Homework 5

Math 324F
Advanced Multivariable Calculus
Due on 13th November 2015

Read sections 16.1, 16,2 and 16.3 from the text.

Problem 16.2.11 (5 points) Evaluate $\int_C xe^{yz}\,dS$ where C is the line segment from $(0,0,0)$ to $(1,2,3)$.

Problem 16.2.33 (5 points) A thin wire is bent into the shape of a semicircle $x^2+y^2=4$, $x\geq 0$. If the linear density is a constant k, find the mass and center of mass of the wire.

(Note that the center of mass of a wire with density $\rho(x,y)$ is located at $(\bar{x},\bar{y})$ where $\bar{x}=\frac{1}{m}\int_C x\rho(x,y)\,ds$, $\bar{y}=\frac{1}{m}\int_C y\rho(x,y)\,ds$, and $m=\int_C \rho(x,y)\,ds$.)

Problem 16.2.44 (5 points) An object with mass m moves with position function $\vec{\gamma}(t)=a\sin(t)\mathbf{i}+b\cos(t)\mathbf{j}+ct\mathbf{k}$, $0\leq t\leq\frac{\pi}{2}$. Find the work done on the object during this period.

Problem 16.3.15 (5 points) Find a function f such that $\nabla f=\mathbf{F}$ and use that to evaluate $\int_C \mathbf{F}\cdot\mathbf{dr}$. where $\mathbf{F}(x,y,z)=yz\mathbf{i}+xz\mathbf{j}+(xy+2z)\mathbf{k}$, C is the line segment from $(1,0,-2)$ to $(4,6,3)$.

Problem 16.3.35 (7 points) Let $\mathbf{F}(x,y)=\frac{-y\mathbf{i}+x\mathbf{j}}{x^2+y2}$.
(a) Show that $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$.
(b) Show that $\int_C \mathbf{F}\cdot\mathbf{dr}$ is not independent of the path. Also show why this does not contradict Theorem 6.
Hint: Are all the hypotheses for Theorem 6 satisfied?

Problem 16.3.21 (3 points) Suppose you are asked to determine the curve that requires the least amount of work for a conservative force field **F** to move a particle one given point to another given point. What should your answer be and why?