Lecture 13: Recurrent Neural Networks

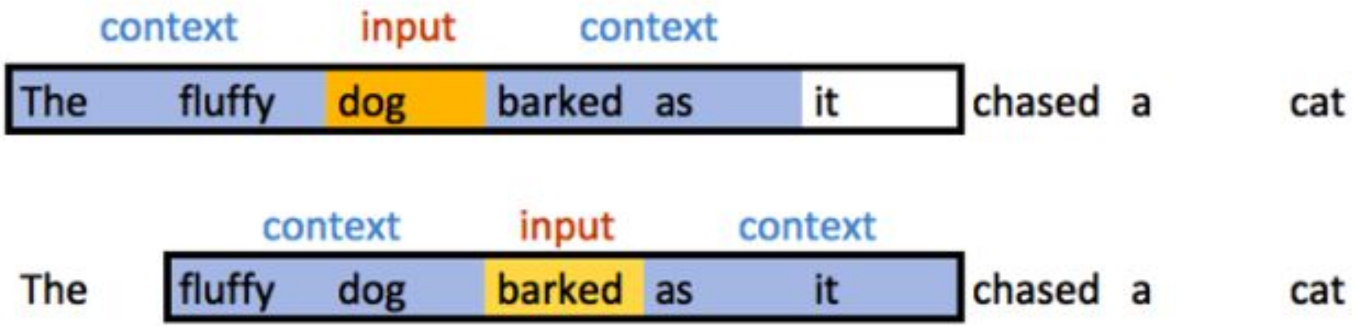
Ivan Provilkov

Outline

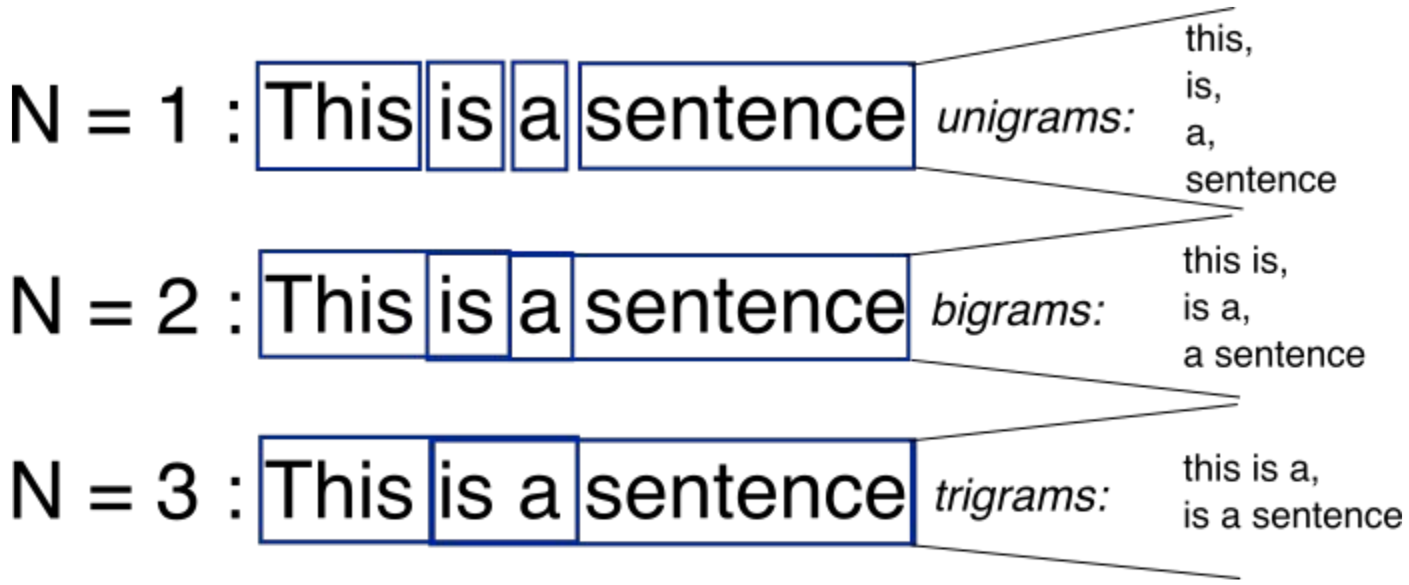
1. Contex idea
2. RNN intuitions
3. LSTM
4. Names generation from scratch
5. Q & A

Optional: Vanishing gradient and attention outro

Words cooccurrences: sliding window



Words cooccurrences: n-grams



RNNs generating...

Shakespeare

PANDARUS:
Alas, I think he shall be come approached and the day
When little srain would be attain'd into being never fed,
And who is but a chain and subjects of his death,
I should not sleep.

Second Senator:
They are away this miseries, produced upon my soul,
Breaking and strongly should be buried, when I perish
The earth and thoughts of many states.

DUKE VINCENTIO:
Well, your wit is in the care of side and that.

Second Lord:
They would be ruled after this chamber, and
my fair nues begun out of the fact, to be conveyed,
Whose noble souls I'll have the heart of the wars.

Clown:
Come, sir, I will make did behold your worship.

VIOLA:
I'll drink it.

Algebraic Geometry (Latex)

Proof. Omitted. □

Lemma 0.1. *Let $\mathcal{C}$ be a set of the construction.*

Let $\mathcal{C}$ be a gerber covering. Let $\mathcal{F}$ be a quasi-coherent sheaves of $\mathcal{O}$-modules. We have to show that

$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

.

Proof. This is an algebraic space with the composition of sheaves $\mathcal{F}$ on $X_{étale}$ we have

$$\mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$$

where $\mathcal{G}$ defines an isomorphism $\mathcal{F} \to \mathcal{F}$ of $\mathcal{O}$-modules. □

Lemma 0.2. *This is an integer $\mathcal{Z}$ is injective.*

Proof. See Spaces, Lemma ??. □

Lemma 0.3. *Let S be a scheme. Let X be a scheme and X is an affine open covering. Let $\mathcal{U} \subset \mathcal{X}$ be a canonical and locally of finite type. Let X be a scheme. Let X be a scheme which is equal to the formal complex.*

The following to the construction of the lemma follows.

Let X be a scheme. Let X be a scheme covering. Let

$$b : X \to Y' \to Y \to Y \to Y' \times_X Y \to X.$$

be a morphism of algebraic spaces over S and Y.

Proof. Let X be a nonzero scheme of X. Let X be an algebraic space. Let $\mathcal{F}$ be a quasi-coherent sheaf of $\mathcal{O}_X$-modules. The following are equivalent

(1) $\mathcal{F}$ is an algebraic space over S.
(2) If X is an affine open covering.

Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of finite type. □

Linux kernel (source code)

```
/*
 * If this error is set, we will need anything right after that BSD.
 */
static void action_new_function(struct s_stat_info *wb)
{
  unsigned long flags;
  int lel_idx_bit = e->edd, *sys & ~((unsigned long) *FIRST_COMPAT);
  buf[0] = 0xFFFFFFFF & (bit << 4);
  min(inc, slist->bytes);
  printk(KERN_WARNING "Memory allocated %02x/%02x, "
    "original MLL instead\n"),
    min(min(multi_run - s->len, max) * num_data_in),
    frame_pos, sz + first_seg);
  div_u64_w(val, inb_p);
  spin_unlock(&disk->queue_lock);
  mutex_unlock(&s->sock->mutex);
  mutex_unlock(&func->mutex);
  return disassemble(info->pending_bh);
}
```

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Proof. Omitted. $\square$

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Proof. See Spaces, Lemma **??**. $\square$

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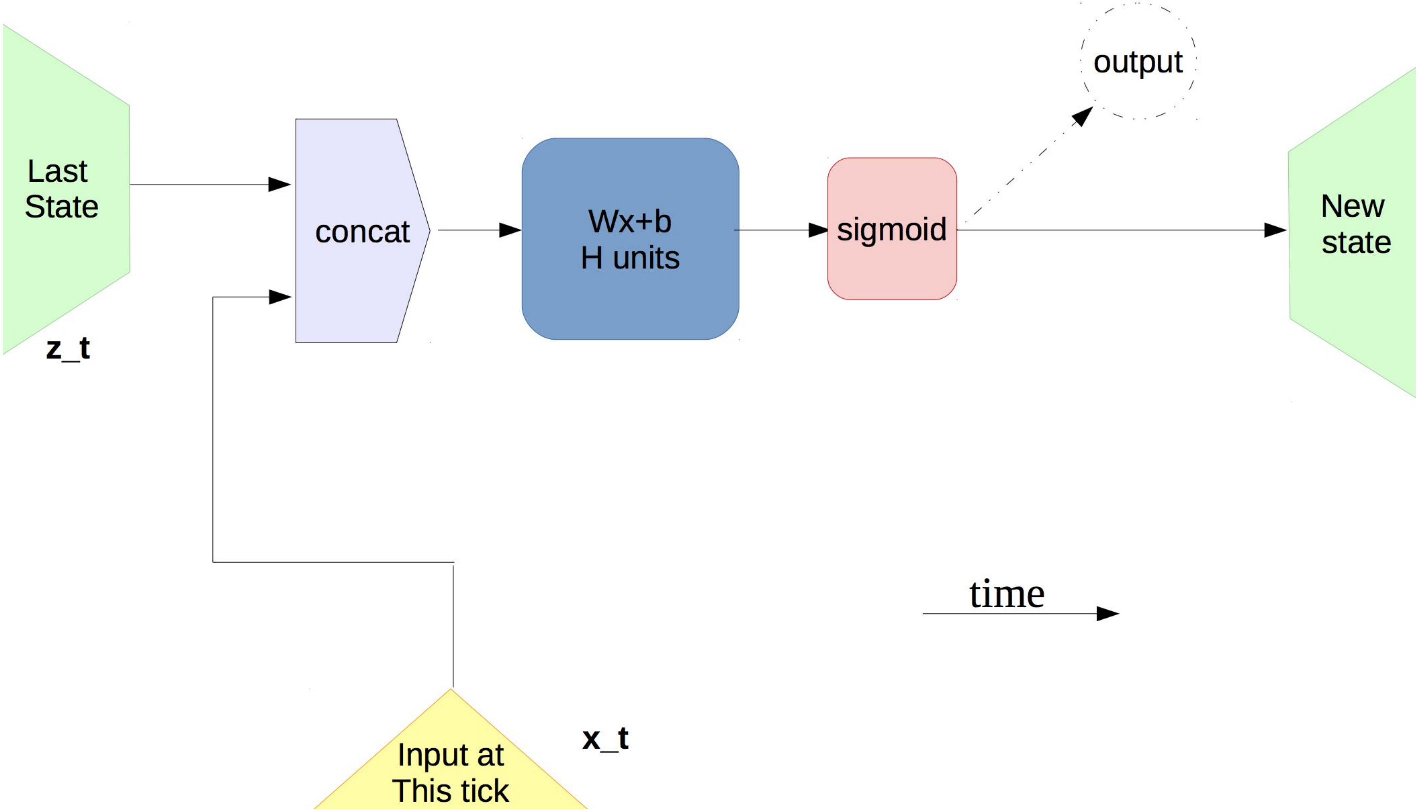
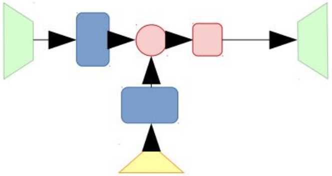
Recurrent neural network

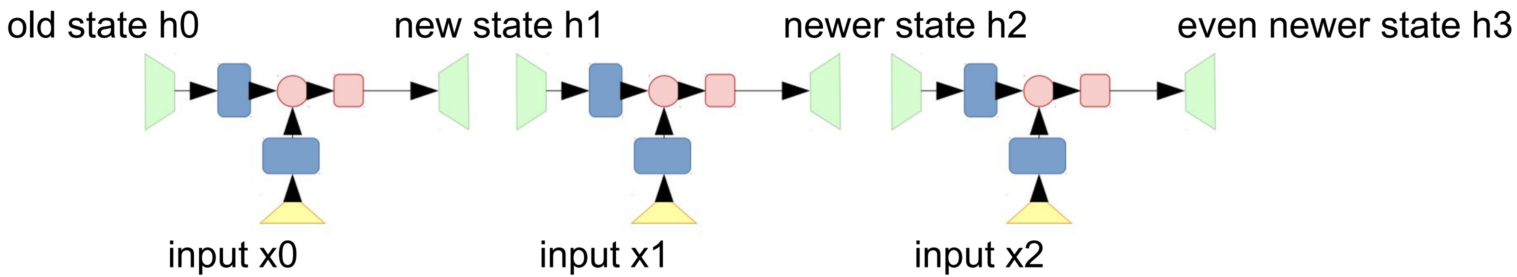


Recurrent neural network

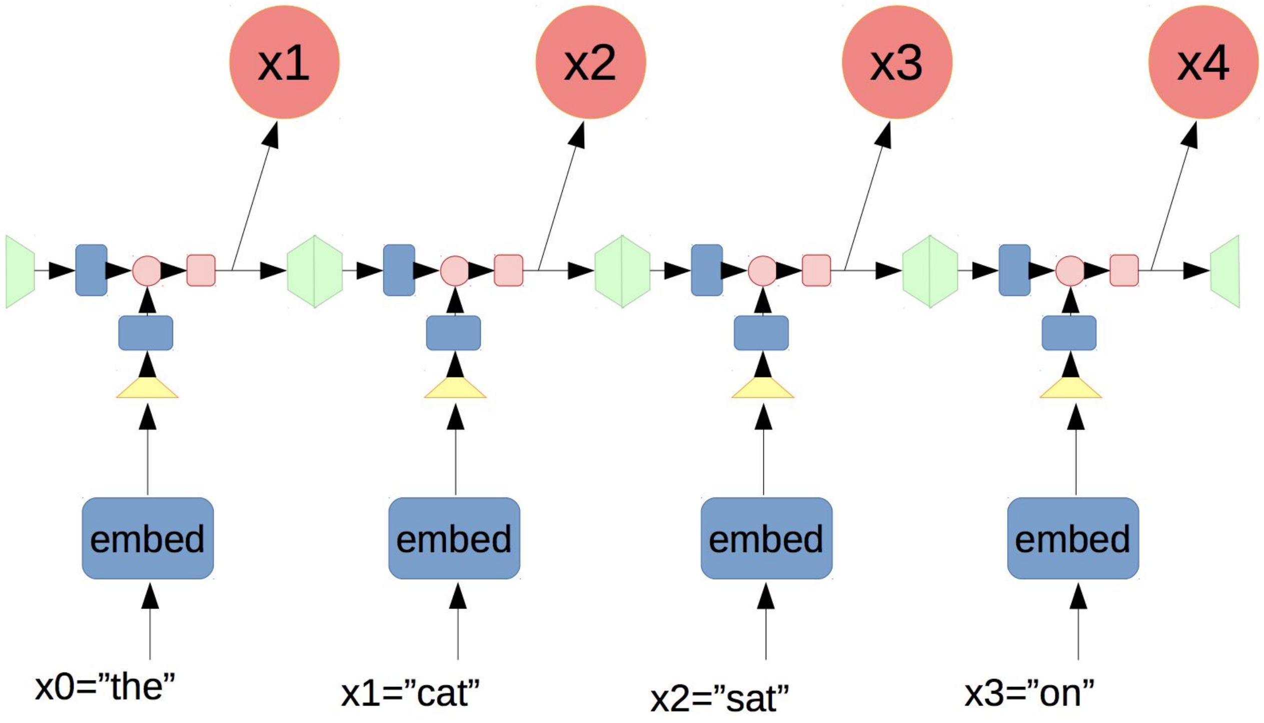


Recurrent neural network

We use same weight matrices for all steps



Recurrent neural network



Now with formulas

$$h_0 = \bar{0}$$

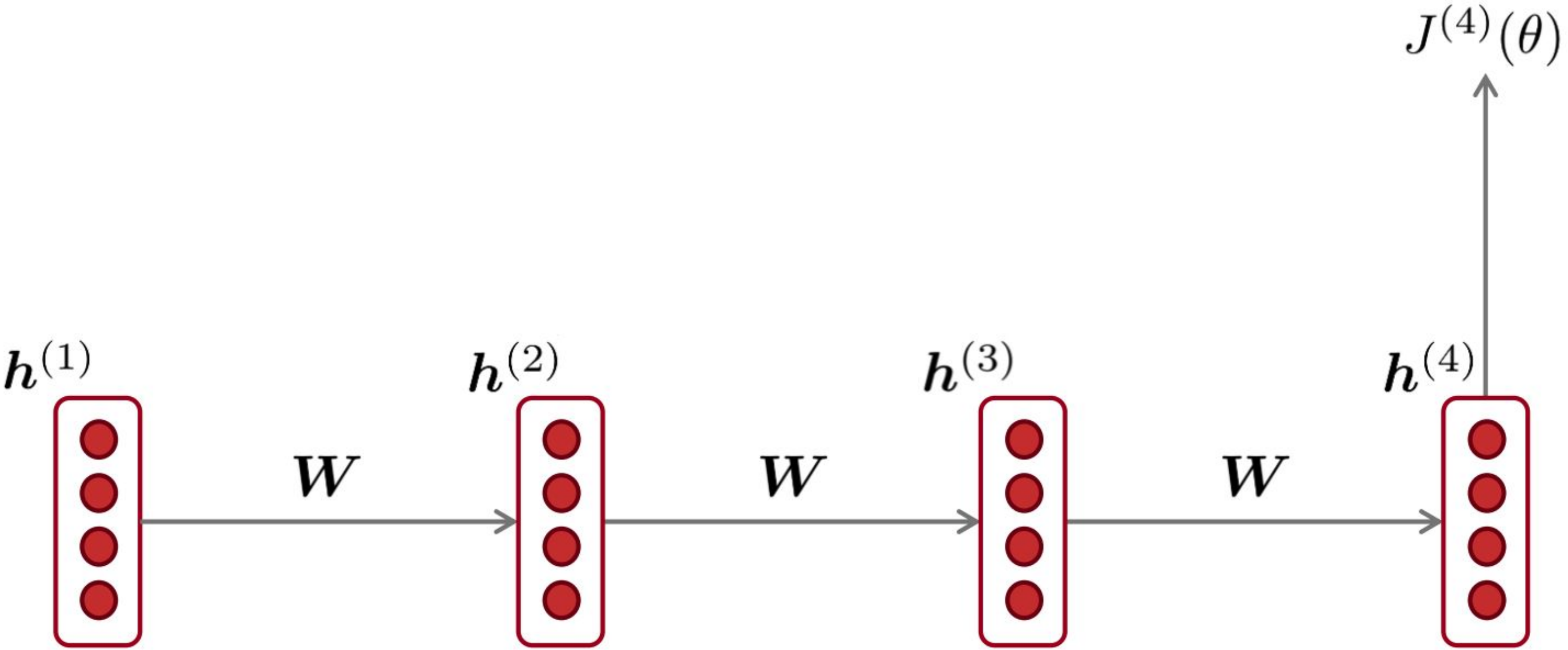
$$h_1 = \sigma\big(\langle W_{\text{hid}}[h_0, x_0]\rangle + b\big)$$

$$h_2 = \sigma\big(\langle W_{\text{hid}}[h_1, x_1]\rangle + b\big) = \sigma\big(\langle W_{\text{hid}}[\sigma\big(\langle W_{\text{hid}}[h_0, x_0]\rangle + b, x_1]\rangle + b\big)$$

$$h_{i+1} = \sigma\big(\langle W_{\text{hid}}[h_i, x_i]\rangle + b\big)$$

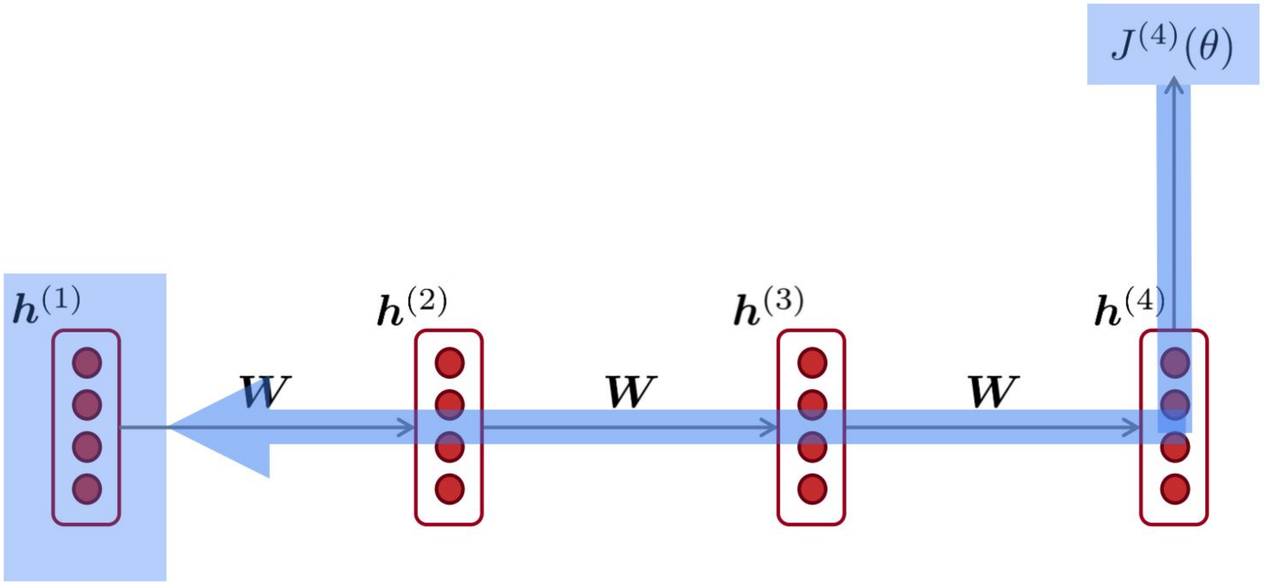
$$P(x_{i+1}) = \text{softmax}\big(\langle W_{\text{out}}, h_i\rangle + b_{\text{out}}\big)$$

Vanishing gradient problem



Based on: Lecture by Abigail See, [CS224n Lecture 7]

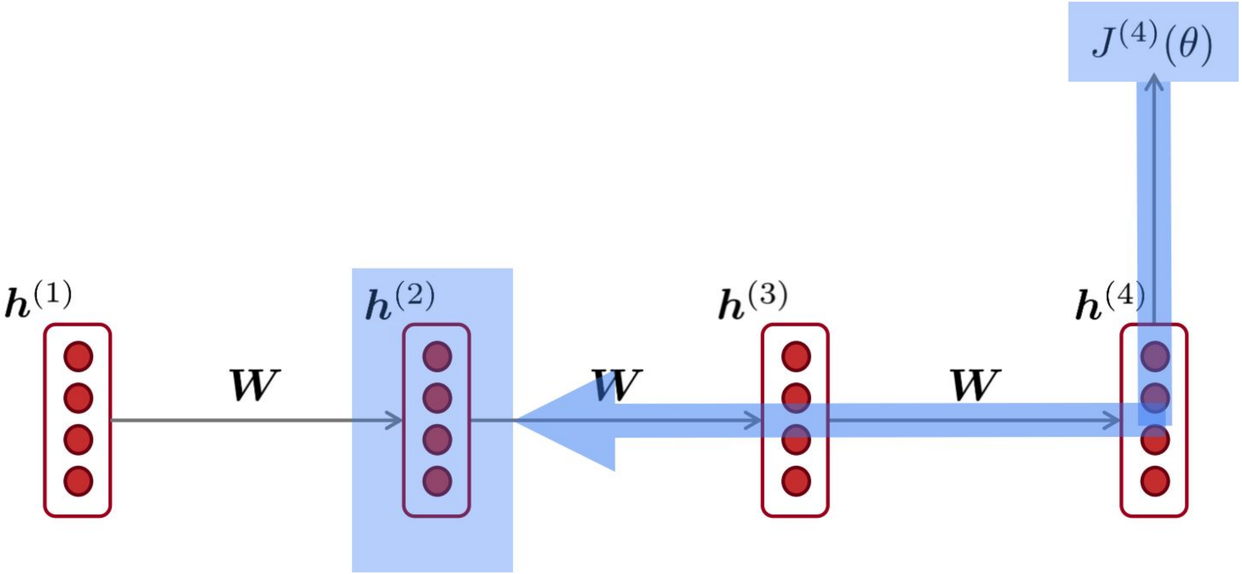
Vanishing gradient problem



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = ?$$

Based on: Lecture by Abigail See, [CS224n Lecture 7]

Vanishing gradient problem



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \frac{\partial \boldsymbol{h}^{(2)}}{\partial \boldsymbol{h}^{(1)}} \times \frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(2)}}$$

chain rule!

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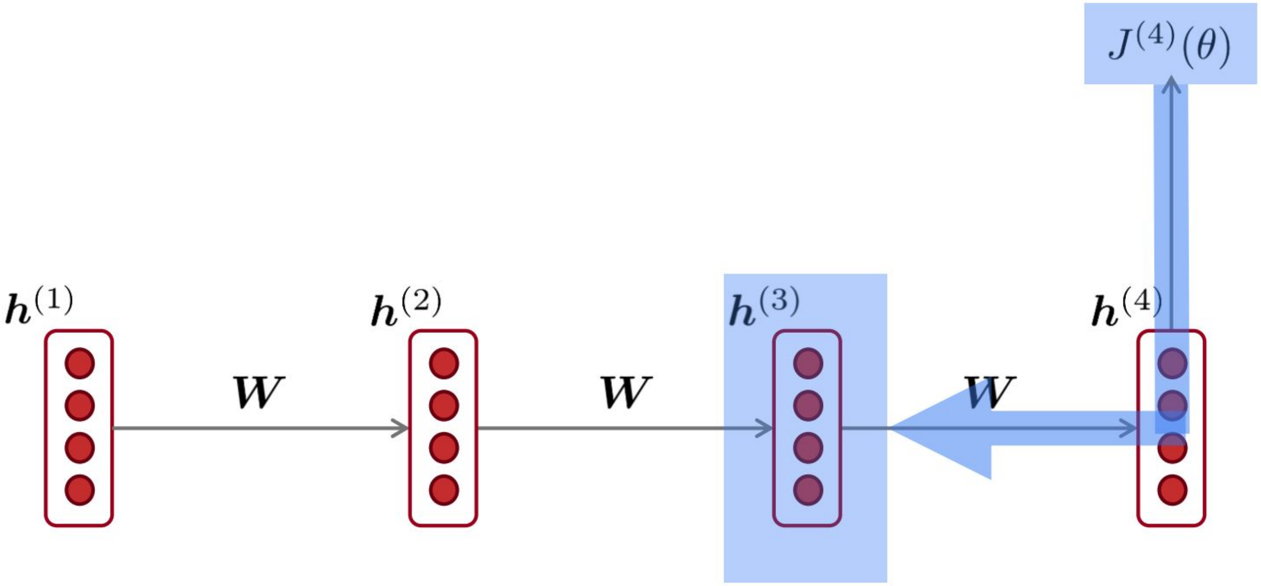
Vanishing gradient problem



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \frac{\partial \boldsymbol{h}^{(2)}}{\partial \boldsymbol{h}^{(1)}} \times \qquad \frac{\partial \boldsymbol{h}^{(3)}}{\partial \boldsymbol{h}^{(2)}} \times \frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(3)}}$$

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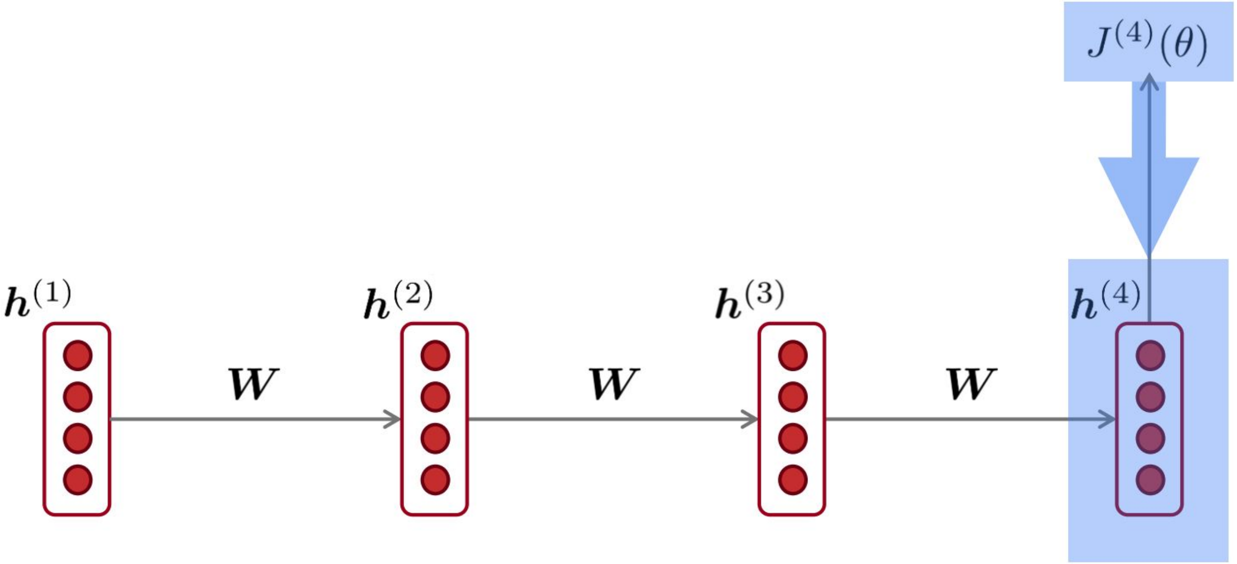
Vanishing gradient problem



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \frac{\partial \boldsymbol{h}^{(2)}}{\partial \boldsymbol{h}^{(1)}} \times \frac{\partial \boldsymbol{h}^{(3)}}{\partial \boldsymbol{h}^{(2)}} \times \frac{\partial \boldsymbol{h}^{(4)}}{\partial \boldsymbol{h}^{(3)}} \times \frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(4)}}$$

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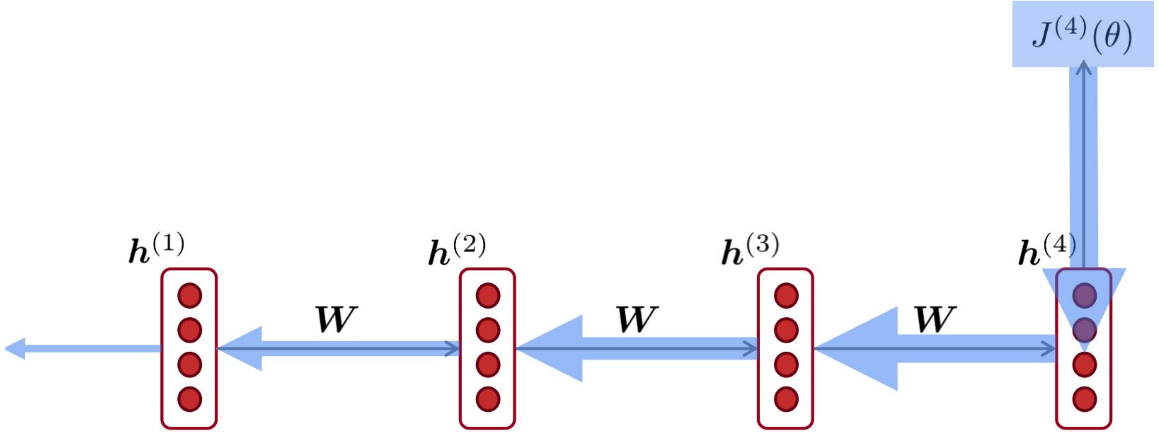
Vanishing gradient problem



Vanishing gradient problem:

When the derivatives are small, the gradient signal gets smaller and smaller as it backpropagates further

$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \boxed{\frac{\partial \boldsymbol{h}^{(2)}}{\partial \boldsymbol{h}^{(1)}}} \times \boxed{\frac{\partial \boldsymbol{h}^{(3)}}{\partial \boldsymbol{h}^{(2)}}} \times \boxed{\frac{\partial \boldsymbol{h}^{(4)}}{\partial \boldsymbol{h}^{(3)}}} \times \frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(4)}}$$

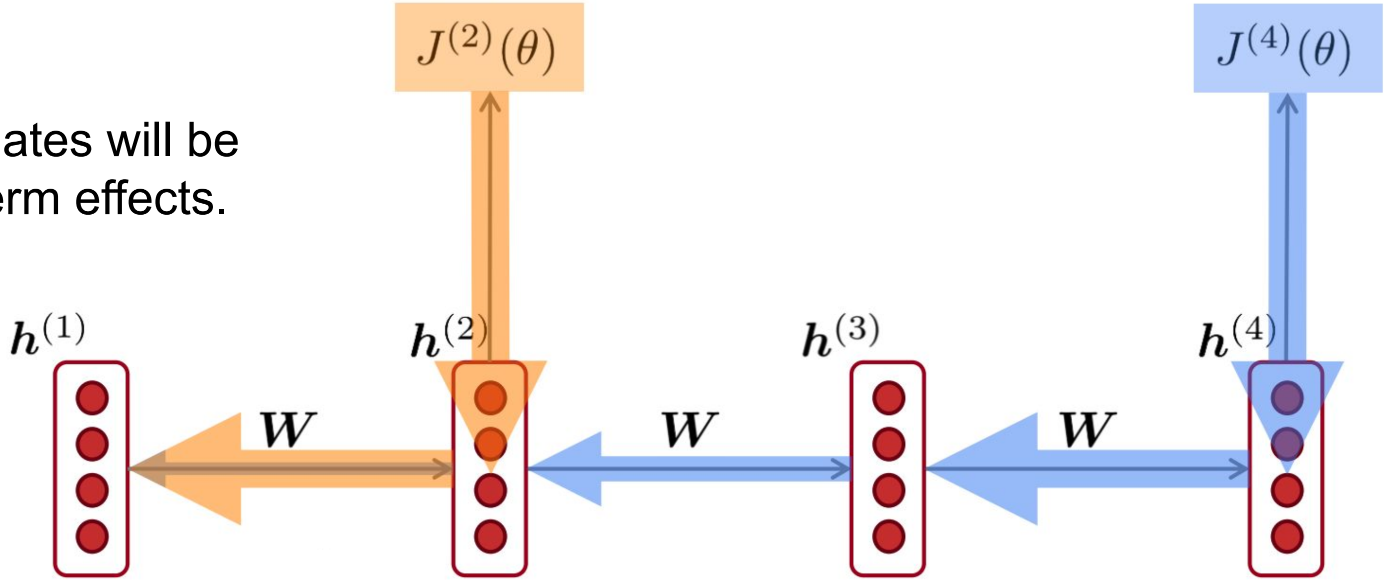
What happens if these are small?

More info: "On the difficulty of training recurrent neural networks", Pascanu et al, 2013
http://proceedings.mlr.press/v28/pascanu13.pdf

Vanishing gradient problem

Gradient signal from far away is lost because it's much smaller than from close-by.

So model weights updates will be based only on short-term effects.



Based on: Lecture by Abigail See, [CS224n Lecture 7]

Exploding gradient problem

- If the gradient becomes too big, then the SGD update step becomes too big:

$$\theta^{new} = \theta^{old} - \overbrace{\alpha}^{\text{learning rate}} \underbrace{\nabla_\theta J(\theta)}_{\text{gradient}}$$

- This can cause bad updates: we take too large a step and reach a bad parameter configuration (with large loss)

- In the worst case, this will result in Inf or NaN in your network (then you have to restart training from an earlier checkpoint)

Based on: Lecture by Abigail See, [CS224n Lecture 7]

Exploding gradient solution

- Gradient clipping: if the norm of the gradient is greater than some threshold, scale it down before applying SGD update

Algorithm 1 Pseudo-code for norm clipping

$$\hat{\mathbf{g}} \leftarrow \frac{\partial \mathcal{E}}{\partial \theta}$$

if $\|\hat{\mathbf{g}}\| \geq threshold$ **then**

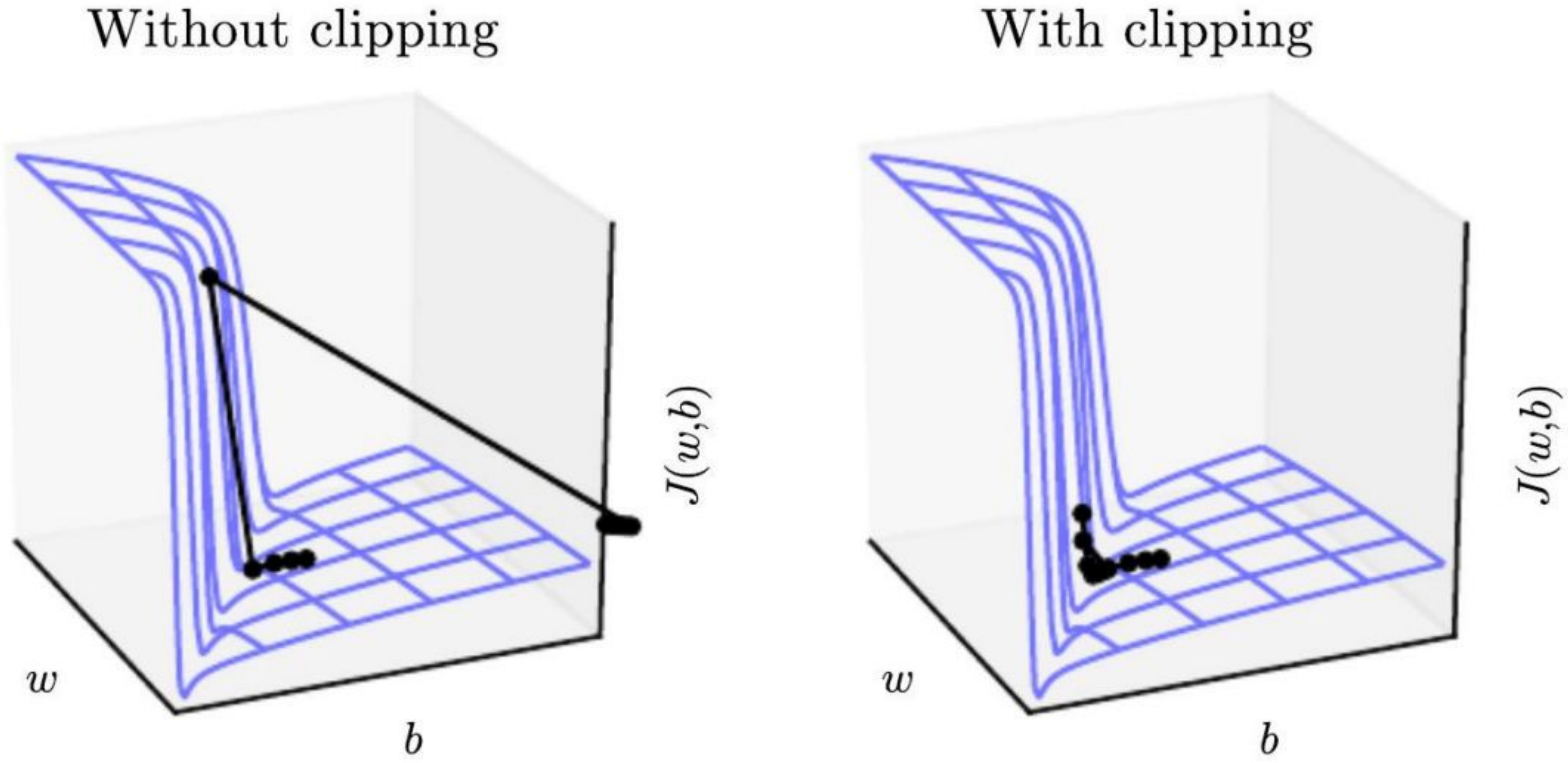
$$\hat{\mathbf{g}} \leftarrow \frac{threshold}{\|\hat{\mathbf{g}}\|} \hat{\mathbf{g}}$$

end if

- Intuition: take a step in the same direction, but a smaller step

Based on: Lecture by Abigail See, [CS224n Lecture 7]

Exploding gradient solution



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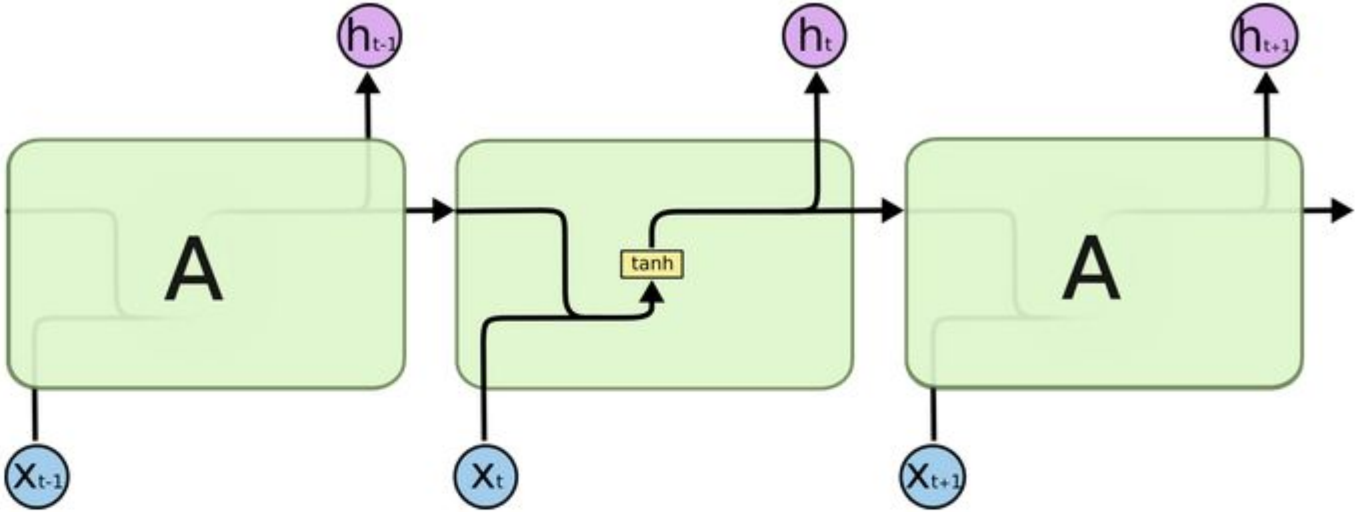
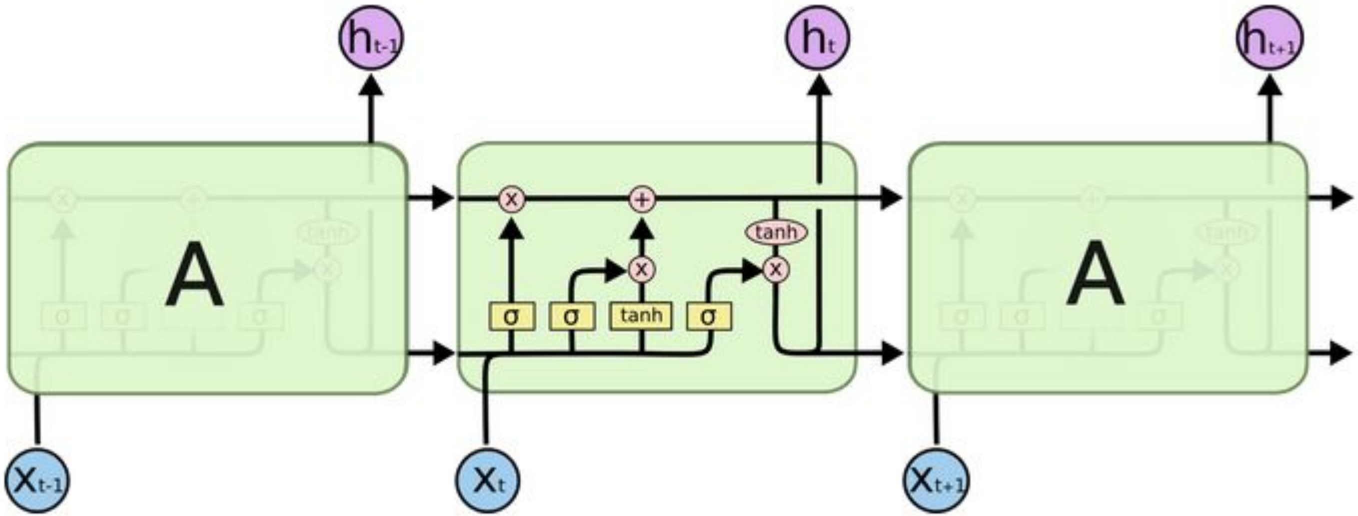
Vanilla RNN



Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTM



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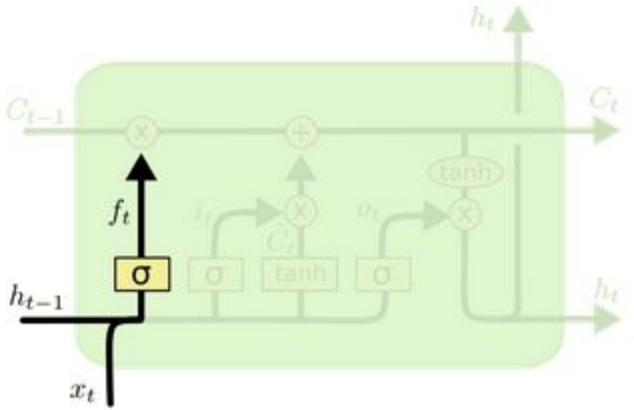
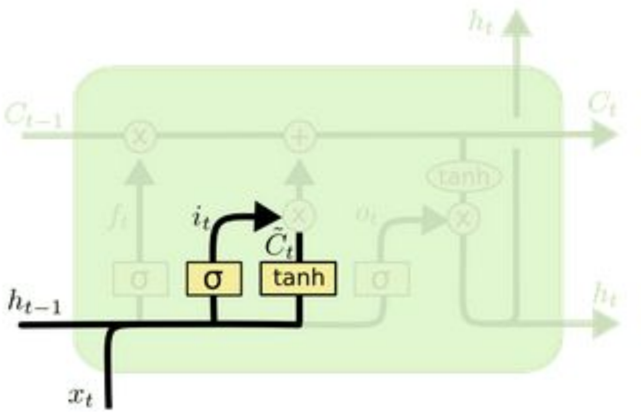
LSTM: quick overview



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$

Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTM: quick overview

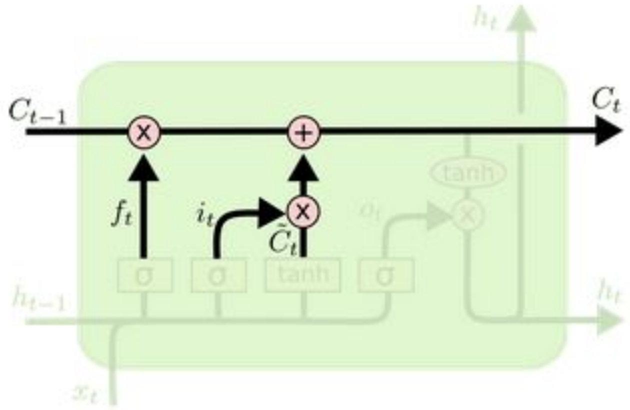


$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

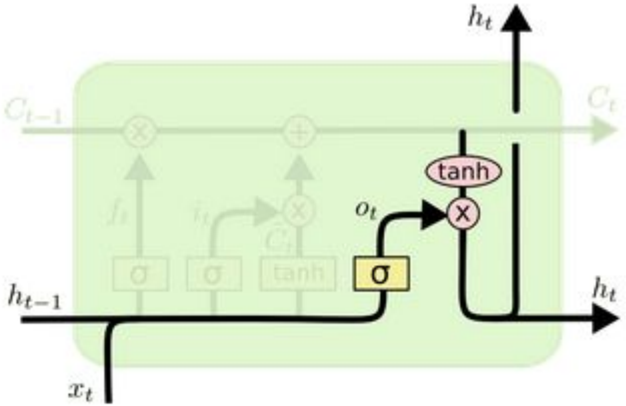
LSTM: quick overview



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTM: quick overview



$$o_t = \sigma\left(W_o\ [h_{t-1}, x_t]\ +\ b_o\right)$$

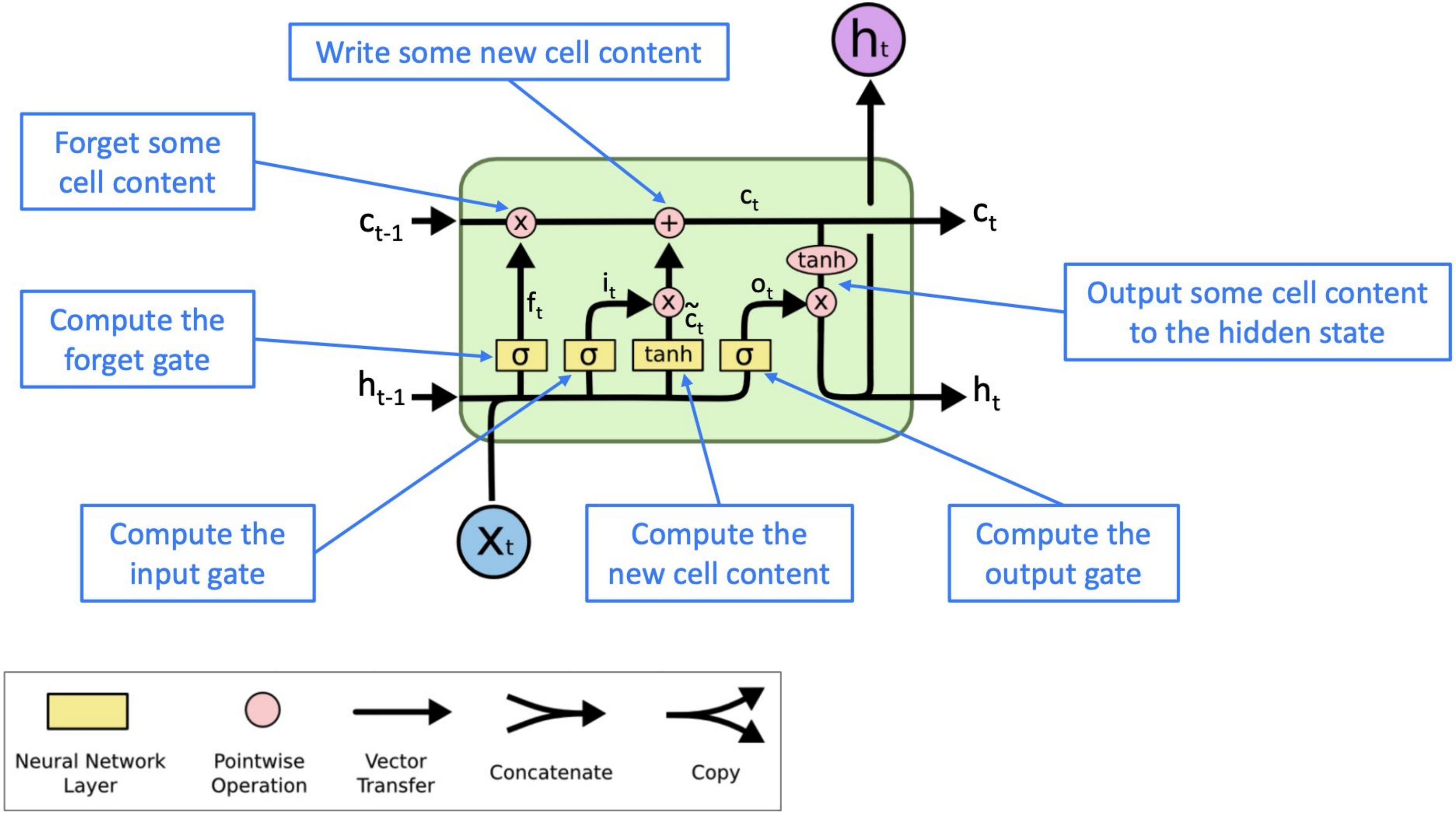
$$h_t = o_t * \tanh\left(C_t\right)$$

Source: https://colah.github.io/posts/2015-08-Understanding-LSTMs/

Tips

- Do not forget about dropout. It really rocks.
- Other regularization approaches are welcome as well (see previous lectures).
- Combining RNN and CNN worlds? *Coming soon*

Vanishing gradient: LSTM



Based on: Lecture by Abigail See, CS224n Lecture 7

Vanishing gradient: LSTM

Sigmoid function: all gate values are between 0 and 1

Forget gate: controls what is kept vs forgotten, from previous cell state

$$\boldsymbol{f}^{(t)} = \boxed{\sigma}\left(\boldsymbol{W}_f \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_f \boldsymbol{x}^{(t)} + \boldsymbol{b}_f\right)$$

Input gate: controls what parts of the new cell content are written to cell

$$\boldsymbol{i}^{(t)} = \boxed{\sigma}\left(\boldsymbol{W}_i \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_i \boldsymbol{x}^{(t)} + \boldsymbol{b}_i\right)$$

Output gate: controls what parts of cell are output to hidden state

$$\boldsymbol{o}^{(t)} = \boxed{\sigma}\left(\boldsymbol{W}_o \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_o \boldsymbol{x}^{(t)} + \boldsymbol{b}_o\right)$$

New cell content: this is the new content to be written to the cell

$$\tilde{\boldsymbol{c}}^{(t)} = \tanh\left(\boldsymbol{W}_c \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_c \boldsymbol{x}^{(t)} + \boldsymbol{b}_c\right)$$

Cell state: erase ("forget") some content from last cell state, and write ("input") some new cell content

$$\boldsymbol{c}^{(t)} = \boldsymbol{f}^{(t)} \circ \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \circ \tilde{\boldsymbol{c}}^{(t)}$$

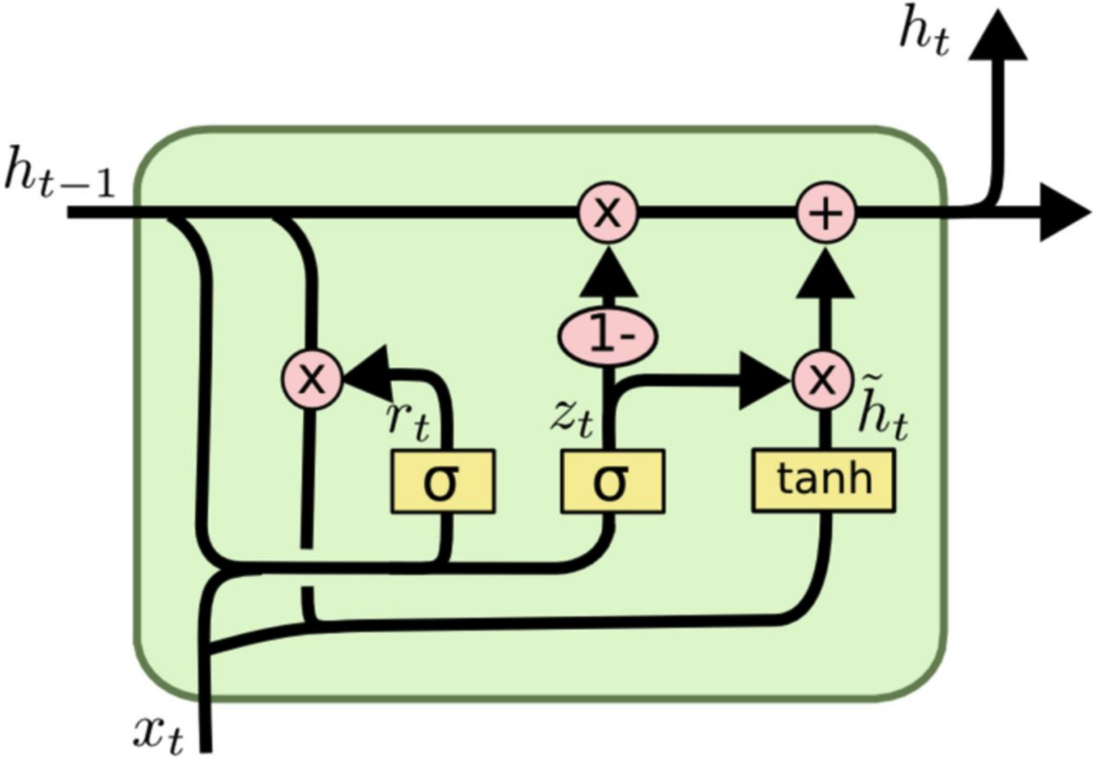
Hidden state: read ("output") some content from the cell

$$\boldsymbol{h}^{(t)} = \boldsymbol{o}^{(t)} \circ \tanh \boldsymbol{c}^{(t)}$$

Gates are applied using element-wise product

All these are vectors of same length *n*

Vanishing gradient: GRU



Vanishing gradient: GRU

Update gate: controls what parts of hidden state are updated vs preserved

Reset gate: controls what parts of previous hidden state are used to compute new content

$$\boldsymbol{u}^{(t)} = \sigma\left(\boldsymbol{W}_u \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_u \boldsymbol{x}^{(t)} + \boldsymbol{b}_u\right)$$

$$\boldsymbol{r}^{(t)} = \sigma\left(\boldsymbol{W}_r \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_r \boldsymbol{x}^{(t)} + \boldsymbol{b}_r\right)$$

New hidden state content: reset gate selects useful parts of prev hidden state. Use this and current input to compute new hidden content.

Hidden state: update gate simultaneously controls what is kept from previous hidden state, and what is updated to new hidden state content

$$\tilde{\boldsymbol{h}}^{(t)} = \tanh\left(\boldsymbol{W}_h(\boldsymbol{r}^{(t)} \circ \boldsymbol{h}^{(t-1)}) + \boldsymbol{U}_h \boldsymbol{x}^{(t)} + \boldsymbol{b}_h\right)$$

$$\boldsymbol{h}^{(t)} = (1 - \boldsymbol{u}^{(t)}) \circ \boldsymbol{h}^{(t-1)} + \boldsymbol{u}^{(t)} \circ \tilde{\boldsymbol{h}}^{(t)}$$

How does this solve vanishing gradient?
Like LSTM, GRU makes it easier to retain info long-term (e.g. by setting update gate to 0)

Vanishing gradient: LSTM vs GRU

- LSTM and GRU are both great
  - GRU is quicker to compute and has fewer parameters than LSTM
  - There is no conclusive evidence that one consistently performs better than the other
  - LSTM is a good default choice (especially if your data has particularly long dependencies, or you have lots of training data)

Rule of thumb: start with LSTM, but switch to GRU if you want something more efficient

Vanishing gradient in non-RNN

Vanishing gradient is present in **all** deep neural network architectures.

- Due to chain rule / choice of nonlinearity function, gradient can become vanishingly small during backpropagation
- Lower levels are hard to train and are trained slower
- **Potential solution(but not actually for that problem)**: dense connections (just like in DenseNet)

Conclusion:

Though vanishing/exploding gradients are a general problem, RNNs are particularly unstable due to the repeated multiplication by the same weight matrix [Bengio et al, 1994]. Gradients magnitude drops exponentially with connection length.

Source: ”Learning Long-Term Dependencies with Gradient Descent is Difficult", Bengio et al. 1994, http://ai.dinfo.unifi.it/paolo//ps/tnn-94-gradient.pdf

Very Deep Backlog

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- **Potential solution**: direct (or skip-) connections (just like in ResNet)

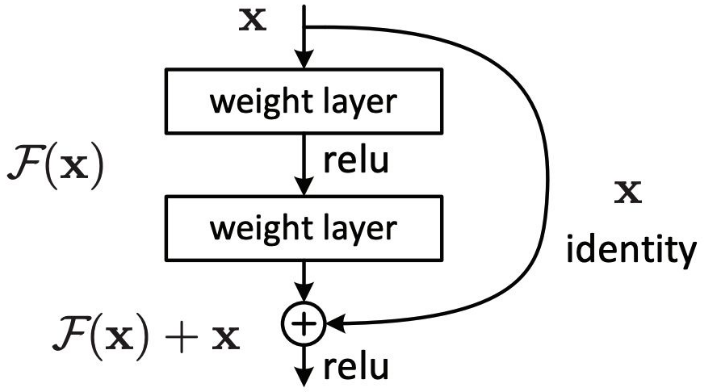

Figure 2. Residual learning: a building block.

Source: "Deep Residual Learning for Image Recognition", He et al, 2015. https://arxiv.org/pdf/1512.03385.pdf

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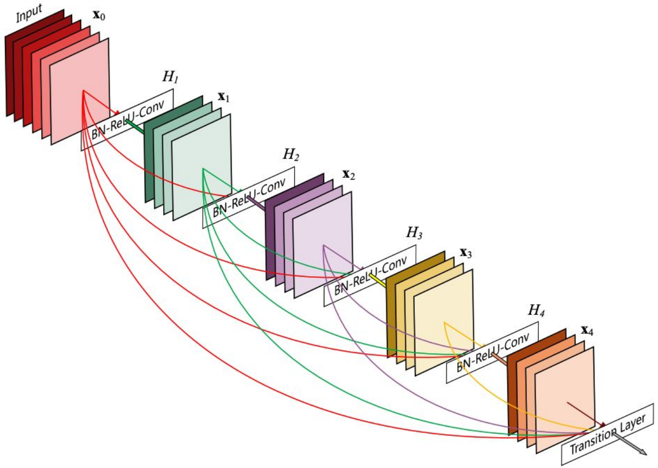
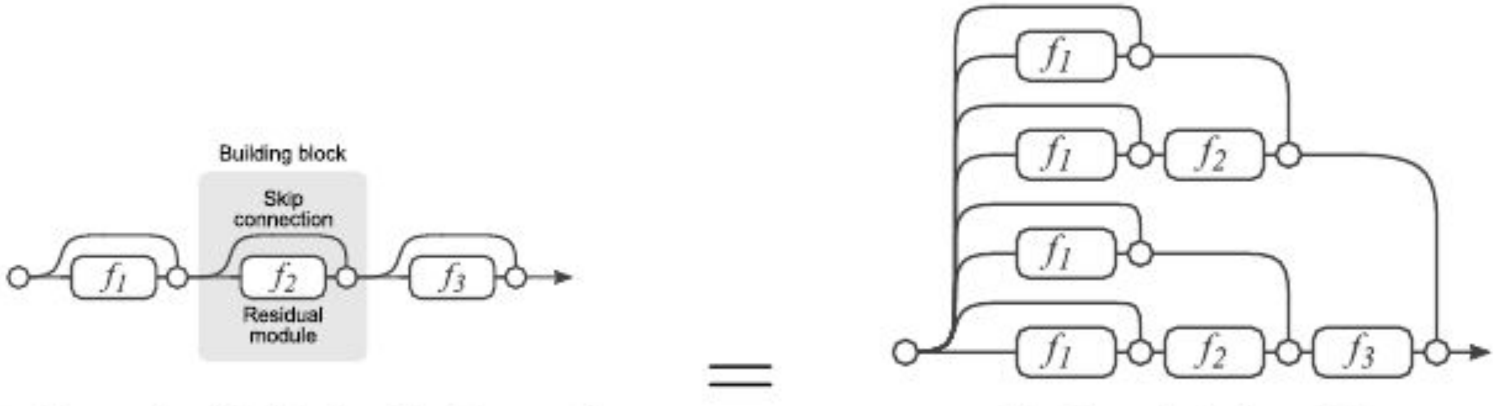


Source: ”Densely Connected Convolutional Networks", Huang et al, 2017 https://arxiv.org/pdf/1608.06993.pdf

Another view on ResNets and vanishing gradient

“**Residual Networks Behave Like Ensembles of Relatively Shallow Networks**”



(a) Conventional 3-block residual network

(b) Unraveled view of (a)

Source: https://arxiv.org/pdf/1605.06431.pdf