**ASSIGNMENT #6**

Analysis of accuracy decrease of tracking in conditions of biased state noise

**Task for laboratory work:** analyze the sensitivity of estimation results obtained by a Kalman filter that doesn't take into account bias of state and measurement noise; develop optimal Kalman filter that takes into account bias of state noise (acceleration).

We've generated a true trajectory of an object motion with biased random acceleration. Parameters of this trajectory:

$$a_{i-1}^{biased} = a_{i-1} + q$$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}^{biased}T^2}{2}$$
$$V_i = V_{i-1} + a_{i-1}^{biased}T$$

Size of trajectory is 200 points.

Initial conditions: $x_1 = 5$; $V_1 = 1$; $T = 1$

Variance of acceleration noise: $\sigma_a^2 = 0.2^2$

Measurements were generated with parameters:

$$z_i = x_i + \eta_i$$

Variance of measurements noise: $\sigma_n^2 = 20^2$

By using the previously developed Kalman filtering algorithm, we obtained estimates of a state vector.

The results of filtering including true trajectory, measurements, filtered estimates of a state vector are presented in fig. 1.

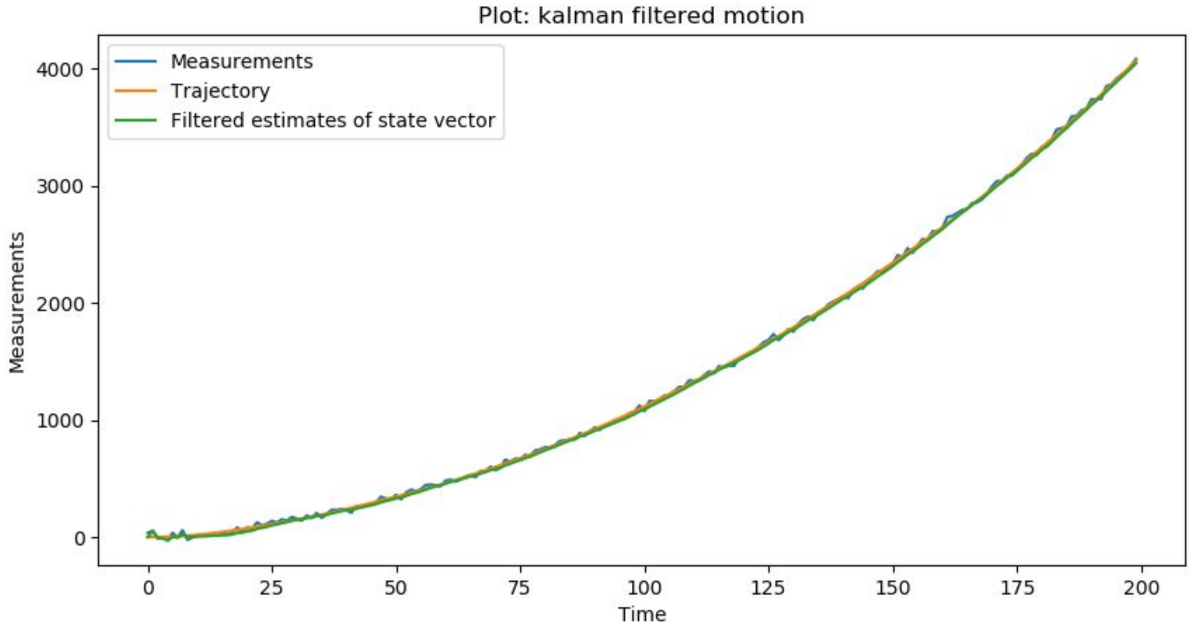


Figure 1

On the plot, it’s quite difficult to analyze the quality of the predicted curve, because of the influence of acceleration with a constant component. We put limitations on axes to make the plot readable. It is presented in fig. 2.

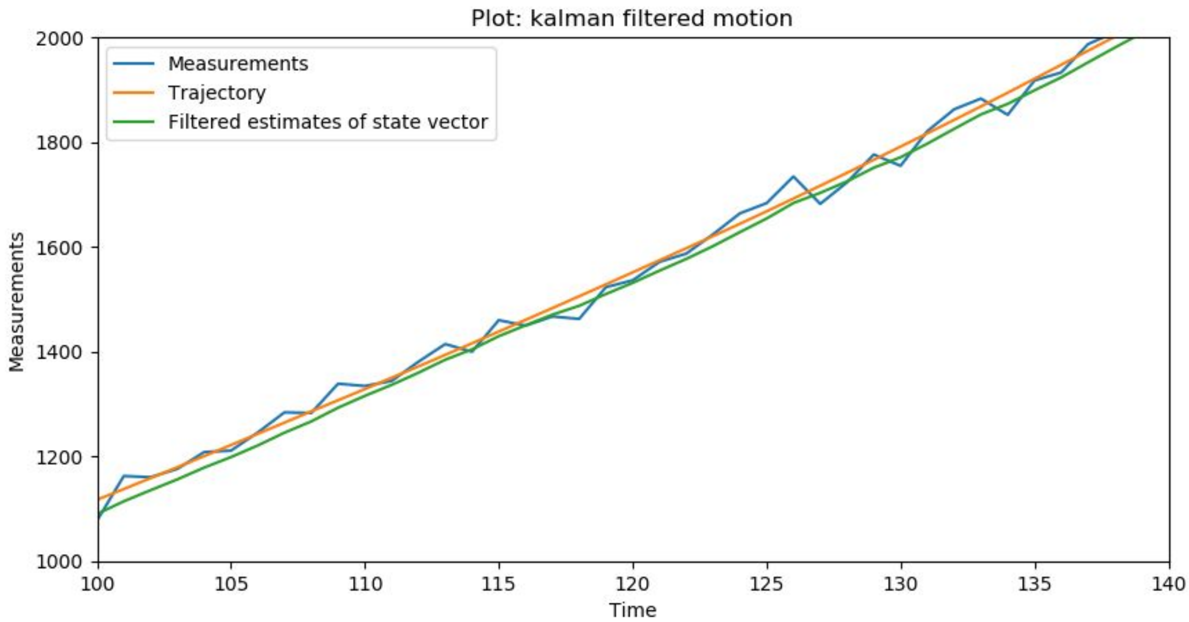


Figure 2

Still, we can’t say anything about the quality of prediction. Filtered estimate repeats somehow true trajectory, but it biased a little down.

To make a decision correctly we have calculated mean-squared error on 500 different measurements. Errors are presented at the plot in fig. 3

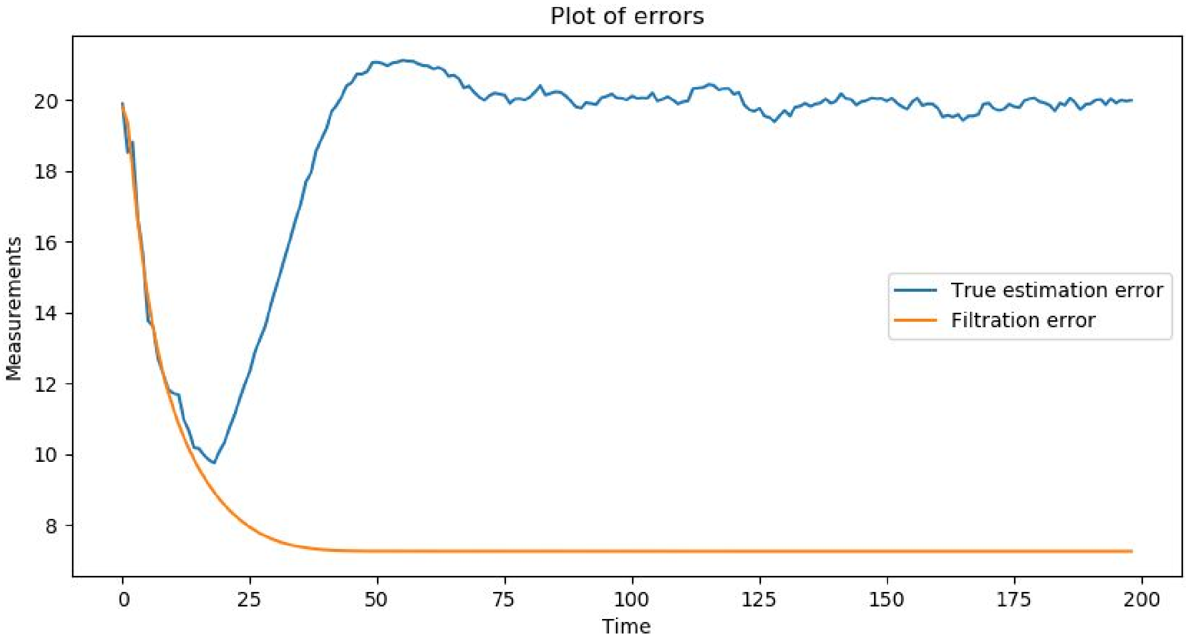


Figure 3

In fig. 3 we finally can confirm that this algorithm is not optimal for these measurements with biased acceleration. The true estimation error after 25 points was increasing to the value of 20 (meters). It biased to top in comparison with the first element of covariance matrix P and has a constant value.

To make our prediction more reliable we should include a value of bias to the Kalman filtration algorithm.

After implementing an upgraded algorithm we've got a true estimation error curve that has a good correlation with a standard deviation (via covariance matrix P). The graph of them is presented in fig. 4.

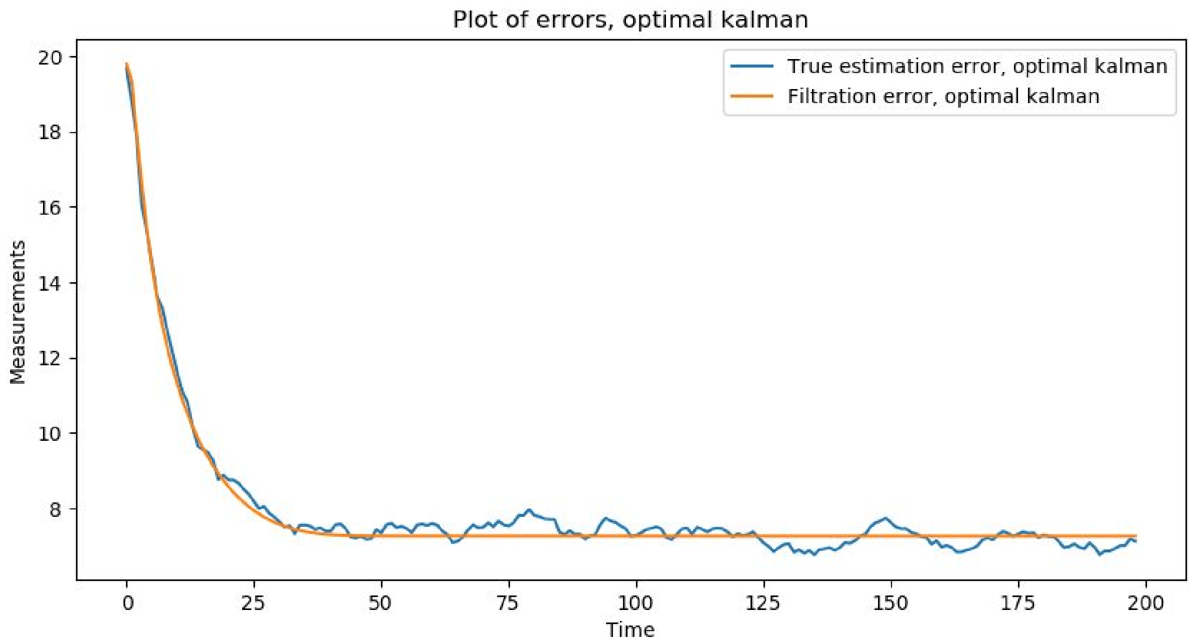


Figure 4

**Conclusions**

During this laboratory work, we’ve learned how to obtain Kalman filtration with biased random acceleration and how this bias influence on estimation error.