MSAI Probability Home Assignment 3
deadline: 28/11/2022 23:59

As announced earlier, grading for HWs consists of points and bonus points. Solving bonus points (indicated with a star) problems is not required, but recommended. Also it will benefit your grade. Roughly you can expect maximum +2 to the grade if you solve bonus problems.

Problem 1. (1 point) Prove that

$$\sum_{j=0}^{k} \binom{n}{j} \binom{m}{k-j} = \binom{m+n}{k}$$

Problem 2. (3 points) Consider two independent random variables $X \sim F_X$ and $Y \sim F_Y$. Find the CDF of random variables $Z_1 = \max(X, Y)$ (it means that for every outcome w we have $Z_1(w) = \max(X(w), Y(w))$, so Z_1 jumps between values of X and Y) and $Z_2 = \min(X, Y)$ (same reasoning applies).

Problem 3. (1 point) An airline overbooks a flight, selling more tickets for the flight than there are seats on the plane (figuring that it's likely that some people won't show up). The plane has 100 seats, and 110 people have booked the flight. Each person will show up for the flight with probability 0.9, independently. Find the probability that there will be enough seats for everyone who shows up for the flight.

Problem 4. (2 points) If $X \sim Bi(n, p)$ and $Y \sim Bi(m, p)$, and $X \perp Y$, what is the conditional distribution of X given $X + Y = r$?

Problem 5^* . (2 points) Find the distribution of a random variable X, which is equal to the number of failures in a series of Bernoulli trials with success probability p, which are carried out not a fixed number of times, but instead until there are r successes.