

MSAI Probability Home Assignment 6-7
soft deadline: 05/12/2024 19:00 Moscow Time
hard deadline: 05/12/2024 19:00 Moscow Time

As announced earlier, grading for HWs consists of points and bonus points. Solving bonus (indicated with a star) problems is not required, but recommended. Solving all homeworks' normal problems correctly will give you a score of 7, solving all homeworks' bonus problems correctly will give you additional 2 points to the score.

Soft deadline is the intended deadline for this homework. Hard deadline is the date and time of homework discussion webinar, where we will discuss solutions to this homework. After solutions are released, no more homeworks are accepted.

Hand-written solutions are accepted if the handwriting is clear enough and scanned with sufficient quality, but LaTeX is always preferable. This homework includes a python task, which can be solved in Google Colab or in a local Jupyter Notebook. It is thus handy to solve everything (both LaTeX and code) in a single Jupyter Notebook.

Problem 1. (2 points) Find the skewness of exponential distribution.

Problem 2. (2 points) Find the kurtosis of the $Be(\frac{1}{2})$ distribution.

Problem 3* .

1. (1 bonus point) Compute the moment-generating function of $Geom(p)$.
2. (1 bonus point) Use the MGF to find expectation.
3. (1 bonus point) Use the MGF to find variance.

Problem 4. (2 points) Let $U \sim U[0, \frac{\pi}{2}]$. Find the PDF of $\sin(U)$.

Problem 5. Let X and Y be i.i.d. uniform on $(-b, b)$.

1. (2 points) Find the probability that equation $t^2 + tX + Y = 0$ has real roots.
2. (1 point) Visualize this probability as a function of b in Python.
3. (2 points) Find the limit of this probability as $b \rightarrow \infty$.

Problem 6. (2 points) Let X and Y be i.i.d. $Exp(\lambda)$, and $T = \log(X/Y)$. Find the CDF and PDF of T .