FEM vs FVM

problema prototipo — eq. in forma forte

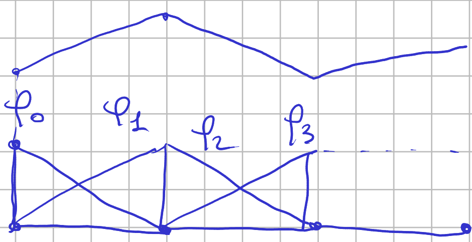
$$\begin{cases} -\operatorname{div}(\nabla u) = f & \text{in } \Omega \\ u = 0 & \text{su } \partial\Omega \end{cases}$$

FEM

$$\underbrace{-\int_\Omega \operatorname{div}(\nabla u)\cdot v \, dx}_{} = \int_\Omega f v \, dx$$

$$\int \operatorname{div}(\nabla u) v \, dx = \cancel{\int_{\partial\Omega} v \, u \cdot n \, ds} - \int_\Omega \nabla u \nabla v \, dx$$

$$\int_\Omega \nabla u \nabla v \, dx = \int_\Omega f v \, dx$$



$$u = \sum_{i=1}^{m} a_i \varphi_i(x)$$

$$Ax = b$$

$$\int_\Omega \varphi_i \varphi_j = \begin{cases} 1 & \text{se } i = j \\ 0 & \text{se } i \neq j \end{cases}$$

$$A_{ij} = \int_\Omega \nabla \varphi_i \cdot \nabla \varphi_j \, dx$$

FVM

$$\begin{cases} \operatorname{div}(\nabla u) = f & \text{su } \Omega \\ u = 0 & \text{su } \partial\Omega \end{cases}$$

$$\int_\Omega \mathrm{div}(\nabla u)\,dx = \int_\Omega f\,dx$$

$$\int_{\partial\Omega} \nabla u\cdot n\,dS = \int_\Omega f\,dx$$

$$\sum_{i=1}^{N_c} \underbrace{\int_{\partial\Omega_i}\nabla u\cdot n\,dS} = \sum_{i=1}^{N_c}\int_{\Omega_i} f\,dx$$

$$\sum_{i=1}^{N_c}\sum_{j=1}^{N_{f_i}} \left[\nabla u_{f_j}\cdot n_{f_j}\right] S_{f_j} = \sum_{i=1}^{N_c} f_i\, V_{c_i}$$

$u, v \qquad u, v \in V^h = \mathrm{span}\left(I_k(x)\right) \ \forall k \in I(\Omega)$

$$I_k(x) = \begin{cases} 1 & \text{se } x \in \Omega_k \\ 0 & \text{altrimenti} \end{cases}$$

$$\sum_{i=1}^{N_c} \cancel{v_i} \int_{\Omega_i} \mathrm{div}(\nabla u)\,dx = \sum_{i=1}^{N_c} \cancel{v_i} \int_{\Omega_i} f\,dx$$