

Implement a new Graph

The BRAPH 2 Developers

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This is the developer tutorial for implementing a new graph. In this Tutorial, we will explain how to create the generator file `*.gen.m` for a new graph, which can then be compiled by `braph2genesis`. All graphs are (direct or indirect) extensions of the base element `Graph`. Here, we will use as examples the graphs `GraphBD` (Binary Directed graph), `MultilayerWU` (Weighted Undirected multilayer graph), `MultiplexBUT` (Binary Undirected multiplex at fixed Thresholds), and `OrdMxBUT` (Binary Undirected ordinal multiplex with fixed Thresholds).

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Implementation of Unilayer Graphs

Unilayer Binary Directed Graph (GraphBD)

We will start by implementing in detail GraphBD, which is a direct extension of Graph. A unilayer graph is constituted by nodes connected by edges.

Code 1: GraphBD element header. The header section of the generator code for `_GraphBD.gen.m` provides the general information about the GraphBD element.

```

1 %% iheader!
2 GraphBD < Graph (g, binary directed graph) is a binary directed graph. ①
3
4 %%% idescription!
5 In a binary directed (BD) graph, the edges are directed and they can be
   either 0 (absence of connection) or 1 (existence of connection).

```

① defines GraphBD as a subclass of Graph. The moniker will be g.

Code 2: GraphBD element prop update. The `props_update` section of the generator code for `GraphBD.gen.m` updates the properties of the Graph element. This defines the core properties of the graph.

```

1 %% iprops_update!
2 %%% iprop!
3 NAME (constant, string) is the name of the binary directed graph.
4 %%% idefault!
5 'GraphBD'
6
7 %%% iprop!
8 DESCRIPTION (constant, string) is the description of the binary directed
   graph.
9 %%% idefault!
10 'In a binary directed (BD) graph, the edges are directed and they can be
    either 0 (absence of connection) or 1 (existence of connection).'
11
12 %%% iprop!
13 TEMPLATE (parameter, item) is the template of the binary directed graph.
14
15 %%% iprop!
16 ID (data, string) is a few-letter code of the binary directed graph.
17 %%% idefault!
18 'GraphBD ID'
19
20 %%% iprop!
21 LABEL (metadata, string) is an extended label of the binary directed graph.
22 %%% idefault!
23 'GraphBD label'
24
25 %%% iprop!
26 NOTES (metadata, string) are some specific notes about the binary directed
   graph.
27 %%% idefault!
28 'GraphBD notes'
29
30 %%% iprop! ①
31 GRAPH_TYPE (constant, scalar) returns the graph type __Graph.GRAPH___.

```

① defines the *graph type* (property `GRAPH_TYPE`): `Graph.GRAPH` (consists of a single layer), `Graph.MULTIGRAPH` (multiple unconnected layers), `Graph.MULTILAYER` (multiple layers with categorical connections between nodes), `Graph.ORDERED_MULTILAYER` (multiple layers with ordinal connections between nodes), `Graph.MULTIPLEX` (multilayer graph where only interlayer edges are allowed between homologous nodes), and `Graph.ORDERED_MULTIPLEX` (multiplex graph that consists of a sequence of layers with ordinal edges between corresponding nodes in subsequent layers).

```

32 %%%% idefault!
33 Graph.GRAPH
34
35 %%% iprop! ②
36 CONNECTIVITY_TYPE (query, smatrix) returns the connectivity type __Graph.
    BINARY___.
37 %%%% idefault!
38 value = Graph.BINARY;
39
40 %%% iprop! ③
41 DIRECTIONALITY_TYPE (query, smatrix) returns the directionality type
    __Graph.DIRECTED___.
42 %%%% idefault!
43 value = Graph.DIRECTED;
44
45 %%% iprop! ④
46 SELFCONNECTIVITY_TYPE (query, smatrix) returns the self-connectivity type
    __Graph.NONSELFCONNECTED___.
47 %%%% idefault!
48 value = Graph.NONSELFCONNECTED;
49
50 %%% iprop! ⑤
51 NEGATIVITY_TYPE (query, smatrix) returns the negativity type __Graph.
    NONNEGATIVE___.
52 %%%% idefault!
53 value = Graph.NONNEGATIVE;
54
55 %%% iprop! ⑥
56 A (result, cell) is the binary adjacency matrix of the binary directed graph
    .
57 %%%% icalculate!
58 B = g.get('B'); ⑦
59
60 B = dedagonalize(B); ⑧
61 B = semipositivize(B, 'SemipositivizeRule', g.get('SEMIPOSITIVIZE_RULE'));
    ⑨
62 B = binarize(B); ⑩
63
64 A = {B}; ⑪
65 value = A; ⑫
66
67 %%%% igui! ⑬
68 pr = PanelPropCell('EL', g, 'PROP', GraphBD.A, ... ⑭
69 'TABLE_HEIGHT', s(40), ...
70 'XSLIDERSHOW', false, ...
71 'YSLIDERSHOW', false, ...
72 'ROWNAME', g.getCallback('ANODELABELS'), ...
73 'COLUMNNAME', g.getCallback('ANODELABELS'), ...
74 );
75
76 %%% iprop! ⑮
77 COMPATIBLE_MEASURES (constant, classlist) is the list of compatible measures
    .
78 %%%% idefault!
79 getCompatibleMeasures('GraphBD')

```

② Graphs have a *connectivity type* (CONNECTIVITY_TYPE): Graph.BINARY (graph with binary, 0 or 1, connections) or Graph.WEIGHTED (graph with weighted connections).

③ Graphs have a *directionality type* (DIRECTIONALITY_TYPE): Graph.DIRECTED (graph with directed edges) or Graph.UNDIRECTED (graph with undirected edges).

④ Graphs have a *self-connectivity type* (SELFCONNECTIVITY_TYPE): Graph.NONSELFCONNECTED (graph without self-connections) or Graph.SELFCONNECTED (graph with self-connections).

⑤ Graphs have a *negativity type* (NEGATIVITY_TYPE): Graph.NONNEGATIVE (graph without negative edges) or Graph.NEGATIVE (graph allowing negative edges).

⑥ The property A contains the supra-adjacency matrix of the graph, which is calculated by the code under icalculate!.

⑦ retrieves the adjacency matrix of the graph, defined in the new properties below.

⑧ ⑨ ⑩ condition the adjacency matrix removing the diagonal elements, making it semidefinite positive, and binarizing it. A list of useful functions is: diagonalize (removes the off-diagonal), dedagonalize (removes the diagonal), binarize (binarizes with threshold=0), semipositivize (removes negative weights), standardize (normalizes between 0 and 1) or symmetrize (symmetrizes the matrix). Use the MatLab help to see additional functionalities.

⑪ preallocates the adjacency matrix that contains the result of the defined graph.

⑫ returns the calculated graph A assigning it to the output variable value.

⑬ Each graph has a panel figure of the cell containing the calculated graph adjacency matrix A.

⑭ PanelPropCell plots the panel for a CELL property with a table and two sliders. It can be personalized with props, e.g., TABLE_HEIGHT (height in pixels), XSLIDERSHOW (whether to show the x-slider), or COLUMNNAME (string list with column names)

⑮ Each graph has a list of compatible measures.

Code 3: **GraphBD element props.** The props section of generator code for GraphBD.gen.m defines the properties to be used in GraphBD.

```

1 %% iprops!
2
3 %% iprop! ①
4 B (data, smatrix) is the input graph adjacency matrix.
5 %%% igui!
6 pr = PanelPropMatrix('EL', g, 'PROP', GraphBD.B, ... ②
7 'TABLE_HEIGHT' , s(40), ...
8 'ROWNAME' , g.getCallback('ANODELABELS'), ...
9 'COLUMNNAME', g.getCallback('ANODELABELS'), ...
10 varargin{:});
11
12 %% iprop! ③
13 SEMIPOSITIVIZE_RULE (parameter, option) determines how to remove the
    negative edges.
14 %%% isettings!
15 {'zero', 'absolute'}

```

① Each graph has a panel figure of the input graph adjacency matrix B.

② PanelPropMatrix plots the panel of a property matrix-like with a table. It can be personalized with props as in ⑫. Here it plots the input graph adjacency matrix B

③ Each graph have different rules that need to be defined: SYMMETRIZE_RULE: symmetrizes the matrix A by the symmetrize rule specified by RULE and the admissible RULE options are: 'max' (default, maximum between inconnection and outconnection), 'sum' (convert negative values to absolute value), 'average' (average of inconnection and outconnection) or 'min' (minimum between inconnection and outconnection) SEMIPOSITIVIZE_RULE: determines how to remove the negative edges and the admissible RULE options are: 'zero' (default, convert negative values to zeros) or 'absolute' (convert negative values to absolute value) STANDARDIZE_RULE : determines how to normalize the weights between 0 and 1 and the admissible RULE options are: 'threshold' (default, normalizes the matrix A by converting negative values to zero and values larger than 1 to 1) or 'range' (normalizes the matrix A in order to have values scaled between 0 and 1 by using a linear function).

Code 4: **GraphBD element tests.** The tests section from the element generator `_GraphBD.gen.m`. A general test should be prepared to test the properties of the graph when it is empty and full. Furthermore, additional tests should be prepared for the rules defined (one test per rule).

```

1 %% itests!
2
3 %% iexcluded_props! ①
4 [GraphBD.PFGA GraphBD.PFGH]
5
6 %% itest!
7 %%% iname!
8 Constructor - Empty ②
9 %%% iprobability! ③
10 .01
11 %%% icode!
12 B = []; ④
13 g = GraphBD('B', B); ⑤
14
15 g.get('A-CHECK'); ⑥
16
17 A = {binarize(semipositivize(dediagonalize(B)))}; ⑦
18 assert(isequal(g.get('A'), A), ... ⑧
19 [BRAPH2.STR ':GraphBD:' BRAPH2.FAIL_TEST], ...
20 'GraphBD is not constructing well.')
21
22 %% itest!
23 %%% iname!
24 Constructor - Full ⑨
25 %%% iprobability! ③
26 .01 ③
27 %%% icode!
28 B = randn(randi(10)); ⑩
29 g = GraphBD('B', B); ⑤
30
31 g.get('A-CHECK') ⑥
32
33 A = {binarize(semipositivize(dediagonalize(B)))}; ⑦
34 assert(isequal(g.get('A'), A), ... ⑧
35 [BRAPH2.STR ':GraphBD:' BRAPH2.FAIL_TEST], ...
36 'GraphBD is not constructing well.')
37
38 %% itest!
39 %%% iname!
40 Semipositivize Rules ⑪
41 %%% iprobability!
42 .01 ③
43 %%% icode!
44 B = [
45 -2 -1 0 1 2
46 -1 0 1 2 -2
47 0 1 2 -2 -1
48 1 2 -2 -1 0
49 2 -2 -1 0 1

```

① List of properties that are excluded from testing.

② checks that an empty GraphBD graph is constructing well

③ assigns a low test execution probability

④ initializes an empty GraphBD graph

⑤ constructs the GraphBD graph from the initialized B

⑥ performs the corresponding checks for the format of the adjacency matrix A: GRAPH_TYPE, CONNECTIVITY_TYPE, DIRECTIONALITY_TYPE, SELFCONNECTIVITY_TYPE and NEGATIVITY_TYPE.

⑦ calculates the value of the graph by apply the corresponding properties function

⑧ tests that the value of generated graph calculated by applying the properties functions coincides with the expected value

⑨ checks that a full GraphBD graph is constructing well

⑩ generates a random graph

⑪ checks the SEMIPOSITIVIZE_RULE on the GraphBD graph.

```

50 ]; (12)
51
52 g0 = GraphBD('B', B); (13)
53 A0 = {[
54   0 0 0 1 1
55   0 0 1 1 0
56   0 1 0 0 0
57   1 1 0 0 0
58   1 0 0 0 0
59 ]}; (14)
60 assert(isequal(g0.get('A'), A0), ... (8)
61 [BRAPH2.STR ':GraphBD:' BRAPH2.FAIL_TEST], ...
62 'GraphBD is not constructing well.')
63
64 g_zero = GraphBD('B', B, 'SEMIPOSITIVIZE_RULE', 'zero'); (15)
65 A_zero = {[
66   0 0 0 1 1
67   0 0 1 1 0
68   0 1 0 0 0
69   1 1 0 0 0
70   1 0 0 0 0
71 ]}; (14)
72 assert(isequal(g_zero.get('A'), A_zero), ... (8)
73 [BRAPH2.STR ':GraphBD:' BRAPH2.FAIL_TEST], ...
74 'GraphBD is not constructing well.')
75
76 g_absolute = GraphBD('B', B, 'SEMIPOSITIVIZE_RULE', 'absolute'); (16)
77 A_absolute = {[
78   0 1 0 1 1
79   1 0 1 1 1
80   0 1 0 1 1
81   1 1 1 0 0
82   1 1 1 0 0
83 ]}; (14)
84 assert(isequal(g_absolute.get('A'), A_absolute), ... (8)
85 [BRAPH2.STR ':GraphBD:' BRAPH2.FAIL_TEST], ...
86 'GraphBD is not constructing well.')
87
88 ...

```

(12) generates an example graph with negative weights

(13) constructs the GraphBD graph from the initialized B with default RULE for SEMIPOSITIVIZE_RULE.

(14) Expected value of the graph calculated by external means

(15) constructs the GraphBD graph from the initialized B with RULE = 'zero' for SEMIPOSITIVIZE_RULE.

(16) constructs the GraphBD graph from the initialized B with RULE = 'absolute' for SEMIPOSITIVIZE_RULE

Implementation of Multilayer Graphs

Weighted Directed Multilayer Graph (MultilayerWD)

We can now use GraphBD as the basis to implement the MultilayerWD graph. The parts of the code that are modified are highlighted.

A multilayer network allows connections between any nodes across the multiple layers, where all layers are interconnected following a categorical fashion.

Code 5: MultilayerWD element header. The header section of generator code for `_MultilayerWD.gen.m` provides the general information about the MultilayerWD element. [← Code 1](#)

```

1 %% iheader!
2 MultilayerWD < Graph (g, multilayer weighted directed graph) is a multilayer
  weighted directed graph.
3
4 %%% idescription!
5 In a multilayer weighted directed (WD) graph, layers could have different
  number of nodes with within-layer weighted directed edges, associated
  with a real number between 0 and 1 and indicating the strength of the
  connection. The connectivity matrices are symmetric (within layer). All
  node connections are allowed between layers.
```

Code 6: MultilayerWD element prop update. The `props_update` section of generator code for `_MultilayerWD.gen.m` updates the properties of MultilayerWD. [← Code 2](#)

```

1 %% iprops_update!
2
3 %%% iprop!
4 NAME (constant, string) is the name of the multilayer weighted directed
  graph.
5 %%% idefault!
6 'MultilayerWD'
7
8 %%% iprop!
9 DESCRIPTION (constant, string) is the description of the multilayer weighted
  directed graph.
10 %%% idefault!
11 'In a multilayer weighted directed (WD) graph, layers could have different
  number of nodes with within-layer weighted directed edges, associated
  with a realnumber between 0 and 1 and indicating the strength of the
  connection. The connectivity matrices are symmetric (within layer). All
  node connections are allowed between layers.'
```

```

12
13 %%% iprop!
14 TEMPLATE (parameter, item) is the template of the multilayer weighted
  directed graph.
15
16 %%% iprop!
17 ID (data, string) is a few-letter code of the multilayer weighted directed
  graph.
18 %%% idefault!
19 'MultilayerWD ID'
```

```

20
21 %% iprop!
22 LABEL (metadata, string) is an extended label of the multilayer weighted
    directed graph.
23 %%%% idefault!
24 'MultilayerWD label'
25
26 %% iprop!
27 NOTES (metadata, string) are some specific notes about the multilayer
    weighted directed graph.
28 %%%% idefault!
29 'MultilayerWD notes'
30
31 %% iprop!
32 GRAPH_TYPE (constant, scalar) returns the graph type __Graph.MULTILAYER__.
33 %%%% idefault!
34 Graph.MULTILAYER
35
36 %% iprop!
37 CONNECTIVITY_TYPE (query, smatrix) returns the connectivity type __Graph.
    WEIGHTED__ * ones(layernumber).
38 %%%% icalculate!
39 if isempty(varargin)
40     layernumber = 1;
41 else
42     layernumber = varargin{1};
43 end
44 value = Graph.WEIGHTED * ones(layernumber);
45
46 %% iprop!
47 DIRECTIONALITY_TYPE (query, smatrix) returns the directionality type __Graph.
    .DIRECTED__ * ones(layernumber).
48 %%%% icalculate!
49 if isempty(varargin)
50     layernumber = 1;
51 else
52     layernumber = varargin{1};
53 end
54 value = Graph.DIRECTED * ones(layernumber);
55
56 %% iprop!
57 SELFCONNECTIVITY_TYPE (query, smatrix) returns the self-connectivity type
    __Graph.NONSELFCONNECTED__ on the diagonal and __Graph.SELFCONNECTED__
    off diagonal.
58 %%%% icalculate!
59 if isempty(varargin)
60     layernumber = 1;
61 else
62     layernumber = varargin{1};
63 end
64 value = Graph.SELFCONNECTED * ones(layernumber);
65 value(1:layernumber+1:end) = Graph.NONSELFCONNECTED;
66
67 %% iprop!
68 NEGATIVITY_TYPE (query, smatrix) returns the negativity type __Graph.
    NONNEGATIVE__ * ones(layernumber).
69 %%%% icalculate!
70 if isempty(varargin)
71     layernumber = 1;
72 else
73     layernumber = varargin{1};

```



```

74 end
75 value = Graph.NONNEGATIVE * ones(layernumber);
76
77 %%% iprop!
78 A (result, cell) is the cell containing the within-layer weighted adjacency
79 matrices of the multilayer weighted directed graph and the connections
80 between layers.
81
82 %%% icalculate!
83 B = g.get('B');
84 L = length(B);
85 A = cell(L, L);
86 for i = 1:L (1)
87     M = dedagonalize(B{i,i});
88     M = semipositivize(M, 'SemipositivizeRule', g.get('SEMIPOSITIVIZE_RULE'));
89     M = standardize(M, 'StandardizeRule', g.get('STANDARDIZE_RULE'));
90     A(i, i) = {M};
91     if ~isempty(A{i, i})
92         for j = i+1:L
93             M = semipositivize(B{i,j}, 'SemipositivizeRule', g.get('
SEMIPOSITIVIZE_RULE'));
94             M = standardize(M, 'StandardizeRule', g.get('STANDARDIZE_RULE'));
95             A(i, j) = {M};
96             M = semipositivize(B{j,i}, 'SemipositivizeRule', g.get('
SEMIPOSITIVIZE_RULE'));
97             M = standardize(M, 'StandardizeRule', g.get('STANDARDIZE_RULE'));
98             A(j, i) = {M};
99         end
100     end
101 end
102 value = A;
103
104
105 %%% igui!
106 pr = PanelPropCell('EL', g, 'PROP', MultilayerWD.A, ...
107 'TABLE_HEIGHT', s(40), ...
108 'XSLIDERLOCK', true, ...
109 'XSLIDERSHOW', false, ...
110 'YSLIDERSHOW', true, ...
111 'YSLIDERLABELS', g.getCallback('ALAYERLABELS'), ...
112 'YSLIDERWIDTH', s(5), ...
113 'ROWNAME', g.getCallback('ANODELABELS'), ...
114 'COLUMNNAME', g.getCallback('ANODELABELS'), ...
115 varargin{:});
116
117 %%% iprop!
118 PARTITIONS (result, rvector) returns the number of layers in the partitions
119 of the graph.
120 %%% icalculate!
121 value = ones(1, g.get('LAYERNUMBER'));
122
123 %%% iprop!
124 ALAYERLABELS (query, stringlist) returns the layer labels to be used by the
125 slider. (2)
126 %%% icalculate!
127 alayerlabels = g.get('LAYERLABELS'); (3)
128 if isempty(alayerlabels) && ~isa(g.get('A'), 'NoValue') % ensures that it's
129 not unnecessarily calculated
130     alayerlabels = cellfun(@num2str, num2cell([1:L:g.get('LAYERNUMBER')]), '
uniformoutput', false); (4)

```

(1) For each layer in MultilayerWD graph the corresponding functions are

applied as in ← Code 2 (8, 9 10)

(2) There are some properties of graph adjacency matrix A that can be used in the gui to make the visualization user friendly. The list of properties that can be used are: ALAYERTICKS (to set ticks for each layer according to the layer number), ALAYERLABELS (to set labels for each layer), ANODELABELS (to set the node labels for each layer)

(3) returns the labels of the graph layers provided by the user

(4) constructs the labels of the layers based on the number of the layer (in case no layer labels were provided by the user).

```

128 end
129 value = alayerlabels;
130
131 %% iprop!
132 COMPATIBLE_MEASURES (constant, classlist) is the list of compatible measures
133
134 %%%% idefault!
135 getCompatibleMeasures('MultilayerWD')

```

Code 7: MultilayerWD element props. The props section of generator code for MultilayerWD.gen.m defines the properties to be used in MultilayerWD. ← [Code 3](#)

```

1
2 %% iprops!
3
4 %% iprop!
5 B (data, cell) is the input cell containing the multilayer adjacency
   matrices.
6 %%%% idefault!
7 {[] []; [] []}
8 %%%% igui! ①
9 pr = PanelPropCell('EL', g, 'PROP', MultilayerWD.B, ...
10 'TABLE_HEIGHT', s(40), ...
11 'XSLIDERSHOW', true, ...
12 'XSLIDERLABELS', g.get('LAYERLABELS'), ...
13 'XSLIDERHEIGHT', s(3.5), ...
14 'YSLIDERSHOW', false, ...
15 'ROWNAME', g.getCallback('ANODELABELS'), ...
16 'COLUMNNAME', g.getCallback('ANODELABELS'), ...
17 varargin{:});
18
19
20 %% iprop!
21 SEMIPOSITIVIZE_RULE (parameter, option) determines how to remove the
   negative edges.
22 %%%% isettings!
23 {'zero', 'absolute'}
24
25 %% iprop! ②
26 STANDARDIZE_RULE (parameter, option) determines how to normalize the weights
   between 0 and 1.
27 %%%% isettings!
28 {'threshold' 'range'}

```

① Same as in ← [Code 3](#) ②

② Same as in ← [Code 3](#) ③

Code 8: MultilayerWD element tests. The tests section from the element generator _MultilayerWD.gen.m. ← [Code 4](#)

```

1 %% itests!
2
3 %%%% iexcluded_props!
4 [MultilayerWD.PFGA MultilayerWD.PFGH]
5
6 %%%% itest!
7 %%%% iname!
8 Constructor - Full
9 %%%% iprobability!
10 .01
11 %%%% icode!

```

```

12 B1 = rand(randi(10));
13 B2 = rand(randi(10));
14 B3 = rand(randi(10));
15 B12 = rand(size(B1, 1), size(B2, 2));
16 B13 = rand(size(B1, 1), size(B3, 2));
17 B23 = rand(size(B2, 1), size(B3, 2));
18 B21 = rand(size(B2, 1), size(B1, 2));
19 B31 = rand(size(B3, 1), size(B1, 2));
20 B32 = rand(size(B3, 1), size(B2, 2));
21 B = {
22     B1          B12          B13
23     B21          B2          B23
24     B31          B32          B3
25 };
26 g = MultilayerWD('B', B);
27 g.get('A_CHECK')
28 A1 = standardize(semipositivize(dediagonalize(B1)));
29 A2 = standardize(semipositivize(dediagonalize(B2)));
30 A3 = standardize(semipositivize(dediagonalize(B3)));
31 A12 = standardize(semipositivize(B12));
32 A13 = standardize(semipositivize(B13));
33 A23 = standardize(semipositivize(B23));
34 A21 = standardize(semipositivize(B21));
35 A31 = standardize(semipositivize(B31));
36 A32 = standardize(semipositivize(B32));
37 B{1,1} = A1;
38 B{2,2} = A2;
39 B{3,3} = A3;
40 B{1,2} = A12;
41 B{1,3} = A13;
42 B{2,3} = A23;
43 B{2,1} = A21;
44 B{3,1} = A31;
45 B{3,2} = A32;
46 A = B;
47 assert(isequal(g.get('A'), A), ...
48 [BRAPH2.STR ': MultilayerWD: ' BRAPH2.FAIL_TEST], ...
49 'MultilayerWD is not constructing well.')

```

Binary Undirected Multilayer Graph with fixed Thresholds (MultiplexBUT)

Now we implement the MultiplexBUT graph based on previous codes GraphBD and MultilayerWD, again highlighting the differences.

A multiplex graph is a type of multilayer graph where only inter-layer edges are allowed between homologous nodes. In this case, the layers follow a categorical architecture, which means that all layers are interconnected.

Code 9: MultiplexBUT element header. The header section of generator code for `_MultiplexBUT.gen.m` provides the general information about the MultiplexBUT element. ← [Code 1](#)

```

1  %% iheader!
2  MultiplexBUT < MultiplexWU (g, binary undirected multiplex with fixed
3  thresholds) is a binary undirected multiplex with fixed thresholds. ①
4
5  %%% idescription!
6  In a binary undirected multiplex with fixed thresholds (BUT), the layers
   are those of binary undirected (BU) multiplex graphs derived from the
   same weighted supra-connectivity matrices binarized at different
   thresholds. The supra-connectivity matrix has a number of partitions
   equal to the number of thresholds.
```

① MultiplexBUT is a child of MultiplexWU graph

Code 10: MultiplexBUT element prop update. The `props_update` section of generator code for `_MultiplexBUT.gen.m` updates the properties of MultiplexBUT. ← [Code 2](#)

```

1  %% iprops_update!
2
3  %%% iprop!
4  NAME (constant, string) is the name of the binary undirected multiplex
   with fixed thresholds.
5  %%% idefault!
6  'MultiplexBUT'
7
8  %%% iprop!
9  DESCRIPTION (constant, string) is the description of the binary undirected
10 multiplex with fixed thresholds.
11 %%% idefault!
12 'In a binary undirected multiplex with fixed thresholds (BUT), the layers
   are those of binary undirected (BU) multiplex graphs derived from the
   same weighted supra-connectivity matrices binarized at different
   thresholds. The supra-connectivity matrix has a number of partitions
   equal to the number of thresholds.'
```

```

13
14 %%% iprop!
15 TEMPLATE (parameter, item) is the template of the binary undirected
   multiplex with fixed thresholds.
16
17 %%% iprop!
18 ID (data, string) is a few-letter code of the binary undirected multiplex
   with fixed thresholds.
19 %%% idefault!
20 'MultiplexBUT ID'
21
```

```

22  %%% iprop!
23  LABEL (metadata, string) is an extended label of the binary undirected
    multiplex with fixed thresholds.
24  %%% idefault!
25  'MultiplexBUT label'
26
27  %%% iprop!
28  NOTES (metadata, string) are some specific notes about the binary
    undirected multiplex with fixed thresholds.
29  %%% idefault!
30  'MultiplexBUT notes'
31
32  %%% iprop!
33  GRAPH_TYPE (constant, scalar) returns the graph type __Graph.MULTIPLEX__.
34  %%% idefault!
35  Graph.MULTIPLEX
36
37  %%% iprop!
38  CONNECTIVITY_TYPE (query, smatrix) returns the connectivity type __Graph.
    BINARY__ * ones(layernumber).
39  %%% icalculate!
40  if isempty(varargin)
41      layernumber = 1;
42  else
43      layernumber = varargin{1};
44  end
45  value = Graph.BINARY * ones(layernumber);
46
47  %%% iprop!
48  DIRECTIONALITY_TYPE (query, smatrix) returns the directionality type
    __Graph.UNDIRECTED__ * ones(layernumber).
49  %%% icalculate!
50  if isempty(varargin)
51      layernumber = 1;
52  else
53      layernumber = varargin{1};
54  end
55  value = Graph.UNDIRECTED * ones(layernumber);
56
57  %%% iprop!
58  SELFCONNECTIVITY_TYPE (query, smatrix) returns the self-connectivity type
    __Graph.NONSELFCONNECTED__ on the diagonal and __Graph.SELFCONNECTED__
    off diagonal.
59  %%% icalculate!
60  if isempty(varargin)
61      layernumber = 1;
62  else
63      layernumber = varargin{1};
64  end
65  value = Graph.SELFCONNECTED * ones(layernumber);
66  value(1:layernumber+1:end) = Graph.NONSELFCONNECTED;
67
68  %%% iprop!
69  NEGATIVITY_TYPE (query, smatrix) returns the negativity type __Graph.
    NONNEGATIVE__ * ones(layernumber).
70  %%% icalculate!
71  if isempty(varargin)
72      layernumber = 1;
73  else
74      layernumber = varargin{1};
75  end

```

```

76 value = Graph.NONNEGATIVE * ones(layernumber);
77
78 %%% iprop!
79 A (result, cell) is the cell containing multiplex binary adjacency
    matrices of the binary undirected multiplex.
80
81 %%% icalculate!
82 A_WU = calculateValue@MultiplexWU(g, prop); ①
83
84 thresholds = g.get('THRESHOLDS'); ②
85 L = length(A_WU); % number of layers ③
86 A = cell(length(thresholds)*L); ④
87
88 if L > 0 && ~isempty(cell2mat(A_WU))
89     A(:, :) = {eye(length(A_WU{1, 1}))};
90     for i = 1:length(thresholds) ⑤
91         threshold = thresholds(i);
92         layer = 1;
93         for j = (i - 1) * L + 1:i * L ⑥
94             A{j, j} = dediagonalize(binrize(A_WU{layer, layer}, 'threshold',
                threshold)); ⑦
95             layer = layer + 1;
96         end
97     end
98 end
99
100 value = A;
101
102
103 %%% igui! ⑧
104 pr = PanelPropCell('EL', g, 'PROP', MultiplexBUT.A, ...
105     'TABLE_HEIGHT', s(40), ...
106     'XSLIDERLOCK', true, ...
107     'XSLIDERSHOW', false, ...
108     'YSLIDERSHOW', true, ...
109     'YSLIDERLABELS', g.getCallback('ALAYERLABELS'), ...
110     'YSLIDERWIDTH', s(5), ...
111     'ROWNAME', g.getCallback('ANODELABELS'), ...
112     'COLUMNNAME', g.getCallback('ANODELABELS'), ...
113     varargin{:});
114
115 %%% iprop!
116 PARTITIONS (result, rvector) returns the number of layers in the
    partitions of the graph.
117 %%% icalculate!
118 l = g.get('LAYERNUMBER');
119 thresholds = g.get('THRESHOLDS');
120 value = ones(1, length(thresholds)) * l / length(thresholds);
121
122 %%% iprop!
123 ALAYERLABELS (query, stringlist) returns the layer labels to be used by
    the slider.
124 %%% icalculate!
125 alayerlabels = g.get('ALAYERLABELS');
126 if ~isa(g.get('A'), 'NoValue') && length(alayerlabels) ~= g.get('
    LAYERNUMBER') % ensures that it's not unnecessarily calculated
127     thresholds = cellfun(@num2str, num2cell(g.get('THRESHOLDS')), '
        uniformoutput', false);
128

```

- ① calculates the graph MultiplexWU calling its parent MultiplexWU.
 ② gets the thresholds to be applied to A_WU.
 ③ gets the number of layers in graph A_WU.
 ④ The new MultiplexBUT graph will have L layers for each threshold applied.
 ⑤ iterates over all the thresholds to be applied
 ⑥ iterates over all the layers in A_WU
 ⑦ binarizes the present layer of the A_WU graph according to the present threshold

- ⑧ Same as in ← Code 2 ②

```

129     if length(alayerlabels) == length(g.get('B'))
130         blayerlabels = alayerlabels;
131     else % includes isempty(layerlabels)
132         blayerlabels = cellfun(@num2str, num2cell([1:1:length(g.get('B'))]), '
uniformoutput', false);
133     end
134
135     alayerlabels = {};
136     for i = 1:1:length(thresholds) ⑨
137         for j = 1:1:length(blayerlabels)
138             alayerlabels = [alayerlabels, [blayerlabels{j} '|' thresholds{i}]];
139         end
140     end
141 end
142 value = alayerlabels;
143
144 %%% iprop!
145 COMPATIBLE_MEASURES (constant, classlist) is the list of compatible
    measures.
146 %%% idefault!
147 getCompatibleMeasures('MultiplexBUT')
148

```

⑨ sets the labels of layers considering the thresholds and the number of layers in each multiplex graph for each threshold

Code 11: **MultiplexBUT element props.** The props section of generator code for MultiplexBUT.gen.m defines the properties to be used in MultiplexBUT. ← [Code 3](#)

```

1
2
3 %%% iprops!
4
5 %%% iprop!
6 THRESHOLDS (parameter, rvector) is the vector of thresholds.
7 %%% igui! ①
8 pr = PanelPropRVectorSmart('EL', g, 'PROP', MultiplexBUT.THRESHOLDS, 'MAX'
    , 1, 'MIN', -1, varargin{:});
9
10

```

① PanelPropRVectorSmart plots the panel for a row vector with an edit field. Smart means that (almost) any MatLab expression leading to a correct row vector can be introduced in the edit field. Also, the value of the vector can be limited between some MIN and MAX.

Code 12: **MultiplexBUT element tests.** The tests section from the element generator _MultiplexBUT.gen.m. ← [Code 4](#)

```

1 %%% itests!
2
3 %%% itest!
4 %%% iname!
5 Constructor - Full
6 %%% iprobability!
7 .01
8 %%% icode!
9 B1 = [
10 0 .1 .2 .3 .4
11 .1 0 .1 .2 .3
12 .2 .1 0 .1 .2
13 .3 .2 .1 0 .1
14 .4 .3 .2 .1 0
15 ];
16 B = {B1, B1, B1}; ①

```

① creates an example MultiplexWU

```

17 thresholds = [0 .1 .2 .3 .4]; ②
18 g = MultiplexBUT('B', B, 'THRESHOLDS', thresholds);
19
20 g.get('A_CHECK')
21
22 A = g.get('A');
23 for i = 1:length(B) * length(thresholds)
24     for j = 1:length(B) * length(thresholds)
25         if i == j
26             threshold = thresholds(floor((i - 1) / length(B)) + 1);
27             assert(isequal(A{i, i}, binarize(B1, 'threshold', threshold)), ...
28                 [BRAPH2.STR ':MultiplexBUT:' BRAPH2.FAIL_TEST], ...
29                 'MultiplexBUT is not constructing well.')
30         else
31             assert(isequal(A{i, j}, eye(length(B1))), ...
32                 [BRAPH2.STR ':MultiplexBUT:' BRAPH2.FAIL_TEST], ...
33                 'MultiplexBUT is not constructing well.')
34         end
35     end
36 end
37

```

② defines some example thresholds

Binary Undirected Ordinal Multiplex Graph with fixed Thresholds (OrdMxBUT)

Finally, we implement the OrdMxBUT graph based on previous codes GraphBD, MultilayerWD and MultiplexBUT, again highlighting the differences. An ordered multiplex is a type of multiplex graph that consists of a sequence of layers with ordinal edges between corresponding nodes in subsequent layers.

Code 13: OrdMxBUT element header. The header section of generator code for `_OrdMxBUT.gen.m` provides the general information about the OrdMxBUT element. ← [Code 1](#)

```

1  %% iheader!
2  OrdMxBUT < OrdMxWU (g, ordinal multiplex binary undirected with fixed
    thresholds) is a binary undirected ordinal multiplex with fixed
    thresholds. ①
3
4  %% idescription!
5  In a binary undirected ordinal multiplex with fixed thresholds (BUT), all
    the layers consist of binary undirected (BU) multiplex graphs derived
    from the same weighted supra-connectivity matrices binarized at
    different thresholds. The supra-connectivity matrix has a number of
    partitions equal to the number of thresholds. The layers are connected
    in an ordinal fashion, i.e., only consecutive layers are connected.
```

① OrdMxBUT is a child of OrdMxWU graph

Code 14: OrdMxBUT element prop update. The `props_update` section of generator code for `_OrdMxBUT.gen.m` updates the properties of OrdMxBUT. ← [Code 2](#)

```

1  %% iprops_update!
2
3  %% iprop!
4  NAME (constant, string) is the name of the binary undirected ordinal
    multiplex with fixed thresholds.s.
5  %%% idefault!
6  'OrdMxBUT'
7
8  %% iprop!
9  DESCRIPTION (constant, string) is the description of the binary undirected
    ordinal multiplex with fixed thresholds..
10 %%% idefault!
11 'In a binary undirected ordinal multiplex with fixed thresholds (BUT), all
    the layers consist of binary undirected (BU) multiplex graphs derived
    from the same weighted supra-connectivity matrices binarized at
    different thresholds. The supra-connectivity matrix has a number of
    partitions equal to the number of thresholds. The layers are
    connected in an ordinal fashion, i.e., only consecutive layers are
    connected.'
12
13 %%% iprop!
14 TEMPLATE (parameter, item) is the template of the binary undirected
    ordinal multiplex with fixed thresholds.
15
16 %%% iprop!
17 ID (data, string) is a few-letter code of the binary undirected ordinal
    multiplex with fixed thresholds.
```

```

18 %%%% idefault!
19 'OrdMxBUT ID'
20
21 %%% iprop!
22 LABEL (metadata, string) is an extended label of the binary undirected
    ordinal multiplex with fixed thresholds.
23 %%%% idefault!
24 'OrdMxBUT label'
25
26 %%% iprop!
27 NOTES (metadata, string) are some specific notes about the binary
    undirected ordinal multiplex with fixed thresholds.
28 %%%% idefault!
29 'OrdMxBUT notes'
30
31 %%% iprop!
32 GRAPH_TYPE (constant, scalar) returns the graph type __Graph.
    ORDERED_MULTIPLEX__.
33 %%%% idefault!
34 Graph.ORDERED_MULTIPLEX
35
36 %%% iprop!
37 CONNECTIVITY_TYPE (query, smatrix) returns the connectivity type __Graph.
    BINARY__ * ones(layernumber).
38 %%%% icalculate!
39 if isempty(varargin)
40     layernumber = 1;
41 else
42     layernumber = varargin{1};
43 end
44 value = Graph.BINARY * ones(layernumber);
45
46 %%% iprop!
47 DIRECTIONALITY_TYPE (query, smatrix) returns the directionality type
    __Graph.UNDIRECTED__ * ones(layernumber).
48 %%%% icalculate!
49 if isempty(varargin)
50     layernumber = 1;
51 else
52     layernumber = varargin{1};
53 end
54 value = Graph.UNDIRECTED * ones(layernumber);
55
56 %%% iprop!
57 SELFCONNECTIVITY_TYPE (query, smatrix) returns the self-connectivity type
    __Graph.NONSELFCONNECTED__ on the diagonal and __Graph.SELFCONNECTED__
    off diagonal.
58 %%%% icalculate!
59 if isempty(varargin)
60     layernumber = 1;
61 else
62     layernumber = varargin{1};
63 end
64 value = Graph.SELFCONNECTED * ones(layernumber);
65 value(1:layernumber+1:end) = Graph.NONSELFCONNECTED;
66
67 %%% iprop!
68 NEGATIVITY_TYPE (query, smatrix) returns the negativity type __Graph.
    NONNEGATIVE__ * ones(layernumber).
69 %%%% icalculate!
70 if isempty(varargin)

```

```

71 layernumber = 1;
72 else
73 layernumber = varargin{1};
74 end
75 value = Graph.NONNEGATIVE * ones(layernumber);
76
77 %%% iprop!
78 A (result, cell) is the cell containing binary supra-adjacency matrix of
    the binary undirected multiplex with fixed thresholds (BUT).
79
80 %%% calculate!
81 A_WU = calculateValue@OrdMxWU(g, prop); ①
82
83 thresholds = g.get('THRESHOLDS'); ②
84 L = length(A_WU); % number of layers
85 A = cell(length(thresholds)*L);
86
87 if L > 0 && ~isempty(cell2mat(A_WU))
88     A(:, :) = {zeros(length(A_WU{1, 1}))};
89     for i = 1:length(thresholds) ③
90         threshold = thresholds(i);
91         layer = 1;
92         for j = (i - 1) * L + 1:i * L ④
93             for k = (i - 1) * L + 1:i * L
94                 if j == k ⑤
95                     A{j, j} = dediagonalize(binarize(A_WU{layer, layer}, 'threshold'
                        , threshold));
96                     elseif (j-k)==1 || (k-j)==1 ⑥
97                         A{j, k} = {eye(length(A{1, 1}))};
98                     else ⑦
99                         A{j, k} = {zeros(length(A{1, 1}))};
100                     end
101                 end
102                 layer = layer + 1;
103             end
104         end
105     end
106
107 value = A;
108
109 %%% igui! ⑧
110 pr = PanelPropCell('EL', g, 'PROP', OrdMxBUT.A, ...
111     'TABLE-HEIGHT', s(40), ...
112     'XYSLIDERLOCK', true, ...
113     'XSLIDERSHOW', false, ...
114     'YSLIDERSHOW', true, ...
115     'YSLIDERLABELS', g.getCallback('ALAYERLABELS'), ...
116     'YSLIDERWIDTH', s(5), ...
117     'ROWNAME', g.getCallback('ANODELABELS'), ...
118     'COLUMNNAME', g.getCallback('ANODELABELS'), ...
119     varargin{:});
120
121 %%% iprop!
122 PARTITIONS (result, rvector) returns the number of layers in the
    partitions of the graph.
123 %%% calculate!
124 l = g.get('LAYERNUMBER');
125 thresholds = g.get('THRESHOLDS');
126 value = ones(1, length(thresholds)) * l / length(thresholds);

```

① calculates the graph OrdMxWU calling the parent OrdMxWU.

② Same as in ← Code 10 ②-④.

③ For each threshold we construct an ordinal multiplex binary undirected graph

④ We need to loop over the layers of A_Wu for each threshold

⑤ In the diagonal of the supra-adjacency matrix we have the layers that are constructed by binarizing A_Wu according to the present threshold

⑥ Consecutive layers are connected

⑦ Non-consecutive layers are not connected

⑧ Same as in ← Code ??

```

127
128   %% iprop!
129   ALAYERLABELS (query, stringlist) returns the layer labels to be used by
      the slider.
130   %%% icalculate!
131   alayerlabels = g.get('LAYERLABELS');
132   if ~isa(g.get('A'), 'NoValue') && length(alayerlabels) ~= g.get('
      LAYERNUMBER') % ensures that it's not unnecessarily calculated
133   thresholds = cellfun(@num2str, num2cell(g.get('THRESHOLDS')), '
      uniformoutput', false);
134
135   if length(alayerlabels) == length(g.get('B'))
136       blayerlabels = alayerlabels;
137   else % includes isempty(layerlabels)
138       blayerlabels = cellfun(@num2str, num2cell([1:1:length(g.get('B'))]), '
      uniformoutput', false);
139   end
140
141   alayerlabels = {};
142   for i = 1:1:length(thresholds)
143       for j = 1:1:length(blayerlabels)
144           alayerlabels = [alayerlabels, [blayerlabels{j} '|' thresholds{i}]];
145       end
146   end
147   end
148   value = alayerlabels;
149
150   %% iprop!
151   COMPATIBLE_MEASURES (constant, classlist) is the list of compatible
      measures.
152   %%% idefault!
153   getCompatibleMeasures('OrdMxBUT')
154
155

```

Code 15: OrdMxBUT element props. The props section of generator code for `OrdMxBUT.gen.m` defines the properties to be used in `MultiplexBUT`. ← [Code 3](#)

```

1
2   %% iprops!
3
4   %% iprop!
5   THRESHOLDS (parameter, rvector) is the vector of thresholds.
6   %%% igui!
7   pr = PanelPropRVectorSmart('EL', g, 'PROP', OrdMxBUT.THRESHOLDS, 'MAX', 1, '
      MIN', -1, varargin{:});
8

```

Code 16: OrdMxBUT element tests. The tests section from the element generator `_OrdMxBUT.gen.m`. ← [Code 4](#)

```

1   %% itests!
2
3   %% iexcluded_props!
4   [OrdMxBUT.PFGA OrdMxBUT.PFGH]
5
6   %% itest!
7   %%% iname!

```

```

8  Constructor - Full ①
9  %%% iprobability!
10 .01
11 %%% icode!
12 B1 = [
13 0 .1 .2 .3 .4
14 .1 0 .1 .2 .3
15 .2 .1 0 .1 .2
16 .3 .2 .1 0 .1
17 .4 .3 .2 .1 0
18 ];
19 B = {B1, B1, B1};
20 thresholds = [0 .1 .2 .3 .4];
21 g = OrdMxBUT('B', B, 'THRESHOLDS', thresholds);
22
23 g.get('A_CHECK')
24
25 A = g.get('A');
26 for i = 1:length(thresholds)
27     threshold = thresholds(i);
28     for j = (i - 1) * length(B) + 1:i * length(B)
29         for k = (i - 1) * length(B) + 1:i * length(B)
30             if j == k
31                 assert(isequal(A{j, j}, binarize(B1, 'threshold', threshold)), ...
32                     [BRAPH2.STR ':OrdMxBUT:' BRAPH2.FAIL_TEST], ...
33                     'OrdMxBUT is not constructing well.')
34             elseif (j-k)==1 || (k-j)==1
35                 assert(isequal(A{j, k}, eye(length(B1))), ...
36                     [BRAPH2.STR ':OrdMxBUT:' BRAPH2.FAIL_TEST], ...
37                     'OrdMxBUT is not constructing well.')
38             else
39                 assert(isequal(A{j, k}, zeros(length(B1))), ...
40                     [BRAPH2.STR ':OrdMxBUT:' BRAPH2.FAIL_TEST], ...
41                     'OrdMxBUT is not constructing well.')
42             end
43         end
44     end
45 end
46

```

① same as in ← [Code 12](#).