MSAI Probability Home Assignment 2
deadline: 23/11/2022 23:59

As announced earlier, grading for HWs consists of points and bonus points. Solving bonus points (indicated with a star) problems is not required, but recommended. Also it will benefit your grade. Roughly you can expect maximum +2 to the grade if you solve bonus problems.

Problem 1. (2 points) A good student solves a problem correctly with probability 0.95, while a bad student — with probability 0.15. What is the minimal number of problems that the test should include so that the probability that good student does not pass the test does not exceed 0.01, and the probability that the bad student passes the test does not exceed 0.1? Passing the test means solving strictly more than half of the problems.

Problem 2. (2 points) If you get a positive result on a COVID test that only gives a false positive with probability 0.001 (true positive with probability 0.999), what's the chance that you've actually got COVID, if

1. (1 point) The probability that a person has COVID is 0.01
2. (1 point) The probability that a person has COVID is 0.0001

Problem 3. (2 points) Alice is trying to communicate with Bob, by sending a message (encoded in binary) across a channel. Suppose for this part that she sends only one bit (a 0 or 1), with equal probabilities. If she sends a 0, there is a 5% chance of an error occurring, resulting in Bob receiving a 1; if she sends a 1, there is a 10% chance of an error occurring, resulting in Bob receiving a 0. Given that Bob receives a 1, what is the probability that Alice actually sent a 1?

Problem 4. (2 points) Show that if events A and B are independent, then

1. (1 point) Events A and $\overline{B}$ are independent
2. (1 point) Events $\overline{A}$ and $\overline{B}$ are independent

Problem 5* . (4 bonus points) You are the contestant on the Monty Hall show. Monty is trying out a new version of his game, with rules as follows. You get to choose one of three doors. One door has a car behind it, another has a computer, and the other door has a goat (with all permutations equally likely). Monty, who knows which prize is behind each door, will open a door (but not the one you chose) and then let you choose whether to switch from your current choice to the other unopened door.

1. (2 bonus points) Suppose for this part only that Monty always opens the door that reveals your less preferred prize out of the two alternatives, e.g., if he is faced with the choice between revealing the goat or the computer, he will reveal the goat. Monty opens a door, revealing a goat (this is again for this part only). Given this information, should you switch? If you do switch, what is your probability of success in getting the car?
2. (2 bonus points) Now suppose that Monty reveals your less preferred prize with probability p, and your more preferred prize with probability $q = 1 - p$. Monty opens a door, revealing a computer. Given this information, should you switch (your answer can depend on p)? If you do switch, what is your probability of success in getting the car (in terms of p)?

Problem 6* . (1 bonus point) Show that if $\mathbb{P}(A|B) = \mathbb{P}\left(A|\overline{B}\right)$ for two non-zero events A and B, then $\mathbb{P}(AB) = \mathbb{P}(A)\,\mathbb{P}(B)$ (i.e. A and B are independent).