Name:______________________________ Quiz Score:____/20

Answer each question completely. Show all work, and explain your reasoning if the work is at all ambiguous.

$\boxed{10}$ 1. Let U be the solid bounded below by $z = \sqrt{3(x^2+y^2)}$ and above by $x^2+y^2+z^2 = 4$. Write an iterated integral that gives the volume of U. (You need not evaluate.)

Cartesian coordinate: $\displaystyle\iint_{x^2+y^2\leq 1} \int_{\sqrt{3(x^2+y^2)}}^{\sqrt{4-x^2-y^2}} 1\, dz\, dA.$

Cylindrical coordinate: $\displaystyle\int_0^{2\pi}\int_0^1\int_{3r}^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta.$

Spherical coordinate: $\displaystyle\int_0^{\pi/6}\int_0^{2\pi}\int_0^2 \rho^2\sin(\phi)\, d\rho\, d\theta\, d\phi.$

$\boxed{10}$ 2. Evaluate the triple integral $\iiint\limits_B (z^3 + \sin y + 3)\, dV$ where B the unit solid sphere $x^2 + y^2 + z^2 \leq 1$

[Hint: Split up the integral and use the fact that volume of the unit sphere is $\frac{4\pi}{3}$].

Splitting up the given integral gives us the sum of the following 3 integrals:

$$\iiint\limits_B z^3\, dV + \iiint\limits_B \sin y\, dV + \iiint\limits_B 3\, dV$$

The first 2 integrals evaluate to 0 since we are integrating an odd function over a region symmetric about the 3 axes. The last integral gives us 3∗vol(B)= 4π. The final answer is thus 4π.

[Symmetries are valuable in complex situations. The key is to think about the problem to be solved before one dives into heads-down calculation.]