

WHAT IS NONCLASSICAL ABOUT UNCERTAINTY RELATIONS?

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Motivation

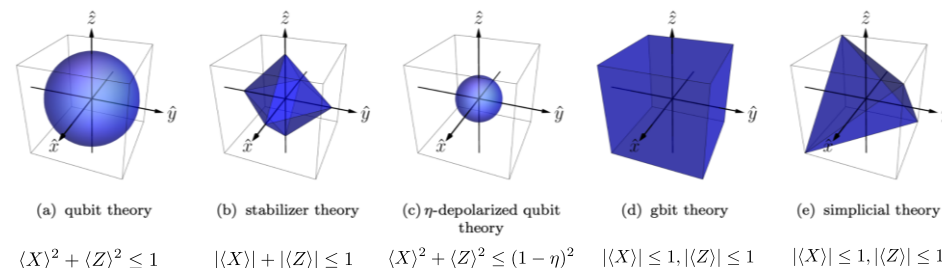
- Uncertainty relations consist of bounds on the joint predictability of outcomes of distinct measurements on a single state.
- The existence of nontrivial uncertainty relations in quantum theory is generally considered to state a departure from the classical worldview.
- However, there exist theories that manifest uncertainty relations but also admit of a noncontextual ontological model.
- Which aspects of uncertainty relations, if any, witness contextuality?

Uncertainty relations

Definition

We consider uncertainty relations describing the tradeoff between the predictabilities of a pair of binary-outcome measurements (e.g., X and Z Paulis). Tradeoffs are in terms of sums of expectation values.

Examples



Noncontextual ontological models

Ontological model of an operational theory

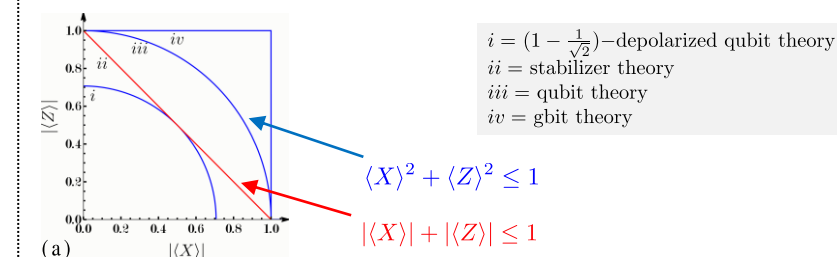
- Each system \longrightarrow ontic state space Λ with ontic states $\lambda \in \Lambda$.
- Preparation $P \longleftrightarrow \mu(\lambda|P)$
- Measurement and outcome $M, y \longleftrightarrow \xi(y|M, \lambda)$
- Predicted statistics $\mathbb{P}(y|M, P) = \sum_{\lambda \in \Lambda} \xi(y|M, \lambda) \mu(\lambda|P)$

Noncontextuality

- Two preparations P, P' are *operationally equivalent*, $P \simeq P'$, if $\mathbb{P}(y|M, P) = \mathbb{P}(y|M, P') \forall M$.
- In a preparation noncontextual ontological model,
 $P \simeq P' \implies \mu(\lambda|P) = \mu(\lambda|P')$.

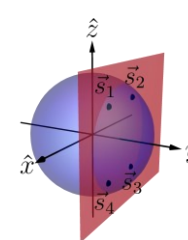
Results

In any operational theory, if one can find a pair of measurements, X, Z , and a state that satisfies the condition of A_1^2 -orbit-realizability, then noncontextuality implies that $|\langle X \rangle| + |\langle Z \rangle| \leq 1$.



Condition of A_1^2 -orbit-realizability

- The state has equal predictability counterparts.
- The state manifest operational equivalences with such counterparts.



For theories where all states satisfy A_1^2 -orbit-realizability our bound is a constraint on the form of the ZX -uncertainty relation within such theories.

Conclusion and future directions

- Given A_1^2 -orbit-realizability, noncontextuality bounds the functional form of the ZX predictability tradeoff below a linear curve.
- The functional form of an uncertainty relation can witness contextuality.
- What is nonclassical about interference phenomena?

References

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- [4] R.W. Spekkens, Phys. Rev. A **71**: 052108 (2005).

