

	$d$	$e$	$S \subset X \subset \mathbb{P}^5$	$W$	$U \subset W$	$Y$
0	42	3	Rational surface of degree 9 and sectional genus 2 with 5 nodes, which is a special projection of the image of $\mathbb{P}^2$ in $\mathbb{P}^8$ via the linear system of quartic curves with 3 simple points and one double point	$\mathbb{G}(1, 4) \cap \mathbb{P}^7 \subset \mathbb{P}^7$	Non-minimal $K3$ surface of degree 21 and sectional genus 18, cut out in $\mathbb{P}^7$ by 5 quadrics and 8 cubics	4-fold of degree 14 in $\mathbb{P}^7$ cut out by 7 cubics
i	14	2	Isomorphic projection of a smooth surface in $\mathbb{P}^6$ of degree 8 and sectional genus 3, obtained as the image of $\mathbb{P}^2$ via the linear system of quartic curves with 8 general base points	$\mathbb{P}^4$	Singular $K3$ surface of degree 10 and sectional genus 7, cut out by 12 quintics and having 8 singular points	4-fold of degree 28 in $\mathbb{P}^{11}$ cut out by 16 quadrics
ii	26	2	Rational scroll of degree 7 with 3 nodes	$\mathbb{P}^4$	Singular $K3$ surface of degree 10 and sectional genus 8, cut out by 12 quintics and one sextic, and having 3 singular points	4-fold of degree 29 in $\mathbb{P}^{11}$ cut out by 15 quadrics
iii	38	2	Smooth surface of degree 10 and sectional genus 6, obtained as the image of $\mathbb{P}^2$ via the linear system of curves of degree 10 with 10 general triple points	$\mathbb{P}^4$	Smooth non-minimal $K3$ surface of degree 12 and sectional genus 14 cut out by 9 quintics	4-fold of degree 20 in $\mathbb{P}^8$ cut out by 16 cubics
iv	26	2	Projection of a smooth del Pezzo surface of degree 7 in $\mathbb{P}^7$ from a line intersecting the secant variety in one general point	$\mathbb{G}(1, 4) \cap \mathbb{P}^7 \subset \mathbb{P}^7$	Non-minimal $K3$ surface of degree 17 and sectional genus 11, cut out in $\mathbb{P}^7$ by 5 quadrics and 13 cubics	4-fold of degree 34 in $\mathbb{P}^{12}$ cut out by 20 quadrics
v	38	3	Rational scroll of degree 8 with 6 nodes	$\mathbb{G}(1, 5) \cap \mathbb{P}^{10} \subset \mathbb{P}^{10}$	Smooth non-minimal $K3$ surface of degree 22 and sectional genus 14, cut out in $\mathbb{P}^{10}$ by 24 quadrics	4-fold of degree 17 in $\mathbb{P}^8$ cut out by 3 quadrics and 4 cubics
vi	14	3	Projection from 3 general internal points of a minimal $K3$ surface of degree 14 and sectional genus 8	Cubic fourfold	Projection from 3 general internal points of a minimal $K3$ surface of degree 14 and sectional genus 8	Complete intersection in $\mathbb{P}^7$ of 2 quadrics and one cubic
vii	14	3	Projection of a $K3$ surface of degree 10 and sectional genus 6 in $\mathbb{P}^6$ from a general point on its secant variety	Gushel-Mukai fourfold in $\mathbb{P}^8$	Smooth minimal $K3$ surface of degree 14 and sectional genus 8	Hypercubic section of a hyperplane section of $\mathbb{G}(1, 4)$
viii	14	5	General hyperplane section of a conic bundle in $\mathbb{P}^6$ of degree 13 and sectional genus 12	Complete intersection of three quadrics in $\mathbb{P}^7$	Smooth non-minimal $K3$ surface of degree 13 and sectional genus 8, cut out by 9 quadrics	Hypersurface of degree 5 in $\mathbb{P}^5$
ix	14	5	General hyperplane section of a pfaffian threefold in $\mathbb{P}^6$ of degree 14 and sectional genus 15	$\mathbb{G}(1, 6) \cap \mathbb{P}^{14} \subset \mathbb{P}^{14}$	Smooth minimal $K3$ surface of degree 14 and sectional genus 8 embedded in $\mathbb{P}^8 \subset \mathbb{P}^{14}$	Hypersurface of degree 5 in $\mathbb{P}^5$
x	38	5	Smooth surface of degree 11 and sectional genus 7, obtained as the image of $\mathbb{P}^2$ via the linear system of curves of degree 12 with one general simple point, 4 general triple points, and 6 general quadruple points	$\mathbb{G}(1, 5) \cap \mathbb{P}^{10} \subset \mathbb{P}^{10}$	Smooth non-minimal $K3$ surface of degree 25 and sectional genus 17, cut out in $\mathbb{P}^{10}$ by 21 quadrics	Hypersurface of degree 7 in $\mathbb{P}^5$
xi	38	3	Projection of an octic del Pezzo surface isomorphic to $\mathbb{F}_1$ from a plane intersecting the secant variety in 3 general points	$\mathbb{G}(1, 3) \subset \mathbb{P}^5$	Non-minimal $K3$ surface of degree 13 and sectional genus 10, cut out in $\mathbb{P}^3$ by one quadric, 9 quartics, and 3 quintics	4-fold of degree 17 in $\mathbb{P}^8$ cut out by 3 quadrics and 4 cubics
xii	38	3	Projection of an octic del Pezzo surface isomorphic to $\mathbb{F}_0$ from a plane intersecting the secant variety in 3 general points (cut out by 10 cubics and one quartic)	$\text{LG}_3(\mathbb{C}^6) \cap \mathbb{P}^{11} \subset \mathbb{P}^{11}$	Non-minimal $K3$ surface of degree 26 and sectional genus 17, cut out in $\mathbb{P}^{11}$ by 30 quadrics	4-fold of degree 18 in $\mathbb{P}^8$ cut out by 2 quadrics and 8 cubics
xiii	14	3	Isomorphic projection of a smooth surface in $\mathbb{P}^7$ of degree 8 and sectional genus 2, obtained as the image of $\mathbb{P}^2$ via the linear system of quartic curves with 4 simple base points and one double point (cut out by 10 cubics and 3 quartics)	Complete intersection of 2 quadrics in $\mathbb{P}^6$	Singular $K3$ surface of degree 14 and sectional genus 8, cut out in $\mathbb{P}^6$ by 2 quadrics and 9 cubics, and having one singular point	Complete intersection of 4 quadrics in $\mathbb{P}^8$
xiv	26	5	Rational scroll of degree 8 with 4 nodes (cut out by 8 cubics and 3 quartics)	$\mathbb{G}(1, 3) \subset \mathbb{P}^5$	Non-minimal $K3$ surface of degree 14 and sectional genus 11, cut out in $\mathbb{P}^5$ by one quadric, 7 quartics, and 2 quintics	Complete intersection in $\mathbb{P}^6$ of a quadric and a quartic
xv	26	5	Surface of degree 13 and sectional genus 11 cut out by 6 cubics and with an ordinary node, which is obtained as a special projection of a minimal $K3$ surface of degree 26 of genus 14	Cubic fourfold	A surface of the same kind as $S$	—
xvi	26	6	Surface of degree 11 and sectional genus 6 cut out by 7 cubics and with 3 non-normal nodes, which is obtained as a special projection of a smooth surface of degree 11 and sec. genus 6 in $\mathbb{P}^6$	$\mathbb{S}^{10} \cap \mathbb{P}^9 \subset \mathbb{P}^9$ , where $\mathbb{S}^{10} \subset \mathbb{P}^{15}$ is the spinorial variety	Non-minimal $K3$ surface of degree 21 and sectional genus 14, cut out in $\mathbb{P}^9$ by 16 quadrics and one cubic	Hypersurface of degree 5 in $\mathbb{P}^5$
xvii	26	5	Smooth surface of degree 11 and sectional genus 7, obtained as the image of $\mathbb{P}^2$ via the linear system of curves of degree 8 with 3 simple base points, 8 general double points, and 2 general triple points (cut out by 7 cubics and one quartic)	$\mathbb{G}(1, 3) \subset \mathbb{P}^5$	Singular $K3$ surface of degree 15 and sectional genus 12, cut out in $\mathbb{P}^5$ by one quadric and 6 quartics, and having 9 singular points	Hypersurface of degree 6 in $\mathbb{P}^5$

TABLE 1. Examples of maps  $\mu : X \dashrightarrow W$  as in diagramm (0.1), where  $[X] \in \mathcal{C}_d$  and  $S \subset X$  admits a congruence of  $(3e - 1)$ -secant rational curves of degree  $e$ .