

Name: _____

Quiz Score: ____/20

Answer each question completely. Show all work, and explain your reasoning if the work is at all ambiguous. Put a box around your answers.

- 10 1. Consider the vector field $\mathbf{F}(x, y, z) = x^2y\mathbf{i} + zy\mathbf{j} + yx^3\mathbf{k}$. Compute $\text{curl } \mathbf{F}$, $\nabla(\nabla \cdot \mathbf{F})$ and $\nabla \cdot (\text{curl } \mathbf{F})$.

$$\text{curl } \mathbf{F} = \nabla \times \mathbf{F} = (x^3 - y)\mathbf{i} - 3y^2\mathbf{j} - 2xy\mathbf{k}.$$

$$\nabla \cdot \mathbf{F} = 2xy + z; \nabla(\nabla \cdot \mathbf{F}) = 2y\mathbf{i} + 2x\mathbf{j} + \mathbf{k}.$$

$$\nabla \cdot (\text{curl } \mathbf{F}) = 0$$

- 10 2. Use Green's theorem to evaluate $\oint_C \sin(x^3) dx + 4x^2 y dy$ where C is the triangle with vertices $(0,0)$, $(2,0)$ and $(2,6)$.

By Green's theorem, $\oint_C \sin(x^3) dx + 4x^2 y dy = \int_0^2 \int_0^{3x} 8xy dy dx = \int_0^2 36x^3 dx = 144.$