Problem Set 3 Solutions

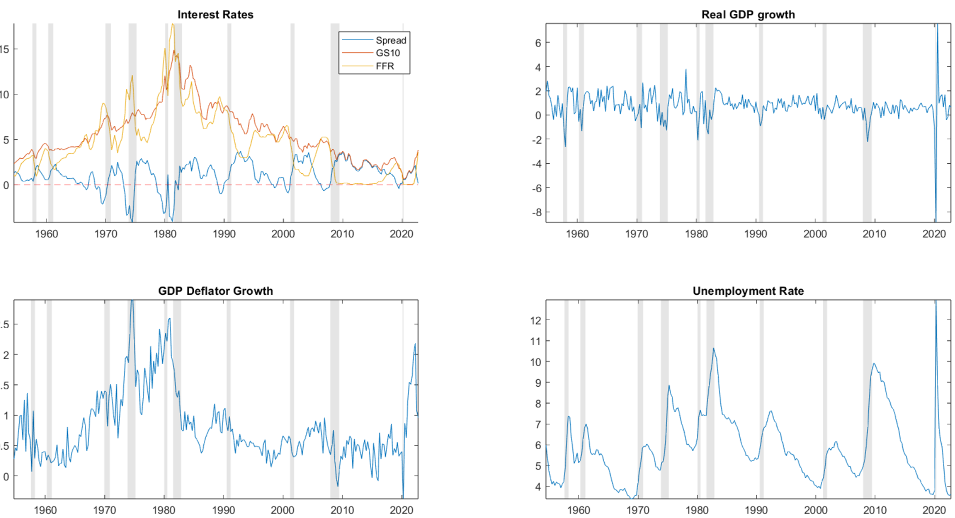
2024

From the FRED database of the St. Louis Fed download data (at quarterly frequencies) for GDP deflator (mnemonic GDPDEF), real GDP (mnemonic GDPC1), the unemployment rate (mnemonic UNRATE), the 10Y government bond (mnemonic GS10) and the federal funds rate (mnemonic FEDFUNDS).

1. Create the spread as the difference between the long and the short rate. Create the growth rates of real GDP and the GDP deflator. Plot the series.
2. Using the growth rates of real GDP, the unemployment rate, the inflation rate, the federal funds rate and the spread estimate a VAR(4).
3. Compute and plot the impulse response functions of the Wold shocks (the Wold representation).
4. Identify, using a Cholesky decomposition, the monetary policy shock as a shock that has no effect on GDP, unemployment and inflation contemporaneously.
5. Plot the (point estimate) impulse response functions and 68% confidence band constructed using bootstrap.

Exercise 1 Solution:

The long rate is given by GS10, the short rate is the effective Federal Funds Rate which the weighted average of the interest rate negotiated for overnight transactions between banks (recall that the target rate is a corridor).



Exercise 2 Solution:

The VAR(4) then looks like the following:

$$\begin{bmatrix} GDP_t \\ UNR_t \\ INF_t \\ FFR_t \\ SPR_t \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} + A_1 \begin{bmatrix} GDP_{t-1} \\ UNR_{t-1} \\ INF_{t-1} \\ FFR_{t-1} \\ SPR_{t-1} \end{bmatrix} + A_2 \begin{bmatrix} GDP_{t-1} \\ UNR_{t-2} \\ INF_{t-2} \\ FFR_{t-2} \\ SPR_{t-2} \end{bmatrix} + A_4 \begin{bmatrix} GDP_{t-3} \\ UNR_{t-3} \\ INF_{t-3} \\ FFR_{t-3} \\ SPR_{t-3} \end{bmatrix} + A_2 \begin{bmatrix} GDP_{t-4} \\ UNR_{t-4} \\ INF_{t-4} \\ FFR_{t-4} \\ SPR_{t-4} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \\ \epsilon_{4_t} \\ \epsilon_{5_t} \end{bmatrix}$$

We have the same regressors in each of the five equation lines which permits the use of our stacking + OLS approach to estimating the VAR. In MATLAB we use our `VAR` and `SUR` functions.

```
% Estimate the reduced-form VAR(4) with finaldata:

finaldata=[GDPC1_gr,UNRATE(2:end),GDPDEF_gr, ...
    FEDFUNDS(2:end),Spread(2:end)];

```

```
% We start from the second observation for the rates since we lost one
% observation by taking diff(log(.)) of GDPC1 and GDPDEF.

% First of all, we have to create the (T-p) x n and (T-p) x np+1 ...
    matrices
% of the SUR representation, i.e. Y and X respectively:

[T,N]=size(finaldata); % # of variables
p=4; % 4 lags
c=1; % including constant

[pi_hat,err]=VAR(finaldata,p,c); % VAR estimation
```

Recall that for the OLS estimation our matrices Y and X look like the following:

$$Y_{(T-p)\times N} = \begin{bmatrix} GDP_{p+1} & UNR_{p+1} & INF_{p+1} & FFRp+1 & SPR_{p+1} \\ GDP_{p+2} & UNR_{p+2} & INF_{p+2} & FFRp+2 & SPR_{p+2} \\ \vdots & \vdots & & & \vdots \\ GDP_T & UNR_T & INF_T & FFRT & SPR_T \end{bmatrix}$$

$$X_{(T-p)\times(1+pN)} = \begin{bmatrix} 1 & GDP_p & UNR_p & INF_p & FFR_p & SPR_p & \dots & GDP_1 & UNR_1 & INF_1 & FFR_1 & SPR_1 \\ 1 & GDP_{p+1} & UNR_{p+1} & INF_{p+1} & FFR_{p+1} & SPR_{p+1} & \dots & GDP_2 & UNR_2 & INF_2 & FFR_2 & SPR_2 \\ \vdots & \vdots & & & & & & & & & \vdots & \vdots \\ 1 & GDP_{T-1} & UNR_{T-1} & INF_{T-1} & FFR_{T-1} & SPR_{T-1} & \dots & GDP_{T-p} & UNR_{T-p} & INF_{T-p} & FFR_{T-p} & SPR_{T-p} \end{bmatrix}$$

Exercise 3 Solution:

To compute the Wold impulse responses we first check whether the VAR is stationary. We check the eigenvalues of the companion form matrix

$$F_{Np\times NP} = \begin{bmatrix} A_1 & A_2 \quad A_3 & A_4 \\ & & \\ I_{(Np-N)\times(Np-N)} & & 0_{(Np-N\times N)} \end{bmatrix}$$

In this case the largest eigenvalue is 0.97027 in absolute value, so the VAR is stable and therefore stationary and we can obtain the Wold representation:

$$y_t = A(L)^{-1} + A(L)^{-1}\epsilon_t = \nu + C(L)\epsilon_t$$

Using the companion form we can compute the matrices $C_0, C_1, C_2...$ which hold the impulse response coefficients we are interested in as

$$
\begin{aligned}
C_0 &= F^0(1:N, 1:N) = I_N \\
C_1 &= F(1:N, 1:N) = A_1 \\
C_2 &= F^2(1:N, 1:N) \\
&\vdots
\end{aligned}
$$

In `MATLAB` we implement this as usual:

```
% Wold representation impulse responses:

% Populate the companion form matrix
BigA=[pi_hat(2:end,:)'; eye(N*p-N) zeros(N*p-N,N)]; % BigA companion ...
    form, np x np matrix

% Check stability
ev = abs(eig(BigA));
evmax = ['The maximum eigenvalue is ', num2str(max(ev)), '.'];
disp(evmax)

hor=20;

C=zeros(N,N,hor+1);

for j=1:hor+1
    BigC=BigA^(j-1);
    C(:,:,j)=BigC(1:N,1:N); % Impulse response functions of the Wold ...
        representation
end

% Alternatively, use the function woldiirf
CC = woldirf(pi_hat, c, p, hor);
```

Finally, we compute bootstrapped confidence intervals, here the 68% bands. We do this using a simple resampling from the residuals we estimated in our VAR. Plotting the Wold IRF we obtain:

For example, here we have that for a 1% point increase in GDP growth (shock u_1) inflation increases by around 0.1% points after around 4-5 quarters, the unemployment rate goes down by 0.4% points, the Federal Funds Rate increases by around 0.6% points and the spread decreases by 0.3% points. Does this seem realistic?

Exercises 4 and 5 Solutions:

Now we are interested in a structural monetary policy shock which is defined here as a shock that has no contemporaneous effect on GDP, unemployment and inflation. The

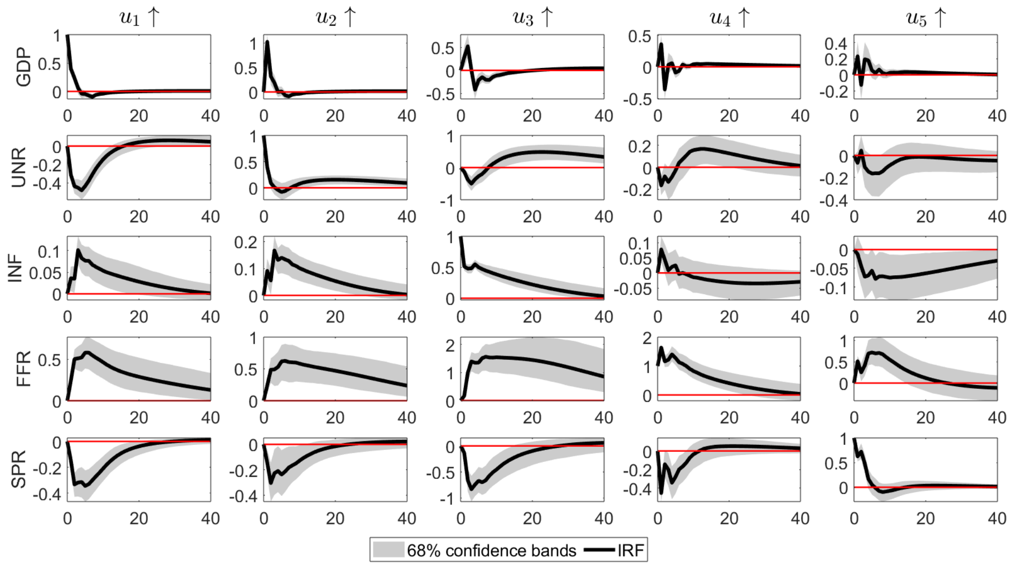


Figure 1: *Wold Impulse Responses*

spread is allowed to be affected by the monetary policy shock. So we will order the VAR as we did before:

$$Y = \begin{bmatrix} GDP \\ UNR \\ INF \\ FFR \\ SPR \end{bmatrix}$$

So we will take the fourth Cholesky shock to be our monetary policy shock. Notice that what we are doing here is called *partial identification* as opposed to *global identification*. This is because we are only making assumptions about one structural shock, when in theory there are 5 of them in this model. About all the others we are silent. Global identification with many shocks is not easy and many restrictions have to fall in place. However, it can add to the credibility of your identification strategy if it describes the other shocks well, too.

To implement this, we compute the variance-covariance matrix of the reduced shocks

$$\hat{\Omega} = \frac{\epsilon_t \epsilon_t'}{T - p - 1 - Np}$$

and then compute the Cholesky factor S of this matrix, s.t. $\hat{\Omega} = SS'$. Then we identify the structural shocks as w_t in

$$y_t = \nu + C(L)SS^{-1}\epsilon_t = \nu + C(L)Sw_t$$

where $Var[w_t] = E[S^{-1}\epsilon_t\epsilon_t' S^{-1'}] = S^{-1}\Omega S^{-1'} S^{-1'} = S^{-1}SS'S^{-1'} = I$. In MATLAB we implement this by postmultiplying the matrices of Wold coefficients we computed above by the matrix S.

```
% Cholesky:

omega=(err'*err)./(T - N*p-1-p); % Estimate of omega
S=chol(omega,'lower'); % Cholesky factorization, lower triangular matrix

D=zeros(N,N,hor+1);

for i=1:hor+1
    D(:,:,i)=C(:,:,i)*S; % Cholesky wold respesentation
end

% Alternatively use the function choleskyIRF
DD = choleskyIRF(C, S);
```

We bootstrap the confidence intervals in a similar manner as before, adding the identification step described above. This gives the following structural IRFs:

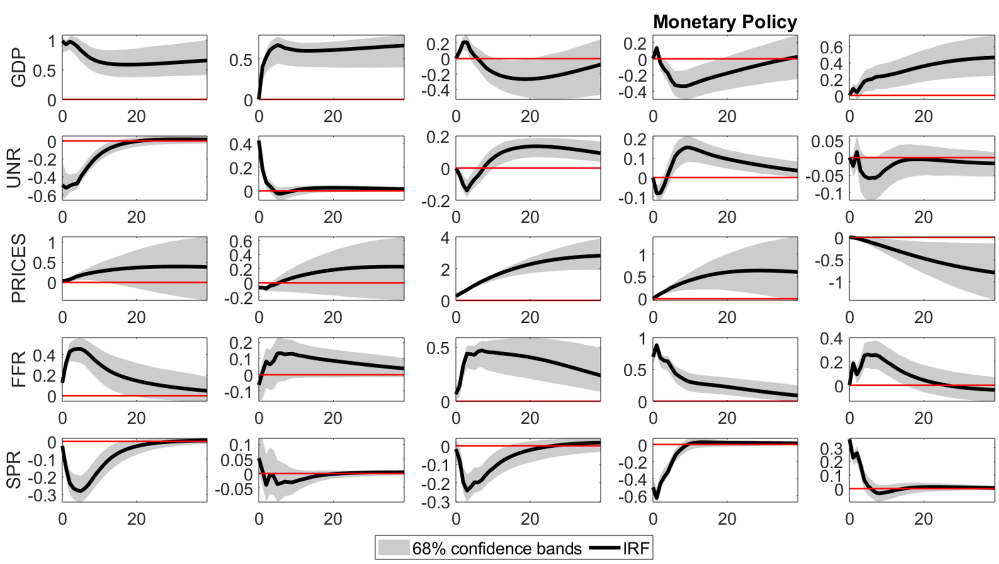


Figure 2: *Cholesky Impulse Responses*

The monetary policy shock increases the Federal Funds rate by around 0.6% points which lowers the spread on impact. GDP growth, inflation and unemployment cannot react on impact as the short rate increases more than the long rate (slope inverts), but we observe a strong price puzzle in the sense that inflation increases significantly following the monetary tightening. GDP growth declines for a brief period but returns to the mean relatively quickly. The unemployment rate increases in the longer run which is counter to the Phillips curve idea that when inflation increases unemployment decreases.