Practice Problems for Midterm

Math 324F
Advanced Multivariable Calculus

1) Find the volume of the solid tetrahedron with vertices $(0,0,0)$, $(0,0,1)$, $(0,2,0)$ and $(2,2,0)$.

Answer: $\frac{2}{3}$

2) Find the volume of the solid bounded by the cylinder $x^2+y^2=4$ and the planes $z=0$ and $y+z=3$.

Answer: 12π

3) Find the volume of one of the wedges cut from the cylinder $x^2+9y^2=a^2$ by the planes $z=0$ and $z=mx$.

Answer: $\frac{2}{9}ma^3$

4) Find the volume of the solid above the paraboloid $z=x^2+y^2$ and below the half cone $z=\sqrt{x^2+y^2}$.

Answer: $\frac{\pi}{6}$

5) Compute $\iiint\limits_E y^2z^2\,dV$ where E is bounded by the paraboloid $x=1-y^2-z^2$ and the plane $x=0$.

Answer: $\frac{\pi}{96}$

6) Compute $\iiint\limits_E yz\,dV$ where E lies above the plane $z=0$, below the plane $z=y$, and inside the cylinder $x^2+y^2=4$.

Answer: $\frac{64}{15}$

7) Evaluate $\iiint\limits_H z^3\sqrt{x^2+y^2+z^2}\,dV$ where H is the solid hemishphere that lies above the xy plane and has center the origin and radius 1.

Answer: $\frac{\pi}{14}$

8) The center of mass $(\bar{x}, \bar{y}, \bar{z})$ of a solid E is defined as follows :

$$\bar{x} = \frac{\iiint\limits_E x\rho(x,y,z)\,dV}{\iiint\limits_E \rho(x,y,z)\,dV}$$

$$\bar{y} = \frac{\iiint\limits_E y\rho(x,y,z)\,dV}{\iiint\limits_E \rho(x,y,z)\,dV}$$

$$\bar{z} = \frac{\iiint\limits_E z\rho(x,y,z)\,dV}{\iiint\limits_E \rho(x,y,z)\,dV}$$

where $\rho(x,y,z)$ is the density distribution of the solid E.

Find the center of mass of the solid tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,2,0)$ and $(0,0,3)$ and density function $\rho(x,y,z) = x^2 + y^2 + z^2$.

Answer: $(\frac{4}{21}, \frac{11}{21}, \frac{8}{7})$

9) Rewrite the integral $\int\limits_{-1}^{1}\int\limits_{x^2}^{1}\int\limits_{0}^{1-y} f(x,y,z)\,dz\,dy\,dx$ as an iterated integral in the order $dx\,dy\,dz$.

**10)* Give 5 other iterated integrals that are equal to $\int\limits_0^2\int\limits_0^{y^3}\int\limits_0^{y^2} f(x,y,z)\,dz\,dx\,dy$.

11) Use the transformation $u = x - y$ and $v = x + y$ to evaluate $\iint\limits_R \frac{x-y}{x+y}\,dA$ where R is the square with vertices $(0,2)$, $(1,1)$, $(2,2)$ and $(1,3)$.

Answer: $-\ln 2$

12) Use the transformation $x = u^2$, $y = v^2$, $z = w^2$ to find the volume of the region bounded by the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$ and the coordinate planes.

Answer: $\frac{1}{90}$

13) Let f be continuous on $[0,1]$ and let R be the triangular region with vertices $(0,0)$, $(1,0)$ and $(0,1)$. Show that $\iint\limits_R f(x+y)\,dA = \int\limits_0^1 u f(u)\,du$

14) Assume all the given functions are differentiable. If $z = f(x,y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ and show that

$$(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 = (\frac{\partial z}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial z}{\partial \theta})^2.$$

15) One side of a triangle is increasing at a rate of 3 cm/s and a second side is decreasing at a rate of 2 cm/s. If the area of the triangle remains constant, at what rate does the angle between the sides change when the first side is 20 cm long, the second side is 30 cm, and the angle is $\frac{\pi}{6}$.

Answer: $\frac{-1}{12\sqrt{3}}$ radians/s

16) Use chain rule to find out the indicated partial derivative for $z = x^4 + x^2 y$, $x = s + 2t - u,\ y = stu^2$;
$\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial u}$ when $s = 4,\ t = 2,\ u = 1$.

Answer: 1582, 3164, -700

17) Find all the points at which the direction of fastest change of $f(x,y) = x^2 + y^2 - 2x - 4y$ is $\mathbf{i} + \mathbf{j}$.

Answer: Any point on the line $y - x = 1$

18) Suppose that over a certain region of space the electric potential V is given by $V(x,y,z) = 5x^2 - 3xy + xyz$.
(a) Find the rate of change of the potential at $P(3,4,5)$ in the direction of the vector $\mathbf{i} + \mathbf{j} - \mathbf{k}$.
(b) In which direction does V change most rapidly at P?
(c) What is the maximum rate of change at P?

Answer: a) $\frac{32}{\sqrt{3}}$, b) 38$\mathbf{i}$ + 6$\mathbf{j}$ + 12$\mathbf{k}$, c) $2\sqrt{406}$

19) Find the equation of the tangent plane and normal line to the surface $xyz^2 = 6$ at $(3,2,1)$.

Answer: a) $x + y + z = 11$, b) $(t + 3, t + 3, t + 5)$

20) Show that every plane that is tangent to the cone $z^2 = x^2 + y^2$ passes through the origin.