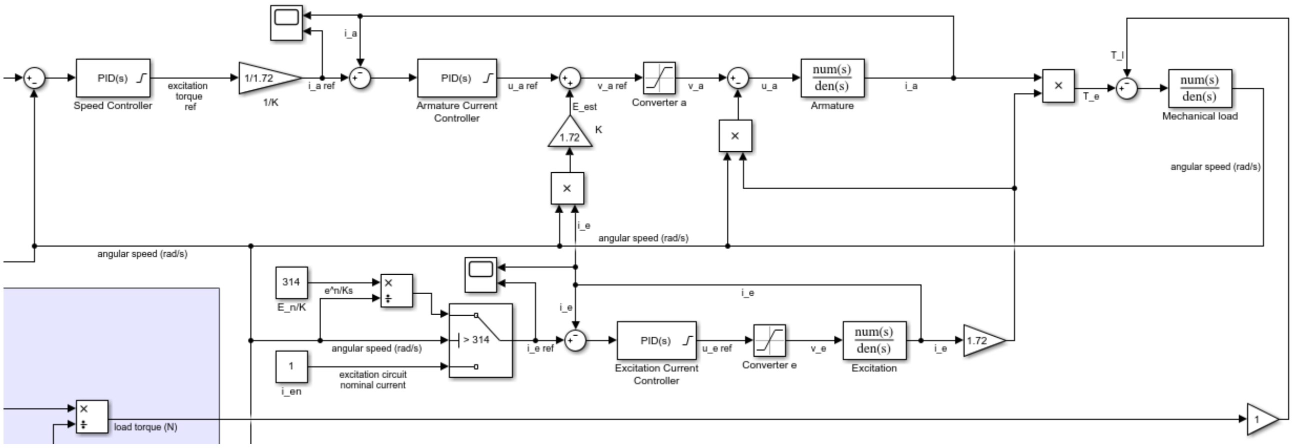

Figure 6: Designed control scheme

The respective plant transfer functions are shown in the Table 2, as well as the k_p and k_i values based on the described settling time τ_s and obtained through the *PIDTool* from *Matlab*.

Loop	Armature Current	Excitation Current	Angular Speed
Transfer function	$\frac{1}{R_a+L_a s} = \frac{1}{0.112+0.000112s}$	$\frac{1}{R_e+L_e s} = \frac{1}{120+120s}$	$\frac{1}{\beta+Js} = \frac{1}{0.977+73.5s}$
τ_s	0.01 s	0.5 s	10 s
Used k_p	0.0224	480	14.7
Used k_i	22.4	480	0.0516

Table 2: Characteristics of the plants and their respective regulators

The difference of at least one decade between the response of the armature and the excitacion current is respected. As for the angular speed (and its consequent linear speed), the requested time condition for the vehicle was to accelerate from 0 to 60 km/h in 25 s, so it was assumed that for reaching the 95% of that speed in 25 s, the settling time τ_s should be 10 s.

Therefore, the following profiles were obtained: the excitation current tracking as shown in Figure 7, the armature current tracking as shown in Figure 8 and the angular speed tracking as shown in Figure 9.

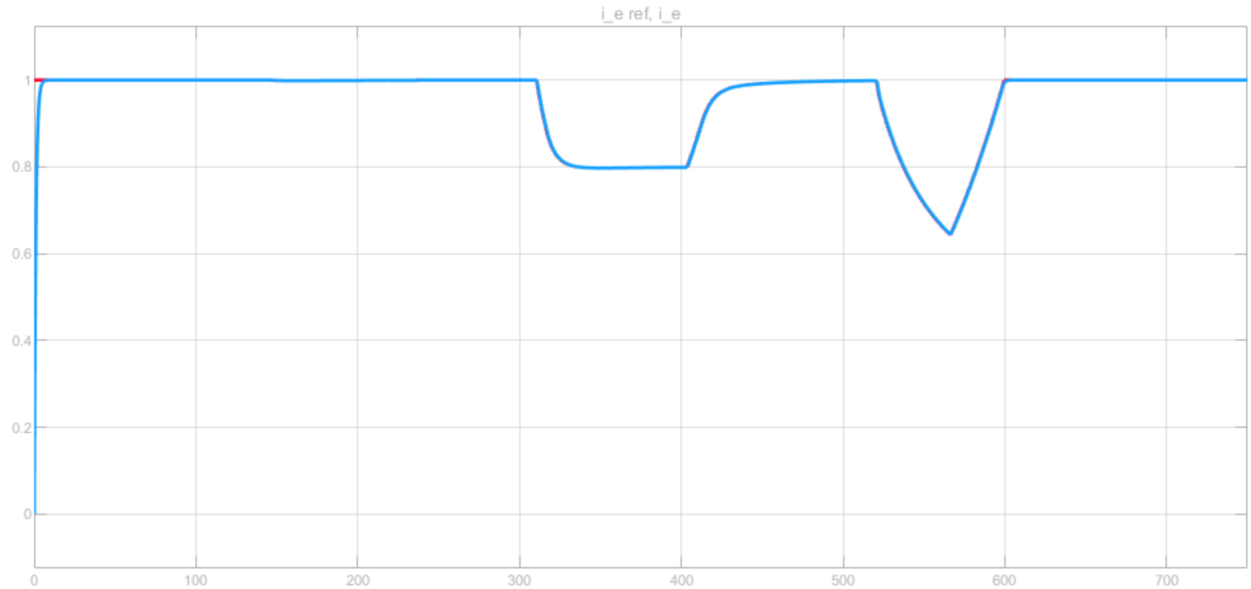

Figure 7: Excitation current profile tracking

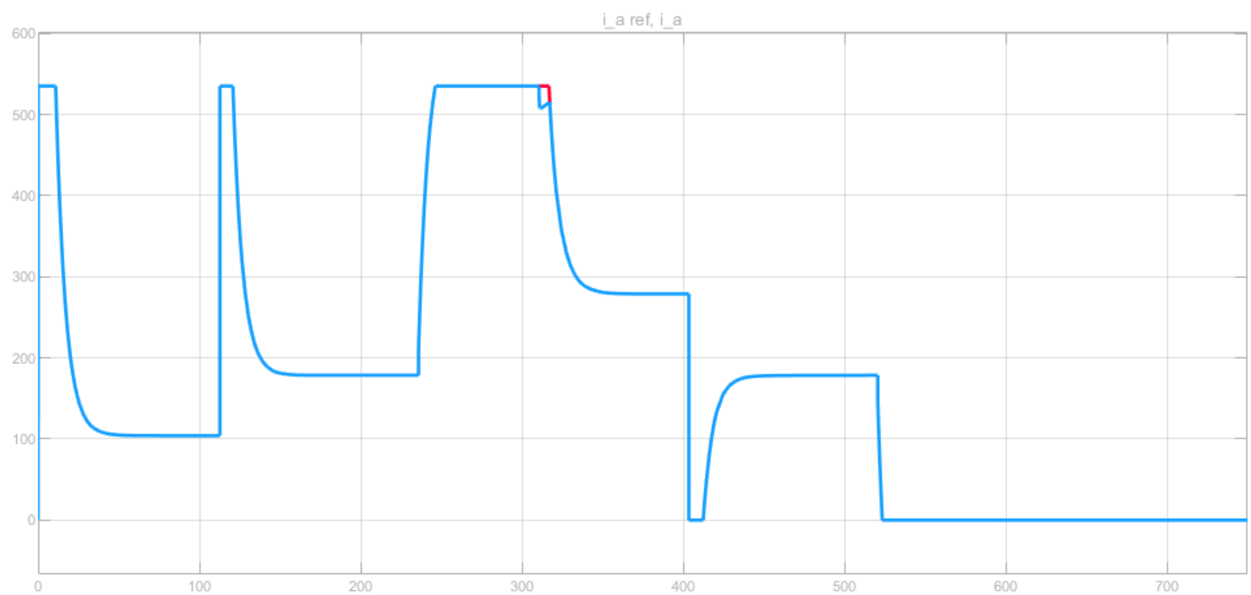

Figure 8: Armature current profile tracking

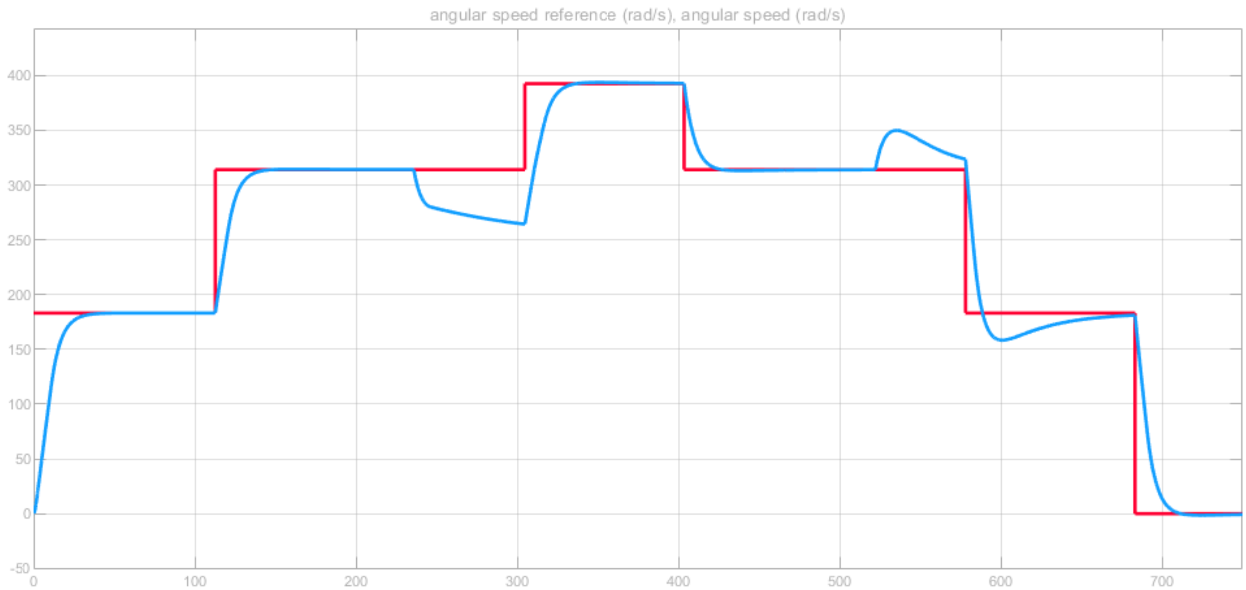

Figure 9: Angular speed profile tracking